SOLUTIONS MANUAL

Basic Engineering Data Collection and Analysis

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Chapter 1: Introduction

Section 1

1. Designing and improving complex products and systems often leads to situations where there is no known theory that can guide decisions. Engineers are then forced to experiment and collect data to find out how a system works, usually under time and monetary constraints. Engineers also collect data in order to monitor the quality of products and services. Statistical principals and methods can be used to find effective and efficient ways to collect and analyze such data.

2. The physical world is filled with variability. It comes from differences in raw materials, machinery, operators, environment, measuring devices, and other uncontrollable variables that change over time. This produces variability in engineering data, at least some of which is impossible to completely eliminate. Statistics must therefore address the reality of variability in data.

3. Descriptive statistics provides a way of summarizing patterns and major features of data. Inferential statistics uses a probability model to describe the process from which the data were obtained; data are then used to draw conclusions about the process by estimating parameters in the model and making predictions based on the model.

Section 2

1. Observational study—you might be interested in assessing the job satisfaction of a large number of manufacturing workers; you could administer a survey to measure various dimensions of job satisfaction. Experimental study—you might want to compare several different job routing schemes to see which one achieves the greatest throughput in a job shop.

2. Qualitative data—rating the quality of batches of ice cream as either poor, fair, good, or exceptional. Quantitative data—measuring the time (in hours) it takes for each of 1000 integrated circuit chips to fail in a high-stress environment.

3. Any relationships between the variables x and y can only be derived from a bivariate sample.

4. You might want to compare two laboratories in their ability to determine percent impurities in rare metal specimens. Each specimen could be divided in two, with each half going to a different lab. Since each specimen is being measured twice for percent impurity, the data would be paired (according to specimen).

5. Full factorial data structure—tests are performed for all factor-level combinations:

Design	Paper	Loading Condition
delta	construction	with clip
t-wing	construction	with clip
delta	typing	with clip
t-wing	typing	with clip
delta	construction	without clip
t-wing	construction	without clip
delta	typing	without clip
t-wing	typing	without clip

Fractional factorial data structure—tests are performed for only some of the possible factor-level combinations. One possibility is to choose the following "half fraction":

Design	Paper	Loading Condition
delta	construction	without clip
t-wing	construction	with clip
delta	typing	with clip
t-wing	typing	without clip

6. Variables can be manipulated in an experiment. If changes in the response coincide with changes in factor levels, it is usually safe to infer that the changes in the factor caused the changes in the response (as long as other factors have been controlled and there is no source of bias). There is no control or manipulation in an observational study. Changes in the response may coincide with changes in another variable, but there is always the possibility that a *third* variable is causing the correlation. It is therefore risky to infer a cause-and-effect relationship between any variable and the response in an observational study.

Section 3

1. Even if a measurement system is accurate and precise, if it is not truly measuring the desired dimension or characteristic, then the measurements are useless. If a measurement system is valid and accurate, but imprecise, it may be useless because it produces too much variability (and this cannot be corrected by calibration). If a measurement system is valid and precise, but inaccurate, it might be easy to make it accurate (and thus useful) by calibrating it to a standard.

2. If the measurement system is not valid, then taking an average will still produce a measurement that is invalid. If the individual measurements are inaccurate, then the average will be inaccurate. Averaging many measurements only improves precision. Suppose that the long-run average yield of the process is stable over time. Imagine making 5 yield measurements every hour, for 24 hours. This produces 120 individual measurements, and 24 averages. Since the averages are "pulled" to the center, there will be less variability in the 24 averages than in the 120 individual measurements, so averaging improves precision.

3. Unstable measurement systems (e.g., instrument drift, multiple inconsistent devices) can lead to differences or changes in validity, precision, and accuracy. In a statistical engineering study, it is important to obtain valid, precise, and accurate measurements throughout the study. Changes or differences may create excessive variability, making it hard to draw conclusions. Changes or differences can also bias results by causing patterns in data that might incorrectly be attributed to factors in the experiment.

Section 4

1. Mathematical models can help engineers describe (in a relatively simple and concise way) how physical systems behave, or will behave. They are an integral part of designing and improving products and processes.

End Chapter Exercises

1. Calibration is most associated with accuracy. A measurement system is accurate if it produces the "true" value of the measured characteristic, *on the average* in the long run. Since calibration centers a measurement system with respect to a known standard, it improves accuracy.

2. There are a total of $3 \times 2 \times 2 = 12$ factor-level combinations:

Level of A	Level of B	Level of C
1	1	1
2	1	1
3	1	1
1	2	1
2	2	1
3	2	1
1	1	2
2	1	2
3	1	2
1	2	2
2	2	2
3	2	2

3. If each alloy specimen is measured for hardness before and after heat treating, the data would be paired (according to specimen).

4. These are paired data, because two measurements of the same characteristic (outside diameter) were made on each spanner bushing. The data are also quantitative.

5. You could choose any number of levels for each factor. In measuring the yield of a chemical process, 3 factors (with 2 levels each) might be Temperature (low vs. high), Catalyst (A vs. B), and Pressure (low vs. high).

6. Typical measurements with a ruler marked in millimeters are as follows:

Trial	Thickness (mm)	# of Pages	Thickness per Page (mm)
1	36.8	513	.0717
2	26.9	372	.0723
3	28.1	399	.0704
4	29.3	421	.0696
5	24.1	343	.0703
6	27.0	386	.0699
7	31.8	454	.0700
8	22.1	310	.0713
9	29.4	412	.0714
10	27.6	392	.0704

7. Assuming that for each spin, heads and tails are *equally likely*, it is not unusual to get results quite different from 10 heads and 10 tails. Using the binomial probability distribution (Chapter 5), the chance of getting ≤ 7 or ≥ 13 heads in 20 spins is about 1 in 4. (Note: For most coins, heads and tails are *not* equally likely when the coin is spun! The distribution of mass on the coin usually favors heads or tails slightly.)

8. (a) Rockwell hardness: multivariate (bivariate), repeated measures (paired), quantitative data. Flatness: univariate, qualitative data.

   (b) There are many possibilities. Possible factors are Vendor, Material, Heating Time, Heating Temperature, Cooling Method, and Furnace Atmosphere Condition. You could choose any number of levels for each factor. If you choose Vendor (1 vs. 2), Heating Time (short vs. long), and Cooling Method (1 vs. 2), the factor-level combinations are given below.

Vendor	Heating Time	Cooling Method
1	short	1
2	short	1
1	long	1
2	long	1
1	short	2
2	short	2
1	long	2
2	long	2

9. It is a good idea to test several such dowels (and average the observed strength) in order to arrive at a value for this "physical constant" because dowels of a given type will not be identical.

10. Let the factor with 3 levels correspond to the "day" a dowel was produced (day1, day2 or day3). Let the factor with 2 levels correspond to the person (person1, person2) that conducts the test. The 6 treatment combinations are:

day1, person1
day2, person1
day3, person1
day1, person2
day2, person2
day3, person2

11. Obtain a large sample of hydrostatic transmissions and record on each transmission the y = lifetime, x1 = piston hardness, x2 = piston diameter, x3 = piston roughness, x4 = bore hardness, x5 = bore inside diameter and x6 = bore surface roughness. Explore any trends amongst these variables and now the observed trends relate to y = lifetime.

In an experimental study, consider 2 levels of piston diameter, 2 levels of piston surface roughness, 2 levels of bore surface roughness, 2 levels of bore hardness and 2 levels of bore diamter (greater than piston diameter). Thus, we have a 2^6 different possible setups, i.e., 6 factors each at 2 levels. Record lifetimes at each of these 64 combinations and explore possible factor effects on lifetime.

12. The average dowel strength would be more precise because an average ($n > 1$) is less variable than a single observation.

13. **(a)** Let factor A be the length of the arm (6 in., 12 in.), factor B be the place of rubber chord attachment (position 1, position 2) and factor C be the angle the arm makes when ithits the stop (angle1, angle2). Thus 8 different launch setups with two launches per setup.

Set-up	combination
1	6 in., position1, angle1
2	6 in., position2, angle1
3	6 in., position1, angle2
4	6 in., position2, angle 2
5	12 in., position1, angle1
6	12 in. position2, angle1
7	12 in. position1, angle2
8	12 in. position2, angle2

Hold the pull-back angle fixed and the ball weight fixed for each of the 8 setup combinations.

(b) $2^5 = 32$ is the minimum number of setups for 5 factors each at 2 levels. A fractional factorial, $2^{5-1} = 16$ setups, would need to be designed if one launch per set-up occurred. If 2 launches per set-up are desired, then a fractional factorial with $2^{5-2} = 8$ setups would need to be designed.

Chapter 2: Data Collection

Section 1

1. *Flight distance* might be defined as the horizontal distance that a plane travels after being launched from a mechanical slingshot. Specifically, the horizontal distance might be measured from the point on the floor directly below the slingshot to the point on the floor where any part of the plane first touches.

2. If all operators are trained to use measuring equipment in the same consistent way, this will result in better repeatability and reproducibility of measurements. The measurements will be more repeatable because individual operators will use the same technique from measurement to measurement, resulting in small variability among measurements of the same item by the same operator. The measurements will be more reproducible because all operators will be trained to use the same technique, resulting in small variability among measurements made by different operators.

3. This scheme will tend to "over-sample" larger lots and "under-sample" smaller lots, since the amount of information obtained about a large population from a particular sample size does not depend on the size of the population. To obtain the same amount of information from each lot, you should use an absolute (fixed) sample size instead of a relative one.

4. If the response variable is poorly defined, the data collected may not properly describe the characteristic of interest. Even if it does, operators may not be consistent in the way that they measure the response, resulting in more variation.

Section 2

1. Label the 38 runout values consecutively, 1, ..., 38, in the order given in Table 1-1 (smallest to largest). Move through the table 2 digits at a time, ignoring numbers between 39 and 00 and numbers that have already been picked.

Sample	Labels	Runout Values	Sample Mean
1	12, 15, 5; 9, 11	11, 11, 9, 10, 11	10.4
2	34, 31, 36, 2, 14	17, 15, 18, 8, 11	13.8
3	10, 35, 12, 27, 30	10, 17, 11, 14, 15	13.4
4	15, 5, 19, 11, 8	11, 9, 12, 11, 10	10.6

The table below shows how the *labels* were chosen:

12159 66144 05091 13446 45653 13684 66024 91410 51351 22772
30156 90519 95785 47544 66785 35754 11088 67310 19720 08379

The samples are not identical. The population mean (the average of all 38 runout values) is 12.63. You can see that samples 1 and 4 do not include any of the higher runout values and samples 2 and 3 do not include many low runout values, so none of the samples are very representative of the population. However, the average of the 4 sample means is 12.05, which is closer to 12.63. Simple random samples are representative "on the average", but any particular simple random sample may not be representative of the population.

3. A simple random sample is not guaranteed to be representative of the population from which it is drawn. It gives every set of n items an equal chance of being selected, so there is always a chance that the n items chosen will be "extreme" members of the population.

Section 3

1. Possible controlled variables: operator, launch angle, launch force, paper clip size, paper manufacturer, plane constructor, distance measurer, and wind. The response is Flight Distance and the experimental variables are Design, Paper Type, and Loading Condition. Concomitant variables might be wind speed and direction (if these cannot be controlled), ambient temperature, humidity, and atmospheric pressure.

2. Advantage: may reduce baseline variation (background noise) in the response, making it easier to see the effects of factors. Disadvantage: the variable may fluctuate in the real world, so controlling it makes the experiment more artificial—it will be harder to generalize conclusions from the experiment to the real world.

3. Treat "distance measurer" as an experimental (blocking) variable with 2 levels. For each level (team member), perform a full factorial experiment using the 3 primary factors. If there is a difference in the way each team member measures distance, then it will still be possible to unambiguously assess the effects of the primary factors within each "sub-experiment" (block).

4. List the tests for Juanita in the same order given for Exercise 1-8. Then list the tests for Tom after Juanita, again in the same order. Label the tests consecutively 1, ..., 16, in the order listed. Use the following coding for the test labels:

Table Number	Test Label
01–05	1
06–10	2
11–15	3
16–20	4
21–25	5
26–30	6
31–35	7
36–40	8
41–45	9
46–50	10
51–55	11
56–60	12
61–65	13
66–70	14
71–75	15
76–80	16

Move through Table D-1 choosing two digits at a time. Ignore previously chosen test labels or numbers between 81 and 00. Order the tests in the same order that their corresponding two-digit numbers are chosen from the table. Using this method (and starting from the upper-left of the table), the test labeled 3 (Juanita, delta, typing, with clip) would be first, followed by the tests labeled 13, 9, 1, 2, 7, 10, 8, 14, 11, 6, 15, 4, 16, 12, and 5. The use of the table is illustrated below.

12159 66144 05091 13446 45653 13684 66024 91410 51351 22772

30156 90519 96785 47544 66785 35754 11088 67310 19720 08379

5. For the delta/construction/with clip condition (for example), flying the same plane twice would provide information about flight-to-flight variability for that particular plane. This would be useful if you are only interested in making conclusions about that particular plane. If you are interested in generalizing your conclusions to all delta design planes made with construction paper and loaded with a paper clip, then reflying the same airplane does not provide much more information. But making and flying two planes for this condition would give you some idea of variability among different planes of this type, and would therefore validate any general conclusions made from the study. This argument would be true for all 8 conditions, and would also apply to comparisons made among the 8 conditions.

6. Random sampling is used in enumerative studies. Its purpose is to choose a representative sample from some population of items. Randomization is used in analytical/experimental studies. Its purpose is to assign units to experimental conditions in an unbiased way, and to order procedures to prevent bias from unsupervised variables that may change over time.

7. Blocking is a way of supervising an extraneous variable. Within each block, there may be less baseline variation (background noise) in the response than there would be if the extraneous variable were not supervised. This makes it easier to see the effects of the factors of interest within each block. Any effects of the extraneous variable can be isolated and distinguished from the effects of the factors of interest. Compared to holding the variable constant throughout the experiment, blocking also results in a more realistic experiment.

