Chapter 6

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Supply Functions Estimated in Demand Studies

SINCE THE TITLE of this paper may appear puzzling, we shall try to justify it, at least in terms of definitions, before proceeding to the principal topic: the usefulness of such equations in studying supply.

DEMAND STUDIES

For a long time, the individual consumer has been postulated to regulate his consumption habits according to some system of ordered preferences so that his demand per unit of time for some particular one of the n commodities available could be expressed as a function of the prices of the n commodities and his "income."

(1) $$D_i = D_i\ (p_1, p_2, \cdots, p_n, I)$$

Under more restrictive assumptions it has been usual to assume that the market demand resulting from the aggregation of such functions is a stable and meaningful relationship in the structure of the economy.

These functions are the principal object of estimation in demand studies.

(2) $$Q_i = Q_i\ (p_1, p_2, \cdots, p_n, y)$$

At an earlier time when these functions were universally estimated from time-series or cross-sectional data, by minimizing the sum of squared deviations of computed quantities from observed quantities during some "sample" or observation period, classification of studies as to whether or not they were demand studies was relatively simple. Although some studies may have considered such questions as how much of the price change from one period to another was due to changes in demand and how much to changes in supply, the estimating phases of the work could be segregated into those devoted to the estimation of the demand functions, and those devoted to the estimation of the supply functions.

The advent (c. 1943) of "simultaneous equations methods" has made such classifications difficult. Titles such as "Econometric Analysis of Supply and Demand," "Statistical Analysis of the Feed-Grain Economy," or "Estimating Elasticities of Supply and Demand" are more common. These titles, although more or less hypothetical, emphasize the problem of trying to define demand studies in such a way that we may speak meaningfully of "supply functions estimated in demand studies." If the estimating techniques are such that a supply function is estimated, the study is likely to have a symmetry in the treatment of supply and demand so that the authors would not be willing to call it a demand study.

We should probably emphasize that the reference to the advent of simultaneous equations methods does not mean that students of demand were unaware of the supply function up to that time. Even the pioneers (H. L. Moore, E. J. Working, H. Schultz, et al.) were aware of the essential role of supply response in generating the observed data, and their treatment of demand may not have suffered so much from "blind analogy" to experimental situations as has been alleged.

THE ROLE OF "OTHER EQUATIONS" IN DEMAND STUDIES

Demand studies intended to estimate the parameters of equation 2 have been criticized because they do not always take explicit precautions to prevent the resulting estimates from reflecting the shape of the remainder of the system which generates the observations.

This argument may be illustrated by reference to a simplified model.

(3) $$Q = a_1 + b_1 P + u$$

(4) $$Q = a_2 + b_2 P + v$$

In estimating the least squares relation between quantity and price, it is not clear (unless additional information is available) whether the regression coefficient will approximate b_1 or b_2 or some combination of the two. This information may come in the form of knowledge or assumptions about the variables u and v which are assumed to be unobservable. The impact of assumptions in this area may be assessed from the relation which shows the expected value of this regression coefficient as a function of the parameters of the model and the distribution of u and v.

(5) $$E\left[b_{qp}\right] = \frac{b_1\sigma_v^2 - (b_1 + b_2)\sigma_{uv} + b_2\sigma_u^2}{\sigma_v^2 - 2\sigma_{uv} + \sigma_u^2}$$

Thus, information indicating that $\frac{\sigma_u^2}{\sigma_v^2}$ is small will assure that the least-squares regression coefficient will approximate the slope of the demand curve. (If $\frac{\sigma_v^2}{\sigma_u^2}$ is small it will approximate the slope of the supply curve.) As long as σ_u^2 is not zero, the estimates will be subject to what has been called <u>Haavelmo bias.</u> It is usually called <u>least-squares bias.</u>

If, in this model, equation 3 is replaced by

$$Q = a_1 + b_1 P + C_1 Y + u' \tag{6}$$

it can be shown (assuming $\sigma_{yu'} = \sigma_{yv} = 0$) that

$$E\left[b_{qp\cdot y}\right] = \frac{b_1\sigma_v^2 - (b_1 + b_2)\,\sigma_{uv} + b_{2u'}^2}{\sigma_v^2 - 2\sigma_{u'v} + \sigma\sigma_{u'}^2}. \tag{7}$$

This is the same as equation 5 with u replaced throughout by u′.

