Chapter 13

C. B. BAKER Interpretation of Regional and Spatial Models

UMEROUS EXAMPLES exist of empiric studies to estimate equilibria among regions in prices and flows of agricultural products. A few such studies particularly relevant for adjustment problems of Midwestern agriculture are summarized below. Limitations of findings imposed by the models and of the data available for use of the models are then examined in light of these adjustment problems. Some extensions of the models also will be suggested, as well as alternate approaches to research relevant to adjustment problems.

#### SPATIAL EQUILIBRIUM MODELS

Agricultural economists long have demonstrated a strong interest in problems of interregional trade.<sup>1</sup> Closely associated are studies in the theory of production location.<sup>2</sup> Isard has shown that theories of trade over space and of location are two views of the same phenomenon. He also shows that important features of leading variants in both trade and location theory can be translated into each other and integrated into the general theory of production economics (11).

Interest has been stimulated by development of programming models by Enke, Baumol, and Samuelson. Enke (6) demonstrated in 1951 that with "a relatively simple electric circuit," a model could be used to generate estimates for a single product in each of several regions, (1) net price and (2) quantity, if any, of exports or imports; and, among regions, (3) aggregate trade and (4) volume and direction of interregion trade. Each region is specified to be separated from other regions only by a transport cost per physical unit of product independent of volume or direction of shipment. Price-quantity relations as well as transport rates are taken as givens.

<sup>&</sup>lt;sup>1</sup>Though a long list might be mentioned, we refer the reader to the last major effort prior to the recent applications of operations research methods, Mighell and Black (14). Farms were budgeted in each of six areas sampled in the Lake States and New England in 1935-36 and 1945-46, the objective being to predict the relative responsiveness of the two areas to likely changes in demand conditions.

<sup>&</sup>lt;sup>2</sup>Again, a rich literature is available in economics generally and in agricultural economics specifically. See especially Isard (11) and Dunn (5). Some features of Dunn's contribution might be used in probing for boundaries and shifters thereof between regions, a problem neglected or "solved" arbitrarily in the studies outlined below.

Baumol (2) and Samuelson (15) formulated the same problem as one of maximizing an objective function, given demand and supply functions in each of three or more regions separated, as were Enke's regions, by transport rates independent of volume or direction of shipment. In all three models production, consumption, and shipments occur at a single point in each region. Subject to obvious empiric limits imposed by this simplification, the models are adaptable to the study of change in any three of the four sets of data already mentioned, given a postulated change in some part(s) of the fourth set of data. To overcome this limitation, Beckmann (3) has developed a model capable of generating equilibria in the presence of continuous change in distributions of production density and transport costs.<sup>3</sup>

Most agricultural applications of spatial equilibrium models have been of adaptations of the Baumol or Samuelson type.<sup>4</sup> Hence we turn to a brief summary of such a model and its properties. In each of two regions we define for a single product an "excess supply function." It relates to prices of the product differences between quantity supplied and quantity demanded at respective prices. Each such quantity difference represents a surplus exportable at the given price from the region. Figure 13.1 is due to Samuelson. On the vertical axis we represent

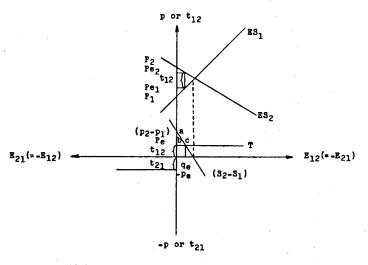


Figure 13.1. Equilibria in prices and quantities: a single product in two markets.

<sup>&</sup>lt;sup>3</sup>It may be something of an anomaly, however, to regard Beckmann's model as one to use in a study of "interregion relations." The power of his model is in its ability to depict an area of continuous change instead of one divided into regions. However, a careful review of the model reveals that the requirements for making it empirically usable are considerable.

<sup>&</sup>lt;sup>4</sup>For an alternative type, see Henderson and Schlaifer (10). This "transportation" model is a special case of the more general model of Samuelson, Baumol, et al, though certain computational advantages make its extension for some problems easier than is the case with the more general model.

monetary rates: positive for quadrants I and II; negative for quadrants III and IV. On the horizontal axis we show quantities of product: positive for quadrants I and IV; negative for quadrants II and III.

 $ES_1$  and  $ES_2$  are excess supply functions in, respectively, regions 1 ( $R_1$ ) and 2 ( $R_2$ ).<sup>5</sup> In Figure 13.1, prices or transport costs are measured vertically; quantities shipped, horizontally. Equilibrium in  $R_1$  ( $R_2$ ) yields  $p_1$  ( $p_2$ ), since at this price supply equals demand and  $ES_1$  ( $ES_2$ ) is zero. The fact that  $p_2$  exceeds  $p_1$  suggests that if interregion trade develops it will consist of shipment from  $R_1$  to  $R_2$ . Whether trade will develop depends on the cost of shipping the product from  $R_1$  to  $R_2$ . Equilibrium is established with product flow  $q_e$  from  $R_1$  to  $R_2$  and prices  $p_{e1}$  and  $p_{e2}$  in  $R_1$  and  $R_2$ , respectively. The difference in price is exactly equal to the cost,  $t_{12}$ , of transporting a unit of product from  $R_1$  to  $R_2$ . Should the prices without trade have been reversed, trade would have been reversed. Should they have differed by less than the transport rate,  $t_{12}$ , no trade would have developed.

