Two Lab Activities Demonstrating Some Physics of High Altitude Balloon Launches

Investigators: Seiji Takemae and Gordon McIntosh

Institution: University of Minnesota, Morris

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Outline

- Learning Objectives
- Background (Dynamics, Free-Body Diagram of Descending Helium Balloon, Archimedes' Principle, and Drag Equation)
- Description of the Lab:
  - Apparatus
  - Procedure
  - Sample Data
  - Numerical Recursion Method for Predicting Descent Time and Terminal Velocity
- Conclusions

Learning Objectives

These activities:

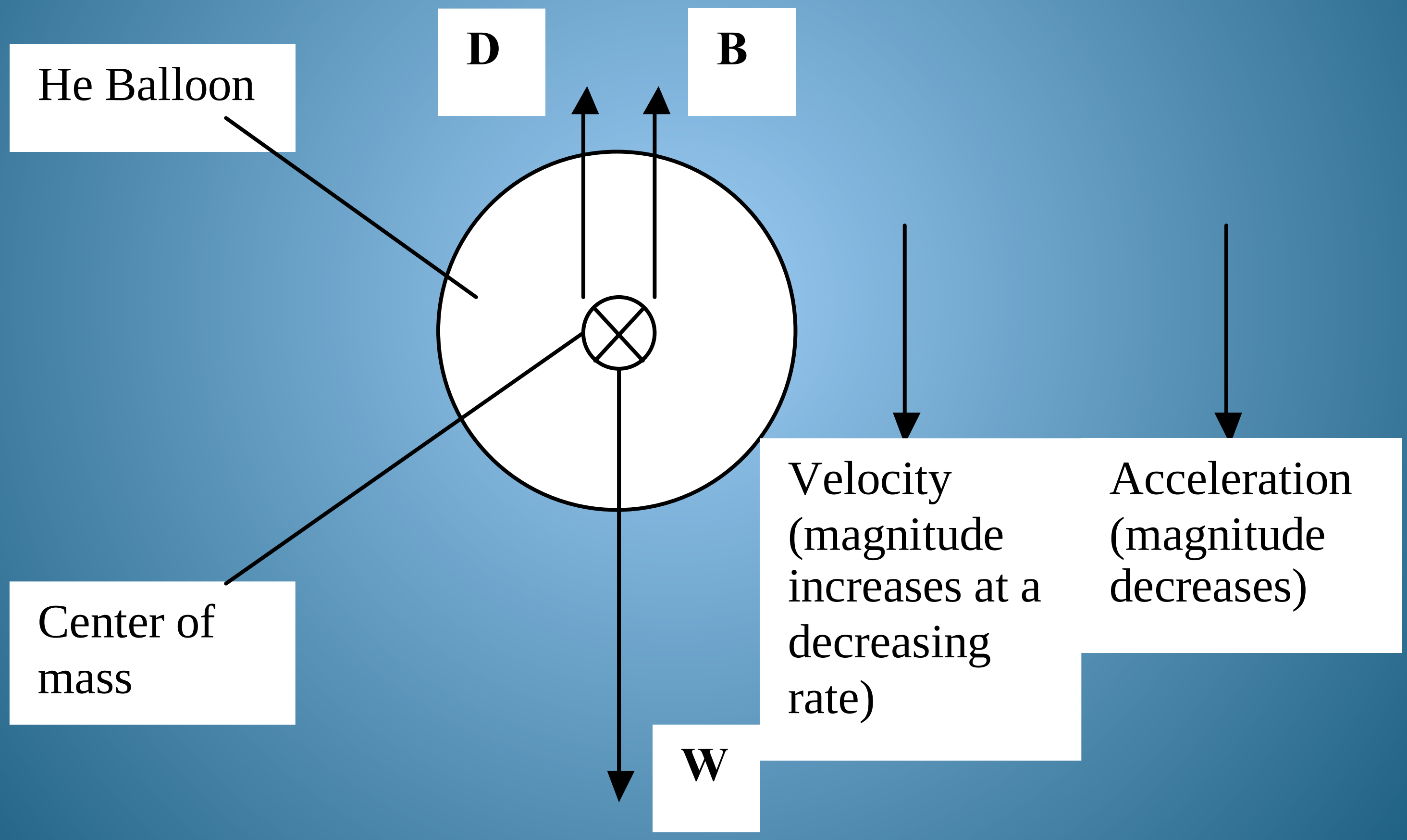
1. Use balloon and parachute drops to illustrate the sum of forces in one dimension.