Even though the ratio $\frac{\sigma_u^2}{\sigma_v^2}$ may not be small enough to cause b_{qp} to be a useful estimate of b_1, it may be that the ratio $\frac{\sigma_{u'}^2}{\sigma_v^2}$ will be small enough to make $b_{qp\cdot y}$ a useful estimate of b_1.

Thus, even in the absence of simultaneous equations methods, the assumed presence of a varying supply function plays a fundamental role in the model for estimating the parameters of the demand function. This must reflect upon the accuracy of the usual designation of demand models fitted by the method of least-squares as uni-equational complete models. Although the estimating procedure is dependent only upon assumptions and observations made upon variables of a single equation, the "other equations" of the model must be assumed to justify these assumptions.

A more usual exposition of the role of the other equations may be illustrated by the following statements which refer to the model consisting of equation 6 and equation 4.

If we are unwilling to make any assumptions about $\frac{\sigma_{u'}^2}{\sigma_v^2}$, but are willing to assume $\sigma_{yu'}$ and σ_{yv} are zero, we cannot fit equation 6 by least-squares because of equation 7. These assumptions will, however, justify fitting.

$$Q = a_{qy} + b_{qy}Y + W_{qy} \tag{8}$$

$$P = a_{py} + b_{py}Y + W_{py} \tag{9}$$

These two planes intersect in a line which also lies in the plane defined by equation 6 and in the plane defined by equation 4. If neither b_1 or b_2 is zero these planes are distinct from those defined by equations 8 and 9. There are, however, an infinite number of planes which contain this line, and we must ask whether there is a way to distinguish equation 4 and equation 6 among this family of planes. Equation 4 can be distinguished. It is the one plane which contains this line and is also parallel to the Y axis. No such restriction holds for equation 6 so that in this model it cannot be distinguished. Thus equation 6 cannot be identified in any formal sense.

These simple examples may serve to illuminate the role of "other equations" in demand studies. This role is usually expressed in the general case as:

> A parameter estimate cannot be identified unless the number of variables effectively present in the system, but absent from the equation in which the parameter is estimated, is at least equal to one less than the number of endogenous variables present in the system.
>
> A parameter estimate can be identified if the coefficients of these excluded variables in the other equations of the system form a matrix of that rank.

The role of the "other" equations is to identify the demand function, and to make possible the estimation of relations in which untenable or unsupportable assumptions seem required for estimates analogous to equation 7.

DEMAND STUDIES AND ECONOMETRIC STUDIES

Reference has already been made to the difficulties of classification. The finished product may be formally the same whether the principle objective of the research is to estimate the functions (2) or whether it is to estimate functions of some other sort. The estimates of any one relation depend upon the structure of the other relations.

The distinguishing characteristics may be the structures permitted in various equations, but I should not like to be adamant on this point. Some illustrations may serve to indicate the differences in structural precision attempted in the various equations.

In 1953 Fox (1) estimated the demand and supply for pork as

$$p = -1.14q + .90y$$

$$q = -0.07p + .77z$$

p = price of pork
q = consumption of pork
y = consumer income
z = the estimate of production that would be arrived at based on predetermined variables alone.

Since the coefficient of p is not significantly different from zero, Fox concludes that this demand function can be fitted by least-squares. It should be noted, however, that this short-run supply elasticity of zero would not be very helpful in appraising supply response and that the structure of the variable z, which is not specified in this exposition, contains all the information on factors which affect supply.

