# Understanding High-Altitude Balloon Flight Fundamentals 

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For effective research and teaching it is vital to understand the physics of balloon flight. Starting with the basic equations of motion in a viscous fluid a relationship is obtained relating the ascent velocity to the balloon size and gas parameters, to the atmospheric parameters, and to the fluid flow viscosity parameters (coefficient of drag and Reynolds number). In addition, the differential thermal heat transfer is important for understanding lift particularly around the tropopause region where the ascending balloon gas is expanding (and cooling) while the external stratospheric temperature is increasing. The detailed equations of motion are fundamentally based on the more complex thermodynamic and fluid dynamic equations with aerodynamic forces and balloon shape changes from a sphere. Theoretical data are compared with several balloon flights where a special internal probe within the balloon is used to measure heat transfer. A dynamic Excel database is available based on these equations and available constants to help predict and understand the balloon flight and the atmospheric environment. From the physical understanding of the balloon physics the ascent rates, fluid properties, and heat transfer can be used for making new measurements and improving STEM teaching.

|  |  |
| :--- | :--- |
| $A_{b}$ | $=$ Surface area of the balloon $\left(\mathrm{m}^{2}\right)$ |
| $A_{c}$ | $=$ Cross sectional area $\left(\mathrm{m}^{2}\right)$ |
| $C_{d}$ | $=$ Coefficient of drag |
| $F_{d}$ | $=$ Force due to atmospheric drag |
| $F_{l i f t}$ | $=$ Lifting force of the gaseous fill in the balloon $\left(\mathrm{N}, \mathrm{lb}_{\mathrm{f}}\right)$ |
| $F L$ | $=$ Free lift $(\mathrm{kg})$ |
| $g$ | $=$ Gravity |
| $h_{c}$ | $=$ Convective heat transfer coefficient |
| $k$ | $=$ Thermal conductivity |
| $M_{a}$ | $=$ Molar mass of air. $(28.96 \mathrm{~g} / \mathrm{mol})$ |
| $M_{g}$ | $=$ Molar mass of gas. Helium: $4.0026 \mathrm{~g} / \mathrm{mol}$, Hydrogen gas $\left(\mathrm{H}_{2}\right): 2.0158 \mathrm{~g} / \mathrm{mol}$ |
| $m_{g}$ | $=$ Mass of gas $(\mathrm{kg})$ |
| $m_{t o t}$ | $=$ Total mass: Includes payload, balloon, and ballast masses |
| $n$ | $=$ Moles = mass $/$ Molar mass |
| $P$ | $=$ Pressure |
| $P_{a}$ | $=$ Atmospheric pressure. $($ atm, Psi $)$ |
| $P_{g}$ | $=$ Pressure of gas. $($ atm, Psi $)$ |
| $\rho_{a}$ | $=$ Mass density of air $\left(\mathrm{kg} / \mathrm{m}^{3}\right)$ |
| $\rho_{g}$ | $=$ Mass density of gas $\left(\mathrm{kg} / \mathrm{m}^{3}\right)$ |
| $Q$ | $=$ Heat |
| $R$ | $=$ Gas constant. $8.206 \times 10^{-5} \frac{\mathrm{~m}^{3} \mathrm{~atm}}{\mathrm{~mol} \mathrm{~K}}$ |
| $T$ | $=$ Temperature |
| $t$ | $=$ Time |
| $t$ | $=$ Temperature of atmosphere. Kelvin $(\mathrm{K})$ and Celsius $\left({ }^{\circ} \mathrm{C}\right)$ |
| $T_{a}$ | $=$ Temperature of gas. Kelvin $(\mathrm{K})$ and Celsius $\left({ }^{\circ} \mathrm{C}\right)$ |
| $T_{g}$ | $=$ Volume |
| $V$ | $=$ Volume of balloon $\left(\mathrm{m}^{3}\right)$ |
| $V_{b}$ | $=$ Velocity of balloon $(\mathrm{m} / \mathrm{s})$ |
| $v_{b}$ |  |
|  |  |

[^0]
## I. Introduction

To improve education and understanding of high-altitude balloon flight it is important to consider the interrelated and fundamental balloon flight physics. Understanding flight physics combines classical dynamics, thermodynamics, electromagnetic (EM) radiation balance, fluid dynamics, and stability to optimize experiments and improve learning in the study of this relatively unexplored troposheric and stratospheric regions. Researches and students have the opportunity to learn much as they relate this environment and technology to the disciplines of physics, engineering, chemistry, astronomy, aeronomy, meteorology, biology, ecology, environmental science, and education.