2. Incorporate air resistance (a velocity dependent force).

3. Are intrinsically interesting to students. He balloons are a novelty.

4. Give students experience using a numerical analysis to solve equations and interpreting graphs.

Free-Body Diagram of Balloon

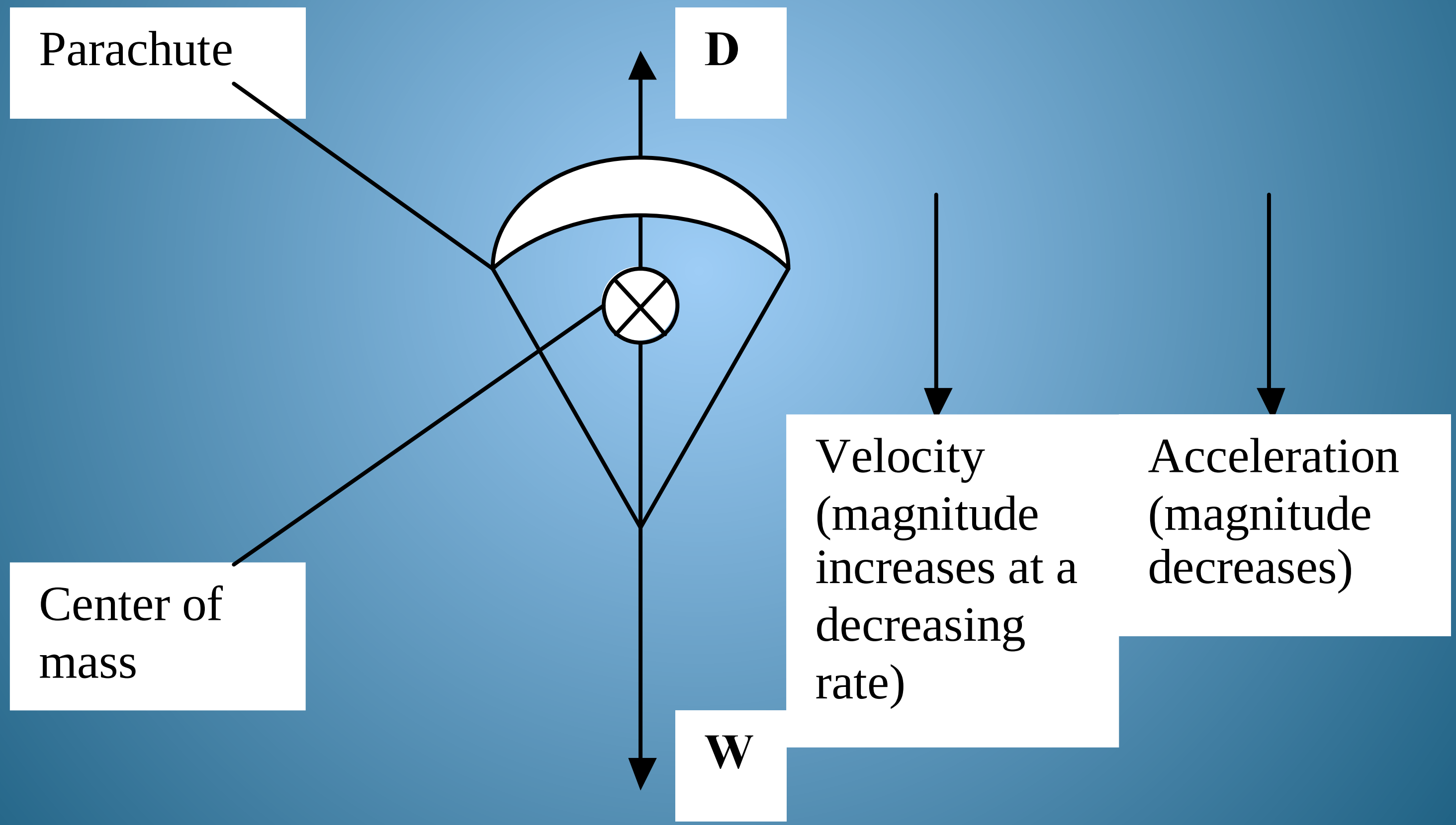


Balloon Equation of Motion

$$a_y = \frac{B + D - W}{\left(m_{balloon+Cu\ \ pellets+He}\right)}$$

$$a_y = \frac{(\rho_{air})\left(\frac{4}{3}\pi r^3\right)g + \frac{1}{2}\rho_{air} A_c C_d v^2 - \left(m_{balloon} + m_{Cu\_pellets} + \rho_{He}\left(\frac{4}{3}\pi r^3\right)\right)g}{\left(m_{balloon} + m_{Cu\_pellets} + \rho_{He}\left(\frac{4}{3}\pi r^3\right)\right)}$$