Ladd and Tedford (3) fitted the demand equation in the model:

$$q_t = a + a_0P_t + a_1P_{at} + b_0y_t + b_1y_{at} + u_1t$$
$$q_t = c + c_0P_t + c_1X_t + u_2t$$

q_t = log per capita meat consumption, time t
P_t = log average retail meat prices, deflated, time t
P_{at} = log average retail meat prices for n (=3, 5 or 9) time periods preceding t
Y_t = log per capita disposable income, time t
Y_{at} = log average per capital income for n (=3, 5 or 9) time periods preceding t
X_t = log meat marketing charges, deflated, time t.

Here we have a relatively elaborate formulation of the demand function to accommodate dynamic characteristics of demand with respect to both price and income. The supply equation was not fitted, but suppose it had been. It is not reasonable to assume that the quantity of meat supplied to stores depends upon retail prices and marketing margins except to the extent that this is a behavior equation for the marketing sector. If it is, then one must wonder why X_t is treated as exogenous. This is a demand study!

An older example, also concerned with the demand for meat, is reported in an appendix to Demand for Meat by E. J. Working (4).

Demand $X_1 = 157.462 - .9135X_2 + .5998X_3 - .5217Y_0 + .0706Y_7 + u_1$

Supply $X_1 = 28.675 + .0356X_2 + .7392Y_4 + .0914Y_6 + u_2$

Income $X_3 = 55.324 + .2049Y_5 + .3033Y_6 + .3158Y_7 + u_3$

X_1 = quantity of meat demanded per capita in a given year
X_2 = real retail price of meat in a given year
X_3 = real disposable personal income per capita in a given year
Y_4 = production of meat per capita
Y_5 = real investment expenditures
Y_6 = time
Y_7 = lagged real disposable personal income per capita.

This study used quantity produced as a predetermined variable in the supply at retail and hence also leaves the question of why these amounts were produced out of consideration.

All of these examples had both expository and empirical objectives so that they should not be regarded as examples of what can be done in demand analysis.

Judge (2) fitted the first and fourth equations in a 12-equation model of demand and supply relationships for eggs. The equations of the model are:

1. The demand for eggs at retail;
2. The supply function for eggs at retail;
3. Demand for eggs by the commercial sector;
4. Supply of eggs by farmers;
5. Demand for meat in the retail market;
6. Supply function for meat in the retail market;
7. Demand for meat by the commercial sector;
8. Supply of meat by producers;
9. Demand for food other than meat and eggs at retail;
10. Supply of other food at retail;
11. Demand for other food by the commercial sector;
12. Supply of other food by farmers.

Is this a demand study? It is certainly not subject to the adverse comments made about some of the other supply estimates. The supply is elaborated even more than the demand in discussing the model.

The supply of eggs by farmers is apparently the equation most closely related to supply response in the usual sense since this is where the "production," as contrasted to "marketing," activities take place.

Let us, therefore, look at the structure of this equation. The following supply equations were obtained by Judge. Since the data used were in logarithms, the coefficients are in terms of elasticities:

(a) Limited information:

$$y_6 = 1.17y_5 + .54y_7 + .002z_3 + .23z_4 - .97z_5 - .78z_6 + 1.67,$$

(b) Reduced form:

$$y_6 = .36y_5 + .54y_7 - .01z_3 + .30z_4 - .44z_5 - .54z_6 + 1.62,$$

(c) Single equation:

$$y_6 = .19y_5 + .01y_7 - .07z_3 + .53z_4 + .01z_5 - .40z_6 + 1.30,$$

Where:

y_6 = index of per capita supply of eggs by farmers;
y_5 = index of prices paid to farmers for eggs;
y_7 = index of prices paid to farmers for meat;
z_3 = time with the origin at 1920;
z_4 = index of prices paid to farmers for eggs lagged one year;
z_5 = index of the cost of the poultry ration;
z_6 = index of the cost of the poultry ration lagged one year.

The author concludes that the limited information estimates are "reasonable" and the signs are "consistent with the economic theory of the firm."

Table 6.1 presents equations (a) and (b) in a way intended to emphasize the effect of eliminating some variables from the other equations.

