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QUADRUPOLE SIMULATIONS OF THERMOGRAPHIC RESPONSES OF IMPACT DAMAGE IN COMPOSITES

William P. Winfree[1], Joseph N. Zalameda,
Patricia A. Howell
NASA Langley Research Center
Hampton, VA

ABSTRACT

The quadrupole method has been shown as a viable technique for rapid three-dimensional thermographic simulations of a delamination. Previous efforts have focused on the simulation of a single delamination in a composite. Realistic impact damage in composites is typically multiple overlapping delaminations at different depths. The quadrupole method is expanded to enable rapid simulation of multiple overlapping delaminations at different depths. Quadrupole simulations are performed by incorporating x-ray computed tomography information on the depth, shape and contact resistance of impact induced delaminations. The simulation results are compared to thermographic measurements on the impacted composite.

Keywords: thermography, quadrupole simulation, composite

NOMENCLATURE

d_i	Location of interface between layers
$\boldsymbol{F}_i$	Laplace transform of flux at d_i
$\boldsymbol{T}_i$	Laplace transform of temperature at d_i
$\boldsymbol{R}_i$	contact resistance at d_i
$\boldsymbol{M}_i^{csch}$	csch transfer matrix for layer i
$\boldsymbol{M}_i^{coth}$	coth transfer matrix for layer i
CT	computed tomography

1. INTRODUCTION

A common method for solving the Laplace transform of the heat equation is the quadrupole method [1,2]. For a one-dimensional heat equation, the quadrupole method has been used extensively for rapid computation of the thermal response of multiple layers to the application of an external heat flux. The method has also been extended for solving the heat equation in two-dimensions and three-dimensions. A primary advantage of the method is it is possible to calculate the thermal response at only the times of interest and each time of interest can be computationally performed in parallel with every other time of interest.

For composites, the one-dimensional quadrupole method has been developed for composites with delaminations by representing each layer as a 2x2 transfer matrix and the interfaces between layers with delaminations as a 2x2 matrix that incorporates a contact resistance corresponding to the reduction in heat flow resulting from a delamination [3]. Multiplication of the matrices representing each layer and interface, results in a 2x2 matrix relating the Laplace transform of the flux and temperature of one surface of a stack of layers to the Laplace transform of the flux and temperature of the other surface. This methodology enables simple calculation of the Laplace transform of the temperature from the Laplace transform to the applied flux. For all but the simplest of configurations, a numerical inversion of the Laplace transform is performed to calculate the temperature.

For two-dimensions or three-dimensions, the heat equation is solved by first representing the temperature and flux in the plane of the layers by a spatial cosine transform [4,5,6]. By substituting this representation into the heat equation, a one-dimensional partial differential equation is obtained for each coefficient of the cosine transform, and the thermal response is found by finding the Laplace transform of each coefficient. If there is no spatial variation in the contact resistance at the interface, calculation of the Laplace transform of each coefficient is relatively simple. However, if there is a spatial variation in the contact resistance, such as is typical for delaminations in composites, solving for the Laplace transform of each coefficient involves a large set of simultaneous equations. To date, only multilayer configurations with one delamination have been simulated. This presentation uses the same methodology to simulate a composite with multiple delaminations.

2. MATERIALS AND METHODS

2.1 Impacted composite

An 18-ply composite flat panel 6.8 cm by 3.8 cm by 0.32 cm was impacted. Based on x-ray CT measurements, the impact resulted in one or two delaminations at 13 different depths in the panel with the total delaminated area for a given depth estimated to vary from 0.06 cm^2 to 1.03 cm^2.

CT images at four locations with delaminations are shown on the left side of Figure 1. All of the CT images are scaled the

[1] Contact author: william.p.winfree@nasa.gov

same. From top to bottom, the slice locations are approximately 0.016 cm, 0.036 cm, 0.052 cm, 0.092 cm below the surface or approximately at the first, second, third and fifth interface between plies. On the right side of the figure are images of the estimated shapes and gap thicknesses for the delaminations based on the CT image on the left. All of the gap images are scaled the same with a maximum for the gap width of 0.005 cm and minimum of zero.