8. Replication is used to estimate the magnitude of baseline variation (background noise, experimental error) in the response, and thus helps sharpen and validate conclusions drawn from data.

9. It is not necessary to know exactly how the entire budget will be spent. Experimentation in engineering is usually sequential, and this requires some decisions to be made in the middle of the study. Although some may think that this is "improper" from a scientific/statistical point of view, it is only practical to base the design of later stages on the results of earlier stages.

Section 4

1. If you regard student as a blocking variable, then this would be a randomized complete block experiment. Otherwise, it would just be a completely randomized experiment (with a full factorial structure).

2. (a) Label the 24 runs as follows:

Labels	Level of A	Level of B	Level of C
1, 2, 3	1	1	1
4, 5, 6	2	1	1
7, 8, 9	1	2	1
10, 11, 12	2	2	1
13, 14, 15	1	1	2
16, 17, 18	2	1	2
19, 20, 21	1	2	2
22, 23, 24	2	2	2

Use the following coding for the test labels:

Table Number	Test Label
01–04	1
05–08	2
09–12	3
13–16	4
17–20	5
21–24	6
25–28	7
29–32	8
33–36	9
37–40	10
41–44	11
45–48	12
49–52	13
53–56	14
57–60	15
61–64	16
65–68	17
69–72	18
73–76	19
77–80	20
81–84	21
85–88	22
89–92	23
93–96	24

Move through Table D-1 choosing two digits at a time, ignoring numbers between 97 and 00 and those corresponding to test labels that have already been picked. Order the tests in the same order that their corresponding 2 digit numbers are picked from the table. Using this method, and starting from the upper-left corner of the table, the order would be 3, 4, 24, 16, 11, 2, 9, 12, 17, 8, 21, 1, 13, 7, 18, 5, 20, 14, 19, 15, 22, 23, 6, 10. The use of the table is shown below.

12159 66144 05091 13446 45653 13684 66024 91410 51351 22772

36156 90519 95786 47544 66735 35764 11088 67510 19720 08379

59069 01722 53338 41942 65118 71236 01932 70343 25812 62275

(b) Treat day as a blocking variable, and run each of the 8 factor-level combinations once on each day. Blocking allows comparisons among the factor-level combinations to be made within each day. If blocking were not used, differences among days might cause variation in the response which would cloud comparisons among the factor-level combinations.

(c) List the 8 factor-level combinations separately for each day. For each day, label the runs as follows:

Label	Level of A	Level of B	Level of C
1	1	1	1
2	2	1	1
3	1	2	1
4	2	2	1
5	1	1	2
6	2	1	2
7	1	2	2
8	2	2	2

For each day, move through Table D-1 one digit at a time ignoring the digits 9 and 0 and any that have already been picked. Order the 8 runs in the same order that the numbers were picked from the table. Starting from where I left off in part (a), the order for day 1 is 5, 3, 8, 4, 1, 2, 6 (which implies that run 7 goes last). For day 2, the order is 5, 1, 8, 7, 2, 3, 6 (which implies that run 4 goes last). For day 3, the order is 1, 3, 2, 7, 4, 5, 8, (which implies that run 6 goes last). The use of the table is shown below.

59069 01722 53338 41942 65118 71236 01932 70343 25812 62275

The plan is summarized below.

Day	Level of A	Level of B	Level of C
1	1	1	2
1	1	2	1
1	2	2	2
1	2	2	1
1	1	1	1
1	2	1	1
1	2	1	2
1	1	2	2
2	1	1	2
2	1	1	1
2	2	2	2
2	1	2	2
2	2	1	1
2	1	2	1
2	2	1	2
2	2	2	1
3	1	1	1
2	1	2	1
2	2	1	1
2	1	2	2
2	2	2	1
2	1	1	2
2	2	2	2
2	2	1	2

Part (a) randomized all 24 runs together; here, each block of 8 runs is randomized separately.

3.

Block	Design	Paper
Tom	delta	construction
Tom	t-wing	typing
Juanita	delta	typing
Juanita	t-wing	construction

4. Focusing on Design, you would want each person to test 2 delta-wing planes and 2 t-wing planes; this would allow you to clearly compare the two designs. You could separately compare the designs "within" each person. If possible, you would want a plan such that this is true for all 3 primary factors, simultaneously. This is possible by using the same design that is used in the second part of Example 2-14:

Person	Design	Paper	Loading Condition
Juanita	delta	construction	with clip
Tom	t-wing	construction	with clip
Tom	delta	typing	with clip
Juanita	t-wing	typing	with clip
Tom	delta	construction	without clip
Juanita	t-wing	construction	without clip
Juanita	delta	typing	without clip
Tom	t-wing	typing	without clip

This design also allows each person to test each Design/Paper combination once, each Design/Loading combination once, and each Paper/Loading combination once.

5. This is an incomplete block experiment.

Section 5

1. A cause-and-effect diagram may be useful for representing a complex system in a relatively simple and visual way. It enables people to see how the components of the system interact, and may help identify areas which need the most attention/improvement.

End Chapter Excercises

1. Label the widgets 1, 2, ..., 491. Chóose the widgets labeled 121, 405, 91, 134, 464, 313, 249, 141.

12159 66144 05091 13446 45653 13684 66024 91410 51351 22772

2. (a) Possible responses: volume of popped corn, number of unpopped kernels, and taste of popped corn.

3. (a) Label the gears 1, 2, ..., 20. Use the following coding for the gears:

Table Number	Gear Label
01–05	1
06–10	2
11–15	3
16–20	4
21–25	5
26–30	6
31–35	7
36–40	8
41–45	9
46–50	10
51–55	11
56–60	12
61–65	13
66–70	14
71–75	15
76–80	16
81–85	17
86–90	18
91–95	19
96–00	20

Move through Table B-1 choosing two digits at a time, until 10 different gears are chosen. These gears will be laid; the remaining 10 gears will be hung. Using this method, the gears to be laid are the ones labeled 3, 20, 13, 9, 1, 2, 7, 10, 8, and 17.

12159 66144 05091 13446 45653 13684 | 66024 91410 51351 22772

(b) This will guard against bias. A naive or convenient method of assignment may surely assign most of the "good" gears to one group and most of the "bad" gears to the other. There is only a small chance that this type of assignment will result from randomization (but it is possible).

4. Water sample has been treated as a blocking variable, with 8 levels. Comparison of the two methods can be made within each sample (block). In addition, by using samples that are quite different, the two methods can be compared under a variety of conditions that might be encountered in practice. Therefore, conclusions made about the two methods based on these data are more generally applicable. The data are paired because the same type of measurement is being made twice on each of the 8 samples.

5. (a) To best compare all three treatments, it would make sense to assign each treatment to 2 widgets in each group. To balance out differences in surface texture *within* each group, it would make sense to do this assignment randomly. This amounts to randomly dividing units in each group of 6 into 3 subgroups of 2. Then assign method A to the 1st subgroup, method B to the 2nd subgroup, and method C to the 3rd subgroup. Do this for each group of widgets. Start at the upper left of Table D-1 and with widget 1 (from the 1st group). Move one digit at a time, putting widgets into subgroups; each digit corresponds to a subgroup number (skip digits that are not 1, 2, or 3). When a subgroup has been filled, ignore that subgroup's digit. When all subgroups of a group have been filled, go on to the next group. Using this method, the following assignments would be made:

Widget	Treatment
1	A
2	B
3	A
4	C
5	C
6	B
7	A
8	C
9	B
10	A
11	C
12	B
13	A
14	B
15	B
16	C
17	A
18	C

Note that you do not need to use the table for the last widget in each group. The use of the table is shown below.

12159 66144 05091 13446 45653 | 13684 66024 91410 51351 22772

32156 90519 95785 47544 66735 35754 11088 67310 19720 08379

(b) Use the following coding for the widgets:

Table Number	Widget
01–05	1
06–10	2
11–15	3
16–20	4
21–25	5
26–30	6
31–35	7
36–40	8
41–45	9
46–50	10
51–55	11
56–60	12
61–65	13
66–70	14
71–75	15
76–80	16
81–85	17
86–90	18

Move through Table B-1 choosing two digits at a time, ignoring numbers between 91 and 00 and those corresponding to widgets that have already been picked. Order the widgets in the same order that their corresponding 2 digit numbers are picked from the table. Using this method, and starting from where I left off in (a), the order would be 12, 18, 11, 17, 10, 14, 7, 15, 9, 2, 1, 4, 8, 5, 6, 3, 13, 16. The use of the table is shown below.

30156 90519 25785 47541 66735 38754 11088 67310 19420 08379

59069 01722 53338 41942 65118 71236 01932 70343 25812 62275

(c) You could run each method-group combination once on each day. This allows treatments to be compared within days, and also within groups. Since there are 2 widgets for each method-group combination, you need to randomly assign one of these widgets to day 1, and one to day 2 (for each method-group combination). Then, for each day, randomize the order in which the 9 runs are made using the same method as in (b). The following pairs of widgets need to be divided between the two days: (1, 3), (2, 6), (4, 5), (7, 10), (9, 12), (8, 11), (13, 17), (14, 15), (16, 18). This can be accomplished by moving through Table D-1 one digit at a time. For each pair, if the digit is odd, the first in the pair goes on Day 1; if the digit is even, the second in the pair goes on Day 1. Using this method, and starting where I left off in part (b), the assignments would be as follows. Day 1: 3, 2, 4, 7, 12, 8, 17, 14, 16. Day 2: 1, 6, 5, 10, 9, 11, 13, 15, 18. The use of the table is shown below:

59069 01722 53338 41942 65118 71236 01932 70343 25812 62275

54107 58081 82470 59407 13475 95872 16268 78436 39251 64247

To randomize the order, temporarily relabel the Day 1 widgets 1–9 in the order listed above. Do the same with the Day 2 widgets. For each day, move through the table one digit at a time, ignoring the 0 digit and digits corresponding to widgets that have already been picked. Order the widgets in the same order that their corresponding 1 digit numbers are picked from the table. Using this method, and starting from where I left off above, the orders (and entire plan) would be:

Day	Original Widget Number	Treatment
1	14	B
1	3	A
1	2	B
1	7	A
1	17	A
1	12	B
1	16	C
1	4	C
1	8	C
2	10	A
2	13	A
2	9	B
2	18	C
2	15	B
2	6	B
2	1	A
2	11	C
2	5	C

The use of the table for randomizing the order within each day is shown below.

54107 53841 82370 58497 13475 96872 16268 78436 39251 64247

(d) Part (b) has a full 3×3 factorial structure with 2 observations per condition. Part (c) has a full 3×3×2 factorial structure with no replication.

6. There are 2×2 = 4 factor-level combinations. Treat batch as a blocking variable, and run each of the 4 combinations 3 times for each batch. Randomize the order of the 24 tests by labeling them as follows:

Labels	Batch	Wall Thickness	Ignition Point Placement
1, 2, 3	1	$\frac{1}{16}$	1
4, 5, 6	1	$\frac{1}{8}$	1
7, 8, 9	1	$\frac{1}{16}$	2
10, 11, 12	1	$\frac{1}{8}$	2
13, 14, 15	2	$\frac{1}{16}$	1
16, 17, 18	2	$\frac{1}{8}$	1
19, 20, 21	2	$\frac{1}{16}$	2
22, 23, 24	2	$\frac{1}{8}$	2

Use the following coding for the test labels:

Table Number	Test Label
01–04	1
05–08	2
09–12	3
13–16	4
17–20	5
21–24	6
25–28	7
29–32	8
33–36	9
37–40	10
41–44	11
45–48	12
49–52	13
53–56	14
57–60	15
61–64	16
65–68	17
69–72	18
73–76	19
77–80	20
81–84	21
85–88	22
89–92	23
93–96	24

Move through Table B-1 choosing two digits at a time, ignoring numbers between 97 and 00 and those corresponding to test labels that have already been picked. Order the tests in the same order that their corresponding 2 digit numbers are picked from the table. Using this method, and starting from the upper-left corner of the table, the order would be 3, 4, 24, 16, 11, 2, 9, 12, 17, 8, 21, 1, 13, 7, 18, 5, 20, 14, 19, 15, 22, 23, 6, 10. The use of the table is shown below.

12159 66144 05091 13446 45653 13684 66024 91410 51351 22772

30156 90519 95785 47544 66735 35754 11088 67310 19720 08379

58069 01722 53338 41942 65118 71236 01932 70343 25812 62275

(You could have also randomized each block of 12 separately.) The resulting data will have a full 2×2×2 factorial structure with 3 observations per condition.

7. (a) Label the widgets 1, 2, ..., 354. Select the widgets labeled 121, 91, 134, 313, and 249. The use of the table is shown below.

12159 66144 05091 13446 45653 13684 66024 91410 51351 22772

(b) Label the 12 experimental runs as follows:

Labels	Level of A	Level of B
1, 2	1	1
3, 4	2	1
5, 6	1	2
7, 8	2	2
9, 10	1	3
11, 12	2	3

Use the following coding for the test labels:

Table Number	Test Label
01–05	1
06–10	2
11–15	3
16–20	4
21–25	5
26–30	6
31–35	7
36–40	8
41–45	9
46–50	10
51–55	11
56–60	12

Move through Table B-1 choosing two digits at a time, ignoring numbers between 61 and 00, and those corresponding to runs that have already been picked. Order runs in the same order that their corresponding 2 digit numbers are picked from the table. Using this method, and starting from where I left off in part (a), the order would be 3, 2, 11, 7, 6, 1, 4, 9, 12, 5, 8, 10. The use of the table is shown below.

12159 66144 05091 13446 45653 13684 66024 91410 51351 32772

30158 80519 96765 47444 86765 35754 11083 87810 18720 00876

59069 01722 53338 41922 86118 71236 01932 70343 25812 62275

8. (a) See Ex. 8, Ch. 1 for the factors and levels. Two possible responses would be flatness and concentricity. Replication dictates that at least one of the 8 factor-level combinations given in ex. 8, ch. 1 be run at least twice. One possibility is to run each factor-level combination twice, for a total of 16 runs.

