

PART IV

*Regional Competition and
Spatial Equilibrium Models*

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Interregional Competition or Spatial Equilibrium Models in Farm Supply Analysis

PROBLEMS in interregional competition are concerned with supply and demand analysis in an appropriate degree of spatial disaggregation from national levels or a practical degree of aggregation of "contiguous" firms.

The viewpoint of the authors is that important problems in interregional competition exist primarily because of our dynamic economic system. In the absence of change, the need to explain the existing pattern of production is not nearly so great. Hence, our concern here will be with methods as they help us analyze important regional changes in physical production possibilities and factor markets. Changes in farming skills and managerial abilities (so called quality changes in factors), as well as changes in technology, are considered to be reflected in production possibilities.

Analysis of changes evident in the economy or in prospect might have either of these two objectives: (1) to predict regional changes in output, income, and product distribution given certain changes in the economic structure, or (2) to specify optimum regional output and product distribution patterns as goals for change in light of current and prospective changes in the economic system. The latter objective, of course, is normative, but it merges into the former as more and more restraints that simulate the actual adjustment process are imposed in the analytical system. Conversely, we would expect analyses of the former type to produce results that would merge into the second as adjustment "frictions" are removed from the economy.

The ideal information for studies in interregional competition would consist of (1) regional supply functions for individual products, (2) supply and demand functions for individual factors, and even (3) regional consumer demand functions for each commodity. Given this information we could better assess the relative advantages of regions in terms of location and transportation costs as well as natural factors. To support these supply and demand functions, we would have basic information

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which ties the relevant regional supply and demand functions to the shifters representing regional changes in (a) production functions, (b) market structures, (c) institutions, and (d) factor prices.

With this information, we could predict eventual distributions of producing among regions as changes in production functions, prices, and institutions occur. But more important, we could also predict (1) the effects of relevant changes on the demands and prices of resources in the various regions and (2) increases and decreases in regional farm income. This "longer-run, *ex ante* outlook" would then be the basis for guiding the decisions of individuals, structuring education and vocational guidance, and providing action elements which would facilitate this change and make it less painful.

Quite obviously, we are not soon to have data conforming to such a general equilibrium system unless the many relevant supply and demand functions are synthesized from other data. It is unlikely even that we will soon have this "system of equations model" in national aggregate such that (1) it will provide any great detail as to individual commodities and resources and (2) will take account of the major "shifters" represented by changes in technology, market institutions, and factor prices.

What tools, then, have we for analyzing the many facets of product supply and factor demand? What tools have we for tying together, region by region, the inevitable changes in technology, factor markets, and institutions? These questions must remain unanswered. The authors have not presumed to "provide the perfect tool." While the alternatives may be discussed, it must be realized that they are imperfect due to similar data inadequacies. Although, conceptually, numerous alternatives are available only those that appear most realistic in terms of the computational task and related problems will be discussed. An attempt will be made to indicate some of the problems and limitations involved in using these methods in empirical work. The authors have interpreted their specific assignment as a thorough discussion of programming models as alternatives in analyzing those supply problems that come under the heading of interregional competition.

INTERREGIONAL COMPETITION MODELS

Before turning to the central theme of this paper we shall (1) review briefly regression methods and some special techniques in the area of activity analysis, and (2) indicate summarily the apparent limitations of these methods as tools for analysis of interregional competition problems.

Regression Analysis

It is assumed the technical aspects of regression methods are covered adequately in other papers and that most of the conclusions and

observations will apply to interregional competition studies. Therefore, comments are more or less general and are concerned mainly with the problem of incorporating change into regression systems.

At a high level of aggregation, regression analysis can provide some "history of the past," but it is doubtful that it can produce anything that adequately accounts for major changes expected in the future. Unless we are interested in interregional analysis only from the standpoint of academic sophistication, the essential task, from the standpoint of education and public policy, is to predict, even if only roughly, (1) where the types of changes mentioned above will occur and (2) the magnitude of these changes. History and historic coefficients are interesting perhaps to farmers as well as to academicians. Ex post information alone, however, is of little value in guiding their decisions relative to change.

The fundamental problem in the use of regression methods is that of "detail;" detail with respect to regional disaggregation and range of observations, but primarily with respect to technological change and regional development. Supply shifters resulting from technological change cannot be ascertained from past observations. Technology is not a "smooth" function of time. True, judgment shifters could be applied to ex post regression functions but this would involve analysis that may best be carried out by other methods.

There apparently is little opportunity to use time series data and regression procedures to account for important changes of the future; changes such as those which caused much of the cotton production to shift to the Southwest, broiler production to become concentrated in the Southeast, dairy production to retreat in the central Cornbelt, and so forth. Hence, it seems that the major reliance must be placed on other methods of analysis. But there is an opportunity to use regression methods to measure the functional relationships between variables; then, use these relationships in other methods.

The Transportation Programming Model

The transportation programming model has been used quite extensively in recent studies dealing with interregional competition in agriculture; at times as a supplementary tool and at other times as the only tool. Although this type of programming has its uses, it has important limitations in studies of interregional competition because of its lack of generality. Specifically, this method is restricted to problems involving a single commodity or commodities that are perfect substitutes in satisfying requirements or demand. Thus, problems dealing with changes in comparative advantage among regions cannot be handled by this method. Furthermore, nonlinear production coefficients cannot be used as satisfactorily in this model as in others. It is best suited to short-run problems. The fundamental nature and possible uses of this programming method can be explained by referring to the following equations. The usual objective in the transportation model is to:

$$(1a) \quad \text{minimize } f(c) = \sum_i \sum_j C_{ij} A_{ij}$$

subject to these constraints

$$(1b) \quad \sum_i A_{ij} = X_i,$$

$$(1c) \quad A_{ij} \geq 0,$$

$$(1d) \quad \sum_i X_i = \sum_j Y_j,$$

in which

X_i = quantity or fixed stock of a homogeneous¹ product available at i -th supply point,

Y_j = quantity of a homogeneous product required at j -th destination,

C_{ij} = unit cost of transporting product from i -th supply point to j -th destination,

A_{ij} = quantity of product transported from i -th supply point to j -th destination.

The transportation method of programming can be best used to solve an allocation problem when: (1) fixed quantities are available at particular locations, (2) fixed quantities are required at particular destinations, (3) one unit of input (product at origin) is transformed into one unit of output (product at destination), (4) the cost of transporting one unit from the origin to the destination is the same for all levels of the activity (A_{ij}), and (5) the sum of the inputs is equal to the sum of the outputs.

A transportation model of the type outlined parallels the short-run market situation in agriculture of a single production period. Specifically, it characterizes the case in which a specific crop has been harvested, storage cannot be expanded, and consumer purchases are invariant for a wide range in prices. Under the assumption of competitive markets, shipments are made as long as transport costs are covered. Market prices are determined by the cost of making the marginal shipments to each market, and the distribution pattern will be such that net revenue will be a maximum for each supply point.²

In the comparative statics sense, the transportation model depicted by equations 1a through 1d, could be used to analyze regional changes in technology and factor prices within the marketing system. For example, it would be possible to express the effects of changes in freight rates or processing costs on the regional distribution of particular products.

