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Chapter 1

Methodological Problems in Fertilizer Use

THE central methodological problem in fertilizer use on a single crop is prediction of the mathematical form and the probability distribution of the response function. This is a task, of course, for various soils, crops, and climatic situations. However, there are other methodological problems which are auxiliary to this central problem. They include: (a) the design of experiments to allow efficient prediction of the response function, and (b) the estimating procedure for predicting the surface and optimum use of nutrients. Since the last two problems are being given detailed treatment in other chapters, this chapter will focus on the fundamental and basic problems which relate to estimating the response functions.

Practical Importance of Knowledge in Response Functions

Although this chapter has the main objective of treating methodological problems in fertilizer economics, some of the practical or applied aspects of these fundamental considerations need to be pointed out. First, greater knowledge of simple, single-variable response functions can encourage greater use of fertilizer. The slope of the response function represents the incremental or marginal yield due to small increases in fertilizer use. The farmer with limited capital needs this information in determining how much fertilizer to apply. Knowledge represented by a response function is more useful than knowledge represented by the mean yield increase of one or two fertilizer (level) treatments.

Suppose a farmer with limited capital can earn $2.50 return on funds spent for other lines of his business (such as tractor fuel, mule feed, crop seed, or hog supplement). He is given information showing that one discrete level of fertilization, 30 pounds of nitrogen, will increase oat yield by 17 bushels. With oats at 70 cents per bushel and nitrogen application costing 18 cents per pound, the total return is $11.90 and the total cost is $5.40, a net of $6.50. However, the return per dollar spent on fertilizer ($11.90 ÷ $5.40) is only $2.20, and the farmer will allocate his scarce funds where he can get $2.50.

Suppose, however, that the farmer is given even three points from a response function showing: the first 10 pounds of N has a marginal yield of 10 bushels; the second 10 pounds has a marginal yield of 5 bushels; and the third 10 pounds has 2 bushels marginal yield. With a unit costing \$1.80, the first 10 pounds returns \$3.89 per dollar invested in fertilizer, and the second returns \$1.95. Hence, since the farmer can realize only \$2.50 elsewhere in his business, he now is encouraged to invest in at least 10 pounds of N. With more detailed knowledge of the response function, he may even invest in 15 pounds. Obviously then, knowledge of the response function, coupled with information on the economics of fertilizer use, can encourage a greater investment in this resource on that great majority of farms with limited capital. (See Chapter 11 for indications of use of these notions in farm planning.)

Knowledge of the response function is equally important for the farmer who considers his crop in the environment of unlimited capital. This is the case of tobacco producers; it is becoming the case of many other farmers. It is known that the optimum or most profitable level of fertilization for these farmers is defined by equation 1 where the term to the left of the equality

$$\frac{dY}{dF} = \frac{P_f}{P_y} \tag{1}$$

is the marginal yield or response and the term to the right is the price ratio (price per unit of fertilizer divided by the price per unit of yield). The marginal yield is the derivative of yield in respect to nutrient; it is the slope of the response function for any particular input level. This is the type of information basic for making recommendations to farmers who seek to maximize profits in a decision-making environment of unlimited capital.

It is obvious that the most profitable level of fertilization changes as the term to the right of the equality changes. (Likewise the optimum level of fertilization will change for the limited-capital farmer previously cited, as the price of crop yield, fertilizer, or any other product or resource for his farm changes.) How much change needs to be made in fertilizer use, as prices change, again depends on the slope of the response function. If the slope changes only slightly over a wide range of fertilizer inputs, the loss (profit depression) from not shifting rates can be great;[1] if the slope changes greatly over a small input range, the farmer may lose but little in not adjusting his rates to price change.

Finally, greater knowledge of the response curve is needed as an aid in farm planning and linear programming, to allow improved predictions of how and where fertilizer fits into the program of the farm as a whole. If numerous points are known for the response curve, each suggested level of fertilization can be treated as an activity or investment opportunity. The optimum level of fertilization relative to (a) other investment alternatives (activities), and (b) complete farm organization can then be predicted. Data in a form for this purpose will generally encourage use of more fertilizer. The reason has been suggested

[1]This statement applies particularly where the previous price ratio was equal to a derivative of the function high (low) on the response function and the new price ratio is equal to a derivative low (high) on the curve.

already: Knowledge of high marginal returns for small fertilizer inputs can specify use of this resource, even by the farmer with very limited funds. This knowledge also will indicate how far in the use of fertilizer the farmer with more funds can profitably go.

The farmer is the only one who can make the decision as to the most profitable quantity of fertilizer to use. Optimum quantity is determined partly by the response function for his particular soil, tempered as it is by previous soil management, weather, insects and pests, and other variables which are both endogenous and exogenous to his decision-making environment. But aside from the purely physical and biological variables of the fertilizer production function, the optimum quantity is as much a function of present nutrient and future (crop) price ratios as it is of the response ratios. Since prices, and even yields, are held with uncertainty, the fertilizer recommendation must conform to the farmer's uncertainty or risk-bearing ability which includes (a) his equity position; (b) his psychological makeup; and (c) other phenomena which cause him to temper the quantity and kinds of the resources which he employs. Refined estimates of the fertilizer response function can help provide the basic data needed to guide these decisions which are unique to each farm.

