PART III

Supply Estimates
Derived from
Individual Farm Data
Chapter 7

Determinations of Supply Functions from Cost and Production Functions

This paper will explore some of the possibilities for using firm level production and cost functions as a basis for estimating agricultural product supply functions. It is further confined to certain aspects of this problem when cross section farm records and surveys are used as basic data for estimation.

It is assumed that the supply response of an industry may be looked upon as the aggregation of individual firm experience and action. Further, cross section estimates of sector or industry relationships are based on an assumption that individual units of observation are relatively homogeneous in certain respects, e.g., with respect to production functions. Compared with time-series analysis, the problem of estimation is one of accounting for spatial heterogeneity rather than changes or shifts in relations over time.

Since the production function is basic to both estimation of cost relationships and of supply functions, some of the theoretical production relationships and their implications for estimation of supply response or functions will be discussed first. The theory in this area is well-developed for static situations, i.e., the analysis abstracts from time as a variable. No attempt will be made to exhaustively present this static theory, but a sketch of some pertinent relationships is made as a basis for further discussion. This theory is presented mostly in the context of certainty of knowledge with respect to prices, quantities of the productive services of factors of production and their relationships to the forthcoming products. There is no well-known and well-developed body of theory in this area beyond this static level. The lack of such theory is, of course, a bottle neck to empirical research. The gaps on the theory side as listed by Bachman and Nerlove (1, pp. 3-4) are:

1. An adequate theory of aggregation of firm supply functions.
2. An adequate theory of behavior under uncertainty.
3. An adequate operational theory of investment for the firm, that is, an empirically useful theory of how so-called fixed factors are varied over time in response to economic forces.
4. A theory of, or at least techniques of measuring, the diffusion of technological changes and their specific effects on the production possibilities open to the firm.
THEORY

Single Product Situation

We suppose that a single product forthcoming in a given period is some function of the input of variable factor services given certain amounts of fixed factor services which may be considered as a group or technical unit. Variable and fixed designate services the use of which respectively affect a change in costs or do not affect costs as output is changed given the period in question (4, p. 12). Relationships between the product and factor services indicate the maximum amount of product forthcoming from any combination of factors (technically efficient production). Accordingly, with appropriate secondary conditions we can assert the economic theorem (12, p. 60) that in order for total cost to be a minimum for any

\[
\frac{\partial Y}{\partial X_1} = \frac{\partial Y}{\partial X_2} = \cdots = \frac{\partial Y}{\partial X_n} = \frac{1}{\lambda}
\]

where \( \frac{\partial Y}{\partial X_i} \) are the partial derivatives of the above mentioned production function and represent the marginal productivities of the variable factor services, \( X_i \). \( P_i \) represent the factor service prices.

Given output, marginal productivity of the last dollar input \( (1/\lambda) \) must be equal in every use. The combination of points of minimum cost for different levels of output is termed the expansion path. A firm that increases production in the least costly manner remains on its expansion path which, especially in the short run, may not be linear. Marginal cost is the addition to total cost brought about by increasing output by one unit while remaining on the expansion path. There is no reason why all firms should have the same marginal cost curves. In fact, it is expected that firms will generally have different short run cost curves. Lacking perfect knowledge in times past and present the firms have been and are being organized in various ways involving different combinations of fixed assets.

The marginal cost curve of the firm under conditions of perfect competition is looked upon as the supply curve of the firm. If marginal cost is above the average variable cost per unit of output the lowest cost at which the firm will offer a given quantity of product is the marginal cost of the corresponding output. Furthermore, if individual firms are ruled by the profit maximization motive the supply curve of the industry is the simple sum of the individual firm supply functions, other things being equal. The ceteris paribus conditions are (1) the firms do not affect their factor markets, i.e., the changes in quantities of factors demanded by the firms of the industry as a result of shifting levels of output do not affect the price of these factors or factor services to the
firms and (2) the number of firms in the industry is given. If the con-
ditions (1) and (2) are not in effect, the static marginal cost curves of
the firms will not sum to the industry supply curve.