¹ The term homogeneous as used here includes perfect substitutes.

² For added details see, Henderson, James M., "A short-run model for the coal industry," *Review of Economics and Statistics*, 38:336-46, 1955.

In actual practice, the transportation model need not be confined to the very narrow orientation suggested by equations 1a through 1d. Certain modifications can be used to give this analytical method a wider range of applicability than has been true thus far in application to agriculture. For example, the inputs, X_i , could be defined as the maximum amount of beef that could be produced in each of a set of regions. The costs, C_{ij} , could be defined as the combined costs of producing beef in the i -th regions and shipping it to the j -th market. Furthermore, if total possible production exceeds the total requirements (that is, $\sum_i X_i > \sum_j Y_j$), a dummy requirement, Y_{j+1} , can be set up so that restraint 1d will be met. In this case, the cost of shipments to the dummy destination would be set equal to zero, or any other constant, since they represent shipments which in practice would not be made. Also, the model can be given a "longer-run flavor" by incorporating costs due to factors involving some fixity. Thus the transportation programming model could be used to define optimal (minimum cost) production or supply patterns as well as distribution patterns, even to the extent of considering some long-run variables.

The transportation model can be used to solve maximum profit problems as well as problems of minimum costs. In this case, market prices, as well as requirements, are considered to be given. Such an analysis could be useful in determining the differential influence of different price levels on regional farm income, but again from the standpoint of one commodity only.

The types of problems in interregional competition best adapted to analysis by the transportation method of linear programming have been summarized briefly. The chief limitations of this method, compared with other programming methods, are these: (1) Only one commodity or group of close substitutes can be considered. The method cannot account for the effects of opportunity costs and competition among commodities as they relate to possible supply relationships. (2) Insufficient flexibility exists in consideration of fixed and variable resources. In summary, the transportation model can handle only very special cases of more general supply or programming problems. It was not designed to do more and the authors have not intended to imply that it was.

The Input-Output Model

Input-output models imply the opposite extreme with respect to non-fixity of resources and possible supply relationships. This is hardly a tool for meaningful analysis of the production response relationships that characterize problems of interregional competition. First, there is no adequate way to account for change aside from recomputing a matrix of interdependence coefficients for each new point in time at which data become available. Even the so-called dynamic input-output model does not express supply functions that reflect important changes in interregional competition.

The conventional input-output table can provide a useful description of certain interrelationships among geographic, commodity, and other separate agricultural sectors for which measurements are available. Similarly, it can specify flows between agriculture and nonagricultural sectors. For a particular period or point in time, and given the particular mix in which the commodities were exchanged, the interdependence coefficients of an input-output model indicate the association between changes in output in one sector, or in final demand, with changes in other sectors. But because of the mathematical properties of this particular model, it is not useful for projecting differential changes in inputs and outputs among regional sectors — supposing that the focus of the analysis is in spatial stratification of the economy.

These properties can best be illustrated by a quick review of the system. Given the available data, we start with the equality in equation 2a

$$(2a) \quad AX = Y$$

In which X is a vector of outputs for regions and commodity sectors within regions; A is a matrix of input-output coefficients with a_{ij}^{km} , an individual element, defining the amount of the k -th output in the i -th region necessary (associated with a projected increase) for one unit of the m -th output in the j -th region; and Y is a vector of final demand for autonomous or consuming sector.³

The main representation of the input-output model is

$$(2b) \quad X = A^{-1}Y.$$

$A^{-1} = C$ is an inverse matrix. Each element, c_{ij}^{km} , indicates the amount by which the k -th output of the i -th region must increase for a one unit increase in the autonomous demand for the m -th product in the j -th region.

✓ A main limitation of input-output models for analyzing output interrelationships characterizing problems in interregional competition is apparent in equation 2b. It is impossible to measure how the changes in supply relations of one region may affect the outputs of other regions. In fact, the method is not one for measuring competitive interactions among sectors. Instead it stresses the complementarity among them. It shows, under a spatial formulation of the model, how outputs of commodities in various regions must be increased together, if the output of or demand for a commodity in a particular region is increased.

³ This representation supposes that an initial flow matrix, M has been constructed. The m_{ij}^{km} element indicates the amount of the k -th output of the i -th region flowing into the m -th commodity sector of the j -th region in the period under study. The elements of A , therefore, are computed as $a_{ij}^{km} = m_{ij}^{km} (X_j^m)^{-1}$. For a "net" model, $a_{ij}^{km} = 0$, where $i = j$ and $k = m$, but all other a_{ij}^{km} are negative or zero. The relationship in (2a) could be expressed also as $(I-A)X = Y$ where I is an identity matrix and flows within a region, which characterize a "gross" model, are assumed.

The ratios between the coefficients in C for the i-th and z-th regions, $(c_{ij}^{km})(c_{zj}^{km})^{-1}$ expresses the "rate of complementarity" of the regions. It shows the "fixed mix" rate at which the k-th outputs of the two regions, i and z, (the ratio of increase of $(\Delta X_i^k)(\Delta X_z^k)^{-1}$ are required to increase with a unit increase in the final demand for the m-th product in the j-th region. Or, if we return to the original requirements matrix, A, the ratio of the two coefficients for the i-th and z-th regions, k-th product, $(a_{ij}^{km})(a_{zj}^{km})^{-1}$ shows the fixed mix rate at which the k-th output of the i-th and z-th regions must increase for a one unit increase in the output of the m-th product of the j-th region. Hence, the system assumes that the commodities produced in the various regions are technical complements, and for any increase in final demand, they must increase in the proportions existing at the time of the observations.

This discussion of the conventional input-output model is in terms of its limitations in studying major problems of interregional competition. The input-output model, of course, has other limitations which are not specific to regional analysis. These limitations include assumptions of constant scale returns, the absence of fixed factors, and others. The problem of "pure scale relationships" for firms is not a major problem when the model is applied to geographic sectors of agriculture. But differential quality of resources such as soil within a region is a problem akin to that of "pure scale" considerations. As output of an agricultural commodity is increased in a region, increments may need to be produced on soils of lower productivity or by farmers of less ability. Furthermore, if all available or suitable land is already devoted to a product, output can be increased only by a more intensive use of resources such as fertilizer on a fixed acreage of land; that is, output is augmented by a nonproportional increase of inputs rather than by "scale" increases. It is not possible to set up an input-output model with "less intensive" and "more intensive" subsectors in one geographic region to allow output to be increased from the latter as demand increases. Outputs from the less intensive and more intensive subsectors would bear the same ratio to each other regardless of the level of demand. This condition, which is forced into quantitative projections by the mathematical properties of the model, is unrealistic.