Test Label	Test Order	Vendor	Heating Time	Cooling Method	Flatness	Concentricity
1		1	short	1		
2		1	short	1		
3		2	short	1		
4		2	short	1		
5		1	long	1		
6		1	long	1		
7		2	long	1		
8		2	long	1		
9		1	short	2		
10		1	short	2		
11		2	short	2		
12		2	short	2		
13		1	long	2		
14		1	long	2		
15		2	long	2		
16		2	long	2		

(b) For the scenario in (a), you should use 16 slips of paper. Each slip corresponds to a run. Order the runs in the same order as their corresponding slips are picked from the hat. Avoid placing the slips into the hat in any special order, and mix the slips well before picking them. All slips should be physically identical so that the selection order is completely random.

9. (a) Using the method in Ex. 7, Ch. 2, select the widgets labeled 121, 596, 614, 405, 91, 134, and 464. The use of the table is shown below.

12159 66144 05091 13446 45653 13684 66024 91410 51351 22772

(b) The use of the table is shown above. Select the widgets labeled 565, 313, 249, 141, 51, 351, and 227. This sample is completely different; there is no overlap.

11. (a) Control the extraneous variable heat by using only one bar for the entire study. This will eliminate any heat-to-heat variability.

(b) Advantage: may reduce baseline variation (background noise) in the response, making it easier to see any difference between the 2 brands. Disadvantage: One heat may not be representative of all such material that the drills would be used on. Controlling it makes the experiment more artificial—it will be harder to generalize conclusions from this heat to others.

(c) Treat heat as a blocking variable. For each of the 3 bars, test 5 drills of each brand. The brands can then be compared within each block (bar).

Chapter 3: Computationally Simple Descriptive Statistics

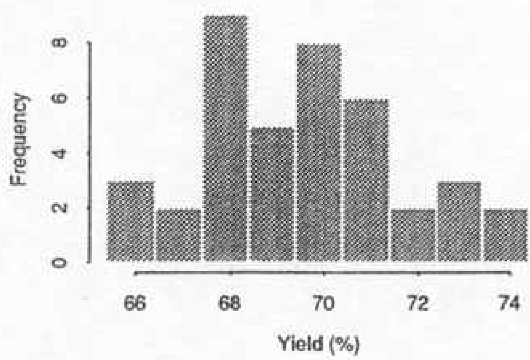
Section 1.

1

```
     :   .   .   .   . . :::.:   :::.: .: ..::.   . . .   :.   .   .
   -+---------+---------+---------+---------+---------+----- Yield (%)
  65.6      67.2      68.8      70.4      72.0      73.6
```

```
Decimal point is
at the colon;
hundredths' place
has been truncated

   65 : 66
   66 : 28
   67 : 2588
   68 : 0023349
   69 : 012355589
   70 : 02466789
   71 : 37
   72 : 0678
   73 : 5
   74 : 2
```

Yield (%)	Tally	Frequency	Relative Frequency	Cumulative Relative Frequency
65.5–66.4	|||	3	.075	.075
66.5–67.4	||	2	.05	.125
67.5–68.4	~~||||~~ ||||	9	.225	.35
68.5–69.4	~~||||~~	5	.125	.475
69.5–70.4	~~||||~~ |||	8	.2	.675
70.5–71.4	~~||||~~ |	6	.15	.825
71.5–72.4	||	2	.05	.875
72.5–73.4	|||	3	.075	.95
73.5–74.4	||	2	.05	1.00



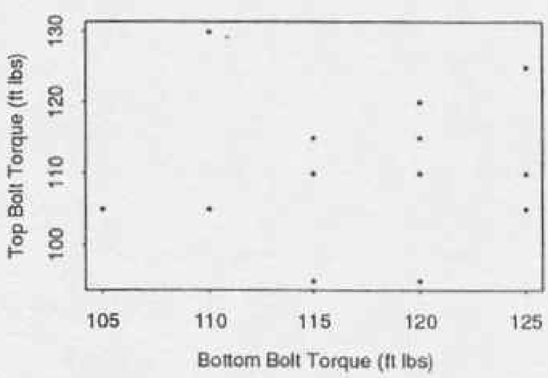
Other choices for the intervals are possible. The plots reveal a fairly symmetric, bell-shaped distribution.

2.

```
Decimal point is 1 place to the right of the colon
Data are rounded to the nearest integer; units are mm

                          8 : 2 :
                            : 3 :
                            : 3 :
                            : 3 :
                          7 : 3 :
                        988 : 3 :
                       1110 : 4 :
                         33 : 4 :
                         44 : 4 :
230 grain                 7 : 4 :              200 grain
                         88 : 4 :
                         10 : 5 :
                          2 : 5 :
                            : 5 :
                          6 : 5 :
                            : 5 : 899
                            : 6 : 0011
                            : 6 : 22333
                            : 6 : 44455
                            : 6 :
                            : 6 : 8
                            : 7 : 0
                            : 7 : 2
```

The plot shows that the depths for the 200 grain bullets are larger and have less variability than those for the 230 grain bullets.

3. (a)



There are no obvious patterns. -20

(b) The differences are -15, 0, ~~20~~, 0, -5, 0, -5, 0, -5, 20, -25, -5, -10, -20, and 0.

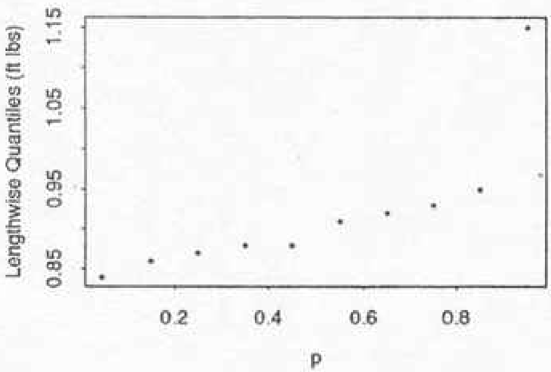
```
                              .
                         :    :
     .    :    .    .    :    :                   .
-+---------+---------+---------+---------+---------+----- Top - Bottom
-30       -20       -10        0        10        20      (ft lbs)
```

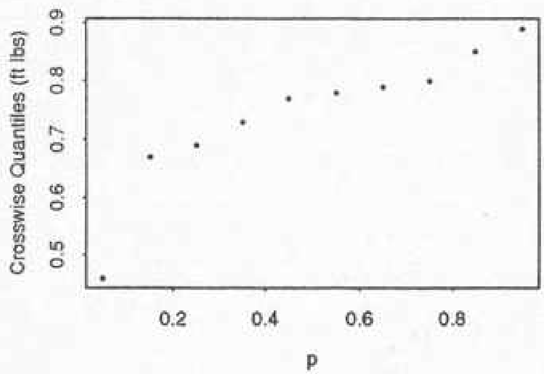
The dot diagram shows that most of the differences are zero or negative and "truncated" at zero. The exception is the 10th piece of equipment, with a difference of 20. This point does not fit in with the shape of the rest of the differences, so it is an outlier. Since most of the differences are zero or negative, the bottom bolt generally required more torque to loosen than the top bolt.

Section 2

1. (a)

i	$\frac{i-.5}{10}$	$Q_L(\frac{i-.5}{10})$	$Q_C(\frac{i-.5}{10})$
1	.05	.84	.46
2	.15	.86	.67
3	.25	.87	.69
4	.35	.88	.73
5	.45	.88	.77
6	.55	.91	.78
7	.65	.92	.79
8	.75	.93	.80
9	.85	.95	.85
10	.95	1.15	.89





For the lengthwise sample:

$Median = Q(.5) = \frac{.88+.91}{2} = .895$

$1st\ Quartile = Q_1 = Q(.25) = .870$

$3rd\ Quartile = Q_3 = Q(.75) = .930$

$Q(.37) = .880$

For the crosswise sample:

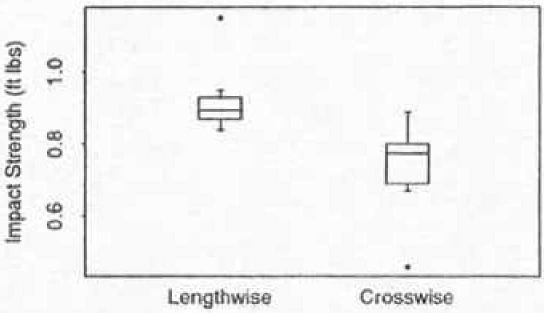
$Median = \frac{.77+.78}{2} = .775$

$Q_1 = .690$

$Q_3 = .800$

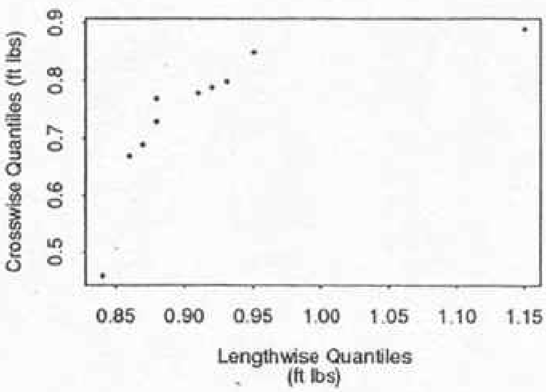
$Q(.37) = (.8)(.73) + (.2)(.77) = .738$

(b)



On the whole, the impact strengths are larger and more consistent for lengthwise cuts. Each method produced an unusual impact strength value (outlier).

(c)



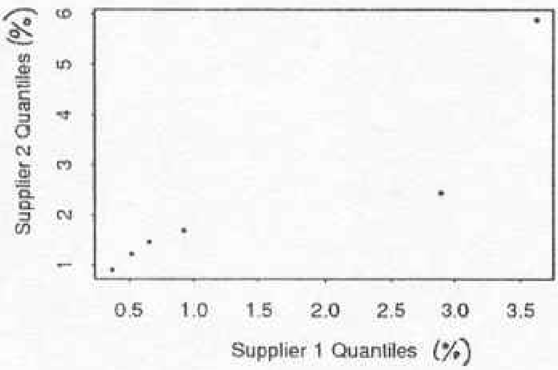
The non-linearity of the Q-Q plot indicates that the overall shapes of these two data sets are not the same. The lengthwise cuts had an unusually large data point ("long right tail"), whereas the crosswise cuts had an unusually small data point ("long left tail"). Without these two outliers, the data sets would have similar shapes, since the rest of the Q-Q plot is fairly linear.

2. Use the $(i-.5)/n$ quantiles for the smaller data set.

i	$\frac{i-.5}{6}$	$Q_1(\frac{i-.5}{6})$	$Q_2(\frac{i-.5}{6})$
1	.08	.37	.91
2	.25	.52	1.22
3	.42	.65	1.47
4	.58	.92	1.70
5	.75	2.89	2.45
6	.92	3.62	5.89

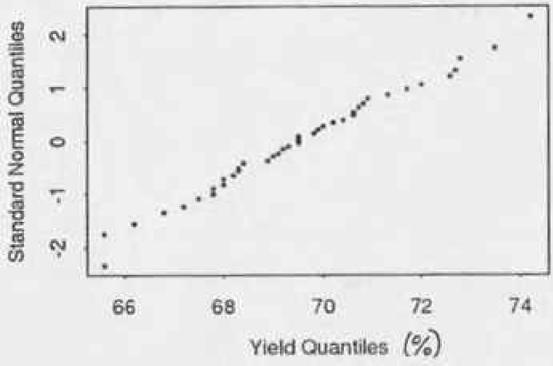
The above quantiles for the Supplier 2 data were obtained by interpolation from the following table.

i	$\frac{i-.5}{8}$	$Q_2(\frac{i-.5}{8})$
1	.06	.89
2	.19	.99
3	.31	1.45
4	.44	1.47
5	.56	1.58
6	.69	2.27
7	.81	2.63
8	.94	6.54



3. Some of the plot coordinates are given in the table below.

i	$\frac{i-.5}{40}$	$Q(\frac{i-.5}{40})$	$Q_{SN}(\frac{i-.5}{40})$
1	.01	65.6	-2.33
2	.04	65.6	-1.75
3	.06	66.2	-1.55
4	.09	66.8	-1.34
5	.11	67.2	-1.23
	⋮		
36	.89	72.6	1.23
37	.91	72.7	1.34
38	.94	72.8	1.55
39	.96	73.5	1.75
40	.99	74.2	2.33



The normal plot is quite linear, indicating that the data are very bell-shaped.

4. Theoretical Q-Q plotting allows you to roughly check to see if a data set has a shape which is similar to some theoretical distribution. This can be useful in identifying a theoretical (probability) model to represent how the process is generating data. Such a model can then be used to make inferences (conclusions) about the process.

Section 3

1.

	$\bar{x}$	$Median$	R	IQR	s
Lengthwise	.919	.895	.310	.060	.088
Crosswise	.743	.775	.430	.110	.120

The sample means and medians show that the center of the distribution for lengthwise cuts is higher than the center for crosswise cuts. The sample ranges, interquartile ranges, and sample standard deviations show that there is less spread in the lengthwise data than in the crosswise data.

2. These values are statistics. They are summarizations of two samples of data, and do not represent exact summarizations of larger populations or theoretical (long-run) distributions.

4. In the first case, the sample mean and median increase by 1.3, but none of the measures of spread change; in the second case, all of the measures double.

Section 4

1. $\hat{p}$ = the proportion of part orders that are delivered on time to the factory floor. $\hat{u}$ = number of defects per shift produced on an assembly line. A measured value of 65% yield for a run of a chemical process is of neither form.

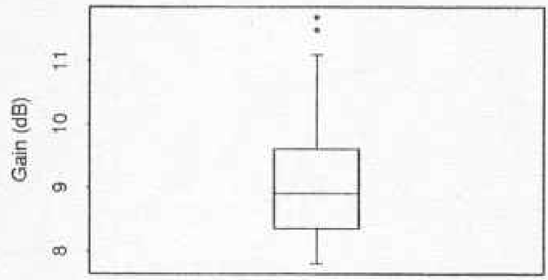
2. $\hat{p}_{\text{Laid}} = \frac{6}{38} = .158$. $\hat{p}_{\text{Hung}} = \frac{24}{39} = .615$. Most engineering situations call for minimizing variation. The $\hat{p}$ values do not give any indication of how much spread there is in each set of data, and would not be helpful in comparing the two methods with respect to variation.

