CHAPTER 9

The Measurement of Changes in Demand: Multiple Correlation

The graphic method of multiple curvilinear correlation is, as it were, an F_2 product. It is an offshoot of an offshoot of the standard mathematical method of linear multiple correlation.

The first offshoot originated fifteen or twenty years ago in the fertile mind of Mordecai Ezekiel.[1] He was working with the problem of curvilinear regressions (curved lines of relationship between different series of data). The simplest form of the standard mathematical method involves the assumption that the data are related in straight-line fashion. If this assumption is not valid—if the regressions are actually curvilinear—the standard method yields inaccurate results; before accurate results can be obtained, the curves must be represented by mathematical equations incorporated in the basic formula.

But the regressions cannot be determined accurately until the nature of the curvature is known, and the nature of the curvature cannot be determined accurately until the regressions are known. Ezekiel broke through this impasse by the method of successive approximations, starting with mathematically determined straight lines and adjusting them by graphic methods.[2]

[1] Mordecai Ezekiel, "A Method of Handling Curvilinear Correlation for any Number of Variables," *Journal of the American Statistical Association*, XIX, No. 148, 1924, p. 441. *See also* the more recent presentation in his book *Methods of Correlation Analysis*, John Wiley & Sons, 1941, Chap. 14.

[2] In his own words: "The linear partial regressions are . . . computed [by the standard mathematical correlation method]. Then the dependent variable is adjusted for the deviations from the mean of all independent variables except one, and a correlation chart, or dot-chart, is constructed between these adjusted values and that independent variable. This provides the basis for drawing in the first approximation curve for the net regression of the dependent variable on that independent variable. . . . The dependent variable is then corrected for all except the next independent variable, the corrected values plotted against the values of that variable, and the first approximation curve determined with respect to that variable. This process is carried out for each independent variable in turn, yielding a complete set of first approximations to the net regression curves. These curves are then used as a basis for correcting the dependent factor for the approximate curvilinear effect of all independent variables except one, leaving out each in turn; and second approximation curves are determined by plotting these corrected values against the values of each independent variable in turn. New corrections are made from these curves, and the process is continued until no further change in the several regression curves is indicated."—Ezekiel, *op. cit.*, Chap. 14, p. 223.

This first offshoot of the standard mathematical method of linear multiple correlation, therefore, was a hybrid, a combination of mathematics and graphics. Then Louis Bean developed the second offshoot.[3] This second offshoot shed the mathematics inherited from its grandparent completely and became entirely graphic. In effect, Bean said, it is a waste of time to fit straight-line curves mathematically, and then modify them graphically, freehand, to make them fit any curvature existing in the regressions. Don't bother with any mathematics, he said; put the regression lines in freehand in the first place, curves and all.

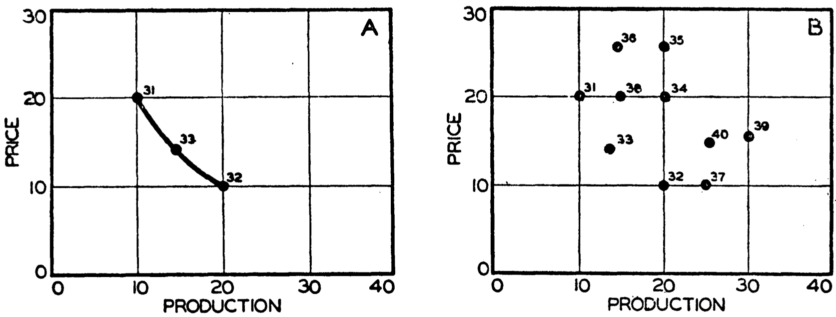


FIG. 41.—Hypothetical price and production data.

But how does one know where to draw in the regression lines right off? The essence of Bean's contribution is his simple answer to this question.

In simple correlation, the dependent variable, for example price, is related to one independent variable, for example production. One simply plots price against production in a scatter-diagram, and draws in the regression line, straight or curved, wherever the dots indicate that it should go. This is illustrated in section A of Figure 41, based upon the first three price and production items in Table 14.

In the simplest case of multiple correlation, the dependent variable price is related to two independent variables, for example production and demand. Two scatter-diagrams are required here —one to show the regression of price on production (or in more everyday language, the influence of production on price) indepen-

[3] L. H. Bean, *Applications of a Simplified Method of Graphic Curvilinear Correlation,* BAE, USDA, April, 1929, mimeo.; and "A Simplified Method of Graphic Curvilinear Correlation," *Journal of the American Statistical Association,* XXIV, December, 1929, pp. 386–97.

dent of the influence of demand on price; and the other to show the influence of demand on price independent of the influence of production on price.

In handling a multiple correlation problem of this sort, the first

TABLE 14
PRICE, QUANTITY, AND DEMAND SCHEDULE
(Hypothetical Data)

Year	Price	Production	Index of Demand
1931	20	10	10
1932	10	20	10
1933	14	14	8
1934	20	20	20
1935	25	20	25
1936	25	15	20
1937	10	25	15
1938	20	15	15
1939	15	30	25
1940	15	25	20

thing to do is to plot the dependent variable price with one of the independent variables, say production, in a simple scatter-diagram. The price and production data from Table 14 are thus plotted in section B of Figure 41.

The dots in this section B are scattered about with no evidence of any relationship. But this may be because the influence of production on price is obscured by the co-existing influence of demand on price. What we want is the *net* influence of production on price—the influence of production on price independent of the influence of demand on price.