In the same vein, the "forced condition" — that as one region increases its output, poultry for example, it will be supplied inputs such as grain from other regions in the same proportions as in the past — is also unrealistic. As positive tools for supply analysis, input-output models have the same limitation as regression models in the sense that they must be restricted to observations of inputs and outputs from the past. They are historic or descriptive analyses or relationships as they have existed, and not as they will or might exist under major structural changes. Regression models, however, are not restricted to fixed mix projections of regional outputs and inputs within the historic framework. The supply functions so defined would allow specification of changes in patterns of output, given an increase in demand, among

regions. This change in pattern could rest partly on differential qualities of soil and additions of such inputs as fertilizer. Even so, the relationships expressed by the regression model could only be those experienced in the past.

The input-output model is superior to the "transportation model" in this respect: it can consider the impact of changes in demand for many products simultaneously. Through the input-output model, we can "trace back" the impact of demand changes on regional and commodity output levels and the quantity of factor inputs needed region by region. More specifically, we can ascertain how, within the limitations of the model as noted, changes in the demand for one commodity will change the demand for others. The transportation model can be used, if we are concerned with one commodity only, to "discover" what possible impact changes in such things as production techniques and shifts or differential rates of population growth would have on production location, the distribution pattern of products, and regional changes in demand for factors.

OTHER MODELS FOR INTERREGIONAL COMPETITION ANALYSIS

We now turn to the main purpose of this paper, which is to outline other programming models that might be used in supply analysis relating to interregional competition. A model will be outlined that might be feasible for analyzing those interregional supply and competition problems related to change. The nature of this approach will be explained and the difficulties inherent in it (or any method that must extend estimates of change into the future) will be indicated. Before outlining the general model let us pause and review some of the more obvious difficulties involved in such applications.

A programming model could be constructed to consider all competing agricultural products, resources, and fixed factors of different regions. It could incorporate ample detail in terms of the types and number of variables. National agriculture, in fact, could be disaggregated to any extent we wished, even to the degree of treating each farm as a separate entity with its own unique set of resources and products of different qualities. Data collection and computational problems limit such a procedure. It is necessary, therefore, to spatially disaggregate agriculture into a "reasonable" number of regions. The term "reasonable" is yet to be defined, although 104 regions have been used for a particular problem. In programming, the computational limit is related more to the number of equations than to the number of variables.

Size limits aside, programming per se does not provide the means of obtaining the necessary coefficients. It is necessary to know or to estimate the production functions (in various forms) before programming can begin. This knowledge must come from production data and scientists who can supply production relationships and other sources.

The product "supply" and "factor" demand functions derived by programming can be no better than the coefficients obtained from other sources; or no better than the restraints incorporated into the model, such as institutions and time.

Regional Producing Units

The model to be outlined considers regions rather than individual farms to be the producing units. Restraints on outputs are those of the region and various programming activities are defined accordingly. Given the concept of a regional producing unit, we would want to define regions so as to include only those farms that have the same supply or response functions.

For example, an area could be defined in which there are N farms, each with a production function for product Y as indicated by equation 3a

$$(3a) \quad Y_i = a_i z_i \quad i = 1, 2, 3, \dots, N$$

in which $z_i = b_{1i} X_1 + b_{2i} X_2 + \dots + b_{ji} X_j + \dots + b_{mi} X_m$ and the X 's are the various factors, the b_{ji} are the various weights that define the least cost factor "mix" as it appears to farmers, and a_i is the transformation coefficient of the i -th farm.⁴ Theoretically, the farm supply function would be given by equation (3b),

$$(3b) \quad P_{y_i} = \frac{P_{z_i}}{a_i} \quad \text{for } y_i \leq m_i$$

in which m_i is the output limit defined by some absolute restraint, such as land, P_{y_i} is the price of the product, and P_{z_i} is the cost of the input mix.⁵

If the ratios $\frac{P_{z_i}}{a_i}$ are equal for all farms, the total supply response for the region can be represented by equation (3c)

$$(3c) \quad P_y = K \quad \text{for } Y \leq \sum_i m_i.$$

Hence, the region can be treated as a single producing unit.

If the ratios $\frac{P_{z_i}}{a_i}$ are not equal, the regional supply response would be characterized as a discontinuous step function such as AB shown in Figure 12.1.

⁴ We have assumed, of course, a Cobb-Douglas type function, which is homogeneous of degree 1. As an approximation of output increases within a limited range, due to expansion by technical units, such an assumption may be quite realistic.

⁵ The particular variables that go into the pricing of aggregate z_i will, of course, depend on the length of run.

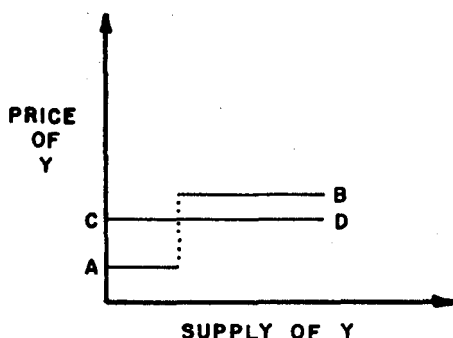


Figure 12.1.

If function AB characterizes the actual one, the use of constant production coefficients to represent an entire region would not be realistic and would assume a regional supply response such as CD in Figure 12.1.

If, in fact, the ratios $\frac{P_{zi}}{a_i}$ in equation 3b are not equal or nearly alike, and if the number of rows in the matrix is not a limiting factor, farms could be stratified, new activities could be defined for each strata with appropriate production coefficients and restraints, and the actual regional production response would be depicted. If the problem is such that the number of rows cannot be increased, the theory of convex sets can be used in some problems to account for intraregional differences in production coefficients. But this matter, which is simply the problem of adjusting the matrix size to the computing facilities, need not be gone into here.

It has not been the intent to imply in the foregoing discussion that only one activity per commodity per region would be considered in the model to be discussed. In this model, any number of activities representing different techniques for the production of the same commodity in each region can be used. These "same product" activities could be of two types: (1) Those representing different activities of the same production function; that is, vectors specifying different factor-output combinations, and (2) activities representing different production functions. If alternative activities are considered for production of a particular commodity in a region, the regional supply response may be similar to that characterized by "step" function AB in Figure 12.1. The lower segments of the step function would represent activities relatively "less efficient" in the use of fixed resources. The converse would be true for the higher segments. Consideration of many production alternatives per product in each region does not mean that farms cannot be represented in aggregate in programming analysis. The essential element for aggregation is that farms are homogeneous in production response. This may mean that they have similar combinations of resources as well as similar production functions.

The authors believe that a model built around a regional producing

unit, with certain adjustments for intraregional differences, is sufficiently realistic for interregional competition analysis; especially in consideration of the goal of analysis of prospective changes in regional variables, the data requirements, computational costs, and general "manageability."

SOME FORMAL MODELS

A regional programming model, one in which restraints and activities are defined relative to regions that make up the national aggregate, can be used to define: (1) the production and resource use pattern for a given set of regional factor prices and regional requirements of commodities; (2) the production and resource use pattern for a given set of regional product requirements when we incorporate activities which depict factor supply functions; and (3) production and resource use patterns when we incorporate activities which depict product demand functions and mesh output with regional demands. Regional "supply" relationships can be ascertained by any of these methods by making appropriate adjustments in output or demand.

