PART II

Regression Analysis of Aggregative, Time-Series Data

Chapter 2

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Time-Series Analysis of the Supply of Agricultural Products

IN A RECENT SURVEY of the problems involved in the analysis of agricultural supply, the author and Bachman have classified these into five major categories: problems connected with (1) the complex structure of production, (2) technological change, (3) aggregation, (4) investment in fixed or quasi-fixed factors, and (5) uncertainty and expectations (1, pp. 9-23). While it is not proposed to repeat here what was said in this survey, all the problems mentioned are encountered in time-series supply analysis. It therefore seems appropriate to delineate them briefly.

The Complex Structure of Production

As is well known, agricultural production in the United States is composed of hundreds of different products and requires scores of different inputs. To a lesser degree the same is true at the level of the individual firm. It is rare to find a farm producing a single homogeneous product and using but a few well-defined factors of production. Furthermore, production of the same commodity may be carried out in a very different manner in one part of the country than in another. In the case of many industrial commodities, the relations among products and factors are relatively simple or can be well approximated by models which neglect many of the interconnections in the structure of production. However, in the agricultural sector, relations among products and factors are both strong and numerous at whatever level from firm to industry we choose to consider (19, especially chaps. 5 and 7). The complex structure of agricultural production leads to serious problems in time-series supply analysis for two reasons. First, time-series are generally short relative to the number of variables which it would be desirable to include in statistical analyses in the light of the complexity of agricultural production; hence, only relatively few may be taken into account. Second, because many time-series, particularly prices, tend to move together over time, the separate effects of even those few variables included frequently cannot be discerned.

Thus, we have of late begun to turn to the study of the production functions of individual firms or to time-series analyses related to geographical regions. While the value of this sort of investigation should not be denied, it should be pointed out that (1) most of the policy uses to which knowledge of agricultural supply is germane require knowledge of the behavior of aggregates over time, and (2) the link between results on a disaggregative level and the knowledge needed for policy purposes is by no means obvious and indeed beset with many problems. In the appendix, one of the ways in which aggregative time-series and other types of analyses can supplement one another and lead to greater knowledge of agricultural supply is pointed out.

Technological Change

Changes in technology are to supply analysis what changes in tastes are to demand analysis. The former are likely to be much more important, both in terms of their frequency and their impact on supply conditions. At a point in time or over a relatively short period, the assumption of unchanging technology may be a good approximation to reality. But over long periods of time, to which time-series supply analyses refer, it is clear that this assumption is a poor one. In time-series supply analyses for individual agricultural commodities, a simple trend has generally been used to take account of the effects of changing technology. In the analysis of the agricultural sector as a whole, and to some extent in the analysis of individual products, Cochrane and others (10, 11, 12, 39) have attempted to analyze the effects which prices and other factors have on the adoption of new technology. There is no doubt that this is an important first step. Further progress may lie in the recognition that the adoption of new technology and its effects on the productive process are closely related to the problems of uncertainty and investment. Because of the complex effects of technological changes on the ways in which individual commodities are produced, there is special need for studies on a disaggregative level.

Aggregation

Since time-series supply analysis generally deals with national aggregates which are of direct interest for policy purposes, it might be thought that aggregation problems play no role. However, this is incorrect for several reasons. As already remarked, the complexity of the structure of production in agriculture frequently leads us to study subsectors, which may be geographic, product, or both. Aggregation problems arise when an attempt is made to use such results for purposes of national agricultural policy. Furthermore, the necessity of confining attention to but a few relevant variables in time-series analysis is itself a form of the aggregation problem. Finally, because much of our knowledge of supply, both theoretical and empirical, is disaggregative, and because of our desire to supplement time-series analysis with such knowledge, the question of the connection between the two arises. The aggregation problems thus encountered are not inherent in time-series analysis, but are clearly related. A few tentative remarks on the question are made in the appendix, where a model which reveals the influence of an unequal distribution of fixed factors and technical knowledge on the elasticity of industry supply is discussed.

Investment in Fixed or Quasi-Fixed Factors

Fixed factors of production form the basis for the traditional distinction between short- and long-run supply response to price, as well as a similar distinction in the theory of costs at the firm level. Of course, fixed factors are not really fixed for all time but can and will be varied in response to economic forces. The study of such variation is the subject of the analysis of investment decisions. Glenn Johnson (24) has emphasized the importance of investment decisions to supply behavior and offers a number of constructive hypotheses about the relation among acquisition costs, salvage values, and expected marginal productivities which would appear to promise increased understanding of the effects of changes in price on resource use and output.

The principal criticism of this author of much of Johnson's recent work on this subject is that he does not pay sufficient attention to the smoothing effects of aggregation and overemphasizes the discontinuities found at the microlevel. However, this is a sin of omission rather than of commission.

The use of distributed lag models in aggregate time-series supply analysis is basically a very simple way of taking account of the effects of fixed factors on supply response (17, 31, 33, 35). It should be noted, however, that the model used is not well-founded on an explicit micro-theory, nor, for this reason, are the results of analyses based on it capable of easy comparison with information about behavior on the level of the individual firm. The use of such models are subject to a number of difficulties in practice: (1) the empirical distributions of lag have been found to be unstable over time (17); (2) these distributions have also been found to imply unreasonably long periods of adjustment; and (3) there are severe problems in separating lagged adjustments of this sort from those resulting from the process of expectation formation (31, pp. 63-65, 236-54).

The simple sort of distributed lag model is particularly ill-adapted to the study of commodities in the production of which so-called quasi-fixed factors are involved. The line between final products and capital goods is not a sharp one. Agricultural outputs and inputs run the whole gamut from farm buildings and machinery, clearly durable capital goods, to fresh vegetables, which are clearly perishable final products. Although inventories of one kind or another play a role in the production of most agricultural commodities, their influence on supply is nowhere so evident as in the production of livestock and livestock products. In addition to feed inventories, livestock may be thought of as intermediate both to strictly capital goods and to final outputs. The multiple roles of livestock in the productive process are well described by Hildreth and Jarrett (22, p. 21).

> A given animal at a given time may be viewed as (a) a finished good, (b) a good in process or (c) a piece of fixed capital. This is perhaps most dramatically apparent for a young heifer, say 16 to 20 months old, of a beef or dual-purpose breed. If the animal has been well fed, she may be immediately marketable as medium or possibly good beef. Alternatively she might profitably be fed intensively for a short period with a consequent increase in weight and possibly in grade. A third alternative would be to retain her in the breeding herd to produce calves (or calves and milk if she were a dual-purpose heifer). Though a narrower range of alternatives exists for most other animals, it is typically true that selling livestock for slaughter reduces the productive capacity of the farm herd. Thus an individual producer or all producers as a group can increase current marketings either by increased feeding and production or by decreasing the productive potential of their herds.

The marked cycles observed in the numbers and prices of beef cattle, hogs, and sheep may be due in part to their special nature (3, 4, 13, 28, pp. 41-82).

D. Gale Johnson (23) has pointed out and analyzed in some detail the close connection between the problem of investment decisions in agriculture and the problem of uncertainty. His discussion is oriented primarily toward policy, but the implications for supply over time in the face of changing uncertainties is clear.

Uncertainty and Expectations

One of the chief problems in the empirical application of economic theory is the problem of specifying the correct, or at least a useful, relation between the constructs of the theory and the variables which can actually be observed. Economic theorists have been aware of this problem for some time, but much of the discussion has resulted in relatively sterile criticism of econometric work. Recently, however, there has been an increased recognition of the problem among econometricians who have consciously made an effort to state the relation between observable variables and theoretical constructs as an explicit part of the underlying theory. In time-series supply analysis one of the forms which this general problem takes is that of specifying the relation between observable events and the prices which farmers expect to receive for their outputs and expect to pay for their inputs. In the production of almost all farm commodities, inputs must be committed to a greater or lesser degree some time before output is realized. A farmer must therefore base his plans not on what he is currently receiving or has received in the past, but on what he thinks he may receive in the future. What a particular farmer thinks, or better what farmers as a group think on the average, is what is relevant to farmers' supply decisions, and is therefore the relevant theoretical construct for supply analysis. But what farmers think is a subjective matter and not directly observable over long periods of time; hence, we are faced in time-series analysis with an extremely difficult problem of the general type discussed here.