In the event that the total demand of the industry for factors of pro-
duction affects the price of various factor services to the individual
firm, the flexibility of factor prices must be incorporated into any
method of aggregation of firm level production response to product
price change. If farmers have imperfect knowledge of price and pro-
duction relationships, or if farmers are under capital rationing pres-
sures, simple summation of marginal cost curves need not lead to in-
dustry supply functions. Similarly, technological change may lead to
unforeseen changes in firm behavior. If the profit maximization mo-
tives of the farmer are qualified by or in competition with household
goals, elements other than the marginal cost functions must also be
considered in aggregating the individual firm actions to obtain supply
response of the industry.

Multiple Product Situation

The theory of production and costs has been extended to include
firms producing more than one product (4). If the products are inde-
pendent in production, i.e., if the production of one product does not af-
flect the costs of producing any other and vice versa, the supply curves
of the products at the firm level may be considered as specified in the
same manner as for the single product firm. The production response
of each product may be considered separately in the case of independ-
ent production. However, there would probably be no incentive for pro-
duction of multiple products in such situations.

Also, in the joint product situation in which the products are forth-
coming in fixed proportion there is no difference between the single
product and joint product situation at the production level. The products
may be combined and considered as a single product for purposes of
analyzing cost and production functions.

The multiple product situation differs from the single product pro-
duction case if products are interdependent because of technical and/or
service price conditions. Technical interdependence occurs if the mar-
ginal productivity of one product is a function of the level of output of
another product and/or the levels of service inputs of other products.
If service prices are not constant the marginal cost of one product may
be affected by the level of service input of other products when cost is
assumed a function of levels of output. It may be noted that these inter-
dependent situations describe products which while independent at cer-
tain levels of production of the products may be interdependent at others.

1 If service prices are assumed constant, technical independence is specified. It is pos-
sible for products to be interdependent in production because of (a) technical interdepend-
ence and/or (b) service prices varying with levels of product output.
Even with constant service prices this is true if certain factor services are fixed. Such a possibility may have implications of importance to short-run and relatively short-run analysis.

A more complete development of the foregoing points may be found in the references cited (7, 11).

**ESTIMATION PROBLEM**

The estimation of supply functions from cross section data consists generally of (1) estimation of individual firm supply relationships and (2) aggregation of individual firm relationships into a supply function for the commodity in the industry or sector of the economy. Attempts to estimate firm supply relationships from cross section data may take either of two directions. In the first, underlying production functions may be estimated and then the firm's cost and supply functions are derived from the estimated production functions. In the second, supply relationships from the firm are obtained from cost relationships estimated directly from financial data of the individual producing units. The first approach has the advantage of being more general. That is, production functions, if sufficiently detailed and with inputs and outputs in physical units, may have alternative prices attached in order to analyze various cost and price situations. Theoretically, this approach permits one to arrive at a set of relations that will remain valid or change in known ways under a wider variety of circumstances than would an approach involving estimation of supply relationships directly from recorded cost data. The direct cost analysis approach has the advantage of requiring a type of data which is usually more easily acquired. Financial records are generally used for income tax filing and other purposes so the data can usually be had by survey or by simple record keeping of a type understood by farmers. However, cost relationships estimated directly from such records reflect specific cost and price situations and are not easily modified to account for changes. The direct cost analysis approach also leads to problems in the case of multiple product firms because of the difficulty of allocating certain fixed costs among the various enterprises.

**Single Product Firm Production Function Approach**

If the output of a single product is considered as a function of certain resource inputs, the conventional procedure of predicting the total output curve or surface as a regression equation may be followed. On the product side, the output may be measured in physical units or as a value product that is a constant multiple of the physical product. Except in controlled experimental situations, the number of possible variables on the input side is too large to permit working with all variables. Hence, researchers have aggregated the factor services (or investments
in factors) into categories on the basis of their being technical complements and near perfect substitutes (9). This categorization has led to the measurement of input categories such as machinery service in value terms. The specification of the input categories and the measurement of the appropriate variables raises the same problems of estimation considered in some detail in the literature concerned with the estimation of production functions for farm management or intra-firm purposes. As has been pointed out elsewhere (6), biases may result from failure to include important variables such as management service. As usual, the problem of multicollinearity will continue to plague researchers trying to obtain production function estimates from cross section data. These problems may be no greater in estimating supply relationships than in the usual production function analysis where they have caused considerable concern. However, few attempts have been made to carry production function estimation at the firm level to supply functions on an industry basis. The implications of these problems for aggregate level analysis are not yet spelled out.