Knowledge of multi-variable response functions also has great practical implications. Anyone knowing the basic principles of production recognizes immediately that the production coefficient for, and the return from, any one input category is a function of the amount and kind of other input categories with which it is combined. The economic potential in, and limits of, any one resource can be determined only by studies which consider numerous input categories as variables. These variables may include different fertilizer nutrients, seeding rates, seed varieties, irrigation, and various other technologies. A fertilizer rate study may show a much lower response curve for one nutrient, if it is varied alone, than if it is varied along with another nutrient. Similarly, a multi-variable response study may be applied productively when a new crop variety, which has a great yield-boosting effect, is discovered. In much of the Midwest higher-yielding varieties have little effect unless used with sufficient fertilizer nutrients. A simple single-variable response study may fail to "lift the lid on yield potential," under new varieties or other developments in technology. Finally, knowledge of isoclines from multi-variable studies provides a practical guide in fertilizer manufacture.

Methodological Problems in Single-Variable Functions

A few practical applications of fundamental fertilizer research have been presented above because (a) the practical problems and their solutions are the main goals of fundamental research and methodological considerations, and (b) fundamental research can result in a greater and more efficient use of fertilizer if it provides refinements for obtaining more practical recommendations for the individual farmers. (Practicality is characterized by recognition of the variables peculiar to each farm, including capital, equity position, risk considerations, and other economic variables, as well as physical and biological variables such as the crop and variety, alternative nutrients, soil conditions, etc.)

In discussing practical applications first, the cart has been put before the horse. The remainder of this chapter will deal with the fundamental science or methodological considerations — in this instance, the "horse."

Form of Single-Variable Function

For research on simple response functions with a single-variable nutrient, for a particular soil and management system, there are two basic methodological problems, viz., (a) the appropriate algebraic form of the response function, and (b) the between-year variability in the production function.

As far as this writer knows and as pointed out by Mason in Chapter 5, there is no biological proof that the fertilizer response function conforms universally to a particular algebraic form of equation. It is likely that the best-fitting form of the fertilizer production function varies by crop, year, soil, or other variables. One algebraic form which has been popular over time with research workers has been the Mitscherlich-Spillman type of function. One form of this function is equation 2,

$$Y = m - ar^F \ . \tag{2}$$

(Another form is shown in equation 2 of Chapter 5.) This function employs specific assumptions about the nature of the response curve: (a) It assumes that the elasticity of response is less than 1.0 over all ranges of fertilizer applications, a condition likely to be encountered in most situations but one which need not hold true universally (some experiments at particular locations show a short range of increasing returns). (b) It assumes that fertilization rates never become so great as to cause negative marginal products (i.e., declining total yields), since yield becomes asymptotic to the limit m. (c) It assumes the condition of equation 3,

$$\frac{\Delta_2 Y}{\Delta_1 Y} = \frac{\Delta_3 Y}{\Delta_2 Y} \cdots \frac{\Delta_n Y}{\Delta_{n-1} Y} \ , \tag{3}$$

namely, that the ratios of successive increments to total yield over all fertilizer inputs are equal. Lastly, (d) the function assumes that where two nutrients are involved, the maximum yield per acre can be attained with a large number of nutrient combinations (i.e., it does not allow the isoclines to converge at the point of maximum yield).

A function which also forces particular assumptions into the predictions, although these are considerably different from the Mitscherlich equation, is the Cobb-Douglas function, listed as equation 4. It does not

(4) $$Y = aF^b$$

assume that the ratios of marginal yields are equal. However, it does assume that the percentage increase in yield is constant and equal to b for all increments of fertilizer. This assumption, illustrated in equation 5 below, may be as realistic as the parallel assumption of the Mitscherlich equation.

(5) $$\frac{\frac{\Delta_1 Y}{Y}}{\frac{\Delta_1 F}{F}} = \frac{\frac{\Delta_2 Y}{Y}}{\frac{\Delta_2 F}{F}} = \cdots = \frac{\frac{\Delta_n Y}{Y}}{\frac{\Delta_n F}{F}} .$$

The Cobb-Douglas equation allows the yield to increase at either a diminishing, constant, or increasing rate, although the response curve can be represented by only one of these and never by a combination. If total yield increases at a diminishing rate, the function assumes negative marginal products and, therefore, that total yield becomes asymptotic to some limit.

Somewhat more flexible functions are the simple quadratic and square root forms indicated respectively as equations 6 and 7 below:

(6) $$Y = a + bF - cF^2$$

(7) $$Y = a + b\sqrt{F} - cF .$$

These equations do not force certain of the elasticity and marginal ratio restraints of the previous equations. Also, they allow the total yield to reach a maximum, followed by negative marginal yields. Equation 6 may apply particularly where a maximum is reached with relatively low fertilization level; equation 7 may apply where marginal yields change rapidly over low fertilization levels but "straighten out" for higher levels, if no other practices or inputs are limitational. But again these functions may have no unique biological base. Is there a unique biological base for response functions?

The research worker makes a biological (and at this stage of knowledge, a subjective) assumption when selecting a particular function. Methodological effort should be devoted to proving either that (a) biological responses do follow particular mathematical forms, or that (b) there is no unique algebraic response function for all situations. The hypothesis followed is that the latter will most likely prove correct. While fundamental greenhouse research may prove the first to have some validity, objective statistical tests may be used to specify which function is most appropriate under field conditions. This methodological problem merits further attention, since every fertilizer recommendation to farmers implies knowledge of the mathematical nature of the response function. Greater knowledge of the response form is needed for most efficient designs. If the mathematical form is known to be a quadratic equation, a Box design may be most efficient (see page 48, Chapter 3). However, another design may be more efficient if the mathematical form proves to be logarithmic or exponential.