This word "independent" is the key to the graphic method. One way to determine the influence of production on price independent of the influence of demand on price is to choose two years in which the values of the demand variable are identical. Any change in price from one of these years to the other then must show the influence of production on price independent of the influence of demand on price, since demand did not change from the one year to the other. A line connecting these two years would then be a preliminary indication or estimate of the influence of production on price independent of the influence of demand on price.

Inspection of Table 14 shows that there are several pairs of years in which the values of the demand variable are identical (within each pair). The years 1931 and 1932, for example, both carry demand values of 10. The dots for these two years may therefore be connected by a light line, as shown in section A of Figure 42.

This is a beginning. If now another pair of years can be found in which the demand values are identical, another line can be drawn in connecting these two years. Two such years are 1934 and 1936. This provides a second estimate of the influence of production independent of the influence of demand. Additional pairs of years, in

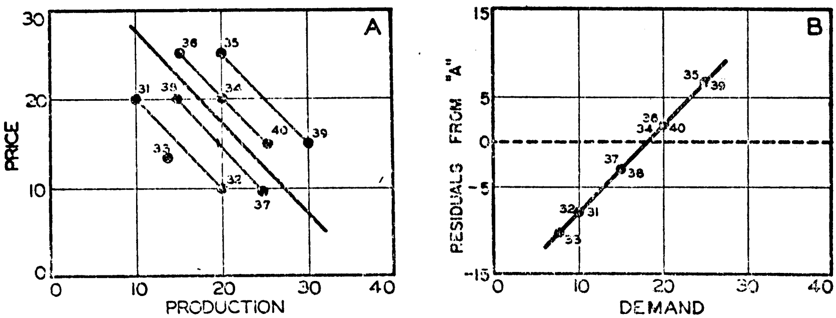


FIG. 42.—Hypothetical price and production data. Net regression curves.

each of which the demand values are identical, may also be connected, providing additional estimates.

By now the chart looks like a piece of prehistoric bedrock with scratches on it showing the direction in which a glacier passed over it. A long heavy line can now be drawn in freehand, passing through the dots with a slope representing the general average slope of various short lines on the chart. It should go through the general mean. This line is shown in section A of Figure 42. It is a first approximation to the net regression line desired.

The determination of the net influence of demand on price is then simple. If the heavy line just drawn in shows the net influence of production on price, the vertical distances of the individual dots above and below this line must show the net influence of demand on price. The way to reveal this net influence clearly is to take these vertical distances or residuals and plot them against demand in a second chart. In this chart, the demand scale runs along the bottom, like the production scale in the first chart. A horizontal line is drawn across the middle of the chart (about half way up). This line is regarded as zero on the vertical scale. The vertical distances of individual dots above or below the preliminary regression line in the first chart are then plotted above or below the horizontal line across the middle of the second chart against the respective demand readings along the bottom of the second chart. A line drawn through the dots in the second chart then represents the net influence of demand on price—net, because the dots were plotted from the regression line on the first chart which showed the net influence of A on X.

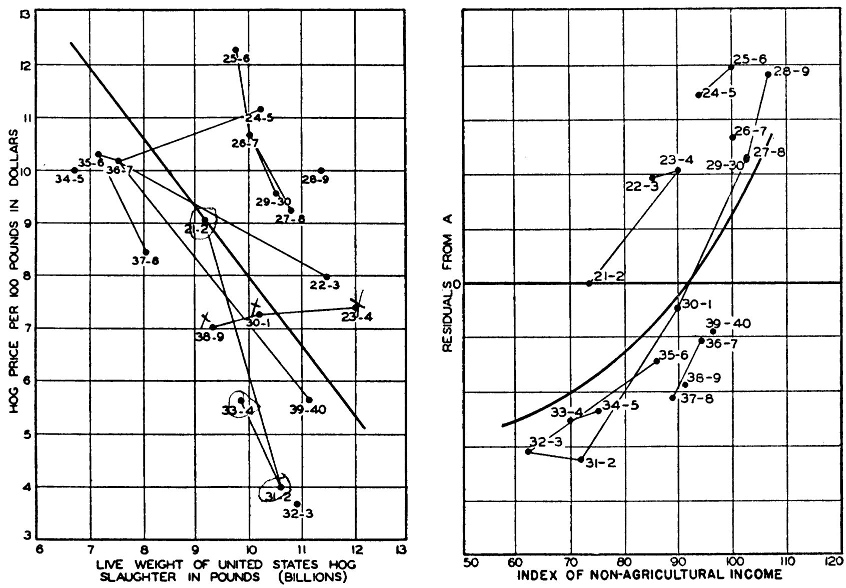
If the first approximation line in the first chart is correct, and if the dependent variable price is completely determined by the two independent variables, production and demand, the line drawn through the dots in the second chart will pass through all of them. This is the situation shown in section B of Figure 42.

If, however, some scatter still remains, either the first approximation line in the first chart was not correctly placed, or one or more additional independent variables need to be taken into account.

