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MULTI-MODE IMAGING IN THE FOURIER DOMAIN FOR FAST RECONSTRUCTION OF CRACK-LIKE DEFECTS

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ABSTRACT

The Total Focusing Method (TFM) was generalized about ten years ago to deal with complex ray paths involving mode conversions and reflections by the specimen interfaces. The resulting multi-mode imaging allows to fully image crack-type defects. More recently, the time-domain Plane Wave Imaging (PWI) has been explored to perform multi-mode reconstructions with a reduced number of transmissions compared to the TFM. In order to further accelerate the imaging process, we propose to combine the multi-mode PWI with a Fourier-domain reconstruction algorithm, inspired by the method developed by J.-Y. Lu. We show that the proposed f-k algorithm provides images of quality equivalent to that of the time-domain PWI, even reducing some artifacts, while decreasing computation times by a factor up to 13.

Keywords: ultrasounds, transducer arrays, multi-mode Plane Wave Imaging, Fourier-domain reconstruction

1. INTRODUCTION

Multi-mode imaging with ultrasonic arrays [1,2] has been introduced as part of the TFM, which consists in synthetically focusing the inter-element impulse signals on every point of the region of interest [3]. The principle has recently been adapted to the PWI and it has been shown that a small number of incident plane waves yield the same image quality, and even reduce imaging artifacts compared to the TFM that requires as many emissions as the number of array elements [4].

In a recent paper, we have demonstrated that reconstruction algorithms in the *f-k* Fourier domain are an efficient way of reducing the number of operations in the image formation process [5]. The *f-k* imaging methods have been widely used in seismic migration, medical ultrasound imaging, and synthetic aperture radar. In [5], we generalized one of them, namely the Lu method [6], in order to deal with usual inspection configurations encountered in NDT: imaging with linear or matrix transducer arrays, contact or immersion testing, imaging of defects located beyond the array aperture. Furthermore, we have demonstrated that the Lu method offers much higher computational performances than time-domain methods, theoretically reducing the number of operations by a factor of $N/3$ compared to the time-domain PWI, where N is number of array elements. We propose here to extend this approach to multi-mode imaging in order to speed up the reconstruction process for crack-like defects.

In this talk, we first present the principle of the multi-mode PWI in the *f-k* domain using a general formalism: an incident plane wave with an arbitrary polarization P_1 is reflected by the backwall in a plane wave of polarization P_2, and backscattered as a sum of cylindrical waves of polarization P_3. Then, imaging equations are given in their explicit form according to the polarizations of the incident, reflected and backscattered waves. We then compare the experimental images of planar defects obtained with the time-domain PWI and the *f-k* method, in 2D with linear arrays, as well as in 3D with matrix arrays.

2. Theory of the multi-mode *f-k* method

Here, we provide the theoretical background relative to the Lu method in multi-mode imaging. The method is based on a forward propagation model, in which the wave polarization is assumed to be arbitrary for each sub-path of the complete back-and-forth path in half-skip mode. The inversion of this forward model yields the imaging equation in the Fourier domain.

2.1 Forward propagation model in half-skip mode

As represented in Fig. 1, we consider a contact linear array transmitting a plane wave of incident angle θ_1 in a steel specimen of thickness H. The plane wave is reflected by the backwall with a reflection angle θ_2. This reflected wave is then back-scattered by the defect, considered as a distribution of point-like reflectors and modeled by a reflectivity function $g(x, z)$. The wave celerity for each sub-path is denoted by c_i, $i = 1,2,3$, and the associated wavenumber is $k_i = \omega / c_i$.

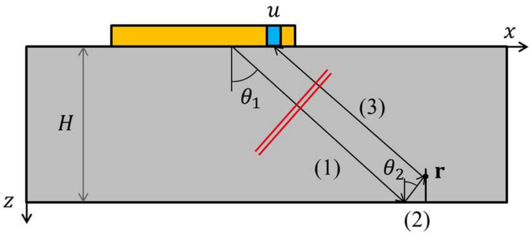


FIGURE 1: HALF-SKIP IMAGING OF A VERTICAL NOTCH WITH A LINEAR CONTACT ARRAY AND A SINGLE PLANE WAVE EMISSION

Following [5], we propose the following forward model to express the spectrum recorded by an element of abscissa u:

$$S(u,\omega) = \int_{-\infty}^{+\infty} \int_{0}^{+\infty} \Phi_2(x,z,\omega) g(x,z) H_0^{(2)}(k_3 \|\mathbf{r}_u - \mathbf{r}\|) \mathrm{d}x\mathrm{d}z \tag{1}$$

