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Chapter 2

Interdisciplinary Considerations in Designing Experiments To Study the Profitability of Fertilizer Use

MORE interdisciplinary cooperation among agronomists, statisticians, and economists is an important need in agricultural research. Fertilization research should be looked at from an agriculturist's viewpoint rather than from the confined viewpoints of the farm management specialist, the soils specialist, the marketing specialist, the mathematical statistician, or the specialist in leguminous nitrogen fixation.

The Economics of Designing Experiments

Economics is concerned with the use of scarce resources in attaining multiple objectives. Experimental designs involving interdisciplinary research involve economic considerations. In designing interdepartmental experiments, some of the objectives pursued are in conflict; other objectives are complementary, i.e., attainment of one objective may make it easier to attain another. Such conflicts and complementarities occur both within and between the sets of objectives commonly of interest to agronomists, economists, and statisticians.

The job of agriculturists in designing an experiment is to approach the "best combination" of objectives in designing a particular fertilization experiment. The best combination of objectives should recognize any existing complementarity. Of course, the best combination of objectives depends on the relative costs of attaining the objectives. Mention of the cost of attaining objectives calls attention to the relationships among research resources and attainment of research objectives.

Pairs of Resources May Be Substitutes or Complements

If substitution is "near perfect," the designer should use the cheaper of the two resources in designing his study; i.e., if two identical fields are available, one for $400 an acre and the other for $350 an acre, he should use the latter. At the other extreme, pairs of resources may complement or contribute to the productivity of each other. For instance, an agronomist and a statistician working together may design an experiment which is superior to the product of either working alone. Their effort is then complementary. If two resources are perfect complements in the sense that they are unproductive used alone, or in only one proportion, the designers should take full advantage of this complementarity. The two research resources should be used in the one proportion.

The difficult problems in selecting research resources arise, however, when resources are neither perfect complements nor perfect substitutes but are, instead, complements over wide ranges and substitutes over narrower ranges. In this case, the designer has to match the added costs of and returns from using another unit of one resource against the added costs of and returns from using another unit of an alternative resource. If a unit of one resource is more productive relative to its costs than another, it is logical to expand its use relative to the other. When research funds for a given experiment are limited, the best experimental design is one which yields equal additional returns for equal additional expenditures on the resources subject to the designer's control. If, as is very unlikely, there are unlimited funds to support the experiment, the best experimental design is one which yields additional returns equal to additional costs for all resources subject to the designer's control.

The designer should also ask himself whether (a) any part, or all, of any of the fixed resources could be disposed of (by sale or transfer to another experiment) at a net return in excess of what it would produce in the experiment under consideration, and whether (b) more of any of the fixed resources can be acquired at a net cost below what it would produce in the experiment. If the answer to either of these two questions is "yes" for a particular resource, the designer should cause the resource to become variable and adjust its use according to the rules previously considered.

In experiments on the economics of fertilization, a high degree of complementarity exists among the services of agronomists, statisticians, and economists. In most fertilizer experiments, agronomic (both in soils and in crops) and statistical training are complementary. And, if the experimental results are to be interpreted economically, the services of an economist complement those of the agronomist and the statistician. Thus, with the exception of a highly technical fertilization experiment intended to yield technical information for noneconomic application, most fertilization experiments can advantageously employ the services of agronomists, statisticians, and economists.

Reconciliation of Objectives

Agronomists, statisticians, and economists, as a result of their different training, comprehend and prefer to pursue objectives which are sometimes conflicting. Also, because research workers are specialists in different organizations or different parts of a given organization, their preferences and objectives may differ still further. These different objectives and preferences have to be reconciled and aggregated into group choices in designing cooperative experiments.

Generally speaking, *the reconciliation and aggregation process is a bargaining one,* with weights assigned to individual and institutional preferences on various bases such as, (a) the amount of resources contributed by the different organizations, (b) the professional repute of the individuals, (c) the democratic procedure of one vote per participant, or (d) the principle of "greasing the wheel which squeaks the loudest." If it were possible to price the objectives separately and produce research on some sort of a free enterprise basis, a free price system might be used instead of a bargaining process in making these decisions. Similarly, consensus or deference to recognized authority would make it unnecessary to use bargaining processes in arriving at these design decisions. But administrative authority is not well enough informed to make these decisions; uniformly recognized professional authorities do not exist and differences, not consensus, as to preferences are the rule, not the exception. Thus, the bargaining process seems inevitable in the committee meetings, Kaffeeklatsches, seminars, and informal cooperative arrangements in which experiments are designed.

