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Chapter 10

Practical Applications of Fertilizer Production Functions

BEFORE practical use can be made of fertilizer production functions, the basic experiments must have first been designed to allow satisfactory estimates of the production surface. Complete factorial experiments, in which every level of one fertilizer is combined with every level of other elements, can be recommended for surface estimation if the required number of treatment combinations is not too large. Where complete factorial experiments require excessive treatment numbers, the composite design appears to offer a promising alternative.[1]

Interaction of Nutrients and Design

In Iowa experiments, fertilizer elements have often interacted to produce an added effect not due to either element alone (2). For positive interaction, as between nitrogen and phosphorus on corn yield, fertilizer rates should go high enough to cause a decrease (or at least a leveling out) of average yield for each nutrient. In table 10.1, average response to N and P_2O_5 has leveled out or started to decline at the heaviest rates. Unfortunately, the "incompleteness" of this experiment makes interaction effects harder to isolate. The average yields of 200, 240, and 280 pounds of P_2O_5 presented in table 10.1 are 97.66, 101.79, and 106.03 bushels, respectively. However, it is hard to tell whether the increase in yield is due to: (a) the heavier level of P_2O_5; (b) the higher levels of N at the 280-pound level of P_2O_5 as compared to lower levels of N at 240 and 200 pounds of P_2O_5; (c) the interaction of N and P_2O_5. The average responses in table 10.1 which are strictly comparable are only those which include *all* levels of the other nutrient, or the 0-, 160-, and 320-pound rates. Therefore, a "complete" factorial experiment would seem preferable to the incomplete type in table 10.1, even though it might be necessary to space some of the fertilizer rates farther apart to keep down the total number of treatments.

Highly significant N · P interaction in the data of table 10.1 was indicated by an F(40, 57 d. f.) of 4.25. The strong interaction is indicated in table 10.1. At low levels of N, yield of corn is restricted despite large P_2O_5 applications. With N at 160 and 320 pounds, response to P_2O_5

[1]See Chapters 3 and 5 for a detailed discussion of these points.

TABLE 10.1 Bushels of Corn per Acre for Varying Levels of Fertilizer on Calcareous Ida Silt Loam Soil in Western Iowa in 1952. (Each Cell Represents the Average of Two Observations) (2).

Pounds of P_2O_5	Pounds of Nitrogen								
	0	40	80	120	160	200	240	280	320
0	15.35	17.85	17.55	24.80	10.75	8.65	11.50	17.25	22.05
40	28.15	71.35			101.50	95.40			79.15
80	26.35		107.45		94.25		119.00		105.50
120	33.05			108.35	107.85			122.05	103.80
160	23.00	88.45	105.35	128.85	123.00	110.60	127.40	133.05	129.15
200	33.95	66.05			119.00	141.25			127.10
240	36.50		102.50		126.70		117.65		137.90
280	29.90			127.35	131.00			135.95	119.45
320	11.60	59.60	100.25	128.60	122.80	132.40	133.40	121.85	123.35

Average Response to Pounds of Nutrients

Pounds	0	40	80	120	160	200	240	280	320
P_2O_5	26.65	60.66	86.62	103.59	104.09	97.66	101.79	106.03	105.27
N	16.19	75.11	90.51	95.02	107.65	97.87	104.25	108.73	103.76

is comparatively great. Likewise, nitrogen response is definitely dependent upon the P_2O_5 application.

Regression estimates of the data in table 10.1 confirm the importance of interaction. A t value of 8.85 for the interaction term in regression equation 10 in Chapter 1 is significant at the .00001 level of probability.

Although many Iowa experiments have shown important interaction between nutrients, what if a factorial experiment is run and no significant interaction is found? The independent responses can then be computed with a considerable gain in efficiency; each level of one nutrient can be used as a replication in estimating the response from the other element (1). Thus, complete factorial experimentation is efficient if treatment effects are independent and experimental error does not increase markedly with large treatment numbers. If treatment effects are not independent, some type of factorial experiment is necessary to estimate treatment interaction.

Practical Application

Assuming that the basic experiment was properly designed and that the estimating function gives a good "fit" and is logically and statistically satisfactory, how can these fertilizer production functions be used?