Reviews of scientific ballooning are presented by Morris (1975) ${ }^{1}$, Brock and Richardson (2001) ${ }^{2}$, Yajima et al. (2009) ${ }^{3}$. Remarkable benefits of Ballooning include: Atmospheric insitu research, Space Weather sensing, testing spaceflight hardware above $99 \%$ of atmosphere, develop new researches with inspiring education with investment the future ${ }^{4}$, remote sensing instruments for earth and space, cargo transportation, communication links, energy platforms, and other benefits. Furthermore scientific ballooning is low cost with short preparation times, has a fast turnaround with reuse capability, is now expanding with the miniaturization of technology, has flexible launch sites, high data rates, heavy payloads, and long flight times compared to rockets.

## II. Balloon Data and Standard Atmosphere Data

To better understand the atmosphere and balloon flight there are a number of very useful software programs for available the include the Atmospheric Properties Calculator, The Totex balloon calculator, and a new Data Base Program being developed at Taylor University.

## A. Atmospheric Properties Calculator

The Atmospheric Properties Calculator (by Aerospaceweb.org, Version 2.1.4, released August 2005) ${ }^{5}$ is based on US Standard Atmosphere ${ }^{6}$ 1976. The Table 1 below is mainly based on the data from this calculator.

Table 1 Representative Atmospheric Properties

|  |  |  |  |  | Atmosphere and Balloon Properties |  |  |  |  |  |  |  |  |  |  |  |  | iv $=5.1 \mathrm{~m} / \mathrm{s}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Altitude | Altitude | Temp | mp | Density | Pressu | ressure | Speed | Dynamic | Kinematic | Air | Collision | Mean Freel | Mole | Thermal | Pressur | ensit | Number | Reynolds | Balloon | Reynold | amina | rbulent |
|  |  |  |  |  |  |  | Sound | Vivcosity | Viscosity | Speed | Frequenc | ath | Volumn | Conductiv | Scal |  | Density | /Unit LengD | Diamet | Number | Bounda Bour | oundary |
| (km) | (ft) | $\left({ }^{\circ} \mathrm{C}\right)$ | K | $\mathrm{kg} / \mathrm{m}^{3}$ | (atm) | (Pa) | (m/s) | ( $\mathrm{Ns} / \mathrm{m}^{2}$ ) | $\left(\mathrm{m}^{2} / \mathrm{s}\right)$ | $(\mathrm{m} / \mathrm{s})$ | $\mathrm{Hz}(1 / \mathrm{s})$ | (m) | m/kmol | J/(msk) | km | km | $1 / \mathrm{m}^{3}$ | 1/m | m |  | cm | cm |
| 0 | 0 | 15 | 288.2 | 1.225 | 1 | 101,325 | 340 | 1.79E-05 | 1.46E-05 | 459 | 6.92E+09 | 6.63E-08 | 23.64 | 0.025 | 8.43 | 10.42 | $2.55 \mathrm{E}+25$ | 3.49E+05: | 1.879 | 6.6E+05 | 1.206 | 4.833 |
