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AN EXPERIMENTAL STUDY OF ONE-WAY MIXING OF GUIDED WAVES IN CIRCULAR CYLINDERS

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ABSTRACT

Long pipes and circular bars are common structural elements. The ability to conduct nondestructive evaluation of such structural elements is thus a critical in maintaining the safe operation of the structures containing such elements. However, such pipes and bars are typically very long, and access to the entire pipe/bar is usually not available. Thus, nondestructive evaluation of local damage along the length direction is a major challenge. To overcome this difficulty, we propose in this paper a method based on the one-way mixing of guided waves to detect the localized damage in pipes/bars. In the experiment, a torsional mode is first generated from one end of a solid bar. After a time-delay, the first symmetric longitudinal mode is also generated from the same end of the bar. Since the longitudinal mode propagates faster, it will catch up with the torsional mode at a location that can be easily calculated based on the group velocities of these two modes and the time-delay between them. When the two primary modes mix, their nonlinear generates a secondary torsional mode, which propagates backwards towards the end of the bar where the two primary waves were generated. This mixed shear wave contains the acoustic nonlinearity parameter of the material in the mixing zone. Therefore, by measuring this mixed shear wave, microstructural damage within the mixing zone can be obtained. Thus, this technique is capable of nondestructively evaluating the material damage along the bar far away from the end where the interrogation signals are injected into the bar.

NOMENCLATURE

$\overrightarrow{k_1}$	wave vector of the primary wave one
$\overrightarrow{k_2}$	wave vector of the primary wave two
$\overrightarrow{k_3}$	wave vector of difference or sum harmonics
ω_1	angular frequencies of primary wave one
ω_2	angular frequencies of primary wave two
C_{p1}^{L}	phase velocities of primary longitudinal waves
C_{p2}^{T}	phase velocities of primary torsional waves
C_{p3}^{T}	phase velocities of difference harmonics

1. INTRODUCTION

Dislocations, precipitates, micro-voids or micro-cracks may lead to the initiation of macro-defects, and finally result in the catastrophic failure of the structure [1]. Therefore, it is vitally important to detect and localize the micro-damage at an early stage. Nonlinear ultrasound has shown high sensitivity to the microstructural changes as compared with traditional linear ultrasound. Considerable experimental measurements have been conducted using the second harmonic of longitudinal waves to detect plastic deformation [2], fatigue damage [3], and degradation of adhesive joints [4]. The method of second harmonic generation has also been extended to waveguides of constant cross-section (i.e. plates [5], rods [6], and rails [7]). For cumulative second harmonic generation, phase velocity matching and non-zero power flow criteria were considered as the necessary conditions for the cumulative second-harmonic generation [8]. If pulses of finite length are used, group velocity matching is also required [9]. However, the conventional second harmonic technique can only measure the average of the acoustic nonlinearity parameters between transmitters and receivers. Thus, it is incapable of characterizing local damage along the waveguide. Further, the method is also vulnerable to extraneous nonlinearities unrelated to material damage such as nonlinearity in the measurement system itself.

Wave mixing offers an alternative method to overcome these shortcomings. By mixing two primary waves of appropriate polarizations and frequencies, a secondary wave might be generated that is associated with the acoustic nonlinearity

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parameter for the material within the zone where the two primary wave mix. Mixing of bulk waves have been used to characterize plasticity [10], corrosions [11] and micro-cracks [12]. Comparing with the bulk waves, the nonlinear mixing of guided waves is considerably more complex due to the dispersive and multi-mode nature of guided waves. For plate-like waveguides, the generation of cumulative second-order harmonic has been studied extensively, e.g. [13].

Mixing of guided waves in circular cross-section bars has also been studied for detecting and localizing microstructural changes. The self- and mutual interactions of guided waves in cylinders are theoretically studied by Lima and Hamilton [14], and Liu et al [15]. Finite element simulations are also conducted to investigate the generation of cumulative second harmonics in cylinders [6, 15]. However, no experimental measurements have been carried out on the nonlinear mixing of torsional and longitudinal modes in circular cross-section bars, which is the focus of the present study. Specifically, this paper presents an experimental study on one-wave wave mixing of guided waves in circular cross-section bars. The aim is to investigate the feasibility of non-destructive detection and localization of micro-damages using the one-way mixing of guided waves in circular cross-section bars.

