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Chapter 6

Agronomic Problems in Securing Fertilizer Response Data Desirable for Economic Analysis

SINCE economic considerations so often determine the adoption of fertilizer use and other production practices, the individuals involved with the making of recommendations for fertilizer use need a workable economic framework within which to make their estimates. In research designed to determine alone the deficiency of a particular crop, the economics are simple, viz., either the quantity of fertilizer used returns a profit or it does not, and therefore, it is or is not recommended under similar conditions. The most serious limitation of this approach is that there is no way to know whether more or less fertilizer would have returned a higher net profit. In more complex experiments involving several rates of a fertilizer element, or several rates of two or more elements in various combinations, the simple economic procedure outlined above is not adequate. Although the simple procedure can be used in successive trials for the several rates and combinations individually, the flexibility and versability of a continuous production function are highly desirable.

Variations by Soils

Fertilizer needs of a given crop vary among the different soils, among fields on the same soil types, and under different levels of management. Because of this, some method of determining the fertilizer requirement of a particular crop on a given field is used. The chemical soil test is commonly used to accomplish this, although techniques employing the composition of plants or plant parts show promise of making better estimates of fertilizer needs possible.

Economic functions for decisions must take into account (a) the soil deficiencies in one or more elements, with the related crop responses to the combinations of these elements, and (b) the changing price ratios among input factors and between these and output. One object of this chapter is to review the types of experiments which are needed in estimating economic levels of fertilizer use. Another objective is to indicate some of the problems involved in conducting these experiments. The manner in which the experimental data might be used will be considered in view of attaining the best possible fertilizer recommendation for the farmer. Financing of research programs is usually a problem.

However, it will be assumed that required financial support exists, and a study of nonfiscal problems will be made.

Previous Work

Mitscherlich (9) and Spillman (14) proposed use of the logarithmic and exponential forms of an equation outlined in Chapters 1 and 5. Others (1, 7, 8, 11) have proposed a number of equations to express crop responses as a function of a single fertilizer nutrient variable. These several equations can be used to determine the optimum rate of fertilizer needed by the usual procedure of equating the first derivative of the particular equation to the price ratio of the input to the output, as indicated by equations 11 and 12 in Chapter 1 (page 12).

Ibach (5, 6) employed the Spillman function to determine the optimum quantity of nitrogen, phosphorus, and potassium by means of successive approximations. This procedure has the advantage of being mathematically simple. The most serious objections to the procedure are: (a) Only part of the data is utilized. (b) Three separate functions need to be written for the responses to the three elements, instead of a single equation with three variables. (c) No provision is made to allow the three elements to interact simultaneously. General limitations to the logarithmic and exponential equations are outlined in Chapters 1 and 5.

Both Mitscherlich (10) and Spillman (14) showed how to extend their single-variable equations to include more variables. More recently Heady et al. (3, 4), and Pesek and Heady (12) have proposed equations in two variables, to express the yield response of crops, which overcome the objections listed above. These investigators have also shown (a) how to employ these equations by simultaneous solution, in arriving at the optimum fertilizer rates and ratios for the experimental conditions under different assumed prices of product and fertilizers, and (b) how fertilizer ratios may change as production is expanded. They have also shown that, under given conditions, maximum yields can be achieved only by a particular combination of the two fertilizer elements. Similar equations in three or more variables can be written to express yield responses when the proper data are available.

Hanway and Dumenil (2) have related the response of corn to nitrogen in Iowa and the test for nitrifiable nitrogen in the soil. The Mitscherlich equation was used to express the response curve; a logarithmic equation was employed to express the response of corn to a given quantity of nitrogen as a function of the nitrogen soil test. These equations were combined to permit evaluation of optimum nitrogen needs for individual fields, and are currently used as the basis for nitrogen recommendations. Of particular interest was the fact that the data on hand for correlation purposes indicated that the experimental results did not deviate significantly from a general nitrogen response curve, and that as the nitrifiable nitrogen in the soil increased, the point of origin needed only to be translated along this response curve. The correlation is based on about 85 nitrogen experiments conducted over a period of 10 years, using those experiments in which there was evidence that the levels of other nutrient elements were adequate, and stand levels averaged 12,000 stalks per acre.