3. Neither type. These rates represent continuous measurements on each specimen; there is no "counting" involved.

End Chapter Excercises

1. (a) Decimal point is at the colon; units are dB

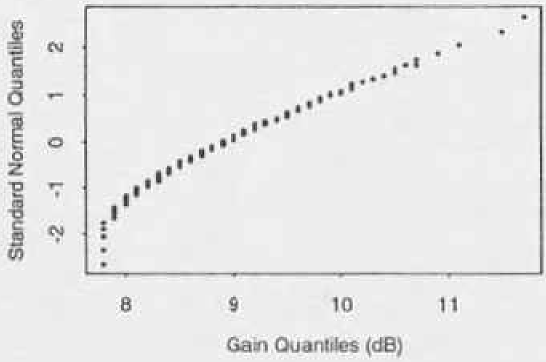
```
 7 : 8888899999
 8 : 00000111112222233333344444
 8 : 5555556666677777788888899999
 9 : 00000011111222223334444
 9 : 55556666777788899
10 : 001112234
10 : 556779
11 : 1
11 : 57
```



The shape is right-skewed. It seems to be truncated on the left. It appears that the manufacturer inspected most (if not all) of the amplifiers, and removed those with gains less than 7.8 dB.

(b) Some of the plot coordinates are given in the table below.

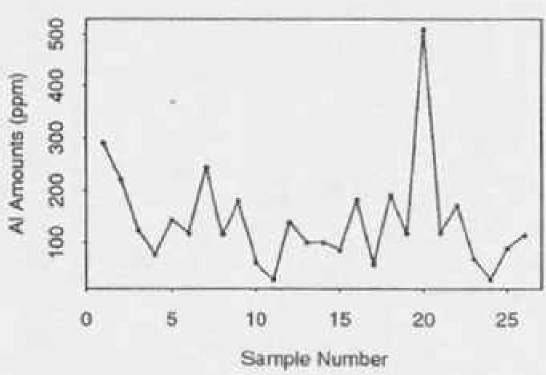
i	$\frac{i-.5}{120}$	$Q(\frac{i-.5}{120})$	$Q_{SN}(\frac{i-.5}{120})$
1	.004	7.8	-2.64
2	.01	7.8	-2.33
3	.02	7.8	-2.05
4	.03	7.8	-1.88
5	.04	7.8	-1.75
	⋮		
116	.96	10.7	1.75
117	.97	10.9	1.88
118	.98	11.1	2.05
119	.99	11.5	2.33
120	.996	11.7	2.64



The plot tails off strongly at the end; the smallest data points would need to be pushed to the left (made smaller) in order for this data set to have a bell-shaped distribution. This agrees with part (a).

(c) Before inspection, the distribution of gains would not be truncated. It seems like it would be symmetric, centered at about 8.5 dB. 7.75 is .75 dB below 8.5. Looking at the right (untruncated) side of the distribution, 9.25 is .75 dB above 8.5. So it may be reasonable to assume that the fraction of amplifiers with gains below 7.75 is about the same as the fraction with gains above 9.25. The fraction of the data above 9.25 is $42/120 = .35$. Since none of the gains were close to 12.2, the fraction of gains within specifications is about $1 - .35 = .65$. (Of course, $p \approx 1$ for post-inspected amplifiers.)

2. (a)



There seems to be a slight downward trend over time, with the exception of the 20th observation, which is unusually high. If there is a trend, this might give an engineer a clue about what is affecting the process. The period in which the trend was observed may coincide with some known change associated with the process. This might enable the engineer to reduce future impurity levels.

(b) Decimal point is 2 places to the right of the colon
Units are ppm

```
0 : 30,30
0 : 60,63,70,79,87,90
1 : 01,02,15,18,19,19,20,25,40,45
1 : 72,82,83,91
2 : 22,44
2 : 91
3 :
3 :
4 :
4 :
5 : 11
```

(c) Even if you ignore the outlier, the distribution is right-skewed. It is not bell-shaped.

(d)

i	$\frac{i-.5}{26}$	$Q(\frac{i-.5}{26})$	$Q_{SN}(\frac{i-.5}{26})$
1	.02	30	-2.05
2	.06	30	-1.55
3	.10	60	-1.28
4	.13	63	-1.13
5	.17	70	-.95
6	.21	79	-.81
7	.25	87	-.67
8	.29	90	-.55
9	.33	101	-.44
10	.37	102	-.33
11	.40	115	-.25
12	.44	118	-.15
13	.48	119	-.05
14	.52	119	.05
15	.56	120	.15
16	.60	125	.25
17	.63	140	.33
18	.67	145	.44
19	.71	172	.55
20	.75	182	.67
21	.79	183	.81
22	.83	191	.95
23	.87	222	1.13
24	.90	244	1.28
25	.94	291	1.55
26	.98	511	2.05

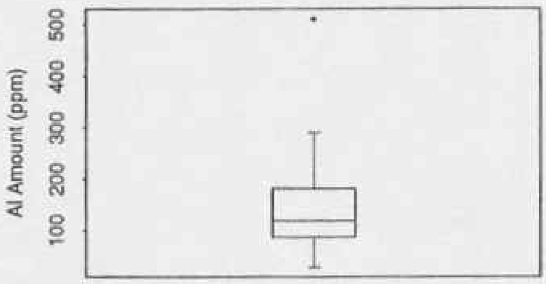
$Median = 119$

$Q_1 = 87$

$Q_3 = 182$

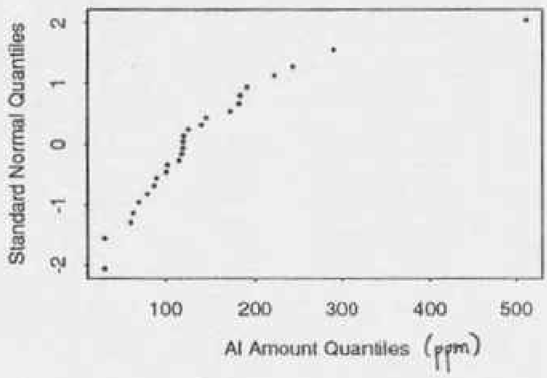
$Q(.58) = \frac{125+120}{2} = 122.5$

(e)



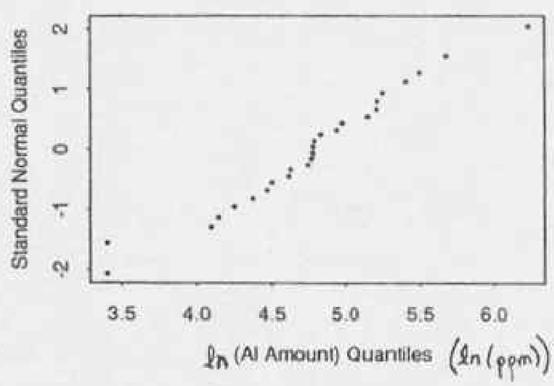
The median is closer to Q_1 than Q_3, and the upper whisker is longer than the lower whisker. Both of these features indicate that the distribution is right-skewed.

(f) The plotting points are given in part (d).



The plot is quite non-linear, indicating that the data distribution is not bell-shaped. The data are more bunched up at low values and more spread out for high values than they would be if they were bell-shaped. This again shows that the distribution is right-skewed.

(g) The first 3 coordinates of the plot are: $(3.40, -2.05)$, $(3.40, -1.55)$, $(4.09, -1.28)$.



The plot is more linear, indicating that the transformed data are more bell-shaped than the raw data set.

(h)

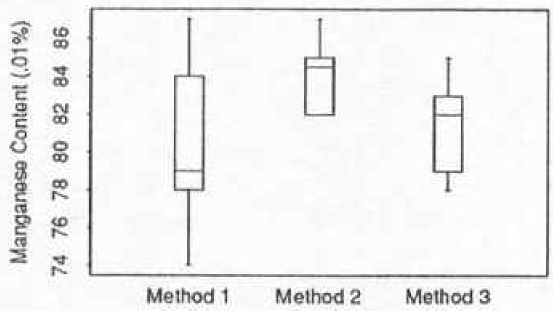
	$\bar{x}$	R	s
Original data	142.65	481	98.20
Transformed data	4.779	2.835	.631

No. Since the transformed data are more symmetric, their mean is not "pulled up" as much as the mean of the original data.

3. (a)

i	$\frac{i-.5}{10}$	$Q_1(\frac{i-.5}{10})$	$Q_2(\frac{i-.5}{10})$	$Q_3(\frac{i-.5}{10})$
1	.05	74	82	78
2	.15	77	82	79
3	.25	78	82	79
4	.35	78	84	81
5	.45	78	84	82
6	.55	80	85	82
7	.65	81	85	82
8	.75	84	85	83
9	.85	85	86	84
10	.95	87	87	85

	Q_1	*Median*	Q_3
Method 1	78	$\frac{78+80}{2} = 79$	84
Method 2	82	$\frac{84+85}{2} = 84.5$	85
Method 3	79	82	83



(b) Method 2 is the most precise, since it produces the least amount of spread. Method 3 is more precise than Method 1. Method 3 is the most accurate, since it comes closest to 80 on the average. Method 1 is more accurate than Method 2.

(c) These would be paired data. 10 specimens, 20 measurements would provide a better comparison. To compare averages under this plan, you would take differences for each of the specimens and look at the average of the 10 differences. Under the other plan, you would average measurements for 10 specimens for each method, and look at the difference between the averages. There will be less variability in the average of the differences than in the difference between the averages, because of the pairing. Less variability results in a sharper comparison of the difference.

4. (a) Units are Ohms

```
                       20 Ohm Nominal Resistors

Decimal point is 1 place to the left of the colon

                           0 :  190 :
                          00 :  191 :
                        0000 :  192 :
                        0000 :  193 :
                          00 :  194 :
                           0 :  195 :
                           0 :  196 :
                             :  197 : 00
   1/4 Watt                  :  198 :               1/2 Watt
                             :  199 : 0
                             :  200 : 0
                             :  201 : 00
                             :  202 : 00
                             :  203 : 0
                             :  204 : 00
                             :  205 : 0
                             :  206 : 0
                             :  207 :
                             :  208 : 0

                                High: 24.4
```

```
                       75 Ohm Nominal Resistors

Decimal point is at the colon

                             : 68 : 6
                             : 69 :
                           9 : 70 :
                           8 : 71 : 7
                       01334 : 72 : 18
   1/4 Watt           556799 : 73 : 289           1/2 Watt
                          24 : 74 : 0268
                             : 75 : 0
                             : 76 : 257
```

100 Ohm Nominal Resistors

Decimal point is at the colon

```
                          16889 :  94 :
                            148 :  95 : 5
                             00 :  96 : 68
                            347 :  97 : 229
   1/4 Watt                  35 :  98 : 57             1/2 Watt
                                :  99 : 2
                                : 100 : 00
                                : 101 :
                                : 102 : 000
                                : 103 : 0
```

150 Ohm Nominal Resistors

Decimal point is at the colon

```
                                : 145 : 0
                                : 146 : 0
                            000 : 147 : 0
                    00000000000 : 148 : 0
                              0 : 149 : 0000
   1/4 Watt                     : 150 : 00             1/2 Watt
                                : 151 : 0
                                : 152 : 0
                                : 153 : 0
                                : 154 : 0
                                : 155 : 0
```

200 Ohm Nominal Resistors

Decimal point is at the colon

```
                               : 192 : 0
                             0 : 193 :
                             0 : 194 :
                           000 : 195 : 0
                        000000 : 196 : 0
                               : 197 : 00
                             0 : 198 :
                           000 : 199 : 0
                               : 200 :
   1/4 Watt                    : 201 : 0            1/2 Watt
                               : 202 : 0
                               : 203 :
                               : 204 :
                               : 205 : 0
                               : 206 :
                               : 207 : 00
                               : 208 :
                               : 209 :
                               : 210 : 0
                               : 211 : 0
                               : 212 :
                               : 213 :
                               : 214 : 0

                               High:  257
```

The $\frac{1}{2}$ watt resistances are generally larger (and closer to nominal on average) and mor spread out than the $\frac{1}{4}$ watt resistances.