As noted, discussion of production function estimation opens a Pandora's box. It is not the purpose of this paper to review these problems in detail. Most of them are well-known to researchers who have tried to estimate a production function. The articles cited offer possibilities for solutions to the problems at least in certain instances.

However, the use of cross section data gives rise to the suspicion that each observation may represent a point on a different production surface, especially since firms are found using different technologies. A possible solution under relatively short run conditions lies in the selection of the cross section samples. If strata are delineated in a manner to represent firms with common technologies and/or other factors likely to affect technology, separate production function estimates may be made for these situations. For example, firms may be classified into strata by size, age of operator, production practices such as use of milking parlors, etc. This stratification procedure, besides aiding in identification of the production functions, may be useful in reducing the biases caused by failure to include variables such as management. One intuitively feels that in such situations management and other nonincluded variables are more likely to cause randomly distributed disturbances. That is, the correlation between management factors and included variables is apt to be relatively less given the extent of the variation in these variables within the homogeneous strata. Among strata, management might be expected to vary in proportion to capital inputs, but when size of firm is specified, and hence the corresponding capital inputs are specified within limits, the correlation will ordinarily be lower.

If in the short run one can assume that the number of firms in each

\[ \text{A number of production function estimation problems were reviewed in Heady, Earl O., and others, eds. (3, 9.) Reference cited by a number of authors in this book may be used to further follow up the problem.} \]
strata will remain relatively constant, the firm supply relations may be estimated and subtotaled for the strata and then added to obtain the supply function of each other stratum. (It is assumed that appropriate sampling techniques are used within strata so that the resulting firm supply function for each stratum is representative for that stratum.)

More specifically, this procedure involves first estimating the outputs for various factor combinations within each stratum by the use of the production function. One generally uses the combinations of factors corresponding to points on the expansion path defined by the production function and expected factor service prices (values). The levels of certain factor service inputs would be fixed according to length of run considerations. Since the relationship of various levels of output to the combined (minimum) value of the factor services required to produce these outputs is the variable cost curve of the firm, the strata marginal costs are obtained by estimating with this function the additional cost that accompanies each added unit of output. Multiplying the successive output levels by the number of firms in each stratum and relating to the corresponding marginal costs leads to a stratum supply curve. To obtain the industry or sector supply curve, the strata output levels corresponding to the specified marginal cost situations are simply added together. An aggregation problem arises here.

It is, of course, likely that the firms as a group will influence factor prices by their combined action. Factor price flexibilities or the group output effect on prices may be taken into account in the single product model. As long as one assumes that no individual firm influences price perceptibly, it is only necessary that the price be specified for the industry or group of firms as a whole for each total (group) output level. Conceptually, one has only to set these prices at their expected level for the output in total and then maneuver the individual strata models to a point on the respective expansion paths that leads to the desired total output for all strata but keeps the marginal cost the same for all firms among strata.

As output levels for the group of firms is changed, marginal costs to the individual firm within strata shift, e.g., upward as suggested in Figure 7.1. OA represents the individual firm share of total output given the initial price and total output specification for the group of firms. OB specifies the individual firm share with a second specification and similarly for OC. It is the sum or aggregation of curves such as FG which becomes the supply curve of the industry or group of firms rather than the aggregation of marginal cost curves per se, as was the case when the factor service prices did not change.

It is possible that even when firms know their production cost relationships and are able to adjust outputs to maximize short run profits, they may choose not to do so. Various reasons are hypothesized for such action, e.g., influence of holding other goals than profit.

