Chapter 8

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*Programming Intra-Farm Normative Supply Functions**

THE PURPOSE of the normative supply function is to describe the optimum relation between the quantity of a product supplied and its price, relative to some given norm. The maximization of farm profits is one possible norm upon which estimates of normative supply functions may be based. It is the norm assumed in the following discussion. The normative supply function is thus an estimate of the optimum supply reaction to product price changes in terms of the stated norm, rather than an estimate of actual supply reactions made by producers to changes in product price (1).

Estimates of normative supply functions become of particular importance in the evaluation of major resource allocation alternatives on individual farms. Although the actions taken by an individual farm usually have little impact upon the industry, the collective actions of all competing farms will have a considerable impact. The optimum allocation of resources on an individual farm will be dependent in part upon (1) what constitutes the optimum allocation of resources on competing farms in the same area as well as in competing areas and (2) the effect this has upon the aggregate supply of the product and the product price. The aggregation of individual farm normative supply functions to form the aggregate supply function provides a means of taking into account the effect of changes in aggregate supply in evaluating the resource allocation alternatives of individual farms when related to the appropriate product demand function. The results obtained by this approach are optimum only in terms of the assumed norm, or set of norms, used in the analysis. However, such results provide a point of reference against which divergent uses of resources and divergent goals or norms can be compared.

The objective of this paper is to discuss the problems of using linear programming in the derivation of individual or intrafarm

*Journal Article 2543 from the Michigan Agricultural Experiment Station. Many of the ideas presented in this paper were evolved in the course of the development of the Lake States Dairy Adjustment Study involving the states of Michigan, Wisconsin, Minnesota, Illinois, and Iowa. The study consists of a group of individual state projects coordinated on an informal basis through the Farm Economics Research Division, ARS, USDA. The many discussions that have taken place among the persons associated with these projects have contributed greatly to the content of this paper. However, the responsibility for the validity and accuracy of the statements made rests entirely with the authors.

normative supply functions used to construct an estimate of the aggregate normative supply function. The aggregate supply function is here regarded as being obtained by the horizontal summation of the supply functions of the individual farms. Hence, the basic unit of inquiry is the individual farm and the main use to be made of linear programming is to derive the normative supply functions of the individual farms. The problems of deriving the normative supply function of an individual farm using linear programming will be considered first. Following this, the problems of deriving the individual farm supply functions as related to construction of the aggregate supply function will be discussed. The main problems connected with the use of linear programming for this purpose are more nearly ones of definition and theoretical development of the problem than they are of programming methodology per se.

THE LINEAR PROGRAMMING PROBLEM

Basically, linear programming is simply a method of solving a particular type of mathematical problem (2). Stated in general form, this problem is,

$$\text{maximize} \quad f(x) = c_1x_1 + c_2x_2 + \dots + c_jx_j + \dots + c_nx_n$$

$$\text{subject to} \quad \begin{aligned} a_{11}x_1 + a_{12}x_2 + \dots + a_{1j}x_j + \dots + a_{1n}x_n &\le b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2j}x_j + \dots + a_{2n}x_n &\le b_2 \\ \vdots \qquad\qquad &\quad \vdots \\ a_{i1}x_1 + a_{i2}x_2 + \dots + a_{ij}x_j + \dots + a_{in}x_n &\le b_i \\ \vdots \qquad\qquad &\quad \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mj}x_j + \dots + a_{mn}x_n &\le b_m \end{aligned}$$

$$\text{and} \quad x_j \ge 0$$

Any empirical problem that can be represented in terms of this general mathematical statement is amenable to analysis by linear programming. Linear programming is thus a general analytical technique that may be usefully adapted to the analysis of a variety of empirical problems. The major problem of using linear programming concerns how its mathematical format is adapted to the conditions of a specific application. To determine this, it is necessary to look to the conditions of the specific problem as it is defined and the relevant body of theory that applies to it. Linear programming only specifies the general mathematical form in which these conditions and the pertinent theoretical concepts must be expressed.