The following sequence will be used for the remainder of the paper. First, models will be explained that parallel items (1) through (3) above. Second, data will be summarized that has been computed for a model of the first type. Finally, some limitations of and difficulties involved in the use of these models will be outlined.

Model A. For this model, prices of the variable factors, fixed resource levels, and regional requirements of commodities are given. We wish to define (1) the regional production and resource use pattern, (2) commodity flows between regions, (3) regional supply prices for commodities, and (4) the prices (imputed returns) of the fixed resources. Items (1) and (2) are derived by the primal solution and (3) and (4) by the dual solution to the programming problem.

The programming objective is to:

$$(4a) \quad \min. f(c) = Y_1 C_1 + \dots + Y_i C_i + \dots + Y_m C_m + T_1 R_1 + \dots \\ + T_i R_i + \dots + T_m R_m$$

in which Y_i is a subvector of product outputs, containing n elements to represent the output levels of n products⁹ in the i -th region. C_i is a subvector of n elements representing the per unit variable costs of the n products of the i -th region. T_i is a subvector of $n \times (m-1)$ elements which represent the export levels of n products for the i -th region. R_i is a subvector of per unit transfer costs for these exports.

Function 4a is minimized subject to these regional restraints:

⁹ As noted before, in some cases two or more of these elements may represent the output levels of the same commodity produced by different methods.

$$(4b) \quad y_i^1 f_{ij}^1 + \dots + y_i^k f_{ij}^k + \dots + y_i^n f_{ij}^n \leq f_{ij}$$

in which y_i^k represents the output of the k -th product in the i -th region, f_{ij}^k represents the per unit inputs of the j -th resource used to produce the k -th product in the i -th region, and f_{ij} is the level of the j -th resource available in the i -th region. There are r inequalities or restraints of type 4b per region.

The interregional restraints, the restraints that tie the system together, are defined by restraint 4c:

$$(4c) \quad \pm y_i^k p_i^k + \sum_{i \neq j} t_{ij}^k + \sum_{i \neq j} t_{ji}^k \geq s_i^k$$

in which y_i^k means the same as in equation 4b, p_i^k stands for the output (+) coefficient of the k -th product in the i -th region (or input (-) for this same activity if it represents an intermediate product, for example feed, t_{ij}^k stands for import or export levels of the k -th product with respect to the i -th region (exports have a negative sign and imports a positive sign), and s_i^k represents a constant which defines the requirements of the k -th product in the i -th region. The elements s_i^k representing intermediate goods are zero. The coefficients of all t_{ij}^k terms are unity by definition. There are n restraints or inequalities of type 4c per region.

The length of run nature of model A can be changed simply by shifting factors from the fixed to the variable class, so long as variable factor prices can be taken as given. In this case, a regional restraint or inequality is removed from the system for each resource shifted from the fixed to the variable class. Factor prices also can be set at particular levels to determine how differential changes in factor earnings would affect the production and distribution patterns, and supply prices of commodities by regions.

As factor prices would seldom be expected to be invariant for significant changes in output levels within regions, it is desirable to incorporate factor supply functions, if these functions can be defined, into interregional competition models.

Model B. For this model, prices of variable factors are endogenous (to be determined). All other variables are either exogenous (given) or endogenous as in model A. The objective of this model is to:

$$(5a) \quad \begin{aligned} \min. f(c) = & F_1 P_1 + \dots + F_i P_i + \dots + F_m P_m \\ & + Y_1 C_1 + \dots + Y_i C_i + \dots + Y_m C_m \\ & + T_1 R_1 + \dots + T_i R_i + \dots + T_m R_m \end{aligned}$$

in which Y_i , C_i , T_i , and R_i have the same meaning as in model A, but some C_i may represent a subvector of zeroes if all resources used by the Y_i activities are either acquired by factor purchasing activities or

produced within a region (intermediate products). If this is the case, Y_i represents a "transformation" subvector; F_i is a subvector of factor use levels containing q elements to represent the level of the i -th factor used at each price level in the i -th region;⁷ P_i is a subvector of factor prices containing d elements, one element for each factor and for each change in the average supply price of that factor.

Restraints of the type 4b of model A are modified for factor purchasing as in restraint 5b,

$$(5b) \quad y_i^1 f_{ij}^1 + \dots + y_i^k f_{ij}^k + \dots + y_i^m - b_{ij}^1 - \dots - b_{ij}^z - \dots - b_{ij}^r \leq 0$$

in which b_{ij}^z represents the quantity of the j -th factor purchased at the z -th price in the i -th region.

Restraints of type 5b are necessary in model B only for purchased factors. Regional resources that are fixed, such as land, would have restraints like 4b for model A.

The optimum solution for model B, that is, when function 5a is minimized, would define a production and resource use pattern that is more realistic than that for model A. This added realism is attained because not all factor prices are taken as invariant. Obviously, changes in the output of predominate crops in a region would result in changes in factor prices. Hence, the incorporation of factor supply functions into the "interregional" competition matrix should produce answers that are more important in regard to regional changes in prospect.

Changes in Demand

Both models, A and B, can be used to define changes in regional production and resource use patterns, given certain changes in regional commodity requirements or "demands" that might be associated with growth or shifts in population. To do this we would simply change the requirement levels in the 4c equations. We could, in fact, "map" the differential changes occurring region by region as one or more of the regional requirements was varied in a continuous manner. Programming routines are available to do this. But when such a procedure is applied to more than a few of the total regions and commodities the computational problem becomes excessive.

Either model A or model B could be used to obtain a more or less general spatial equilibrium solution if regional demand functions were available by commodities. The procedure, though complex, would be as follows: A particular level of output would be selected for each commodity required in each region. A minimum cost solution for supplying these requirements would be obtained. The dual solution would give the

⁷ In this formulation, the factor supply function is partitioned into segments and one average price is taken for each segment. In other words, price increases due to increased levels of factors use are approximate. The greater the number of segments used, the more appropriate will be the average factor price.

supply prices associated with the regional requirements assumed. The supplies and associated prices would then be compared with price-quantity relationships denoted by the demand functions to ascertain whether the markets were in equilibrium; that is, "supply price" would be equal to "demand price" for the quantity supplied. If the markets were not in equilibrium, new supply levels would be selected that were in the direction of equilibrium. For example, consider a particular market and product, with the supply price at \$1.50 and the quantity supplied at 100,000 units, as given by the first programming solution. The demand price for this quantity is \$1.25. Hence, to move toward equilibrium, 90,000 units might be selected for the next round of programming. Of course, if the supply price were below the demand price, the target output would be increased in the next programming matrix. Such a procedure would be followed until all markets were in balance (that is, all price-quantities had converged).