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3 For a mathematical derivation in the static case of the individual firm supply function from a Cobb-Douglas type production function, see Bachman and Nerlove (1), pp. 39–41.
maximization. If the suggestions for deriving a supply function are used, various adjustment factors might be devised. Similarly, the effects of risk and uncertainty modify any such analysis. Nevertheless, it is felt that until more general theories of firm behavior under risk and uncertainty and conflicting goals are devised, approaching supply analysis through static classic production functions may offer worthwhile insights to the researcher and policy maker. Differences between estimates made with the static model and the supply responses in reality may be analyzed or rationalized in the light of such factors as risk and uncertainty as a first step toward more adequate analysis of the effects of these factors.

In the long run, changes in the factors by which strata are specified may become of major interest. Technological change cannot be ignored as an important factor in supply analysis. Since there is no well-developed body of theory upon which to draw here the problems of estimation are going to be more difficult. One might suggest, as does Klein (10, pp. 236-40), that cross section data might be taken over time to form a time series of cross section data. Such a procedure might permit the introduction of such techniques as distributed lags (5) or possibly analysis of covariance (8) and other techniques. It must also be recognized that a related problem is the changing form of fixed assets within individual firm that takes place both with or without changing technology.
Elementary theory indicates that the supply curve for an industry is, under certain conditions, the sum of the individual firm marginal cost curves. Thus, it appears possible to estimate the cost curves directly from the cross section firm records rather than starting with the underlying production function. The differences between the cost-output positions of the cross section of firms as shown by appropriate financial records might be considered as data from which to estimate a total cost function. However, even when the firms operate with the same production functions, it is not necessary that they have the same marginal cost curves. Marginal costs depend upon the level and distribution of fixed costs, or rather upon the nature of the fixed factors and their levels within a firm, i.e., upon length of run under consideration. Some way of grouping the firms according to the nature of the marginal cost function is needed. In other words, homogeneous fixed plants must exist before a cross section would indicate points on the same variable cost structure.

If one aggregates all variable costs and estimates the function the costs are of output and fixed cost category levels, an assumption is made that farmers do combine resources to produce on the expansion path of the firm (at minimum cost per level of output). Otherwise, some hybrid cost curve is obtained which would be difficult to interpret meaningfully. In practice firms vary rather widely in their fixed asset structure. In production function estimation the differences in services flowing from such fixed factor situations were taken into account in the cross section estimate by considering such factors as variable from farm to farm and hence within the individual firm, perhaps on a long run basis. Since marginal cost functions will vary depending upon the fixed asset structure, one could either (1) stratify the data to obtain comparable fixed asset structures and estimate individual cost functions for each or (2) include more than one fixed cost category in the model, each with varying levels, just as was done with factor services in the production function model. In a competitive model the production function may theoretically have per unit values attached to the services and any one of the categories made a function of output and the other categories. Elementary theory indicates the total cost function as the inverse of the production function which has undergone a value transformation on the input side. This is the same as working with a production function in which value inputs are used. Hence, the detailed cost analysis approach comes back to the ordinary production function analysis approach.

Most agricultural products are produced on multiple product farms. Interest in the single product firm cases is somewhat limited. It has been presented in some detail to give an insight into the overall problem and because historically cross sectional estimation of production functions for multiple product agricultural firms has seldom proceeded beyond the use of single product variations.
Multiple Product Firm Production Function Approach

The multiple product case, while of most general interest, is unfortunately the most involved and difficult to handle. Some sort of interdependence of the products in production or marketing exists or there would be little incentive of firms to produce more than one product. The nature of this interdependence affects the possibilities and methods of estimation that must be used to discover the production functions and related supply response estimates. Although few attempts have been made in agriculture to connect production functions to supply analysis directly, most firm level production function estimates have been based on multiple product firm data. However, the interdependence problem has not been satisfactorily handled in general. Researchers have often tried to avoid the issue by either selecting firms as nearly homogeneous in their output complex as could be obtained and then aggregating the product outputs in terms of gross income, or they have divided the product outputs into several relatively homogeneous categories such as livestock and crops and then fitted separate production functions for these categories using various accounting procedures to allocate an appropriate share of the input services to each output category. Once output and input categories have been designated, estimation has proceeded as in the single product case, usually by least squares regression fits of an equation. The usual choice has been a variation of the Cobb-Douglas production function.