In estimating an intrafarm normative supply function, the general approach would be to construct a mathematical model of the farm in terms of the above mathematical format. Since the criterion is the maximization of farm profits, the f(X) would represent the total profits of the farm. The x_j would represent the level of the various production alternatives or activities to which the farm's resources may be allocated. The c_j would represent the profit per unit of each activity. The a_{ij} would represent the resource requirements per unit of each of the activities. The b_i would represent the quantity of each of the various resources available to the farm or such other restrictions as limit the use of resources on the farm. As the mathematical model consists of a series of linear equations, nonlinear relationships such as diminishing marginal physical productivity or decreasing marginal rates of substitution must be handled by approximating the relationship by linear segments. The mechanics of dealing with these and similar problems in constructing a linear programming model representing the resource allocation possibilities of a farm have been treated in considerable detail in several recent references (4).

To derive an estimate of the normative supply function for one of the products that could be produced by the farm, the various solutions to this model would be obtained over a range of values of the appropriate c_j, using the procedure of variable price programming. The quantity of the product produced at each price level assumed can be readily obtained from these solutions of the model to form the supply function (9).

SCOPE OF THE INDIVIDUAL FARM MODEL

There are a number of major conceptual problems involved in the construction of this mathematical model of a farm. The first of these concerns the scope of production alternatives to be considered. This becomes primarily a matter of problem definition. In any analysis of overall farm organization, the range of production alternatives considered is likely to be rather large.

At a given point in time, a farm operator will have a certain stock of resources under his control making up the farm's current production organization. At the same time, he will have before him an array of possible alternative courses of action that he might take with respect to the organization of these resources to produce income in subsequent production periods. One possible course of action would be to leave the current organization of the farm unaltered. Other alternatives range from completely disbanding the farm and directing the use of the resources to nonfarm alternatives, to making any of a number of possible investments to expand or modify the farm's existing organization. Some of the possible investments may consist simply of purchases of single period inputs — inputs completely utilized within a single production period — which would involve only minor changes in the farm organization. Other investments might consist of major changes in multiple period inputs. These inputs provide services over a series of production periods, and could involve organizing the farm around entirely new lines of production or adoption of entirely different production technologies. Given that profit maximization is the principal criterion of choice from among these alternatives, there is no theoretical basis for restricting consideration of allocation alternatives to some subset of this full range of possibilities. That is, at any point in time, there is no reason why action cannot be taken to alter any aspect of the farm organization should it be profitable to do so and provided that means for accomplishing the change are available (6).

Although it is true that reorganizations requiring construction and equipping of a new set of buildings will take somewhat more time to accomplish than a reorganization such as changing the composition of a ration, this is not the type of time consideration that is of main concern in a normative analysis of optimum farm organization from the standpoint of profit maximization. The purpose of the analysis is simply to determine from the array of alternatives what changes, if any, should be made in the organization of the farm at a particular point in time to maximize the profits accruing to that farm's managerial unit over subsequent production periods. Whether or not a particular alternative would be included in the optimum organization of the farm would depend upon (1) the current asset structure and organization of the farm, (2) the current investment required to adopt the alternative, and (3) the expected future pattern of price relationships.

In constructing the programming model for this type of problem, the list of activities to be included would theoretically cover all alternatives germane to the agriculture of the particular farm's locale. The list would not only include the various products that might be produced in the area, but also the various technologies by which they might be produced.[1] Furthermore, the techniques of production considered would include those currently being used in the area as well as those known and commercially feasible but not as yet generally adopted. Obviously, such an all-inclusive analysis would be extremely difficult to carry out. Not only would the size of the programming model be near prohibitive but the available data are not likely to be sufficiently reliable to justify attempting to distinguish between alternatives in such extreme detail. Therefore, the exact list of activities making up the model will necessarily be arrived at by arbitrary decisions based on the analyst's judgment and knowledge with respect to the problem. However, the above reasoning with respect to the scope of the model is nonetheless valid and provides the conceptual basis for designating appropriate activities for the type of problem outlined.

The relevant restraints to be incorporated into the model as well as the activities will depend partly upon the amount of detail that can be "reasonably justified" in considering the organization of a farm.

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[1] When a set of alternatives or activities pertains to production of the same product, those having a greater requirement for all inputs per unit of output need not be included in the model. Such alternatives would be technically inferior and would never be included in an optimum solution.

Again, this is a matter of judgment. Conceptually, at least, one could go to the point of breaking each possible farm enterprise into tasks such as seedbed preparation, row-crop cultivation, grain harvesting, corn harvesting, etc., and designating every different way of performing a task as a separate activity. By the same token, the capacity of each distinct type of machine or facility could be designated as a separate restriction or equation in the model. The feasibility of this depends partly upon whether or not meaningful differences of this magnitude are reflected by the data and partly by the resources available to carry out the analysis.