Uncertainty in agriculture has primarily been discussed in connection with farm management problems (25, 36). It is clear that if uncertainty affects how farmers <u>ought</u> to behave with respect to investment, farm organization, and production plans, it must also affect how they <u>do</u> behave, although perhaps in not quite the same way. This is just the other side of the economic coin. In this light, uncertainty raises issues for time-series supply analysis which go beyond the problem of relating theoretical constructs to observable variables. Technological developments, changes in market organization and structure, and government policies have altered the impact of uncertainty upon supply decisions. However, the problem of most immediate concern remains the one first mentioned; without some sort of workable solution to this problem, the other elements cannot be brought into the picture in a truly meaningful way.

The relation between the problems of uncertainty and investment decisions has already been indicated. What is less fully realized is how this relation can play a fundamental role in time-series supply analysis. One can appreciate the relation best in the context of the supply of livestock and livestock products because of their intermediate nature between true capital goods and final outputs and because of their greater degree of specificity. Ladd (27) and Breimyer (3) have pointed out in this connection, that the problem of specifying relevant expectational variables is greatly complicated in the case of livestock products because supply decisions can be made at many points of time, and decisions at many previous points in time affect current alternatives.[1]

A long paper could be devoted to each of the five problem areas discussed above. To attempt to delve further into all these areas would obviously be impossible within the scope of this paper and would furthermore infringe upon areas discussed elsewhere in this book. The rest of this discussion will be restricted to topics (4), investment in fixed and quasi-fixed factors, and (5), uncertainty and expectations. A discussion of specific techniques or achieved results will be minimized. Emphasis will be on needed areas of future research. Examples will be given of what the author believes to be fruitful ways of looking at the problems involved.

[1] This point is nowhere better illustrated than in the discussion of the British pig cycle by Coase and Fowler (8, 9).

INVESTMENT IN FIXED AND QUASI-FIXED FACTORS

In this section, the question of how the presence of fixed factors of production produces lagged adjustment in supply and the difference between the short- and long-run will be examined. As indicated elsewhere (33), recognition of this difference is crucial to successful time-series supply analysis. Bachman and the author have discussed the relation between the investment problem and the problem of technological change elsewhere (1). A simple model is presented in the appendix which relates the fixed factor problem to the aggregation problem in connection with the question of how to use cross section information to check a time-series study. Perhaps the most serious omission in the present discussion is the lack of any consideration of the relation between the problems of uncertainty and investment which D. Gale Johnson (23) has cogently argued.

As indicated previously, many forms of fixed and semifixed inputs are used in the production of agricultural commodities. These range from things such as barns or tractors, which may clearly be treated as capital goods fixed in the short run, to an 18-month-old sow or a stock of seed. The sow or the seed may be considered as a final output or as capital goods to be used in the production of more of the same. Because capital goods consist of a tremendous variety of different things with greatly different durabilities, and enter the productive process in many different ways, simplification is both necessary and conceptually difficult.[2] For this reason, subsequent remarks are confined to a model in which the question of simplification does not arise by assumption.

The closest connection between supply analysis and investment in fixed factors appears in conjunction with the distinction between the long and short runs. To see how this occurs, it is useful to examine a simple example recently presented by Smith (41). Let us consider a firm which produces a single homogeneous output with two homogeneous factors of production, one a current input and the other a capital input. Suppose the capital input is like a "one-hoss shay" — it requires no maintenance and disintegrates at the end of a fixed period, L. We suppose that the current input may be measured continuously by some real number x_1 and that the stock of the capital input may also be so measured by X_2. For the moment we beg the question of whether the capital input is divisible or, if so, to what degree. Following Smith (41, p. 66), we write the production function, assumed to be continuous and differentiable, as

$$y = f(x_1, X_2) \tag{1}$$

where y is the continuous output of the productive process, x_1 is the continuous current input, and X_2 is the physical stock of the

[2] For some of the theoretical issues involved see Robinson (38). Griliches (16) gives an excellent account of the issues which must be faced on a practical level.

replaceable capital good.[3] We assume $f(x_1, X_2) = 0$ if either x_1 or $X_2 = 0$.

Suppose that the firm attempts to minimize the "current" cost of producing a given output (how output is determined is discussed later) given its production function and the prices it must pay per unit of the current inputs, w_1, and the fixed factor, W_2.[4] Under these circumstances and when discounting occurs continuously, Smith shows that current costs are

$$C = w_1 x_1 + \frac{r W_2 X_2}{1 - e^{-rL}}, \tag{2}$$

where r is the rate at which the future is discounted and L is the length of life of the fixed equipment. Minimization of equation 2 subject to equation 1 yields the conditions

$$\left\{ \begin{aligned} w_1 - \lambda \frac{\delta f}{\delta x_1} &= 0 \\ \frac{r W_2}{1 - e^{-rL}} - \lambda \frac{\delta f}{\delta X_2} &= 0 \end{aligned} \right. \tag{3}$$

in addition to equation 1, where λ may be interpreted as marginal cost, i.e., $\lambda = \dfrac{\delta C}{\delta y}$.

If equations 1 and 3 are satisfied for a given output, the firm is in long-run equilibrium. The value λ^0, which may be obtained by solving equations 1 and 3 for λ in terms of $w_1, \dfrac{r W_2}{1 - e^{-rL}}$, and the given output y is the <u>long-run marginal cost</u> for the output y. If the firm sells its output in a competitive market at a price p, it will be in long-run equilibrium only if

$$p = \lambda^0 . \tag{4}$$

[3] Actually there is some question about whether the stock of fixed productive factors should be included explicitly in the production function. Smith takes the position that both stocks of capital goods and flows of current inputs should be included. He says, "The direct objects of adjustment or action parameters of the firm are (1) the current inputs to current production, and (2) the physical stocks of the various kinds of capital goods employed. ...The distinguishing character of capital goods is simply that their presence, in the form of physical stocks, is required in order for production to take place. ...the inclusion of all current inputs in the production function permits one to account for the economizing consequences of variations in equipment utilization through the latter's impact upon the consumption of current inputs." (41, pp. 65-66.) The classical position, best expressed in Carlson (6), is that only flows of services should be included. In the appendix stocks and flows of the services of fixed factors are treated as parameters in the production function. I do not believe that these various positions are contradictory, but rather that one may be appropriate for one problem and another for a different problem.

[4] Current cost is taken to be the constant outlay stream per period which "...has a present value equal to that of all future cost outlays over the firm's planning horizon." (41, p. 67.) In order to compute this value a rate of discount is necessary. Smith does not, nor shall this paper, discuss what rate of discount should be used nor how it is determined.

Since λ^0 is a function of w_1, ${}^{r}W_2/(1 - e^{-rL})$ and y, equation 4 may be rewritten as

$$(5) \qquad y = S^0 \left(p, w_1, \frac{r\,W_2}{1 - e^{-rL}}\right) .$$

S^0 is the long-run supply function for this particular firm. It shows output supplied at a price p, given the prices of the current input and the capital input, the discount rate, and the length of life of the capital equipment. The important thing to note is that the long-run quantity supplied depends on the discount rate and the length of life of capital equipment, neither of which have generally played a role in time-series analysis.

The above formulation, however, cannot be applied directly to time-series supply analysis for the following reasons: (1) We do not generally observe firms on their long-run supply functions but only on their short-run functions. These differ according to the length of run considered and, as Smith shows, according to the nature of the physical capital involved, whether it is divisible, if so to what extent, and whether it has a resale value. (2) The formulation applies to the individual firm, and time-series analysis generally deals with industry aggregates. Firms in the industry will generally have equipment of various ages even if they all face the same prices and use the same rate of discount.

Let us suppose that (1) the capital good is perfectly indivisible, (2) it must be replaced in toto; and (3) current prices are expected to continue indefinitely into the future by all firms, each of which applies the same rate of discount, r, to the future. We also suppose that the production function f is the same for all firms. The relation between the short- and long-run supply functions for the individual firm may now be analyzed as follows: Let the firm's capital stock be of age A. For $A \neq L$ capital stock is fixed and unalterable at the level X_2. For this stock, it will choose x_1 to satisfy the equation

$$(6) \qquad w_1 = p \frac{\delta f}{\delta x_1} .$$

For very low prices, p, this may imply $x_1 = 0$, which means the firm is out of business. Given X_2, equations 1 and 6 may be used to eliminate x_1. Replacing X_2 by its numerical value and rewriting the result, we could obtain output as a function of p and w_1

$$(7) \qquad y = S' (p, w_1) .$$

S′ is the short-run supply function of the individual firm. Its relation to the long-run supply function is as follows: Suppose that prices are the same at the end of A years (when the capital stock suddenly disintegrates) as they were at the time initially considered. Then the firm would have the option of repurchasing an amount X_2 of the capital input and continuing in business, or of not repurchasing and going out of business. The matter would be decided on the basis of whether equation 1 and the following equations could be satisfied on the basis of an amount X_2 of the capital good:[5]

$$(8)\quad \begin{cases} w_1 = p \dfrac{\delta f}{\delta x_1} \\[2ex] \dfrac{r W_2}{1 - e^{-rL}} \leqslant p \dfrac{\delta f}{\delta X_2}. \end{cases}$$

In general there will be some price p_m for which the equality in the second of equations 8 just holds. This is the lowest product price consistent with repurchase and is a decreasing function of W_2, an increasing function of r, and a decreasing function of L and $\frac{\delta f}{\delta X_2}$.[6]

Let p'_m be the price at which the firm would discontinue business in the short-run. This price must, in general, be lower than p_m, the price at which the firm would discontinue business in the long-run and will be an increasing function of w_1. The short-run supply curve, given w_1 is perfectly elastic at p'_m; above p'_m, it is less than perfectly elastic but not perfectly inelastic. The long-run supply curve, given W_2, r, and L, is perfectly elastic at $p_m \geqslant p'_m$ and identical with the short-run curve thereafter.