| 2 | 6,562 | 2 | 275.2 | 1.0066 | 0.785 | 79,540 | 333 | $1.73 \mathrm{E}-05$ | 1.71E-05 | 448 | 5.56E+09 | 8.07E-08 | 28.78 | 0.024 | 8.06 | 9.95 | $2.09 \mathrm{E}+25$ | $3.49 E+05 i$ | 2.006 | 7.0E+05 | 1.351 | 5.2591 |
| 4 | 13,123 | -10.98 | 262.2 | 0.8194 | 0.609 | 61,707 | 325 | 1.66E-05 | 2.03E-05 | 438 | 4.41E+09 | 9.92E-08 | 35.35 | 0.023 | 7.68 | 9.49 | $1.70 \mathrm{E}+25$ | $3.49 \mathrm{E}+05$ ! | 2.148 | 7.5E+05 | 1.520 | 5.744 |
| 6 | 19,685 | -23.96 | 249.2 | 0.6601 | 0.466 | 47,217 | 316 | 1.59E-05 | 2.42E-05 | 427 | 3.47E+09 | $1.23 \mathrm{E}-07$ | 43.88 | 0.022 | 7.31 | 9.02 | $1.37 \mathrm{E}+25$ | $3.49 \mathrm{E}+05$ ! | 2.305 | 8.0E+05 | 1.718 | 6.294 |
| 8 | 26,247 | -36.93 | 236.2 | 0.5258 | 0.352 | 35,666 | 308 | 1.53E-05 | 2.90E-05 | 416 | 2.69E+09 | 1.55E-07 | 55.09 | 0.021 | 6.93 | 8.56 | $1.09 \mathrm{E}+25$ | $3.49 \mathrm{E}+05$ | 2.479 | 8.7E+05 | 1.954 | 6.922 |
| 10 | 32,808 | -49.9 | 223.3 | 0.4135 | 0.262 | 26,547 | 300 | 1.46E-05 | 3.53E-05 | 404 | $2.06 \mathrm{E}+09$ | 1.97E-07 | 70.05 | 0.020 | 6.56 | 8.10 | 8.60E+24 | $3.49 \mathrm{E}+05{ }^{\text {i }}$ | 2.677 | 9.3E+05 | 2.237 | 7.651 |
| 12 | 39,370 | -56.5 | 216.7 | 0.3119 | 0.191 | 19,353 | 295 | 1.42E-05 | 4.56E-05 | 398 | $1.53 \mathrm{E}+09$ | 2.60E-07 | 92.85 | 0.020 | 6.37 | 6.37 | $6.49 \mathrm{E}+24$ | $3.49 \mathrm{E}+05{ }^{\text {i }}$ | 2.930 | 1.0E+06 | 2.661 | 8.658 |
| 14 | 45,932 | -56.5 | 216.7 | 0.22 | 0.1 | 14,186 | 295 | 1.42E-05 | 6.24 | 398 | 1.12E+09 | 3.57 | 127 | 0.020 | 6.37 | 6.37 | 4.74E+24 | 3.49E+05! | 3.243 | 1.1E+06 | 3.275 | 1 |
| 16 | 52,493 | -56.5 | 216.7 | 0.1665 | 0.102 | 10,335 | 295 | 1.42E-05 | 8.54E-05 | 398 | 8.15E+08 | 4.88E-07 | 173.99 | 0.020 | 6.37 | 6.37 | $3.46 \mathrm{E}+24$ | 3.49E+05! | 3.595 | 1.3E+06 | 4.035 | 11.562 |
| 18 | 59,055 | -56.5 | 216.7 | 0.1217 | 0.075 | 7,599 | 295 | 1.42E-05 | 1.17E-04 | 398 | 5.96E+08 | 6.68E-07 | 238.10 | 0.020 | 6.38 | 6.38 | $2.53 \mathrm{E}+24$ | $3.49 \mathrm{E}+05$ | 3.991 | $1.4 \mathrm{E}+06$ | 4.973 | 13.384 |
| 20 | 65,617 | -56.5 | 216.7 | 0.0889 | 0.055 | 5,573 | 295 | 1.42E-05 | 1.60E-04 | 398 | 4.35E+08 | 9.14E-07 | 325.77 | 0.020 | 6.38 | 6.38 | $1.85 \mathrm{E}+24$ | $3.49 \mathrm{E}+05$ | 4.431 | 1.5E+06 | 6.129 | 15.4 |
| 22 | 72,178 | -54.58 | 218.6 | 0.0645 | 0.04 | 4,053 | 296 | 1.43E-05 | 2.22E-04 | 400 | $3.17 \mathrm{E}+08$ | 1.26E-06 | 448.99 | 0.020 | 6.44 | 6.26 | $1.34 \mathrm{E}+24$ | 3.49E+05 | 4.932 | $1.7 \mathrm{E}+06$ | 7.619 | 18.024 |