Free-Body Diagram of Descending Parachute

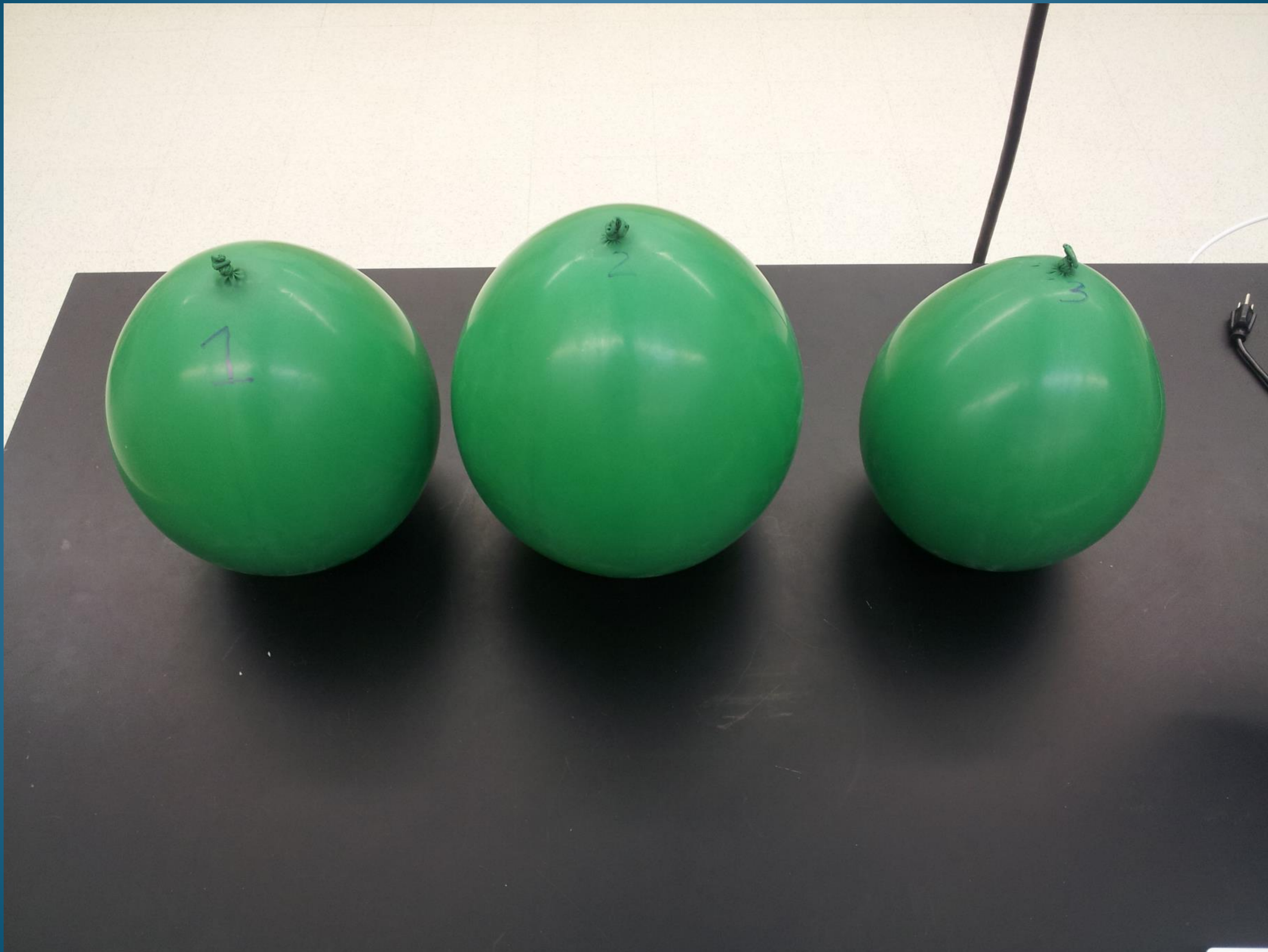


Parachute Equation of Motion

$$a_y = \frac{D - W}{m_{parachute} + m_{attached\ mass}}$$

$$a_y = \frac{\frac{1}{2}\rho_{air} A_s C_d v^2 - \left(m_{parachute} + m_{attached\_mass}\right) g}{m_{parachute} + m_{attached\_mass}}$$