The curve  $(S_2 - S_1)$  is a locus of points representing the vertical differences  $(P_2 - P_1)$  at scheduled quantities,  $E_{12}$ . Thus it shows in terms of joint equilibria the same information as do  $ES_1$  and  $ES_2$ . Ordinates of the curve T are transport rates between regions. At  $q_e$ ,  $(S_2 - S_1)$ equals T. Trade at  $t_{12}$  would develop so long as the negatively sloped  $(S_2 - S_1)$  intersects T to the right of the vertical axis. Should the intersection occur between  $p_e$  and  $-p_e$  no trade would occur between regions. Indeed the gain from trade is given by the area of triangle abc, the difference between total gain oacq<sub>e</sub> and transport cost obcq<sub>e</sub>. The fact that this gain can be formulated as a function to be maximized makes possible a normative approach to problems of interregion trade.

$$s = a_s + b_s p + u$$

where  $a_s$  and  $b_s$  are constants in the supply relation and u is a source of random variation in s. The quantity demanded is given similarly by

$$d = a_d - b_d p + v.$$

The excess supply at any given price is thus given by

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(3)

(4)

$$= a_{s} + b_{s}p + u - (a_{d} - b_{d}p + v)$$
  
=  $a_{s} + b_{s}p + u - a_{d} + b_{d}p - v = (b_{s} + b_{d})p + a' + w$ 

where a' represents the collection of constant terms and w the net effect of u and v. Differentiating relation 3 with respect to p we get as the slope of the excess supply function in the ith region:

$$b_{FS} = b_s + b_d$$

Thus, to assume  $\mathbf{b}_s$  equal to zero is equivalent to assigning to  $\mathbf{b}_{ES}$  a minimum of likely values.

<sup>&</sup>lt;sup>5</sup>For models in which supply for the ith region is taken as fixed, the slope ES<sub>i</sub> depends solely on the slope of the demand curve in the ith region. With supply functionally related to price, it can be shown that the slope of the excess supply curve is simply the algebraic sum of the slopes of the supply and demand functions.

We define for a given region an "excess supply" (ES) as the difference at a given price (p) between quantity supplied (s) and quantity demanded (d). The quantity supplied is given by

We digress here to point out that the function T could reflect a transport rate  $t_{12}$  that declines with volume of shipment,  $E_{12}$ . The general result would be an increased quantity of interregion trade. Should the transport rate from  $R_2$  to  $R_1$  differ from the transport rate from  $R_1$  to  $R_2$ , the values of  $p_e$  and  $-p_e$  (sign neglected) would differ. So long as  $p_2$  exceeds  $p_1$  in pretrade equilibrium, the only consequence of the latter extension is to vary the price differential that can exist without inducing trade between regions.

The gain from interregion trade is a nonlinear function of trade volume. That is, the area of triangle abc increases or decreases with respect to shifts in the curve  $(S_2-S_1)$  at a rate that is not constant with respect to changes in  $E_{12}$ . Moreover, the extension of the two-region problem to a problem of n regions is not obvious. However, Samuelson (15) shows that at a maximum of gain, the following relation holds between any pair of regions  $R_i$ ,  $R_i$ :

 $-t_{ij} \leq (p_j - p_i) \leq t_{ji}$  (i, j = 1...n)

where  $t_{ij}(t_{ji})$  is the rate of transport cost from  $R_i(R_j)$  to  $R_j(R_i)$  and  $p_i(p_j)$  is the price of product in  $R_i(R_j)$  at an interregion equilibrium. The problem then may be visualized as one of finding price differences that will maximize total gain from interregion trade, subject to transport costs between regions.<sup>6</sup> Solution to this problem is equivalent to a solution that minimizes the sum of transport costs subject to equilibrium price differences.

The major assumptions imposed on investigations of interregion competition in which spatial equilibrium models are applied are summarized here. There are, first of all, assumptions common to models generally, arising from attempts to simulate real phenomena with either scale replicas or, still more abstractly, variables and relations among variables. Since applications of spatial equilibrium models have employed programming techniques, they employ the special assumptions common to programming models (4): (1) linearity (in space-related variables), (2) divisibility (in quantities of product flow), (3) additivity, i.e., imports in  $R_j$  are independent (other than in terms of transport costs and demand) of imports in regions other than  $R_j$  and (4) finiteness (of all possible divisions into which a space might be divided, only n divisions are considered, among which k export and (n-k) import).

Because spatial equilibrium models are specialized examples of programming models, still further assumptions are involved. Of greatest relevance for adjustment problems are the following: (1) within regions, spatial homogeneity with respect to opportunity cost in terms of nonoptimized products,<sup>7</sup> and a sum of deviations from the mean of

<sup>&</sup>lt;sup>6</sup> It is also required that over all regions, total supply equals total demand, unless allowance is made for accumulation or depreciation of stocks.

<sup>&</sup>lt;sup>7</sup>A related assumption is that resources are assumed immobile as among regions. However, this assumption seems not particularly limiting since resources are easily conceived as products.

quantity-weighted transport cost at each regional basing point, equal to zero; (2) between regions, homogeneity in elasticity of supply of optimized product with respect to price(s) of nonoptimized product(s); and (3) extension to regional entities of normative behavior otherwise ascribed to decision makers at levels of firms and sovereign public bodies.

#### AGRICULTURAL APPLICATIONS IN SINGLE PRODUCT STUDIES

Judge and Wallace (12) used an adaptation of the Samuelson model to estimate an equilibrium marketing pattern for slaughter beef under production and consumption conditions in 1955. For each of 21 regions in continental United States, beef supply was taken as given and perfectly inelastic with respect to price of slaughter beef. Demand in each region was estimated by the following equation:

 $Y_{Bi} = 78.3543 - 1.0529 x_{1i} + 0.6509 x_{2i} + 0.0303 x_{3i}$ 

where for region i,

 $Y_{B_i}$  is per capital consumption of slaughter beef in pounds.

 $X_{11}$  is retail price of beef in cents per pound.

 $X_{2i}$  is price of pork in cents per pound.

 $X_{3i}$  is disposable income per capita in dollars.