However, if intermediate products were not involved, a general spatial equilibrium solution could be more easily worked out if regional market prices were taken as given and the objective criterion were that of maximizing regional profits. If this approach were used, regional outputs could be programmed independently. This would mean a drastic cut in matrix size for programming. The amounts of each product supplied to the different markets would be added together to determine whether markets were in equilibrium; that is, the quantities supplied could be sold at the prices assumed. If markets were not in equilibrium, a new set of prices would be selected which would encourage outputs in the general direction of equilibrium. The programming routine would be repeated again and again with new sets of prices until all markets were brought into balance. In the maximum regional profit approach to general equilibrium we are asking the question: "At these prices, what quantities would be supplied?" In the minimum-cost approach we are asking the question: "At what prices would these quantities be supplied?" The general equilibrium solution would be the same in either case, but the time involved and the matrix size would determine the procedure to be used.

Technology or Other Changes in Supply Structure

The foregoing discussion has emphasized production and output response resulting from changes or shifts in demand. In summary, the general types of programming models outlined could be used as follows: (1) Either a cost minimizing or profit maximizing model could be used to define the optimum pattern of outputs, given the level of national or regional demands for the relevant commodities. (2) Using a cost minimizing model, demand restraints could be varied to trace out the pattern of outputs and their associated supply prices. (3) Using a profit maximizing model, prices could be varied to trace out patterns of supply responses in individual regions and for the national aggregate.

These models are not restricted to analysis of the demand side alone. They can also be used for analysis of changes on the supply side. If analyses were restricted to the demand side, time would have provided sufficient observations, so that very likely this general class of problems could be best solved by regression analysis. But dramatic changes are occurring in production and programming models probably have greatest applicability in analyzing possibilities in regional output response when important changes occur in production functions and institutions. (If changes relating to supply functions originated only through changes in factor prices, it is possible that time series data would provide sufficient observations for sole reliance on regression models.)

If our purpose is to project differential changes in production patterns due to changes in production functions or institutions, we might follow about this procedure: First, we would program to define regional production patterns given the current state of technology, using any of the models or approaches outlined above. Next, we would set up a new programming matrix, in which substitutes would be made for the c_i^k , f_{ij}^k , p_i^k and other elements as affected by changes in production functions, institutions, factor supply or restraints, etc. The general computational procedure as outlined above would then be repeated. The results would allow us to specify how output responses, production patterns, supply prices, and imputed factor returns would be altered if (1) projected coefficients were employed, (2) producers attempted to approximate maximization of certain objectives, and (3) certain conditions in respect to restraints of fixed resources were to prevail.

For such analyses, we would suppose that for the sake of simplicity and manageability of computations in consideration of the size of problems visualized, a comparative statics or "snapshot of time" approach would be used. That is, solutions would be computed for each set of production coefficients, factor prices, and institutional restraints. By this arrangement, the variables in the first set of solutions would not be related to those of a later period in the vein of a true dynamic model. It is not that programming disallows this approach, but it becomes computationally cumbersome for a problem involving more than a few regions and commodities.

Difficulties and limitations abound for regional programming models, just as they do for any other empirical approaches that can now be used. The authors are, because of some experience in trying to manipulate them, perhaps as aware of them as anyone else. Later, some of the difficulties inherent in the application of such models will be summarized. But now, some empirical results generated by a regional programming model are presented. These results entail the types of limitations noted here and elsewhere.⁸ The example is not proposed as one illustrating the optimum degree of detail and empirical

⁸Heady, Earl O., and Egbert, Alvin C., "Programming regional adjustments in grain production to eliminate surpluses," *Jour. Farm Econ.*, 41:718-33, 1959.

adequacy. It is presented only as a relatively simple example (although certainly a difficult example in terms of data assembly and computational routine) of the general types of data that can be generated by regional programming models. The emphasis in this model is on regional production patterns. As outlined previously, another step will produce an expression of supply functions and factor demand relationships. The model to be explained employs those techniques existing at the time the study was initiated. Further analysis is being made to define production patterns under potential changes in technology and demand. Also, project plans call for the incorporation of livestock production, but under a greater degree of aggregation, into the model.

AN EMPIRICAL MODEL

The model and results presented below required several man years of data assembly, model construction, and computing. This initial investment would be required for most regional models of some detail. However, once the data have been assembled and converted to appropriate form, professional time spent can be less for other phases of the analysis. Only a few of the possible analytical phases or steps are outlined below.

Nature of the Model

The analysis deals with wheat and feed grains. The objective function for programming was similar to function 4a. Three products were considered for each region: food wheat, feed wheat, and a feed grain composite consisting of corn, oats, barley, and grain sorghums weighted by the average relative acreage of each planted in the particular region. One hundred and four production regions were considered. To make the analysis manageable, the consumption regions were limited to ten. Thus there were three elements in each of the 104 Y_i subvectors and 27 elements in each of the 104 T_i subvectors of functions 4a. Each element, c_{ij}^k , of the subvector C_i , represented the per unit cost of producing the k -th grain in the i -th region and included costs that were due to labor, power, machinery, seed, fertilizer, and related inputs. Land and overhead costs were not included in this cost calculation. Each element, r_{ij}^k , of the subvector R_i , represented the per unit freight cost of transporting the k -th grain from the i -th production region to the j -th consumption region.

Only one resource restraint (F_i in the 4b inequality) was used for each region. This was the maximum acreage planted to the five grains in the last eight years. Because feed wheat was considered as a substitute for other feed grains, only two demand restraints (inequality 4c) were needed for each consumption region. S_j^1 was an estimate of the food wheat disappearance in the j -th consumption region in 1954,

