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MODELING AND SIMULATION FRAMEWORK TO INVESTIGATE HEAT GENERATION AROUND SURFACE CRACK DURING VIBRO-THERMOGRAPHY

G. Kolappan Geetha, A. K. Singh, D. Roy Mahapatra[1]
Department of Aerospace Engineering, Indian Institute of Science, Bangalore 560012, India

ABSTRACT

In this paper, a generalized modeling and simulation framework is discussed to investigate heat generated at the crack surface during vibro-thermography. Modeling framework includes (i) a recently developed coupled thermo-elastic strain-rate dependent heat generation model in the bulk of the material, and (ii) due to the clapping contact between the crack faces during vibro-thermography. Simulation results are analyzed considering a flat plate with a semi-ellipsoidal crack. Transient heat generation behavior near the crack region is analyzed. Dynamic contact models are used to capture the non-linear harmonics during crack interaction and its contribution towards heat signature. Simulation results are obtained by incorporating the model in a finite element scheme.

Keywords: Vibro-thermography, ellipsoidal crack, clapping contact, nonlinear harmonics, thermo-elastic heat generation

1. INTRODUCTION

Vibro-thermography is widely used in non-destructive inspection of structural components. Thermo-acoustics or thermo-sonics is one type of vibro-thermography phenomenon where a part of high frequency waves gets converted into thermal energy and its signature can be detected as infrared radiation from the surface of the object. The high frequency waves are typically generated by acoustic or ultrasonic transducers. The nature of wave energy interaction and conversion due to the defect is the most important aspect to be used in the design and analysis of thermal signature. The temperature distribution in the specimen is imaged by an infrared imaging camera [1-3]. Heat is generated differently in the region of defects as compared to surrounding continuous uniform regions. This enables vibro-thermography to become a potentially attractive non-destructive method for detecting different kinds of defects, both in metallic and composite structures. Defects in metallic structure include hidden corrosion [4, 5], cracks [6, 7], poor adhesion of coating [8] etc. Defects in composite structure include delamination, disbond, impact damage, voids and inclusions [9, 10] [11] etc. Though the technique shows promising results in detecting defects, a very little is understood about the accuracy and resolution to resolve the defects. Nevertheless, more quantitative and systematic work is still needed for this method to become a fully reliable non-destructive testing technique. This study proposes a further step in this direction and its objective is to (a) model the heat generation near the crack surface during vibro-thermography and (b) understand the contribution of heat signature at the crack tip due to various harmonics.

The paper is organized as follows. Section 2, discuss about the modeling framework for the heat generation in the bulk of the material and clapping contact between the crack faces. Section 0, discusses about the simulation results obtained on a flat plate with a semi-ellipsoidal crack

2. MATHEMATICAL MODEL

To the best of author's knowledge, the earlier reported studies for vibro-thermography uses a semi-empirical model [12, 13], where the heat generation is entirely due to out-of-plane vibration, without accounting the effect of in-plane displacement components. In this work, the authors have employed a recently developed generalized three-dimensional coupled thermo-elastic heat generation scheme which is established through thermodynamics. In this scheme, each component of strain is analytically correlated with the heat generation in the solid. The detailed thermoelastic heat generation model in the bulk of material can be seen in the reference [14]. Following the conventional notations [15], the first law of thermodynamics is given by

$$\rho \ddot{u} = \dot{\varepsilon}_{ij}\sigma_{ij} + \frac{1}{V}\frac{\delta Q}{dt} = \dot{\varepsilon}_{ij}\sigma_{ij} - r - q_{i,i} \qquad (1)$$

where U is the specific internal energy, q_i is the heat flux vector and r is the heat supply per unit time and unit volume, σ_{ij} and ε_{ij} are the stress and strain tensor, respectively. The second law of the Clausius-Duhem inequality is given by

$$\rho \dot{s} - \frac{r}{\theta} + \frac{q_{i,i}}{\theta} - \frac{q_i\theta_i}{\theta^2} \geq 0 \qquad (2)$$

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[1] Contact author: droymahapatra@iisc.ac.in

where s is the specific entropy (state function) and θ is the absolute temperature. The state functions U and s are expressed in terms of another state function Helmholtz free energy (ψ).

$$\psi = U - s\theta \Longrightarrow \dot{U} = \dot{\psi} + \dot{s}\theta + s\dot{\theta} \tag{3}$$