A basing point was selected for each region. Truck and rail costs between basing points were estimated. The lower of the two was used to reflect transport costs between regions.

In Figure 13.2 we show the general results that were obtained in shipments that minimize transport costs. For each region the top number indicates the amount of slaughter beef "produced"<sup>8</sup> in millions of pounds. The following "E" numbers indicate either exports from the region (E) or imports into the region (-E), also in millions of pounds. Subscripts refer to regions of either origin (for -E) or destination (for E). Following the E number(s) is the total regional consumption of slaughter beef in equilibrium. The last number is the price of slaughter beef consistent with the interregion system of production, consumption, and shipments. The shipments are so organized that transport costs are minimized subject to the separate demand and supply functions for each region and the constraint that total production for the U.S. equals total consumption for the U.S. within the year.

The Iowa-Nebraska region produces the largest supply of beef and also the largest export of beef. In total supply it is followed closely by the Illinois-Indiana region. However, the Illinois-Indiana region produces only a small export that goes entirely to the deficit region of

<sup>\*</sup>The product is beef slaughter cattle in carcass weight.

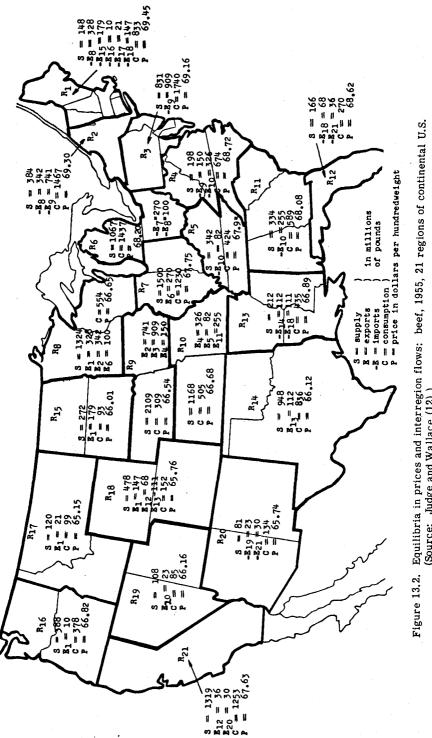
Michigan and Ohio. The Iowa-Nebraska region exports to regions centered on (in order of importance) Philadelphia, New York, and Roanoke, Virginia. It furnishes the Philadelphia region its sole supply of imports. It furnishes the New York market with more than two-thirds of its imports, the remainder being supplied from the Minnesota-Wisconsin region. Exports to the Virginia-West Virginia-North Carolina region are relatively small. This market is also shared. The Kansas-Missouri region furnishes it with more than twice the volume supplied by the Iowa-Nebraska region. The Kansas-Missouri region also ships a small volume to the Kentucky-Tennessee region, otherwise selfsufficient in beef.

The prices shown in Figure 13.2 are equilibrium prices in the restricted sense permitted by the model. In each region it is the price that equates the given supply with the amount consumed within the region plus or minus the amount imported or exported, where the latter is consistent with a minimum total of interregion transport cost. It is interesting to compare these with prices observed in the regions in 1955. Should they correlate highly one might conclude that (1) transport costs are important as criteria of interregion organization and are well represented despite the simplifying assumptions already referred to; and (2), as a corollary of (1), that the substitution relations not taken into account are not as important as is commonly assumed by agricultural economists. Unfortunately, prices for 1955 are not available for such a comparison. However, they are for the years of 1947 and 1952. The squared correlation coefficients are less than 0.50.

Thus, only a small fraction of geographic price variation is explained by shipments that minimize transport costs. Yet the phenomena of interregion specialization are shown to be important by assuming, with observed supply, the consequences of no shipments. In deficit regions prices become high, while in the surplus regions the product sells at low prices. In the New England region the price goes to more than \$5.00 per pound. In the Iowa-Nebraska region the price drops to seven cents per pound. Such variations as these would generate supply response in each region. However, regional specialization is of evident importance.

These results are of considerable interest in terms of adjustment problems of individual farms and areas. They illustrate that all regions are affected by a change of "givens" in any one. Thus, what happens to consumer income in the Philadelphia region is vitally important to cattle feeders in Iowa-Nebraska. The population of this importing region is 20,213,000. Hence, an increase or decrease of \$1.00 in per capita disposable income increases or decreases the demand for beef by 6.1 million pounds (= .0303 x 20,213,000). More than half of the variation traces back to the Iowa-Nebraska region. In turn, increased or decreased demand for beef from this region reverberates throughout the interregion system.

The assumption of fixed supply restricts the empiric significance of the results to a short time run. The authors ascribe to it a one-year



(Source: Judge and Wallace (12).)

significance. This may be a reasonable assumption for slaughter cattle. Few production alternatives in cattle feeding can be changed radically within a year's time. However, even in a one-year period, the feeding rate and weight to which cattle are fed can be varied. Also, since alternatives to cattle feeding vary considerably among regions, one would hardly expect the price elasticity of supply to be the same among regions. Thus, though in the aggregate a fixed supply might be a fairly reasonable assumption, a supply fixed <u>in each region</u> might not be.

Another limitation is imposed by the aggregation represented in a 21-region division of continental U.S. Such aggregation is a necessary characteristic of a regional study.<sup>9</sup> Yet it requires an arbitrary assumption that for product(s) investigated, all production, consumption, export, and import occur at basing points of the regions. Even for a single product the selection of a basing point is more or less arbitrary because of limitations of data. Where several products are involved the problem of selecting an appropriate basing point becomes extremely complex even from a conceptual viewpoint. Data requirements force an arbitrary selection.