| 24 | 78,740 | -52.59 | 220.6 | 0.0469 | 0.029 | 2,938 | 298 | 1.44E-05 | 3.07E-04 | 402 | $2.32 \mathrm{E}+08$ | 1.73E-06 | 617.08 | 0.020 | 6.50 | 6.32 | 9.76E+23 | $3.48 \mathrm{E}+05$ | 5.484 | 1.9E+06 | 9.455 | 20.94 |
| 26 | 85,302 | -50.61 | 222.5 | 0.0343 | 0.022 | 2,229 | 299 | 1.45E-05 | 4.24E-04 | 403 | 1.70E+08 | 2.37E-06 | 845.51 | 0.020 | 6.57 | 6.38 | $7.12 \mathrm{E}+23$ | $3.48 \mathrm{E}+05$ | 6.092 | 2.1E+06 | 11.708 | 24.296 |
| 28 | 91,864 | -48.62 | 224.5 | 0.0251 | 0.016 | 1,621 | 300 | 1.46E-05 | 5.84E-04 | 405 | $1.25 \mathrm{E}+08$ | 3.24E-06 | 1,155.10 | 0.020 | 6.63 | 6.44 | $5.21 \mathrm{E}+23$ | $3.48 \mathrm{E}+05$ | 6.761 | 2.4E+06 | 14.469 | 28.1 |
| 30 | 98,425 | -46.64 | 226.5 | 0.0184 | 0.012 | 1,216 | 302 | 1.48E-05 | 8.01E-04 | 407 | 9.22E+07 | 4.41E-06 | 1,573.30 | 0.020 | 6.69 | 6.50 | $3.83 \mathrm{E}+23$ | $3.48 \mathrm{E}+05$ i | 7.495 | $2.6 \mathrm{E}+06$ | 17.845 | 32.56 |
| 32 | 104,987 | -44.66 | 228.5 | 0.0136 | 0.009 | 912 | 303 | $1.49 \mathrm{E}-05$ | 1.10E-03 | 409 | $6.82 \mathrm{E}+07$ | 5.99E-06 | 2,136.80 | 0.020 | 6.76 | 6.56 | $2.82 \mathrm{E}+23$ | $3.48 \mathrm{E}+05$ i | 8.302 | $2.9 \mathrm{E}+06$ | 21.966 | 37.627 |
| 34 | 111,549 | -39.41 | 233.7 | 0.0099 | 0.007 | 709 | 306 | 1.51E-05 | 1.53E-03 | 413 | $5.03 \mathrm{E}+07$ | 8.22E-06 | 2,929.40 | 0.021 | 6.92 | 6.39 | $2.06 \mathrm{E}+23$ | $3.48 \mathrm{E}+05$ i | 9.227 | $3.2 \mathrm{E}+06$ | 27.370 | 43.775 |
| 36 | 118,110 | -33.87 | 239.3 | 0.0073 | 0.005 | 507 | 310 | 1.54E-05 | 2.13E-03 | 418 | 3.74E+07 | 1.12E-05 | 3,990.70 | 0.021 | 7.08 | 7.08 | $1.51 \mathrm{E}+23$ | 3.48E+05: | 10.233 | 3.6E+06 | 33.966 | 50.78 |
| 38 | 124,672 | -28.33 | 244.8 | 0.0054 | 0.004 | 405 | 314 | 1.57E-05 | 2.93E-03 | 423 | $2.79 \mathrm{E}+07$ | 1.51E-05 | 5,397.20 | 0.022 | 7.25 | 6.70 | $1.12 \mathrm{E}+23$ | $3.48 \mathrm{E}+05$ | 11.320 | $3.9 \mathrm{E}+06$ | 41.933 | 58.697 |
| 40 | 131,234 | -22.8 | 250.4 | 0.004 | 0.003 | 304 | 317 | 1.60E-05 | 4.01E-03 | 428 | 2.10E+07 | 2.03E-05 | 7,248.90 | 0.022 | 7.42 | 6.86 | $8.31 \mathrm{E}+22$ | $3.48 \mathrm{E}+05$ | 12.495 | 4.3E+06 | 51.520 | 67.626 |
|  |  |  | Most | mns | ased on A | Atmospheric | eric Prop | operties Calcu | culator (U.S. | S. Standa | dard Atmo | here 1976 | 6), Not for | or commerc | aluse |  |  |  |  |  |  |  |
|  |  |  | Last th | ree colu | ns based | d on balloo | on velo | ocity of 5.1 m |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