The use of the experiments above is pointed out not only to show how the soil test can and should be integrated with response data, but also to indicate the value of using data already collected (see Chapter 8 on estimation of functions from soil tests). A unique advantage of utilizing data collected over a past period of years is the fact that they include a sample of climatic variations not possible to achieve in any number of experiments in any two-year, three-year, or other short period. Since it is still necessary to predict fertilizer needs for average weather conditions, fertilizer response data over a period of years are invaluable.

The Multi-variable Response Function

Numerous experiments with two or more variables have been conducted. However, only recently has any successful attempt been made in expressing two variables in a general response equation and in applying principles of production economics to them. Yet there is a strong need to know the general nature of the multi-variable fertilizer response equations for various crops because they are the basis for fuller utilization of past and future fertility experiments within an economic framework. The number of combinations of soils, soil fertility levels, fertilizer grades, prices, and responses is so great that it seems improbable that rapid progress can be made in the absence of such empirical equations.

Nutrient Interaction

The approach by successive approximation by single-variable equations is possible under some particular conditions, but is likely to be inadequate under others because of the interaction of nutrients in producing yield increases. Interactions of nitrogen and phosphorus, phosphorus and potassium, lime and phosphorus, and population level and nitrogen are commonly observed. Interactions of three factors of production even appear frequently enough to merit attention in developing equations. It is necessary to utilize these interactions to take full advantage of the soil fertility, the applicable alternative fertilizer combinations, and varying price ratios of fertilizer elements and products in making the best fertilizer recommendation possible.

Although interactions are most often recognized when two or more of the nutrient elements are in low supply in the soil, it is quite probable that they also exist at higher fertility levels. They are less frequently identified at these higher levels because they are of smaller magnitude and the experiments usually are not precise enough to detect them, and because other independent factors limit the potential yield increases.

Requirements in Experiments

Keeping in mind the curvilinear nature of the normal response curve, and the remarks made above, it is possible to write the minimum terms required in a production function. For two variables it would be:

$$(1)\qquad Y_2 = a + b_1F_1 + b_2F_1^n + b_3F_2 + b_4F_2^n + b_5F_1F_2,$$

where Y_2 is the yield, F_1 and F_2 are the two fertilizer elements, n is an exponent other than one, and b_1 through b_5 are constants. If only the response to fertilizer is considered, a becomes zero, and Y_2 becomes Y_2'. The equation may also require an additional term such as $b_6F_1^nF_2^n$.

For a three-variable production function the equation would be:

$$(2)\qquad Y_3 = a + b_1F_1 + b_2F_1^n + b_3F_2 + b_4F_2^n + b_5F_1F_2 + b_7F_3 +$$

$$b_8F_3^n + b_9F_1F_3 + b_{10}F_2F_3 + b_{11}F_1F_2F_3,$$

where F_3 is the third fertilizer variable and b_1 through b_{11} are constants. It may also be necessary to include such terms as $b_{12}F_1^nF_3^n + b_{13}F_2^nF_3^n + b_{14}F_1^nF_2^nF_3^n$.

The next problem is one of determining the combinations of treatments which give data satisfactory for equations, and of allowing ample measure of "goodness of fit." Remembering that data containing X points can be fitted with an equation with X - 1 constants, and the multiple correlation coefficient, R, will be equal to one, it is apparent that a two-variable experiment must have at least 7 treatments, and a three-variable experiment must have a minimum of 12. These experiments might be 4 x 2 and 3 x 2 x 2 factorials, respectively. While the former would provide one degree of freedom for deviation from regression, the latter would provide no measure of deviation from regression since the equation would pass through all points and there would be a "perfect" fit. Further objections to such small experiments arise from the fact that one variable in the former, and two variables in the latter could be fitted with linear terms. It is generally accepted that responses are curvilinear and provisions for this should be made in the design of the experiment. Therefore, the minimum types of experiments for two and three variables are 3^2 and 3^3 factorials, respectively, provided the general nature of the production function is known.