The problem is not one of eliminating bargaining decisions in designing experiments. Instead, it is one of improved bargaining leading to design decisions. Such decisions can be improved *first* by appealing for agricultural statesmanship, rather than by encouraging competition among departments of institutions or among institutions. Agricultural research statesmanship, rather than destructive competition for personal position and prestige among individuals or ill-advised loyalty to one discipline among those serving agriculture, will lead to cooperative research which solves the problems of agriculture. A *second* important way of improving decisions on experimental design is to increase the knowledge of the designer (whether an individual or a committee) about (a) the nature and importance of objectives held by different research organizations, different disciplines, and different individuals, (b) the nature of different research resources and their usefulness in attaining the objectives listed in (a), and (c) research techniques or methods of value in using the resources considered in (b), to attain the objectives considered in (a).

In the remainder of this chapter, fertilization experiments in general will first be considered. Following this, special problems of making economic interpretation of data secured from fertilizer experiments will be considered along with the desirable characteristics of experimental data from the standpoint of economic analysis. Finally, a recently designed Michigan experiment will be reviewed. This outline will permit emphasis of two principle methods available for improving decisions on experimental design. They are (a) use of agricultural statesmanship, and (b) use of more knowledge about objectives, research resources, and research methods.

Specification of Function for Investigation

Most fertilization experiments involve investigation of a set of functional relationships such as that represented by equation 8 in Chapter 1.

This generalized function is, of course, too complex and extensive to be handled with the intellectual and physical resources of any research organization. Hence, the first step is to restrict the general area of investigation to a manageable size or number of input categories. This is commonly done in two ways. First, autonomous subfunctions within the function are isolated for study. The word autonomous here means that outcomes within the subfunction are not influenced by events in the remainder of the function. Choices among alternative subfunctions depend on the preferences of individuals and agencies and upon the comparative productivity of research resources in such alternatives. If, as is generally the case, such autonomous subfunctions are still too large to work with, controls have to be imposed on certain of the variables to limit further the realm of inquiry. Here the conflicting ends are "generality" and "accuracy." For given resources, the study can cover a larger subfunction with a low degree of accuracy or a smaller subfunction with greater accuracy. The designer must decide how much of one he is willing to sacrifice in order to get the other.

To illustrate the above two steps, consider the problem of setting up a fertilizer experiment within a generalized function, including all possible products, inputs, and associated technologies. This function could be cut down to, e.g., a corn, oats, and clover rotation which can be presumed to be independent of other rotations. This, however, would still require a very large experiment. There is almost an infinity of inputs to consider — land with all its variation, labor, nitrogen, different sources of phosphorus, potash, machinery, different technologies, varieties of oats, cultural practices, etc. If an attempt were made to study all of these factors at once, the resources required for the project would be spread very thinly, and only very inaccurate results (i.e., those with great variance) would be secured.

Restriction of scope can be attained by the imposition of controls, both selective and experimental. Here, many individual and organizational preferences must be considered. One agronomist may be particularly interested in corn over the cornbelt, while a cooperating colleague may be endeavoring to become a national authority on planting and fertilization practices for small grains. The experiment station director may know that agricultural leaders favor investigation of corn fertilization on a soil type within one state. Hence, all of these kinds of preferences and others, along with the conflict between generality and accuracy, enter into the series of negotiations leading to the final choice.

The final choice might involve, for example, (a) restricting the experiment to a rotation on the given soil type to include: (i) given varieties of corn, oats, and clover, (ii) given cultural practices, and (iii) given levels of available K_2O, and (b) restricting the experiment further to N and P_2O_5 as the primary variable inputs to be studied in application to corn only.

This last step would narrow the realm of inquiry to a consideration of only the following subfunction:

$$(1) \qquad Y_c = f'(N, P_2O_5 \mid \text{oats, clover, } K_2O, X_f \ldots X_n) + u \;.$$

This function reads as follows: the yield, Y_c, of a given variety of corn is a function of the amount of N and P_2O_5 applied to corn grown in a C-O-CL rotation with K_2O and other inputs X_f . . . X_n (such as, soil type, oats variety, clover varieties, cultural practices, etc.), fixed at specified conditions, or levels.