The relevant restrictions for a programming model to be applied to the analysis of the overall organization of a farm might be classified into three broad groups: (1) The resource restrictions; (2) the institutional restrictions; and (3) the technical restrictions.

Resource Restrictions

The resource restrictions refer to the physical resources of the farm, such as land, labor, capital, equipment, and facilities. A separate equation would be included in the model for each distinct resource. The general rule of thumb applies here that resources which are perfect or near perfect substitutes in all uses can be treated as the same resource and be combined into a single equation in terms of their least common denominator. For example, all feed grains could be combined into corn equivalents on the basis of TDN. Resources that are imperfect substitutes or perfect compliments would have to be dealt with in separate equations. Resources such as labor provide a continuous flow of services over a production period and are used in varying amounts by a number of different activities at different times within the period. They are most appropriately handled by designating separate equations relating to the use of the resource in the different subparts of the production period. The initial level of each of the resource restrictions, the b_i, would be the current stock of each resource that is on the farm. Except for the capital equation, the construction and use of these equations in the model would follow the usual lines of the many applications made of linear programming to the analysis of the resource allocation problems of farms (4).

As the problem posed concerns not only how the present resources of the farm should be used but also whether or not the level and form of the assets should be altered, the level of the restraint for each equation would not be regarded as absolutely fixed at their initial level. The model would be constructed to allow for either the purchase or sale of any resource, depending upon whether or not it would be profitable to do so. This is accomplished by including in the model a purchase activity and a sale activity for each separate resource (8). Whether or not a resource such as a farm building has any sale value is a question of empirical fact. If its sale value is zero, that is, it has no value other than in direct use on the farm, its selling activity would become the usual slack or disposal activity with $C_j = 0$ that appears in the programming model when the initial restriction is stated as an inequality.

The capital equation would express the amount of capital required per unit of each of the activities, just as in the case of any of the other resource equations. However, as the analysis deals with new capital investments in multiple period resources as well as the possible disinvestment in existing assets, some unique problems arise. The initial level of the capital restriction, b_i, would represent the amount of liquid assets currently available to the farm exclusive of credit. The model would include a capital sale activity that would account for the opportunity cost of capital in nonfarm uses. Purchase of capital would involve the use of available sources of credit expressed in the model as capital borrowing activities. To the extent to which different sources of credit are available at different interest rates and/or involving different repayment schedules, separate activities can be defined to take this into account. The capital equation thus deals with the allocation of both the liquid assets of the farm and any capital that the farm may find profitable to acquire from credit sources. The coefficients of the capital equation would express the total capital requirement per unit of an activity for both single or multiple period inputs. The capital coefficients thus reflect the full cost at the current point of time of introducing an activity into the farm organization and operating it over one production period.

As all resources are regarded as potentially variable in this formulation of the problem, the availability of capital and credit constitutes the principal resource limitation to the organization of the farm. The availability of credit is based largely upon the entrepreneurs equity in his assets. The slope of all factor supply functions to the farm is generally regarded as being zero. In instances in which this assumption does not hold, an upward sloping factor supply function can be approximated by a step function incorporated into the model (5). A separate activity and a separate equation would be required for each step in the function. As in the case illustrated in Figure 8.1, there would be three activities, X_1, X_2, and X_3, for which the corresponding Factor prices would be C_1, C_2, and C_3. There also would be three equations stating the range in factor purchases over which each price applies and for which the corresponding restrictions would be b_1, b_2, and b_3. This construct probably would be most pertinent in the case of credit when there are a number of possible sources of credit which differ in terms of the interest charge, repayment rate, and quantity available.