But one of the most important problems at present is to determine the production function. Two measures of the "goodness of fit" are the multiple correlation coefficient, R, and the deviation from regression. There seems to be no good figure for the number of degrees of freedom needed to give a reliable estimate of the mean square for deviation from regression. However, a number five or six times as great as the number of constants in the function would appear to be reasonable. Under these conditions a 6^2 or 7^2 factorial would be needed for an experiment in two variables, and a 4^3 or 4 x 4 x 5 factorial for three variables. These are about the minimum to satisfy the requirements of the statistical manipulations. When replicated twice, these experiments would contain 72 to 160 plots or require an area of about 0.9 to 2.0 acres in a corn experiment and would require up to one ton of high analysis fertilizer material. As hay or small grain plots, the area would need to be about one-fifth as large and would require proportionately less fertilizer.

In addition to the above, there are other characteristics which are desirable in experiments. The range of treatments should be wide enough to allow a diminishing total product at the higher treatments. It is not likely that the complete range of treatments on a soil low in fertility can be made with fewer than seven or more levels of the fertilizer elements without making the increments too great, especially at the lower levels. It is quite apparent that an experiment of 7^3 factorial would be about four acres in size and contain each treatment only once. Where the complete range cannot be included the lower range should be studied.

Some provision should be made to evaluate the residual effect of the fertilizer in terms of currently applied fertilizer. High rates of fertilizer are not usually expended in a single year, especially on heavy textured soils, and the residual value of even moderate applications is high. The value of the residual fertilizer should be measured in fertilizer equivalent during the second season as well as in terms of product yield. This gives a better evaluation of the residual effect because it cancels much of the seasonal influence. To include this phase in the study, the experiment will need to be made even larger.

Physical Problems

The agronomist conducting these experiments is faced with the need of experiments of ever-increasing size. The primary problem then is the size of the experiment and location of the experiment on a relatively uniform site. A uniform soil area is required in order that the results can be properly interpreted. Any soil survey map shows that soil types usually do not occur in large uniform areas. If the limits on soil survey maps were homogeneous, representing a particular soil type, the situation would not be so difficult. However, even on either side of the soil boundary line, there is a transition strip which is not representative of either soil type and cannot be used. With a heterogeneous system the variance will increase and interpretation and extension to other soils will be more complicated.

In any large area of a given soil type, other natural variations can cause nonuniform conditions. The depth of topsoil and the degree of erosion vary at different positions in the field. The slope can change by several percent and, furthermore, the aspect may change (i.e., in case of many soil types the north, south, east, and west slopes may be present and bring about variations in crop yields). When topography changes, internal drainage conditions, fertility, rainfall retention, evaporation, insolation, and soil temperature may also change. All of these variations contribute to errors of measurement which may become prohibitively large. In some cases, with small replicated experiments, as much as one-half of the total sum of squares has been due to replications. Consider what would happen to the precision in an experiment in which one replication was as large or larger than a small experiment such as the one above. It makes little difference whether the variance is measured by analysis of variance or deviation from regression, it remains if soils are not homogeneous.

Large areas of a given soil type or management system do not necessarily remain intact but are often broken up by man's activities. Parts of the area may be on different farms, in different fields, or divided by present or past crop boundaries. Differential treatments with respect to liming, fertilizing, manuring, crop removal, drainage, and tillage further subdivide the natural soil areas. Terracing, stripcropping, and contouring not only reduce the area into small strips, often too small for large experiments, but also present difficulties in laying out an experiment, with the consequence that such fields are often omitted from consideration as experimental sites.

In order that the experiment may be useful, it must be conducted in a field which will respond to fertilizer. If it responds to only one element, or only slightly to all elements, it is essentially worthless in describing the production function. Fortunately, enough work has been done with the soil test to make possible the selection of a field which will respond with a rather high degree of certainty. The lower the fertility and the greater the potential response, the more valuable is the experiment in determining with confidence the important characteristics of the production equation. Hence, fertility level is a primary consideration.