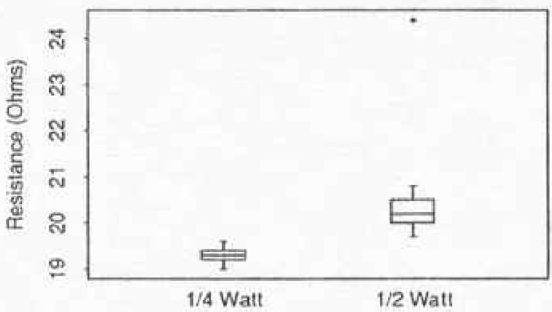
(b)

i	$\frac{i-.5}{15}$	$Q_{\frac{1}{4},20}$	$Q_{\frac{1}{4},75}$	$Q_{\frac{1}{4},100}$	$Q_{\frac{1}{4},150}$	$Q_{\frac{1}{4},200}$
1	.03	19.0	70.9	94.1	147	193
2	.10	19.1	71.8	94.6	147	194
3	.17	19.1	72.0	94.8	147	195
4	.23	19.2	72.1	94.8	148	195
5	.30	19.2	72.3	94.9	148	195
6	.37	19.2	72.3	95.1	148	196
7	.43	19.2	72.4	95.4	148	196
8	.50	19.3	72.5	95.8	148	196
9	.57	19.3	72.5	96.0	148	196
10	.63	19.3	72.6	96.0	148	196
11	.70	19.3	72.7	97.3	148	196
12	.77	19.4	72.9	97.4	148	198
13	.83	19.4	72.9	97.7	148	199
14	.90	19.5	73.2	98.3	148	199
15	.97	19.6	73.4	98.5	149	199

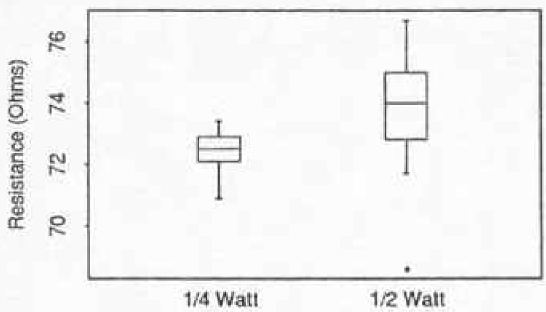
i	$\frac{i-.5}{15}$	$Q_{\frac{1}{2},20}$	$Q_{\frac{1}{2},75}$	$Q_{\frac{1}{2},100}$	$Q_{\frac{1}{2},150}$	$Q_{\frac{1}{2},200}$	Q_{SN}
1	.03	19.7	68.6	95.5	145	192	-1.88
2	.10	19.7	71.7	96.6	146	195	-1.28
3	.17	19.9	72.1	96.8	147	196	-.95
4	.23	20.0	72.8	97.2	148	197	-.74
5	.30	20.1	73.2	97.2	149	197	-.52
6	.37	20.1	73.8	97.9	149	199	-.33
7	.43	20.2	73.9	98.5	149	201	-.18
8	.50	20.2	74.0	98.7	149	202	.00
9	.57	20.3	74.2	99.2	150	205	.18
10	.63	20.4	74.6	100.0	150	207	.33
11	.70	20.4	74.8	100.0	151	207	.52
12	.77	20.5	75.0	102.0	152	210	.74
13	.83	20.6	76.2	102.0	153	211	.95
14	.90	20.8	76.5	102.0	154	214	1.28
15	.97	24.4	76.7	103.0	155	257	1.88

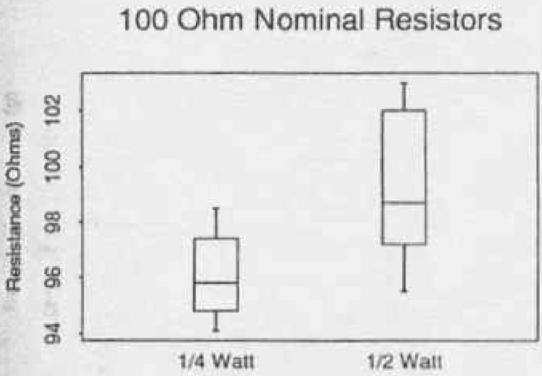
Sample	Q_1	*Median*	Q_3
$\frac{1}{4}$, 20	19.2	19.3	$(\frac{5}{7})(19.4) + (\frac{2}{7})(19.3) = 19.37$
$\frac{1}{4}$, 75	$(\frac{5}{7})(72.1) + (\frac{2}{7})(72.3) = 72.16$	72.5	72.84
$\frac{1}{4}$, 100	94.83	95.8	97.37
$\frac{1}{4}$, 150	148	148	148
$\frac{1}{4}$, 200	195	196	197.43
$\frac{1}{2}$, 20	20.03	20.2	20.47
$\frac{1}{2}$, 75	72.91	74	74.94
$\frac{1}{2}$, 100	97.2	98.7	101.43
$\frac{1}{2}$, 150	148.29	149	151.71
$\frac{1}{2}$, 200	197	202	209.14

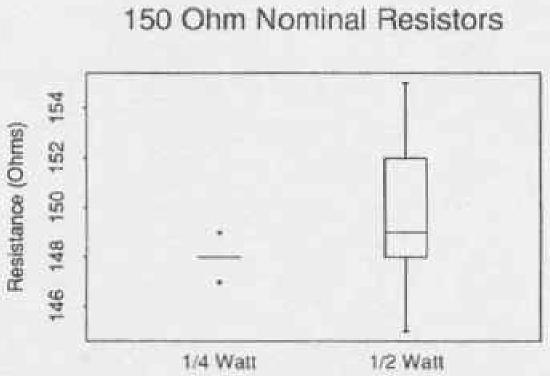
20 Ohm Nominal Resistors

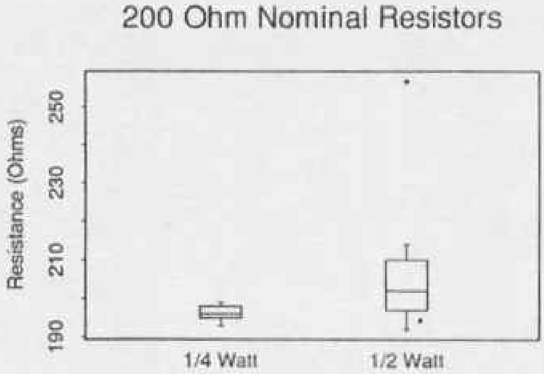


75 Ohm Nominal Resistors

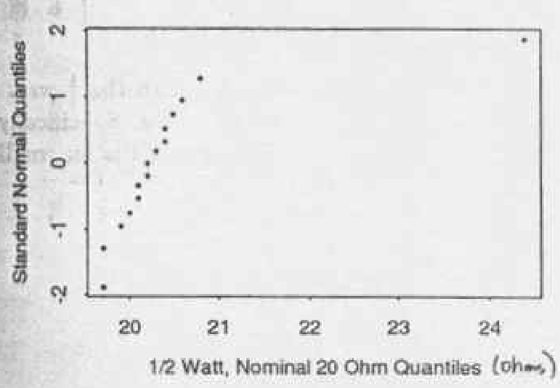


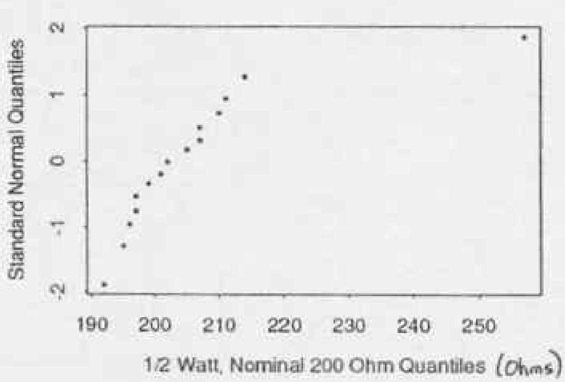






(c) The plotting points can be found in part (b).





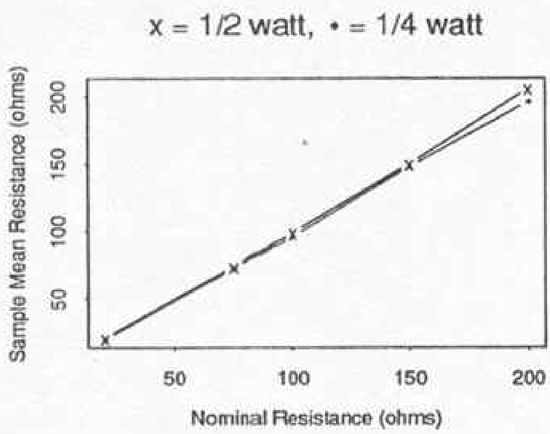
Both of these plots are relatively linear, except for one large outlier. One would tend to overestimate the fraction of resistances near the nominal value, since there will occasionally be an exceptionally large value which a bell-shaped distribution does not account for.

(d)

Sample	$\bar{x}$	s
$\frac{1}{4}$, 20	19.27	.16
$\frac{1}{4}$, 75	72.43	.61
$\frac{1}{4}$, 100	96.05	1.44
$\frac{1}{4}$, 150	147.87	.52
$\frac{1}{4}$, 200	196.20	1.82
$\frac{1}{2}$, 20	20.49	1.13
$\frac{1}{2}$, 75	73.87	2.08
$\frac{1}{2}$, 100	99.11	2.33
$\frac{1}{2}$, 150	149.80	2.83
$\frac{1}{2}$, 200	206.00	15.53

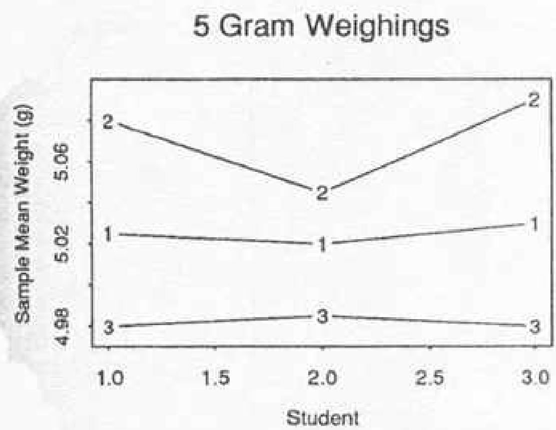
Yes. For each nominal resistance, the sample mean for the $\frac{1}{2}$ watt resistors is larger than the sample mean for the $\frac{1}{4}$ resistors, and the sample standard deviation for the $\frac{1}{2}$ resistors are larger than the sample standard deviation for the $\frac{1}{4}$ resistors.

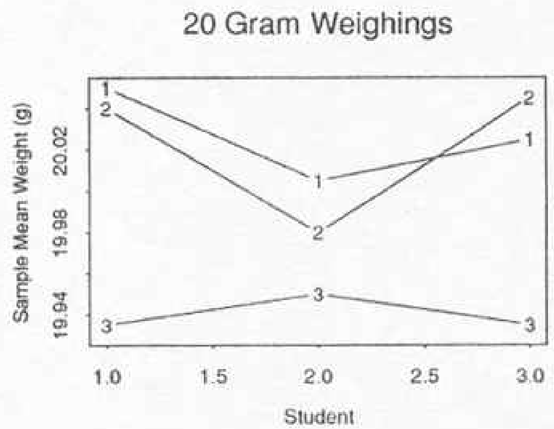
(e)

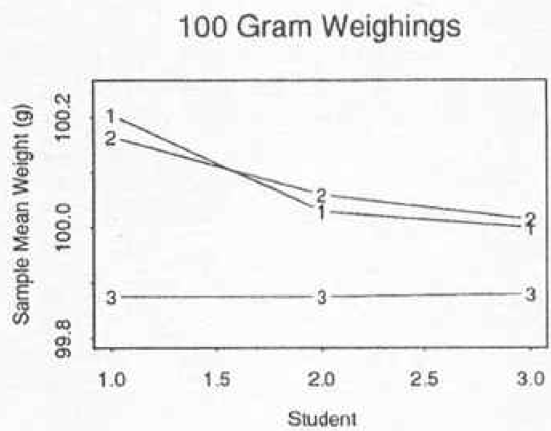


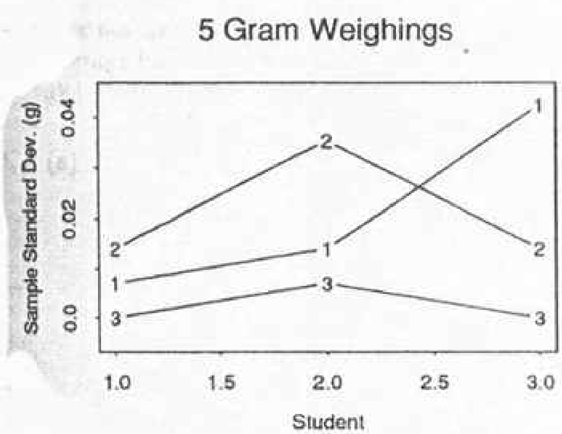
It shows that the $\frac{1}{2}$ watt resistors have generally higher resistance than the $\frac{1}{4}$ watt resistors, but the difference is not consistent across nominal resistances. Specifically, it seems like the difference is greater for larger nominal resistances than it is for smaller nominal resistances.

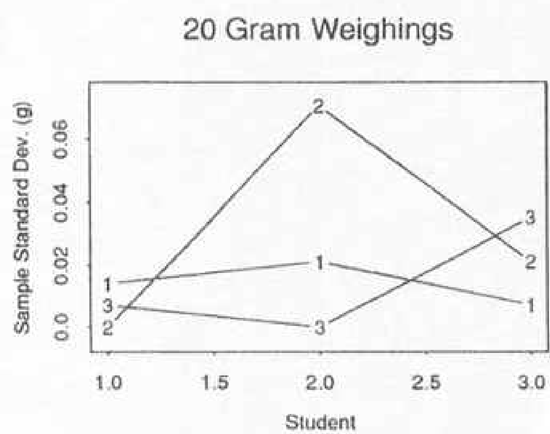
5.


5 Gram Weighings
Sample Mean Weight (g)
4.98
5.02
5.06
1.0
1.5
2.0
2.5
3.0
Student

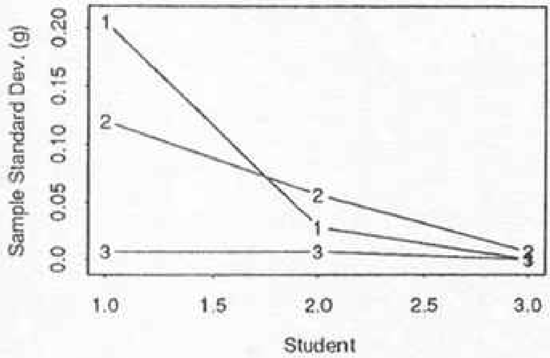

20 Gram Weighings
Sample Mean Weight (g)
19.94
19.98
20.02
1.0
1.5
2.0
2.5
3.0
Student


100 Gram Weighings
Sample Mean Weight (g)
99.8
100.0
100.2
1.0
1.5
2.0
2.5
3.0
Student


5 Gram Weighings
Sample Standard Dev. (g)
0.0
0.02
0.04
1.0
1.5
2.0
2.5
3.0
Student


20 Gram Weighings
Sample Standard Dev. (g)
0.0
0.02
0.04
0.06
1.0
1.5
2.0
2.5
3.0
Student

100 Gram Weighings



Overall, it seems that Scale 3 is more precise than the other 2 scales. Comparison of student precision is less clear-cut. Scale 3 is less accurate than the other 2 scales; it reads lower than the other 2 scales, and this is true for all 3 students. Scale 1 may be a little more accurate than Scale 2. There is some indication that Students 1 and 2 read higher than Student 3, but this seems to apply only to scales 1 and 2, and is only true for the 100g weight. Variation in repeated measurements of the same weight on the same scale by the same student (repeatability) seems to be slightly less than variation due to different students (reproducibility), but both sources of variation are on the same order of magnitude.