## B. Totex Balloon Data ${ }^{7}$

Table 2 Reference Totex Balloon Data

| Balloon Weight (gr) | $\mathbf{2 0 0}$ | $\mathbf{3 0 0}$ | $\mathbf{3 5 0}$ | $\mathbf{4 5 0}$ | $\mathbf{5 0 0}$ | $\mathbf{6 0 0}$ | $\mathbf{7 0 0}$ | $\mathbf{8 0 0}$ | $\mathbf{1 0 0 0}$ | $\mathbf{1 2 0 0}$ | $\mathbf{1 5 0 0}$ | $\mathbf{2 0 0 0}$ | $\mathbf{3 0 0 0}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Diameter at Release (cm) | 117 | 123 | 125 | 130 | 133 | 142 | 146 | 150 | 157 | 179 | 185 | 195 | 212 |
| Volume at Release (cu.m) | 0.83 | 0.97 | 1.03 | 1.1 | 1.22 | 1.5 | 1.63 | 1.76 | 2.01 | 2.99 | 3.33 | 3.89 | 4.97 |
| Gross Lift (gr) | 960 | 1110 | 1185 | 1335 | 1405 | 1720 | 1870 | 2020 | 2310 | 3440 | 3830 | 4470 | 5720 |
| Nozzle Lift (gr) | 760 | 810 | 835 | 885 | 905 | 1120 | 1170 | 1220 | 1310 | 2240 | 2330 | 2470 | 2720 |
| Payload (gr) | 250 | 250 | 250 | 250 | 250 | 250 | 250 | 250 | 250 | 1050 | 1050 | 1050 | 1050 |
| Recommended Free Lift (gr) | 510 | 560 | 585 | 635 | 655 | 870 | 920 | 970 | 1060 | 1190 | 1280 | 1420 | 1670 |
| Rate of Ascent (m/min) | 320 | 320 | 320 | 320 | 320 | 320 | 320 | 320 | 320 | 320 | 320 | 320 | 320 |
| Diameter at Burst (cm) | 300 | 378 | 412 | 472 | 499 | 602 | 653 | 700 | 786 | 863 | 944 | 1054 | 1300 |
| Volume at Burst (cu m) | 14.1 | 28.3 | 36.6 | 55.1 | 65.1 | 114.2 | 145.8 | 179.6 | 254.3 | 336.5 | 440.5 | 613.1 | 1150.3 |
| Bursting Altitude (km) | 21.2 | 24.7 | 25.9 | 27.7 | 28.4 | 30.8 | 31.8 | 32.6 | 33.9 | 33.2 | 34.2 | 35.4 | 37.9 |

## (http://ukhas.org.uk/guides:balloon data)

"Its worth noting that the data above is based on the use of Hydrogen rather than Helium. The Balloon Gross Lift is the lift generated by the volume of Gas. Hydrogen has a density of about $0.09 \mathrm{~kg} / \mathrm{cu} \mathrm{m}$ and air about 1.2 $\mathrm{kg} / \mathrm{cu} \mathrm{m}$ at normal atmospheric pressure - generating a lift of a bit over $1.1 \mathrm{~kg} / \mathrm{cu} \mathrm{m}$. Helium has a density of about $0.17 \mathrm{Kg} / \mathrm{cu} \mathrm{m}$ - generating a lift of a bit over $1.0 \mathrm{Kg} / \mathrm{cu} \mathrm{m}$. Subtract the Balloon Weight plus the Payload weight form the Gross Lift to give the free lift. The Recommended Free Lift gives a Rate of Ascent of $320 \mathrm{~m} / \mathrm{min}$ (a bit over 1000 ft per minute). Heavier payloads can be carried than the values above - this will either reduce ascent rate, burst altitude (if the balloon is further inflated to compensate) or both. The following spreadsheet allows you to calculate the affect on burst altitude and ascent rates of various levels of fill and payload. To use: choose the balloon size and fill in the payload weight - then adjust the launch diameter until you get the desired ascent rate (normally about $320 \mathrm{~m} / \mathrm{min}$ ) - the volume at launch will tell you how much gas you will need." guides:burst3.xls by Steve Randall

## B. Balloon Launch and Burst Estimator ${ }^{7}$

Table 3 Modified Balloon Fill and Burst Estimator Totex Balloon Burst Estimator - by Steve Randall- Modified Voss June 2012



## C. Taylor Balloon Flight Spreadsheet ${ }^{8}$

The final Balloon Calculator is the Data Base Spreadsheet under development at Taylor University (Ramm and Voss, This publication, 2012). It includes many of the gas filling requirements and flight related equations of motion as described.