It appears from physical considerations that an experimenter should not expect to find suitable areas which exceed two acres in size, for use in a single experiment. Even so, uniform areas of some soil types as large as one-half acre are relatively few.

Miscellaneous Considerations

The geographical distribution of soil areas within a political or experimental unit presents problems in communications and control. Even in a small state such as Iowa, where the experiment station is centrally located, it is necessary to travel 200 miles to reach certain soil types. Experiments this distance from headquarters may not receive sufficient attention.

In addition to the above considerations, the field selected must be accessible both from the standpoint of location on a good public road and a good farm road where required. Means for controlling insects, diseases, and weeds must be present on the farm or within easy reach, since experiments may fail as a result of any one of these factors. Varieties capable of responding fully to the fertilizers with optimum plant populations have to be used. Poor varieties might lead to faulty conclusions about potential responses.

Plant population is one variable which may be studied; it is a factor which can influence the results markedly. Some crops with low stands respond less to fertilizers than crops with high stands. High stands on low levels of fertilizer treatments may affect the production adversely. There is always a question as to whether the stand on individual plots should be adjusted so as to fit expected performance (i.e., to adjust yields for stand). If these adjustments are made, the amount of labor needed will probably be doubled. For some crops, stands are less important. In other cases such as hay and some grains, stands already established are used for the experiments. Selection then is possible to meet the requirements of the trial.

Problems in Public Relations

Aside from the physical problems related to soils and their distribution, there are also problems in public relations. The research agronomist must depend upon some local person to help locate the possible sites for his experiments. Usually the local representatives of the college such as the extension directors or county agricultural agents are asked to help. For best results, it is important that the particular individual be highly interested in the type of work to be done, and that he understand the magnitude of the resources which will be expended in performing the work. Since he has to know what characteristics are wanted in the site, he should have an understanding of soil types and soil fertility in general. Unfortunately, a minority of county extension directors in Iowa, and possibly in other states, are graduates in agronomy, and real interest and good training in agronomy are needed. Finally, the local man must be willing to keep close contact with the experiment and the cooperator and to help answer the questions which always arise in the farmer's mind when the research agronomist is not available.

The cooperating farmer should have an interest in and an understanding of experimentation as a means of learning more about his own and his neighbors' problems, in addition to operating a farm with an acceptable area for the experiment which will be in the desired crop in a given year. He should have some willingness to accept a little inconvenience. Even though the investigator makes an effort to avoid interference with the normal operations on the farm, he is not always successful, and the farmer's understanding in the matter is very welcome. Usually the inconvenience is no more than having to plow, plant, or harvest in another field before the one in which the experiment is located, but even such small items could cause difficulties with some farmers. Inconvenience should be avoided where possible because a reflection on the experiment station would be inevitable. The man best suited to "size up" the cooperator is probably the county extension director.

In some cases, the experiment may cause a real or apparent loss in total product to the farmer. Usually this presents no special problems. Experience has been that the farmer was willing to accept a small loss on the check plots, low treatments for the increase in yield on the other plots, and the residual value of the fertilizer. Where this has not been acceptable, a reimbursement in cash or in kind has been satisfactory. Any additional operations requested in the execution of the experiment should be contracted with the farmer, and the farmer should be remunerated for any large removal of product which will not be returned. Oral agreements have been completely satisfactory in Iowa.

The seasonal nature of agronomic field experiments often presents problems in initiating and completing experiments because of weather. The experimenter must draw on experience to plan his program. He must be able to complete it in normal seasons.

The final problems in conducting these experiments relate to the personnel available at the experiment station to distribute the fertilizer, make notes, sample, harvest, and record the information. Well-trained technicians usually are not available to perform the operations. Much of the work has to be done by the researcher and the remainder by untrained persons. It is extremely difficult to instill into a day laborer the necessity for the careful and precise performance of all operations.

Some Solutions to Problems

The primary problem, which sets apart experiments designed to study the form of the production equation, is that of probable size and subsequent selection of sites. All the other problems are shared in common by smaller experiments which, of course, also have an important place in eventually providing sound bases for making the best possible fertilizer recommendations to farmers. The problem is to keep as many treatments as possible, provide for residual comparisons, and still keep the experiment within reasonable limits of size.