6.

i	$\frac{i-.5}{12}$	$Q(\frac{i-.5}{12})$	$Q_{SN}(\frac{i-.5}{12})$
1	.04	47	-1.75
2	.12	48	-1.17
3	.21	52	-.81
4	.29	86	-.55
5	.38	110	-.31
6	.46	116	-.10
7	.54	122	.10
8	.62	145	.31
9	.71	149	.55
10	.79	172	.81
11	.88	172	1.17
12	.96	194	1.75

$Q_1 = \frac{52+86}{2} = 69$

$Median = \frac{116+122}{2} = 119$

$Q_3 = \frac{149+172}{2} = 160.5$

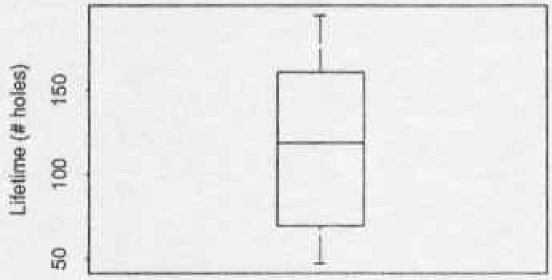
$IQR = 91.5$

$\bar{x} = 117.75$

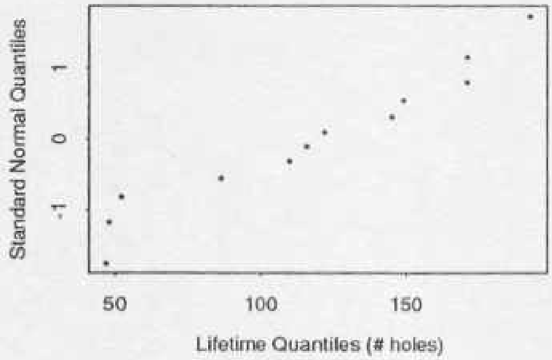
$s = 51.08$

$R = 147$

Box plot:



Normal plot:



The normal plot is roughly linear, indicating that the data are roughly bell-shaped. The average lifetime of the drills tested was about 118 holes, but the lifetimes ranged from 47 to 194 holes. There were no outliers.

7. (a)

i	$\frac{i-.5}{8}$	$Q(\frac{i-.5}{8})$	$Q_{SN}(\frac{i-.5}{8})$
1	.06	-.0005	-1.55
2	.19	-.0005	-.88
3	.31	-.0005	-.50
4	.44	0	-.15
5	.56	0	.15
6	.69	0	.50
7	.81	0	.88
8	.94	.0005	1.55

$Q_1 = -.0005$

$Median = 0$

$Q_3 = 0$

(b) $\bar{x} = -.000125$

$s = .000354$

(c) On the average, Student B's measurements were larger than Student A's.

(d) $(-.0005, -1.55)$.

8. (a)

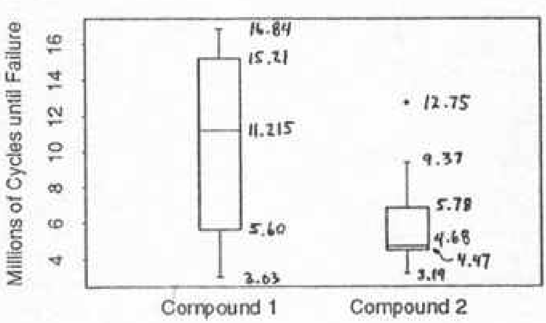
i	$\frac{i-.5}{10}$	$Q_1(\frac{i-.5}{10})$	$Q_2(\frac{i-.5}{10})$	$Q_{SN}(\frac{i-.5}{10})$
1	.05	3.03	3.19	-1.64
2	.15	5.53	4.26	-1.04
3	.25	5.60	4.47	-.67
4	.35	9.30	4.53	-.39
5	.45	9.92	4.67	-.13
6	.55	12.51	4.69	.13
7	.65	12.95	5.78	.39
8	.75	15.21	6.79	.67
9	.85	16.04	9.37	1.04
10	.95	16.84	12.75	1.64

$Q(.84) = (.9)(16.04) + (.1)(15.21) = 15.957.$

(b) $(3.03, -1.64)$, $(5.53, -1.04)$.

(c)
```
Decimal point is at the colon
Units are millions of cycles

                        0 :  3 : 2
                          :  4 : 35577
                       56 :  5 : 8
                          :  6 : 8
                          :  7 :
                          :  8 :
Compound 1             39 :  9 : 4              Compound 2
                          : 10 :
                          : 11 :
                       59 : 12 : 7
                          : 13 :
                          : 14 :
                        2 : 15 :
                       08 : 16 :
```

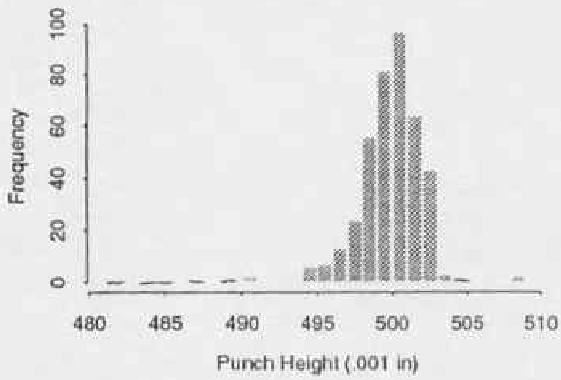
(d) $Median_1 = \frac{9.92+12.51}{2} = 11.215$, $Median_2 = \frac{4.67+4.69}{2} = 4.68$.



(e) $\bar{x}_1 = 10.693$, $s_1 = 4.819$, $\bar{x}_2 = 6.05$, $s_2 = 2.915$.

(f) The lifetimes of bearings made with Compound 1 are generally longer than those made with Compound 2, but there is more variability in the lifetimes of bearings made with Compound 1 than in the lifetimes of those made with Compound 2.

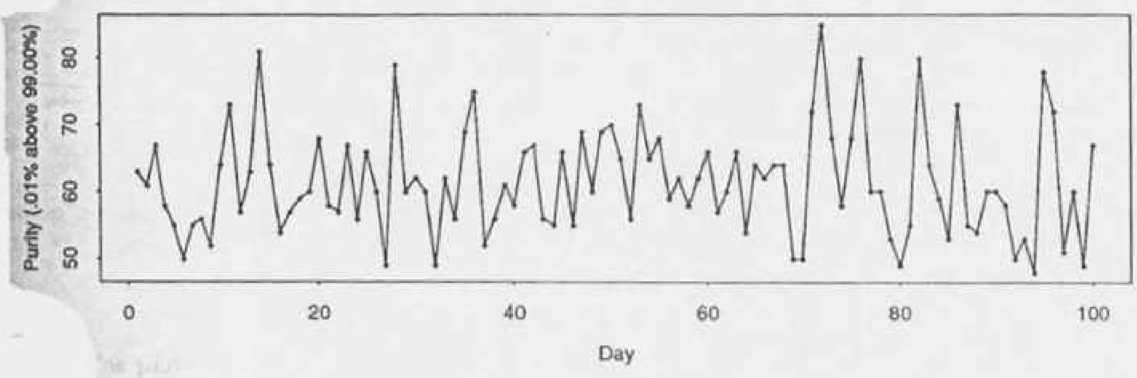
9. (a) $\bar{x} = 500.24$, $s = 2.60$, $Median = 501$, $IQR = 3.0$, $R = 27.0$. The following histogram is easily constructed using the data as given.



The histogram shows that the distribution is truncated on the right. It appears that the supplier has inspected the punches and removed almost all which had punch heights greater than .505 inches or less than .495 inches. It seems that the supplier's equipment is not capable of meeting the specifications, because there would be a fair number of punches outside of specifications if no inspection were done.

(b) If a cut piece of material has hole diameters with a large amount of variability, it may be difficult to further process the piece. The punched holes may be filled with another part that can be made to have a uniform diameter. It may be easy to change the mean diameter of the filling part to accommodate the mean punch height, but if there is too much variability in punch heights, it will be impossible to make the filling part so that it fits consistently.

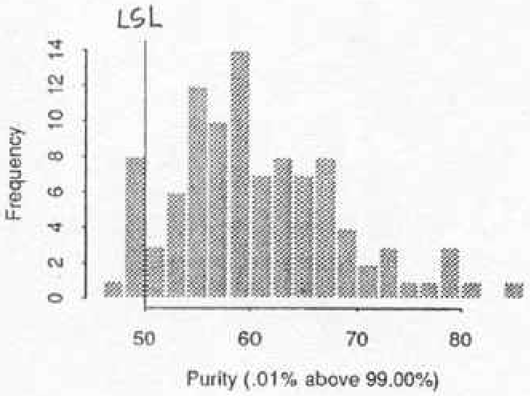
10. (a)



There do not seem to be any obvious time trends. Early detection of a trend could allow the manufacturer to search for any problems with the process before it starts producing unacceptable product. An observed trend may be linked to some known change associated with the process, allowing engineers to better control the process.

(b) Decimal point is 1 place to the right of the colon
Units are .01% above 99.00%

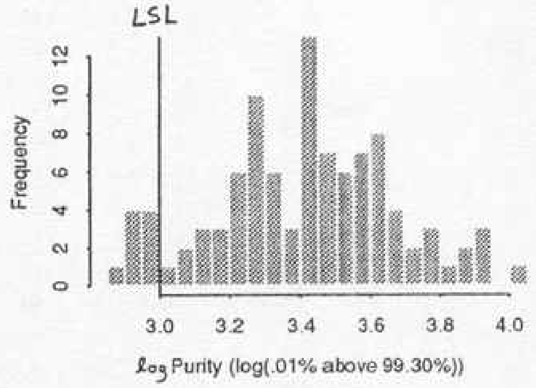
```
4 : 89999
5 : 000122333444
5 : 5555556666667777888888999
6 : 000000000001122222233444444
6 : 556666677778888999
7 : 022333
7 : 589
8 : 001
8 : 5
```



The distribution is right-skewed.

(c) Decimal point is 1 place to the left of the colon
Units are log(.01% above 99.30%)

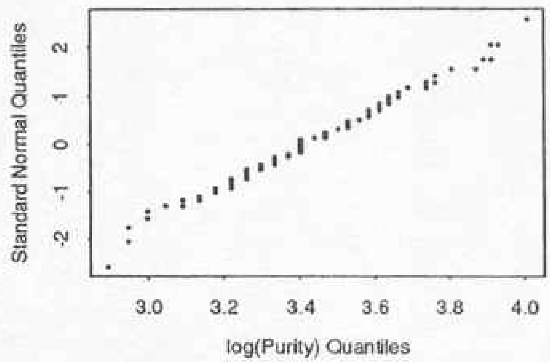
```
28 : 9
29 : 4444
30 : 0000499
31 : 444888
32 : 222222666666
33 : 0000333333777
34 : 000000000003377777
35 : 003333336688888
36 : 111144446669
37 : 44666
38 : 179
39 : 113
40 : 1
```



The distribution of the transformed data is much more bell-shaped.

(d) Some of the plot coordinates are given in the table below.

i	$\frac{i-.5}{100}$	$Q(\frac{i-.5}{100})$	$Q_{SN}(\frac{i-.5}{100})$
1	.005	2.890372	-2.58
2	.02	2.944439	-2.05
3	.02	2.944439	-2.05
4	.04	2.944439	-1.75
5	.04	2.944439	-1.75
	⋮		
96	.96	3.891820	1.75
97	.96	3.912023	1.75
98	.98	3.912023	2.05
99	.98	3.931826	2.05
100	.995	4.007333	2.58

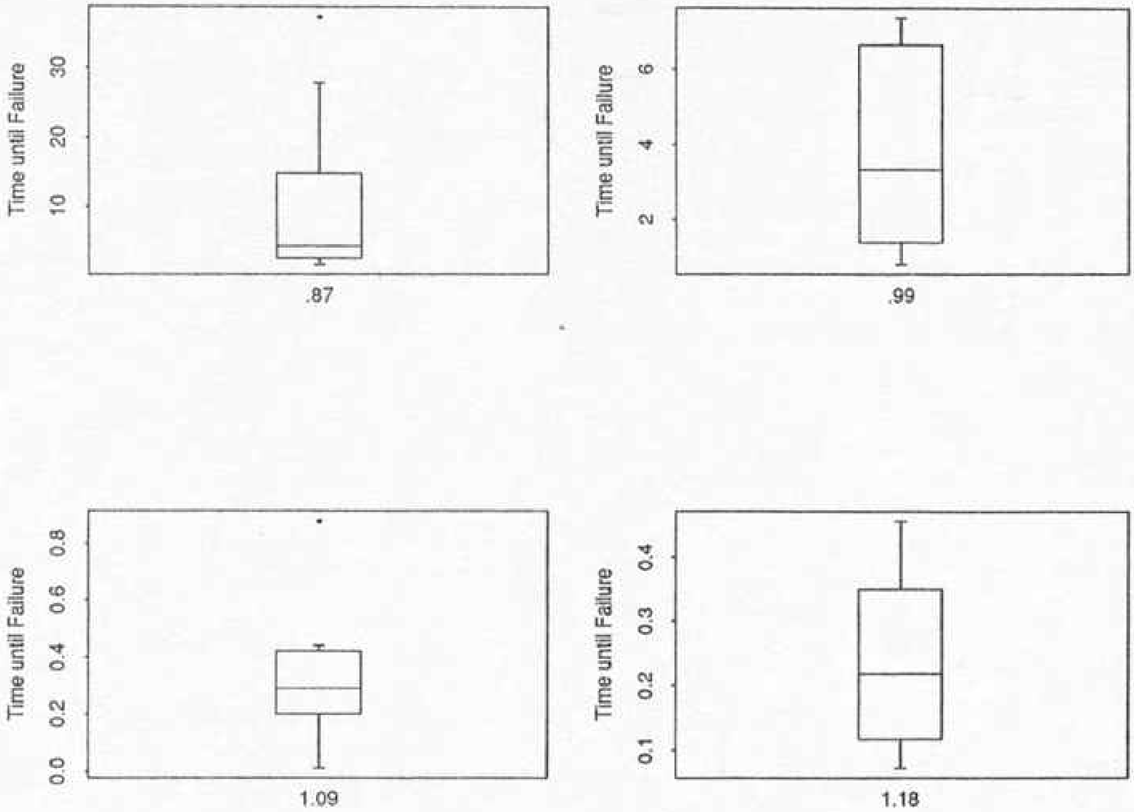


The plot is quite linear, indicating that the distribution of transformed values is bell-shaped.