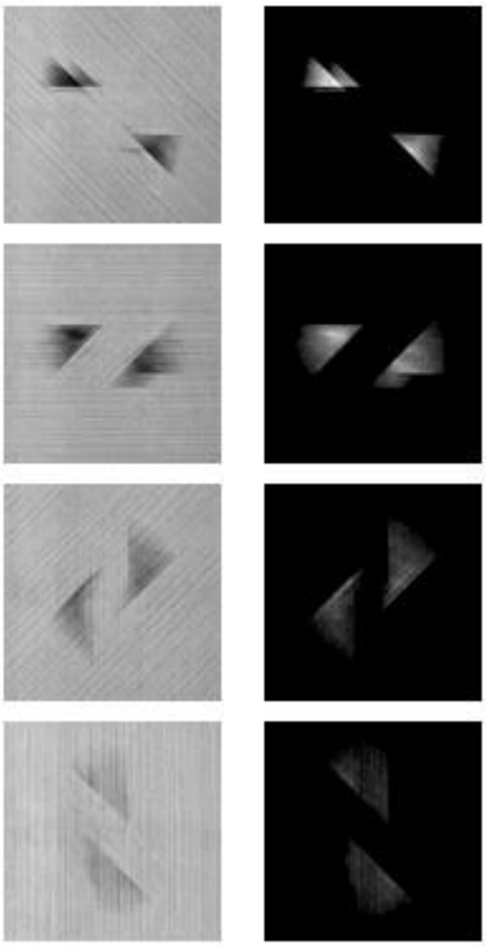

FIGURE 1: CT IMAGES AT FOUR DEPTHS INDICATING SIGNIFICANT DELAMINATING ON THE LEFT AND THE GAP WIDTH ESTIMATED FROM THE CT IMAGE ON THE RIGHT.

2.2 Thermography Measurements

The composite surface closest to the first CT slice shown in figure 1 is heated with a flash lamp. The thermal response of the composite specimen was performed with a commercial flash thermographic measurement system. The flash duration has been measured to be approximately 0.008 second. Since the earliest thermal responses of interest occur approximately one tenth of a second after the heat pulse, this is a good estimate of the impulse excitation. The thermal response was measured with a focal plane array infrared imager detector that operates in the 3-5 micrometer wavelength band. The approximate size of a pixel in the thermography images is 0.0250 cm. The imager digitized the thermal response at 60 hertz. The clearest delineation of delaminations is in the first second after the flash.

2.3 Outline of Computational Methodology

Calculation of the thermal response of a composite with delaminations at multiple depths focuses on the calculation of flux at the interfaces of the layers. If there is a stack of n layers, there are n+1 interfaces including the two surfaces at the top and bottom of the stack with each of the interfaces being at a depth d_i, where the index is the number of the interface. The Laplace transform of the temperature and flux at each interface can be represented by a set of cosine series coefficients which are written in a matrix form as $\boldsymbol{T}_i$ and $\boldsymbol{F}_i$. If there is a contact resistance at the interface, there is a discontinuity in the temperature such that the temperature on the one side of the interface($\boldsymbol{T}^+_i$) and the temperature on the other side ($\boldsymbol{T}^-_i$) are related by

$$\boldsymbol{T}_i^+ = \boldsymbol{T}_i^- + \boldsymbol{R}_i \cdot \boldsymbol{F}_i \qquad (1),$$

where $\boldsymbol{R}_i$ is a matrix representation of the spatial variation of the contact resistance at the surface. The expression for $\boldsymbol{T}^+_i$ in terms of $\boldsymbol{F}_{i-1}$ and $\boldsymbol{F}_i$ is

$$\boldsymbol{T}_i^+ = \boldsymbol{M}_{\mathrm{i}}^{csch} \boldsymbol{F}_{i-1} - \boldsymbol{M}_{\mathrm{i}}^{coth} \cdot \boldsymbol{F}_i \qquad (2),$$

and the equivalent expression for $\boldsymbol{T}^-_I$ is

$$\boldsymbol{T}_i^- = \boldsymbol{M}_{i+1}^{coth} \cdot \boldsymbol{F}_i - \boldsymbol{M}_{i+1}^{csch} \cdot \boldsymbol{F}_{i+1} \qquad (3),$$

where the transfer matrices are given elsewhere [6]. Equations 1-3 are combined to find $\boldsymbol{F}_i$ in terms of $\boldsymbol{F}_{i-1}$ and $\boldsymbol{F}_{i+1}$.