Unexplained Residuals

The "u" introduced in equation 1 stands for variations of actual yields from the functional relationship specified in (1) above. In practice, the u's are always, partially, functions of more or less uncontrolled and unstudied variables, such as lack of uniformity in soil types, variations in weather, and disease or insect infestation. So long as the u's behave substantially as though they are randomly and independently distributed with respect to the studied variables, they can be "averaged out" with statistical procedures. For instance, the method of least squares may be applied to secure estimates of equation 1 which minimize the sum of the squared deviations in the Y_c's. This procedure is appropriate so long as the u's can be interpreted as due to errors in measuring the Y_c's, or as random stochastic movements in the function, either with or without antecedent causes.

Another practical requirement is that the u's be small enough for the estimates of Y_c to be usable. At this point the objectives of the statistician and agronomist may come in conflict. Trained in estimating procedures, and perhaps charged by the experiment station director with responsibility for the statistical accuracy of estimates based on the data produced by the experiment, the statistician desires accuracy. Ordinarily, the agronomist does too, but not at the expense of what he may consider undue restriction of his work and expensive randomization and control procedures.

In investigating equation 1, the statistical conditions required with respect to the u's may be secured, in part at least, by (a) procedures which reduce errors in measuring X_j and Y_c, (b) controls on non-studied inputs and factors, and (c) procedures designed to randomize the incidence of unstudied and uncontrolled variables in the experiment and, hence, of the u's generated by those variables. Examples of the first set of procedures are doublechecking and the measurement of nutrients in the soil as well as those applied. The imposition of controls was illustrated above. Plot layouts to randomize the distribution of soil differences between plots are a common example of the third set of procedures. Decisions on such procedures must be made early in the experiment. As an earlier step, the total amount of resources to be devoted to the experiment has to be determined and allocated among such competing ends as: number of plots, measurement accuracy, search for uniform fields, etc. After the number of plots is determined, its use in producing accuracy versus generality must be determined.

Desirable Characteristics of Experiments for Economic Interpretation

To this point, the discussion has been general. It applies to purely agronomic experiments as well as to experiments to be interpreted economically. Experimental data to be used in agronomic analysis, however, may or may not possess certain desirable characteristics for economic interpretation. It is important that the nature of characteristics which are desirable for economic analysis be known *before* fertilization experiments expected to yield data of economic significance are designed. The nature of these desirable characteristics can be seen most clearly by examining the uses which an economist may wish to make of the data.

The first required modification of concepts used to this point, if economic analysis is to be carried out, is the introduction of input prices, P_{x_j}, and output prices, P_{y_i}, to produce a profit equation of the form:

$$(2) \qquad g(P_{y_1} Y_1 \ldots . P_{y_n} Y_n : X_1 P_{x_1} \ldots . X_m P_{x_m}) = \pi .$$

When narrowed down to manageable size, as previously done by isolation of an autonomous subfunction and imposition of selective and experimental controls, the following type of subfunction is secured:

$$(3) \quad g'(Y_c P_c, NP_n, P_2O_5\, P_{P_2O_5} \mid \text{oats, clover, } K_2O, X_f, \ldots, X_m) = \pi .$$

Application of maximization procedures (as taught in any elementary calculus course) to equation 3 or portions thereof, permits location of such economic optima as the quantity of Y to produce maximum profit and the least-cost combination of N and P_2O_5 to use in producing that amount of Y.

Corresponding applications also permit determination of how these optima shift with price changes. The laws of growth, of the minimum, or of diminishing returns (which are highly interrelated and are investigated by agronomists and economists alike) tend to assure the second order conditions necessary to locate these optima. The most important economic optima tend to occur on the function where the

$$\frac{\partial Y_c}{\partial X_i} > 0 \quad \text{are decreasing.}$$

As an example, when P_2O_5 is constant, $\frac{d\pi}{dN} = 0$ defines the most profitable amount of nitrogen to use with the constant amount of P_2O_5. Under ordinary competitive conditions $\frac{d\pi}{dN} = \frac{dY_c}{dN} P_{y_c} - P_n$. Thus, an estimate of $\frac{dN_c}{dN}$, which is the slope of equation 3 in the Y_cN dimension, is important to the economist attempting to ascertain the most profitable amount of input N to use.