## III. Balloon Experiment for Understanding Balloon Fundamentals

It is very difficult to measure temperature in a changing radiation environment (UV, Visible, IR) as the pressure and density vary by $99 \%$ and the IR and UV radiations change by orders of magnitude. Special care must be taken to increase convective flow for heat transfer to the sensor while keeping the sensor thermal mass and energy dissipation low. In addition, low emissivity and absorptivity coatings must be used on the sensor to minimize energy transfer to and from the environment.


Fig. 3-1 Test HARP Balloon Flight 282 to study the Temperature profile inside of the balloon using a series of internal temperature sensors on a boom. Boom is overlaid on Balloon for illustration.

Balloon Flight 282 was launched on June 9, 2012 at 9:00 AM to investigate the internal and external temperatures of the flight. The external temperatures where monitored using a calibrated and certified Met unit using an exposed thermistor. Another external temperature sensor based on a diode junction was floated about 3 cm from the POD for verification inter-calibration.

On the internal 70 cm boom three diode temperatures were located as indicated in Fig. 3-1. Near the Top sensor an additional IR sensor was positioned to measure the balloon thin film temperature. The IR sensor field-of-view was 80 degrees and faced the top of the balloon. The boom POD is shown in Fig. 3-2 and collected all of the internal data at four samples per second for wireless transfer and internally stored in a memory stick.


Fig. 3-2 internal temperature boom for insertion into nozzle.


Fig. 3-3 Temperature POD with calibrated MET unit (with plastic protective cover) and diode temperature sensor in white plastic hollow ball with holes for convection.

Over the past year three balloon flights have now been flown with the internal temperature boom to better understand the heat transfer and temperature variations within the balloon. These data are then used to check the model simulations to see if the physics is properly accounted for to explain the observed rate of assent and radiation balance.


Fig. 3-4 Flight 282 at beginning of launch showing four PODS and the parachute with a enclosed 100 g balloon. The internal probe boom is not visible but is located inside the balloon.


## IV. Balloon Fundamental Motion

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## A. Basic forces and Lift

A diagram of the vertical forces on a balloon is shown in Figure 4-1 assuming a stationary atmosphere (no vertical winds). The upward buoyant force or Gross Lift is $F_{B}=\left(m_{a}-m_{g}\right) * g$ where $m_{a}$ is the balloon fill equivalent atmospheric mass that is displaced by the balloon gas mass, $\mathrm{m}_{\mathrm{g}}$, and $\mathrm{g}=9.8 \mathrm{~m} / \mathrm{s}^{2}$ is the acceleration of gravity. Hydrogen gas at STP has a density of $0.09 \mathrm{~kg} / \mathrm{cu} \mathrm{m}$ and air a density of about $1.2 \mathrm{~kg} / \mathrm{cu} \mathrm{m}$ which generates a lift of about $1.1 \mathrm{~kg} / \mathrm{cu} \mathrm{m}$. The balloon gross mass, $\mathrm{m}_{\mathrm{G}}$, includes the mass of the rubber balloon, $\mathrm{m}_{\mathrm{b}}$, plus the mass of the payload, $\mathrm{m}_{\mathrm{p}}$.so that $\mathrm{m}_{\mathrm{G}}=\mathrm{m}_{\mathrm{b}}+\mathrm{m}_{\mathrm{p}}$.

The Nozzle Lift is $F_{B}-m_{b} g$. The Free Lift is the net lift with the payload attached, $\mathrm{F}_{\mathrm{L}}=\mathrm{F}_{\mathrm{B}}-\mathrm{W}=\mathrm{F}_{\mathrm{b}}-\mathrm{m}_{\mathrm{G}}$. g .