2. MATERIALS AND METHODS

Circular cross-section bars or cylinders will be used in the measurements. The bars are made of Al6061-T6 aluminum alloy. The materials properties are C_L = 6445.1m/s and C_T = 3094.9m/s. These were measured experimentally by the authors. The study consists of the following steps:

1. Obtain the dispersion curves
2. Select suitable mode triplets for the measurements
3. Generate torsional mode at one end of the bar
4. Generate longitudinal modes at one end of the bar
5. Generate both torsional and longitudinal modes at the same end of the bar
6. Receive torsional wave at the same end of the bar where the two primary modes are generated

3. PRELIMINARY RESULTS

3.1 Selection of the suitable mode triplets

Based on the theoretical analysis, the nonlinear mixing of torsional and longitudinal waves can lead to the generation of cumulative second order harmonics when the phase matching and non-zero power flux criteria are satisfied. Specifically, primary torsional modes T(0,1) can be generated from the interaction of the longitudinal mode L(0,*n*) and the torsional mode T(0,1), where *n* denotes the mode order of the Longitudinal wave. Further, the wave vectors of these three waves must satisfy,

$$\vec{k}_1 + \vec{k}_2 = \vec{k}_3 \tag{1}$$

where $\vec{k}_1$ and $\vec{k}_2$ present the wave vectors of primary waves, $\vec{k}_3$ is the wave vector of the difference or sum frequencies. For the one-way mixing of guided waves in circular cross-section bars, the relationship between the frequencies and phase velocities can be written as,

$$\frac{\omega_1}{C^L_{p1}} - \frac{\omega_2}{C^T_{p2}} = \frac{\omega_2 - \omega_1}{C^T_{p3}} \tag{2}$$

where ω_1 and ω_2 are the angular frequencies of primary waves, C^L_{p1}, C^T_{p2} and C^T_{p3} present the phase velocities of primary longitudinal wave, primary torsional wave and mixed secondary wave, respectively. We note that C^T_{p2} equals to C^T_{p3} since the torsional mode T(0,1) is non-dispersive. The crossing points in **FIGURE 1** present the selected mode triplets on the dispersion curve of a 6061-T6 aluminum alloy cylinder with the diameter of 6mm. The L(0,1) mode at f_1=292KHz and T(0,1) at f_2=250KHz are chosen as the primary waves because single L(0,1) and T(0,1) modes at relative low frequencies can be easily excited.

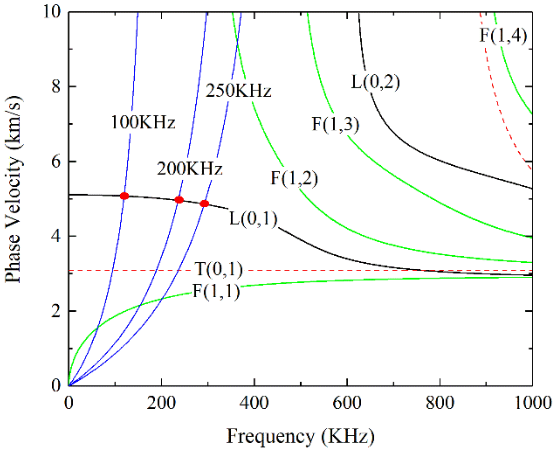

FIGURE 1: THE SELECTED MODE TRIPLETS ON THE DISPERSION CURVE OF A 6061-T6 ALUMINUM ALLOY CYLINDER WITH THE DIAMETER OF 6MM

3.2 Experimental excitation of the torsional waves

A transducer consisting of two thickness-poled d_{15} piezoelectric half-rings is used as the transducer to generate the primary torsional mode and as the receiver to record the mixed secondary torsional mode. The transducer is glued onto the left end of the bar. To check the integrity of the transducer and the measurement system, a 20-cycle tone burst at 250KHz was generated by a high-power gated amplifier and then fed into the transducers. The generated pulse propagated towards the right end of the bar, was then reflected back towards the left end of the bar. The reflected signal is then recorded by the same torsional wave transducer on the left end of the bar. **FIGURE 2** shows typical time-domain signals received by the torsional wave transducer. The multiple packets are due to the multiple reflections of the torsional mode from the two ends of the bar, from which, the shear wave velocity of the material is calculated as 3094.7m/s.

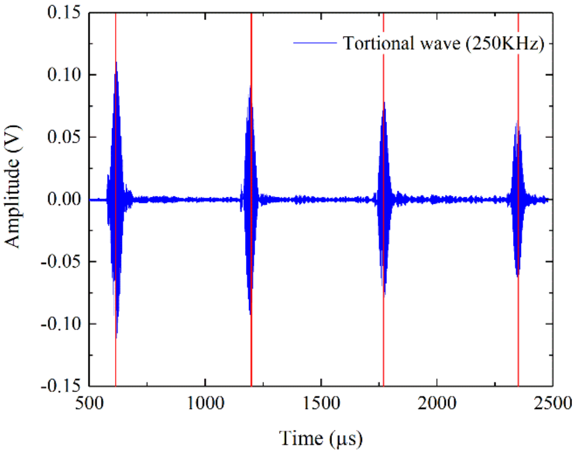

FIGURE 2: A TYPICAL TIME-DOMAIN SIGANLS RECEIVED BY THE TORSIONAL WAVE TRANSDUCERS

4. CONCLUSION

Preliminary measurements have been conducted for developing a one-way mixing technique in a circular cross-section bar. Specifically, a single torsional mode has been successfully generated by using a torsional wave transducer consisting of two thickness-poled piezoelectric half-rings.

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