Suppose, however, that the economist's interest is somewhat more complex. He may desire to find the best (most profitable) combination of N and P_2O_5 in producing a given amount of Y_c. The condition

$$\frac{\dfrac{\partial Y_c}{\partial N}}{\dfrac{\partial Y_c}{\partial P_2O_5}} = \frac{P_n}{P_{P_2O_5}}$$

defines the least-cost combination of N and P_2O_5 to use in producing the amount of Y_c under consideration. As the

$$\frac{\partial \pi}{\partial N} = \frac{\partial Y_c}{\partial N} P_{Y_c} - P_n \text{ and } \frac{\partial \pi}{\partial P_2O_5} = \frac{\partial Y_c}{\partial P_2O_5} P_{Y_c} - P_{P_2O_5}$$

are the slopes of equation 3 in the Y_cN and $Y_cP_2O_5$ dimensions, respectively, also, slopes are crucial to the economist attempting to ascertain the most profitable (least-cost) combination of N and P_2O_5 to use in obtaining a given yield (Y_c = a constant) of corn. These steps parallel those of equations 11 through 14 in Chapter 1.

If the economist is considering the problem of a farmer with a given amount of money to spend on N and P_2O_5 then, instead of fixing Y_c, the relationship

$$\frac{\dfrac{\partial Y_c}{\partial N}}{\dfrac{\partial Y_c}{\partial P_2O_5}} = \frac{P_n}{P_{P_2O_5}}$$

is solved simultaneously with

$P_nN + P_{P_2O_5} P_2O_5 = C$ (the amount of money which can be spent on N and P_2O_5), to determine N and P_2O_5. These values for N and P_2O_5 can, in turn, be substituted in equation 3 to determine Y_c.

In both this and the previous instance involving Y_c = a constant, the productivity of N may depend on the amount of P_2O_5 present (and vice versa) and the study should be designed so that the estimates of $\frac{\partial Y_c}{\partial N}$ and $\frac{\partial Y_c}{\partial P_2O_5}$ can reflect such relationships.

When the economist desires to determine the most profitable amounts of N and P_2O_5 to use and of Y_c to produce, he sets $\frac{\partial \pi}{\partial N}$ and $\frac{\partial \pi}{\partial P_2O_5}$ equal to zero, and solves simultaneously for N and P_2O_5. Having secured N and P_2O_5 in this manner, he then substitutes them in equation 3 and solves for Y_c. Alternatively, the optimum combination of N and P_2O_5 and the optimum level of Y_c can be solved in the manner of equations 11 through 14 in Chapter 1. As in the previous cases, $\frac{\partial \pi}{\partial N}$ and

$\frac{\partial \pi}{\partial P_2O_5}$ involve estimates of $\frac{\partial Y_c}{\partial N}$ and $\frac{\partial Y_c}{\partial P_2O_5}$, the slopes of equation 3 in the Y_cN and $Y_cP_2O_5$ dimensions, as the crucial values to be determined from the fertilization experiment.

Consideration of more complex subproduction functions involving more than two inputs reveals that, in each instance, $\frac{\partial Y_c}{\partial X_j}$ turns out to be crucial in estimating the most profitable quantities of Y_c to produce, and of the inputs X_j. The same is true if Y_c is fixed, or if the money which can be spent on the variable inputs is limited.

If the subfunction being investigated involves two products, Y_c and Y_L (corn and a legume) with the amount of Y_c produced affecting the productivity of resources used in producing Y_L (and vice versa), these influences should be measured and reflected in the estimates of the $\frac{\partial Y_i}{\partial X_j}$. In such subfunctions, an additional problem of determining the most profitable combination of Y_c and Y_L exists. Y_c and Y_L are in the most profitable combination and amounts when the following equations hold simultaneously:

(4a) $$\frac{\partial \pi}{\partial N(Y_c)} = 0$$

(4b) $$\frac{\partial \pi}{\partial P_2O_5(Y_c)} = 0$$

(4c) $$\frac{\partial \pi}{\partial N(Y_L)} = 0$$

(4d) $$\frac{\partial \pi}{\partial P_2O_5(Y_L)} = 0$$

where $\partial N(Y_c)$ stands for a change in the amount of N used in producing Y_c, as contrasted to a change in N used in producing Y_L, which is written $\partial N(Y_L)$, or a change in P_2O_5 used in producing Y_c, which is written $\partial P_2O_5(Y_c)$. After solution of (4a), (4b), (4c), and (4d) for $N(Y_c)$, $N(Y_L)$, $P_2O_5(Y_c)$, and $P_2O_5(Y_L)$, these values can be substituted into equation 3 to determine the most profitable amounts of Y_c and Y_L to produce.

