Chapter 5

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Recursive Programming and Supply Prediction*

FOR ANALYSIS AND PREDICTION of aggregative production, it seems evident from past theoretical and empirical research that at least the following interrelated categories must be considered:

1. The interdependence of outputs using common inputs;
2. Technological change;
3. Planned or programmed policy actions;
4. Changes in both acreage and yield components in field crop production;
5. Uncertainty;
6. Demand, supply, and price interactions;
7. Adjustment over time;
8. The aggregate supply of production inputs;
9. Rates of investment in factors fixed in the short run; and
10. Regional specialization and competition.

Econometricians, in their use of multiple regression and simultaneous equation techniques have made considerable progress in accommodating variables and relations which reflect interdependencies among these phenomena. Yet there are certain fundamental difficulties in these techniques which send one in search of different, more suitable methods. This paper is an account of such a quest.

*The research on which this paper is based was begun while the author was research assistant at the Harvard Economic Research Project. It was continued while he was Teaching Fellow in the Economics Department at Harvard University and later while a member of the staff of the Farm Economics Research Division, ARS. At this writing, the author is on military leave from the latter organization. The specific contents of this paper have profited particularly from the comments of Professors James M. Henderson, Louis Lefeber, and Wassily W. Leontief, all of Harvard University; Hendrick S. Houthakker, Stanford University; and Dr. Glen T. Barton, Farm Economics Research Division, ARS.

SUPPLY EQUATIONS

Perhaps the simplest of all field-crop supply models is an equation which relates the acreage of a crop in a given year to its own price lagged one year. The simplest form in which this equation can be expressed is

$$X(t) = \alpha\ p(t-1)\ . \tag{1}$$

Suppose an acreage allotment is imposed on the crop. This specifies a constraint on the acreage of the crop which can be expressed

$$X(t) \leq a(t)\ , \tag{2}$$

in which a(t) is the acreage allotment in the year t. If $\alpha\ P(t-1) > a(t)$, i.e., if the acreage predicted by relation 1 is greater than the allotment, the two relations are inconsistent. This inconsistency can be removed if relation 1 is made into an inequality like that of relation 2. However, the system is now underdetermined.

The manipulation of instrumental or policy variables is not the only cause for the intrusion of inequalities in supply systems. A more fundamental cause consists of constraints on output arising from factors of production fixed in the short or long run. An example of the latter is an over-all land constraint in a developed region or country.

Suppose only two field crops are grown in a developed region whose acreages are $X_1(t)$ and $X_2(t)$, respectively. Assuming a lagged price supply relation as before, but including the competing crop's price as well as its own, a typical supply system might be written

$$\begin{aligned} X_1(t) &= \alpha_1\ p_1(t-1) + \alpha_2\ p_2(t-1) \\ X_2(t) &= \beta_1\ p_1(t-1) + \beta_2\ p_2(t-1)\ . \end{aligned} \tag{3}$$

On the basis of economic theory, it would be expected that $\alpha_1 > o$, $\alpha_2 < o$, $\beta_1 < o$, $\beta_2 > o$, i.e., that acreage increases with an increase in <u>own</u> price while it decreases with increases in a competitor's price.

The overall land constraint assumed for this example is

$$X_1(t) + X_2(t) \leq \bar{x}\ . \tag{4}$$

But now a situation analogous to the first example arises. Only if equation 4 holds as a strict inequality — i.e., only if part of the land is idle — can both equations of relation 3 hold. Again, the over-determinancy of the supply system could be avoided by making inequalities of the acreage lagged price relations.

If this is done, however, the model is underdetermined as before. Some kind of mechanism must be added if one is to decide in a meaningful way which of the two supply equations holds whenever the over-all land constraint holds. The mechanism which will resolve problems of this kind is the optimizing principle of economics. Rather than trying to force it on supply relations like those of the examples listed, it would seem to be more appropriate to follow the theory of production and to use it to derive supply relations from the underlying technical structure of production.

The suggestion that this principle be applied to predictive problems of supply is a little foreign to usual practice. Ordinarily, one attempts to estimate aggregative supply relations themselves without explicit reference to production structures and their choice mechanism. Even when this is done, the optimizing principle plays a role in the evaluation of the results. Thus, it is by means of this principle that one arrives at the conclusion that the response to "own price" will be positive while that to a competing commodity will be negative (7). Consequently, the explicit application of optimization is not as radical an innovation for supply response as it may at first appear.

The important problem is not whether it should be used but rather how it can be used without grossly misrepresenting the simple decision processes governing farm behavior. The attempt to solve this problem leads to a synthesis of time-series analysis and linear programming versions of production theory. It is to such a synthesis that the rest of this paper is devoted. We shall call it recursive programming.

A SIMPLE RECURSIVE PROGRAMMING MODEL: FLEXIBILITY IN CHANGING OUTPUT PATTERNS

An important application of linear programming to the problem of aggregative supply prediction is due to Professor James M. Henderson (3). The ingenuous innovation on which it rests is the specification of what we shall call flexibility constraints. These constraints specify that in any one year only a limited change from the preceding year's production can be expected. This hypothesis is based on the conglomerate of forces which lead to caution by farmers in altering established production patterns. Primary among them are uncertainty of price and yield expectations and restriction on the aggregative supply of production inputs. In short, they are the same factors which underpin Nerlove's adjustment equations (8). During this discussion, we shall split off the factors whose capacities are fixed in the short run for separate treatment. At this point, it will be supposed that the flexibility coefficients contain them as components.