The test to determine whether additional variables are needed will also show whether the first approximation line was correctly placed. It consists in taking the residuals from the second chart (the vertical distances above and below the net regression line) and plotting them in red or in some other distinguishing manner, above or below the first approximation regression line in the first chart. (Or this first regression line may be traced on a new clean sheet of graph paper, with the same scales as the original chart). If they fall uniformly about this first approximation line, then a second approximation is not required; what is needed is one or more additional variables. If, however, the dots do not fall uniformly about the first approximation line, but have a different curvature or slope, this indicates that the first approximation line was incorrectly drawn. A second approximation line must be drawn, passing as nearly as possible through the red dots, and the residuals of the *original black dots* from that line plotted in a new second chart (or in different color in the old one) against demand. This may show that the regression line in the second chart needs to be revised. The process is repeated back and forth until the scatter is reduced to the minimum. If some scatter still remains, it means that the study should be extended to include one or more additional variables. In that case the residuals remaining in the second chart should be plotted against a third variable, and so on.

EMPIRICAL ILLUSTRATION

The preceding example, based upon hypothetical data with straight-line relationships and perfect correlation among the variables, serves to illustrate the principles of the graphic method under the simplest conditions. A second illustration based upon actual



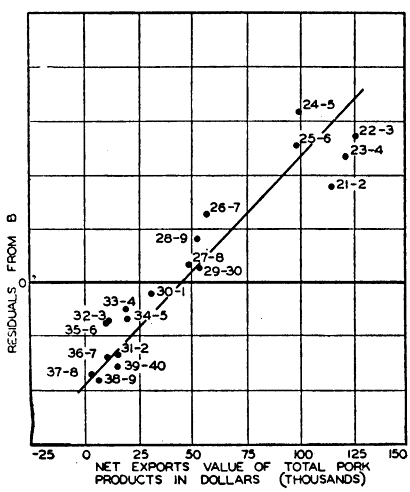


FIG. 43.—Relations between weighted average hog prices and (A) live weight of hog slaughter, and (B) total nonagricultural income, and (C) value of net exports of pork products.

empirical data of a more complicated kind will now be used. In this illustration there are three independent variables, one or two of the regressions are curved, and the correlation is not perfect. The data for this illustration are given in Table 15.

TABLE 15

UNITED STATES PRICES OF HOGS, NONAGRICULTURAL INCOME, VALUE OF EXPORTS OF PORK PRODUCTS, AND TOTAL HOG SLAUGHTER

Year	Average Hog Price (Weighted) Oct.–Sept. *	Total Live Weight of Hogs Slaughtered Oct.–Sept. †	United States Nonagricultural Income Oct.–Sept. ‡	Value of Net Exports of Pork Products July–June§
	Dollars	*Millions of lbs.*	*Calendar Year 1926 = 100*	*$(000)*
1921–22	9.06	9,156	73.6	114,490
1922	7.98	11,440	85.0	126,739
1923	7.41	12,013	90.3	122,344
1924	11.18	10,258	93.8	98,690
1925	12.29	9,776	99.9	96,009
1926	10.70	10,009	101.8	63,571
1927	9.24	10,823	102.9	47,880
1928	10.01	11,343	106.7	53,148
1929	9.57	10,530	102.9	57,962
1930	7.28	10,200	89.9	32,663
1931	4.05	10,625	71.8	14,826
1932	3.68	10,918	61.8	10,856
1933	5.64	9,872	69.9	18,294
1934	10.00	6,742	75.1	18,981
1935	10.32	7,191	86.0	8,923
1936	10.18	7,538	94.2	−986
1937	8.41	8,089	88.1	−408
1938	7.03	9,311	91.1	5,985
1939–40	5.63	11,142	96.4‖	14,309

* *Livestock, Meats and Wool Market Statistics and Related Data, 1938,* AMS, USDA, 1939, p. 77. *Livestock, Meats and Wool, Market Review and Statistics* (weekly), 1940.

† *Livestock, Meats and Wool Market Statistics and Related Data, 1938,* AMS, USDA, 1940, p. 79.

‡ "Demand, Credit, and Prices," *1941 Agricultural Outlook Charts,* BAE and AMS, USDA, 1940, p. 7.

§ *Foreign Crops and Markets* (mimeo), USDA, August issue each year, *Livestock Meats and Wool, Market Review and Statistics* (weekly), 1940.

‖ 10 month average.

The dependent variable, hog prices, is plotted first against the independent variable, hog slaughter (i. e., quantity), as in the previous illustration. The results are shown in section A of Figure 43. Here, as in the other case, the dots are scattered all over the page;

they do not fall around any clearly defined demand curve. Evidently the demand curve must have shifted up and down (or sideways) during the period so that the slope of the curve is obscured.

What is needed is some measure of these shifts or changes in demand. Several such measures are available. One of the best is the index of national nonagricultural income, which is published monthly by the Bureau of Agricultural Economics in *The Agricultural Situation.* This index may be used as the second independent variable.

The pairs of years in which the values of the index of demand (nonagricultural income) are nearly identical may then be located, and the two years in each pair joined by light connecting lines. In this case there seem to be several chains or groups of more than two years; the years in each group can be connected in a series. One such series is 1933–34, 1931–32, 1921–22, and 1934–35. Another is 1923–24, 1930–31, and 1938–39. Still another is 1925–26, 1926–27, 1927–28, and 1929–30.

