46th Annual Review of Progress
in Quantitative Nondestructive Evaluation
QNDE2019
July 14-19, 2019, Portland, OR, USA

QNDE2019-1234

ACOUSTOELASTIC CHARACTERIZATION OF CONCRETE PRISMS VIA TORSIONAL VIBRATION PHENOMENA

Agustin Spalvier[1], Leandro Domenech, Gonzalo Cetrangolo
Universidad de la República
Montevideo, Uruguay

John S. Popovics
University of Illinois at Urbana-Champaign
Champaign, IL, USA

ABSTRACT

Concrete has been traditionally modeled as a linear elastic material when subjected to low stresses, and non-linear for higher compressive stress, showing a softening trend. Conversely to the traditional model, recent research has shown that even under small compressive stress levels, concrete behaves non-linearly following a hardening behavior. This behavior can be modeled in 1D by adding a "non-linear" parameter β_G*. This ongoing investigation has the objective of characterizing the* β_G *parameter for concrete specimens subjected to low compressive stresses using vibration phenomena. Three prisms of different concrete mixture designs are studied. The prisms are subjected to four loading and unloading cycles in steps of approximately 1 MPa, up to 6 MPa. The fundamental torsional frequency is measured at each loading/unloading step. The parameter* β_G *is computed for every loading/unloading cycle of each prism. The experimental results agree with the theoretical model. For all specimens, parameter* β_G *was found to be lower for the first loading cycle, between 90 and 110, and between 120 to 170 for the remaining loading/unloading cycles.*

Keywords: acoustoelasticity, concrete, monitoring, NDE, nonlinear.

NOMENCLATURE

$f_{T,0}$	fundamental torsional frequency at undeformed state
$f_{T,\varepsilon}$	fundamental torsional frequency at strain ε
τ	shear stress
G_0	transverse modulus at undeformed state
G_ε	transverse modulus at strain ε
β_G	nonlinear parameter
γ	shear strain
ε	uniaxial strain (compression = positive values)

1. INTRODUCTION

Concrete holds a remarkable balance between its cost and mechanical properties. This fact has made it the most used construction material in the world. Relevant civil structures — such as dams, tunnels, bridges and skyscrapers— are usually built with concrete.

To design reinforced concrete structures, concrete material has been traditionally modeled as linear and elastic when subjected to low stresses, and as non-linear for higher compressive stresses, showing a softening trend [1-3]. However, conversely to the traditional model, recent research has shown that under small compressive stresses concrete behaves non-linearly, following a hardening behavior [4-7].

This behavior could be modeled in 1D by adding a "non-linear" parameter β_G. The objective of this ongoing investigation is to characterize β_G for concrete prisms subjected to low compressive stresses using torsional vibration phenomena. An accurate determination of the non-linear parameters would enable the determination of concrete stresses using fully non-destructive testing techniques.

2. MATERIALS AND METHODS

2.1 Theory

Our experiments consisted in applying a dynamic stress field (torsional vibration) onto a pre-stressed (pre-deformed) rectangular prism —in which one of the dimensions is greater than the other two—. The existing stress field corresponds to an external quasistatic load —along the direction of length— and the dynamic field to a torsional vibration of the prism. The quasistatic stress field creates displacements much greater than the torsional vibrations. Thus, we assume that the material behaves nonlinearly when subjected to the quasistatic stresses but linearly for the vibration. Given these considerations, the solid's 1D constitutive equation for our elastic problem is

$$\tau = G_0(1 + \beta_G\varepsilon)\gamma, \tag{1}$$

---

[1] Contact author: aspalvier@fing.edu.uy

where τ is the shear stress, γ is the shear strain, ε is the uniaxial strain, G_0 is the transverse modulus in the undeformed state, and β_G is the nonlinear parameter. Equation (1) shows that the transverse modulus G_ε affecting the vibrational phenomena is stress dependent. From the definition of (instantaneous) transverse modulus we obtain that and by neglecting the geometric non-linear effects in the frequency of vibration and considering that, for a specific state of uniaxial deformation, the square of the torsional frequency of vibration is proportional to the transverse modulus, we conclude that