What happens when we aggregate firms with equipment of different ages? Suppose that all firms have the same production functions and face the same prices, and let $\pi(A)$ be the proportion of firms with equipment age $A = 0, 1, \cdots L$. If the product price is less than p'_m, no firm will produce; hence, the industry supply curve is perfectly elastic at p'_m. If the product price is between p'_m and p_m, the short-run industry supply function is a weighted average of the short-run individual firm supply functions; the weights are determined by the proportions $\pi(A)$. However, at a price less than p_m no firm will repurchase equipment when that which it has wears out; hence, the long-run industry supply function must be perfectly elastic at p_m. Given a price less than p_m, the quantity supply will gradually drop off by a proportion depending on the distribution of firms with equipment of different ages. This can only stop when either industry output is zero or price has risen to p_m along the demand curve. This process is illustrated in Fig. 2.1. $D_1 D_1'$ is the demand curve after a shift has occurred.

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[5]Note that for the purpose of this example we are continuing to regard X_2 as continuously variable in the definition of f, even though we regard it as discretely variable in the problem at hand. The inequality reflects the latter; if the cost of X_2 is greater than the value of its marginal product at a price p, it will not pay to repurchase. If it is less it will. The possibility of setting up another plant, i.e., essentially another firm with X_2 of the fixed factor is disregarded for the moment.

[6]The proof of these propositions is elementary.

(Originally the demand curve passed through the point S′.) $p'_m SS'$ is the short-run industry supply function. The decline in output the first period, $y_1 - y_2$, is such that as a proportion of industry output, $\frac{y_1 - y_2}{y_1}$, it equals the proportion of firms with equipment age L - 1, and so on for subsequent periods. The gradual decline in output only ceases when industry output is y_m, at which output the price as determined by the demand curve is p_m. At this price producers are just indifferent as to whether they replace the capital equipment or not. By allowing different shifts in the demand curve, e.g., D_2D_2', D_3D_3' or D_4D_4', we can trace out the various short-run supply schedules, $p_2p_2'\ p_2''\ p_2'''$, $p_3p_3'\ p_3''\ p_3'''$, etc., appropriate to different periods of adjustment.

When the product price is above p_m, under the assumptions existing firms have the incentive to open new plants, i.e., purchase another unit of the capital good, and new firms will enter the industry. Output must increase until price falls to p_m along the demand curve. It follows that the short-run industry supply curve is perfectly elastic at the price p_m past the point S′. Thus the over-all one-period short-run industry supply function is given by the curve $p'_m SS'\ S''$.

In the long run, the industry supply function is perfectly elastic at a price p_m for the same reason it is perfectly elastic in the short run for upward shifts in demand. The long-run industry supply function is the curve $p_m\ S'\ S''$. As can be seen from the figure, for downward shifts in demand we have the usual fan of short-run industry supply functions emanating from S′ (except in this case they are bent lines with corners on the long-run supply function because of the discontinuities in age of equipment). These short-run curves approach the long-run curve from below.

Had we relaxed the assumption that the production function was the same for each firm, we would have found that the prices p_m and p'_m were different for different firms. In this case the adjustment pattern would be more complex, depending on the joint distribution of age and prices p_m and p'_m. In general, the long-run industry supply function would no longer be perfectly elastic. It would still be true, however, that upward adjustments would be instantaneous and downward would not be. Still further relaxation of the assumptions to restrict the purchases of new equipment and freedom of entry in the short run would produce less than instantaneous upward adjustments. A secondhand market for the capital good would induce faster downward adjustments. It is conceivable that a simple asymmetric distributed lag model could represent the phenomenon, at least approximately.

The main conclusion of this analysis is one which should be obvious from economic intuition: namely, that upward and downward adjustments are likely to be asymmetric in the short run. Black (2) and Cassels (7) suggested long ago that this might be the case.[7] However,

[7]It should be noted, however, that Halvorson (17) did not find a statistically significant difference in the case of milk.

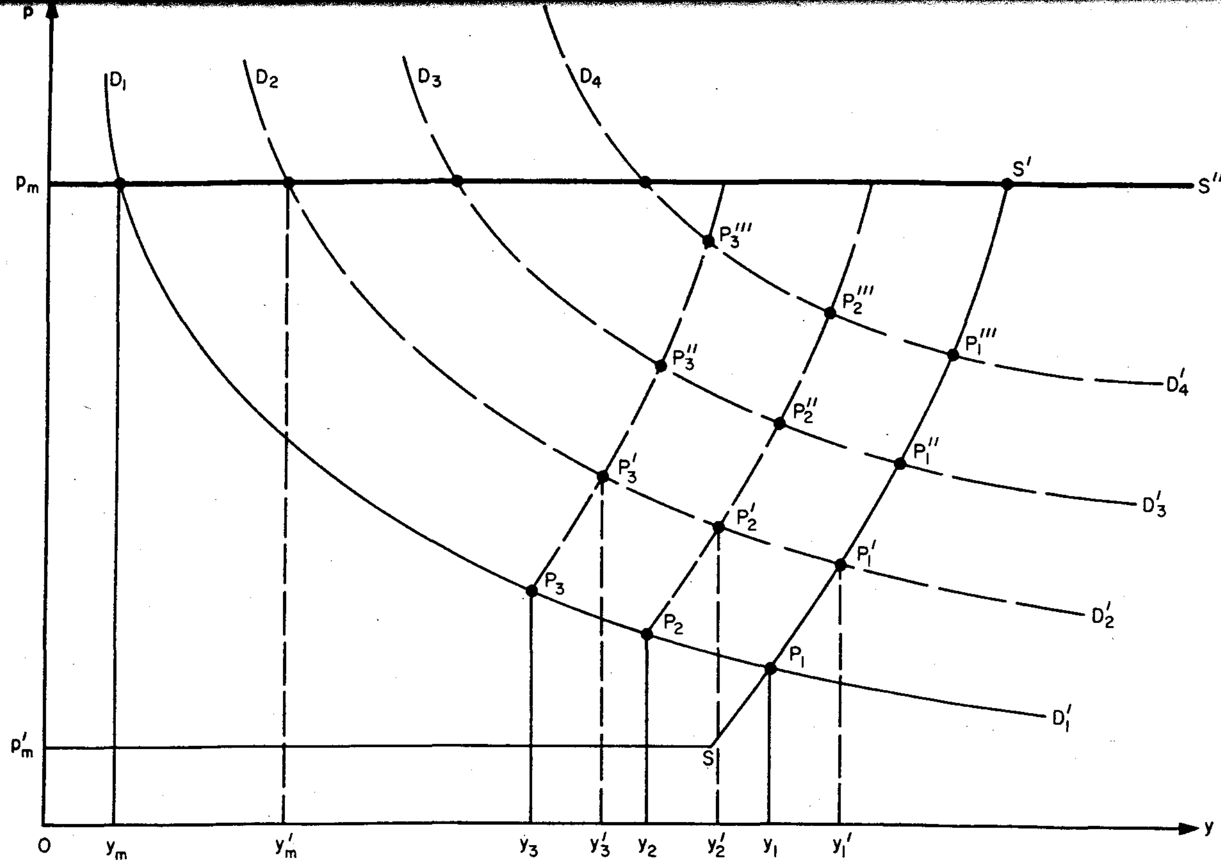

Figure 2.1. Illustrative derivation of short-run supply schedules.

it is hoped that the simple model discussed here will be a first step towards an engine of analysis which will permit the development of more realistic dynamic models of supply behavior which can be used in time-series supply analysis. Even the model discussed, as simple as it is, suggests several reasons why the distribution of lag in adjustment might be unstable over time. As noted above, the relation between the short- and long-run curves depends on (1) the durability of the capital good, and (2) the age distribution among firms. Over time the first is likely to change; but the second is almost certain to, because as the industry expands the average age will decline, and as it contracts, it will increase.