The flexibility constraints can be expressed in dynamic notation as follows:

$$
\begin{array}{lrcl}
 & X_1(t) \qquad\qquad & \leq & (1+\bar{\beta}_1)\, X_1(t-1) \\
 & X_2(t) & \leq & (1+\bar{\beta}_2)\, X_2(t-1) \\
(5) & -X_1(t) \qquad\qquad & \leq & -(1-\beta_1)\, X_1(t-1) \\
 & -X_2(t) & \leq & -(1-\beta_2)\, X_2(t-1)
\end{array}
$$

in which $X_1(t)$ and $X_2(t)$ have the same meaning as before and in which we shall call the β's flexibility coefficients.

The first equation of relation 5 asserts that the acreage of the first crop will not exceed the previous year's acreage plus some proportion of its determined by the upper flexibility coefficient $\bar{\beta}_1$. Equation 3 of relation 5 asserts that the acreage of the first crop must not be less than an amount determined by the lower flexibility coefficient, β , and the preceding year's acreage. Equations 2 and 4 of relation 5 have the same meanings, respectively, for the second crop. (This example follows the preceding one, assuming that two crops only are grown in the region or country in question.)

The over-all land constraint (relation 4) should also apply here, further limiting the possibilities for change. Together with the inequations of relation 5 this gives a total of five constraints on change in output patterns. These five constraints form a system of linear nonhomogeneous difference inequations.

Now let $\pi_1'(t)$ and $\pi_2'(t)$ be the expected per acre net returns to the first and second crops, respectively. The system consisting of relations 4 and 5 can be resolved by an appropriate application of the optimizing principle, thus

$$\text{(6)} \qquad \text{maximize } \{\pi_1'(t)\ X_1(t) + \pi_2'(t)\ X_2(t)\}$$

subject to relations 4 and 5. That is, choose $X_1(t)$ and $X_2(t)$ so that total net returns are as great as caution and fixed factors will allow. The flexibility constraints are now seen to enclose the profit motive in a web of dynamic adjustment.

A fundamental theorem asserts that the solution to a linear programming problem is such that the number of constraints which hold as equalities is just equal to the number of nonzero variables. Translated into recursive programming language, this means that a supply system is governed by exactly as many dynamic equations as there are positive variables selected by the optimizing principle.

In our example, at least two variables must be positive because of the lower bounds, relations 2 and 4 of equation 5. As there are only two variables in the system for time t, we know that two equations will govern the behavior of the system over time. Which two there will be for any time period will depend upon which is greater, $\pi_1'(t)$ or $\pi_2'(t)$, and upon the relative magnitudes of the five constraints.

To actually obtain the solution, we must begin at a base period t=o. The initial conditions are then X(o). Then as the $\pi_1'(t)$ and $\pi_2'(t)$ are formed (exogeneously so far), a linear programming problem becomes available for each period that can be solved by the usual techniques.

A change in the equations which "govern" the system is called a phase change and the period of time during which the same equations hold a phase. The operation of this system over time will tend, in general, to exhibit multiple phases (6). During a given phase, simple first-order difference equations will determine the time paths of acreage.

The solution to such an equation is

$$y(t) = \lambda^{(t-t_j)}\, y^{(t_j)} \tag{7}$$

in which $y^{(t_j)}$ is the value of y(t) holding in the time period just prior to a phase change.

To visualize how time paths of acreage might appear, suppose that with each new year, the first crop is expected to be the more profitable ($\pi_1'(t) > \pi_2'(t)$,), and that net returns from both crops are positive. Suppose also that the acreage of the first crop is much smaller than that of the second, and that there is some idle land.

The following phases are a possible outcome.[1]

$$\begin{array}{ll} \text{Phase I} & \\ X_1(t) = (1+\bar{\beta}_1)^t\, X_1(o) & (t=1,\ldots,t_1) \\ X_2(t) = (1+\bar{\beta}_2)^t\, X_2(o) & \\ \text{Phase II} & \\ X_1(t) = (1+\bar{\beta}_1)^t\, X_1(t_1) & (t=t_1+1,\ldots,t_2) \\ X_2(t) = \bar{X} - X_1(t) & \\ \text{Phase III} & \\ X_1(t) = \bar{X} - X_2(t_2) & (t=t_2+1,\ldots) \\ X_2(t) = (1-\beta_2)^t\, X_2(t_2) & \end{array} \tag{8}$$

A graphic representation of these phases is shown in Figure 5.1.

In phase I, the acreage of idle land is sufficient to allow both crops to increase at the maximum rate allowed by caution and growth in the aggregate supply of factors. In phase II, the over-all land constraints render inconsistent the maximal growth of both crops, and the less profitable crop merely takes up the slack. Finally, in phase III, the maximal rate of growth demands that more land be released from crop 2 than farmers are willing to release, so that the maximal abandonment rate for the relatively unprofitable alternative dominates supply response.

A linear program has a dual solution, as well as the primal solution discussed above. The dual variables express (in this example) the marginal net revenue productivities of unit changes in the constraints. Call $\rho_i(t)$, $i=\bar{X}, \bar{\beta}_1, \bar{\beta}_2, \beta_1, \beta_2$, the dual variables for land, the upper flexibility and lower flexibility constraints, respectively. The dual results are

[1] In general, the phases will depend upon net returns, the initial conditions, and the flexibility coefficients.