To obtain a generalized three dimensional coupled thermo-mechanical model, consider an inelastic material

$$\psi = \psi(\theta, \varepsilon_{ij} - \varepsilon_{ij}^{ie}, k_a); \quad \sigma_{ij} = \rho \frac{\partial \psi}{\partial \varepsilon_{ij}} + \sigma_{ij}^{\nu} \tag{4}$$

where $\varepsilon_{ij} - \varepsilon_{ij}^{ie}$ is the state variable for inelastic behavior, $k_a$is the internal variable or hidden variable, σ_{ij}^{ν} denotes the viscous stress. Further expressing $\frac{\partial Q}{dt}$ in terms of specific heat capacity at constant strain ($\varepsilon_{ij} = const$) gives $\frac{\partial Q}{dt} = \rho C_p \dot{\theta}$. For a thermo-elastic material $s = -\partial\psi/\partial\theta$, and $\sigma_{ij} = \rho \frac{\partial \psi}{\varepsilon_{ij}}$. Neglecting inelastic effects, internal variables for a thermoelastic material and on simplification gives $-\theta \frac{\partial^2 \psi}{\partial \theta^2} = C_p$. The temperature gradients exist in the system due to heat flow and the simplest constitutive equation that relates the heat flux vector q_i and the temperature gradient $\theta_{,j}$is a linear relation given by Fourier's law

$$q_i = -k_{ij}\theta_{,j} \tag{5}$$

For an isotropic material the viscous stress is given by

$$\sigma_{ij}^{\nu} = -\mu \dot{\varepsilon}_{ij} \tag{6}$$

where μ is tensor representing coefficient of viscosity. For thermoelastic materials, expanding free energy $\psi(\theta, \varepsilon_{ij})$ in a Taylor series about the reference state where $\theta = \theta_0$ and $\varepsilon_{ij} = 0$, and simplifying gives coupled thermo-elastic equation for an isotropic material.

$$\rho C_p \dot{\theta} = \nabla(\mathrm{k}\nabla\theta) - 3K\alpha T(\dot{\varepsilon}_{11}^2) + \mu_{\perp}(\dot{\varepsilon}_{11}^2 + \dot{\varepsilon}_{22}^2 + \dot{\varepsilon}_{33}^2) + 2\mu_{||}(\dot{\varepsilon}_{12}^2 + \dot{\varepsilon}_{23}^2 + \dot{\varepsilon}_{13}^2) \tag{7}$$

where K is the bulk modulus of elasticity, α is the coefficient of thermal expansion. $\mu_{\perp}$and $\mu_{||}$ denotes the components of coefficient of viscosity in axial and shear direction, respectively. The contribution towards heat generation due to axial and shear strain rate terms are given by$\mu_{\perp}(\dot{\varepsilon}_{11}^2 + \dot{\varepsilon}_{22}^2 + \dot{\varepsilon}_{33}^2)$ and $\mu_{||}(\dot{\varepsilon}_{12}^2 + \dot{\varepsilon}_{23}^2 + \dot{\varepsilon}_{13}^2)$, respectively. This model correlates the effect of in-plane and out-of-plane polarized vibrations.

Heat generation in cracks is primarily generated from clapping or rubbing of contact regions along the crack faces. Two types of contact forces are been induced in the model due to the displacement caused by the ultrasonic excitation. They are normal contact force and tangential contact force. The normal contact forces are induced in the longitudinal direction of wave propagation. The normal contact force is given by

$$f_n = p_n - K_p \Delta u; \qquad \Delta u \le 0 \tag{8}$$

$$f_n = p_n e^{K_p/p_n} \Delta u; \qquad \Delta u \ge 0 \tag{9}$$

where Δu is the relative displacement between the crack faces, f_n is the normal contact force, p_n is the contact pressure estimation at $\Delta u = 0$ and K_p is the penalty stiffness. The tangential contact forces are induced in the transverse direction of wave propagation. The frictional force in tangential direction is given by

$$f_t = K_t u_t \quad ; \quad K_t u_t \le \mu f_n \tag{10}$$

$$f_t = \mu f_n \quad ; \quad K_t u_t > \mu f_n \tag{11}$$

3. RESULTS AND DISCUSSION

To investigate the heat generation during vibro-thermography, a calibration sample with a surface crack in the form of a semi-ellipsoidal geometry is chosen whose schematics and dimensions are shown in Figure 1.