Apparatus


1
2
3

Procedure

- Filled 17” balloons with helium and attached set of weights.
- Attached similar set of weights to an 18” parachute.
- Dropped these objects from a known height.
- Measured descent time and compared to theoretical calculation of descent time.

Sample Data from Helium Balloon Drop

Measurement of Descent Time

Data for Balloon 1

- C_d = 0.47 (sphere)
- ρ_{air} = 1.20 kg/m^3, ρ_{He} = 0.179 kg/m^3
- $m_{balloon+payload+He}$ = 0.03798 kg
- Equatorial Radius: 0.140 m
- Polar Radius: 0.152 m
- Average Radius: 0.146 m
- Volume: 0.0129 m^3 (sphere)
- Descent Height: 6.01 m
- Measured Descent Time: 2.30 s (average of 10 trials)

Theoretical Calculation of Descent Time and Terminal Velocity

Problem

- We cannot use

$$v_y(t) = v_{0y} + a_y t$$

$$y(t) = y_0 + v_{0y} t + \frac{1}{2} a_y t^2$$

because the acceleration is not constant.

- We used a numerical method to find a descent time.

Recursive Numerical Method

Chose step size: Δt = 0.050 s. In one step from t_n to t_{n+1},

$$t_{n+1} = t_n + \Delta t$$

$$v_{y,n+1} = v_{y,n} + a_y\left(t_n, v_{y,n}\right)\Delta t$$

$$y_{n+1} = y_n + v_{y,n}\,\Delta t + \frac{1}{2}\,a_y\left(t_n, v_{y,n}\right)(\Delta t)^2$$

Initial Conditions

(1) At $t_0 = 0$, $y_0 = 0$, $v_{y,0} = 0$,

$$a_y(t_0, v_{y,0}) = \frac{B - W + D(v_y(t_0))}{m_{balloon+Cu\ \ pellets+He}} \approx -3.77 m/s^2$$

Step to t_1

(2) At $t_1 = t_0 + 0.050\ s = 0.050\ s$

$$v_{y,1} = v_{y,0} + a_y(t_0, v_{y,0})0.050s \approx -0.187 m/s$$

$$y_1 = y_0 + v_{y,0}0.050s + \frac{1}{2}a(t_0, v_{y,0})(0.050s)^2 \approx -0.00468m$$

$$a_y(t_1, v_{y,1}) = \frac{B - W + D(v_y(t_1))}{m_{balloon+load+He}} \approx -3.73m/s^2$$