To find the flux at every interface, an initial guess for the flux at every interface is estimated using a one-dimensional model. Starting at the interface closest to the source, the combination of equations 1-3 are applied to update the values for the flux at each interface until it converges on a solution. Convergence to within 0.01% typically occurs after one iteration.

3. RESULTS AND DISCUSSION

A quadrupole simulation was compared to a finite element simulation for the same configuration of overlapping contact resistance profiles at different depths. The assumed thermal properties for the composite were a thermal conductivity of 9.7e4 erg/cm/K/sec, a heat capacity of 1.2e7 erg/gm/K and a density of 1.6 gm/cm^3, which results in a thermal diffusivity 0.005 cm^2/sec. The agreement between the two different methods was better than 2%, which is better than the typical signal to noise due to material variations in composites.

To compare the quadrupole simulations to experimental data, the gap thicknesses were estimated from the CT and a contact resistance map was calculated by dividing the gap thickness map by the thermal conductivity of air (2624 erg/cm/K/sec). The depth of the interfaces, estimated from the CT data, were 0.018 cm, 0.035 cm, 0.053 cm and 0.090 cm for the first four interfaces respectively. Only the first 3 depths of the delaminations are obvious in the simulations and the measured response.

A comparison of the simulation result and the measured response is shown in figure 2. In the figure, the measured response at four times are shown on the left and the simulation results at the same four times are shown on the right. The times after flash heating from top to bottom are 0.1 sec, 0.2 sec, 0.3 sec and 0.5 sec.

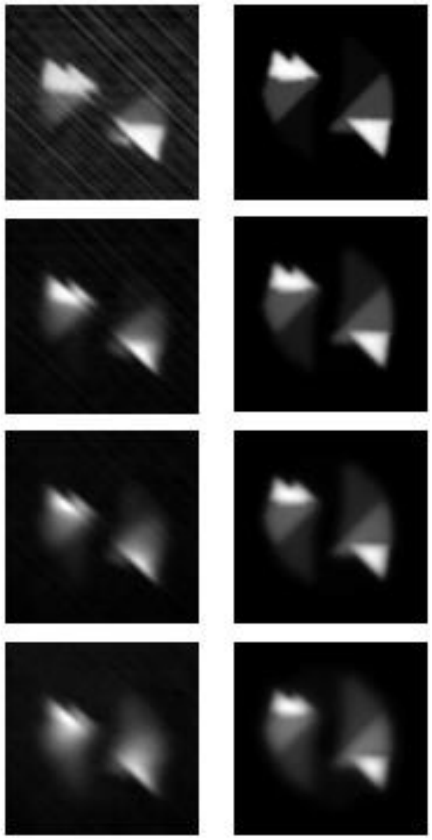

FIGURE 2: COMPARISON OF THE THERMOGRAPHY MEASUREMENTS AND THE SIMULATION RESULTS.

As can be seen from the figure, for early times, the shapes of the indications and the contrast between and regions with no delaminations are approximately the same. The agreement is not as good for later times, when the responses from the delaminations close to the surface tend to become much smaller in the measured responses than in the simulations. Assuming the in-plane diffusivity of the composite is ten times greater than the surface normal diffusivity does not produce significantly better agreement. For the simulations shown, the in-plane diffusivities are assumed to be equal to the surface normal diffusivity.

A second possibility investigated was the gap thicknesses estimated from the CT data is larger than the actual gap spacing, however, decreasing the gap spacing by a factor of two does not significantly improve the agreement. A third possibility being considered is the gap is not a clean separation of two plies, rather the gap has points of contact between the two sides. Assuming a wider gap has less contact, the resulting contact resistance does not have a linear dependence on estimated gap width, as is assumed for a simple model. Future efforts will investigate this possibility.

4. CONCLUSION

A method for simulating the thermographic response of a composites with multiple delaminations at multiple depths has been developed. The method was applied to simulating the thermographic response of a composite with impact damage. From CT data, maps of the contact resistance at different depths in the composite were generated. These maps were incorporated into the simulation which produced good agreement with the early response, then poorer agreement at later times. Different models estimating the contact resistance from the CT data are being considered to improve the agreement at later times.

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