Institutional Restrictions

The institutional restrictions would reflect those that bear upon the

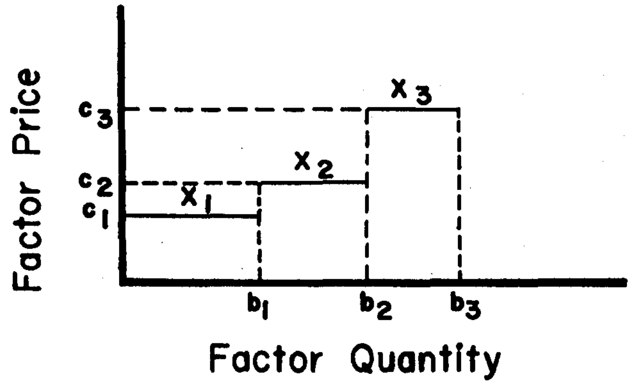

Figure 8.1. Step factor supply function.

organization of the farm and arise from the institutional setting within which the farm must operate. The most obvious of these would be crop acreage limitations or marketing quotas imposed by government programs. Depending upon how they are established, these limitations may or may not be altered by increasing or decreasing the land area in the farm. Contractual obligations would be another type of restraint in this group. Limits on the availability of credit from various sources and of various types might also be regarded as among the institutional restraints.

Technical Restrictions

The technical limits within which the farm must be organized are expressed here as the technical restrictions. For example, it is not advisable, in certain areas at least, to plant field beans on the same field two years in a row because of disease problems. Therefore, in areas in which field beans are a cropping alternative, one technical restraint would be that the acreage of field beans in any one year could not exceed 50 per cent of the total cropland acreage of the farm.[2] The basis for specifying such limits may be the soil characteristics, topography, or disease problems of the particular locale. The number and nature of this type of restraint required in any particular programming model will depend upon the technical conditions pertaining to the particular farm programmed.

Where activities may be combined in vertical sequence in the production process and more than one possible combination exists, a

[2]This type of limitation may also be taken into account by specifying crop activities in terms of rotations. Rotation activities involving field beans in two successive years would not be included or would reflect the low yields on the second year of field beans making it a relatively unprofitable alternative.

series of technical restraints are required to express the nature of the relationship between the activities. This is the case when the output of one activity may become an input for one or more other activities. A common example of this is when feed crops produced on the farm may be either sold or used as inputs for livestock activities. Here, a separate equation would be required for each distinct feed category. The output of the feed from the crop activities would be expressed as a negative input in the appropriate equation. The use of the feed by other activities would be handled in the same equations in the same manner as any other input requirement. The same kind of situation arises when enterprise alternatives are separated into tasks and activities are defined for each alternative way in which a particular task may be performed. In this case, the output of the activity is not a product in the usual sense but a condition that is a necessary prerequisite to the performance of other tasks occurring later in the production process. However, the way in which it is handled, in terms of specifying equations in the model, is exactly the same as in the case of intermediate products, where inventories may exist in which the initial level of the restraint would be the inventory of the product currently on hand.

CONSTRUCTION OF THE PROFIT EQUATION

The construction of the profit equation for this type of programming model presents some problems as the model takes into account possible changes in the quantity of both single- and multiple-period resources used on the farm. If all multiple-period resources were regarded as fixed at their current levels and only the purchase of single-period resources were considered in the model, construction of the profit equation would be relatively simple. The profit equation would include expenditures on all single-period resources used during the production period and all revenue from the sale of products during the same production period. The costs associated with the multiple-period resources that have been assumed to be fixed in quantity, for example taxes, repairs, etc., would not be included in the profit equation as they would not be affected by the pattern of organization that may be adopted.[3]

As the production processes involve some lapse of time between the initiation of production and the output of the product, the maximization of profits will be made with reference to some specific span of time. The choice of time span is an arbitrary choice, although the calendar year is customarily used as the accounting period. Any other span of time could just as well be used. However, the calendar year does conform reasonably well to the cycle at which operations are

[3] For convenience these fixed costs may be entered into the profit equation in a lump as a negative constant so that profits (gross returns less fixed and variable costs) of the optimum organization would be obtained directly in solving the model.

repeated in agricultural production processes. Using the calendar year as the accounting period, all expenditures, revenues, and resource requirements would be stated in terms of this 12-month interval and profits would be maximized in terms of that interval.