Distribution of Response Functions

Conventionally, fertilizer recommendations are made as if the response or regression coefficients were single-valued. It would be convenient if farmers' decisions could be made in this framework of certainty in respect to both prices and yield increments. Unfortunately this is not true. A methodological problem arises in providing response information which recognizes that risk and/or uncertainty must be incorporated into farmers' decisions: The farmer is not faced with a single response function but with a distribution of response functions. He recognizes this situation and makes his decisions accordingly. Incorporation of risk-uncertainty and probability concepts into fertilizer research and recommendations would aid him in these decisions.

The problem can be brought into focus by viewing fertilizer response in the manner of the generalized production function represented by equation 8. Yield response (Y) is represented as a function of

$$(8) \qquad Y = f\,(F_1 \mid F_2 \ldots F_n, X_1, X_2 \ldots X_n \parallel Z_1, Z_2 \ldots Z_n)$$

fertilizer nutrients F_1 through F_n and other types of inputs (practices represented by X_1 through X_n and Z_1 through Z_n). The last two categories of inputs (X_i and Z_i) are denoted by soil type, nutrients already in the soil, seed variety, cultural practices, number of cultivations, seeding rate, moisture of particular weeks, temperature at critical times, and other variables (resource inputs) which affect yield. In this case a single bar follows F_1, denoting that nutrient F_1 alone is the input in the production function which is variable or which can be controlled. All variables between the single and double bars, F_2 through X_n, are endogenous to the decision-making environment, (can be controlled by the farmer or decision-maker) but are held fixed for the particular production period (i.e., crop year). These represent seeding rates, number of cultivations, application of particular nutrients in fixed levels, etc. To the right of the double bar are variables, such as weather, which are exogenous to the decision-making framework and cannot be controlled by the farmer. These exogenous variables vary within and between seasons. Hence, the response curve for the single variable F_1 will take on a different height and slope with each change in the exogenous variables. The result is a distribution of response functions such as shown in figure 1.1. The most likely hypothesis is that the response functions are normally distributed. There have been suggestions, however, that this is not the case, at least over a period of a few years (the span usually relevant in a farmer's decisions). In case the response curves are not normally distributed, the mean may be represented by the dotted line in figure 1.1 and is above the mode (the "most probable"

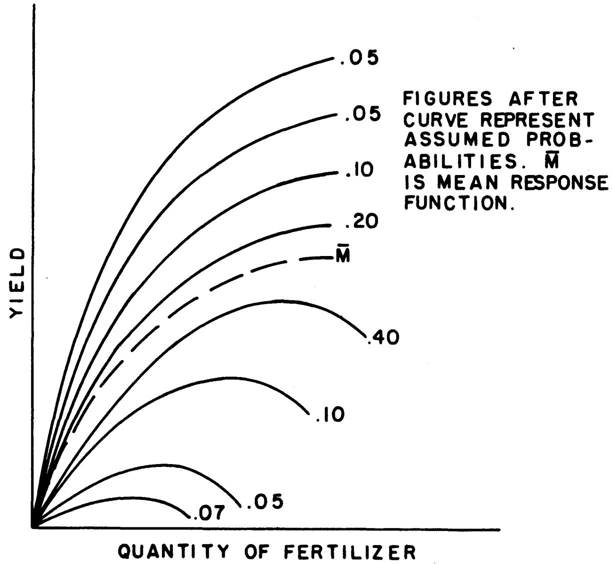


Fig. 1.1 — Possible effect of weather variations on the distribution of fertilizer production functions.

curve of any one year). (The mean might also fall below the mode, depending on the skewness of the distribution.)

How should the farmer make decisions when the response curve varies between years? Even though the distribution of functions might be established (and hence conform with Knight's (2)[2] risk concept), the curve of any particular year represents uncertainty. The answer depends on the individual farmer and his ability to bear risk as characterized by his capital, his equity position, and his aversion for risk. If he is a conservative individual with little capital and a low equity, he may wish to take few or no chances. In this case he may, in effect, count on the lowest possible response function and apply fertilizer accordingly. Using this type of "uncertainty precaution" (discount system), he feels assured that the probability is in favor of outcomes better than expected, and that there is slight chance of outcomes worse than predicted.[3] Undoubtedly, this type of uncertainty precaution causes farmers to use fertilizer in quantities smaller than conventionally recommended.

The farmer in a better capital position and with less risk aversion may make decisions on the basis of model response expectations. He

[2]Numbers in parentheses which appear in sentences refer to reference citations listed at the end of each chapter.

[3]Regardless of the decision and the outcome, the farmer is always faced with the possibility of two kinds of errors. First, he may assume "the best" and act accordingly. If he is wrong, he may be penalized by a depression of profits greater than if he had anticipated "the worst." Secondly, he can assume "the worst" and act accordingly. If he is wrong, his profits will be less than if he had used an alternative expectation and planned for "the best."

wishes the greatest probability of success in expectations and plans. He will, of course, never be 100 per cent correct. He will apply too little fertilizer for maximum profits in good years and too much in poor years.