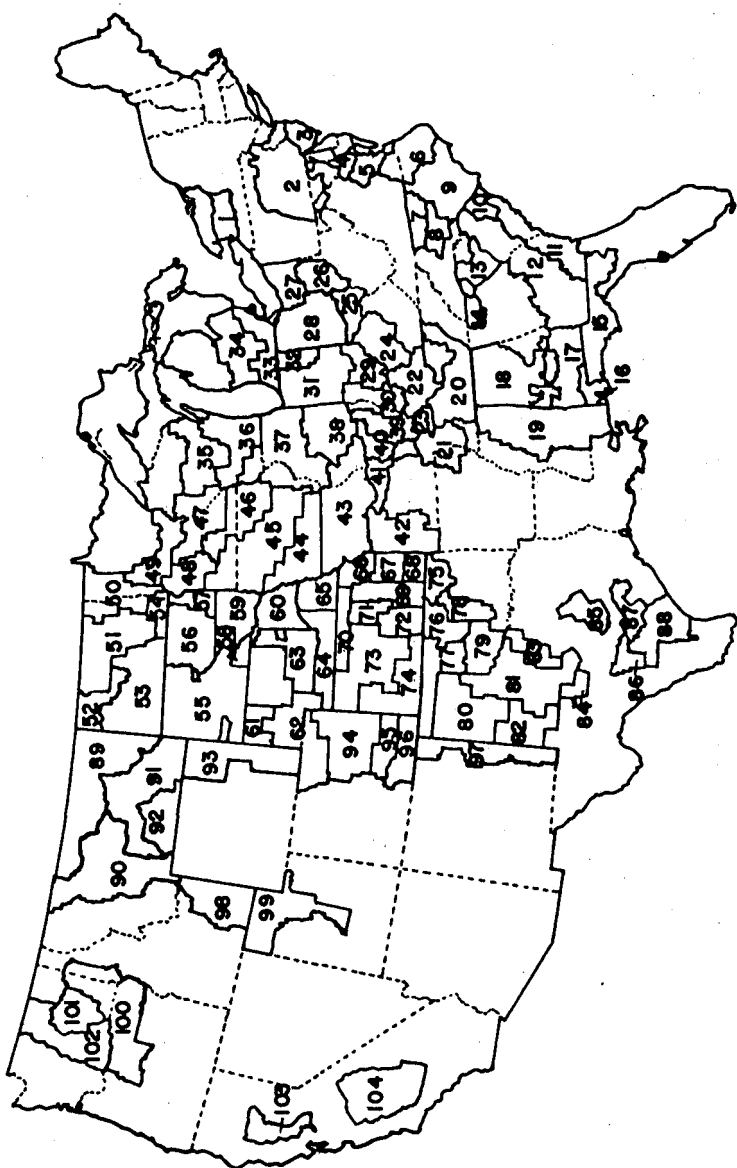


Figure 12.2. Production Regions.

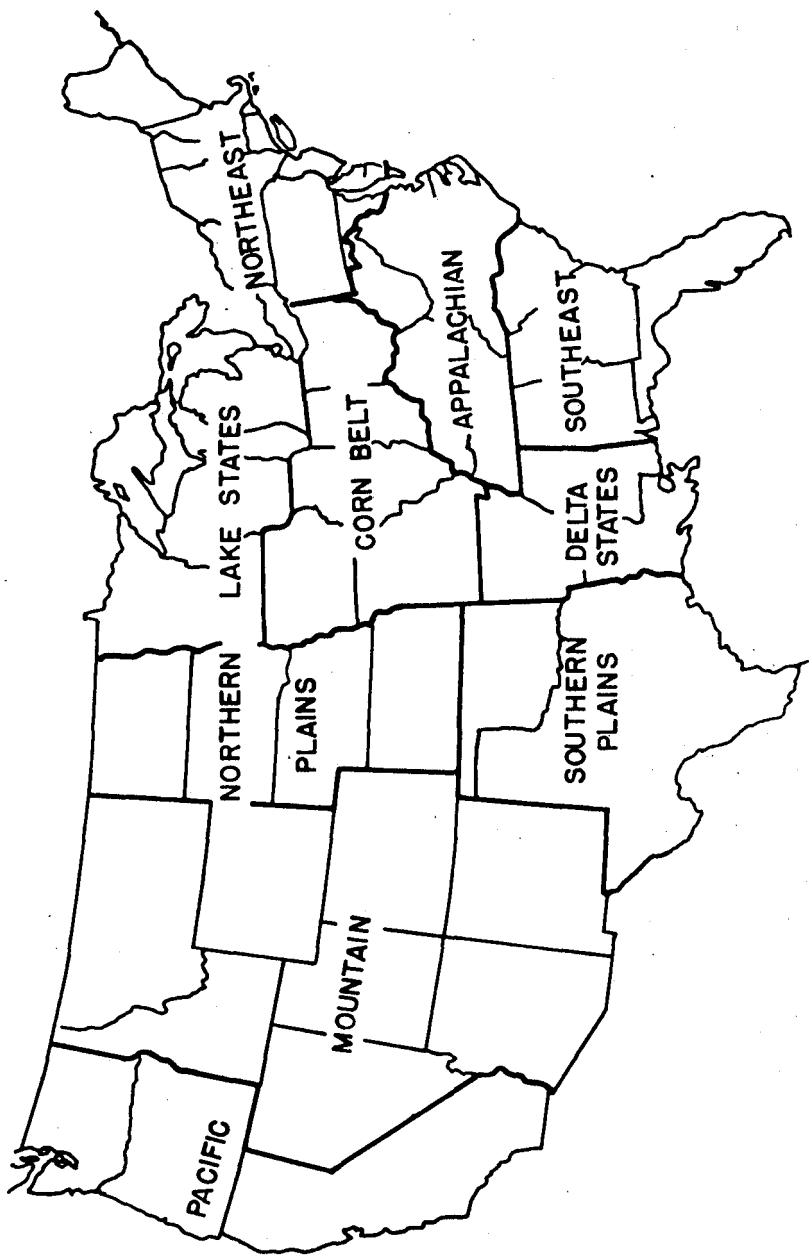


Figure 12.3. Consumption Regions.

Table 12.2. Feed Grain Shipments

Origin	Destination										Total exports
	N. east	App.	S. east	Lake States	Corn Belt	Delta States	N. Plains	S. Plains	Moun.	Pac.	
						(1,000 bushels) ^a					
Northeast		--	--	--	--	--	--	--	--	--	--
Appalachian	--		--	--	--	--	--	--	--	--	--
Southeast	--	--		--	--	--	--	--	--	--	--
Lake States	--	--	--		--	--	--	--	--	--	--
Corn Belt	296,391	118,419	198,923	--		--	--	--	--	--	613,733
Delta States	--	--	--	--	--	--	--	--	--	--	--
Northern Plains	--	--	--	--	--	129,167		20,806	--	74,266	224,239
Southern Plains	--	--	--	--	--	--	--	--	--	--	--
Mountain	--	--	--	--	--	--	--	--	--	--	--
Pacific	--	--	--	--	--	--	--	--	--	--	--
Total Imports	296,391	118,419	198,923	--	--	129,167	--	20,806	--	74,266	837,972

^aIn corn equivalents

As shown in Table 12.2, only the Corn Belt and the Northern Plains are feed grain exporters. But again, historical deficit areas are shown as importers; namely, the Northeast, Appalachian, Southeast, Delta States, Southern Plains, and Pacific. The shipments from the Northern Plains are primarily from the Corn Belt fringe in Nebraska. This fact makes the feed grain flow pattern more plausible than it might appear to be on the basis of Table 12.2.

Although Tables 12.1 and 12.2 make it appear as though there is little interdependence or competition between these ten broad areas, such is not the case. For one reason, interdependence is built into the model; for another, shifts in regional requirements would change significantly these regional flows.

As mentioned before, one objective of this analysis was to obtain supply prices in each of the ten consumption regions. These prices are shown in Table 12.3. One apparent "inconsistency" in the prices shown in this table is the feed grain supply price, at 1954 price levels, shown for the Mountain States. Here, historically, the price of feed grain has been about the highest in the United States. But perhaps a price of 69 cents is not so unreasonable for this area when considered in light of

Table 12.3. Wheat and Feed Grain Supply Prices, by Regions

Region	Wheat price	Feed grain price
	(dollars)	(dollars)
Northeast	1.48	1.05
Appalachian	1.53	1.16
Southeast	1.39	1.24
Lake States	1.11	0.88
Corn Belt	1.04	0.67
Delta States	1.22	0.98
Northern Plains	0.73	0.65
Southern Plains	1.21	1.08
Mountain	0.77	0.69
Pacific	1.28	1.12

the fact that an "output brake" was imposed by the model because of a wheat and feed grain economy with current overcapacity. Hence, if the historical wheat areas in the Mountain States were "squeezed out" of wheat production by market forces, feed grain production (in this case barley) could easily be expanded. Consequently, the price of feed grains would decline. Aside from this, the regional supply prices shown in Table 12.3 appear to be reasonable, at least in the relative sense.