## B. Drag Force

When the Balloon system is first launched it quickly accelerates to a constant upward speed based on force balance between the Free Lift force and the drag force, $F_{L}=F_{D}$.

$$
\begin{equation*}
\text { The Drag Force, } F_{D}=1 / 2 C_{D} A_{b} \rho_{a} v^{2} \text {, } \tag{1}
\end{equation*}
$$

where $A_{b}$ is the cross sectional area of the balloon $=\pi D^{2} / 4, \rho$ is the atmospheric density, v is the balloon upward velocity, and $C_{D}$ is the Drag constant of proportionality and is usually about 0.3 as indicated in Table 2. To understand this relationship you can use conservation of energy.

Drag force on the balloon surface $=F_{D}{ }^{*} \Delta z$ proportional to the Kinetic Energy imparted to air $=1 / 2 \mathrm{~m}_{\mathrm{a}} \mathrm{v}^{2}$ where $\Delta \mathrm{z}$ is a vertcal increment of height. If we multiply and divide the KE side by $\mathrm{V} / \mathrm{V}$, where V is the Volume of the displaced air, $V=(A)(\Delta z)$, we get that $F_{D} * \Delta z \propto 1 / 2(V / V) m_{a} v^{2}=1 / 2 \Delta z A \rho_{a} v^{2}$. Therefore, $F_{D}=1 / 2 C_{D} A_{b} \rho v^{2}$.

This formula can also find the velocity during parachute descent by taking A as the area of the parachute, $\mathrm{A}_{\mathrm{p}}$ (ref). The descent terminal velocity can be found by the requirement that the drag force must equal the total parachute and payload weight: $m_{G} g=F_{D}=1 / 2 C_{D} A_{b} \rho_{a} v^{2}$. Solving for v gives the falling terminal velocity, $\mathrm{v}_{\mathrm{T}}$,

$$
\begin{equation*}
\mathrm{v}_{\mathrm{T}}=\left(2 \mathrm{mg} /\left(\mathrm{C}_{\mathrm{D}} \rho \mathrm{~A}\right)\right)^{1 / 2} \tag{2}
\end{equation*}
$$

Where the Drag Coefficient, $\mathrm{C}_{\mathrm{D}}$, is between 0.5 to 0.8 (Seifert and McIntosh, 2011) ${ }^{9}$ for the parachute.
The Drag constant, $C_{D}$, is a function of the Reynolds number, $\operatorname{Re}=v D \rho_{a} / \mu$, where $\mu=$ dynamic viscosity. From Fig. $4-2^{10}$ the Coefficient of Drag for a balloon (see Table 1) is in the region of interest shown was the flow can change from laminar to turbulent. Understanding the flight dynamics and atmospheric parameters during a flight can help to indirectly measure $\mathrm{C}_{\mathrm{D}}$. The two dimensional flow past a cylinder is easier to visualize as shown in fig. 4-3 and is claimed to be very similar to the three dimensional flow around a sphere.

Coming up with clever ways to video capture the aerodynamic flow in the in-flight high-altitude balloon laboratory at high altitudes would greatly help to understand the flow behavior and drag coefficient. Evaluation of the Aerodynamic Differences of a Balloon and a Sphere Using Computational Fluid Dynamic Modeling in Fluent software has been demonstrated by Scholes (2011) ${ }^{11}$ and the MS thesis is available as a PDF. The results of this study show that the drag on a realistic balloon shape is not statistically different from a sphere although only small balloons at low altitudes were investigated with diameters less than 1 m and Reynolds numbers about $10^{4}-10^{5}$.


Fig. 4-2 The Coefficient of Drag, $C_{D}$, is primarily dependent on the Remolds Number, Re, and the condition of the balloon surface. The balloon region of interest is shown in red. The dip between points 4 and 5 indicate the transition from a Laminar to a turbulent change in flow regime. Blue data taken from NASA.gov in reference.


Fig. 4-3 Example of Laminar (region 4) and Turbulent (region 5) of the flow regime shown in Fig. 4-2 for a cylinder. Data from NASA.gov.