Data on the distribution of the production function are lacking in most locations. To fill this gap in the farmer's decision-making environment, time sequences of fertilizer experiments are needed, with all endogenous variables (soil, seeding rate, previous management, etc.) held constant over a period of years. The exogenous variables then would be reflected in the distribution of functions, which would be useful in recommendations to, and decisions by, farmers. There is some preliminary indication that farmers believe the fertilizer response to "reflect the best yield to be expected" and, therefore, that deviations from this quantity are likely in the direction of lower yields.[4]

Information is needed to show whether the fertilizer functions are normally distributed and to indicate to farmers that "better incomes" are just as probable as "lower outcomes." But most important, this type of information would provide the decision-making basis for farmers who must use different plans because of variations in their ability to assume uncertainty. Table 11.3 (page 169) provides some insight into the need for variability data for farm planning.

Carry-over and Alternative Rates in Succeeding Years

Under the research needs outlined above, level of fertilization would be a variable handled similarly in a series of years. The focus here is on the distribution of functions, due to weather and other variations, without regard to: (a) carry-over effects or (b) the results of alternative fertilization rates in succeeding years. However, both of the latter are needed if fertilizer is to become a resource used to its full economic potential.

Leaching is great in parts of the Southeast and carry-over response is unimportant in economic decisions. In some localities, however, carry-over responses are important. Information on these residuals can increase the quantity of fertilizer used. With carry-over effects in years following the one of application, the optimum level of fertilization can be determined by equating the discounted value of marginal responses with the discounted value of marginal costs of each fertilizer increment. The value of the marginal response for any fertilizer input (i.e., the j-th input) then becomes, as shown in equation 9, the sum of the marginal response values

$$V_j = \sum_{i=n}^{i-1} \frac{R_i}{(1+r)^i} \tag{9}$$

[4]This statement is based on a survey of farmers' expectations being conducted by the writer.

divided by the discount coefficient. For example, suppose the third increment of fertilizer gives a response of 8 bushels in the first year, 4 bushels in the second year, and 2 bushels in the third year. The price of the crop is $1 and the farmer's discount rate is 10 per cent. With discounting for yearly periods, the present value of the sequences of yield response is:

$$\frac{\$8}{(1+.10)} + \frac{\$4}{(1+.10)^2} + \frac{\$2}{(1+.10)^3} = \$12.08 .$$

Without knowledge of residual responses, the first-year discounted marginal value of the third input is only $7.27. Obviously, then, more fertilizer will be used where residual effects exist and are made known to farmers. Knowledge of residual effects can reduce uncertainty considerations if the farmer knows that even though weather of the first year is bad, probabilities are high for getting a large residual effect in following years. He then will not be so timid about using fertilizer.

Finally, residual response functions allow farmers to discount fertilizer returns to fit their own particular capital and uncertainty situations. The magnitude of the discount rate should differ with each farmer. On the one hand, it will be a function of the alternative returns on capital in other parts of the farm business; the beginning farmer may discount at 40 per cent while the wealthy, established farmer may discount at 4 per cent. On the other hand, the magnitude of the discount rate will be a function of the subjective price and yield uncertainty in the farmer's mind. By supplying information on time sequences of yield responses, the research worker aids the farmer in using the fertilizer to fit his own unique circumstances.

A final phase of time should be mentioned. It is the effect of rate of fertilizer application in previous years on the response function in subsequent years. How much difference is there in the response function for corn this year on fields which received respectively 20, 40, 60, and 80 pounds of nitrogen last year?

Nature of the Production Surface

In order to be systematic, we have discussed single-variable functions or curves first. In following this procedure, the cart is placed before the horse. The reason is that one cannot know which single-variable curve is the appropriate one to predict unless he knows or assumes something about the response surface itself. Hence, he turns to the concepts and methodological problems involved in production functions involving two or more variables. Of course, what has been said about appropriate biological or algebraic forms of functions, about the distribution of the fertilizer response function, and other time considerations also applies to functions involving two or more variables.

When more than two nutrients can be variable for a single crop, two economic problems are involved: (a) the least-cost combination of nutrients for any given yield level, and (b) the most profitable level of fertilization, considering the nutrient combinations, which yields the lowest cost for each yield level. These decisions must be made by both the farmer with limited capital and the farmer with unlimited capital. If he has unlimited capital, then the optimum level of fertilization and the optimum combination of nutrients are simultaneously attained when the partial derivatives for both nutrients are equated with the crop/nutrient price ratio for each.

Using data for an Iowa corn experiment (1), for example, we have the two-variable response functions in equation 10. Using prices of \$1.40 per bushel for corn, 18 cents per pound for nitrogen, and 12 cents per pound for phosphorus,

$$Y = -5.68 - .316N - .417P + 6.35\sqrt{N} + 8.52\sqrt{P} + .341\sqrt{NP}, \tag{10}$$

the partial derivatives to equal the price ratios in equations 11 and 12 are set. From these, one solves for the quantities of the two nutrients in equations 13 and 14. Given this particular function, the optimum level of fertilization and combination of nutrients include 142.5 pounds of N and 156.5 pounds of P_2O_5.