where $\mathbf{r}_u = (u,0)$ is the position of the receiver, $\mathbf{r} = (x,z)$ the position of a scatterer, $H_0^{(2)}$ is the Hankel function of second kind describing the cylindrical back-scattering, and

$$\Phi_2(x,z,\omega) = A(\omega) e^{-iH(k_1 \cos\theta_1 + k_2 \cos\theta_2) - i\, k_2 (x \sin\theta_2 - iz \cos\theta_2)} \tag{2}$$

is the wavefield spectrum reflected by the backwall, where $A(\omega)$ is the emission spectrum. The forward model (1) can be inverted by means of the Weyl's identity, which expresses the Hankel function as a sum of plane waves [7]:

$$H_0^{(2)}(k_3 \|\mathbf{r}_u - \mathbf{r}\|) = \int_{-\infty}^{+\infty} \frac{e^{ik_u(x-u) + iz\sqrt{k_3^2 - k_u^2}}}{\sqrt{k_3^2 - k_u^2}} \mathrm{d}k_u. \tag{3}$$

Injecting (3) and (2) into (1), and applying a Fourier transform along the lateral direction yields the imaging equation

$$G(k_x, k_z) = \frac{\sqrt{k_3^2 - k_u^2} e^{ik_1 H \cos\theta_1 + ik_2 \cos\theta_2}}{A(\omega)} S(k_u, \omega), \tag{4}$$

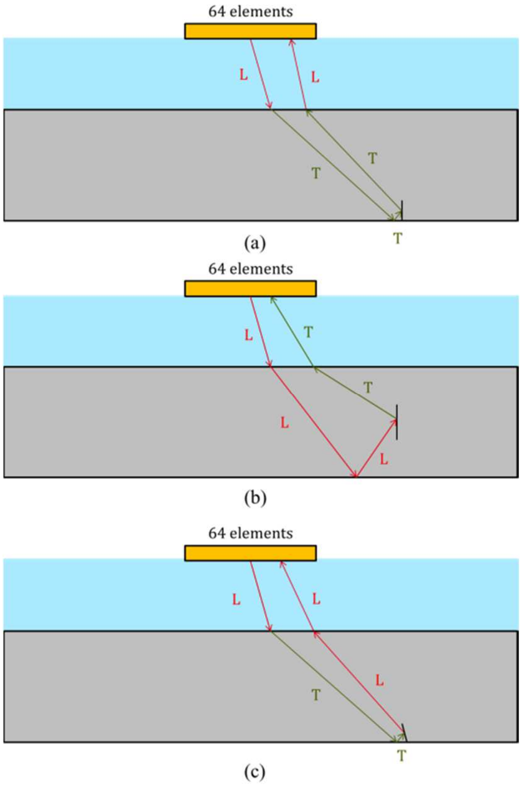
with

$$\begin{cases} k_x = k_u + k_2 \sin\theta_2 \\ k_z = \sqrt{k_3^2 - k_u^2} - k_2 \cos\theta_2 \end{cases} \tag{5}$$

Equation (5) defines the spectral transformation from the signal frequencies (k_u, ω) to the image spatial frequency grid (k_x, k_z). $G(k_x, k_z)$ is the 2D Fourier transform of the reflectivity function and is interpreted as the image spectrum. In practice, if M plane waves illuminate the medium with different incidence angles, (4) and (5) give M estimations $G_m(k_x, k_z)$ of the reflectivity of the medium in the spatial frequency domain, and $IFFT_{2D}\{\sum_{m=1}^{M} G_m\}$ generally provides higher quality images. As the polarization of waves are arbitrary in the theory, the imaging algorithm is valid for any half-skip mode, denoted by $P_1P_2dP_3$ where P_i = L or T are the polarizations of the incident, reflected and backscatterred waves, and "d" stands for the scattering by the defect.

3. RESULTS AND DISCUSSION

In order to assess the ability of the Lu method to image and characterize planar defects, experimental data were recorded with an immersion linear array (64 elements operating around 5 MHz), and by transmitting plane waves into a ferritic steel block (thickness 30 mm) featuring three different notches (see Fig. 2): a breaking vertical notch (A) of length 5 mm; a vertical notch (B) of length 10 mm located at the mid-thickness of the sample; a breaking notch (C) of length 10 mm and tilted of 15°.