The above example involving two outputs Y_c and Y_L, and two inputs N and P_2O_5, is easily generalized to "n" outputs and "m" inputs. In this generalized form, the same conclusion holds, i.e., *the crucial estimates required to determine high profit points, least cost combinations of inputs, and high profit combination of outputs are the estimates of* $\frac{\partial Y_i}{\partial X_j(Y_i)}$, such estimates to reflect interactions among the Y as well as among the X_j.

The economist's strong preference for accurate estimates of the

$\frac{\partial Y_i}{\partial X_j(Y_i)}$ where $\frac{\partial Y_i}{\partial X_j(Y_i)}$ are positive and decreasing may come into sharp conflict with the interests of agronomists in the early stages of interdepartmental negotiations on the design of fertilization experiments. The agronomist, after many earlier negotiations with statisticians, has a strong preference for accuracy in estimations yields for some combination of fertilizer nutrients; the economist has, for reasons expressed above, a strong preference for accuracy in estimates of $\frac{\partial Y_i}{\partial X_j(Y_i)}$. The agronomist is led to seek replications at points on the surface while the economist is led to seek less replication and more "spread" of the observations over the surface. These two objectives, while competitive over a narrow range, are also quite complementary over wider ranges since the standard error of estimate for yields is a component of the standard error for $\frac{\partial Y_i}{\partial X_j(Y_i)}$. In fact, an experimental design yielding low standard errors for $\frac{\partial Y_i}{\partial X_j(Y_i)}$, can be made to yield as low or even a lower standard error of estimate for Y_i than one in which the standard error of $\frac{\partial Y_i}{\partial X_j Y_i}$ is high. When the agronomist sees these complementarities and opportunities for cooperation, it is a relatively short step toward agreement and the presentation of a unified research proposal backed by personnel from both areas of work.

Alternative Agronomic Objectives and Linear Programming Determinations

Other objectives of agronomists, while not always complementary with those of economists, are seldom in sharp conflict. This is especially true if the need for full use of fixed research resources is considered, as well as the need for economy in the use of variable, or "out of pocket," research resources. For instance, fertilizer placement and tillage practices can be tested in subseries within a design with only a small increase in variable costs and probably no increase in fixed or overhead costs.

Another consideration involving slopes of function should be mentioned here. Some persons argue that economic interpretations of fertilization data can be made on a comparative budget and/or on a linear programming basis which does not require estimates of the $\frac{\partial Y_i}{\partial X_j(Y_i)}$ from continuous production functions. This is, of course, true. In such procedures profits are computed for each discretely estimated point on the relevant subproduction function for which an *estimate* of yield is available. Comparison of profits among such points permits the economist to determine the most profitable among them, as discrete opportunities. While these procedures do not make direct use of $\frac{\partial Y_i}{\partial X_j(Y_i)}$ estimates, they locate the "best" point by comparing finite difference between points. The smaller these differences, the more accurately the "best" point can be located. Thus, regardless of whether or not the economic analysis is to be based on point estimates or on estimates of derivatives from continuous functions, experimental observations should yield information on a multiplicity of points on that area of the surface where the derivatives are positive and decreasing.

Another point of similarity should be noted in the data requirements of economic analyses based on *point* versus *continuous function* estimates. In both instances, the "best" amounts of the different fertilizers to use vary with prices of the inputs and of the output. These variations occur in areas of the function where decreasing increments in yields result from equal successive increments in the variable inputs. This mutual characteristic of the different methods of economic interpretation further increases the desirability of having yield information over large areas of the surface, or on a multiplicity of points on the surface. Thus, we note again that the same complementarity which exists between the agronomist's desire for a low standard error of estimate for yields and the economist's desire for a low standard error of $\frac{\partial Y_i}{\partial X_j(Y_i)}$ also exists between the desires of (a) the budgeter or linear programmer on the one hand, and (b) the continuous function analyst on the other.