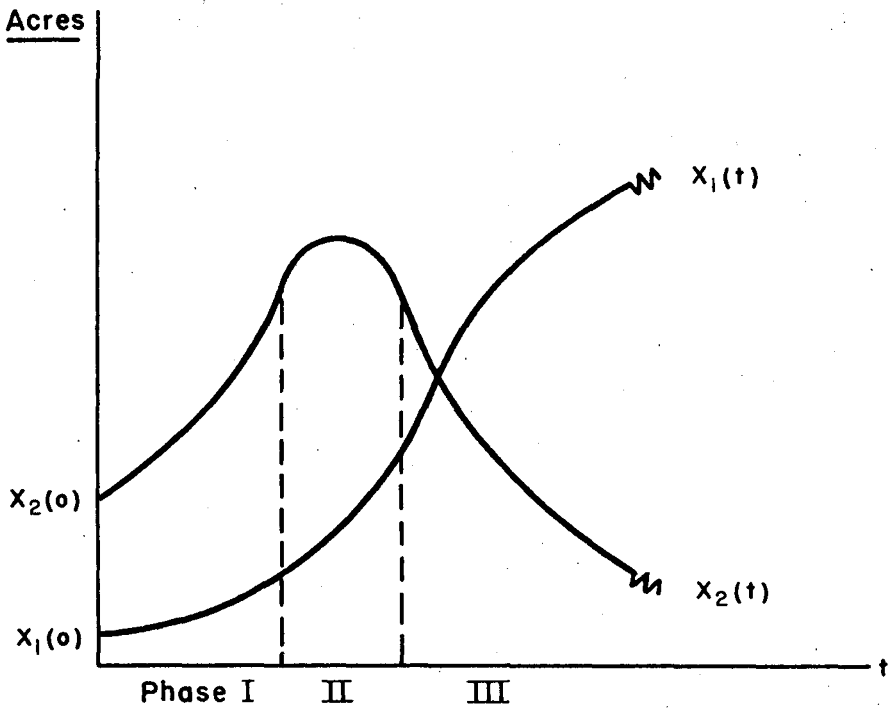

Figure 5.1. Acreages of both crops expand geometrically until land supplies are exhausted. After this, willingness of farmers to specialize is governed by the upper bound on the first crop (in Phase II) and the lower bound on the second crop (in Phase III).

(9)

Phase		I	II	III
$\rho_{\bar{x}}(t)$	=	o	$\pi_2'(t)$	$\pi_1'(t)$
$\rho\bar{\beta}_1(t)$	=	$\pi_1'(t)$	$\pi_1'(t) - \pi_2'(t)$	o
$\rho\bar{\beta}_2(t)$	=	$\pi_2'(t)$	0	o
$\rho\beta_1(t)$	=	o	o	o
$\rho\beta_2(t)$	=	o	o	$\pi_1'(t) - \pi_2'(t)$.

Notice that in phase III the marginal return to the lower flexibility constraint for the second crop is positive. This illustrates how lower bounds can be made to reflect the unwillingness of farmers to abandon too rapidly relatively unprofitable alternatives in the face of uncertainty.

The optimizing principle in this application does not imply that long-run or even short-run optima are obtained. Rather, it expresses the empirical fact that when farmers change, they cautiously improve their economic positions according to their current uncertain expectations.

The example above may be said to be an "open" model — open with respect to output and input prices. No mechanism was allowed for determining net-return expectations. In aggregative models, this openness is a drastic limitation, for neither net returns, nor their expectations, can be assumed to be independent of past prices and, therefore, of past output. It is to this interdependence we shall now turn.

NET RETURN EXPECTATIONS AND INTERACTION WITH AGGREGATIVE DEMAND

Net returns are a function of prices and of outputs, inputs, and the technical structure of production. Thus, net returns can be expressed as

$$\pi_i(t) = p_i(t)\, y_i(t) - C_i(t) \tag{10}$$

in which $p_i(t)$, $y_i(t)$ and $C_i(t)$ are the actual price, the yield, and the cost (which is a function of input prices and technical coefficients) for the i^{th} crop in the year t. We could submit net returns to some kind of expectation model, for example, Nerlove's price-expectation model (7). As it seems unlikely that farmers have much notion of what their "long-run equilibrium price" is (even conceding that such a price exists), it may be advisable to use a simple function of past net returns. This can be done while still preserving the properties of the analysis presented so far. What is more important, output plans are independent of current demand, a result that would not be true if no lag were presumed. In the latter case, we would have a model which would represent a region as a monopolist who had complete knowledge of his demand curves and not an agglomerate of atomistic sellers.

Of course, the simplest expectation function is obtained when net-return expectations are equal to the preceding year's actual net returns $[\pi'(t) = \pi(t-1)]$. As this is sufficient to illustrate the generality of recursive programming, we shall hypothesize the validity of this model.

Suppose that the demand structure for our two commodity regions is

$$\begin{aligned} Y_1(t) &= a_1 p_1(t) + a_2 p_2(t); \\ Y_2(t) &= b_1 p_1(t) + b_2 p_2(t), \end{aligned} \tag{11}$$

in which $Y_1(t)$ and $Y_2(t)$ are the demands for production and in which, according to the theory of consumption, we would expect a_1 and b_2 to be negative and a_2 and b_1 to be positive. As with expectations, this model is chosen because it is just sufficient for our present purpose. Suppose further (for simplicity) that yield is constant for each crop: $y_1(t) = y_1$ and $y_2(t) = y_2$ all t. Then, if the market is free to clear itself,

$$\begin{aligned} Y_1(t) &= y_1 X_1(t); \\ Y_2(t) &= y_2 X_2(t). \end{aligned} \tag{12}$$

Substituting relation 12 into relation 11 and solving for $p_1(t)$ and $p_2(t)$, the following expressions could be obtained:

$$(13)\qquad \begin{aligned} p_1(t) &= a_1' X_1(t) + a_2' X_2(t);\\ p_2(t) &= b_1' X_1(t) + b_2' X_2(t), \end{aligned}$$

in which the coefficients of the X(t)'s are determined by the coefficients of relation 9 and the yields. This closes the model with respect to output price, though not with respect to costs. The latter could be treated similarly, but to avoid further complexities let it be supposed that $C_1(t)$ and $C_2(t)$ are constant over time. This gives the closure needed to develop explicit dynamic solutions for acreage, price, and marginal returns over time.