(a)

(b)

(c)

FIGURE 2: IMAGING OF NOTCHES UNDER A WATER/STEEL PLANE INTERFACE USING THE TTdT (A), LLdT (B) AND TTdL (C) HALF-SKIP MODES

The Lu method was implemented on Matlab, as well as the time-domain PWI [4], which serves here as a reference method. Although the proposed method presented here for a contact array, it can be generalized to immersion inspections by proceeding in a similar way as in [5]. Both codes were run on an Intel Xeon 3.30 GHz CPU and 16 GB of RAM. In order to obtain the best images of a given notch, the angular range and the polarization of the incident plane waves, as well as the relevant reconstruction mode, were chosen with the SEE estimator [8]. The half-skip modes $P_1P_2dP_3$ and incidence angles θ_1 for the three notches are listed below (the angular step is 1°):

- A: TTdT mode with incidence angles $35 \leq \theta_T \leq 45$;
- B: LLdT mode with incidence angles $20 \leq \theta_L \leq 30$;
- C: TTdL mode with incidence angles $40 \leq \theta_T \leq 55$.

Figure 3 displays the reconstructed images with the time-domain PWI (left) and with the Lu method (right). All images have the same size and resolution, i.e. 20 × 20 mm² and 512 × 512 pixels, and are centered around the lateral position of the corresponding notch. It can be seen that the two methods provide very similar images, with an excellent signal-to-noise ratio greater than 30 dB. The computation times in Matlab to form PWI and Lu images with 11 plane waves are displayed in Table 1. We notice that the Lu method is faster than PWI for resolutions greater than 256^2 pixels, and up to 13 times faster for 1024^2 pixels. More importantly, the number of pixels has a small effect on the reconstruction times in the f-k domain, which remain less than 2 seconds.

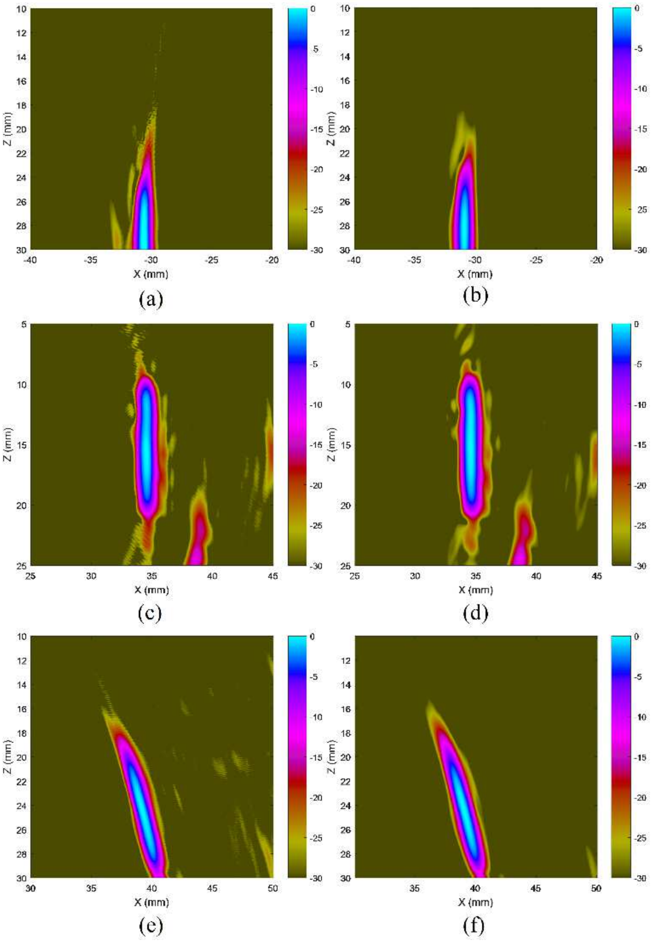


FIGURE 3: EXPERIMENTAL IMAGES OF NOTCHES WITH THE PWI (LEFT) AND THE FOURIER-DOMAIN (RIGHT) METHODS: (A, B) TTdT, (C,D) LLdT, AND (E,F) TTdL HALF-SKIP MODE

Nb of pixels	128^2	256^2	512^2	1024^2
t_{PWI} (s)	0.27	0.94	5.96	24.93
t_{Lu} (s)	0.79	0.90	1.1	1.98
t_{PWI}/t_{Lu}	0.34	1.05	5.42	12.6

TABLE 1: RECONSTRUCTION TIMES IN MATLAB (S) AND RATIOS BETWEEN THE TIME-DOMAIN PWI AND THE LU METHOD ACCORDING TO THE NUMBER OF PIXELS

4. CONCLUSION

In this communication, we have presented a reconstruction algorithm in the Fourier domain to image crack-like defects with reduced computation times. Compared to the time-domain PWI, the proposed method leads to very similar images, but the use of the Fast Fourier Transform (FFT) algorithm allows to speed up the reconstruction process by a factor up to 13 for images of high resolution (1024^2pixels). The theory remains valid for full-skip reconstruction modes and 3D imaging with matrix arrays, which will be demonstrated through other experimental results.

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