A heavy straight line is then drawn in through the dots with approximately the average slope of these light lines. In the estimation of this average slope, each light line should be given an importance proportional to its length. The reason for this is that if the dots in one pair are only an inch apart, let us say, and one of the dots has been pulled up or down by some other influence a distance of half an inch, the slope of the light line connecting the pair will be very much affected. But if the pair had been four inches apart, the half inch displacement of one of the dots would have only a slight effect on the slope of the line connecting them. The longer the light line, the more likely it is to show the net influence of the variable correctly; the longer lines, therefore, should be given more weight than the shorter.[4]

This suggestion is made merely in the interests of speed. The more nearly correctly the demand curve is drawn in, the less revi-

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[4] A. G. Hart makes an interesting suggestion here. He points out that the demand for most products is continually changing with the passage of time and therefore that any index of general demand is likely to measure changes in the demand for a particular product most accurately for years that are close together (in time) and least accurately for years that are far apart (in time). Accordingly, he advocates connecting only the dots for the years (when national income was similar) that are close together in time, or at least giving them the greater importance. This suggestion is particularly apt in this case, where one of the variables (exports) shows a strong downward trend throughout. If this procedure is followed, all of the positively-sloping lines in Figure 31A disappear.

sion will be required, and the more quickly will the whole job be finished. But there is no question of principle involved. The same final result will be attained no matter what slope is given to the first approximation to the demand curve.[5] The successive adjustments of the line, described below, will bring it into its proper place eventually, no matter how it was drawn in in the first place. The method is not delicate, unstable, or tricky. Recently a student worked through his first analysis of this kind in some haste, and fell into a considerable error; he confused the first independent variable with the second, and believing that the curve should have a positive rather than a negative slope, drew it in that way in spite of the evidence of the light lines to the contrary. The slope was 90° wrong —as far wrong as it is possible to make it. But the successive adjustments of the curve gradually rotated it into its proper position, and his final results were the same as those of others; it merely took him longer to reach them.

PLOTTING THE RESIDUALS

To return to Figure 43: The residuals from the heavy line drawn in freehand are now plotted against the second variable, total nonagricultural income. The results are shown in section B of Figure 43. The dots in this section fall along a path (the long curved line and the short lines are not put in until later) with a positive slope, but the pathway is rather broad. Either the line in section A must have been incorrectly placed, or else an additional independent variable needs to be brought into the picture.

The standard procedure for testing this, as explained earlier, is to draw a line through the dots in section B of Figure 43 and plot the residuals from that line back against the line in section A. With a little practice, however, even this short-cut method can be cut still shorter. We can simply *imagine* the line drawn through the dots in section B, and observe which dots fall above, and which fall below, the imaginary line. When this is done, it is clear, for example, that the dots for the years from 1921–22 to 1925–26 are all high. Now if the dots for those years in section A all fell in the left or right side of the chart, that would show that the line on that side of section A should be raised (which would have the effect of lowering the dots in section B).

[5] If the sample is small, the correlation low, or the intercorrelation high, the results from different trials may be only approximately identical. But the differences between them would be small relative to the standard error.

Inspection shows, however, that the dots from 1921–22 to 1925–26 range pretty well over most of section A. There is some tendency for the dots to lie in the right-hand two-thirds of that section, but other years (for example 1927–28 and 1928–29, and 1931–32 and 1932–33) lie along with them and would be pulled down also (in section B) if the right-hand end of the line in section A were raised.

The preliminary conclusion may be reached, therefore, that the line in section A is correctly placed with respect to the two independent variables A and B. The scatter in section B, therefore, means that an additional variable needs to be considered. What additional variable should be used? The prices or production of some competing product such as beef, some index of export demand, or what?

The proper procedure is to plot the residuals from a line drawn in through the dots in section B against each of the additional variables that might logically be expected to have an influence on the price of hogs. This procedure can be short-cut, also; it can be done mentally. The residuals from an imaginary line drawn through the dots in section B can be mentally compared with the values of each of the additional variables considered while they are still in tabular form. Only the variables which show some correlation with the residuals need to be considered. In this case, the only variable that meets this test, out of the several investigated, is net exports. This is given in the last column of Table 15. Pairs of years in each of which the values of this variable are nearly identical may then be joined. There is some difficulty in finding pairs of years in this case, because of the marked downward trend in the data over most of the period, but these dots may be joined: 1922–23, 1923–24, and 1921–22; 1924–25 and 1925–26; 1928–29 and 1929–30; 1932–33, 1931–32, 1933–34, and 1934–35; 1935–36 and 1938–39, and 1936–37 and 1937–38. A line may then be drawn in freehand with the average slope of these short lines, as shown in section B.

In this case, the short lines give some indication that the regression is slightly curvilinear, not straight (the short lines are steeper in the upper right-hand part of the chart than in the lower left-hand part). There is nothing sacred about a straight line; as a matter of fact a straight line on arithmetic paper is a curved line on logarithmic paper. There is nothing inherently superior in a straight line except that it is easier to draw. One might have expected the line to curve the opposite way, but it does not. It is interesting to see that a BAE study of consumption by different income groups shows the same curvature for pork as this one.[6]

The residuals from the curved line are then plotted against the data showing net exports, with the result shown in section C.

This does not constitute by any means a complete analysis of hog prices. Other factors have been affecting the price of hogs over the period considered. Vegetable oils have been used in increasing quantities, and this has reduced the demand for lard. This downward trend in the demand for lard has paralleled the downward trend in the net export of pork products. In the analysis, all of the decline in hog prices not explained by the first two independent factors (hog slaughter and nonagricultural income) is attributed to the decline in net exports; actually, part of the decline no doubt results from the increasing competition from vegetable oils. The effect of this "intercorrelation" between two independent factors is discussed in more general terms later in this chapter.