$$\frac{\partial C}{\partial N} = -.316 + \frac{3.1756}{\sqrt{N}} + \frac{.1705\sqrt{P}}{\sqrt{N}} = \frac{.18}{1.40} \tag{11}$$

$$\frac{\partial C}{\partial P} = -.417 + \frac{4.2578}{\sqrt{P}} + \frac{.1705\sqrt{N}}{\sqrt{P}} = \frac{.12}{1.40} \tag{12}$$

$$N = 142.48 \text{ lbs.} \tag{13}$$

$$P = 156.45 \text{ lbs.} \tag{14}$$

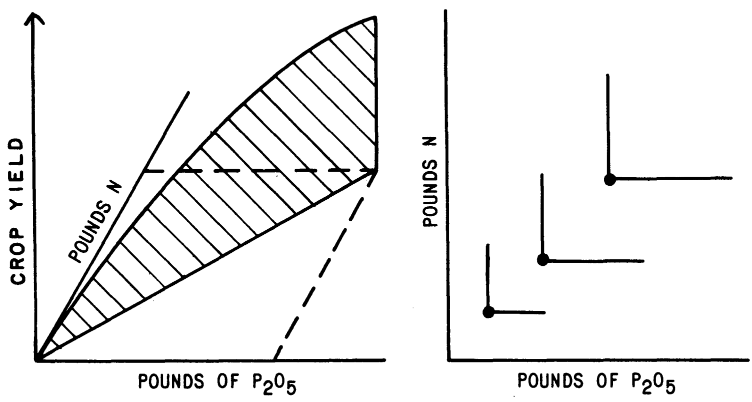
Even if the farmer has limited capital and cannot push fertilization to the point that the value of the last increment of yield is just equal to the cost of the last increment of fertilizer, he still needs to know the least-cost combination of nutrients for the particular yield to be attained. The least-cost combination is determined by equating the marginal rate of substitution of the two nutrients (the derivative of one nutrient in respect to the other with yield considered constant at a specific level) with the nutrient price ratio. Using the response function of equation 10, equation 15 is obtained, which defines the marginal rate of substitution between N and P_2O_5. Setting this equation of substitution rates to equal

$$\frac{dN}{dP} = \frac{-.8348\sqrt{PN} + 8.5155\sqrt{N} + .3410N}{-.6323\sqrt{PN} + 6.3512\sqrt{P} + .3410P} = \frac{.12}{.18} \tag{15}$$

the $\frac{P}{N}$ price ratio of $\frac{.12}{.18}$, it is determined that for a 50-bushel yield, the least-cost nutrient combination includes 11.8 pounds of N and 24.3 pounds of P_2O_5; for a 100-bushel yield, the least-cost fertilizer ratio includes 79.3 pounds of N and 101.6 pounds of P_2O_5.[5]

The Nature of Yield Isoquants and Fertilizer Isoclines

The question of nutrient substitutability is now raised and, hence, the nature of the fertilizer production surface. Some concepts assume that nutrients are not substitutes in attaining a given crop yield. Liebig's classical *Law of the Minimum* assumed, for example, that the fertilizer yield surface reduces to a "knife's edge" as shown in figure 1.2. Higher yields can be attained only if higher rates of fertilization follow some limitational nutrient ratio. This also is the assumption employed in the so-called practical information which pictures crop production in the vein of a barrel, wherein yield cannot be raised above the shortest stave, namely, a particular fertilizer nutrient.



Figs. 1.2 and 1.3 — Production surface and yield isoquants for nutrients which are technical complements.

Now, for every yield surface, there is a corresponding map of yield isoquants or contours.[6] For the Liebig response surface, the yield isoquants take the form suggested in figure 1.3. Both nutrients are limitational in the sense that increasing one alone (a) neither reduces the

[5]In addition to knowing the least-cost nutrient ratio for a specified yield, the farmer with limited capital needs to use this information to determine the return per dollar invested in fertilizer as compared to other alternatives. This information will aid him in determining how much to invest in fertilizer.

[6]If the yield response for two nutrients is pictured as a surface or "hill" on a 3-dimensional diagram, it can be reproduced in 2-dimensional form just as a hill is reproduced by the soils expert on a topographical map, as a set or family of contours. Each contour represents a given yield level and the points on it represent the various nutrient combinations which allow attainment of this specified yield level. The yield contour, showing all possible combinations of nutrients allowing its attainment, is termed a yield isoquant (equal quantity).

amount of the other required to produce the given yield, or (b) increases the level of yield. This is denoted by the fact that the isoquant forms a 180-degree angle. However, if it is assumed that addition of one nutrient, without change in the other, causes toxic or other effects reducing total yield, the isoquants reduce to a single point consistent with the corner of the angles in figure 1.3.

However, a strict Liebig type of production surface is the exception rather than the rule. Otherwise agronomists would not have (or have been able to have) successfully conducted a relatively large number of single-nutrient experiments. Perhaps it is true that such distinct nutrients as nitrogen, P_2O_5, or K_2O do not substitute in the chemical processes of the plant (although close substitution may hold true for elements such as Na and K). However, availability of one nutrient may affect the ability of the plant to utilize other nutrients. Hence, in any case where variation of one nutrient, with another fixed at specific levels as in figure 1.4, results in different response curves, substitution does take place in the sense that different nutrient combinations can be used to attain a given yield. For example, if a 10-bushel response is attained with 20 pounds of N and 120 pounds of P_2O_5, with 60 pounds of N and 90 pounds of P_2O_5, or with 120 pounds of N and 40 pounds of P_2O_5, the given response can be attained with various nutrient combinations. It may be stated that nutrients are substitutes, at least at the level of farm decision-making.[7]

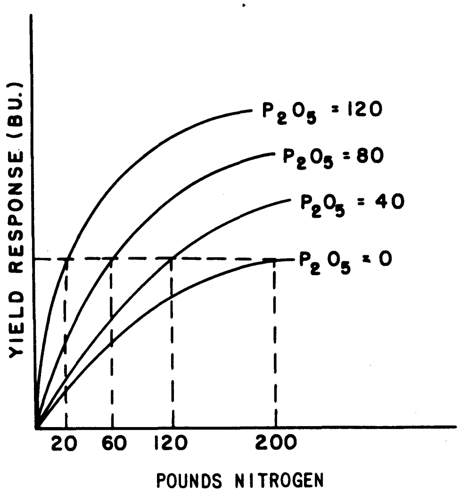


Fig. 1.4 — Yield resource curves for nitrogen with P_2O_5 fixed at different levels.