It has been suggested that livestock are quasi-fixed factors of production. The analytical problems encountered are similar to those encountered in connection with the more usual types of durable goods with secondhand markets. The author hopes to present a theoretical framework for discussing this type of problem as well as a more complete over-all framework in the near future.

UNCERTAINTY AND EXPECTATIONS

The Notion of a Certainty Equivalent

In a recent survey of problems of uncertainty in relation to farm planning, Hildreth (21) distinguishes between probalistic and game theoretic approaches to the problem of decision making under uncertainty. The value of the latter appears at present to lie chiefly in normative applications at the level of the individual firm; but the former, particularly in connection with the recent reintroduction of the notion of certainty equivalence, may be quite useful in the analysis of observed behavior. Hicks (20) suggested that behavior under uncertainty could be studied by constructing, for each of various situations involving uncertain expectations, a related situation involving expectations held with certainty. Hart (18) rejects the certainty equivalent construct on the grounds that the most important behavioral manifestations of uncertainty, such as the maintenance of liquidity, postponement of decisions, and restriction of investment in specialized fixed capital, can be explained only by rather peculiar certainty equivalence models. It is clear, however, that in any application of the theory of behavior under uncertainty to time-series analysis of supply, it must be possible to summarize factors influencing behavior in a few variables. Thus, if several future prices, whose values are uncertain, are supposed to influence present supply behavior, it must be possible to take account of these effects by one or two variables for each price, e.g., the anticipated mean value and variance of the distribution of each.[8] Note that these need not be directly observable.

[8] It is interesting to note that even in normative applications of statistical decision theory, which is at present the most highly developed theory of optimal behavior under uncertainty, the need has been felt for such a reduction to more manageable proportions. See Reiter (37).

It is fortunate that Reiter (37), Simon (40), and Theil (43) have recently been able to demonstrate that a problem of decision making under uncertainty can, in some circumstances, be replaced by a much simpler problem involving many fewer parameters and which, in fact, can be thought of as a problem in decision making under certainty. A simple example will illustrate the principles involved. Suppose we are given a producer who produces a single homogeneous commodity. The producer must decide in advance how much to produce, q, at a time before that at which he can sell the commodity at a price, p_0, per unit. Furthermore, suppose that the total costs of producing q units depend upon the prices of factors, p_1 and p_2, which are also unknown at the time the decision is to be taken, i.e., total costs are

$$C\,(q,\, p_1,\, p_2)\ .$$

Let the prices, p_0, p_1, and p_2 have a joint (subjective) probability distribution $F(p_0, p_1, p_2)$. One possible formulation of the producer's decision-making problem under uncertainty is the problem of maximizing the mean value (or expected value in a statistical sense) of net revenue, i.e.,

$$\text{(9)}\qquad E\,R(q,\, p_0,\, p_1\,,\, p_2) = \int \{p_0 q - C\,(q,\, p_1\,,\, p_2)\}\, d\,F(p_0,\, p_1\,,\, p_2)\ .$$

From the standpoint of statistical decision theory, the solution of this problem in its general form requires knowledge of the entire distribution function $F\,(p_0\,,\, p_1\,,\, p_2\,)$. Suppose, however, that the cost function is a function of the form

$$\text{(10)}\quad C\,(q,\, p_1\,,\, p_2) = a_0 + a_1 q + a_2 q^2 + b_1 q\, p_1 + b_2 q\, p_2 + b_{12}\, q\, p_1\, p_2\ .$$

In this case the problem of maximizing the mean value of net revenue reduces to a simpler problem which Reiter (37) calls the <u>surrogate</u> of form 9, i.e., maximize the expression

$$\text{(11)}\qquad q\, \bar{p}_0 - \{a_0 + a_1\, q + a_2\, q^2 + b_1\, q\, \bar{p}_1 + b_2\, q\, \bar{p}_2 + b_{12} q\, \sigma_{12}\}$$

with respect to q, where $\bar{p}_0$, $\bar{p}_1$, and $\bar{p}_2$ are the means of the distribution of prices $F(p_0,\, p_1\,,\, p_2)$ and σ_{12} is the covariance of p_1 and p_2.[9]

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[9] This follows from the fact that if C is of the form 10,

$$ER\,(q,\, p_0\,, p_1, p_2) = q \int p_0 dF - \{a_0 \int dF + a_1 q \int dF$$
$$+ a_2 q z \int dF + b_1 q \int p_1 dF + b_2 q \int p_2 dF + b_{12} q \int p_1 p_2 dF\,.$$

Reiter (37) shows in general that, if x is a vector of decision variables, such as the quantity to be produced, and y is a vector of random elements, such as future prices, and if the function f(x,y), the expected value of which is to be maximized, is of the form

$$f(x,y) = \sum_{i=1}^{n} A_i\,(x)\; B_i\,(y)\,,$$

The problem of maximizing form 11 with respect to q has the solution

$$(12) \qquad q = \frac{\bar{p}_0 - a_1 - b_1\bar{p}_1 - b_2\bar{p}_2 - b_{12}\sigma_{12}}{2a_2} .$$

and this is the solution of the original problem 9 when C is of the form 10. Thus, the certainty equivalent problem to 9 is 11, and the uncertain prices p_0, p_1 and p_2 are replaced by their certainty equivalents; namely the means of the distribution of prices and the covariance of p_1 and p_2, in this equivalent problem.

Theil (43) has shown that if the following conditions are satisfied: (1) The function to be maximized is a quadratic in the decision variables and uncertain variables, and (2) actions planned to be taken in the future do not affect the present values of the uncertain variables, then the certainty equivalents of the uncertain variables are simply the means of their distribution. In our simple example, Theil's assumption would be fulfilled if b_{12} were zero. While the assumption that the function to be maximized is a quadratic is certainly quite restrictive, it has merit as an approximation because it allows us to replace each uncertain variable by only one certainty equivalent variable.[10] In subsequent discussion, it will be assumed that Theil's conditions are fulfilled and therefore mean by certainty equivalent the mean value of the (subjective) probability distribution of the uncertain variable.

To this point the discussion has concerned a single decision maker, but in time-series supply analysis we are concerned with the collective results of decisions by a large number of individuals. Thus, in order to apply the theory of certainty equivalence to a group, we must adopt the Marshallian device of the representative firm, i.e., a possibly hypothetical firm whose certainty equivalent expectations, if held by all other firms, would result in the observed group behavior. The certainty equivalents of the representative firm may be thought of in some sense as group averages, but it is clear that in so doing a host of aggregation problems are raised. A discussion of these, however, would take us beyond the scope of this paper.

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then there is a simpler surrogate problem; namely to maximize

$$G(x_1 Z) = \sum_{i=1}^{n} A_i(x)\, Z_i$$

where $\qquad Z = (Z_1 \cdots Z_n)$ and $Z_i = \int B_i(y)\, dF(y)$.

Theil's results (43) for quadratic decision functions f(x,y) are a special case of Reiter's more general formulation in which it can be shown that the surrogate problem involves only the means of the distribution F (y).

[10]Reiter (37) points out that this approximation is better for normative applications of the theory than for positive, because, although the decision based on the surrogate problem may lead to a value of the function to be maximized not far from the true maximum, the values of the decision variables chosen may be very different. This is a basic difficulty inherent in the use of certainty equivalents in time-series supply analysis.

Summary. The device of introducing a certainty equivalent to an uncertain variable appears both useful and necessary in time-series supply analysis. To justify the use of a single certainty equivalent for each uncertain variable, it is necessary to make the following restrictive assumptions: (1) Group behavior can be adequately explained by treating it as the behavior of a single representative and hypothetical decision maker; and (2) the representative decision maker behaves as if he maximizes the expected value of a function which is quadratic in the decision variables and the uncertain variables. Theil's further assumption that future plans do not affect the present values of uncertain variables is hardly restrictive for the present purpose, although in the case of national planning it may be because of the effects of announced plans on the public.

Models of Expectation Formation

The certainty equivalent expectations held by the representative firm are both subjective and aggregative. They are not necessarily observable. The problem in time-series supply analysis is to construct an empirically useful hypothesis which relates these expectations to observable variables. In essence, this means constructing a model of expectation formation. This section will consider three types of models: (1) extrapolative, (2) adaptive, and (3) rational. The latter is actually a broad class of models of expectation formation stemming from the recently proposed "Rational Expectations Hypothesis" (29, 30). The rational expectations model is intimately related to the over-all model of behavior formulated. For that reason, the concluding section of this paper will be a discussion of rational expectations in a simple cobweb model.