A further limitation attaches from the assumption that variation in transport rate is independent of direction from the basing point.<sup>10</sup> Casual observation suggests that the actual structure of transport rates is far different. Not only are the rates not continuous but they also differ by direction. While directional differences might be taken into account, to do so would multiply the computational problems already large for such models as these and larger still on attempts to improve the region-aggregate assumptions.<sup>11</sup>

We return finally to the fixed supply assumed for each region. This assumption presumes not only that in the aggregate the marginal cost of output increases with output at the same rate in each region (infinitely in the above example), but that any scale economies that might exist in producing slaughter cattle are exploited equally in all regions. Otherwise, the difference in horizontal position of marginal cost curves will alone create a difference in price elasticity of supply at given price. With slope constant with respect to shifts, the elasticity coefficient is decreased (increased) with an increase (decrease) in quantity. Since the economic position of the cattle feeding enterprise varies between farms and between regions we suspect large differences in the aggregate supply elasticities.

There may be reason to suspect that in many corn-belt farm organizations, cattle feeding is coming to play an important supplemental role (1). Should this hypothesis be confirmed, the corn belt supply function for fed cattle and the shifters strategic to such a function would differ from functions found in regions where cattle feeding is dominantly "competitive." In supplemental enterprises one would expect less

<sup>10</sup>It is commonly assumed too that transport rates are independent of the product. Yet actual rate structures are a complex of product-direction-distance interrelations.

<sup>&</sup>lt;sup>9</sup>See footnote 4 above.

<sup>&</sup>lt;sup>11</sup> For an attempt with a 104-region system, see Heady and Egbert (9).

response to changes in product price ratios and stronger relations between production levels and inputs in which household values are important determinants.

The programming model yields interesting results from changes introduced in basic data and the postulates. We have already referred to the effects of barring all interregion shipments. The authors also show the effects on regional prices and interregion shipments of an increase and a decrease (of 20 percent) in transport costs; and changes in regional income, population and supplies to represent conditions of 1947, 1952 and projected for 1963, the latter considering, alternatively, (1) increase in population, production and income, at rates similar to those observed between 1947 and 1955, (2) increase in population and income, production fixed at 1955 levels, (3) increase in population and income, production per capita (within regions) fixed at 1955 levels, and (4) increase in population and production, per capita incomes (within region) fixed at 1955 levels. Transport costs were held constant throughout at 1955 rates.

The 20-percent increase (decrease) in transport costs decreased (increased) interregion shipments by only about half of one percent, accentuating (dampening) the interregion price variation as compared with the 1955 equilibria. The combination of lower income and high level beef production in 1947 led to equilibria with lower prices generally and a smaller volume of interregion shipments than found at 1955 equilibrium. Relatively low production in 1952 led to higher prices in equilibrium and a slightly lower volume of interregion movement of beef. Projection (1) converted Kentucky-Tennessee into a surplus region. Interregion shipments increased by 34 percent and prices remained about the same as 1955 equilibrium prices. Projection (2) yielded increased prices in all regions and some changes in interregion flows. California changed from a surplus (in slaughter cattle "production") to a deficit region importing the total surplus of Washington-Oregon and Utah-Nevada and most of the surplus from the Montana-Idaho region. Projection (3) increased prices generally and altered the surpluses and deficits by small amounts but changed no interregion shipment patterns. Projection (4) resulted in lower prices and converted Kentucky-Tennessee into a surplus region.

As a by-product of the various solutions the model generates an estimate of the amount by which the transport cost must be reduced between any pair of regions to induce, <u>ceteris paribus</u>, a flow of product from the region of surplus to the region of deficit. The <u>ceteris paribus</u> assumption is, of course, something of an anomaly in a programming model. The very strength of the model lies in its capacity to encompass simultaneous change and (synthetically) to produce a logically valid estimate of results from their joint effects.

Besides introducing explicitly only a limited number of the phenomena that are in fact related to geographic price variation and interregion product flow, the model postulates a behavioral relation in the objective function to be minimized. Hence, departures in behavior from such an optimizing postulate furnishes still another reason for the relatively low value of  $r^2$ . The surprising result is that it is in fact as large as it is in the two years observed.

In a comparable study of slaughter hogs, Judge and Wallace (13) report equilibria among the 21 regions outlined in Figure 13.3 in prices and interregion shipments, given 1955 supplies in each region and demand for pork in region i according to  $Y_{\rm pi} = 50.9829 - 1.1917 x_{1i} + 0.3468 x_{2i} + 0.0359 x_{3i}$  where for the ith region

 $Y_{p}$  is the per capita consumption of pork in pounds.

 $X_1$  is the price of pork in cents per pound.

 $X_2$  is the price of beef in cents per pound.

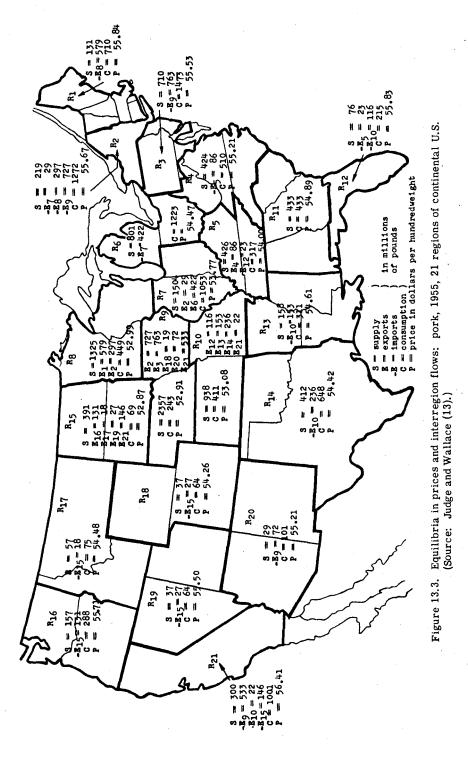
X, is per capita disposable income in dollars.