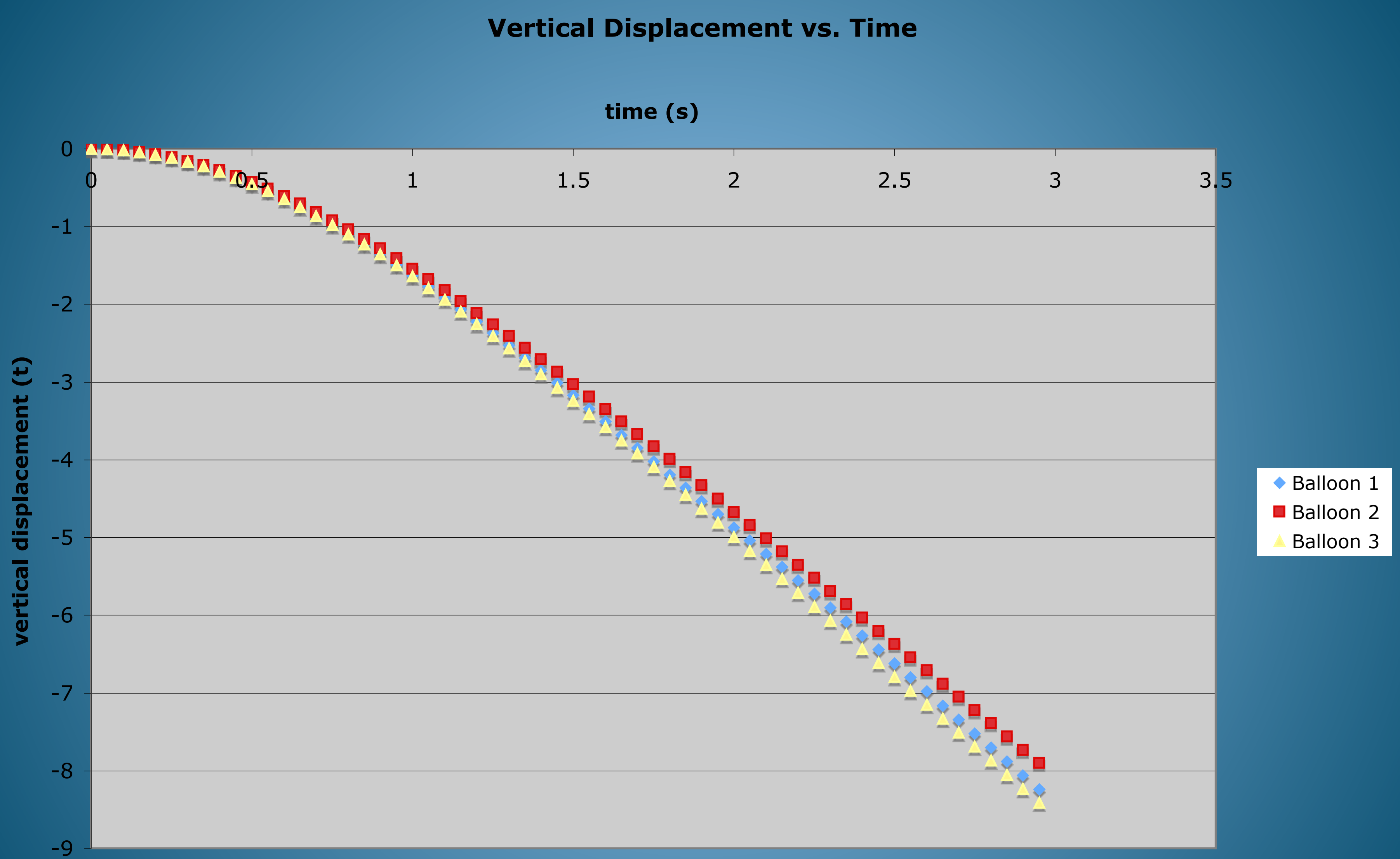
Step to t_2

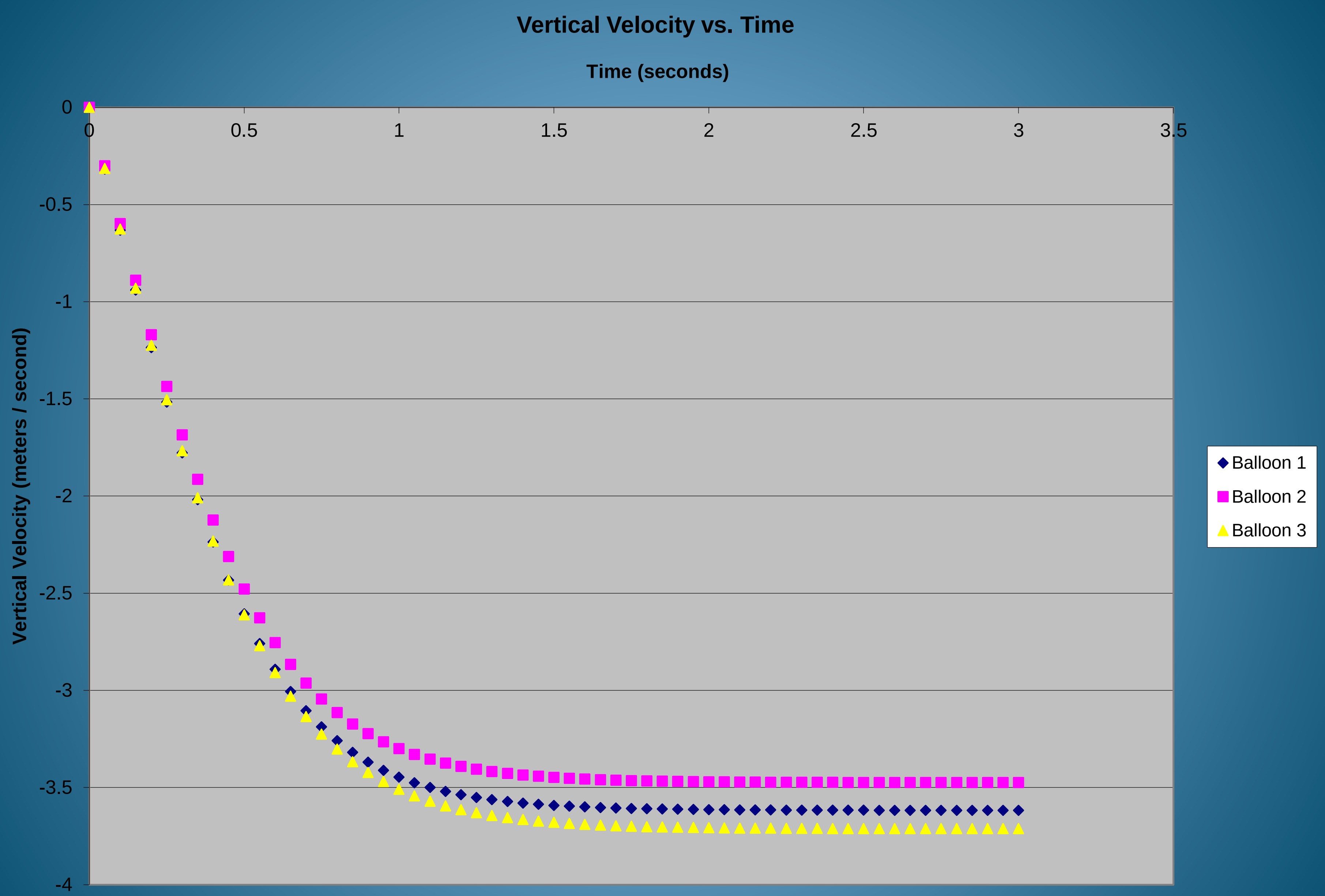
(3) At $t_2 = t_1 + 0.050$ s $= 0.100$ s

$$v_{y,2} = v_{y,1} + a_y(t_1, v_{y,1})0.050s \approx -0.374 m/s$$

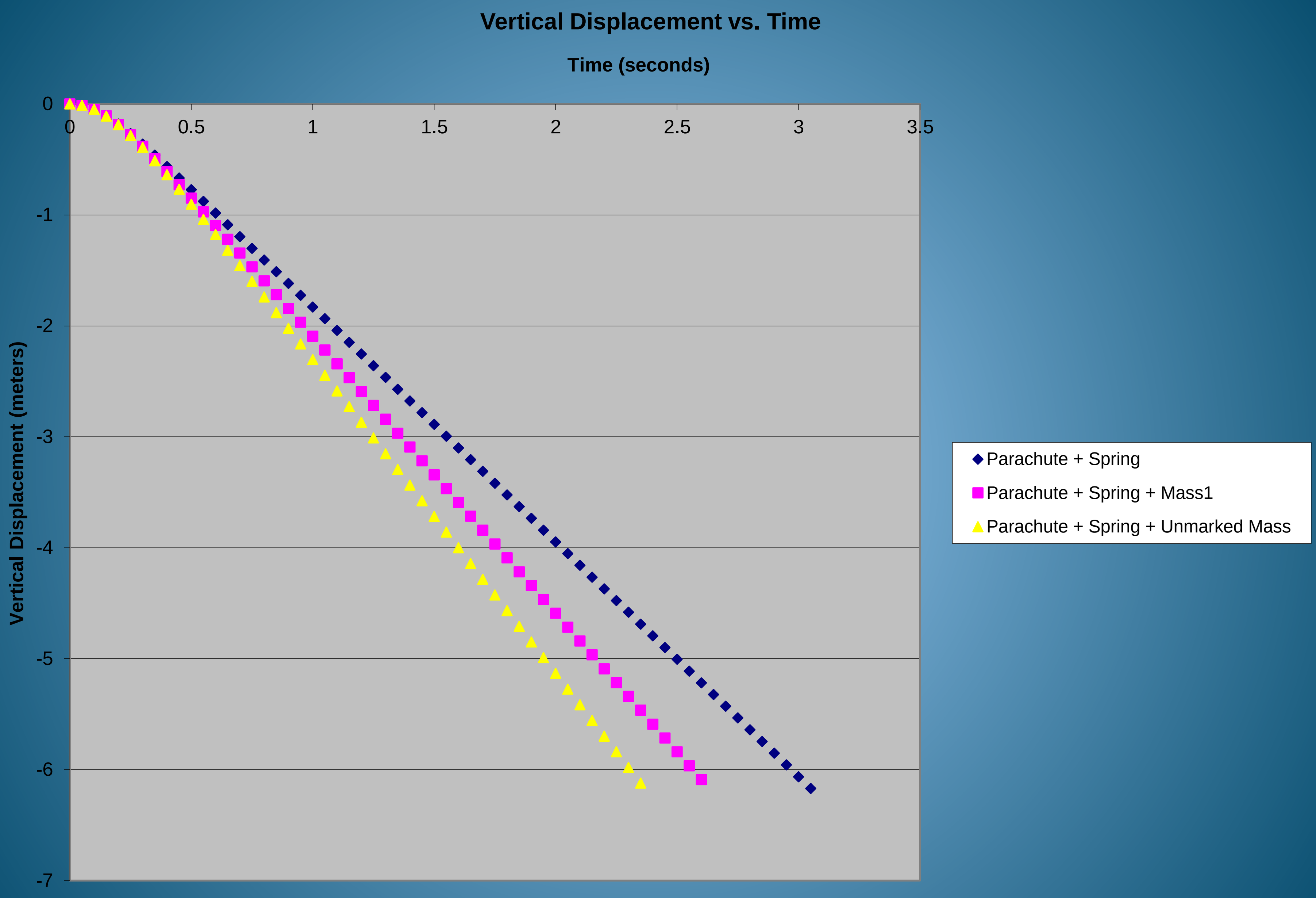
$$y_2 = y_1 + v_{y,1}0.050s + \frac{1}{2}a(t_1, v_{y,1})(0.050s)^2 \approx -0.0187m$$