Extrapolative

The classical approach in agricultural supply analysis is to suppose that expectational variables can be directly identified with some one past actual value of the variable to which the expectation refers. For example, the supply of an agricultural commodity at a future time depends on its price expected at that time. It might be assumed that this expectation is the current value of price, so that supply is simply related to lagged price. An extension of this approach, due to Goodwin (15), is to suppose that expected price in period t is actual price in t-1 plus (or minus) a fraction of the change in price from period t-2 to t-1:

(13) $$p_t^* = p_{t-1} + \alpha(p_{t-1} - p_{t-2}),$$

where p_t^* is the price expected in period t. Note that the term "expected price" is used for convenience in place of the term "certainty equivalent price for the representative firm." Elsewhere, the author has called the model generating expected prices of the form 13 the "intermediate model" and has discussed the estimation of supply functions when such a model is assumed (31, pp. 199-200). Muth (30) terms the expectation generated by form 13 "extrapolative." The classical model of expectation formation is a special case of form 13 when $\alpha = 1$.[11]

Adaptive

Expectations based on the extrapolative model do not forecast actual events very well. Furthermore, such expectations are theoretically unsatisfying in that they are determined by only two past items of experience and neglect information which we may feel is contained in other items of past experience. To develop a model of expectation formation useful in time-series analysis, the model must be kept relatively simple. A model, due originally to Cagan (5), which has greater intuitive appeal than the extrapolative model is the adaptive expectations model. According to it, expectations are revised periodically by some portion of the error between last period's expectation and what actually occurred. To use the previous example and notation,

$$p_t^* - p_{t-1}^* = \beta(p_{t-1} - p_{t-1}^*), \tag{14}$$

where β is called the coefficient of expectations, it can be shown that this model leads to a representation of expected price as a geometrically weighted moving average of past prices[12]

$$p_t^* = \beta p_{t-i} + (1-\beta)\beta p_{t-2} + (1-\beta)^2 \beta p_{t-3} + \cdots. \tag{15}$$

Muth (29) has shown that adaptive expectations are optimal if the time series to be forecast is the result of two kinds of random components, one lasting a single time period and the other lasting through all subsequent time periods. Following Friedman (14), Muth calls these respectively the transitory and permanent components of the time series. The sense in which the forecasts are optimal is that they either give the means of the distribution of the actual values of the series or are a least-squares approximation to them. The former is the case when the permanent and transitory components are not statistically

[11] Goodwin (15) points out that the coefficient α can be considered as an average of the coefficients of individual subgroups of producers weighted by the elasticities of their respective supply functions.

[12] See Nerlove (32) where the problems of estimation are also discussed at some length.

independent;[13] and the latter is the case when the permanent and transitory components are independent. The second case can be illustrated by the following example: A price at time t is divided into two components $\bar{p}_t$ and η_t, respectively the permanent and transitory components,

$$p_t = \bar{p}_t + \eta_t . \tag{16}$$

If the transitory component is assumed to have mean zero and finite variance $\sigma_\eta{}^2$; and they are statistically independent of one another, i.e. $E\,\eta_t\,\eta_s = 0,\ t \neq s$; and if the changes in the permanent component are statistically independent of each other and of the transitory components, then form 15 is a least-squares estimate of the mean of the distribution of actual prices, provided β is a certain function of the relative variances of the transitory component and the changes in the permanent component (29, pp. 6-8). It is also true that the forecast for all future periods is the same, although the standard error of forecast becomes larger in the more distant future.

These results show that for certain kinds of time series, adaptive expectations are optimal in the sense of being good forecasts. Insofar as good forecasts are useful to the farmers, adaptive expectations are rational ones for such time series; but it is very doubtful that the time series of agricultural prices are of the random character necessary for this to be true. The reason is simply that agricultural prices are generated in large part by an economic mechanism; they are only in part the result of stochastic forces. It is highly doubtful that they can be well represented solely by the appropriate purely stochastic scheme.

Rational

Muth's discovery that adaptive expectations are optimal only under rather restrictive assumptions which can only doubtfully be applied to economic phenomena has led him to a formulation which he calls the "rational expectations hypothesis" (30). In some instances this does result in adaptive expectations, but in others it does not. From the standpoint of economic theory, the rational expectations hypothesis is the most attractive hypothesis concerning the formation of expectations which has been formulated to date and which is sufficiently simple to be

[13] When the actual value, for example, p_t, is a weighted sum of independent shocks, ϵ_t, $\epsilon_{t-1}, \cdots$, of the form

$$p_t = \epsilon_t + \beta \sum_{i=1}^{\infty} \epsilon_{t-i}$$

where $$E\epsilon_t = 0,\ E\epsilon_t^2 = \sigma^2 < \infty \text{ and } E\epsilon_t\epsilon_s = 0,\ t \neq s .$$

In this case, a weighted moving average of the form (7) gives the mean value of the distribution of p_t. Furthermore, this is true for all subsequent periods, $t + 1, t + 2, \cdots$. In short, p_t^* is an estimate of the permanent component and independent of the future date for which the forecast is made. See Muth (29), pp. 2-5.

used in connection with time-series analysis. It merits a detailed description.

Stated in concise form, the rational expectations hypothesis " . . . is that, expectations being informed predictions of future events, are essentially the same as the prediction of the relevant economic theory." Superficially the hypothesis thus stated might appear grossly unreasonable. After all, farmers and businessmen are not economists, let alone econometricians; how then could they formulate the appropriate economic model and use it in the best way possible to forecast future events? On closer examination, however, it appears to the author that this hypothesis is far more reasonable than it first sounds.

First, the rational expectations hypothesis does not require that every farmer or businessman formulate a correct and relevant economic model. Economists cannot even do that! What it does require is that the representative firm behave as if it had made predictions on the basis of the same economic model used by the economist to analyze industry behavior. It implies expectations which are constructs of the same nature as "certainty equivalents," "adaptive expectations," and "supply functions" — indeed almost any other economic concept. Furthermore, the expectations thus generated will be entirely consistent with the economic model used and will have the additional advantage of not assuming less rationality in the formation of expectations than in other forms of economic behavior. If one is prepared, for the purposes of a predictive model, to assume that on the average producers maximize profits, it does not make sense to assume that they err greatly in making forecasts on the average, or at least err more than the model used to predict their behavior.[14] The rational expectations hypothesis an attractive one from the aesthetic standpoint and because of its consistency both with general economic theory and the particular economic model underlying the statistical analysis undertaken.

Second, if expectations were not rational, at least on the average, then insofar as our economic model approximates reality we should tend to find a small group of individuals, whose expectations are better than those of the rest, gradually driving the others out of business. This is essentially the same argument used to support the hypothesis of profit maximization under conditions of competition: Those who do not maximize do not survive; therefore, those who survive must achieve maximum profits on the average.

Third, insofar as this argument is unconvincing, Muth shows that it is possible to introduce elements of irrationality into the picture. Such deviations from rationality are, of course, unimportant when they are unsystematic. This is what we mean when we speak of rational expectations "on the average." But if the deviations are systematic, biased expectations may result. Muth (30, pp. 17-19) gives an example of how

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[14] This, in fact, has been the chief criticism leveled at the cobweb theorem, namely that it rests on an extreme assumption of irrationality in expectation formation but assumes rationality in other aspects of behavior. A simple cobweb model will be discussed in the light of the rational expectations hypothesis.

such biases may be introduced. The approach of introducing errors in expectations explicitly as deviations from rationality has the advantage of making clear exactly how elements of irrationality enter the picture and precisely what effects they have on behavior, although introducing them in any way tends to destroy some of the simplicity inherent in the rational expectations hypothesis.

The acid test of any hypothesis is whether it proves useful in explaining actual behavior, not what it assumes and what it does not assume. The rational expectations hypothesis has only recently been proposed, and so faced the test of application to only a very limited extent. However, what limited evidence has been brought to bear tends to support the rational expectations hypothesis: Simple cobweb models which are based on extrapolative or adaptive expectations suggest that we should observe negative serial correlations in prices and cycles of relatively short duration. Both predictions are contradicted by experience. On the other hand, as Muth (30, pp. 32-39) shows, simple cobweb models based on rational expectations suggest that prices will exhibit positive serial correlation and cycles longer than three or four production periods (depending on which way cycles are measured).[15]

Application of the Rational Expectations Hypothesis to a Simple Cobweb Model

One of the attractive features of the rational expectations hypothesis is that the character of the implied expectations depends on the entire model. This feature makes the hypothesis difficult to explain out of a particular context. In this section, the results of applying the rational expectations hypothesis to a simple cobweb model are examined. These results and their derivation are reported by Muth (30, pp. 13-17) although the exposition here is thought to be somewhat clearer than Muth's, perhaps because it is less concise.