INCOME-PRICE CURVES

The curve shown in the upper left-hand section of Figure 43 is an approximation to a demand curve, subject to the reservations given in Chapter 6 (pp. 64–68). What is the curve in the upper right-hand section? It has a positive slope like a supply curve. Is it a supply curve?

It is not. A supply curve shows the relation between price and quantity produced. But the scale along the bottom of the present chart shows a measure of demand (nonagricultural income) not of production. The curve shows the relation between changes in nonagricultural income and the price of hogs. It is an income-price curve, not a price-quantity curve.

This sort of curve is useful for price predicting purposes. If the prediction is made that national income will change by a given amount, the effect of that change in income upon hog prices can be read off the chart and used, in the light of estimates of production, to predict hog prices.[7]

[6] *The National Food Situation*, BAE, USDA, July, 1942, p .18.

[7] For a heroic attempt to apply graphic multiple correlation analysis to the prediction of "postwar conditions in domestic and foreign markets," *see* Hans Staehle, "Relative Prices and Postwar Markets for Animal Food Products," *Quarterly Journal of Economics*, LIX, No. 2, February, 1945, pp. 237–79.

APPRAISING THE RESULTS OF GRAPHIC PRICE ANALYSES

The validity of price analyses of this sort should be judged by three criteria, (1) the closeness of fit of the dots about the lines of net regression, and the conformity of the results (2) with economic theory, and (3) with what is already known about the characteristics of the commodity. Neither one test alone is adequate. An analysis that yielded a positively sloping demand curve, no matter how closely the dots lay about it, would at least call for further investigation, if not rejection; so would an analysis that yielded a demand curve with the expected negative slope but a wide scatter of the dots. A careful worker tests his analyses by these three criteria, subconsciously, as he goes along.

Some controversy has recently arisen over the question whether the investigator should follow "the procedure of determining in advance to fit a specific type of curve or set of curves," or should "more nearly allow the data to determine the shape of the curves by fitting a set of curves of minimum residuals."[8] Most of this controversy seems unnecessary; no sensible person follows either procedure alone.[9] It is easy to carry the spirit of determining what to do in advance so far as to make it unnecessary to make the analysis at all, if the investigator already knows all he will permit himself to learn from it. On the other hand, Bean exposed himself to later criticism because in his earlier work he let his curves follow his data too closely to conform well with economic theory. It would seem sufficient, if not superfluous, now that the pioneer stage is passing, to warn price analysts to use their heads as well as their tools.

In presenting their results, some workers show a final chart in which the prices estimated from the regression lines are plotted along with the actual prices in an ordinary time chart (with the price scale running up the side and time along the bottom). This shows nothing about the amount of the difference between the estimated and actual

[8] W. Malenbaum and J. D. Black, "The Short-Cut Graphic Method: an Illustration of 'Flexible' Multiple Correlation Techniques," *Quarterly Journal of Economics*, LII, November, 1937, p. 66. *See also* comments by Bean and Ezekiel, on "The Use of the Short-Cut Graphic Method of Multiple Correlation," *Quarterly Journal of Economics*, LIV, February, 1940, p. 318.

[9] If Yntema had determined in advance to fit a markedly curved marginal cost curve to the data from the United States Steel Corporation, similar to the curves that have been accepted in economic theory (*see* the results of his research, obtainable free from the U. S. Steel Corporation) he would have had to reject the evidence of the data that the curve actually is rather flat. He would have let his preconceptions close his eyes to new knowledge.

prices that is not already shown in the final regression chart. The scatter of the dots about the line in the final regression chart is the same as the "scatter" of the actual prices about the estimates in the time chart.

It is worth while for purposes of further analysis, however, to plot the residuals from the final regression chart against time treated as an additional variable. This will reveal any serial correlations (cycles or waves) in the residuals, and any trends that may exist. It may be that the residuals gradually rise, or fall, with the passage of time, or show some sort of cyclic movement. If so, plotting them against time will reveal it. If the residuals do rise or fall, the investigator may be tempted to use time as an additional independent variable. But that would be a mistake. Time of itself does not cause residuals to rise or fall; some variable associated with time does it, and the variable itself should be isolated and used, not merely time as such. Otherwise the results may be disastrous, since a variable such as the displacement of horses by tractors that moves in one direction over a certain period of time may cease to move, or reverse the direction of its motion, in another.

THE ORDER OF PRESENTATION

The results attained will be the same no matter what the order in which the variables are worked through.[10] The results will be attained more quickly if the variables are taken in the order of their importance, for in that case the first approximation lines are most likely to be accurate. But the lines will be the same whatever the order of the variables may be.

For purposes of presentation, however, one order may be better than another. In some cases attention is being focused in the discussion upon the effect of one particular variable, for instance quantity, upon price. In that case it may be advisable for presentation purposes to use that variable last (even if it is the most important) because the average nontechnical reader is more convinced by the close scatter of the dots around the final variable than by the wider scatter around the earlier ones, though there is actually no real

[10] Warren C. Waite, "Some Characteristics of the Graphic Method of Correlation," *Journal of the American Statistical Association,* XXVII, No. 177, March, 1932, p. 68; and Mordecai Ezekiel, "Further Remarks on the Graphic Method of Correlation, a Reply to 'Some Characteristics of the Graphic Method of Correlation,'" same Journal, No. 178, June, 1932, p. 183.

difference between them. In an earlier publication,[11] variables similar to those shown in Figure 43 were used, but in a different order from the order adopted here. The chief item of interest in that case was the elasticity of the demand for hogs. Accordingly, the variables were taken in such order that livestock slaughter was used last; the elasticity was then shown directly by the line drawn through the dots in the last chart.