Returning to phase II, there is a corresponding phase for prices. The reader can verify that it is

Phase II

$$(14)\qquad \begin{aligned} p_1(t) &= (a_1' - a_2')\ (1 + \bar{\beta}_1)^{t-t_1} X_1(t_1) + a_2'\bar{x}\\ p_2(t) &= (b_1' - b_2')\ (1 + \bar{\beta}_1)^{t-t_1} X_1(t_1) + b_2'\bar{x} \end{aligned}$$

Similarly, price movements can be found for any phase.

By means of relation 10, these price movements can be converted to expected returns. Thus, though they have not exact knowledge of it, farmer expectations follow an inexorable law which is based on the aggregative demand functions for their products.[2] At some place in the course of phase III, for example, net returns will reverse their relation; crop 2 will become less desirable to produce, and farmers will begin a response to the changed price expectations by transferring land from crop 2 to crop 1, thus reversing the former trend. The effect of this process on prices is shown in Figure 5.2. The price lines cross before the end of a phase because of the lag in expectations and the role of costs and yields. The dual variables can also be expressed as functions of time. For example, in phase II equation 14 can be substituted into equation 10; for $i = 1$ and $i = 2$.

The following results seem most important. First, prices and acreages, ergo, net returns, marginal revenues, and outputs undergo multiple phases in which rates of change over time change in each phase. Second, the phases begin to repeat themselves. This is called phase periodicity[3] and the results tend to resemble dampened sine and cosine curves! Third, phases occur in which output of a commodity may increase while its price is falling!

[2] Needless to say, this law is inexorable in a statistical sense. In stochastic processes, dynamic laws determine not variable values but rather their probability distributions over time. The rather complicated stochastic processes underlying recursive programming have not been explored very fully as yet. The term "dynamic law" is still used in its stochastic sense.

[3] Again, these may be stochastic laws.

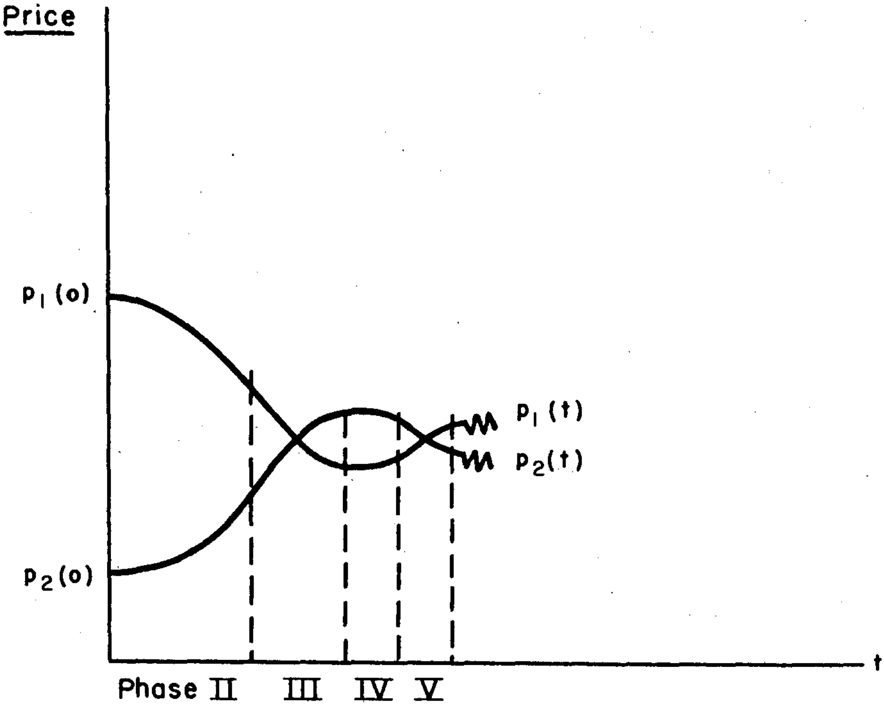

Figure 5.2. The time paths of output prices in this hypothetical example resemble dampened sine and cosine curves whose periods become shorter with the passage of time.

The implication of the first result is that the elasticity of supply is not a very stable parameter for prediċing response over time. The second result is the attainment of a multivariate cobweb cycle, which is likely to be highly stable (for crops) because of the quick change in phase when relative returns change. The third explains the enigma of the downward sloping supply curves sometimes obtained with time-series data! This result stems from the lag in expectations linked through the reaction of output on demand to a production structure with a finite number of alternatives.

This explanation of the inverse supply relation over time is consistent with the Marshallian, positively sloped, short-run supply curve. Such curves are obtained by holding constant everything except the price of a given commodity. The latter is varied continuously over a wide range to obtain the relation between output plans and price for a given time period. This same type of relation can be obtained with this model, which, for a given time period, is a straightforward linear programming problem. "Price mapping" or "parametric programming" is the technique which gives the desired supply functions. These functions will, of course, be step functions which increase discretely. However, the analysis (and synthesis) of this section reveal that the only conditions under which such curves have any real meaning is when the supply system is relieved of the influence of demand on price. Hence, Marshall's purely theoretical construct is useful operationally only for predicting the effect of artificial prices, such as those created by law. In the market, prices and production must be determined by dynamic laws derived from technical and demand structures (1).

INVESTMENT, CAPACITY, AND TECHNOLOGICAL CHANGE

To treat the relation of aggregative investment to output, the constraints on production expansion can be split into two components, one expressed by the flexibility constraint and the second by capacity constraints. The former expresses the reluctance of farmers to specialize too rapidly in a given product. The latter expresses farmers' unwillingness and inability to invest in any particular method of production at a rate greater than some maximum. This inability may come from limitations which are imposed by the rate of expansion of farm machinery and related industries or from external credit rationing. The former might be expressed as internal credit rationing.

