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Chapter 9

Evaluating Response to Fertilizer Using Standard Yield Curves

THE term "standard" curve implies acceptance of some particular hypothesis or yield function for evaluating response to fertilizer. For purposes of this presentation, the standard curve technique based on the exponential function will be used. But his in no way implies satisfaction with the current state of knowledge as to appropriate yield functions for fertilizer (cf. Chapters 1, 5, and 6).

Standard yield curves based on the exponential function are prepared from a table of values of $1\text{-}R^x$, in which R, a fixed ratio of successive increments in yield, has been assigned a specific value, 0.8 in this instance. Each value of x (unit of fertilizer) is associated with a specified value of $1\text{-}R^x$. The higher the value of $1\text{-}R^x$ the nearer the curve approaches maximum. On the standard yield chart, M, maximum yield, is coincident with the top of the $1\text{-}R^x$ scale, that is, when $1\text{-}R^x = 1.0$. A large number of standard yield curves should be prepared by anyone who uses this form of the graphic method. The curves will have different shapes by varying the scale on the x axis, but for each tabulated value of x the value of $1\text{-}R^x$ is always the same. Instead of finding the value of R that represents best fit to the data, R is standardized and a fit is obtained by varying the size of a unit of application. Each standard curve is based on a different size of unit. The decimal fraction $1\text{-}R^x$, when multiplied by 100, represents the percentage of maximum yield.

Given an adequate set of standard yield curves, some part of one of them can be found to describe yield responses to fertilizer from good rate experiments. This may be considered to be a fair statement, but it might well be added that it is true provided the exponential function fits the data. Fitting a standard curve to the reported yield is done by plotting the latter and overlaying on a standard yield curve. Usually the obvious choice is between 2 or 3 portions on one or two curves. From that point further refinement can be attained by recording plus and minus deviations of reported yields from the curve. A little practice will enable one to locate a fit at which the sum of the plus and minus deviations approximates zero.[1] If a good "tool kit" of standard curves has been

[1] A forthcoming U. S. Department of Agriculture publication, "A Graphic Method of Interpreting Response to Fertilizer," includes a more complete description of the method.

prepared, results from this process can be made rather quickly to approach the accuracy found by more precise time-consuming methods.

The constant M is read on the overlay at the point at which the yield scale coincides with the value 1.0 on the $1\text{-}R^x$ scale of the standard yield curve. The $1\text{-}R^x$ value for any yield read from the curve is obtained as y/M. Two readings from the curve provide two $1\text{-}R^x$ values, and the x value of each of these is found in the table used in preparing the standard yield curves. The number of pounds of fertilizer that represents the range between the two yields read from the curve, divided by the difference between the two x values, results in the number of pounds per unit required to obtain the fit, when R = 0.8.

Example

M = 119 bushels, read from standard yield chart,

	y	$1\text{-}R^x = y/M$	Units of x (from table)
y_{120} lb. =	92 bu.	.77311	6.65 = n+a
y_0 lb. =	17 bu.	.14286	0.69 = n, "soil content"
		Difference	5.96 = a, applied portion

Size of unit, $u_a = 120/5.96 = 20.13$ lbs.

a, in units = lbs. applied/u_a

x value in table = n^{+a} in units. $y = M\,(1\text{-}R^x)$.

Graphic Versus Mathematically Fitted Curves

Some indication of the approach to accuracy that can be obtained by this method is indicated by results obtained when used in analyzing three 12-rate experiments involving nitrogen on irrigated corn. Results are compared with those obtained from use of the mathematical solution for least squares suggested by Stevens (2). These comparisons are presented in table 9.1. Results obtained by the two methods are equivalent for purposes of recommendations. This is true whether all 12 rates were used, or whether only 5 or 6 rates distributed over the range were used in fitting the curves. In the Oregon and Nebraska experiments, rates were carried to 320 pounds of N per acre; in the Washington experiment, to 520 pounds. In all instances the curves become decidedly flat at the higher rates.

In 3 of the 6 comparisons between the graphic and the Stevens methods, differences in yields at the most profitable rates were smaller than 1 standard error in the yield at MPR on the Stevens fitted curve. Sums of squared residuals are approximately twice as great for the graphic as for the Stevens fitted curve. Obviously, if the problem were one in which precision rather than a basis for field recommendations were needed, the mathematically fitted curve would be necessary. But if the exponential equation is suited to the data, the graphic method of fitting the curve is useful for those who need reliable answers quickly.

TABLE 9.1. Comparison of Results from Graphic and Mathematical Solutions for Least Squares Fit When Applied to Three 12-Rate Experiments Involving Nitrogen on Irrigated Corn (Based on Unpublished Data Supplied by Soil and Water Conservation Research Branch, ARS, in Cooperation with Indicated State Experiment Stations)

Experiment and Method of Analysis	Number of Rates[a]	Most profitable rate	Yield at MPR	$s_{\hat{Y}}$ at MPR	Σe^2	M	R	s_M	s_R	Return per Acre at MPR	
		(Pounds)	(Bushels)	(Bushels)		(Bushels)		(Bushels)		(Dollars)	
Ontario, Oreg., 1952											
Graphic	6	168	140.7	-	85.4	-	-	-	-	173	-
Stevens	6	151	139.0	5.0	75.0	-	-	-	-	173[b]	173
Graphic	12	168	142.0		270.3	150.1	0.75544	-	-	175	-
Stevens	12	164	141.7	1.7	254.8	150.2	.75000	6.7	0.0204	175[b]	175
Hardy, Nebr., 1952											
Graphic	6	136	119.1	-	76.4	-	-	-	-	147	-
Stevens	6	153	119.9	1.4	27.3					146[b]	145
Graphic	12	135	119.0	-	331.3	123.9	.63653	-	-	148	-
Stevens	12	164	121.6	1.4	161.7	127.8	.70652	5.0	.0520	146[b]	146
Prosser, Wash., 1953											
Graphic	5	233	137.9	-	121.3	-	-	-	-	159	-
Stevens	5	267	140.9	2.7	52.8	-	-	-	-	157[b]	156
Graphic	12	231	135.3	-	520.0	145.3	.89660	-	-	155	-
Stevens	12	269	140.2	2.0	307.2	152.7	.83504	6.5	.0435	156[b]	155

[a] When only 5 or 6 rates were used, they were the same for each method and were scattered over the range.