INTERCORRELATION AND GROSS AND NET REGRESSION

Whenever there is any correlation between two independent variables,[12] intercorrelation, it is called, the average slopes of the light lines connecting pairs of years in each of which the values of the next variable are equal will be flatter or steeper than the slope of the group of dots as a whole. Where this happens, the demand curve should be drawn in with reference only to the light lines, not to the group of dots as a whole. For the objective is to ascertain, not gross regression (i. e., simple regression, the regression of X on A[13]) but net regression (i. e., partial regression, the regression of X on A independent of its regression on other variables). The group of dots as a whole shows the gross regression of the dependent variable X upon the first independent variable A; but what we are trying to find is the *net* regression of X upon A after the influence of other independent variables has been taken into account.

A more detailed statement of what gross and net regression are may be helpful here. Regression means, roughly, dependence; we may speak of the dependence of X on A, but it is more direct to speak of the influence[14] of A upon X. When statisticians speak of the regression of X upon A, they mean, in everyday language, the influence of A upon X. Now the gross influence of A upon X is actually the gross influence of a rather extended phrase, namely "A and everything else correlated with A by chance or otherwise."

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[11] *See* analysis in Geoffrey Shepherd and Walter Wilcox, *Stabilizing Corn Supplies by Storage*, Iowa Agr. Exp. Sta., Bul. 368, 1937, p. 337.

[12] This sounds like a contradiction in terms, for one might think that if two variables were correlated, they could not be independent. But that would be a mistake. Two variables may be completely independent, completely free of any causal relationship to each other, and yet show some degree of correlation, either because they are both influenced by a third variable, or merely by chance.

[13] Where X = the dependent variable, and A, B, etc., = the independent variables.

[14] The word influence is more accurate than the word effect. An influence may be more or less offset by another influence; this is frequently the case in economics. An influence is exerted, but not necessarily registered, whereas an effect is not an effect until it is registered, i.e. effected.

And this phrase is not merely a qualification seldom required; it is the rule, rather than the exception. In our economic world, so characterized by interdependence, it is only rarely that A is not correlated with other variables that have an influence upon X.

It is indeed impossible, practically speaking, to show *the* net influence of A upon X. What we speak of as the net influence of A upon X, with the net influence of B taken into account, is actually the net influence of "A and everything else correlated with A by chance or otherwise, *except* B." What we speak of as the net influence of A upon X, with the influence of B and C taken into account, is actually the net influence of "A and everything else correlated with A by chance or otherwise, except B and C." And so on for additional variables.

In a world full of complex interrelationships, therefore, successive net regressions, as more and more independent variables are taken into account, should be expected to be different, not only from the gross regression, but from each other. They may even be different in sign. One almost hesitates to use concrete illustrations, for so many other intercorrelations are involved than the one selected for the illustration. But consider the net influence of hog prices upon corn prices, before the influence of hog numbers has been taken into account (and assuming that there are no changes in the general price level to complicate the picture). Hog prices are negatively correlated with hog numbers. If the net influence of hog numbers is greater than the net influence of hog prices, then the net influence of hog prices alone before hog numbers are taken into account would be more than offset by the effect of hog numbers, and would appear actually negative. But the addition of hog numbers as an additional variable would change the influence of hog prices (change the slope of the hog price regression curve) to its proper sign, positive.

This means that absolute net relationships are unattainable, because we can only ascertain absolute net influence if we take all other influences into account—literally hundreds of them. And this is a practical impossibility. But from a practical point of view, absolute net influences can be closely approximated. Economic reasoning and published studies in the field of agricultural economics both indicate that serviceably accurate results can be attained in most cases by the use of a relatively small number of variables. Practically all of the published studies use only two or three independent variables. While interdependence is ubiquitous, its quantitative importance diminishes rapidly after the most influential variables have been taken into account; and these most influential variables are usually few in number.

Some of the problems of intercorrelation are not as baffling as they appear at first sight. A concrete case will illustrate this. An analysis of the United States average farm price of corn[15] shows that the elasticity of the demand for corn is about 0.65. The bulk of this corn is No. 3 Yellow. But if the price of No. 2 Yellow corn were used as an additional independent variable, the elasticity of the demand for No. 3 corn would become almost infinitely great; that is, the regression of No. 3 Yellow corn prices on corn production would be practically a horizontal straight line. This results from the fact that the price of No. 2 Yellow corn is so highly intercorrelated with the dependent factor that there is not much left over for the other independent variables to explain.

What does this result mean? What is the real or true elasticity of demand for No. 3 Yellow corn—is it 0.65, or is it practically infinity?

The answer is, both. In both cases, the coefficient of elasticity shows what happens to prices when production changes, "the other independent variables being held constant," as it is often expressed, or more accurately, "independent of the accompanying variation of the other variables."[16] If the price of No. 2 Yellow corn were in actual fact "held constant," it is clear that changes in corn production would have very little effect on the price of No. 3 Yellow corn. Or to use the more accurate phrase above, there is very little fluctuation in the price of No. 3 Yellow corn independent of the fluctuation in the price of No. 2. When the price of No. 2 corn is included as one of the variables, the regression of the price of No. 3 corn should be practically zero, as in fact it is. This is merely an extreme illustration of the fact that the addition of another independent variable changes the so-called "net" regression of the dependent on the independent variables whenever (as usually happens) the additional variable is correlated with any of the other independent variables.