## C. Gas Law equations

For an ideal gas, $\mathrm{PV}=\mathrm{nRT}$, where P is the pressure, V is the balloon volume, n are the number of moles of lifting gas, R is the Gas constant $\left(\mathrm{R}=8.31\right.$ joules/(mole $\left.{ }^{\circ} \mathrm{K}\right)$ ), and T is the internal balloon temperature. If we multiply both sides this equation by the average molecular mass of air $\left(\mathrm{M}_{\mathrm{a}}=28.97 \mathrm{~kg} / \mathrm{mole}\right)$ we can solve for the density of air as, $\rho_{a}=M_{a} \mathrm{P}_{\mathrm{a}} /\left(\mathrm{R} \mathrm{T}_{\mathrm{a}}\right)$. We can also solve for the volume of the balloon if we use the fill gas parameters designated by the subscript $\mathrm{g}: \mathrm{V}_{\mathrm{b}}=\mathrm{m}_{\mathrm{g}} \mathrm{R} \mathrm{T}_{\mathrm{g}} /\left(\mathrm{M}_{\mathrm{g}} \mathrm{p}_{\mathrm{g}}\right)$. $\mathrm{V}_{\mathrm{b}}$ is a function of the balloon radius when approximated to the volume of a sphere, $4 / 3 \pi r^{3}$.

## D. Equations of motion

By applying Newton's law, $\mathrm{F}=\mathrm{ma}$, the dynamic motion of the balloon can be nicely solved using a second order differential equation with certain approximations (Bachman). For steady state, $a=0$, the sum of forces are zero with the drag force equaled by the lifting force ( $\mathrm{F}_{\mathrm{L}}=\mathrm{F}_{\mathrm{D}}$ ). In this case the steady state ascent rate can be solved as

$$
\begin{equation*}
\mathbf{V}_{\mathbf{b}}=\sqrt{\frac{\pi D^{3}\left(\rho-\rho_{B}\right) g / 6-\left(m_{B}+m_{P}\right) g}{C_{D} \rho \pi D^{2} / 8}} \tag{3}
\end{equation*}
$$

Using Table 1 to plug in realistic values (see Bachman ${ }^{12}$ ) the vertical speed of the balloon is on the order of $5 \mathrm{~m} / \mathrm{s}$. For many flights the balloon continues to rise over 30 km at nearly a constant rate of $5 \mathrm{~m} / \mathrm{s}$ (about $1000 \mathrm{ft} . \mathrm{min}$ ) even though many parameters are changing (density, balloon size, drag, temperature) and we will look at this next with regard to energy balance.


Fig. 5-1 1 Previous Balloon Flight 280 with internal temperature boom. The three boom internal temperature monitors are near each other while the external temperature is somewhat warmer in the troposphere below 6 km . A striking temperature difference id observed above 12 km when the external temperature stops falling at the Lapse rate. The balloon is likely cooler because of continued adiabatic expansion.

## V. Balloon Heat Transfer

Ascent Top, Mid, Low, Ext. Temp


- Ext Temp
- Top Temp
- Mid Temp
- Low Temp


Fig. 5-2 Heat going into and out of Balloon film and lifting gas. Adapted from Yajima et al. 2004, Fig 2.35.

In Fig. 5.2 adapted from Yajima ${ }^{3}$ the component heat transfer is shown as function of altitude. Note that the blue line is the adiabatic expansion of the lift gas and that it is nearly constant as the balloon ascends. Also note the solar energy inputs and the large increase if IR radiation and absorption as the balloon ascends.

## VI. Ascending Motion

The assent profiles depends strongly on the temperature reversal at the tropopause (about 15 km ) when the temperature changes from falling in the troposphere to increasing in the stratosphere. The rate of ascent equation (3) shows that when the adiabatic expansion continues in the stratosphere while the external temperature increases with altitude. Depending on the solar angle the solar heating can also accelerate the balloon with increasing altitude.


Fig. 6-1 Plot shows four balloon assent rates for different times. Note the pronounced "knee" observed on the 5/2/2009 launch at $40,000 \mathrm{ft}$.

## VII. Future Work and Education

Additional work is planned for making more measurements of the internal temperature variations within the balloon and the balloon film surface. More accurate emissivity's and absorptivity's are required with a better understanding of the IR, visible and UV heating radiation. Student education is greatly enhanced with a good understanding of the balloon physics and when ascent data profiles, fluid flow behavior, and radiation heat transfer can be quantitatively compared with models.

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