When the purchase or sale of resources having different lengths of useful life are considered in the analysis, construction of the profit equation becomes somewhat more complicated. When the useful life of a resource extends over several accounting or production periods, charging its full purchase price or crediting its full sale value to a single accounting period in most instances would mean that profits would be maximized by selling all assets of the farm. If the concern is only with maximizing the profits obtained in the current production period, the sale of all the farm assets probably would be the optimum course of action when there are assets that can be sold. This raises the question as to what constitutes the appropriate length of planning period. Is it a single production period or a series of production periods into the future? To some extent, this question goes beyond the bounds of a normative analysis of farm organization in terms of profit maximization and gets into consideration of individual values and preferences, as well as other characteristics associated with the individual such as age, family status, etc. As it is conceived here, the normative analysis of farm organization completely abstracts from these aspects of the problem related to the human factor. The analysis is concerned specifically with the choice of the most profitable set of alternatives as of the present point in time in relation to the technological and institutional circumstances surrounding the individual farm. Under these conditions, the appropriate approach is to seek to maximize profits of the immediate production period because the results that can be achieved in subsequent production periods will depend partly upon how effectively the farm is organized in the preceding production periods.[4] Therefore, the model still would be cast in terms of the single production period or calendar year, even when considering the purchase and/or sale of resources of widely different lengths of useful life. However, the price of the multiple-period resources must be pro rated over the series of production periods in which their services are available.

The cost of a multiple-period resource that is borne in a single production period would be related to the proportion of all the stock of services provided by the resource that are utilized in the single production period. An estimate of this cost may be obtained by deducting the estimated scrap value of the resource at the end of its productive life from the total initial investment and dividing the remaining value

[4] This presumes that exploitive practices which would impair the productivity of the resources in future production periods are not to be considered. Under some circumstances, such practices may be justified but this aspect of the problem has not been taken into account here. See Heady (3).

by the estimated number of years of useful life.[5] The sale of a multiple-period resource would be treated similarly. Using these "annualized" prices for multiple-period resources, the profit equation would be stated as before with one exception. Any additional costs incurred as a result of resource ownership (for example, property taxes) would also need to be included in the coefficient of the profit equation. These costs would be added to the purchase price of the multiple period resource. They would also be added to the sale price of a resource.

The basic assumption implicit in this statement of the profit equation is that the prices of all inputs and all product prices hold over all production periods extending into the future. Under this assumption, the optimum choice of the alternatives would apply not only to the current production period but also to each subsequent production period, provided the profits are not reinvested in the farm, growth resulting from capital accumulation. Derivation of a normative supply function from this model involves considering different assumptions as to the price of a particular product over this series of production periods relative to factor prices and the prices of other products. This supply function describes optimum adjustments in resource allocation and farm organization to price relationships at a particular point in time, the present, under the assumption that the farm's profits are to be maximized. Further, the nature of this supply function is related directly to the present asset structure and organization of the farm. If the purhcase and selling prices of all resources are equal, that is, there are no price discontinuities, this normative supply function would be perfectly reversible and would apply equally well to all production periods extending into the future. With an increase in product price, the farm would move up this normative supply function. With a decline in price, the reverse movement would occur along the same function.

In the presence of price discontinuities, the supply function no longer would be reversible and if there has been any change in the organization and asset structure of the farm in the meantime, there would be a unique normative supply function for each subsequent production period (6). Having made an adjustment in resource use to one set of price relationships, the conditions under which adjustments can be made to changes in that set of price relationships will have been altered. When the price obtained from the sale of an asset is less than its original cost by more than the value of the services used, a reverse movement in a price of the same magnitude that made one course of action profitable would not return the farm to the same position it held prior to any price change. Once having made an investment, it may be profitable to continue using it in the face of substantial price declines because of the loss that would be sustained by disinvesting or by failing

[5]This procedure is an approximation of the appropriate distribution of the costs of the multiple-period resources. The duration of the expected price, rate of obsolescence and rate of use, as well as other factors will affect the period of time over which this cost should be pro rated.

to use the asset.[6] It should be clearly understood that the normative supply function describes optimum adjustments to alternative product price levels as of a particular point in time and does not relate to adjustments made over time. Over time, the nature of the normative supply function of the farm changes because of changes made in the farm organization, as well as because of technological developments.

At any one point in time, there is a single normative supply function that describes the optimum adjustment of a farm organization to different levels of a product price. The price level of the product has implications with respect to the use of both single-period and multiple-period resources. However, whether or not changes in multiple-period resources will be profitable will depend as much upon the length of time the price remains at a particular level as upon the price level per se. Large investments in very durable resources may be profitable with small changes in price relationships of long duration. On the other hand, only changes in the use of single period resources may be profitable if the price change is of short duration, regardless of whether it is of large or small magnitude.