Economists carrying out continuous function analyses sometimes are devotees to certain functions. For instance, prior knowledge that one will predict a Cobb-Douglas, Spillman, or linear function creates the desire for special designs; i.e., a Cobb-Douglas analyst may want to avoid all zero rates of application since the $\log 0 = -\infty$. However, because of the current lack of knowledge of which function best fits the data, it appears desirable to avoid designs which confine the analysis to a particular function, unless resource limitations restrict the analyst to one of the simpler functions.

Methods of Attaining Desirable Characteristics for Economic Analysis

The objectives outlined above are attained in designing experiments by:

A. Ascertaining on the basis of existing information the range of combinations of X_j's for which the $\frac{\partial Y_j}{\partial X_i(Y_j)} > 0$ and decreasing and concentrating experimental observations on these combinations.

B. Securing observations for a sufficient number of combinations in the area defined in (A) to give the economist flexibility in selecting functional forms if he elects to use continuous functions or, if he elects not to use continuous functions, confidence that he has data on sufficient alternatives to make the relevant discrete comparisons. It should be recognized that while this may reduce the number of replications which can be made with given resources for any one combination there are complementarities between the desire of accuracy in Y_i estimates and accuracy in $\frac{\partial Y_i}{\partial X_j(Y_i)}$ estimates. This requirement insures that data on the interactions among the X_j's will be available.

C. Allocating experimental observations among the possible combinations of the X_j in such a way as to minimize the linear correlations among terms whose coefficients are likely to be estimated; i.e., if $Y_C = A + b_1X_1 + b_2X_1X_2 + b_3X_2 + b_4X_1^2 + b_5X_2^2$ is likely to be fitted, an experimental design which minimizes (with due consideration to the cost of minimization) the linear correlations among X_1 and X_1X_2 or betweem X_1 and X_2, or X_1 and X_1^2, etc., is desirable. Minimization of the intercorrelations among the variables whose coefficients are to be estimated reduces the standard errors of the estimated coefficients.

D. Allocating experimental observations among the possible combinations of the X_j's in such a way as to increase the standard deviation of the terms whose coefficients are likely to be estimated; i.e., if $Y = aX_1^bX_2^{b^2}$ is to be estimated linearly in the logarithms, then $\sigma_{\log Y}$, $\sigma_{\log X_1}$, and $\sigma_{\log X_2}$ should be kept large or, alternatively, if $Y = a + b_1X_1 + b_2X_2X_1 + b_3X_3^2$ is to be estimated, then σ_{X_1}, $\sigma_{X_1X_2}$, and σ_{X_3} should be kept large.

E. Controlling or measuring the influence of the Y_i's on each other's functional relationships with the X_i. This can be done if all but one of the Y_i is held constant or, if more than one of the Y_i is to be studied, by (a) measuring the by-products of each Y_i studied and the influence of these by-products on the production of the other Y_i, or (b) by simply letting the separate functions for the Y_i reflect the levels at which the other Y_i are produced. If by-products and/or "by-losses" involving humus, biologically fixed nitrogen, soil, nutrient removal, soil structure, erosion, etc., can be measured and incorporated into the functions, this is probably the preferable solution. Simply letting the separate functions for the Y_i reflect the levels at which the other Y_i are produced may cause estimates of the productivity of one or more of the applied nutrients to reflect either by-product losses or gains.

F. Maximizing, with available resources and in view of direct and opportunity costs, the number of observations made.

There are at least two important sets of interrelationships to be kept in mind in using the above methods. First, the objectives, both economic and noneconomic, being sought are in some instances competitive or conflicting while, in other instances, they are complementary with attainment of either or both of two objectives making it easier to attain the other. Second, the methods of agronomists, statisticians, and economists are in some instances competitive but are in many instances complementary.

Agronomic Methodologies

However, considering the interrelationships among objectives of economists and agronomists in some detail, certain agronomic methodological developments should be mentioned. The mechanization of plot work is extremely important in lessening some of the competitive aspects of objectives of experimental designs. In effect, agronomists are substituting especially adapted or constructed machines for much of the labor previously used in hand-weighing and measuring fertilizers and in hand-harvesting and measuring the crops produced. Use of such equipment calls for larger lanes and turning areas. Thus, these new technologies make it "profitable" to substitute both capital and land for labor in the research process.