Suppose, for example, that the first commodity can be produced by either of two methods. The first of these has been introduced in the recent past and as yet accounts for only a small portion of current practice. Let $X_1^I(t-1)$ and $X_1^2(t-1)$ be the actual capacities in number of acres utilized during the year (t-1). Let $I_1^*(t)$ and $I_2^*(t)$ be maximal investment patterns potentially observable during the year (t). Now let α_1 and α_2 be the investment coefficients in the two capacities, respectively. Now suppose that maximal potential investment can be related to the immediate past levels of capacity utilization by

$$
\begin{aligned}
I_1^*(t) &= X_1^1(t) - X_1^1(t-1) \leq \alpha_1 X_1^1(t-1); \\
I_2^*(t) &= X_1^2(t) - X_1^2(t-1) \leq \alpha_2 X_1^2(t-1),
\end{aligned}
\tag{15}
$$

Expressed as inequalities, these relations determine the maximal potential rate of investment. Predicted capacity in either process is thus constrained by the relation

$$
X_1^i(t) \leq (1 + \alpha_i)\, X_1^i(t-1), \quad (i=1,2). \tag{16}
$$

The new dynamic production model including both kinds of constraints can be written as:

$$
\max \{\pi_1^1(t-1)\, X_1^1(t) + \pi_1^2(t-1)\, X_1^2(t) + \pi_2(t-1)\, X_2(t)\} \tag{17}
$$

subject to

$$
\begin{aligned}
X_1^1(t) + X_1^2(t) + X_2(t) &\le \bar{X} \\
X_1^1(t) + X_1^2(t) &\le (1+\bar{\beta}_1)\,[X_1^2(t-1) + X_1^2(t-1)] \\
X_1^1(t) &\le (1+\alpha_1)\,X_1^1(t-1) \\
X_1^2(t) &\le (1+\alpha_2)\,X_1^2(t-1) \\
-X_1^1(t) - X_1^2(t) &\le -(1-\beta_1)\,[X_1^1(t-1) + X_1^2(t-1)] \\
X_2(t) &\le (1+\bar{\beta}_2)\,X_2(t-1) \\
-X_2(t) &\le -(1-\beta_2)\,X_2(t-1)
\end{aligned}
$$

Actual investment patterns are then predicted by the model. Omitting demand functions to simplify the argument and returning to phase III conditions, the change in capacity might follow the time paths shown in Figure 5.3. It is presumed that for the period considered, expectations are such that $\pi_1^2(t) > \pi_1^1(t) > \pi_2(t)$, that is, that the second (newest) way of producing crop 1 is most profitable, while the older method is more profitable than production of the second crop.

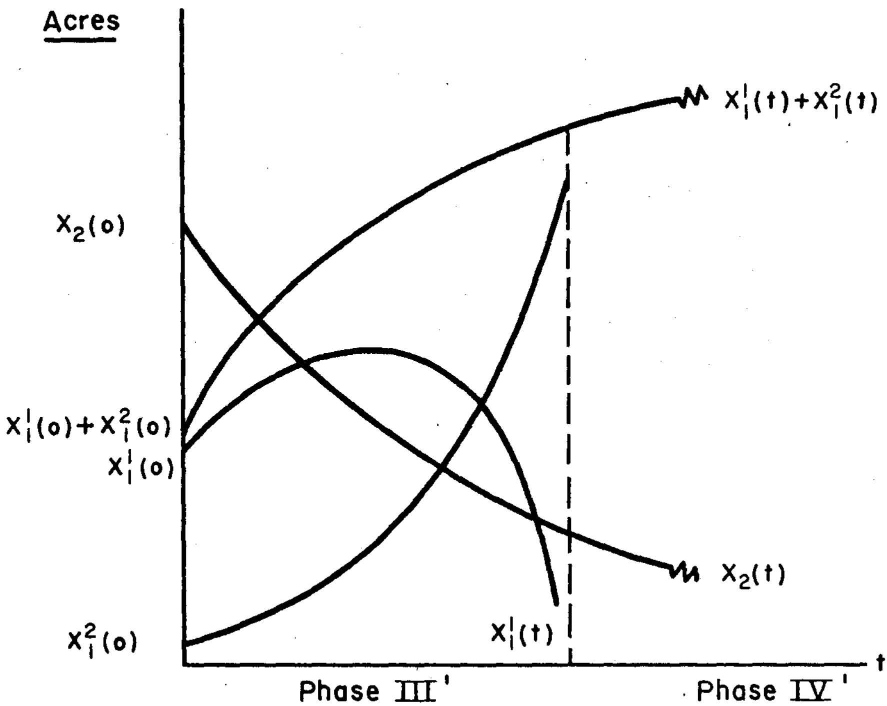

Figure 5.3. The acreage of crop 1 handled by the older method increases but eventually decreases until the method is entirely abandoned by Phase IV'. Acreage of crop 1 grown under the new method rapidly replaces acreage devoted to the old method until Phase IV' when uncertainty and other forces constrain aggregate production of the crop.

Here are the main results. First, actual capacity expansion (investment) and abandonment are predicted simultaneously with production patterns. Aggregate production is constrained by the forces acting on the rate of change in output patterns and by the forces which determine maximal potential growth. Second, it is likely that when the capacity of a superior production process is small, investment may occur in a relatively inferior process until sufficient growth has taken place in the former (phase III). Finally, after investment has proceeded long enough in the superior process, not only may capacity of the inferior process be abandoned at an increasing rate, but the unwillingness to alter output patterns beyond a certain rate will prevent investment in a superior process from achieving its maximal potential rate (phase IV). Thus variables may move on paths devious to their "long-run equilibrium" positions as calculated by relative profits alone.