[7]These statements need, of course, to be conditioned in terms of plant composition and quality.

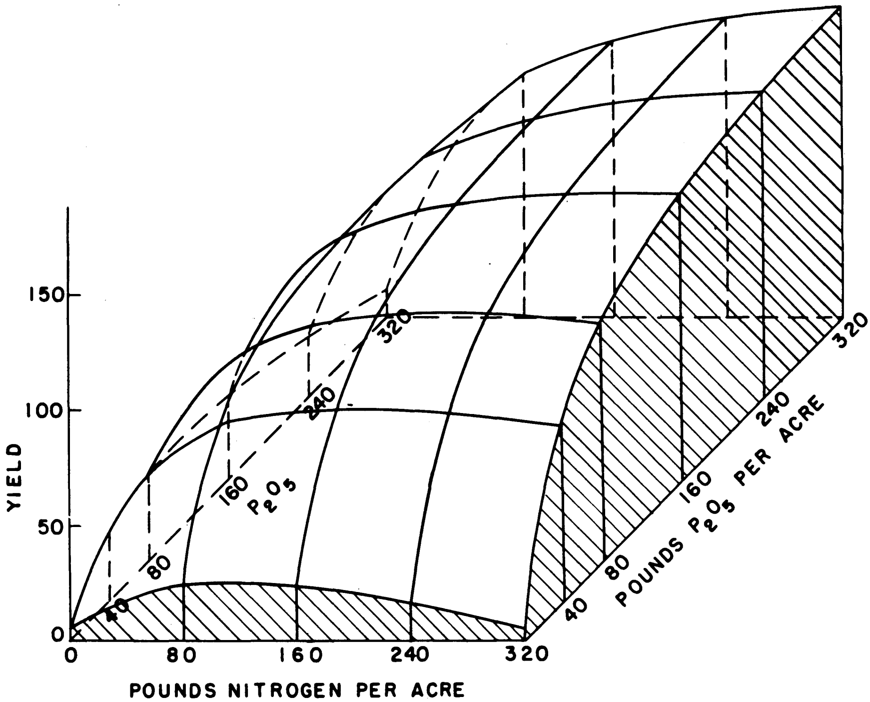


Fig. 1.5 — Predicted yield surface for corn. Source: Pesek, Heady, and Brown, Iowa Agr. Exp. Sta. Bul. 424.

The response surface for many crops and soils is more likely to parallel that shown in figure 1.5 for N and P_2O_5 on corn in western Iowa, or some modification of it (1). The corresponding family of yield isoquants is shown in figure 1.6. At high levels, the isoquants bend sharply to a purely vertical position at the "upper" end and to a purely horizontal position at the "lower" end. At these points of infinite and zero slope, respectively, the nutrients actually do become limitational or technical complements in the sense of Liebig; increase of one nutrient alone, at the vertical and horizontal points of the curves, will not result in reduction of the amount of the other nutrient, with yield remaining at the specified level or addition to the total yield. (Yield may actually be reduced if one nutrient is increased while the other is held constant at the level indicated at the points of infinite or zero slope.) However, between the two points of complementarity, the curves have a negative slope, denoting that they are substitutes in the sense that addition of one nutrient reduces the quantity of the other nutrient required to attain

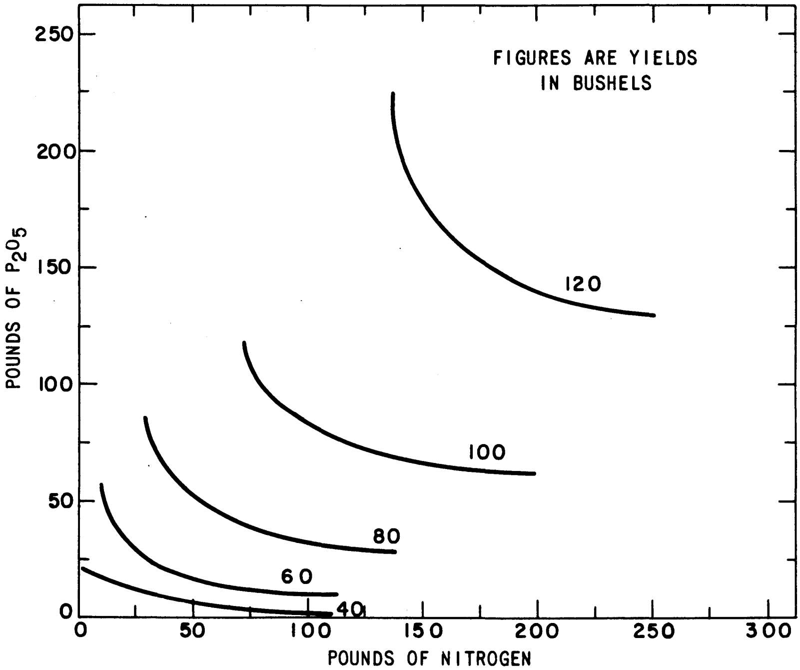


Fig. 1.6 — Predicted yield isoquants for corn (from Fig. 1.5).