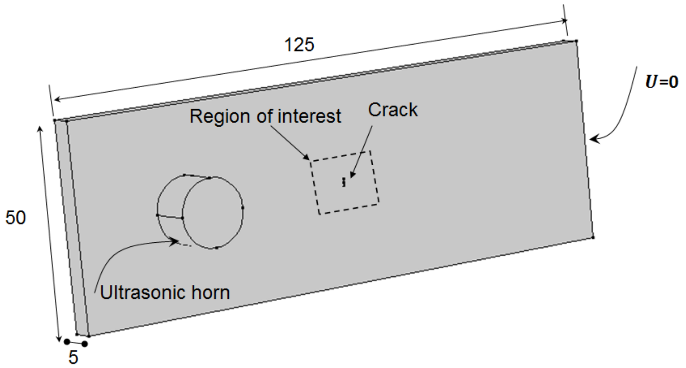

FIGURE 1: Time averaged strain energy for varying degrees of freedom.

The convergence of the time domain simulation is ensured by satisfying the Courant-Friedrichs-Lewy (CFL) condition. For convergence, quantities of interest considered are displacement responses at various points in the sample. But, responses at different point may not give overall perspective of the convergence. In order to give the spatial and time aspect bring together we introduce one more quantity that is time averaged strain energy (see Figure 2). Expression for time averaged strain energy is given by

$$\overline{U} = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} \iiint_V \frac{1}{2} \{\sigma\}^T \{\varepsilon\} dV\, dt \tag{12}$$

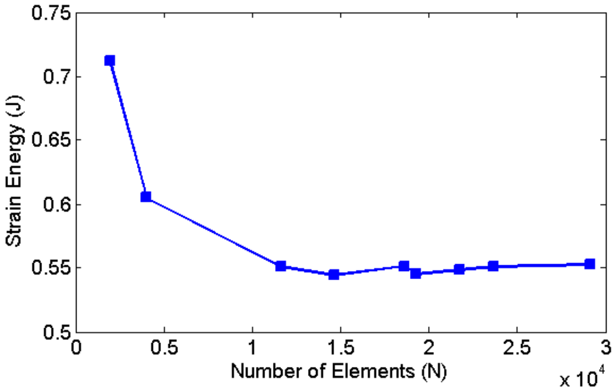

FIGURE 2: Time averaged strain energy for varying degrees of freedom.

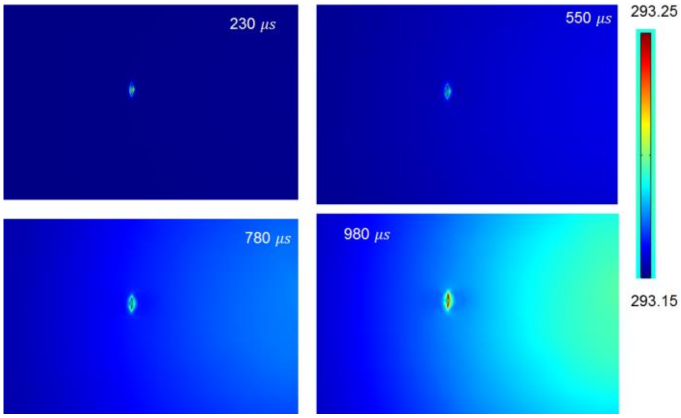

FIGURE 3: Spatio-temporal heat flux obtained in the region of interest.

To validate the developed heat generation model, simulations are performed on the calibration sample. A spatio-temporal temperature distribution at different time stamps are analyzed in the region of interest as marked in Figure 1. The spatio-temporal distribution gives a better insight about the propagation of the transient heat flux in the region of crack. The spatio-temporal thermal patterns at different time stamps are shown in Figure 3. The ultrasonic horn excites the target structure at 30 kHz. The images were obtained at the surface where emissivity $\xi = 1$.

4. CONCLUSION

In this paper, a generalized modeling and simulation framework is developed to investigate heat generation at crack surface during vibro-thermography. The phenomenon of heat generation around the crack during vibro-thermography is modeled for ultrasonic horn excitation. The heat generation scheme uses (i) a recently developed coupled thermo-elastic strain-rate dependent heat generation model in the bulk of the material and (ii) clapping contact between the crack faces during vibro-thermography. A parametric study is further performed for varying crack orientation, depth and planar dimensions. The developed model effectively captures the non-linear dynamics at the crack tip due to clapping contact.

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