Regions producing a surplus in equilibrium are restricted to the Midwest and the Kentucky-Tennessee region. The Iowa-Nebraska region produces the largest total supply and exports a larger percentage of the supply than do any of the other surplus regions. In percentage of supply exported, the Dakota region ranks a close second, followed by the Minnesota-Wisconsin region. However, the total supply from the Dakota region is smallest among surplus regions and the Minnesota-Wisconsin region is surpassed in total by the Illinois-Indiana region. Among surplus regions, the Illinois-Indiana region ranks highest in percent of supply consumed within the region.

It is interesting to note that, as expected, the direction of shipment varies too among the surplus-producing regions. The Iowa-Nebraska region ships east and west. The Dakota's ship exclusively west; Minnesota-Wisconsin and Illinois-Indiana, exclusively east; Kentucky-Tennessee, south and east; and Kansas-Missouri, to Florida, the midsouth and to California. The squared correlation coefficient for equilibrium and actual prices again is less than 0.50. Thus of total variance, among regions, less than half is explained by interregion shipments that minimize transport costs.

The assumption of fixed supply is likely more damaging in the pork study than in the beef study. To counter this the authors provide a quarterly analysis. On this basis the Kentucky-Tennessee region exports less to the West Virginia-Virginia-North Carolina region and more to Florida. The Illinois-Indiana region sends more to New York and less to Michigan-Ohio. Minnesota-Wisconsin ships less to New York and more to Michigan-Ohio. Iowa-Nebraska exports slightly less to California. Kansas-Missouri ships less to Florida and more to West Virginia-Virginia-North Carolina and more to California. Alabama-Georgia-South Carolina become slightly surplus (first quarter) shipping a small quantity to Florida.

By introducing changes analogous to those introduced in the beef



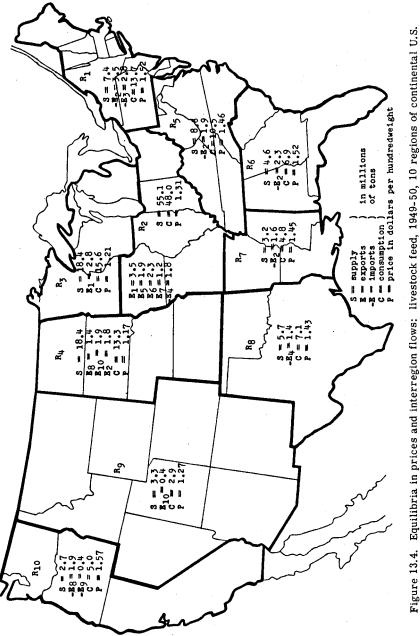
investigation, similar sorts of "adjustment" changes are induced.<sup>12</sup> Interpretations are conditioned for hogs by the same properties of the analytical model as were observed for the beef study. The major phenomenal difference was taken into account by the estimation by quarters of equilibria in region prices and interregion product flows.

Other "single-product" studies include the pioneering study by Fox (7) in which he established interregion equilibria in prices and flows of livestock feed. For each of the ten regions in the United States, shown in Figure 13.4, feed consumption was related to price of feed in a price-dependent demand function where feed production and livestock price were taken as given. As a gross equilibrium condition, aggregate feed consumed in all ten regions was required to equal aggregate feed produced. No net change was permitted in feed inventories. Between each of all possible pairs of regions a transport rate for a bushel of corn was estimated from freight charges by mileage blocks observed in a sample of 1950 ICC waybills. For each region all consumption and production were assumed to occur at a single point. Fox then estimated from an aggregate demand function for feed for the United States a demand function for each region, based on the proportion of United States grain-consuming livestock produced by that region.

Then, feed supplies given for each region and completely nonresponsive to price change, an interregion flow was sought such that no individual could make a profit by (further) shipping from one region to another. This is attained by following these rules: (1) "If one region ships to another region, the prices must differ by the amount of the intervening transportation costs;" and (2) "if two surplus regions ship to the same deficit region, the difference between equilibrium prices in the surplus regions will be equal to the difference between their freight rates to the deficit region." Results are shown in Figure 13.4 for demand and supply conditions as of 1949-50. Prices shown are different by the transport rate between the region of origin and the region of destination. All data are in terms of corn equivalent. The squared correlation coefficient relating actual prices and prices generated by the model is 0.49.

Though no specified objective relation is used explicitly by Fox, properties of his model are similar to those of the model used by Judge and Wallace. Thus all the limitations so far noted apply to these results as well. In addition, the aggregation implied by the 10-region division of continental United States renders more tenuous the assumptions based on (1) production, consumption, exports and/or imports from a single point interior to each region and (2) homogeneity within regions. Yet certain advantages attach to the use of fewer regions: the greater relevance of transport costs (compared with nontransport costs) when shipping centers are farther separated, and the lesser computational

<sup>&</sup>lt;sup>12</sup> In the pork study, a 36-region model also was estimated. In a second study on beef (as yet unpublished), quarterly models were used, but yielded results little different from those obtained with the annual model.



(Source: Fox (7).)

requirements when using the fewer regions. Fox reported solutions obtained "...with ease...with no more exotic equipment than a desk calculator, supplemented by the investigator's judgment."