Consider a simple model of a market containing a supply equation,

$$q_t = a + b\,p_t^* + u_t \tag{17}$$

where q_t is the quantity supplied, p_t^* is the expected price and u_t is a random residual which reflects variations due to such factors as weather, technology, or other variables exogenous to the market in question. Note that the u_t need not be distributed independently of t,

[15] Breimyer (4, p. 765) suggests the hog cycle (trough to trough) may have a period between five and six years, and (3, p. 3) that the cattle cycle (trough to trough) may have a period between thirteen and seventeen years. In neither case do these periods correspond in any reasonable way with the "period of production" or "gestation period" which is much, much shorter than half the period of the cycle. Of course, as Goodwin (15) pointed out, coupling of two or more dynamic markets of the cobweb type may lead to irregular cycles and cycles longer than the individual markets would exhibit in isolation. However, it seems unlikely that such large discrepancies between theory and experience could be explained by coupling.

i.e., it need not be true that E $u_t u_s = 0$, $t \neq s$. Indeed, this assumption would generally be inappropriate. The model also contains a demand equation, which we will assume to be exact:

$$q_t = c + d\, p_t . \tag{18}$$

Under these circumstances the equilibrium price, $\bar{p}$, and quantity, $\bar{q}$, are obtained by setting $u_t = 0$, $p_t^* = p_t = \bar{p}_t$ and $q_t = \bar{q}_t$:

$$\begin{aligned} \bar{p} &= \frac{c - a}{b - d} \\ \bar{q} &= \frac{bc - ad}{b - d} . \end{aligned} \tag{19}$$

Suppose the disturbance term u_t can be written as the weighted sum of independently and normally distributed random variables ϵ with zero means and common variance σ^2:

$$u_t = \sum_{i=0}^{\infty} w_t \, \epsilon_{t-i} . \tag{20}$$

If $w_0 = 1$ and $w_i = 0$ for $i = 1, 2, \cdots$, $u_t = \epsilon_t$ and the u_t are, therefore, independently distributed. On the other hand, more realistically, if the weights for previous periods are not all zero, u_t is serially correlated. By equating q_t in equations 17 and 18, replacing (c - a) by (b - d) $\bar{p}$ from its value in equation 19 and rearranging terms, we find that the deviation of the actual price from the equilibrium price is the following function of the deviation of the expected price from the equilibrium and the independent disturbances ϵ_t:

$$p_t - \bar{p} = \frac{b}{d}(p_t^* - \bar{p}) + \frac{1}{d} \sum_{i=0}^{\infty} w_i \, \epsilon_{t-i} . \tag{21}$$

For simplicity, let p_t' and $p_t^{*\prime}$ represent the deviations $p_t - \bar{p}$ and $p_t^* - \bar{p}$ respectively. Then

$$p_t' = \frac{b}{d} p_t^{*\prime} + \frac{1}{d} \sum_{i=0}^{\infty} w_i \, \epsilon_{t-i} . \tag{21$'$}$$

According to the rational expectations hypothesis, stated in precise form, expectations are " . . . distributed . . . about the prediction of the theory," given the same information available to the decision makers (30, p. 3). Thus, if expectations are rational, their average value, $p_t^{*\prime}$, must equal the mean value of the distribution of the actual value of price, p_t', <u>given past events.</u> The past in this model is summarized in the values of ϵ_{t-1}, ϵ_{t-2}, $\cdots$, which are not directly observable. Thus, under the rational expectations hypothesis, we must have

$$p_t^{*\prime} = E(p_t' \mid \epsilon_{t-1}, \epsilon_{t-2}, \cdots)$$

(22) $$= \frac{b}{d} p_t^{*\prime} + \frac{1}{d} E\left(\sum_{i=0}^{\infty} w_i \epsilon_{t-i} \mid \epsilon_{t-1}, \epsilon_{t-2}, \cdots\right)$$

$$= \frac{b}{d} p_t^{*\prime} + \frac{1}{d} \sum_{i=1}^{\infty} w_i \epsilon_{t-i}$$

since $E(\epsilon_t \mid \epsilon_{t-1}, \cdots) = E(\epsilon_t) = 0$, by assumption. It follows that

(23) $$p_t^{*\prime} = \frac{1}{d-b} \sum_{i=1}^{\infty} w_i \epsilon_{t-1},$$

i.e., the expected price for the t^{th} period is a weighted sum of past (unobservable) random shocks. The problem of relating these expectations to observable variables remains to be solved.

First, it is noted that if the u_t are statistically independent, $w_0 = 1$ and $w_i = 0$, for, $i = 1, 2, \cdots$. Therefore,

(24) $$p_t^{*\prime} = 0$$

i.e., the expected price p_t^* is just the equilibrium price $\bar{p}$. In other words, under the rational expectations hypothesis with statistically independent shocks in the supply curve and a fixed demand curve, all variations in supply are caused by the random shocks. Variations in supply should exhibit no pattern and the observed elasticity of supply with respect to any observed price should be zero.

If the shocks u_t, however, are not statistically independent, more realistic results are obtained. To analyze this case, we express actual prices p_t' as a function of current and past values of ϵ_t by substituting equation 23 in equation 21′.

(25) $$p_t' = \frac{w_0}{d} \epsilon_t + \frac{1}{d-b} \sum_{i=1}^{\infty} w_i \epsilon_{t-i}.$$

Equation 25 is a linear difference equation in ϵ_t and can be solved for ϵ_t in terms of current and past values of p_t'. Thus, the general solution to the problem of expressing $p_t^{*\prime}$ in terms of observable variables, would be to replace $\epsilon_{t-1}, \epsilon_{t-2}, \cdots$ by their values in terms of p_{t-}', $p_{t-2}' \cdots$. However, instead of proceeding in this direction, which is not very enlightening, it is preferable to introduce more explicit assumptions about u_t at this point. Suppose, as is reasonable, the random shocks ϵ_t have a permanent and a transitory effect on u_t: The full value of the current shock ϵ_t is reflected in the current value of u_t. A positive fraction of this shock, for example β, permanently affects u_t. The remainder, $1-\beta$, has no effect in subsequent periods; it is transitory. Then u_t may be written

$$u_t = \epsilon_t + \beta\,\epsilon_{t-1} + \beta\,\epsilon_{t-} + \cdots$$
$$(26) \qquad = \epsilon_t + \beta \sum_{i=1}^{\infty} \epsilon_{t-i}\,.$$

Comparison with equation 20 reveals that

$$(27) \qquad \begin{aligned} w_0 &= 1 \\ w_i &= \beta,\ i = 1, 2, \cdots . \end{aligned}$$

Hence, from equation 25 we have

$$(28) \qquad p'_t = \frac{1}{d}\epsilon_t + \frac{\beta}{d-b}\sum_{i=1}^{\infty} \epsilon_{t-i}\,;$$

hence,

$$(29) \qquad \epsilon_t = d\,p'_t - \frac{d\,\beta}{d-b}\sum_{i=1}^{\infty} \epsilon_{t-i}\,.$$

This is a difference equation in ϵ_t of a complicated sort, but instead of solving it directly we proceed as follows: Note that by equations 27 and 28 we may write expected price as

$$(30) \qquad p_t^{*\prime} = \frac{\beta}{d-b}\sum_{i=1}^{\infty} \epsilon_{t-i}\,.$$

By equation 28, therefore,

$$(31) \qquad p'_t = \frac{1}{d}\epsilon_t + p_t^{*\prime}\,.$$

It follows that

$$(32) \qquad \epsilon_{t-i} = d\,(p'_{t-i} - p^{*\prime}_{t-i})\,.$$

Substituting equation 32 in equation 30 and lagging one period, we have

$$p^{*\prime}_{t-1} = \frac{d\,\beta}{d-b}\sum_{i=1}^{\infty}(p'_{t-1-i} - p^{*\prime}_{t-1-i})$$

$$(33) \qquad = \frac{d\,\beta}{d-b}\sum_{i=1}^{\infty}(p'_{t-i} - p^{*}_{t-i}) - \frac{d\,\beta}{d-b}(p'_{t-1} - p^{*}_{t-1})$$

$$= p_t^{*\prime} - \frac{d\,\beta}{d-b}(p'_{t-1} - p^{*\prime}_{t-1})\,.$$

Since the equilibrium price cancels out when a difference of primed variables is taken, equation 33 may be rewritten in the form

(34) $$p_t^* - p_{t-1}^* = m\,(p_{t-1} - p_{t-1}^*)$$

where $$m = {}^{d\beta}/_{d-b} \quad .$$

Since $d < o$, $b > o$, and $o < \beta \leq 1$, $o < m \leq 1$. Equation 34 is just the equation which generates adaptive expectations. Thus adaptive expectations are also rational ones in this particular case. Note, however, because of the particular interpretation of the coefficient of expectations, m, the equilibrium must be stable.[16]

When other variables such as income are introduced in the demand equation or a random shock is added to it, the situation becomes more complicated. Muth indicates that rational expectations in such cases will depend on other observable variables in addition to past prices. Although the author has not yet worked out examples of this sort, he has no doubt that such examples would show rational expectations in more realistic cases to be quite different from the simple kind of adaptive expectations used in previous work on supply.