[b] Calculated from Stevens' fitted curve at MPR on graphic curve.

The last column of table 9.1 shows returns per acre above cost of fertilizer at the most profitable rate as determined from the graphic curve and from the Stevens fitted curve. Then, using the constants of the Stevens fitted curve, the return above cost of fertilizer was calculated for the rate indicated as most profitable on the graphically fitted curve. The differences are negligible.

Results for Two Nutrients in a Factorial Design

Methods have not been developed for simultaneous solution for values of the constants of the exponential equation when two or more independent variables are involved. For combinations of independent variables, results are obtained by using constants derived by fitting each regression curve at specified levels of each of the other variables. Thus, in calculating yields for a production surface, this equation is used under the assumption that the rate (R) of response to a nutrient is the same at different levels of the other nutrients. Results are shown for this equation applied in this way to three factorial experiments. Results obtained in these instances are compared with results from use of the quadratic square-root equation used by Heady and Pesek.[2]

Comparisons are shown in tables 9.2 and 9.3. Table 9.2 shows the sums of squared residuals explained by regression, and the coefficients of correlation resulting from use of the exponential and quadratic square-root equations as applied to three 9 x 9 partial factorial experiments.

TABLE 9.2. Sums of Squared Residuals Explained by Exponential and Quadratic Square-Root Equations as Applied to Three 9 x 9 Partial Factorial Experiments

Experiment	Sums of Squares				Coefficients of Correlation	
	Total	Treatments	Explained by Regression			
			Exponential[a]	Quadratic Square Root	Exponential	Quadratic Square Root
Corn	242,707	233,811	222,927	222,899	0.9764	0.9764
Alfalfa	29.80	26.75	20.78	22.98	.8694	.9229
Red Clover	17.85	13.66	9.69	11.52	.8425	.9184

[a]Based on constants derived from only 17 of the 57 treatment combinations, as no simultaneous solution is available for the equation. The SS are computed for all 57 treatment mean yields.

[2]See Heady and Pesek (1). When N and P are the independent variables, the quadratic square-root equation used by these authors is written as:

$$y = a + b_1 N + b_2 P + b_3 \sqrt{N} + b_4 \sqrt{P} + b_5 \sqrt{NP} .$$

TABLE 9.3. Sums of Squares Reported from Calculated Yields

	Entire Surface		Deducting "0" and "320" Layers	
Experiment	Exponential	Quadratic Square Root	Exponential	Quadratic Square Root
Corn	5,442	5,485	2,296	2,366
Alfalfa	3.27	1.77	.75	.81
Red clover	1.98	1.40	.64	.62

Only 17 of the 57 treatment combinations were used in finding constants of the exponential equation, because of lack of method for simultaneous solution.

In contrast with the exponential equation, which approaches though theoretically does not reach the calculated maximum, the quadratic square-root equation has merit in the ability to calculate reduced yields after the maximum has been reached.

Sums of squares of deviations reported from calculated yields are shown in table 9.3 for the entire production surface and for the more relevant portion — after deducting deviations at the extremes, 0- and 320-pound levels in these experiments. As indicators of reliability of results for use in recommending rates of application, deviations assume importance primarily around the section of the surface that includes combinations reasonably close to those found to be most profitable. A high percentage of the deviations occurred in the "fringe" area of the surface. If the deviations that occur there are deducted, there is no difference in the sums of squared residuals resulting from use of the two equations. Of course, as mentioned by Mason in Chapter 5, few persons would use this procedure because of the high degree of subjectivity involved. In this sense, the quadratic equation would appear to be the best fit.

Differences in yields calculated by the two equations at the most profitable combination were small in relation to the standard error of the yield at MPR as determined by the quadratic square-root equation. Differences in returns per acre above the cost of fertilizer were substantially less than the value of the units represented by one standard error of the yield.

The many facets of these comparisons are discussed more fully in the reference cited.[3] These few illustrations are presented merely to indicate comparisons based on the three experiments with no wish to imply definite conclusions. Certainly one job of methodological research that might well be undertaken is that of finding a simultaneous solution for values of the constants of the exponential equation when two or more independent variables are involved. Also, it would be well if someone

[3]A graphic method of interpretive response to fertilizer, op cit.

would determine the nature of the distribution of the constants of the exponential equation, so that standard errors computed from the one-variable form of this equation could be predicted with more certainty. Standard errors are now based on the assumption of normal distribution.

References Cited

1. HEADY, E. O., and PESEK, J., 1954. Fertilizer production surface with specification of economic optima for corn grown on calcareous Ida silt loam. Jour. Farm Econ. 36:466-82.

2. STEVENS, W. L., 1951. Asymptotic repression. Biometrics 7:247-67.

PART IV

Application of Data

- Simple Nomographs
- Farm Planning
- Budgeting
- Linear Programing
- Price Considerations