Marginal rates of substitution and isoquants can be computed as illustrated in Chapter 1 and give relevant economic information. It is also possible to present in one chart the economic information needed for decision-making. In figure 10.1, added isoquants and isoclines have been derived for equation 10 in Chapter 1. Along the isocline labeled $P_n = 1.5\ P_p$, 1 pound of nitrogen produces the same yield as would 1.5 pounds of P_2O_5. Therefore, when the price of nitrogen is 1.5 times the price of P_2O_5, the N and P nutrient combination should fall on the isocline labeled $P_n = 1.5\ P_p$. (The isocline can be thought of as the optimum "fertilizer mix" curve).

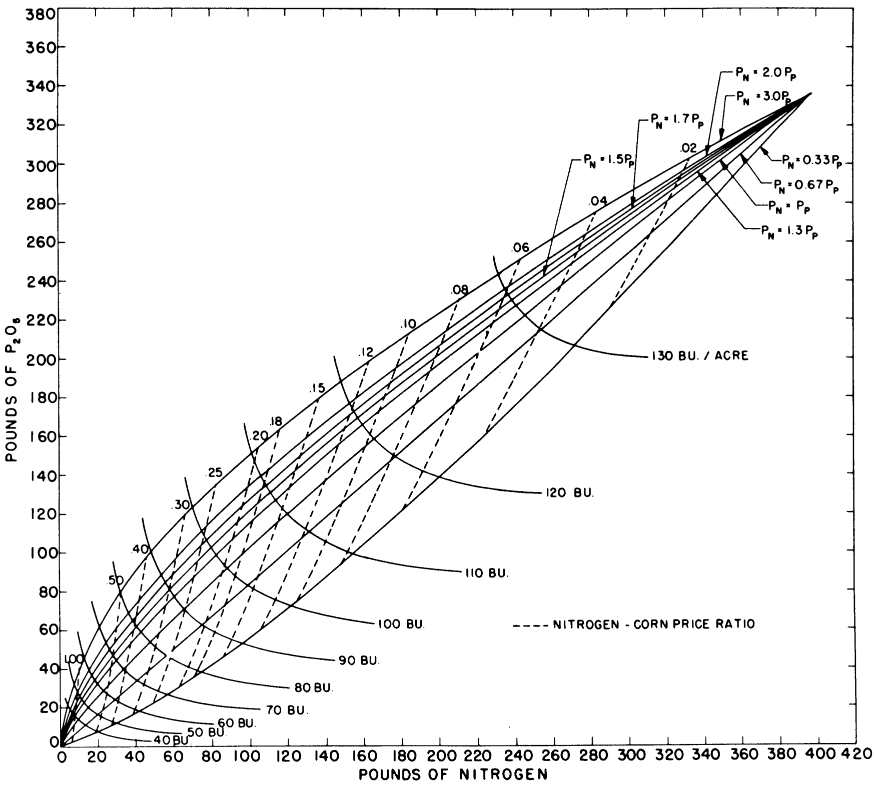

Fig. 10.1 — Yield isoclines and isoquants for corn on Ida-Monona soil, Iowa. Optimum rates are indicated by dashed lines representing the nitrogen-corn price ratio.

Figure 10.1 was designed to provide a simple method of determining the most profitable rates of nitrogen and phosphorus to apply to corn. (Construction of the chart required calculus, but its use by farmers or extension personnel requires only short division). The following price situation illustrates use of the chart:

Price of corn	\$1.00 per bu.
Price of element N	0.15 per lb.
Price of available P_2O_5	0.10 per lb.

The price relationship of nitrogen to phosphorus is $P_n = 1.5\ P_p$. Therefore, the line leading from the lower left corner to the upper right corner (labeled $P_n = 1.5\ P_p$) is chosen. This line gives the optimum nitrogen-phosphorus combinations for all levels of production when nitrogen is 1.5 times as expensive as P_2O_5. To find how far to go on this line, it is necessary to determine the nitrogen-corn price ratio. In this case, $P_n/P_c = 0.15$. Therefore, the line labeled $P_n = 1.5\ P_p$ is followed until the dashed line labeled 0.15 is reached. Then by dropping straight down from this point, a reading of about 125 pounds of nitrogen is obtained. Likewise, by reading straight to the left from the 0.15 point on the fertilizer mix line (isocline), about 142 pounds of P_2O_5 is indicated. (When the price ratios are not exactly those of the chart, it is possible to interpolate between the nearest values given).