Since the study is concerned with determining a regression equation, the stress might be placed upon getting the largest possible number of points in the equation, rather than upon extreme precision in determining any one of fewer points. Hence, a large randomized factorial experiment without replication may be employed. Table 6.1 presents some results from three different experiments involving nitrogen and phosphorus fertilizers for corn, and phosphorus and potassium fertilizers for alfalfa and red clover. There were 57 treatments replicated twice. The data presented indicate that the variation as measured by deviations from regression agreed well with the results obtained among plots treated alike. As long as the number of degrees of freedom for deviations from regression is large compared to the number of constants in the equations fitted, it is not likely that the researcher will be misled with respect to the apparent precision of his experiment. For this reason it is felt that a nonreplicated factorial experiment is a definite possibility for purposes of studying production functions.

To supply a comparison for the residual study in the following year, a number of plots without treatment may be randomized among the factorial treatments. Since the equivalent residual effect does not usually exceed half of the initially applied quantity, the number of treatments can be fewer than in the original experiment. For example, in a 10^2

TABLE 6.1. Comparison of the Variation in Three Experiments as Determined by Deviations from Regression[a], and Among Plots Treated Alike

Crop	Source of Variation	d.f.	M.S.	F. Ratio[b]
Corn 1st year	Deviations from regression	51	215	1.38
	Among plots treated alike	57	156	
Corn 2nd year	Deviations from regression	51	123	1.28
	Among plots treated alike	57	96	
Corn Total	Deviations from regression	51	404	1.50
	Among plots treated alike	57	270	
Red Clover	Deviations from regression	51	215,406	.86
	Among plots treated alike	57	250,856	
Alfalfa	Deviations from regression	51	298,782	1.44
	Among plots treated alike	57	207,302	

[a] $Y = a + bF_1 + cF_1^{\frac{1}{2}} + dF_2 + eF_2^{\frac{1}{2}} + fF_1^{\frac{1}{2}}F_2^{\frac{1}{2}}$.

[b] "F" values for 50 and 55 degrees of freedom are 1.58 and 1.90 for 5 percent and 1 percent points respectively. cf. Snedecor (13).

Source: Heady, et al. (3)

factorial the previous year, perhaps a 4^2 or 5^2 factorial will be sufficient to cover the range of residual effects. It may be desirable to replicate these in the second year so it will be necessary to include 32 or 50 blank plots respectively for the above example. This means that in the first year there will be that many check plots. These extra plots have an advantage in the first year because they provide a good estimate of the check yield, and form an independent estimate of a in equations 1 and 2. Even when residual studies are not to be made it might be advisable to carry several check plots.

Some may not be willing to discard all replication but may want a wide range of coverage with their treatments. In this case it may be enough to replicate 5^2 factorial treatments of a 7^2 for larger factorial experiment and 3^3 factorial treatments in a three-element factorial experiment. The treatments would occur in a completely randomized experimental design; provision for the residual study above could still be included. The size of the experiment would be increased, but there would be an independent estimate of variance for checking the deviations from regression, and the precision of the replicated observations would be improved.

Another alternative in keeping the experimental area within limits and still retaining other desirable characteristics, such as including replication, is to discard treatments which are very likely to be outside the range of economic substitution (3, 4, 12). In a 10^2 nitrogen x phosphorus factorial, for example, it may be possible to discard some treatments in which the nitrogen rate is high and the phosphorus is low and also some in which the opposite is true. Possibly 30 of the 100 treatments may be omitted without serious consequences, and the same applies to experiments in three or four variables. Before proceeding in this manner it is important to have some knowledge of the probable outcome.

A final alternative is the one reported by Heady et al. (4), in which certain treatment combinations were left out in a systematic manner throughout all the factorial combinations, but all treatments retained were replicated. Where a fairly good prediction of the results is not possible this procedure has some advantage over the one above, as was indicated in the red clover and alfalfa experiments reported by these investigators.