The foregoing model, cast in terms of maximizing farm profits of one production period, does not handle the question of the effect of changes in price level of different duration. To handle this type of question using linear programming, it is necessary to construct a different type of model. The principal change in the model would be that a duplicate set of restraints and activities would be defined for each of a series of production periods. Each set would refer to the use of resources and the production of product in a particular production period. The profit equation would be constructed as before, except that it would take into account the profits resulting in each of the time periods and profits would be maximized for the total series of production periods included in the model. The duration of particular price is taken into account by specifying the product price separately in each production period. This type of programming model is sometimes referred to in the literature as a "dynamic" programming model (7). The supply function derived from this model would be a three dimensional relationship, in which alternative magnitudes and durations of price changes would be considered. The usual concept of a supply function considers only the magnitude of a price change. Basically, its interpretation would still be the same as for the normative supply function derived from the first programming model that was outlined. It would describe the optimum adjustment that a farmer would make at specified points in time in the use of resources under various price conditions differing as to level and duration.

[6] The same reasoning would apply to declines in the market value of an asset held by a farm apart from the question of price discontinuities.

THE AGGREGATE

Up to this point, the discussion has been in terms of using programming procedures for deriving a normative supply function for a single farm independently of any consideration of its relationship to other farms with which it competes. As agricultural production takes place under conditions approaching perfect competition, the actions of one farm alone will have little impact on any other farm or on the group of farms as a whole. However, all farms in the group face similar problems of farm organization and resource allocation and their collective action will have a considerable impact upon the conditions facing the group as a whole. From the standpoint of maximization of farm profits, it is necessary to obtain some estimate of the effect of the collective actions of the group of farms on product price. If this can be obtained, the optimum organization of a particular farm under a particular set of circumstances can be more closely specified. This requires that the aggregate supply function be related to the demand function for the product.

The aggregate supply of a product at any given price would be the sum of the outputs of the individual farms at that price. Similarly, the aggregate normative supply function would be the sum of the individual farm normative supply functions. An estimate of the aggregate normative supply function for a product from an area or region may be obtained by first deriving the individual farm normative supply functions of a group of representative farm situations from that area or region. Then, by attaching appropriate weights to each of the individual functions, an estimate of the aggregate function can be made. The weights attached to each individual function should reflect the relative importance of the individual farm in the population of farms to which the aggregate supply function is to apply. In this approach to the estimation of an aggregate normative supply function, the designation of the representative farm situations becomes of crucial importance because these situations form the description of the base from which any change in organization is made. To a large extent, this base determines the nature of the normative supply function. The weights and the description of the farms would be obtained from a sample survey. The farms to be included in the analysis should include not only those currently producing the product for which the function is to be derived but other farms operating in the area as well. With a rise in price of the product, it may be profitable for some farmers who are not currently producing the product to enter into its production. Even without a price increase, it may be that production of the product is a profitable alternative for more farmers than are currently producing it.

Since the aggregate supply function is a weighted sum of the representative farm supply functions, the conceptual problems of constructing and applying the programming model are no different. The programming still is done with respect to individual farms or representative farms. However, in deriving the aggregate supply, the concern is primarily with structural changes that have occurred or are expected to occur in the industry. These types of changes tend to be permanent, as do their effect on prices. To the extent that this is true, the static programming model first described rather than the time-dated or dynamic model would be adequate for deriving the individual farm supply functions that are to be aggregated. That is, the assumption that the pattern of prices over future production periods is constant rather than fluctuating or of some other cyclical nature does no great violence to the validity of the analysis.

When programming an individual farm, the supply of inputs as compared with the farm's demand for these inputs is of relatively minor importance because the individual farm demand represents such a small part of the total demand. In dealing with aggregate relationships, this is no longer true. This becomes particularly important with respect to the land input that is fixed locationally. A static programming model which considers all inputs as variable so long as their marginal value product lies above their purchase price or below their sale price could give the result that it is profitable for all farmers to buy some amount of land. If this occurred, the aggregate supply function would overestimate supply because it would be based upon the condition that all farmers have purchased land when in reality there is none available to be bought. To avoid these problems of aggregate inconsistencies with respect to resources where this type of situation does exist, it may be necessary to fix the supply of the resource that is available to the representative farm at its current level. To do this, however, is in contradiction to the basic objective of determining the optimum allocation of resources and farm organization.