Determination of corn yield for the indicated inputs of N and P_2O_5 is easily made from the yield lines or isoquants. For the preceding optimum inputs, a yield of about 113 bushels is predicted. At \$1 per bushel, a total value of about \$113 per acre is estimated. Gross return and fertilizer cost figures are given below:

Value of corn	\$113.00
Cost of nitrogen	18.75
Cost of P_2O_5	14.20
Margin over fertilizer	80.05

Although the dashed lines in figure 10.1 represent various nitrogen-corn price ratios, they also include the phosphorus-corn price ratio. For example, the intersection of the dashed line labeled 0.15 with the isocline labeled $P_n = 1.5\ P_p$, was found by the optimum solution from equations 11 and 12 in Chapter 1.

Solving equations 11 and 12 from Chapter 1 simultaneously, N = 124.7 and P = 141.8 are used as the point of intersect of the dashed line labeled 0.15 with the fertilizer mix line labeled $P_n = 1.5\ P_p$. The rest of the points on the chart are located in the same way.

Alternative Solutions

How should such a chart as figure 10.1 be used by agronomists in making recommendations to farmers? Figure 10.1 is based upon empirical results which would apply only to farmers who had calcareous Ida silt loam soil with the same general fertility level as the experimental field. If the farmer does have a similar soil, recommendations can be made directly from the chart as in the preceding sample. However, several alternatives to the "optimum" probably should be presented to the farmer. If the price of nitrogen is expected to remain 1.5 times the price of P_2O_5, all alternatives should be selected from the fertilizer mix line marked $P_n = 1.5\ P_p$. For example, the farmer in the preceding price situation might be so restricted on capital that he could apply only \$12 worth of fertilizer per acre. The "restricted" optimum can still be located for the farmer by laying a straight-edge from 120 pounds on the P_2O_5 axis to 80 pounds on the nitrogen axis. (Any point lying along the straight edge would represent \$12 worth of fertilizer for N at 15 cents per pound and P_2O_5 at 10 cents per pound). The intersection of the straight-edge with the isocline labeled $P_n = 1.5\ P_p$ at about 60 pounds of P_2O_5 and 40 pounds of nitrogen is the best that can be done with \$12 worth of fertilizer per acre. A yield of approximately 79 bushels is estimated from figure 10.1 for these inputs of N and P_2O_5.

Value of corn	\$79.00
Cost of N	6.00
Cost of P_2O_5	6.00
Margin over fertilizer	67.00

Although the farmer's margin over fertilizer cost in the above restricted case is reduced by about \$11 per acre as compared to the first "optimum" inputs, farmers may still be justified in "holding back" because of capital shortage or uncertainty. Farmers probably should be permitted to choose their own "optimum" from a range of alternatives presented to them by extension agronomists. Thus, if a farmer could realize \$1.50 on each dollar of his limited capital invested in some other enterprise, he could roughly equalize this return with his last dollar expended for fertilizer.

For a tenant farmer who pays one-half share rent and must bear all fertilizer cost, a different optimum solution is indicated for the preceding price situation. For the tenant, the relevant nitrogen-corn price ratio would be $\frac{0.15}{0.50} = 0.30$. At the most, it would pay him to apply only about 58 pounds of N and 81 pounds of P_2O_5, which would result in a predicted yield of about 90 bushels per acre.

If some new technique should alter the nitrogen-phosphorus price ratio, another isocline (fertilizer mix line) should be chosen. For example, if nitrogen and P_2O_5 were the same price per pound, production should be expanded along the isocline labeled $P_n = P_p$ in figure 10.1.

Application to Alfalfa

A similar production "map" in figure 10.2 could supply relevant economic information to a farmer growing alfalfa on Webster loam soil in north central Iowa. If alfalfa is assumed to be \$20 per ton, P_2O_5 is 10 cents per pound, and K_2O is 8 cents per pound, the fertilizer mix line labeled $P_k = 0.8\ P_p$ should be selected. Following this isocline until the dashed line labeled 0.005 (representing the phosphorus-alfalfa price ratio) is reached, an optimum input of about 21 pounds of K_2O and 69 pounds of P_2O_5 is indicated. A yield of about 3.15 tons is predicted by the isoquants for this input of K and P.