Applications

Once the general form of the production equation is known and the important terms are determined on different soil types and under a range of weather conditions, the results may be employed to utilize other data. Because small experiments will always be less costly and easier to conduct under all conditions, they will be more numerous. As pointed out by Mason in Chapter 5, particular designs of smaller experiments can be formulated which will give the highest efficiency for any particular regression equation which applies. Actually, data for a large number of such experiments are already available. There may be several approaches to the problem after the production functions are determined.

To become useful, crop response data will have to be correlated with a soil test and plant tissue test (or a combination of these or other tests) to evaluate the soil fertility or the resources already on hand. It is as unreasonable to recommend fertilizers for a farmer, without some estimate of his soils' initial fertility, as it is to recommend ten extra dairy cows without knowing how many he already has, whether he can house them, or whether he is even interested in dairying. Too much fertilizer is recommended on this basis already. It it were not for the fact that fertilizers return 100 percent or more on the investment as compared to the 20 percent or less returned by some other enterprises, much more care would be exercised in making the recommendations.

For example, suppose that interest lies in soils which are deficient only in nitrogen and phosphorus, and also that there is interest in making recommendations for average operators. Suppose, too, that the smaller experiments have been on fields with average operators, and that the general fertilizer response function for this area is known. Let it be further assumed that four fertility levels of nitrogen and phosphorus are recognized by soil test: very low, low, medium, and high. There are, therefore, 16 possible nitrogen-phosphorus fertility situations which might occur in the area. One procedure would be to pool the results from all experiments conducted on soils in each of the 16 categories, and fit the production function to each group of pooled data. Since there is no longer interest in studying the production function, but rather in applying it, the experiments need not be large factorials, but they should be factorials. With the equations fitted, the optimum levels of fertilizer to use in individual cases could be determined by the procedures outlined by Heady et al. (4). The second possible procedure would be to relate the response to phosphorus and nitrogen fertilizers and the soil test levels for these two elements, using the pooled data from all the experiments. This pooled data would also be used to express the response part of the production function which might take the form of

$$Y' = c_1 N + c_2 N^n + c_3 P + c_3 P^n + c_4 NP, \tag{3}$$

where N and P represent the pounds of N and P_2O_5 applied and c_1 to c_5 are constants.

This is the response surface starting with a soil that has no nitrogen or phosphorus available. To express the response on soils with nitrogen and phosphorus already present to a given degree, the response predicted by the equation needs to be decreased by the yield proportional to the fertility already in the soil. Under these restrictions both N and P in the above equations have two components: (a) the soil component, and (b) the fertilizer component designated by subscript s and f. Since the plant cannot distinguish between them, $(N_s + N_f)$ and $(P_s + P_f)$ can be substituted for N and P in the terms. But N_s and P_f are functions of the soil tests for these two nutrients which are designated as α and γ, and will give responses proportional to their magnitude with proportionality constants k and m. By making these substitutions, setting n equal to 2, and expanding, the equation becomes:

$$y = c_1 k\alpha + c_1 N_f + c_2 (k\alpha)^2 + 2c_2 k\alpha N_f + c_2 N_f^2 + c_3 m\gamma + c_3 P_f + c_4 (m\gamma)^2 + 2c_4 m\gamma P_f + c_4 P_f^2 + c_5 k\alpha m\gamma + c_5 N_f P_f, \tag{4}$$

where y is the yield increase on a particular soil.

Taking partial derivatives of y with respect to N and P and equating each to the price ratio of N to y and P to y respectively, the following is derived:

$$dy/dN = c_1 + 2c_2 (k\alpha + N) + c_5 P = \frac{X_N}{X_y}, \text{ and} \tag{5}$$

$$dy/dP = c_3 + 2c_4 (m\gamma + P) + c_5 N = \frac{X_P}{X_y}, \tag{6}$$

where X is the price of factor indicated by the subscript.

In this case k and m are evaluated experimentally along with the other constants. A simultaneous solution of both equations will yield the optimum rates and ratio of N and P under specified prices and given soil test values.

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