This substitution tends to increase the overhead or fixed cost of an experiment but reduces the per-unit costs of adding plots to the design. It also makes possible an increase in the number of experimental observations. The increase in observations involves only a small increase in cost, with the advantage of spreading the fixed cost over more plots. Thus, designs are becoming increasingly possible whereby the agronomists can supply economists with the kinds of data needed for economic interpretations.

The work-simplification methods developed at Michigan State University can be mentioned as examples of techniques which make more elaborate experiments possible. One device is a fertilization attachment for corn planters, a mechanism both accurate enough for experimental work and for reducing the fertilizer cleaning work in moving from one plot to another. Another device is a one-row mounted corn picker which makes it possible to pick one row without knocking down adjacent rows. Accurate calibration of fertilizer drills also makes it possible to vary rates of application from plot to plot without hand measurement and weighing. Also, an accurately calibrated fertilizer drill on a garden tractor makes it possible to side-dress corn rapidly and efficiently. While machine work may be somewhat less accurate than hand work (though this is debatable if reliable labor is hard to get), reduced costs make it possible to offset these inaccuracies (if they exist) with more and larger plots. So promising are these developments that many experimental procedures need a thorough work-simplification study. The accuracy of machine work needs an equally thorough statistical evaluation.

An Example

The reconciliation process in designing an experiment for studying the economics of fertilization can be well illustrated with an example from Michigan. For some years there has been a rather close cooperation between members of the Agricultural Economics staff and the staff of the Soils Department. Also, there has been a fair interchange of graduate students, as well as a number of seminars and informal sessions. Thus, personnel involved have known and understood each other and, in general, there exists an environment favorable to agricultural statesmanship.

After some preliminary meetings, a decision was made to develop a joint project between the two departments to study the economics of fertilizing some of the major Michigan crops. Six people from the various departments actively designed the experiment. Statisticians, while not project members, were consulted and used, both directly and indirectly.

Decisions had to be made on: (a) crops to be fertilized, (b) range of fertilizer nutrients to be studied, and (c) soil types to be studied. The problem had to be confined to portions of an autonomous subfunction in order to make the problem manageable.

Preliminary discussions of objectives of the two departments and of the Michigan farmers tentatively indicated that three subprojects should be developed. The first of these was concerned with a corn, oats, wheat, and alfalfa-brome rotation on Miami silt loam, one of south central Michigan's upland soils. Another subproject dealt with corn under continuous cultivation on the Brookston series. The third dealt with the fertilization of pasturage on one of the pasture soils of north central Michigan.

Further consideration of the relative importance of these three studies and of the cost of doing experimental work at the different locations considerably modified the tentative conclusions. For instance, the pasture experiment was dropped because it was too far away from the campus to be conducted economically, and the pasture fertilization problem was less important to Michigan farmers than further strengthening of the continuous corn experiment. Also, it was decided to carry out the corn, oats, wheat, alfalfa-brome rotation on a soil in the Fox series because of the difficulty of getting a sufficiently homogeneous field of Miami soil. It was found that, after preliminary soil tests, the continuous corn experiments on Brookston would have to be moved to a more northern county from the county in which it was originally planned to locate them. The farmers in the original area had already fertilized the soil to such a high level that the response to fertilizer would be of little significance for economic analysis.

The continuous corn experiment on Brookston soil will be considered below. In this experiment, it was decided that each of the three nutrients would be applied at seven different levels including the zero rate of application. It was judged by the agronomists involved that these rates would fall mainly in the area where $\frac{\partial Y_c}{\partial X_i} > O$, and decreasing.

The seven levels are presented in table 2.1.

TABLE 2.1. Rates of Fertilizer Application, Continuous Corn Experiment, Brookston Soil, Michigan, 1953

	Rate						
Nutrient	0	1	2	3	4	5	6
N	0	20	40	80	160	240	320
K_2O	0	20	40	80	160	240	320
P_2O_5	0	40	80	160	320	480	640

It was also possible to incorporate into the design, work of special interest to the agronomists (Fig. 2.1). Thus, the plots running up the main diagonal were replicated and split into two parts. On one-half of each plot in one replication, a different method of fertilizer placement was employed. This made it possible for one agronomist involved to gain certain information in which he was particularly interested. It should also be noted that all the plots were large enough to be split in subsequent years to absorb similar supplementary projects having to do with, e.g., type of fertilizer, variety of corn, planting rates, and various other cultural practices. Soil tests were made for each plot to enable both economists and agronomists to study the effects of difference in soil fertility on yields as well as accumulation of fertilizer residuals.