The regional producing units, the kind and quantity of grain produced by each, and the imputed returns per acre of grain land are shown in Table 12.4. The reader can obtain a more vivid mental image of the specified regional production pattern if the data shown in Table 12.4 are related to Figure 12.2. He should conceive that grain production is absent in much of the Southeast, the Delta States, North Dakota, eastern Kansas, and the Mountain States, and in some other scattered areas. This production pattern represents an "ideal regional distribution of production" arising from "pure" interregional competition in two farm products given invariant consumption at a point in time, in this case 1954.

Furthermore, when interpreting these results the reader should remember that (a) spatial production patterns implied by Table 12.4 were computed under the assumption of techniques (that is, technical coefficients) equal to the average of each region and (b) the coefficients are constant within the delineated regions. Locational variations from the coefficients used would mean that some acreages in the regions not designated for production would "remain" in grain production and some acreages in regions designated for production would be "withdrawn."¹¹

The returns per acre of grainland or imputed rents shown in Table 12.4 are actually the maximum net return achievable in each region from the given production opportunities when shipments are made only to the most profitable market (or markets). Production and distribution take place only when the market price (Table 12.3) will at least cover unit supply costs. The regions shown in Table 12.4 for which no production is listed represent areas in which 48.5 million acres are usually planted to grain. To summarize, the data presented in Tables 12.1 through 12.4 characterize a production and distribution pattern that would be expected in a purely competitive economy — one of restricted nature by definition of the model and the data that went into it.

As suggested previously, many modifications can be made in the model to investigate the effects of regional changes in prospect. With adequate funds and computing facilities and time, we could add competing and complementary commodities such as soybeans, cotton, beef, pork, etc., and tie these all together in one huge matrix. We could add factor supply equations if we had them and, looking to the future, production coefficients could be modified to account for technical innovations in prospect. Product demand equations would be used to define a

¹¹ For details on limitations of the analysis, see Egbert and Heady, *op. cit.*

Table 12.4. Specific Grain Production Levels and Imputed Returns to Grain Land in Production Regions

Region	Wheat production (1,000 bushels)	Feed grain production (1,000 bushels)	Imputed rent (dollars)
1	16,197	--	10.49
2	--	97,567	10.88
3	--	19,189	15.82
4	--	13,075	14.35
5	--	7,434	12.00
6	--	20,029	9.53
7	--	--	--
8	--	--	--
9	--	--	--
10	--	5,780 ^a	1.92
11	--	--	--
12	--	--	--
13	367	7,665 ^a	0.12
14	--	--	--
15	--	--	--
16	2,087	--	8.40
17	14,856	--	4.64
18	24,058	--	3.66
19	--	--	--
20	--	24,651	1.44
21	8,971	8,771	4.87
22	--	36,399	9.41
23	--	8,342	8.85
24	--	11,089	8.98
25	--	27,833	4.68
26	--	17,770	1.15
27	27,691	--	0.78
28	--	233,287	6.46
29	--	--	--
30	--	72,903	8.30
31	--	222,916	6.86
32	26,847	--	10.52
33	5,804	48,348	2.40
34	63,850	--	1.10
35	20,494	--	1.10
36	18,941	61,318	8.83
37	--	356,616	8.72
38	--	231,170	14.30
39	--	39,025	4.01
40	19,630	--	1.36
41	14,756	--	2.14
42	--	--	--
43	57,690	82,317	3.64
44	--	153,258	5.01
45	--	403,933	8.53
46	--	157,062	7.63
47	--	84,314	5.93
48	--	73,085	10.01
49	--	30,795	3.79
50	--	89,054	3.52
51	--	--	--
52	--	--	--

^aWheat used for feed

Table 12.4. (Continued)

Region	Wheat production (1,000 bushels)	Feed grain production (1,000 bushels)	Imputed rent (dollars)
53	--	--	--
54	--	--	--
55	--	31,182	0.27
56	--	68,643	1.29
57	--	--	--
58	--	17,430	0.82
59	--	94,739	3.87
60	--	113,013	6.64
61	5,667	--	2.18
62	10,764	35,771 ^a	0.20
63	--	--	--
64	28,104	--	2.46
65	--	148,798	4.03
66	--	--	--
67	--	--	--
68	--	--	--
69	--	--	--
70	17,098	--	0.06
71	18,212	--	0.40
72	37,617	--	2.76
73	72,121	--	1.60
74	3,795	74,266	2.12
75	--	--	--
76	31,628	4,309 ^a	6.20
77	--	35,114	3.95
78	--	5,841 ^a	1.60
79	20,898	--	4.95
80	--	76,302	15.04
81	14,561	--	3.51
82	--	16,449	5.98
83	3,063	--	2.99
84	--	1,088	3.54
85	--	7,871	5.35
86	--	1,388	1.25
87	--	5,673	2.75
88	--	12,359	10.09
89	--	58,300 ^a	1.18
90	--	0.346 ^a	0.00
91	--	--	--
92	--	--	--
93	--	--	--
94	--	--	--
95	--	--	--
96	--	--	--
97	--	--	--
98	--	--	--
99	--	--	--
100	79,077	--	10.59
101	5,712	56,911	9.31
102	--	7,083 ^a	6.01
103	6,896	--	5.82
104	--	30,643	15.87
Total	677,452	3,548,514	

^aWheat used for feed

general spatial equilibrium pattern. Thus a "longer-run *ex ante* outlook" on American agriculture would be provided to guide the decisions of farmers and those concerned with aiding farmers in the adjustment process.

PROBLEMS AND LIMITATIONS IN THE USE OF INTERREGIONAL PROGRAMMING MODELS

Most of the problems or limitations in the use of interregional programming models fall into three classes: size, data, and research resources. The most important of these, as noted earlier, is research resources. If we had sufficient resources, the other problems would disappear; and we should include human resources as well as funds. But because resources are limited, the other two limitations are highly relevant. Hence, we discuss some of the problems relating to them.

Relatively few variables in an interregional programming model generates a mammoth coefficient matrix. Although only 104 production regions, each with three activities, and ten consumption regions were considered in the wheat and feed grain problem just described, the resulting matrix was of a $124 \times 3,120$ order. With a matrix of this size, "bookkeeping" alone is a significant problem. In addition, it leads to difficulties in finding and financing computing facilities of adequate capacity. But future advances in computing technology may easily take care of most of the size problems.

Problems relating to data for interregional competition studies may be most significant for some period ahead. Much information on farm production functions or input-output coefficients is needed. A large part of that currently available is very fragmentary, incomplete, or out of date. The problem of defining and obtaining coefficients for approaching technology is especially important. If farm production functions were available by type of farm, managerial level, and soil class, data relating to these variables could be incorporated in the matrix, even to the extent of taking into account changes in scale returns. There is even the possibility of using programming "gimmicks" to consider such things as discontinuities of inputs and interactions between activities or enterprises.