The use of independent estimates of the separate production function by enterprise is theoretically feasible, as pointed out by Beringer (3), if the production functions are technically independent. In practice, as noted before, separating the products in the multiproduct firm for the purpose of estimating individual production functions leads to accounting-allocation problems on the input side. It is difficult to determine how much of a feed floor service or building service to allocate to the production of each enterprise or product. On the other hand, treating the entire complex or portion of the products as a group leads to an index number problem.

An example of this problem can be illustrated. If two products, A and B, are aggregated into a single value product category, one can easily obtain biased estimates of the production coefficients, especially labor. For, if the cross section sample is one in which considerable substitution of the two products has occurred one (B) may be a larger user of labor than the other (A). The use of labor may be correlated with the substitution of this product (B) for the other (A). If prices of the products are such that product B leads to lower total value from the same resources, it is possible as B increases in the cross section data, other resources constant, that gross income decreases; i.e., those farms with other resources comparable but having more labor may tend to substitute B for A. A cross section estimate of the production function will often show negative labor coefficients in this case although the addition of a unit of labor in the production of either product may be
positive and the marginal value product of a unit of labor used in B
higher than the corresponding marginal value product in A.

Intra-firm analysis information of considerable interest can often
be obtained when this problem is avoided by limiting the analysis to
farms with relatively the same proportions of A and B in production.
In supply analysis it is the substitution effects among the products that
may be of major concern. The supply function of B cannot be con­sidered separately from possible substitution possibilities between A
and B.

A possible solution to such problems is suggested by Klein (10, pp.
226-36). He uses a production function model in which he includes the
various output categories. Some are designated as independent varia­bles for purposes of regression estimation. In the example he gives,
a Cobb-Douglas variation is used. As he points out, a theoretically
unacceptable relation between the products under pure competition, i.e.,
diminishing marginal rates of substitution, occurs with this model. He
suggests the possibility of other equations, indicating that he has tried
one which at least permits a constant marginal rate of substitution
among products.

Multiple Product Firm Direct Cost Analysis Approach

Again, a direct cost analysis approach might be tried. Total cost
functions may be derived in the multiple product case. Average cost
functions are not meaningful, but marginal cost function derivation is
possible. However, such costs are subject to the conditions implied by
the output levels of the other products. One might, therefore, set out to
estimate the total costs of a production process as a function of the out­
put levels of the various products. Suppose a cross section of account­
ing records supplies the basic data. Presuming that an appropriate
mathematical model has been chosen to represent the relationships in­
volved and that it has been successfully fitted, marginal costs of any
product given specific levels of others may be calculated as a first de­
rivative of cost with respect to output.

Assuming that the firms have the same cost structure given the
same factor prices, cross section estimates of the total cost function
are based upon inter-firm differences in cost and output. If all firms
were operating efficiently they should all be at the same point in their
cost relationships. Only a point estimate could be achieved. To identify
the cost structure, it must further happen that the firms differ with re­spect to output positions for some reason that does not prevent them
from having the same cost structure. If firms had and have different
expectations, such a situation might exist. Some firms might overpro­duce and others might underproduce relative to the “optimum level.”
Under these conditions inter-firm differences would reflect differences
in the total cost structure of a nature found within individual firms.
However, when estimating such cost functions from cross sectional data
one may not even then be approximating the desired cost function because many firms may have had ex ante expectations which led to factor service combinations other than on the expansion path. It would be difficult to identify the real cost functions.

Beside difficulties in meeting all the foregoing assumptions, cost curves developed from cross section cost data reflect a given set of cost conditions (factor service prices and production function). At best, short-run firm supply curves may be obtained by examining marginal cost of a particular product. Difficulties arise when analysis is to be made for a length of run which permits the factor service prices to vary as a function of the output levels.