Theoretically, the optimum allocation of resources among farms is attained when the marginal value product of all resources are the same for all farms using the same resource. To fix the quantity of the resource at its initial level is not likely to allow this condition to be achieved. However, even under these more restrictive assumptions with regard to the extent to which the quantity of certain resources employed can be changed, estimates of the marginal value product are obtained which do allow for some evaluation of the ability of different farms to compete for the same resource. The farms having the higher marginal value products would be capable of outbidding those having the lower marginal value products. The shortcoming of this approach is that no information is obtained concerning how the marginal value product is affected as additional units of the resource are obtained nor how the organization and product output of the farm will be altered as a greater or lesser quantity of the resource is utilized.

The difficulty at the aggregate level arises from the competition among farms for a given supply of a resource. In the case of the land resource, this competition is for a supply that is essentially fixed in physical quantity in the aggregate but not necessarily fixed for the individual farms using the land. Theoretically, from a purely normative standpoint, the individual farmers would exchange land among themselves, bidding up the market price to the point at which the price of land and the marginal value product of land for each farm are equal. To determine this point, it would be necessary to derive the individual farm's demand curve for the resource, in this instance, land. This could be done with the programming model by varying the price of land and finding the optimum solution to the model at each price. Having derived the individual farm normative demand curves for land, an aggregate demand curve could be obtained by aggregating the individual farm demand curves. The equilibrium price would occur at the point where the supply curve for the resource intersects the demand curve. The demand curves for the resource would be affected by the price level of the product as this is one of the determining factors of the marginal value productivity of the resource. Hence, for each product price there would be a different resource demand curve. It would appear that an iterative process would be involved in actually determining what the equilibrium resource price and pattern of allocation would be.

SUMMARY

In general, the use of linear programming for deriving estimates of individual farm normative supply functions has the advantage of permitting examination of resource use alternatives in considerable detail. At the farm level, specific techniques of production, the use of specific resources, and the production of specific products can be taken into account quite readily, depending upon the amount of detail one wishes to build into the model. The principal benefit derived from this is that it allows movement from the micro- to the macro-level of analysis without loss due to gross aggregation of inputs and outputs. The method has the disadvantage of being somewhat cumbersome to handle when dealing with relationships that are curvilinear rather than linear. Curvilinear relationships can only be approximated by linear segments requiring the specification of a large number of activities and, in some instances, additional equations as well. One example of the difficulties encountered as a result of assuming linear input-output relationships arise with respect to labor. In using linear programming, it is necessary to assume that the resource requirement per unit of output remains constant at all output levels for any particular method of production. In many types of agricultural enterprises, there is a minimum overhead labor requirement which does not increase as output is increased. Thus, the per unit labor requirement declines with increasing output. Economies of scale cannot be handled well in the programming model and can lead to biases in the results.

The assumption of perfectly divisible factors and products present few problems except where investments are considered as with the use of programming outlined here. Investments in such things as buildings and machinery occur in large lumps. When transitions from one type of facility to another are considered in a single model, results are often obtained which would imply the partial use of two different types of production techniques for producing the same product. One possible example would be a solution that shows part of a milking herd being handled under a parlor system and the rest being handled under a stanchion system. Problems such as this can be handled by introducing one system in each of two different models, then comparing the results obtained from the separate models.

In deriving aggregate relationships by aggregating individual farm relationships, a compromise must often be struck between the detail and adequacy with which the microanalysis is carried out as compared with the macro-level analysis. As the detail with which representative farm situations are differentiated is increased, the number of individual farm supply functions that must be programmed is increased also. If major emphasis is upon the aggregate relationships, somewhat less precision may possibly be tolerated with respect to these micro-level problems. However, if major emphasis is upon the micro-level or farm management analysis, they become of much greater concern. These are questions that depend primarily upon the definition of the problem and the objectives of the research.

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Discussion

THERE ARE two distinct problem areas involved in deriving an aggregate supply function based on individual farm supply functions. The first of these concerns appropriate procedures for determining the individual farm supply functions. The second problem area concerns the appropriate means of aggregation.