11. (a)

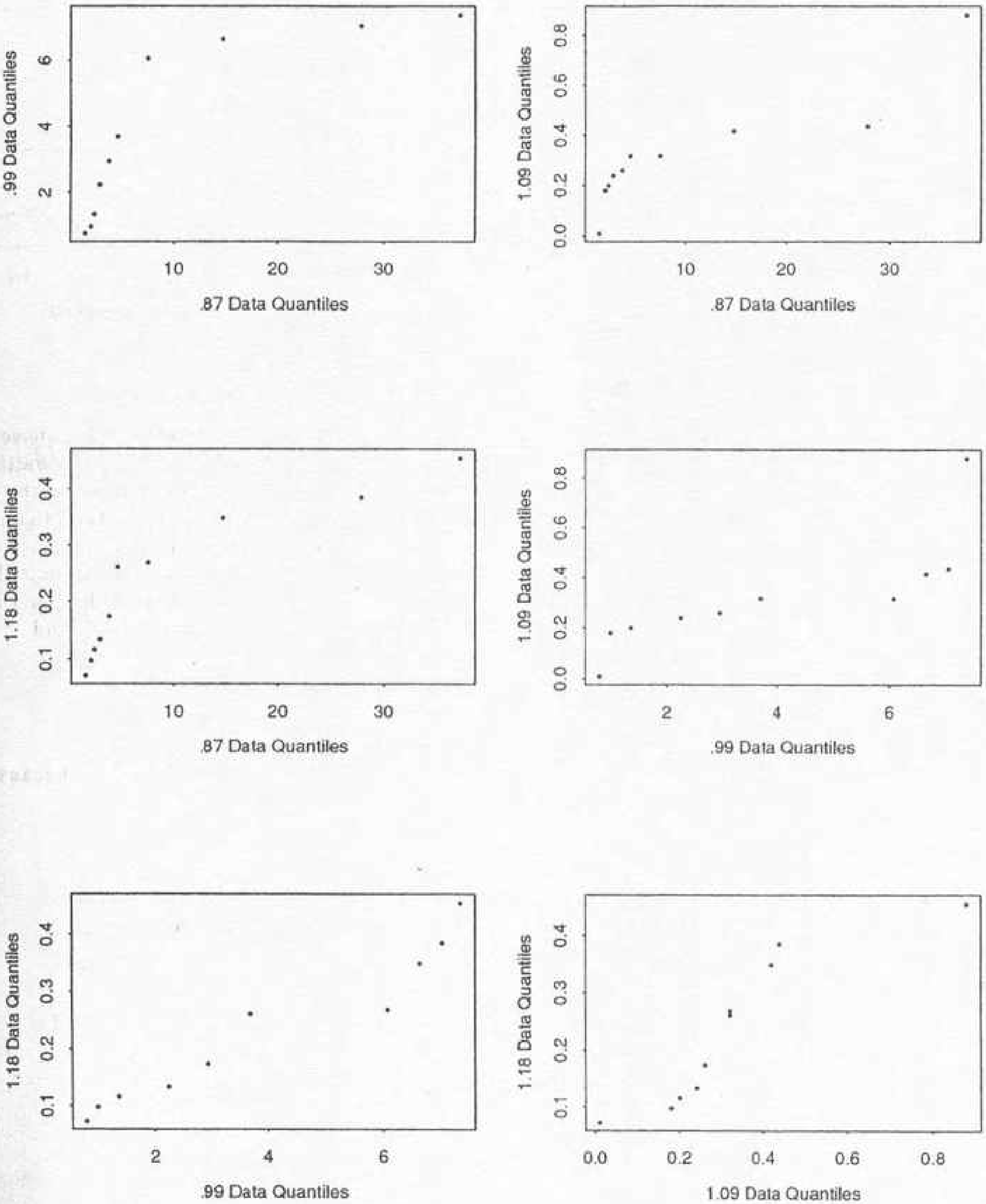
i	$\frac{i-.5}{10}$	$Q_{.87}(\frac{i-.5}{10})$	$Q_{.99}(\frac{i-.5}{10})$	$Q_{1.09}(\frac{i-.5}{10})$	$Q_{1.18}(\frac{i-.5}{10})$
1	.05	1.67	.80	.012	.073
2	.15	2.20	1.00	.180	.098
3	.25	2.51	1.37	.200	.117
4	.35	3.00	2.25	.240	.135
5	.45	3.90	2.95	.260	.175
6	.55	4.70	3.70	.320	.262
7	.65	7.53	6.07	.320	.270
8	.75	14.70	6.65	.420	.350
9	.85	27.80	7.05	.440	.386
10	.95	37.40	7.37	.880	.456

If the box plots are drawn on the same scale, the shapes are hard to see.



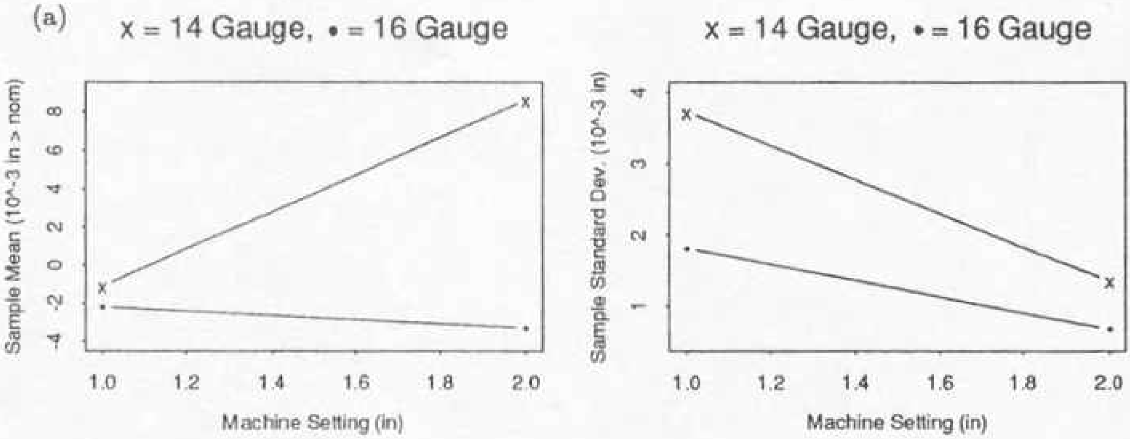
Based on the locations of the medians relative to the quartiles, all 4 distributions seem to be right-skewed, and so are roughly similar in shape. It is difficult to clearly see the shape of any distribution based on only 10 data points.

(b)



Relative to the .99 distribution, the .87 distribution has a longer right tail. If it were not for the outlier from the 1.09 distribution, its shape would be similar to the 1.18 and .99 distributions. The shape of the 1.18 distribution is similar to the shapes of the .87 and .99 distributions.

12. (a)



(b) The 14 gauge, 2 inch nominal strips are the farthest from nominal (and are above nominal), although the machine seems to consistently cut 16 gauge strips to widths below nominal for both settings. The 1 inch 14 gauge strips are close to nominal on the average, but these have more variability than the other types of strips. Overall, the 14 gauge strips seem to have more variability than the 16 gauge strips, and this is more pronounced for the 1 inch than the 2 inch setting. According to the sample means, the machine should be set at about 1.0022 inches for 1 inch nominal/16 gauge strips, 1.0011 inches for 1 inch nominal/14 gauge strips, 2.0033 inches for 2 inch nominal/16 gauge strips, and 1.9914 inches for 2 inch nominal/14 gauge strips.

13. (a)

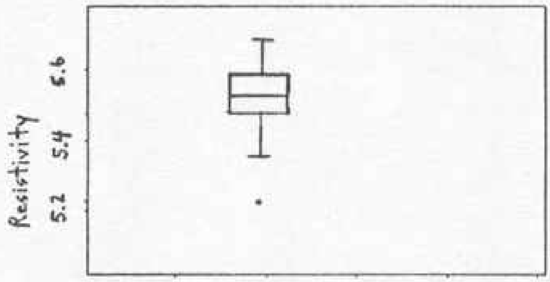
```
           .              .       .      . . : .  .  . .        .
 -+---------+---------+---------+---------+---------+----- Resistivity
 5.20      5.30      5.40      5.50      5.60      5.70
```

i	$\frac{i-.5}{12}$	$Q(\frac{i-.5}{12})$	$Q_{SN}(\frac{i-.5}{12})$
1	.04	5.22	-1.75
2	.12	5.37	-1.17
3	.21	5.45	-.81
4	.29	5.51	-.55
5	.38	5.52	-.31
6	.46	5.53	-.10
7	.54	5.53	.10
8	.62	5.55	.31
9	.71	5.58	.55
10	.79	5.60	.81
11	.88	5.62	1.17
12	.96	5.69	1.75

$Median = 5.53$

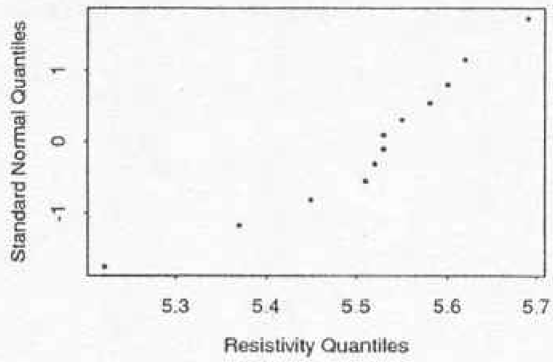
$Q_1 = \frac{5.45+5.51}{2} = 5.48$

$Q_3 = \frac{5.58+5.60}{2} = 5.59$



$\bar{x} = 5.514$, $s = .123$. Both plots reveal an outlier.

(b) The plotting positions are given in part (a).



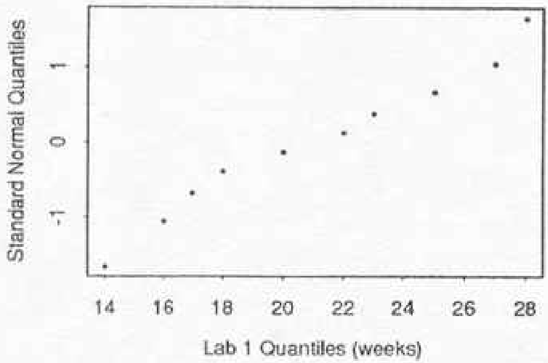
The plot is roughly linear, but it would be more linear if the lower-left point were pushed to the right. This means that lower tail of the distribution is longer than it would be for bell-shaped data.

14. (a)

i	$\frac{i-.5}{10}$	$Q_1(\frac{i-.5}{10})$	$Q_2(\frac{i-.5}{10})$	$Q_{SN}(\frac{i-.5}{10})$
1	.05	14	27	-1.64
2	.15	16	28	-1.04
3	.25	17	29	-.67
4	.35	18	29	-.39
5	.45	20	29	-.13
6	.55	22	30	.13
7	.65	23	31	.39
8	.75	25	31	.67
9	.85	27	33	1.04
10	.95	28	34	1.64

$Median = \frac{20+22}{2} = 21$, $Q_1 = 17$, $Q_3 = 25$, $Q(.64) = (.9)(23) + (.1)(22) = 22.9$.

(b) The plotting positions are given in part (a).



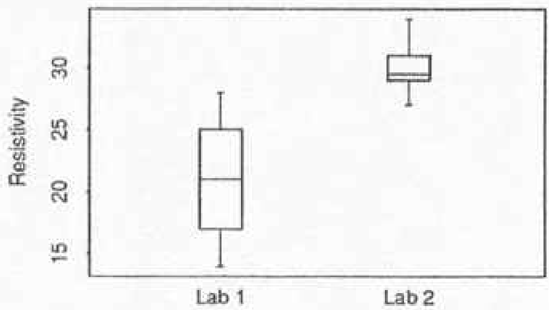
The plot is fairly linear, but there is some indication that the data distribution has shorter tails than a bell-shaped distribution (the smallest and largest points would need to be "moved out" to make a perfectly straight line).

(c) $\bar{x} = 21$, $R = 14$, $s = 4.78$.

(d) Decimal point is 1 place to the right of the colon

Lab 1					Lab 2
	4	1			
	678	1			
	023	2			
	578	2	78999		
		3	01134		

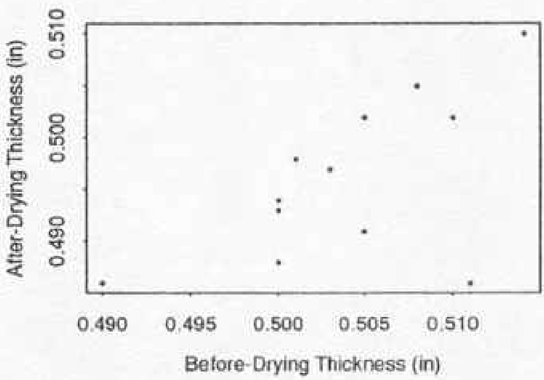
(e) For the Lab 2 data: $Median = \frac{29+30}{2} = 29.5$, $Q_1 = 29$, $Q_3 = 31$.



(f) Both plots show that there is less spread in the Lab 2 data, so it produced the more precise results.

(g) No. You would need to know the "true" long-run average lifetime of these types of specimens in order to determine which lab is more accurate.

15. (a)



There is a strong positive correlation between before-drying thickness and after-drying thickness. This relationship may be important in predicting the after-drying thickness from the before-drying thickness. It would help the manufacturer produce dried boards closer to .500 in. thick.