Also noteworthy at this point is that even in the simple case discussed here, the estimation techniques proposed for models based on adaptive expectations are inappropriate in the case of rational expectations, despite the fact that rational expectations turn out to be of the adaptive form. The reason for this is that the residuals u_t in equation 17 are serially correlated in a way different than that assumed in the development of the estimation procedure. If the recommended procedure (31) is used, the estimates of the elasticity of supply and of the coefficient of expectations will generally be biased. Suitable estimation procedures can be developed along lines suggested by Klein (26), but they will not be discussed here.

In conclusion, it may be said that rational expectations are difficult to find even for very simple economic models. This does not mean, however, that they are not worth finding. They have the property of being entirely consistent with the economic model into which they are introduced. The little qualitative evidence developed supports the rational expectations hypothesis. There is clearly a need for more evidence of a quantitative character.

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[16] See Nerlove (31), where it is shown that the condition

$$1 - b/d < 2/m$$

must be satisfied for stability. Since $2/\beta$ is obviously greater than 1, by our assumptions, the condition is always satisfied for $m = d\beta/d\text{-}b$. The fact that the equilibrium is always stable, however, does not preclude cycles because of the effect noted by Slutzky (42). These will generally be of an irregular character and substantially longer than the two period cycle predicted by the ordinary cobweb theorem.

CONCLUSIONS

The main conclusion of this paper is that there is the need for much more preoccupation with theory and less with estimation techniques in agricultural supply analysis. The foregoing discussion is, of course, greatly restricted in scope, not only to the areas of uncertainty and investment, but within those areas to a few simple models.

In connection with the problem of investment in fixed and quasi-fixed factors, a simple model showing how the relation between the short- and long-run industry response to price was determined by the presence of fixed factors of production was analyzed. Despite its simplicity the model suggested several ways in which time-series supply analyses should be modified. The need for further and more complete theoretical research is apparent.

In connection with the problem of uncertainty, it has been suggested that the theoretical notion of certainty equivalent is essential to time-series supply analysis. A more tentative suggestion has been that the rational expectations hypothesis, recently proposed by Muth, may be a very fruitful one for time-series analysis. This hypothesis implies that expectations depend on the theoretical model employed and are therefore as good as the model and no better. Thus economic theory plays a crucial role.

These remarks are not meant to imply that we should all concentrate on theory for the next few years and give up the task of measuring supply elasticities. Theoretical developments can only be fruitful in the context of real problems; empirical investigations suggest new theoretical approaches as well as vice versa. One may conclude that time-series supply analysis is greatly in need of more and better theory. Estimation techniques are only a means to an end. If there has been some tendency in the past to let techniques of estimation dictate the theory or the problems studied empirically, it should be corrected. Only in this way can the full interplay of theory and practice leading to a better understanding of supply behavior be realized.

APPENDIX: FIXED FACTORS AND AGGREGATION

The classical distinction between extensive and intensive margins of production is closely related to the differences among firms in their endowments of fixed factors and technical knowledge. Suppose we have knowledge about a number of "typical" farms hog-dairy and hog-cash grain, for example), and can deduce optimal production of hogs for these farms at various combinations of pork, dairy, and feed prices. We still know relatively little about the effects of changes in pork prices on aggregate hog production with unchanged dairy and feed prices or with specified changes in these prices. The reason is simply that we do not know what part of the aggregate supply response occurs on those farms which have that combination of fixed factors which makes them "typical" hog producing farms and what part occurs on farms which have combinations which make them "atypical."

In this appendix, a simple model suggesting how cross-section data can be used to estimate a short-run industry elasticity of supply is discussed. The purpose of this model is not to suggest a practical method of doing this, but to exhibit the connection between the fixed factor problem and the aggregation problem.

As noted in the text, the question of whether to include fixed factors in the production function is moot. For the present purpose, which concerns short-run response to price, it is convenient to include only variable inputs in the production function. Consider an industry, F, which is a population of firms, f. The question to be answered is: Can we, by taking a sample of firms in F at a point in time, determine the short-run industry supply function? The answer is yes in the simple case discussed below.

Suppose that each firm in the industry produces a single homogeneous output using two variable factors of production. Furthermore, suppose that each firm expects the prices it actually receives for its product and pays for its variable inputs and under these circumstances it acts in such a way as to maximize the return to its fixed factors of production. Let

p_0 = the price f receives for its product

p_i = the price f pays for variable input i, i = 1, 2

x_{of} = the quantity of output f produces

x_{if} = the quantity of input i f uses, i = 1, 2

and

$$y_{of} = p_0 x_{of}$$
$$y_{if} = p_i x_{if}$$

(Note that the prices paid and received are assumed to be the same for every firm.) We assume that each firm has a production function relating variable inputs to output of the same form, but that the parameters of this function differ from one firm to another reflecting the fact that different firms are possessed of differing amounts of technical knowledge and fixed factors:

$$x_{0f} = (a\, u_{0f})\, x_{if}^{\;a_1 u_{1f}}\, x_{2f}^{\;a_2 u_{2f}} \tag{35}$$

where the terms u_{of}, u_{1f} and u_{2f} reflect differences in the parameters of the production functions.[17] Under the assumptions each firm

[17] This type of production function has been used in connection with the problem of relative economic efficiency (34). The properties and meaning of the function are discussed more fully in this paper.

$$R = p_0 x_{0f} - p_1 x_{1f} - p_2 x_{2f} \tag{36}$$

maximizes the return to its fixed factors subject to equation 35. If this is done perfectly, we must have

$$a_i u_{if} = y_{if}/y_{0f}, \quad i = 1, 2, \tag{37}$$

for all f.[18]

The supply function for the individual firm can be shown to be

$$x_{0f} = K_f \left[p_0^{a_1 u_{1f} + a_2 u_{2f}} \; p_1^{-a_1 u_{1f}} \; p_2^{-a_2 u_{2f}} \right]^{\frac{1}{1 - a_1 u_{1f} - a_2 u_{2f}}} \tag{38}$$

where K_f is a function of $a_0 u_{0f}$, $a_1 u_{1f}$, and $a_2 u_{2f}$.[19] Since the prices paid and received are assumed to be the same for every firm, the quantity supplied by a particular firm differs from that of any other firm because of differences among firms in their possession of fixed factors and technical knowledge, i.e., only because the u's differ among firms. Let x_0 be the quantity supplied by the industry and let $\varphi(u_0, u_1, u_2)$ be the joint density function of the distribution of the u's among firms, then the short-run industry supply function may be written

$$x_0 = \int_{u_0} \int_{u_1} \int_{u_2} x_{0f} \, \varphi(u_0, u_1, u_2) \, du_0 \, du_1 \, du_2 . \tag{39}$$

Equation 39 is the short-run supply function because the fixed factors reflected in the u's are not allowed to vary when prices change. For simplicity, x_{0f} has been written in equation 39 rather than the expression on the right-hand side of equation 38. Replacing x_{0f} in equation 38 by this shows the dependence of industry supply on the prices and the distribution of the u's.

If we knew the production function for every firm in the industry, it would be a simple matter to determine the industry supply function. Knowledge of the production function for each firm could be obtained from a knowledge of the ratio of payments to variable factors to gross revenue for each firm by equation 37 and

$$a_0 u_{0f} = \frac{x_{0f}}{x_{1f}^{\,y_{1f}/y_{0f}} \; x_{2f}^{\,y_{0f}/y_{0f}}} \tag{40}$$

[18] This is a well-known result for production functions of the general Cobb-Douglas form. The assumption of this form is what makes the subsequent discussion as easy as it is. Production functions of other forms could be used, but then the analysis would be much more complex. This is one reason for regarding the model discussed here as illustrative only.