If the farmer is limited on capital or is a tenant, he may be rational in not applying as much fertilizer as indicated by the phosphorus-alfalfa

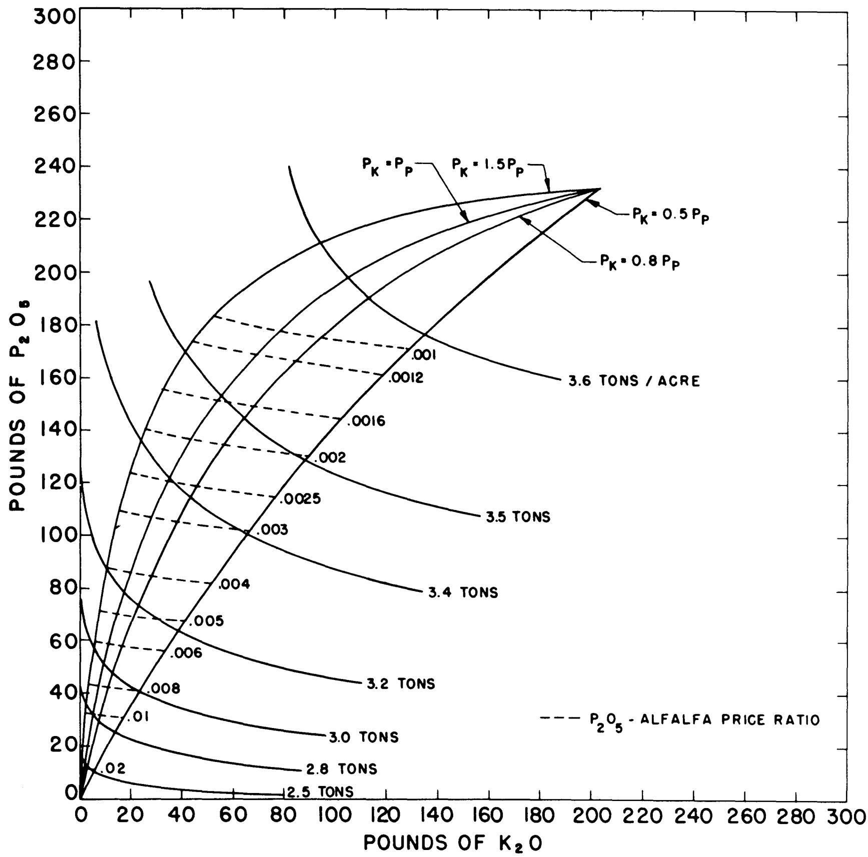

Fig. 10.2 — Yield isoclines and isoquants for alfalfa on Webster soil, Iowa. Optimum rates are indicated by dashed lines representing the phosphorous-alfalfa price ratio.

price ratio. Whatever the level of production, however, it should be obtained along the isocline representing the appropriate phosphorus-potash price ratio.

Limitations of Existing Data

One limitation of the procedure followed by the use of figures 10.1 and 10.2 is that it does not include residual or carry-over response in the second year for fertilizer applied in the first. The residual problem is analyzed for corn by Heady, Pesek, and Brown (2) and could be incorporated into figures 10.1 and 10.2. However, residual response in the second year may partly reduce the response to new fertilizer applications in the second year. Consequently, the first year's response alone may be a fair approximation of the response to be obtained year after year, including the next year's residual response. Of course, more empirical information regarding fertilizer response in succeeding years is needed.

Another limitation to recommendations made from production functions such as those represented by figures 10.1 and 10.2 is that the recommendations are based on a single year's result. Response on the same soil type could be much different in another year under different growing conditions. Confidence interval limits can be set up which may be "narrow" for last year's experiment, but these confidence limits do not really apply to next year's crop — which is what the farmer cares about. Also, even when the farmer has the same soil type, his fields will seldom be of exactly the same fertility level as that of the experimental plot.

A possible solution to these related problems is to apply the principle of continuity between experiments as well as within. Thus, a number of experiments run on the same or similar soil types but with varying fertility levels could be "pooled." A more general production function could be obtained in this manner, and it would include soil test measurements as variables as well as fertilizer applications. Then, results of the farmer's sample soil test could be "plugged into" the general production function to predict expected fertilizer response. Another advantage to such a procedure would be that if experimental results over a number of years were "pooled," an estimate of response variability could be obtained which would relate to *next* year's crop.

Before more general estimating functions can be tried, results of many factorial experiments, along with soil tests, must first be accumulated. Meanwhile, results of individual experiments could be utilized where applicable through use of charts such as figures 10.1 and 10.2.

References Cited

1. COCHRAN, W. G., and COX, G. M., 1950. Experimental Designs. Wiley, New York, pp. 124-27.
2. HEADY, E. O., PESEK, J. T. and BROWN, W. G., 1955. Crop response surfaces and economic optima in fertilizer use. Iowa Agr. Exp. Sta. Res. Bul. 424.