CORRELATION AND CAUSATION

One must clearly keep in mind the difference between correlation and causation. Two series may be highly correlated, and this corre-

[15] Shepherd and Wilcox, *op. cit.,* p. 341.

[16] H. A. Wallace and G. W. Snedecor, *Correlation and Machine Calculation,* Official Publication, Iowa State College, 1931, p. 56.

lation may be used (wrongly) to demonstrate that the one is the cause of the other. But actually the causation may run the other way; or there may be no causal relationship between the two whatever.

About all that can be deduced from a correlation coefficient is that the higher the coefficient the more likely it is that the relationship between the two variables is not due to chance, but is due to some definite relationship, such as cause and effect, between them. For data that are random in character, the standard statistical tests of significance put this statement in precise numerical form.[17] Since most economic time series are not random, however, these tests of significance have only a restricted validity in economics. This matter is discussed at some length in Chapter 13.

As to the nature of the relation between two variables, a correlation coefficient gives no answer. The relation may be one of cause and effect, as in the case of corn production and corn prices (though the correlation shows nothing as to which is cause and which is effect). Or both variables may respond to a third causal factor, as when the prices of two unrelated agricultural products are both affected by industrial prosperity or depression. Or the relationship may result entirely from chance.

ADVANTAGES AND LIMITATIONS

The graphic method has several advantages over the standard method of mathematical correlation analysis, and several limitations. The advantages will be considered first.

1. The graphic method enables the investigator to see just what he is doing. With the mathematical method, he merely feeds the data into the machine and comes out with some numerical coefficients. He does not know without additional testing whether his multiple correlation coefficient, for example, is 0.8 rather than some higher figure because the relationships are curvilinear, because one or two exceptional years were far out of line, or because additional variables are needed. But with the graphic method, he can see just what the curvilinearity is, just how many and which years are exceptional, and whether additional variables are needed, or not. These are ponderable advantages.

These advantages have been arithmetically demonstrated by

[17] *See,* for example, George W. Snedecor, *Statistical Methods,* The Iowa State College Press, 1946, Table 7.3, p. 149.

Elmer Working.[18] He set up four different pairs of variables and plotted each pair in a scatter-diagram to show the relation between each pair of items. The four scatter-diagrams are shown in Figure

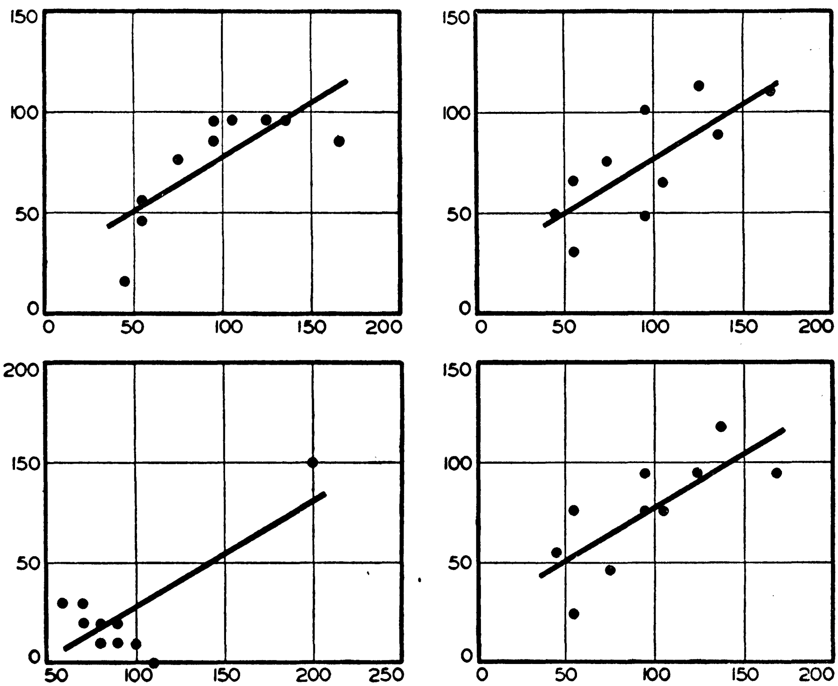

FIG. 44.—Linear regression lines fitted to four pairs of variables. Hypothetical data provided by Elmer Working.

44. Two of the relationships shown are curvilinear, one is linear, and one is based upon a very abnormal distribution. Yet the mathematical coefficients—standard deviations and correlation coefficients—are practically identical. This example illustrates how graphic methods would protect the investigator from errors he might not otherwise have discovered (although the fact that the correlations in this example are "significant" but just barely "highly significant" should put him on his guard).

[18] E. J. Working, "Graphic Methods in Price Analysis," *Journal of Farm Economics*, XXI, No. 1, February, 1939.

2. In the second place, the graphic method usually saves a good deal of time and energy. "In many cases where problems of the same number of observations and variables were treated by both methods, the graphic method proved just as accurate but consumed from one-fourth to one-third of the time."[19] The greatest saving comes in connection with rather short series, say from fifteen to twenty items, such as are common in economic price analysis. The work calls for no more training—in fact, probably for less training—on the part of the worker than is required for the formal mathematical method, and it requires no more complicated computing machinery than a pencil and ruler. In a large research department with plenty of mechanical computing equipment, this consideration is not very important; but in other cases it is.