McKee and Loftsgard specifically limit their discussion to the first of these problem areas. Their discussion is further restricted to the use of linear programming for quantifying the individual farm supply function. The matter of supply function aggregation does come into play, but in a somewhat different context than posed here. The authors do not consider alternative procedures for aggregating individual supply functions, but simply state that "the aggregate supply function is here regarded as being obtained by the horizontal summation of the supply functions of the individual farms." Likewise, they make no attempt to either specify the applications for which such an aggregate function would be used or to evaluate the overall usefulness of the application. Rather, the authors in essence consider the matter of aggregation through a "two-stage question procedure." First they ask the question: "What should be considered in developing an appropriate programming model which is designed to yield a normative supply function for an individual farm?" This is followed by the question: "How should this programming model be altered if you know in advance that the resulting farm supply functions for representative farms are to be aggregated?"

The major part of the McKee-Loftsgard paper is devoted to the first question, and two general programming models are outlined. Both would use the variable price programming technique to specify the optimum profit output levels of a given commodity under varying prices. Although the matter is not completely clear, I gather that both would use the same planning period — a period which is short enough to allow no change in technology or institutional circumstances, but long enough to allow for the purchase of multiple-period inputs. In this connection, I wonder if the authors are serious about their suggestion to allow the purchase of all resources, limited only by capital and borrowing capacity. With only one effective limiting resource (borrowing capacity), the programmed supply function rests upon an extremely narrow base.

The difference in the two models presented involves the length of time over which farm profits are to be maximized. The first model (termed the static model) would maximize profits for a single production period. It assumes that product and factor prices will hold constant long enough to justify the purchase of multiple-period inputs. The second model (termed the time-dated model) would maximize farm profits for each of a sequence of production periods. The latter model avoids the assumption of constant prices over time, for here both price levels and durations would be allowed to vary.

In answer to their second question, the authors rule in favor of the static model when the objective is aggregation of individual farm supply functions. They point out the possibility of a discrepancy between aggregate factor use as prescribed and aggregate factor supply, with land as a specific example.

Neither of the two models outlined are spelled out in detail. Instead, the authors describe in a general way the considerations involved in the formulation of the basic programming matrix. These "considerations" include the choice of process alternatives, the choice of restraints, and the formulation of the profit equation. But the specific nature of the appropriate programming model (as the title leads one to anticipate) is said to be largely a matter of arbitrary judgment. To be sure, the discussion of "considerations" is useful in itself. The authors are to be commended for laying out many of the considerations involved, even though they may be a bit frightening. However, research analysts find little solace in arbitrary judgment as a guideline. If we are not to be told what specific programming model is appropriate, we would at least like to know the criteria for determining propriety. This the authors have accomplished only in part.

At this point I would temper my criticism by indicating my general agreement with the authors' conclusion that "if the major emphasis is upon the aggregate supply relationship, possibly somewhat less precision can be tolerated with respect to the micro-level problems."

My remaining comments relate to the matter of supply function aggregation _per se_. Essentially, I question the usefulness of aggregating normative individual farm supply functions. In so doing, of course, I also question the need for formulating programming procedures designed to yield individual farm supply functions _for aggregative purposes_. My doubts stem from several sources.

First, the prime reason for aggregation is to take into account the effect of collective action upon the optimum resource allocation of the individual farm. But this collective action is not the cumulative result of strict adherence to the profit maximizing norm. To make such an assessment it is more important to know what all farmers _would_ do rather than what they _should_ do under various price levels.

Second, it is illogical to attempt relating aggregate demand and supply for a single commodity back to the individual farm when it is assumed implicitly that prices of all alternative products remain constant.

Finally, I suspect the possibility of a real dilemma as regards the process of aggregation. Let me use a highly oversimplified illustration. Suppose we are interested in deriving an aggregate supply function for milk in Wisconsin and Michigan. Farm A is representative of all farms in Wisconsin while Farm B is representative of all farms in Michigan. We have programmed the optimum profit level of milk production under various milk prices for each farm. These output levels are based on the price for milk at Farm A and at Farm B. Now can we expand each by its appropriate weight and add the supply schedules horizontally? We cannot, for this assumes that Michigan and Wisconsin milk prices are equal. That is, the same milk price must hold in both states at a given point in time to make horizontal summation of the weighted supply schedules valid. Likewise, we cannot attach some historical price differential which is based at least in part upon past area production patterns (as opposed to transportation differentials from some major market), for the production patterns are now variable. Thus there appears an element of circularity; supply depends upon price, but price depends upon supply. This is just one of the aggregation problems to be hurdled.

In view of the above, it may be well to stop and ask how and why normative individual farm supply functions would be aggregated before becoming concerned about programming procedures which are appropriate for the aggregative objective.