Production functions or coefficients are needed for the marketing sectors of the system as well as for farms in interregional competition models. A dearth of information now prevails. Reliable input-output data in the marketing sectors are difficult to acquire, even when resources are adequate.

If size, data, and research resources did not place limitations on models dealing with interregional competition, could we "exactly" describe the production and distribution pattern, and the general supply situation of 10 or 20 years ahead? Very probably we could not. Technological changes and innovations of the future could be forecast only imperfectly. Even though all measurable economic variables could be

"captured," present and future, in one great system, the human psyche or actions of people themselves may distort the "best laid" projections. Hence, the "most exact" programming models might be in error in much the same way that extrapolation into the future with regression models could produce erroneous projections.

Certainly, programming is not the "perfect tool" for interregional competition analysis. However, the authors believe, that given the current state of the arts, it offers one of the better means for analyzing the prospective effects of major changes now occurring in the economy and those now appearing on the horizon. They believe that compared with other formal empirical tools available, it has more inherent flexibility for accounting for change and for considering fixed factors and different lengths of run. Its major limitations arise from data requirements and in defining restraints that are consistent with those of the real world.

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Discussion

IT IS CLEAR that the impact of technology, organizational innovation, economic growth, and demand shifts have not been uniform among regions within this country. This being the case, analyses based on national aggregates do not provide an adequate basis for measuring the effects of structural changes, nor for evaluating the price, income, and efficiency implications of alternative policy and program proposals. The renewed interest of agricultural economists in the structure of interregional competition is commendable and the Egbert and Heady paper is a significant contribution to the growing literature in the area. The Egbert-Heady review of the characteristics and limitations of alternative interregional competition models will prove to be particularly helpful to other researchers and students of the problem.

There are three major conclusions explicitly or implicitly expressed in the Egbert-Heady paper. These are:

1. Programming models presently offer the most effective means of analyzing major structural changes in the economy.
2. The major impediment to the effective application of the various programming models is a critical deficiency of data and computing facilities.
3. Programming models provide a suitable framework for incorporating available coefficients into models approximating general equilibrium systems.

The discussion which follows is primarily devoted to an analysis of the first two of these propositions.

Problems relating to the inclusion of "shifters" in regression models has long been recognized and has been analyzed in other papers presented at this conference. For this reason, as well as the greater flexibility of the programming models, I tend to agree that programming models may offer a superior means of analyzing change. At the same time, the manner of evaluating structural change in static programming models is far from obvious. Clearly other tools have failed to provide the basis for predicting major structural changes, such as the rather recent shifts in the broiler industry. However, it is not clear that programming models could have done better.

Egbert and Heady conclude that transportation models lack generality and must play a minor role in interregional competition studies. Specifically, the transportation model is concerned with spatial allocation of a single commodity when production and consumption for each region are predetermined. The problem is to minimize transportation costs in satisfying the predetermined regional bill of goods. We might accept the single commodity limitation, but the independence of supply, demand, and prices postulated by the transportation model is not a tenable assumption.

The Enke-Samuels-Berkman spatial equilibrium model may be superior to the pure transportation model in that product price differentials between regions and total consumption within regions are endogenously determined, given predetermined production, along with demand relationships for each region. Thus this model enables the investigator to examine the implications of changes in supply, shifts in demand variables and parameters, and changes in transportation or processing costs for equilibrium shipments and prices. An alternative formulation is to take total consumption in each region as a predetermined variable with supply to be determined endogenously.

Input-output models emphasize the interdependence of the various sectors of the economy. This is obviously a desirable feature and an important advantage over the transportation model and its preoccupation with a single commodity. However, input-output models relate to historical periods and their application to analyses of structural changes is yet to be demonstrated. Thus Egbert and Heady are probably correct that the more general programming models offer a more promising approach to the relevant question at hand.

Perhaps without exception, economists engaged in empirical research decry the lack of "suitable" data, time, and computing facilities. In the case of generalized programming models such as those described by Egbert and Heady, data and computing requirements are indeed imposing. Such models require data essentially in the form of supply, demand, and price parameters by regions. Nevertheless, it would appear that important conceptual problems must be solved before the nature of the data needs for programming are specified and developed.

It should be clear that data, computational, and conceptual problems are interdependent. Empirical models are compromises between incomprehensible and unmanageable but realistic detail and understandable

and manageable simplification. In the case of interregional competition problems, simplification must be the rule rather than the exception. I would hypothesize that larger matrices and larger computers are not the answer. Rather, researchers must develop models with relatively high degrees of aggregation which are capable of reflecting the magnitudes of the most relevant variables.

All would agree that supply relationships should be built into inter-regional programming models. But are we interested in normative or predictive relationships? In a like vein, what length of run is most relevant? Similar questions could be raised with respect to demand. However, work to date seems to suggest no differences as to length of run in demand relationships.

Perhaps the proposed use of the results may dictate the nature of the supply relationship desired. If interest is centered on economic efficiency or normative product-resource flows given pure and perfect competition, as appears to be the case in the Egbert-Heady work, normative supply relationship would appear appropriate. On the other hand, if one is interested in providing guides for program evaluation or administration, the predictive models may be more suitable. Actually both the normative and predictive estimates will be necessary if we are to adequately measure the gains and losses implied by alternative programs. Also the "no man's land" between normative and predictive models may offer a fertile field for investigation.

A conceptual problem of first order importance relates to the quantity to be maximized. We have traditionally limited ourselves to criterion functions consistent with efficiency models assuming pure and perfect competition. We must broaden the scope of our research if we are to play a significant role in policy evaluation and program guidance. We need to measure the effects of alternative institutional restraints, including market or supply controls, on the total and the farm economies and regions within the farm economy. I would hypothesize, for example, that agricultural economists have little concept of the nature of intra-industry and inter-regional income transfers which would result from a program of comprehensive supply controls. In fact, the notion of supply controls is rivaled only by the free or flexible price idea as being the most over-advocated, under-analyzed proposition of our time.

Historically, demand relationships have apparently been much more stable than supply relationships. Thus it is generally conceded that our knowledge of demand is superior to our knowledge of supply and that the critical void is the area of supply. Nevertheless, most available demand coefficients are of the *ceteris paribus* nature, while in the real world other things simply do not remain the same with a change in one price or quantity. Such an assumption may be tenable in dealing with a single commodity or group of commodities, however the crucial problems in agriculture relate to total supply-demand relationships.

Egbert and Heady are fully aware of the limitations of the empirical results which they present. Yet the rather simple model presented

does indicate the tremendous complexity and suggests the substantial understanding that could be derived from a more general model. Also it should be clear from the Egbert-Heady paper that the various approaches to supply analysis can be highly complementary when results from various forms of analyses are incorporated into a generalized programming model so as to allow a general equilibrium analysis. Nevertheless, it is likely that over the near term partial equilibrium analyses such as the Egbert-Heady approach would seem to offer a fruitful field for further research.