### AGRICULTURAL APPLICATIONS IN MULTIPLE PRODUCT STUDIES

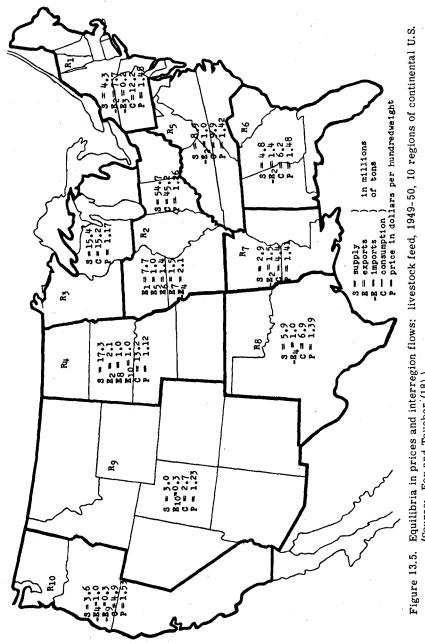
As is well known, there exists for resources in agriculture competition between products within regions as well as between regions with respect to single products. It is interesting to include in our survey such attempts as have been made to extend analyses of spatial equilibria to the multiple-product case. We report at the outset that such an extension has been made only under extremely limiting conditions. Indeed the only examples so far reported entail either complementary products or joint products.

An example of the former is found in an extension of the Fox model made by Fox and Taeuber (8). Under conditions approximated in 1949-50 they established (joint) equilibria among regions in both feed and livestock. Initially Fox took as given, in the regional demand for feed, regional livestock production and price of livestock. In the later approach Fox and Taeuber related regional demand for feed to the regional prices of feed and livestock and to regional livestock production. In turn, regional livestock production was related to regional prices, given the human population and disposable income per capita. Livestock supply was related to prices of both livestock and feed. The supply of feed was assumed fixed by region.

By following the same rules used in Fox's simpler model, Fox and Taeuber generated an equilibrium in regional prices, consumption, and (livestock) production. These and the interregion flows are shown in Figure 13.5. Unfortunately, the feed supplies, given for each region, differ slightly from those used in the previous model. Hence a direct comparison is not possible between equilibrium results. It appears that supplies generally are smaller in the later model. However, it seems clear that by introducing the livestock variable in demand and supply, the following results are obtained: (1) the interregion volume of feed shipments is reduced, (2) feed prices generally are lowered, and (3) the interregion price variation is reduced. All would be expected from introducing livestock feeding as an alternative to export in surplus regions and from the reduced relative importance of transport costs in livestock as compared with feed.

Snodgrass and French (16 and  $17^{13}$ ,) using (explicitly) a transportation model solved for a shipping pattern that minimized transport costs for milk among the 48 continental states, given 1953 supplies and

<sup>&</sup>lt;sup>13</sup> This article contains also references to preceding literature relating to transportation models.



<sup>(</sup>Source: Fox and Taueber (18).)

demands in each state.<sup>14</sup> Then a population increase was projected for each state to represent 1965 conditions. Projected increase in milk production was allocated to each state in proportion to its 1953 contribution to the 1953 total. The new transport cost-minimizing solution yielded results little different from those found for 1953. Michigan switched from a surplus to a deficit state. New Hampshire and Maine switched from deficit to surplus. Relatively more milk was shipped west and total transport costs increased, with increases in total milk shipped and the greater distances involved in shipment. The squared correlation coefficient for actual and equilibrium prices for 1953 is 0.44.

To illustrate the effects of market restrictions, the authors assumed an increase in transport costs imposed by the (importing) states of Rhode Island, New Hampshire, Florida, Alabama, Georgia, Connecticut, Massachusetts, North Carolina, Oregon, New Jersey, Virginia, Pennsylvania, California, and New York. The result was to change Michigan into an exporting state (shipping to New Jersey) and to attract supplies from new sources for Connecticut (from Vermont), for South Carolina (from Wisconsin) and for Virginia (from Wisconsin). The total transport bill increased by 9.5 percent, a considerable result from a relatively small change by a few states. The result would, in the "real world," be modified by production responses in the states imposing the market restrictions and by the chain of reactions set off thereby.

Adjustment implications are shown also by increments to cost from increasing production in each of the regions (states, for the models so far described). Considering milk in the aggregate, transport costs would be increased by more than \$3.00 were production to be increased by one "unit" (i.e., 10,000 hundredweight) in Minnesota, Wisconsin, Iowa, North Dakota, South Dakota, Nebraska, or Missouri. On the other hand, transport costs would be increased by less than \$1.00 were production to be increased by one unit in California, Nevada, Oregon, Florida, or Arizona. The former are surplus and the latter deficit states. Discontinuities in programming solutions limit these estimates, of course, to a range of adjustment with unknown limits. The only estimate that is certain is for one unit. Yet the effect of transport costs is evident.

As the milk product is disaggregated into its marketable components, differences in the cost increments diminish. To increase production of nonfat dry milk solids would increase transport costs by \$2.00 per unit only for two states (Minnesota and Wisconsin) while the increase would be less than \$1.00 also only in two states (New York and Vermont). Differences between high and low are reduced likewise for other components of milk, as the bulk of the product is reduced.

Snodgrass and French adapt the disaggregated models to estimate a regional distribution of processing facilities that would minimize the

<sup>&</sup>lt;sup>14</sup> Note that use of this model requires that within-region demand must be taken as given in terms of quantity. Thus the model does not allow for within-region consumption response to price change induced by inter-region shipments.

sum of transport and processing costs. For each region the processing cost of a milk component is added to transport cost that separates it from each of the other regions. The distribution of processing facilities (and hence product flow) is changed much as would be expected by an increase in transport cost. However, since the milk components are processed in different proportions in the different regions, other changes also occur. One of these is a further reduction in the differ-ences between regions<sup>15</sup> in cost increments consequent to increase of output. In this case the increase was in processing output of each of the various milk components.