Technological change can be split into three components — invention, innovation, and diffusion. While invention and innovation appear still to belong to historical analysis, technological change, insofar as it is a diffusion process, is solidly within the boundary of economics itself (1). Suppose that in the year t=o, the second process for producing crop 1 was innovated. The capacity $X_1^2(o)$ was an historical fact which could not have been predicted, but the diffusion process is expressed now by the same theory under which general investment patterns were predicted. <u>Diffusion is an investment process</u>. The growth of knowledge is simply an added component acting on internal and external credit rationing. Its effect may not differ vastly from "normal" investment processes, which probably always contain a knowledge component. If this is true (as we suspect), we need not search too far outside economics for exotic theories of technological change. Further, its effects on output response are traced by considering production decisions as determining investment and capacity abandonment simultaneously with changing output patterns.

PLANNING OVER TIME, REGIONAL COMPETITION, AND OTHER GENERALIZATIONS

While the optimizing principle is the criterion of micro-economic action, it is applied to the regional unit. This is done in a way which does not truly optimize economic action for a region but rather reflects the time distribution of aggregative response to current average expectations. The model has nothing to say about which production units will change in a given year, but only that specific proportions of the region's resources will be reallocated by a corresponding proportion of the region's producers with the passage of time. Such proportions could be interpreted as probabilities of change for the allocation of individual resource units. The peculiarities of individual decision criteria are subsumed in statistical averages.

The model presents a similar attitude toward planning over time.

If a given process continues to be relatively attractive as an investment opportunity for some extended period of time, the model predicts a growing rate of investment in it. Again, the individual peculiarities of planning are subsumed. Whatever the varied time horizons among producers may be, the model projects investment for the region, indicating that particular budget limitations and time horizons will lead over time to growing aggregative investment.

For certain applications, however, it is likely that planning over time should be accounted for more explicitly. In doing this, we should not like to sacrifice the rather realistic picture of sequential decision making developed so far. Dynamic programming, as it is currently applied, derives the time distribution of production and investment as the result of a single optimizing decision. Aggregative economic processes, however, do not terminate after some finite period of time in achieved terminal objectives. The ubiquitous presence of uncertainty, the accumulation of knowledge, and the play of more or less fortuitous events prevent such grandiose scheming.

The model can be generalized to include planning over time, but in a way that would preserve the yearly reevaluation of production and investment plans. For this purpose, consider a time horizon of two periods. Relations 4 and 5 are still adequate to express possibilities in the region for the first (imminent) time period. For the second (future) production period for which production and investment plans are projected, a second set of relations is required. It consists of the relations of 4 and 5 advanced one time period. The resulting 10 restrictions on production and capacity change form the following recursive programming system:

$$\max \left\{ \pi_1'(t+1)X_1(t+1) + \pi_2'(t+1)X_2(t+1) + \pi_1'(t)X_1(t) + \pi_2'(t)X_2(t) \right\} \tag{18}$$

subject to

$$\begin{array}{llllll}
X_1(t) & + \; X_2(t) & & & \leq & \bar{x} \\
X_1(t) & & & & \leq & (1+\bar{\beta}_1)X_1(t-1) \\
 & X_2(t) & & & \leq & (1+\bar{\beta}_2)X_2(t-1) \\
-X_1(t) & & & & \leq & -(1-\beta_1)X_1(t-1) \\
 & -X_2(t) & & & \leq & -(1-\beta_2)X_2(t-1) \\
 & & X_1(t+1) & + \; X_2(t+1) & \leq & X \\
-(1+\bar{\beta}_1)X_1(t) & & X_1(t+1) & & \leq & 0 \\
 & -(1+\bar{\beta}_2)X_2(t) & & X_2(t+1) & \leq & 0 \\
(1-\underline{\beta}_1 X_1(t) & & -X_1(t+1) & & \leq & 0 \\
 & (1-\underline{\beta}_2) X_2(t) & & -X_2(t+1) & \leq & 0
\end{array}$$

in which $\pi_1'(t+1)$ and $\pi_2'(t+1)$ are expected net returns for the future period. The latter might be linked through an expectation model to a demand structure to obtain a closed system. While production and investment in time t is conducted with an eye for the future, the plans made for t+1 may be changed as a new plan is generated. For a given year t, the plan is a dynamic linear programming problem of the usual kind, but it is dynamic not only in the Hicks sense, but also in the Frisch-Samuelson sense (4). This methodology can be summarized in Leontief's words (5):

> "...[An] economic [process] is...a continuing, unending process the path of which is determined by a never-ending sequence of choices. Particularly important for this point of view is the fact that the explicit time-horizon of each one of these successive choices is much shorter, in principle infinitely shorter, than the span of time covered by the dynamic process as a whole. Thus while each step...satisfies certain maximizing conditions, the sequence as a whole does not.

A dynamic process of regional competition can be formulated too. For illustration, suppose there are two regions. Disregarding time-horizon and demand aspects of the model, two sets of relations, 4 and 5, one for each region, might be specified, with the variables labeled with superscripts I or II for the first or second region. Apart from demand, the regions might be interrelated through the growth in the regional capacities of short-run fixed factors and labor. Thus the farm labor force and investment in machines would flow in the direction of highest marginal returns as reflected in the dual variables. Augmented in this way, the model is

$$(19) \quad \max \{\pi_1^{I}(t)X_1^{I}(t) + \pi_2^{I}(t)X_2^{I}(t) + \pi_1^{II}(t)X_1^{II}(t) + \pi_2^{II}(t)X_2^{II}(t)\}$$