[19] This result is derived in the appendix to Bachman and Nerlove (1).

But, of course, in any realistic situation it is unlikely that we will have this information for any but a small sample of firms. If this is the case, equation 39 suggests that we can still estimate the short-run industry supply function provided we can estimate the parameters of the density function $\varphi(u_0, u_1, u_2)$. This is true because the unknown u's are "integrated out" in equation 39.

Suppose that the u's have a joint lognormal distribution, i.e., if

$$w_{if} = \log u_{if} \text{ for } i = 0, 1, 2$$

then

$$(41) \quad \varphi(u_0, u_1, u_2) = \frac{\sqrt{1\Gamma^{-1}1}}{(2\pi)^{3/2}} \, e^{w_0 + w_1 + w_2} \, e^{-\frac{1}{2}(w_0 \, w_1 \, w_2)\,\Gamma^{-1}\begin{pmatrix} w_0 \\ w_1 \\ w_2 \end{pmatrix}}$$

where Γ is the variance-covariance matrix of the w's and the means of the w's are assumed to be zero.[20] It follows from equation 39 that the short-run industry supply function is

$$(42) \quad x_0 = \int_0^\infty \int_0^\infty \int_0^\infty \frac{x_0 \, e^{w_0 + w_1 + w_2} \sqrt{1\Gamma^{-1}1}}{(2\pi)^{3/2}} \, e^{-\frac{1}{2}(w_0 w_1 w_2)\,\Gamma^{-1}\begin{pmatrix} w_0 \\ w_1 \\ w_2 \end{pmatrix}} dw_0 dw_1 dw_2$$

where x_0 is written as a function of the w's rather than the u's:

$$(43) \quad x_{0f} = K' \left[p_0^{a_1 e^{w_{1f}} + a_2 e^{w_{2f}}} \, p_1^{-a_1 e^{w_{1f}}} \, p_2^{-a_2 e^{w_{2f}}} \right]^{\frac{1}{1 - a_1 e^{w_{1f}} - a_2 e^{w_{2f}}}}$$

Thus, leaving aside the difficulties of integrating a complex expression as appears in equation 42, an estimate of the short-run industry supply function can be obtained from a sample of firms if the observed variables in the sample can be used to estimate Γ, the variance-covariance matrix of the w's.

Suppose we have a random sample of N firms from F and know for each of them the values x_0, x_1, x_2, y_0, y_1, and y_{2f}. Let

[20] The assumption of zero means can always be made, since the a_i, $i = o, 1, 2$ can always be chosen so that

$$E \log a_i u_{if} = E \log a_i + E w_{if} = \log a_i + E w_{if} = \log a_i.$$

$$Z_{if} = \log (y_{if}/y_{of}), \; i = 1, 2,$$

and

$$Z_{of} = \log\left(\frac{x_{of}}{x_{1f}^{\,y_{1f}/y_{of}} \; x_{2f}^{\,y_{2f}/y_{of}}}\right).$$

Now consider the sample variances and covariances of the Z's

$$(44) \qquad s_{ij} = \frac{1}{N} \sum_{f=1}^{N} (Z_{if} - \overline{Z}_i)(Z_{jf} - \overline{Z}_j)$$

where

$$\overline{Z}_i = \frac{1}{N} \sum_{f=1}^{N} Z_{if}.$$

It can be shown that s_{ij} is the maximum-likelihood estimate of σ_{ij}, where σ_{ij} is the population variance or covariance between w_i and w_j,[21] i.e.,

$$\sigma_{ij} = E\, w_i\, w_j .$$

It follows that

$$(45) \qquad G = \| s_{ij} \|$$

is the maximum-likelihood estimate of Γ, so that replacing Γ^{-1} by G^{-1} in equation 42 yields the maximum-likelihood estimate of the short-run industry supply function.

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[21] The proof is somewhat complicated and will not be given here.

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Discussion

MARC NERLOVE has provided us with a very useful survey of the major problems involved in the time-series analysis of supply functions for agricultural products. The quality of the essay is outstandingly high — it is rigorous and it presents important issues. In the past few years, Nerlove has made important contributions to the study of the supply of agricultural products; the present paper provides a strong basis for the expectation that such contributions will continue into the future.

Of the many important points discussed by Nerlove, I will comment on only two. One is the certainty equivalent and its rehabilitation, and the other is the rational expectations hypothesis.

The certainty equivalent has a very considerable appeal to the economist. This appeal rests in large part on the fact that if the certainty equivalent represents a useful simplification, the statistical analysis of important problems becomes possible. If we assume that the entrepreneur maximizes expected utility and does not place rather specific restrictions upon the forms of various functions, it is at once evident that if we are to explain that entrepreneur's behavior in an uncertain situation we must know a great deal. Even if we knew both the utility function and the cost function, and both were known with certainty by the observer, the required knowledge about the entrepreneur's expectations concerning prices of input and output in various time periods presents a staggeringly complex analytical and empirical problem. If one drops the apparently unreasonable assumption (at least in the case of agriculture) that the cost function is known with certainty, one greatly increases the complexity of the situation.

Thus there is little wonder that economists, as rational beings, should search for simplifications that could reduce an insuperable task to one of manageable proportions. Some may argue that the use of the certainty equivalent in time-series analysis represents a high degree of simplification and may not present as accurate a representation of behavior as we would like. However, the use of certainty equivalents makes the analysis manageable and we can only find out its usefulness through actual empirical work.

In adopting the certainty equivalent in time-series analysis of agricultural supply functions, it must be recognized that there is always a possibility that farmers are confronted with a change in circumstances such that their behavior cannot be adequately predicted by the restrictive assumptions utilized. I am convinced, and I see nothing in Nerlove's presentation inconsistent with the position I take, that many of the investment decisions made by farmers cannot be adequately analyzed within the framework of the certain equivalent. If this is a valid position, what are its implications to the use of certainty equivalents in time-series analysis of supply responses? If the expected probability distributions of prices do not change appreciably over the relevant time period, I doubt if significant difficulties would arise. However, if there is a change to a price support system that really worked in the sense that price variability was reduced significantly, investment decisions would be modified even if the expected mean price remained unchanged. The change in investment would affect the cost functions and other variables entering decision functions. Thus it is likely that in the case of a period involving a significant change in the factors affecting price formation, the simplifying assumption of certainty equivalents may be an unsatisfactory one. It must be noted that such changes, at least in some cases, are quite obvious to the researcher, and gross mistakes should be avoidable.

A time-series analysis of supply will usually (hopefully) cover a period of two or three decades. During a period of that length, it is not at all unlikely that the income and wealth positions of farmers will increase by as much as 50 to 100 percent. As a result, it is possible that many farmers' attitudes about risk or uncertainty bearing may change during the period. The decisions made in response to given probability distributions of prices may not be the same at the end of the period as at the beginning. I suspect, though I doubt if I could present the evidence to support it, that many farmers in 1960 were much more willing to undertake ventures with a wide range of possible outcomes than farmers with the same general characteristics falling in the same relative wealth position in 1935 or 1940. However, it should be noted that I know of no other model that adequately takes account of the relationships between income or wealth and the various decision variables.

The rational expectations hypothesis is an inherently appealing one, since it rests on the presumption that economic theory is meaningful and important. The idea is new and it appears that it is worthy of serious attention, consideration, and application. Since I must play the role of a critic, I feel I must introduce at least several possible difficulties.

The first is that competition in agriculture need not result in profit maximization by most farmers. This is true because a large fraction of the resources used do not have a contractual price and "losses" can be absorbed for a considerable period of time as reduced returns to owned resources and perhaps even by resources rented on a share basis. As more and more of the resources used in agricultural production are purchased, the forces of competition are more likely to result in profit maximization as a condition for survival.

The second comment is not in any real sense a criticism of the rational expectations hypothesis, but it is an indication of the difficulties that may be involved. The example may be taken from corn and hog price relationships. Let us assume that price expectations for corn are uniform among all producers and have a small variance due to the price support and storage programs. The application of the rational expectations hypothesis is confronted with the rather serious difficulty that the supply function for corn is very elastic over a rather wide range. Thus the supply function for hogs is much more elastic than it would be in the absence of a price support and storage program for corn. The more elastic the supply function for a product, the greater will be the effect on planned output of a given change in expected price. If these simple statements are approximately correct, it means that the expected prices derived on the basis of the rational expectations hypothesis are likely to be subject to large errors due to the combination of very elastic supply and relatively low price elasticity of demand.

The above comments about the possible difficulties of utilizing the rational expectations hypothesis apply, with equal or greater force, to any other price expectations model. Perhaps the greater relevance of the comments is to problems of achieving price stability.