The saving of time is greatest where the regressions are curvilinear. With the graphic method, it is just as easy to put in a curve freehand as a straight line. But with the mathematical method, the process is complex and laborious. First, the investigator must run the regular straight-line multiple correlation computation. Next, he must test each regression mathematically for curvilinearity. If it exists, he has then to select the mathematical curve most nearly appropriate to the data, add one or more extra terms to his equation, and run the multiple correlation computation again. Finally, he comes out with the coefficients expressing the relationships numerically.

3. Economic data do not usually follow any mathematical formula. They do not arrange themselves, like snowflakes, in geometrical patterns. There is no reason, for example, that the demand curve for wheat should be a straight line, a parabola, a hyperbola, or any other mathematical curve. It is determined by the physiological reactions of consumers, the distribution of income, the effect of different volumes on the intervening charges between producer and consumer, and so forth. The total effect of these things is unlikely to follow any simple mathematical formula.

Accordingly, mathematical curves cannot be expected to fit economic data very accurately. The final result is merely a compromise between the characteristics of the curve chosen by the investigator, and the characteristics of the data. The investigator has to use his judgment in selecting the type of curve to fit, and his

[19] R. G. Hainsworth, *Graphic Methods Used in Presenting Agricultural Economics to the Public*, BAE, USDA, 1938, mimeo., pp. 7–8.

choice of curve determines in considerable part the results he gets.[20] The mathematical method, therefore, is not purely objective; it has a large element of subjectivity in it. The results obtained depend to a considerable extent upon the mathematical curve selected.

The graphic method is similarly subjective. The curves are drawn in freehand, and some judgment is required, as for example in deciding how much weight to give one or two extreme items. No two investigators will draw two curves in exactly alike, any more than they will agree on the exact length of a bar they may measure. So exact tests of significance, standard errors, correlation coefficients, betas, etc., cannot be computed. This appears to be a fairly important weakness. Yet tests of significance are so inapplicable to economic time series that it is doubtful whether the weakness is as great as it seems.

With mathematical methods, if two different investigators choose the same mathematical curves, their results will agree out to as many decimal places as they may wish. When the differences resulting from choosing different mathematical curves may run into whole numbers, however, the identicality of results out to several decimal places is more misleading than confirmatory. For mathematical straight lines or curves are unlikely to fit the data any more exactly than two graphic workers fit their data freehand. And the inexactitude of the freehand line is at least clearly shown in the charts, while that of the mathematical method is covered up by figures running with a great profession of accuracy out to several decimal places.

Comparisons between the graphic and mathematical methods have sometimes approached the proportions of a controversy over the relative merits of the two. To the writer, such a controversy appears rather superficial. Practically all of the real issues involved in the use of either method—representativeness of sample, serial correlation, intercorrelation, multiplicative relationship, etc.—are common to both. Any careful user of the mathematical method would use scatter-diagrams (i. e., make an informal use of graphic methods) in deciding whether to use straight lines or curves in his formulae; for him, the graphic method is a useful exploratory tool. Conversely, any graphic worker who wished to take the time could well go ahead after he had completed his graphic analysis and

[20] This is clearly revealed by the twelve different mathematical curves that have been fitted to corn price and production data by different investigators, shown in G. F. Warren and F. A. Pearson, *Interrelationships of Supply and Price,* Cornell Univ. Agr. Exp. Sta., Bul. 466, 1928, pp. 122–23.

express his results in mathematical form. The differences of opinion as to the merits of the two methods then reduce merely to differences in the emphasis to be given to each. The mathematical statistician regards the graphic method as an exploratory tool, useful in preparing the way for mathematical analysis; the economic statistician, on the other hand, is inclined to regard the graphic analysis as the main job, and publish the mathematical coefficients in a footnote.[21]

This difference in emphasis is nothing to provoke serious controversy. It results primarily from the differences in the kind of data with which mathematicians and economists generally work. The graphic method is most useful in problems: (1) Where the number of items is small, not over twenty or thirty for example; with longer series, the labor of plotting may be as great as the labor of computing the coefficients mathematically. (2) Where the number of variables is small, say three or four; with a larger number, the process of working back and forth becomes complicated. (3) Where the correlation is rather high; this reduces the judgment required in drawing in the curves. These conditions are frequently met in economic problems, and this is probably the reason why the graphic method has been used so widely by agricultural economists.

[21] During the discussion following the presentation of two papers on the graphic method of correlation at the meetings of the American Farm Economics Association at New Orleans on December 27–29, 1940, Don Anderson objected to the term "exploratory" if that meant trying out a large number of different variables and selecting those that merely showed a high correlation with the dependent variable. Warren Waite had pointed out that such a procedure applied to numerous purely random series, such as consecutive numbers out of a telephone directory, would result in fairly high but purely chance correlations. Waite replied that he agreed with Anderson's objection to that sort of procedure. He thought the word "exploratory" should be replaced by the word "preliminary."

The present author has no objection to either term. To him, "exploratory" in this case does not mean selecting a few series from a large number taken at random, but selecting on *a priori* grounds the series that clearly has a causal connection with the dependent variable (corn production, for instance, if the dependent variable is corn prices) and exploring the elasticity and curvature of the relationship by graphic methods. This paves the way for the selection of the approximate mathematical curves to be used with the mathematical method.

For a more detailed treatment of some of the questions discussed in the present chapter, *see* Richard J. Foote and J. Russell Ives, *The Relationship of the Method of Graphic Correlation to Least Squares*, BAE, USDA, 1940, mimeo.