Table 6.1. Comparison of Limited Information and Reduced-form Estimates of Supply Elasticities

Elasticity of supply with respect to	Limited information (equation a)	Reduced-form (equation b)
Egg price	1.17	.36
Livestock price	.54	.54
Egg price lagged	.23	.30
Feed cost	-.97	-.44
Feed cost lagged	-.78	-.54
Time	-.002	-.01

These have been several examples of supply functions estimated in demand studies or studies in which the study of demand is at least as important as the study of supply. They are included, not as being typical, but only as illustrations.

Limitations or possible limitations include:

1. Inadequate specification of the supply function, resulting in estimates which may be useless for supply studies and the improper specification of the supply function may have serious implications for the demand functions to be estimated.
2. Sector aggregate or average orientation contrasted to firm orientation which is appropriate for some objectives of "supply research."
3. The use of "consistent" estimating procedures in lieu of approximate identification and least-squares may have led to an inappropriate distribution of effort between the formulation and computation phases of research.

In supply studies for policy purposes sector aggregate orientation may be appropriate. Even in these cases, the econometric approach (or demand study) has not been as helpful as one could wish because of the low state of information on the factors which determine supply response and their applicability. Most people who work with these functions will probably say that their supply functions turned out to be less "reliable" or less "reasonable" than the demand functions.

We continue to hope that synthetic supply estimates generated from engineering and accounting data may replace statistical supply functions in demand studies. This hope may be held more strongly because of ignorance of the pitfalls along that road.

REFERENCES

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Discussion

WEST DIRECTED his paper toward an evaluation of the usefulness of systems of equations involving estimates of both supply and demand functions. To achieve this end, he found it necessary to differentiate between "demand" and "demand-supply" studies. Essentially, the criterion used to differentiate between such studies was the type of estimating technique employed. He writes: "If the estimating techniques are such that a supply function is estimated, the study is likely to have a symmetry in the treatment of supply and demand so that the authors would not be willing to call it a demand study." Presumably, if the estimating techniques do not result in the estimation of a supply function, the research would be considered a demand study. While this classification is useful for West's purpose of pointing out the interrelationships between demand and supply studies, the more general application of this classification is questionable. If there should be substantive reason for classifying supply and demand studies, it would appear more reasonable to do so on the basis of their primary orientation or objectives rather than on the basis of empirical technique. If the primary objective of a study was the estimation of a demand function, it would be found desirable (for identification or other purposes) to employ a model involving a supply function and it would seem logical to identify it as a "demand" study. Parallel but opposite cases would be identified as "supply" studies. Those with the dual objective of estimating both demand and supply functions would be identified as "demand - supply" studies. This would avoid the difficulty of classifying demand (supply) studies in which the supply (demand) function is not empirically estimated but rather assumed or taken as given.

West's paper contains one rather subtle point which appears to be of extreme importance to a group of researchers working in the supply response area. This point is the necessity of explicitly recognizing the demand factor when attempting to estimate supply functions. Apparently, some early attempts to estimate demand functions were not completely satisfactory because the "supply blade of the scissors" was not adequately incorporated into the models used. In attempts to come up with more satisfactory demand estimates, researchers have moved in the direction of more elaborate models which take this factor into account. The similarities between problems encountered in estimating supply functions and demand functions suggest that researchers on the supply side can profit greatly from the lessons learned by their predecessors working on demand studies. One of the most important of these lessons is that every scissors has two blades and that a description of one blade tends to be meaningless unless it involves the relationship of the pair. West has made a worthwhile contribution by calling this fairly obvious, yet easy to overlook, notion to the attention of this conference.

In his evaluation of supply functions estimated in demand studies, West indicates that such functions may be limited in usefulness because of inadequate or improper specification of the supply function, their aggregate nature and an inappropriate distribution of effort between formulation and computation. He does not elucidate on these possible limitations but leaves one with the impression that he is fairly pessimistic about the usefulness of supply functions estimated in demand studies. It would have been helpful if West would have given us his thoughts concerning (a) the kinds of problems upon which such supply function estimates might shed light, (b) more specific ways in which the usefulness of such estimates might be increased and (c) additional pitfalls which have been encountered in demand studies which might be wise to circumvent in supply studies.