In a final investigation, Snodgrass and French take into account the variation between states in feed and labor costs in milk production. A regional distribution of production minimizing the sum of transport, processing and these production costs yields the following results. All manufacturing milk is produced in Minnesota and Wisconsin, wherein no fluid milk is produced. Fluid milk is produced in only eight states:

- 1. New York, shipping to Massachusetts, Rhode Island, Connecticut, New Jersey, Pennsylvania, Delaware and Maryland
- 2. Iowa, shipping to Arkansas, Oklahoma, Colorado, Nebraska, Kansas and Texas
- 3. Ohio, shipping to Virginia and West Virginia
- 4. Indiana, shipping to Tennessee and Kentucky
- 5. Alabama, shipping to Florida and Georgia
- 6. South Carolina, shipping to North Carolina
- 7. California, shipping to Nevada and Arizona
  - 8. Utah, shipping to New Mexico

As the authors themselves noted, the conversion of Iowa into a surplus producer of fluid milk requires a heroic willingness to forget about opportunity costs imposed by production alternatives other than milk. These are imperfectly reflected in the production costs taken explicitly into account. Also, and closely related to this point, the intra-region heterogeneity would differ among regions so as to impose limits to dairy production that would vary if opportunity costs were taken explicitly into account.

In all models, consumption is taken as given. Hence the interregion reorganizations of production and/or processing assume no consequences in quantities sold within regions. Such a restriction is important. Given a change of transport costs from either a change in form of product or from including processing costs, one would expect a general decline in interregion shipment and an increase in interregion price

<sup>&</sup>lt;sup>15</sup> To facilitate computations, the regions were reduced to 24 in the adaptation.

variation. These models do not permit these results. The final demands in each region are taken as given so that the only change allowed is in the directions and relative quantities of interregion product flows.

With comparable demand assumptions, Heady and Egbert (9) have developed a model more elaborate from an empiric viewpoint than any so far reviewed. Among 104 United States Bureau of Census subregions they sought, with three models, an equilibrium allocation of land and land-related resources to feed wheat, food wheat, and nonwheat feed grains under demand conditions of 1954. In two of the models the equilibrium was solved for in minimizing the sum of regional costs. In the first, costs included, by region, the unit costs from labor, power, machine, seed, fertilizer and "related inputs." In the second, land rent was added. In each case the minimization was subject to two sets of restraints. The first consisted of a land restriction for each of the 104 subregions. The land supply was set for each subregion at the largest acreage used for these crops in the eight years prior to 1954. Two added restrictions were necessary to equate total annual production (for feed grains and for wheat, respectively) with total annual consumption.

In a third model, activity units for feed and food wheat and for feed grains were priced in each region according to a historically determined price relation, assuming that transportation costs were thus accounted for. Then the equilibrium was sought by maximizing a "revenue" function, comprised of the sum of price-quantity products in each region, summed over all 104 regions, subject to restrictions similar to those of the first model. The essential difference in this from the first model lies in the reflection of transport costs in product prices.

The results are as follows. In the model that minimized costs exclusively of land rents, all grains were withdrawn from such marginal areas as southeastern Colorado, eastern New Mexico, northern Utah, eastern Wyoming, southeastern Montana as well as fringe areas in Texas, Nebraska, Wisconsin, Michigan, Oklahoma, Missouri, Kansas, New York and certain areas in the southeast. When land rents were included, food wheat was restored to Montana and to southwest Missouri (from feed wheat). But all grains were withdrawn from the Oklahoma panhandle and from Pennsylvania.

When transport costs were taken into account, the changes were more dramatic. Food wheat was restricted largely to the Dakotas and to Minnesota and Wisconsin. Wheat areas of Nebraska and the Pacific northwest (including Montana) were shifted from food to feed wheat. Yet even here, it is interesting to note, no substantial change was found necessary for many large and stable areas producing wheat and feed grains.

All these models take into explicit account space ordered phenomena ordinarily abstracted from by models used to study agricultural adjustment. Hence they provide a view of agriculture that differs fundamentally from views provided by other models. The applications so far made are severely restricted by the use of extremely large regional aggregates (e.g., Fox); by sorely naive assumptions on demand and supply of product, in aggregate and in regional distribution (e.g., Snodgrass and French); and by fragmentation in terms of products demanded by computational complexities met in a model that would take account of inter-product as well as interregion relations (see comments by Heady and Egbert). Use of results from the models in suggesting action on adjustment problems must be conditioned further by the fact that they have a normative orientation. Limitations imposed by this property differ according to (1) the value judgments of the adviser and/or (2) how well the normative postulate(s) accord with actual behavior in the adjustment units.

#### **REGIONS AS ADJUSTMENT UNITS**

Unless we damage severely ordinary use of the term, we must ascribe to a region the spatial attribute of contiguity. When we speak therefore about the corn-belt region we denote a contiguous space north and east of the middle part of the United States below the western end of the Great Lakes. In contrast, the corn producing industry includes, in addition to parts of (most) farms in the corn belt that produce corn, parts of farms elsewhere that produce corn. The region designation is descriptive merely in terms of a high percentage of farms found within its boundaries. However, criteria for fixing the boundaries for regions are exceedingly vague.

Isard (11, chapter 1) describes the development of a region as a "nucleation," abetted by the gregariousness of human nature, and nurtured by economies provided by such an aggregation that are external to individual action systems. A restricted variety of such economies are external economies made available to firms by financing institutions, labor centers, service agencies, etc., that evolve in the process of "nucleation." As suggested by Isard, the aggregates can even, under certain conditions, develop new "decision foci" and hence new action systems. Further possibilities of economies are created. But as growth continues a retardation occurs in the rate at which the external economies are created. Indeed a cursory view suggests that a stage of external diseconomies has been reached for firms and consumers in many metropolitan areas. Dispersal then occurs subject to costs of transport, diminishing returns and the nonuniform spatial distribution of resources.