(maintain) the given yield.[8] Furthermore, the curvature or slope of the isoquant changes, denoting that increasing quantities of the nutrient being added are necessary to offset constant decrements of the nutrient being replaced.

An important methodological problem in fertilizer research is that of obtaining more information on the slope and degree of curvature of the yield isoquants. If the slope changes only slightly and its length between the points of complementarity (i.e., the vertical point on the "upper" end and the horizontal point on the "lower" end) is great, the nutrients can be classed as "good" substitutes (i.e., "poor" complements). If the curvature is sharp (i.e., the slope changes rapidly) and the range between complementary points is narrow, the nutrients are poor substitutes (i.e., "good" complements). Now it is just as important to know that nutrients are "good" substitutes as it is to know that they are "good" complements. Perhaps too much research and too many recommendations have supposed that nutrients are only good complements. Given the meager knowledge which exists, the specialist making recommendations can suggest specific nutrient ratios with less burden on his

[8]For other alternatives in fertilizer production surfaces and isoquant maps, see (1).

conscience (and less profit depression to the farmer if the expert is not entirely correct) if he knows that substitution is "good" over a wide range.

If the slope of the isoquant is relatively constant over most of its range, and if this slope does not deviate greatly from the magnitude of the price ratio, a large number of nutrient combinations give costs and profits of fertilization which are quite similar. Here, again, the expert making fertilizer recommendations need not let his conscience be bothered greatly if he recommends a particular ratio such as 20-20-0 rather than a 10-20-0. However, if the curvature changes greatly between the complementary points and if the slope at either one or both ends deviates considerably from the magnitude of the price ratio, the expert needs to give particular heed to his recommendations on nutrient ratios. He will want to consider price ratios; he will want to consider the effects of nutrient prices on the optimum nutrient combination and the optimum fertilization level. The optimum nutrient combination will change with yield level, if the slopes of the yield isoquants differ greatly as successively higher yields are attained. Under these conditions, the recommendation on nutrient ratios should differ between farmers (a) who have funds for only low fertilization levels, and farmers (b) who have unlimited capital and can use higher fertilization ratios. Similarly, if slopes between isoquants change greatly with higher yields, the nutrient ratio will need to be changed as the price of the product changes (and higher or lower yield levels are profitable), even if the nutrient price ratio remains unchanged. The extent to which these facets of economics need to be incorporated into fertilizer recommendations depends on the nature of the production surfaces and isoquant maps. While they are fundamental science aspects of agronomic phenomena, knowledge is still too meager to determine where, and the extent to which, these considerations become important.

Fertilizer Isoclines

The slopes of isoquants change (i.e., the marginal rate of substitution between nutrients) as higher yields are attained. However, slope or substitution rate changes must be defined in a particular manner. They must be in reference to a fixed ratio of nutrients such as that illustrated in figure 1.7. The straight lines, A and B, passing through the origin, denote that nutrients are held in fixed ratios at higher fertilization levels. Changes in slopes or substitution rates on successive isoquants, in relation to needs for different nutrient ratios at varying yield levels, are measured at the point of intersection of the fixed ratio lines and the yield isoquants. If the slope of the isoquants were identical at all points where they are intersected by a fixed ratio line, the same fertilizer mix would be optimum for all yield levels. If the slope changes along a fixed ratio line, the nutrient ratio which is optimum for one yield level is not also optimum for another yield level.

A concept with perhaps greater application and more fundamental

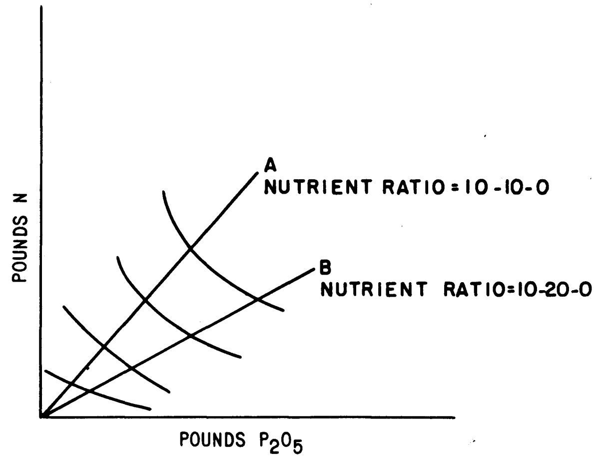
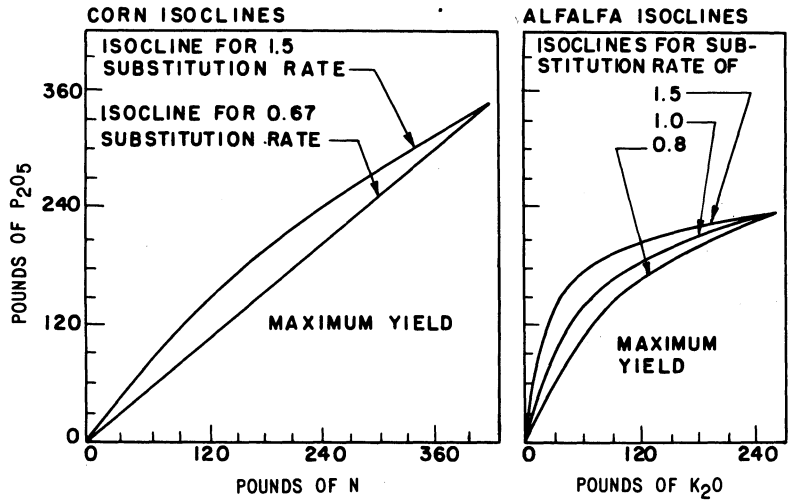


Fig. 1.7 — Isoclines showing equal nutrient ratios in relation to yield isoquants.

importance than the fixed ratio line is the fertilizer yield isocline. An isocline map exists for every fertilizer production surface. An isocline is a line connecting all points of equal slopes or substitution rates on a family of isoquants. In other words, it connects all nutrient combinations which have the same substitution rates for the various yield levels. There is a different isocline for each possible nutrient substitution rate. Of course, if the fertilizer production surface is of the Liebig knife-edge type illustrated in figure 1.2, the map reduces to a single isocline, denoting a zero substitution rate.