The marginal cost curves for individual products can be specified only if the interrelationships of costs among the products are specified. It is difficult to assess the impact of changes in service prices upon these curves, since one assumes that the firms tend to operate upon the expansion paths designating service combinations for each product and also upon the expansion path indicating an optimum combination of products. Any cost complex is a shifting function of the underlying physical production function relations of substitution and the changing factor service prices. Hence, this approach has very definite limitations, especially beyond the short run.

The foregoing notes are not intended as a complete coverage of the problem under discussion, but it is hoped that they will serve as basis for productive discussion in this workshop.

REFERENCES


Discussion

KEHRBERG has done a commendable job of bringing together in a brief, concise manner the theory and methods of determining supply functions by means of production functions and cost analysis approaches. He has outlined procedures involved and discussed objectively both the strong points and the limitations of these approaches. I found no particular area in which to disagree with his analysis, but as I studied the paper one thing kept bothering me—how can this theory be adapted to an actual farm supply response study?

Kehrberg's paper is based on static, profit maximizing, equilibrium theory in which the firms have virtually complete knowledge. This he acknowledges. It therefore provides a good starting point in conceptualizing the problem, but I was a little disappointed that he did not try to bridge the gap between profit maximization theory under complete knowledge and the more realistic decision making behavior under uncertainty conditions that must be resolved in empirical farm supply response studies. I realize that this is no slight task. Perhaps I should have been satisfied with the contribution Kehrberg made. (I feel it definitely was a contribution.) But any exploration into this area, no matter how fragmentary, would have been of considerable value as meat for discussion in this workshop. It must be remembered that we are dealing with farmers who do not have complete knowledge and are not necessarily motivated by the usual concept of the profit maximizing goal.

In the multiple product firm, Kehrberg indicates that unless there was product interdependency because of technical and/or service price conditions "there would probably be no incentive for production of multiple products." His assumption of complete knowledge rules out multiple product organization as a means of combating uncertainty (income security maximizing goal for example). However, he ignores the resource adaptability limitation (for example, land characteristics which prevent producing continuous corn in certain areas) that would necessitate a multiple product organization or idle resources, even with perfect knowledge.

I agree completely with Kehrberg that it is necessary to classify farms into strata based on size of business, "level" or combinations of...
technology, or other criteria to aggregate individual supply functions. This is true, I believe, no matter how the data for estimations are collected. I would like to suggest another set of criteria for stratification, or at least point out situations under which farmers would possibly react differently to price changes.

Kehrberg's paper, and most of the others I have examined, dwell primarily on the situation in which the farmer is now producing the designated commodity or commodities in approximately the amount and proportions he has in the past and will for a certain period in the future (if product and factor prices remain constant). But what about the farmers who are in the process of expanding the production of a certain commodity, or those in the process of cutting back? It is certainly conceivable that their supply functions would be considerably different from those of farmers in a more or less equilibrium position. We also need to determine how much production is called forth by designated price increases from producers not currently producing the commodity, as well as the converse. The input structure, particularly investments, affecting the ease of getting into and out of production of a certain commodity is a major consideration in this regard. There is, of course, wide variation among commodities. Technological changes have probably caused a considerable decrease in the number of "inners andouters," particularly for certain livestock enterprises, but it is still a factor. Probably more important than the "inners and outers" in this entire question of noncurrent producers are the interregional aspects—the possibility of entering the production of a given commodity.

Still another consideration in this stratification process concerns the level of production in relation to "possessed" capital resources. I believe different supply responses exist, even assuming that farmers are combining resources in the manner that will maximize profit if (1) expansion requires no additional investment, (2) additional investment can be accomplished using accumulated capital, and (3) additional investment requires expanded use of credit.

The discussion on the relative merits and weaknesses of using a production function versus a direct cost analysis approach is very well presented. The inadequacies of the records kept by many farmers for use in cost estimation and allocation cannot be overstressed. Differences in farmer evaluations of noncash costs is a major hurdle to be considered.

One other point that might be open for question is the effect of stratification on management inputs. This would depend, among other things, on the basis of stratification selected.

There is very little question but that Kehrberg fulfilled the objectives outlined in his opening paragraphs. My only regret is that he did not try to penetrate the area of developing production and supply functions under conditions of less than perfect knowledge.