In a given stage of development, production differences among regions are partly summed up in the law of comparative advantage. Insofar as the law describes space-related differences in production, it is reflected in the functional relations of Figure 13.1. But Figure 13.1 reflects the combined effect of all determinants of excess supply within regions. Hence the "exportable surplus" of a given region is made to depend on the determinants of demand as well as those of supply for each of the various regions included in the model. Such determinants as are introduced in the models reviewed are restricted to per capita income and to population. While these are important, they by no means exhaust the relevant demand shifters within regions and hence shifters of excess supply of given regions.

Finally, when we view regions as adjustment units we note Isard's "decision foci." Instead of serving merely to reflect any space-related advantages apparent in (historical) data, opportunities for adjustment are created by the development of groups capable of aggregate action in behalf of regions. The results are expressed partly through external economies for firms already in an area by attraction of new firms into an area. Otherwise regional action may be expressed in such ways as regulation of selected aspects of product markets (e.g., market orders) or factor markets (e.g., zoning regulations). Production alternatives are varied, added restraints are imposed (or removed) and resource requirements are affected. Many of these changes are noneconomic or only partly economic in origin. All result from some type of group action and depend on the existence of some sort of region.

#### SUMMARY

In describing the studies selected above we already have suggested some of the properties of spatial equilibrium models that seem especially important in conditioning the interpretations permitted of their results. This discussion is summarized in the following four points.

1. For all applications it was assumed that the supply of product was fixed in the aggregate and for each of the regions among which equilibria were sought. In the aggregate the assumption seems well founded on an annual basis for the products so far studied. It is sensible to take as given, for example, a regional and total feed supply and to assume that neither will vary within a year in response to change in its price.

It may even be sensible to suppose the aggregate will not vary much next year in response to this year's price. However, it is heroic indeed to assume that the response for a given product will be zero next year - or even the same among regions. Supply elasticities will vary among regions for the many reasons cited in this conference. Especially important, however, are (a) the scale of farms producing the product, (b) the degree of specialization in its production and (c) products competing on the region's farms for resources required in its production.

It may be pointed out that the models estimated are "one-year" models. This is a time period particularly convenient for observation and analysis in agriculture. However, in interpreting results generated by the models, the adjustment-problem solver is led to draw implications for "next-year." He is little interested in "this year" except for its helpfulness in predicting (albeit with severely defined conditions) the course of events next year.

2. With respect to related products, the models applied were considerably less than satisfactory. They assumed, as a matter of fact, that for commodity x the elasticity of supply with respect to price of non-x is either zero (by ignoring all non-x) or is determined completely by a complement (e.g., the method of Fox and Taeuber). This point is closely related to the first and would not be especially bothersome except for the requirement that the cross elasticities be the same among regions. For most agricultural products, it is elementary to note that this is not likely to be a very good assumption.

Thus it seems necessary to take inter-product competition into account. The brief account by Heady and Egbert precluded a careful appraisal of their method and findings. We note with fascination the tremendous computational problem imposed by matrices "of the order  $104 \ge 316$ " even though only three products were taken into account in their essentially simple set of models.

3. As in any classification scheme, certain homogeneity properties are assumed for "regions." The point here is subtle in the use of spatial equilibrium models. When implications are drawn from equilibrium product flows or from the "marginal costs" of adjustments reported (e.g.) by Snodgrass and French, an implicit assumption is required for problem-solving purposes that the adjustment can proceed with comparable resource limits in each of the various regions.

A closely related comment might be injected here to indicate a possible direction of extension of the models. Aside from joint equilibria in feed and livestock, all resources were assumed immobile among regions (though perfectly mobile within regions!).

4. Transport costs were assumed either zero from the internal basing point to all points within a region or to yield a zero sum of deviations from a quantity-weighted mean within the region. Again, with a properly selected basing point, such an assumption may be a fairly good one. Yet it seems anomalous in a method that depends on the relative importance of transport costs as the basis for its use.

In conclusion we add that it would be easy at this stage to be negative regarding the application of spatial equilibrium models to studies of agricultural adjustment problems. We have no desire to be so interpreted. Indeed the applications so far made have been ingenious and have shed considerable quantitative light on an area heretofore reserved for (at best) qualitative analysis. To say that we need to refine the models is, therefore, hardly a negative criticism.

We add only one note of pessimism. It may be entirely possible that the agricultural economist is awakening to the quantitative importance of space-ordered comparative advantages at the very time that technological changes are reducing their importance relative to comparative advantages oriented to management differences and relative to advantages induced for regions by group action not captured in the relations so far included in the models.

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# Discussion

BAKER'S OBJECTIVE, as I interpret it, was to summarize and evaluate current regional and spatial equilibrium models appropriate for application to adjustment problems of midwestern agriculture. He further restricted his area by emphasizing interregional trade and production location problems, and by categorizing theories of trade over space and of location as essentially the same phenomenon. His general discussion of spatial equilibrium models draws almost exclusively on programming models as applied to problems of interregional trade. In these examples the differentiating variable factor was the transport rate based on an arbitrary "centralized" point, independent of volume or direction of shipment.

Baker presents a description and analysis of representative studies in the area. I believe he has systematically and thoroughly traced the evolutionary development of regional and spatial equilibrium models relating to trade in midwestern agriculture.

In the process of formulating a critique of this paper, I think we would, logically, attempt to evaluate (1) his review of the literature, including his recognition of the limitations of existing work in its application to the problem being discussed, (2) his view of the pertinent adjustment problems which are appropriately conceived at a regional level, (3) his selection of properties of the programming models for discussion, and (4) the extent and validity of his suggestions to improve existing models or to suggest alternate research approaches to adjustment problems.

I believe Baker has covered this assignment with his characteristic thoroughness. His detailed presentation and analysis of pertinent studies and the many bibliographical references provide an excellent background source for those interested in the application of regional and spatial models.