The isocline is also an expansion path, showing the least-cost and highest-profit combination of nutrients to use as higher yield levels are attained under a given price ratio for nutrients. In other words, it indicates whether the same nutrient ratio should be recommended and used regardless of the yield to be attained. Chapter 10 illustrates practical uses of this concept. Isoclines can be straight lines, such as A and B in figure 1.7. In this case they become identical with a fixed ratio line and the least-cost nutrient ratio will be the same for all yield levels. The expert need not inquire about the yield level to be attained when he makes his recommendation. However, an isocline map composed entirely of straight lines (fixed ratios) is very unlikely and perhaps impossible. Under maps of this nature, the isoclines would never converge but, instead, would spread farther apart at higher yield levels. Therefore, straight-line isoclines would indicate no limit to total yield level. Limits in total production exist only if the isoclines converge to the point of maximum yield and, therefore, are curved rather than straight (see Chapter 6 for other details on this point).

Isocline maps may take on many different forms. Little is known about them, and their nature can be established only by basic research. All isoclines for a given production surface may be bent in the same



Figs. 1.8 and 1.9 — Isoclines for corn and alfalfa showing convergence to maximum yield.

direction and none may be linear. Alternatively, one may be nearly straight while those above and below it bend in opposite directions. Different isocline maps, based on research in Iowa (1), are shown in figures 1.8 and 1.9. The two for corn, covering likely limits in price ratios for nutrients, are quite straight, with a slope relatively close to 1:1, denoting that recommendations of a constant nutrient combination may not deviate far from least-cost ratios for all yield levels. (Cognizance of the slight curvature in recommendations might cause more bother than savings in cost would merit.) In the case of the alfalfa data, however, the relevant isoclines bend rather sharply, suggesting that the least-cost nutrient ratio for one yield level may differ considerably from that for another yield level.

Two isoclines can be called "ridgelines" (see figure 10.1, page 153). They correspond to all points in figure 1.5, where the slope of the surface changes from positive to negative (i.e., the tops of the ridges denoting zero marginal responses). The ridgelines denote the points on successive yield isoquants where the nutrient substitution rate becomes zero. Since they denote technical complementarity of nutrients, they might appropriately be given the term "Liebig lines" because these are the limitational conditions which Liebig had in mind in his law of the minimum. The ridgelines (Liebig lines) converge, along with the other isoclines, at the point of maximum yield where nutrient substitution also is impossible.[9]

If (a) the ridgelines are not far apart, (b) the isoclines within their boundary are fairly straight, and (c) the yield isoquants for a particular

[9]The isoquant at the point of maximum yield reduces to a single point.

yield have only a slight curvature, with slopes not too different from the nutrient price ratio; several nutrient ratios, within the boundaries of the ridgelines, will give costs which are only slightly different (although only one will denote the least-cost ratio). If (a) the ridgelines are "sprung far apart," (b) the isoclines "bend sharply," and (c) the isoquants "curve greatly" away from the price ratios, the saving from changing nutrient ratios along an isocline can be quite considerable. Only basic research can indicate the frequency and extent of different isocline maps. The situation likely varies with soil, crop, year, and other variables.

Information of this nature not only has methodological importance but also practical significance. Therefore, the full economic potential of fertilizer use will be uncovered only by multi-variable response research. This is true since, as production economics logic has long suggested, the productivity of any one resource always depends on the level of input for other resources. While much of the logic is illustrated with two variables, analysis should be extended to variables which include other nutrients, seeding rates, moisture, quantities of nutrients already in the soil, soil type, and others. In other words, one should view the production function in the generalized form of equation 8. It is not inconceivable that soil typing and classification might be relative to the fertilizer production function. For example, with other inputs specified, economic distinction need not be made between soils where marginal response for parallel fertilizer inputs are the same. While they may be complex, steps to incorporate this concept into fertilizer research might obviate the need for considering experiments at isolated locations and in particular years as unrelated facts.

At the outset it was stated that the paramount methodological problem was that of the mathematical form of the fertilizer production function. Experimental designs and estimating procedures are auxiliary problems to it but at the same time are the foundation tools for establishing the mathematical characteristics of the function, at a given point in time and over time. To what extent is replication necessary when interest is in prediction of the response curve or function and the standard error which attaches to it, rather than the mean differences between treatment? Supposing that yield distributions are heteroscedastic in respect to variance; under what conditions would recommendations differ among regression lines predicted with nonreplicated treatments and means of treatments based on replications? What experimental designs allow both statistical and economic efficiency in estimate of complete surfaces, including isoquants, isoclines, and ridgelines? Is it unlikely that responses for different fertilizer inputs follow in the manner of a continuous function, and that other estimating procedures are necessary? There are hypotheses in respect to the answers of some of these questions; however, lack of time and space prevents the unraveling of their logic.

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