$$f_{T,\varepsilon}^2 = f_{T,0}^2(1+\beta_G\varepsilon), \tag{2}$$

where $f_{T,\varepsilon}$ and $f_{T,0}$ are the measured fundamental torsional frequencies at strain ε and 0 (undeformed), respectively, and G_ε and G_0 are the transverse moduli associated to an existing deformation ε and 0 (undeformed). Equation (2) allows extracting the nonlinear parameter β_G by carrying out an experiment and measuring the torsional fundamental frequencies and the uniaxial deformation.

2.2 Experimental procedure

The experimental campaign comprised three prisms of 15 x 15 x 60 cm³, each casted from a different concrete mixture design. Table 1 contains the properties of each mixture measured using the average of three cylinders tested at day 28 after casting.

TABLE 1: Measured properties of the three mixture designs.

	Mixture 1	Mixture 2	Mixture 3
Density $\left(\frac{kg}{m^3}\right)$	2272	2377	2364
Compressive strength (MPa)	30.7	47.9	29.6

The prisms were subjected to four loading and unloading cycles in steps of approximately 1 MPa, up to 6 MPa, in uniaxial compression. The fundamental torsional frequency of each prism was measured at every loading/unloading step.

A compressive load was applied onto the prisms using a hydraulic jack reacting against a rigid steel frame. Two neoprene pads of 2 cm thickness were used to isolate the prisms from the steel loading platens and the frame. The applied load was measured using a load cell. The strains produced in the prisms were measured using a pair of strain gauges attached on two opposing faces, along the direction of the applied load. At each loading step, a torsional dynamic excitation was exerted by applying five impacts onto the prism with an instrumented hammer. The prims' dynamic response was measured with a triaxial accelerometer and various uniaxial accelerometers.

3. RESULTS AND DISCUSSION

Figure 1 shows the results of the measured torsional frequency of prism 1 with respect to the measured compressive strain during four loading/unloading cycles (the first two loading and unloading steps, less than 1 MPa, are not shown). Figure 1 depicts the positive correlation between compressive strain (or stress) and frequency of vibration. The loading and unloading curves of cycles 2 to 4 are almost overlapped, whereas the first loading curve is not. The first loading curve has a lower slope than the rest, showing that the nonlinear behavior is larger for cycles 2 to 4 than for cycle 1 during loading. We consider that this behavior is due to damaged occurring during loading cycle 1, but no additional damage occurred during cycles 2 to 4, given that the maximum historical load was not exceeded.

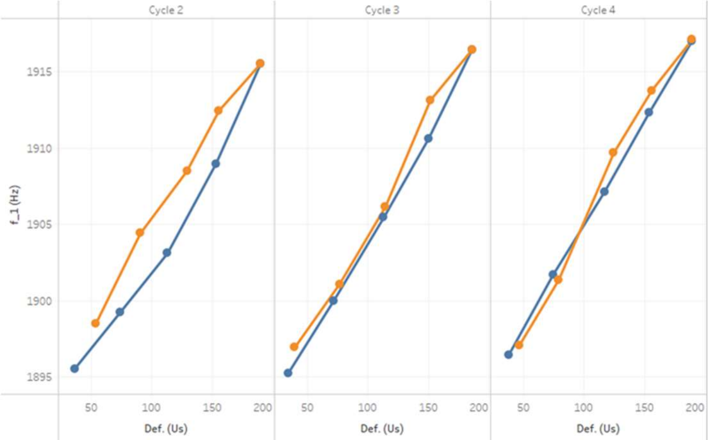

FIGURE 1: Fundamental torsional frequency $f_{T,\varepsilon}$ of prism 1 with respect to compressive strain ε during loading (blue) and unloading (orange) cycles 2 to 4.