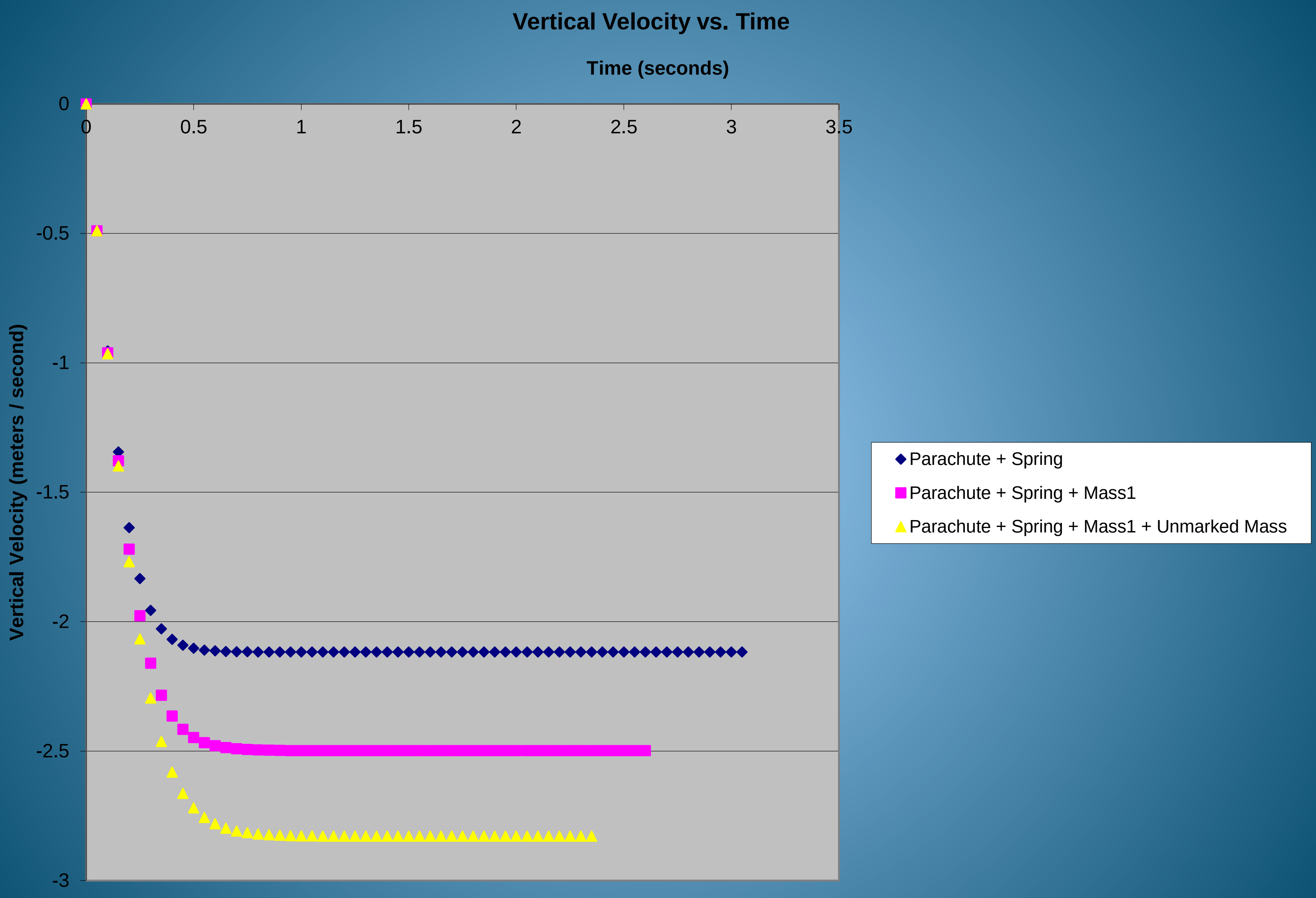
$$a_y(t_2, v_{y,2}) = \frac{B - W + D(v_y(t_2))}{m_{balloon+load+He}} \approx -3.70 m/s^2$$