subject to

$$
\begin{array}{lllllll}
X_1^{I}(t) & + \; X_2^{I}(t) & & & & \leq & \bar{X}^{I} \\
X_1^{I}(t) & & & & & \leq & (1+\bar{\beta}_1^{I})X_1^{I}(t-1) \\
 & X_2^{I}(t) & & & & \leq & (1+\bar{\beta}_2^{I})X_2^{I}(t-1) \\
-X_1^{I}(t) & & & & & \leq & -(1-\beta_1^{I})X_1^{I}(t-1) \\
 & -X_2^{I}(t) & & & & \leq & -(1-\beta_2^{I})X_2^{I}(t-1) \\
 & & X_1^{II}(t) & + & X_2^{II}(t) & \leq & \bar{X}^{II} \\
 & & X_1^{II}(t) & & & \leq & (1+\bar{\beta}_1^{II})X_1^{II}(t-1) \\
 & & & & X_2^{II}(t) & \leq & (1+\bar{\beta}_2^{II})X_2^{II}(t-1) \\
 & & -X_1^{II}(t) & & & \leq & -(1-\beta_1^{II})X_1^{II}(t-1) \\
 & & & & -X_2^{II}(t) & \leq & -(1-\beta_2^{II})X_2^{II}(t-1) \\
X_1^{I}(t) & & + \; X_1^{II}(t) & & & \leq & (1+\alpha_1)[X_1^{I}(t)+X_1^{II}(t-1)] \\
 & X_2^{II}(t) & & + & X_2^{II}(t) & \leq & (1+\alpha_2)[X_2^{I}(t-1)+X_2^{II}(t-1)] \; .
\end{array}
$$

This points a way to analysis of the well-known relation between regional competition and technological change.

The applied linear programmer is familiar with the rich variety of production relations which can be accommodated in the linear programming framework. An important generalization for this aggregative model of production response would be to include distinct processes representing several levels of fertilizer application for each basic technological process or "type." If a relation which would determine aggregative fertilizer stocks (purchases for a given year, for example) could be established, the yield component could be subjected to the same analysis as the acreage component of production.

Like other empirical techniques, the generality of recursive programming is determined in practice by a judicious compromise among logical structuring, data availability, and the research budget.

ESTIMATION PROCEDURES

Although simple examples have yielded interesting, theoretical results, it remains to be seen whether an operational tool exists. Given that they are both meaningful and relatively stable, can parameters of a recursive programming model be estimated? The dynamic nature of the model can be invoked to answer the question in the affirmative. The approach to be suggested is closely related to familiar time-series analysis, but it involves some unfamiliar techniques and problems.

Consider the simple model of section 3. During phase I the regional acreages of the two crops follow two simple equations. Therefore, time-series estimates of aggregate acreages can be used to estimate the coefficients $\bar{\beta}_1$ and $\bar{\beta}_2$. Notice, however, that the coefficients β_1 and β_2 cannot be estimated with data from this period. In phase II, time-series data for the first crop can be used to increase the efficiency of the $\bar{\beta}_1$ estimate, but no additional information can be added to the estimate of $\bar{\beta}_2$. In phase III, time-series data for the second crop can be used to estimate β_2.

In summary, the progress of regional production has revealed sufficient information to permit estimation of the upper flexibility coefficients of the two crops and of the lower flexibility coefficient of one of them. Remaining *unidentified* is the coefficient β_1 which determines the lower flexibility constraint for the first crop. Thus, the model introduces a new kind of *identification problem*.

Having information with which to estimate some of the coefficients is quite different from knowing how to use it. We cannot know exactly which phases actually hold over time. Consequently, two distinct sets of hypotheses are involved. Given the set of structural inequalities defining the dynamics of the model (which are, of course, hypotheses too) one must first *guess* which equations actually determined the system for particular periods of time. Second, using the usual time series techniques (least squares, perhaps) one must estimate the parameter (or parameters) of each equation so "identified." A "good" guess can be made by a study of relative net returns and of the data on acreage and production in the region, and with the help of an intimate knowledge of the region's economic conditions.

Having obtained estimates of some of the parameters in this way, one returns to some initial date and begins the model running as described above. If the optimizing principle selects the same phases as those guessed, and if the model estimates explain a fairly large percentage of the total variation in the several variables of interest, then the model's hypotheses appear to be useful approximations of reality. On the basis of this test, future projections could be made and revised with the passage of time to accommodate the latest information and newly revealed structural relations.

Recursive programming does not replace existing statistical methods but rather performs a synthesis between them and explicit choice criteria and modifies the sphere within which their application is valid.

CURRENT APPLICATIONS[4]

A recursive programming model has been developed by the author for the analysis and prediction of production from 1940 to 1959 in one of the major cotton production regions, the Delta area of Mississippi. The study includes eight commodities, four technological "stages," three soil classes, and four fertilizer levels. The production structure is represented by 103 distinct processes and 38 dynamic inequalities. Among these are investment constraints for each technological stage, capacity constraints for regional labor and fertilizer, over-all stocks of the three soil classes, and cropland acreage constraints reflecting acreage allotments.

With respect to prices, the model is open. Closure in this sense could not be achieved at this level of aggregation because of the national character of demand for Delta commodities. This shortcoming is not serious historically. In recent times, production has been independent of demand because of the incidence of high price supports. The model will generate estimates of production, acreage, and average yields for each commodity, investment patterns and rates of diffusion of existing and newly innovated technological stages, regional land, and other input utilizations. All parameters have been estimated and model predictions are currently being derived. As all time-series information used for

[4] The applications described in this section are being conducted in the Farm Economics Research Division, ARS. The structuring of the Delta model was conducted with the close cooperation of production experts both in Washington and at the Delta Branch Experiment Station at Stoneville, Mississippi. It was during the course of this empirical investigation that many of the important theoretical characteristics of recursive programming became evident. Among those to whom the author is particularly indebted are E. L. Langsford and Grady B. Crowe, both of the Farm Economics Research Division, ARS. The results of this investigation will be reported at a later date.

estimation came from 1954 or earlier, estimates from 1955-59 are true predictions. Consequently, some idea of the model's predictive power as distinct from its explanatory utility can be acquired.