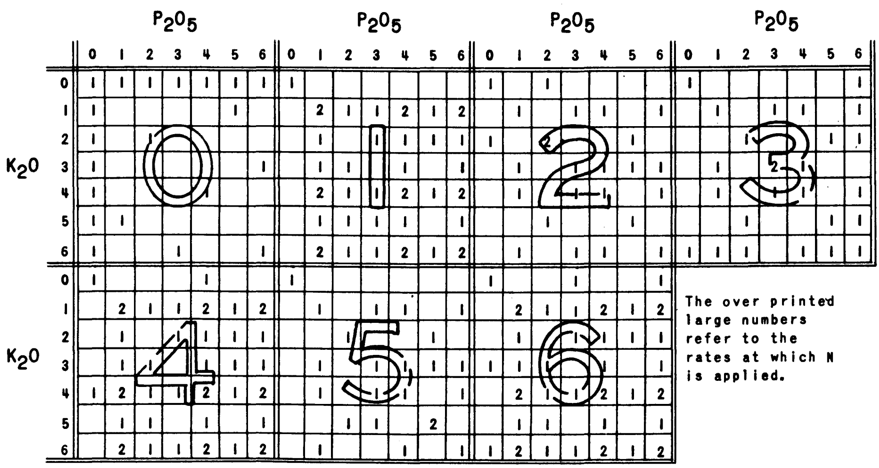

Fig. 2.1 — Schematic presentation of continuous corn experiment, Michigan State University, 1953.

A 7 x 7 x 7 experiment involves 343 different plots, if none of the plots were replicated. Several members of the committee had interests in replication of certain of the plots. For instance, the economists were interested in replication of the O, O, O plots for a number of reasons one encounters in fitting various alternative functions. The agronomists were also interested in having a replicated 3 x 3 x 3 factorial. After provision was made for 11 repetitions of the O, O, O plot and a replicated 3 x 3 x 3 factorial, it was obvious that project resources were inadequate (even after cancellation of the pasture experiment) to permit separate plots for each of the 343 cells in the design. It was decided, therefore, that the observations which could be afforded would be scattered throughout the sample space so as to keep the standard deviations for the three fertilizer nutrients large and to minimize the correlations coefficients among the three fertilizer nutrients applied.

Plans were made to control unstudied variables and to randomize the influences of those which could not be controlled. Controls were imposed in selection of the field and parts thereof as well as in selection of workers and equipment. Within the portion of the field selected by our soil classification expert as one being homogeneous, plot locations were randomized.

At this point a member of the Soils Department took active participation in the project and indicated to the economists that there were advantages of work simplification procedures in research. Hence, the number of plots were expanded somewhat. Some of the extra plots were scattered over the surface to be estimated. Others, however, were used to secure more information about the relationships between yields and each fertilizer nutrient considered separately with zero amounts of the other nutrients applied. The distribution of plots, while probably not ideal for fitting a given function, would give considerable flexibility in selecting functions for analytical purposes. The last requirement appears advantageous in view of certain modifications, which were developed at Michigan State University, in fitting modified Cobb-Douglas functions which are asymmetric and nonconstant, and have elasticities capable of reflecting more than one stage of a production function.

It is not claimed that the ultimate in experimental design has been secured. It is felt, however, that a moderately good job has been done in taking into account the various objectives of economists and agronomists. Experimental designs were used which reflect, rather satisfactorily, group choices (i.e., recognizing the wants, preferences, and objectives of the people and organizations concerned). The economists are pleased; the agronomists feel they will secure more than ample returns for their investment in the project. And both the experiment station administrators and the National Fertilizer Association administrators were favorable to financing the project. It has been shown that when representatives from various fields of work join forces and agree on a mutually advantageous research program to serve agriculture, such a program receives high priority in the minds of administrators charged with using research resources efficiently.

PART II

Fundamental Design and Prediction Problems

- ► **Alternative Designs**
- ► **Appropriate Functions**
- ► **Continuous Functions and Discrete Models**
- ► **Estimational Procedures**