Vertical Displacement vs. Time
time (s)
0
0.5
1
1.5
2
2.5
3
3.5
0
-1
-2
-3
-4
-5
-6
-7
-8
-9
vertical displacement (t)
Balloon 1
Balloon 2
Balloon 3



Vertical Velocity vs. Time
Time (seconds)
0
0.5
1
1.5
2
2.5
3
3.5
0
-0.5
-1
-1.5
-2
-2.5
-3
-3.5
-4
Vertical Velocity (meters / second)
Balloon 1
Balloon 2
Balloon 3


Vertical Displacement vs. Time
Time (seconds)
0
0.5
1
1.5
2
2.5
3
3.5
0
-1
-2
-3
-4
-5
-6
-7
Vertical Displacement (meters)
Parachute + Spring
Parachute + Spring + Mass1
Parachute + Spring + Unmarked Mass



Vertical Velocity vs. Time
Time (seconds)
Vertical Velocity (meters / second)
0
0.5
1
1.5
2
2.5
3
3.5
0
-0.5
-1
-1.5
-2
-2.5
-3
Parachute + Spring
Parachute + Spring + Mass1
Parachute + Spring + Mass1 + Unmarked Mass

Balloon Results

	Balloon 1	Balloon 2	Balloon 3
Total Mass (kg)	0.05847 $\pm$ 0.00001	0.06638 $\pm$ 0.00001	0.07434 $\pm$ 0.00001
Drop Distance (m) (Experimental)	6.01 $\pm$ 0.08	6.01 $\pm$ 0.08	6.01 $\pm$ 0.08
Descent Time (s) (Experimental)	2.30 $\pm$ 0.06	2.43 $\pm$ 0.07	2.37 $\pm$ 0.06
Descent Time (s) (Theoretical)	2.35	2.40	2.30
Terminal Velocity (m/s) (Theoretical)	-3.54	-3.44	-3.66

Parachute Results

	Parachute w/ Spring	Parachute w/ Spring + 0.010 kg Mass	Parachute w/ Spring + 0.020 kg Mass
Total mass (kg)	0.02535 ± 0.00001	0.03530 ± 0.00001	0.04526 ± 0.00001
Drop Distance (m) (Experimental)	5.68 ± 0.08	5.68 ± 0.08	5.68 ± 0.08
Descent Time (s) (Experimental)	3.30 ± 0.13	2.82 ± 0.12	2.48 ± 0.04
Descent Time (s) (Theoretical)	2.95	2.55	2.30
Terminal Velocity (m/s) (Theoretical)	-2.12	-2.50	-2.83

Improvements

- Improve knowledge of the parachute C_d.
- Attach a GPS locator to the balloon.
- Hydrogen is much cheaper than helium and can be used with reasonable caution.

References

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