A study of aggregate production response in the irrigated Far West has been initiated.[5] Plans call for tailoring the recursive programming method for this purpose. In addition, a projected study is the interrelation of several major cotton-producing regions in a dynamic interregional competition model.

The considerable flexibility of the approach lends itself to analysis and prediction at the regional level. Its broad structure, some essentials of which have been presented here, can accommodate all the inventive ingenuity brought to it for any particular application. Its union of statistical methodology and production theory seems to promise much for the analysis and prediction of aggregative supply of agricultural and other commodities.

PROOF OF THE PUDDING

It is too early to pass judgment on the empirical usefulness of recursive programming for the study of production response. At this stage only its promise can be described. It is an operational tool constructed to reflect production structures and to simulate explicitly the aggregative implications of decision processes at the firm level. While there is (as yet largely undeveloped) a theory of statistics by which estimates and hypotheses can be evaluated formally, the most attractive feature of recursive programming is its direct relation to the theory of production. Its foundation is not an esoteric theory of statistical decisions, but rather a highly plausible theory of economic action.

In addition to the lack of extensive empirical testing and a well-developed statistical theory, a thorough exploration of the bias of applying a micro-decision criterion at an aggregative level is lacking. Any study of aggregation must begin with a theory of the firm. Recursive programming seems to be well-suited to the job. The fact that certain other statistical methods are not derived from some explicit production structure does not exempt them from aggregation problems. Rather it implies that even the highest correlations do little to illuminate their essentially obscure micro-structural foundations.

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Discussion

DAY HAS PRESENTED an interesting application of linear programming to regional analysis. As he suggests, the empirical applications now being conducted will serve as a useful evaluation of the technique. Although I have no specific arguments to raise regarding his analysis, I do have one or two general remarks.

The aggregate supply of agricultural products can be traced to production on individual farms. Variations in the quantities of output produced on individual farms are often attributed to such factors as:

1. Technological developments causing changes in costs or output.
2. Variations in prices of agricultural commodities, including the expected duration of a price change.
3. Changes in institutional factors, including credit, tenure arrangements, and farm programs.

Moreover, in any given year, a farmer might base his decision to grow a crop on one or more of the following criteria:

1. The adaptability of his soil to a given crop. (Thus, farmers tend to think of "corn" or "bean" land.)
2. His ability to grow a certain type of crop. (A farmer tends to grow a crop he has had good luck with in the past. This is, in part, a quest for income security.)
3. The type of specialized equipment available to him.
4. The amount of operating capital available.
5. Expected occurrence of disease, weeds, and insects.

6. Expected prices after harvest time.
7. Type of storage available.
8. Necessity to meet fixed annual payments.
9. Planned livestock program.
10. Ability to withstand possible losses.

Other factors could be listed. There are a multiplicity of forces, all affecting supply partially and none affecting supply completely, which must be considered.

Reflection on the above criteria will illustrate the load which must be carried by the flexibility coefficients. A listing of factors determining farmers' decisions to invest or disinvest, not included here because of space limitations, would similarly illustrate the duties of the investment coefficients. Therefore, I feel the crux of Day's method lies in the estimation of the flexibility and investment coefficients. Because of their importance, I would have liked a more detailed discussion of their estimation.

As Day suggested, the meaning of the coefficients should be more fully explored. At first glance, one wonders if the coefficients can adequately estimate the effects of the many forces affecting supply. Upon reflection, however, a coefficient reflecting an aggregate rate of adjustment appears to have considerable utility. Obviously, we can never hope to quantify all the forces which influence production decisions on individual farms. It is not clear that such quantification, even if possible, would supply the answers sought. On the aggregate level, we do not need detailed knowledge of the supply response on individual farms, but rather we need a general knowledge of the supply response of all farmers in the region. Because of interactions within groups of farmers and the effects of aggregate supply and demand, the summation of individual farm responses may not be equal to the regional response. Thus, the flexibility coefficients, representing an over-all response for a region, could reflect effects of forces not apparent at the farm level.

It would also be useful to know the stability of the coefficients with respect to both time and technology. If the rate of adoption of a certain type of technological change, such as mechanization, is found to follow a characteristic trend through time in a given region, the effects of new machinery developments on the supply of the region could be predicted.

Interpretation of the flexibility and investment coefficients poses another, more general problem. Apparently our empirical techniques and available data often do not lend themselves to the estimation of our well-known economic parameters. Thus, we should study the available techniques and data with a view towards estimating parameters which are meaningful in a dynamic setting and useful for prediction and policy decisions. These parameters may not always be the familiar ones presented in classrooms and textbooks. The flexibility coefficients appear to be of this type. We must be careful, however, not to limit our thinking to problems or techniques which can be solved using known data sources and electronic computers.

The linear programming model used by Day is, in a predictive sense, fundamentally deterministic. This means that at any given point in time, it has a single solution. The question I would like to ask is: Should the predictions of analytical models aggregated on a regional level be regarded as completely determined or should they be presented in a probability framework? At what level of aggregation should the deterministic approach be used? Certainly, output prediction at the farm level can only be asserted with some probability. If supply predictions are eventually stated in terms of probabilities, continual effort would be needed to evaluate changes in the probabilities caused by changes in technology and other factors.

The present workshop was stimulated by problems in supply response. The immediate problem in agriculture is that of supply control. However, we should also be prepared to deal with other supply problems as they might arise. One very important problem both now and in the future is that of policy implementation. Thus, I would have liked to have added one or more additional papers to the program of this workshop. They would deal with problems of implementing policies which are founded in economic logic and validated by empirical analysis.