To compare the nonlinear behavior between the different prisms, the fundamental torsional frequency results were normalized by dividing them by the value $f_{0,Fit}$. The value $f_{0,Fit}$ corresponds to a Y-intercept parameter obtained from applying a linear regression to the results of squared frequencies with respect to the measured strains. The resulting values are therefore the "squared frequency increments". Figure 2 shows the squared frequency increments of the three prisms, for the loading cycles. Figure 2 shows that even though the mixture designs of the prisms were different, they behaved similarly; in other words, all prisms behaved with similar nonlinear parameters β_G.

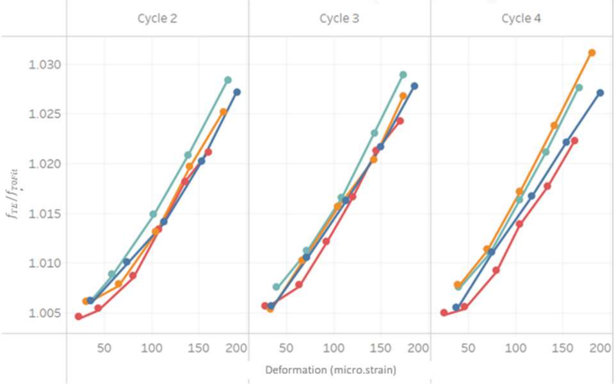

FIGURE 2: The square of the fundamental torsional frequency increment of prisms 1 through 3 with respect to compressive strain ε during loading cycles 2 to 4.

Figure 3 presents the calculated values of β_G. The β_G values of cycles 2, 3 and 4 computed for the three prisms were consistent. Only prism 3 showed a clear difference of β_G results between loading and unloading cycles. All β_G values were between 120 to 170 for the remaining loading/unloading cycles.

FIGURE 3: Results of β_G for each prism and each loading cycle 2 through 4.

4. CONCLUSION

The following conclusions can be drawn:

- Experimental results agree with the theoretical model showing a clear positive correlation between compressive strains (stress) and fundamental torsional frequency.
- The nonlinear parameter β_G was computed for each loading/unload cycle.
- For all prisms, values of β_G corresponding to the first loading cycle, between 90 and 110, were clearly lower than the ones computed for the remaining loading/unloading cycles, with β_G values between 120 and 170.

ACKNOWLEDGEMENTS

This research was partially supported by Centro de Innovación en Ingeniería, Uruguay, by Schmidt Premoldeados S.A. and by Comisión Académica de Posgrado, Universidad de la República, Uruguay.

We thank our colleagues Gonzalo Moltini, Alina Aulet and Andrés Fernández for their assistance during the experimental campaign.

REFERENCES

[1] Mindess, S., Young, J.F., and Darwin, D., 2003, *Concrete*, Pearson Education, Upper Saddle River, NJ.

[2] ACI Committee 318, 2018, *Building code requirements for structural concrete:(ACI 318-14)*, Farmington Hills, MI.

[3] CEN, 2004, *Eurocode 2: Design of concrete structures – Part1-1: General rules and rules for buildings*, Brussels.

[4] Payan, C., Garnier, V., Moysan, J., and Johnson, P.A., 2009, "Determination of third order elastic constants in a complex solid applying coda wave interferometry," Applied Physics Letters, 94**(1)**, pp 011904-1–011904-3.

[5] Lundqvist, P., and Ryden, N., 2012, "Acoustoelastic effects on the resonance frequencies of prestressed concrete beams-short-term measurements," NDT&E International, 50, pp 36-41.

[6] Bompan, K.F., and Haach, V.G., 2018, "Ultrasonic tests in the evaluation of the stress level in concrete prisms based on the acoustoelasticity," Construction and Building Materials, 162, pp 740-750.

[7] Spalvier, A., Bittner, J, Evani, S.K., and Popovics, J.S., 2017, "The stress-induced surface wave velocity variations in concrete," 43rd Annual Review of Progress in QNDE, AIP Conf. Proc. 36, pp 080010-1–080010-8.