(b) The differences are .004, .003, .007, .004, .006, .006, .008, .003, .012, .025, .014, .003. $\bar{x}_d = .0079$, $s_d = .0064$. The ideal before-drying thickness would be about $.500 + .0079 = .5079$ in., since the boards shrink by about .0079 in. Variability in after-drying thickness would be reduced, since it is correlated with before-drying thickness. Some would remain though, since there is variability in the differences (as measured by s_d).

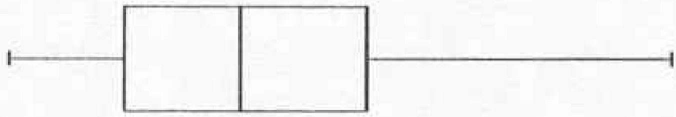
16. (a) $\bar{x} = 5.381$, $s = 2.462$.

(b)

i	$\frac{i-.5}{20}$	$Q(\frac{i-.5}{20})$	$Q_{SN}(\frac{i-.5}{20})$
1	0.02	1.73	-2.05
2	0.08	2.47	-1.41
3	0.12	2.83	-1.17
4	0.18	3.20	-0.92
5	0.22	3.20	-0.77
6	0.28	3.57	-0.58
7	0.32	3.93	-0.47
8	0.38	4.30	-0.31
9	0.42	4.67	-0.20
10	0.48	5.03	-0.05
11	0.52	5.03	0.05
12	0.57	5.40	0.18
13	0.62	5.77	0.31
14	0.68	6.13	0.47
15	0.72	6.50	0.58
16	0.78	7.23	0.77
17	0.82	7.60	0.92
18	0.88	8.33	1.17
19	0.92	9.43	1.41
20	0.98	11.27	2.05

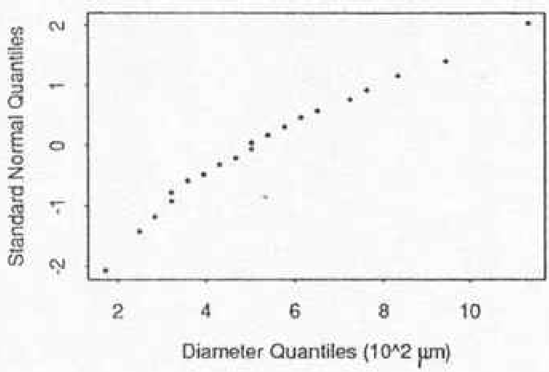
$Q_1 = \frac{3.20+3.57}{2} = 3.385$, $Median = 5.03$, $Q_3 = \frac{6.50+7.23}{2} = 6.865$.

```
    .  .  . :  .  .  . ..  :  .  .  .  .  .    .  .     .      .          .
-----+---------+---------+---------+---------+---------+-  Diameter
    2.0       4.0       6.0       8.0      10.0      12.0  (10^-2 µm)
```



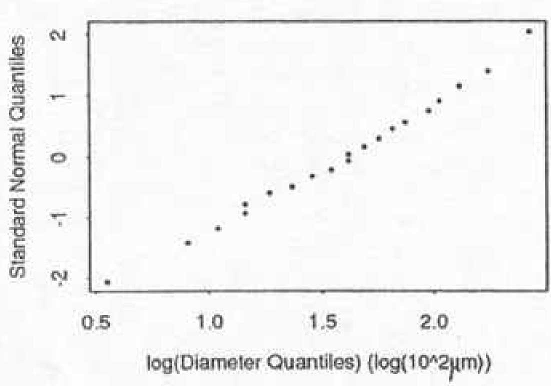
(c) The distribution is slightly right-skewed.

(d) The plotting positions are given in part (b).



To make this plot more linear, you would need to move the smallest data points to the left, so the distribution has a shorter left tail than a bell-shaped distribution.

(e) The first 3 coordinates of the plot are (.548, −2.05), (.904, −1.41), (1.04, −1.17).



This plot is more linear than the one in (d), indicating that the transformed data are more bell-shaped.

17. (a)

i	$\frac{i-.5}{13}$	$Q_A(\frac{i-.5}{13})$
1	.04	79.97
2	.12	79.98
3	.19	80.00
4	.27	80.02
5	.35	80.02
6	.42	80.02
7	.50	80.03
8	.58	80.03
9	.65	80.03
10	.73	80.04
11	.81	80.04
12	.88	80.04
13	.96	80.05

i	$\frac{i-.5}{8}$	$Q_B(\frac{i-.5}{8})$
1	.06	79.94
2	.19	79.95
3	.31	79.97
4	.44	79.97
5	.56	79.97
6	.69	79.98
7	.81	80.02
8	.94	80.03

For Method A:

$Median = 80.03$

$Q_1 = (\frac{6}{8})(80.02) + (\frac{2}{8})(80.00) = 80.015$

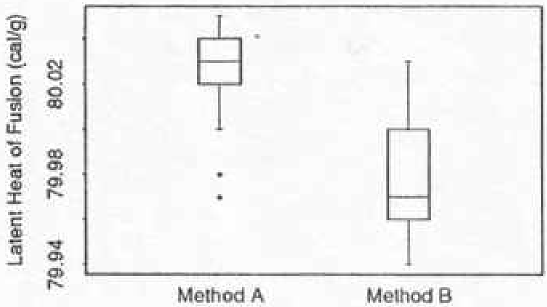
$Q_3 = 80.04$

For Method B:

$Median = 79.97$

$Q_1 = \frac{79.95+79.97}{2} = 79.96$

$Q_3 = \frac{79.98+80.02}{2} = 80.00.$



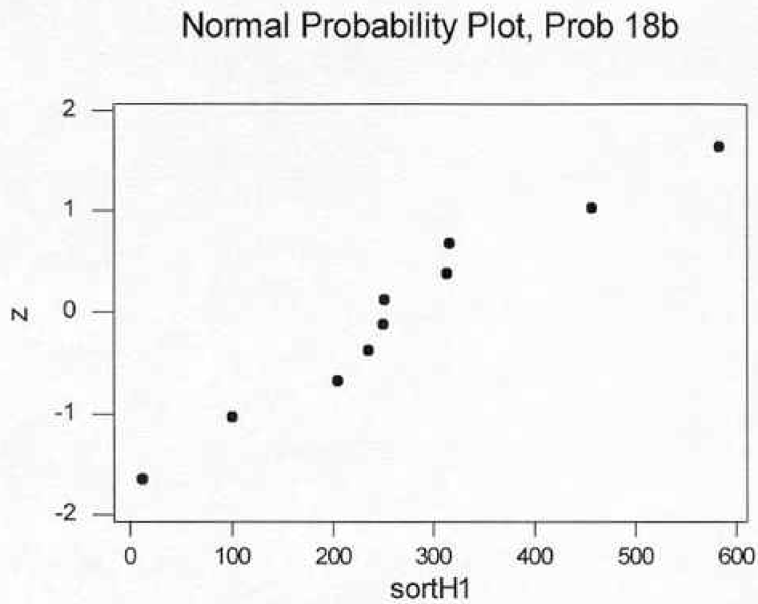
There does not seem to be any important difference in the precisions of the two methods, but Method A generally produced larger values than Method B. Since there is some fixed, true, theoretical latent heat for the fusion of ice, at least one of the methods must be somewhat inaccurate.

(b) $\bar{x}_A = 80.021$, $s_A = .024$, $\bar{x}_B = 79.979$, $s_B = .031$. The sample standard deviations are similar, as reflected by the similar magnitudes of spread in the boxplots. $\bar{x}_A > \bar{x}_B$, as reflected by the location of the boxes on the boxplot.

18(a) p = .25 implies i = (.25)(10) + .5 = 3[rd] ranked value = 204 = 1[st] quartile
p = .5 implies i = (.5)(10) + .5 = 5.5 ranked value = 249.5 = median
p = .75 implies i = (.75)(10) + .5 = 8[th] ranked value = 315 = 3[rd] quartile
p = .62 is .7 of the way from .55 to .65, thus Q(.55) + .7(Q(.65) - Q(.55)) = 294.1

(b)

(i-.5)/10	z
0.05	-1.64485
0.15	-1.03643
0.25	-0.67449
0.35	-0.38532
0.45	-0.12566
0.55	0.12566
0.65	0.38532
0.75	0.67449
0.85	1.03643
0.95	1.64485



It seems the distribution is approximately normal for Heat 1.

(c) $\bar{x} = 271.8$ and s = 163.2

(d)

0	11
1	00
2	04,49,50
3	13,15
4	57
5	84

(e)

H2			H1
	0	11	
96,89,70,63	1	00	
67,16,06	2	04,35,49,50	
59,50,49	3	13,15	
	4	57	
	5	84	

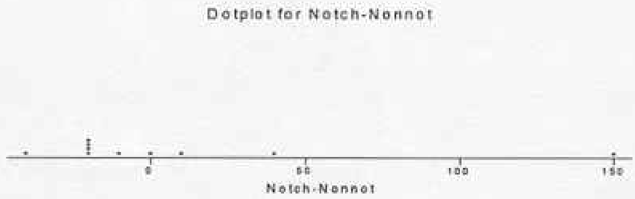
H1 is more disperse.

(f) H1: Q1 = 204, Q2 = 249.5, Q3 = 315, IQR = 111
H2: Q1 = 189, Q2 = 211, Q3 = 349, IQR = 160
H3: Q1 = 185, Q2 = 228, Q3 = 289, IQR = 104

Amongst all Heats, Heat 1 fatigue lives seem to be centered (approximate average fatigue life) at the largest value and Heat 2 fatigue lives seem to be the most disperse (most variable).

(19)(a) Notch minus Non-Notch outside diameter measurements can be accurately evaluated with the current data.

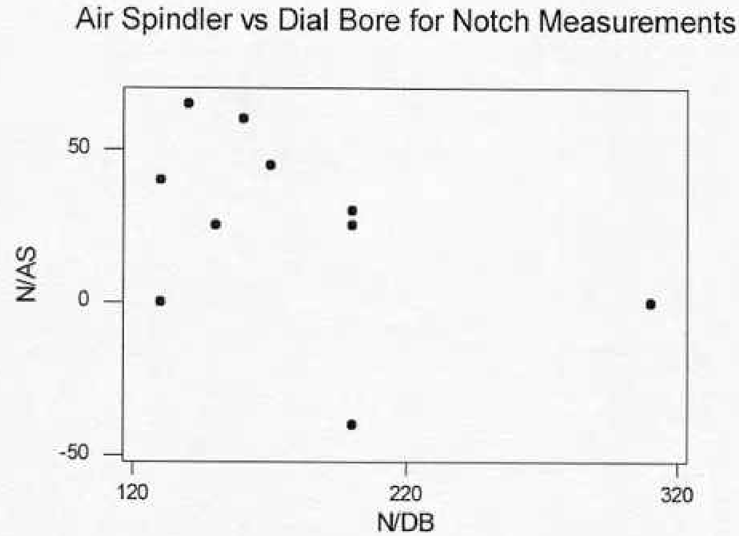
(b) Notch-NonNotch data
-20
10
-40
150
40
-10
-20
0
-20
-20



The notch measurements seem to be less than the non-notch measurements when using a Dial Bore (see that most dots are negative).

Q1 for the notch minus non-notch data is -20. Q2 for the notch minus non-notch data is -15 and Q3 for the non-notch data is 10. The IQR is 30. Thus, the box plot shows the differences are mostly negatively with a few positive differences. A non-zero mean for these data suggests there is an important difference in the diameter measurements for notch vs non-notch sleeves using the Dial Bore. A large variability of these differences suggests something else besides notch (or non-notch) is affecting the measurements.

(c) Yes, the plot suggests there is a relationship.



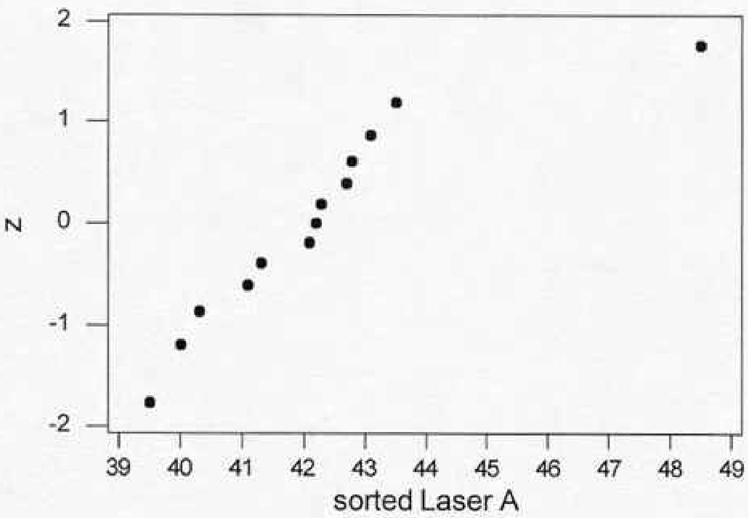
Large deviations above nominal for Dial Bore are associated with very small or negative deviations from nominal for the Air Spindler measurments.

(d) To determine which of the two gages is most precise, the same sleeve needs to be measured at least twice by both gages for the same "type" of sleeve (notch or non-notch). The range (R) for each gage on the same sleeve could be calculated and compared. This could be repeated for several sleeves. The average of all Rs for each gage could then be compared. The gage with the largest average R would be the gage with less precision.

20 (a) The first quartile, Q1 = .75(41.1 - 40.3) + 40.3
The 2nd quartile, Q2 = sample median = 42.2
The third quartile, Q3 = .25(43.1-42.8) + 42.8 = 42.875
The 37th quantile is .3099(42.1 - 41.3) + 41.3 = 41.548

(b) No, the distribution does not appear to be normally distributed. One angle is larger than it should be if the hole angles were all coming from the same normal distribution.

Normal Probability Plot 20(b)



(c) The sample average angle is 42.262, the sample range is 9 and the sample standard deviation is 2.246.

(d)

Laser A		EDM
.5	39	
.3, 0	40	
.3, .1	41	
.8, .7, .3, .2, .1	42	
.5, .1	43	.2, .4
	44	0, .2, .4, .6, .6, .6, .7
	45	.3, .3, .5
	46	.1
	47	
.5	48	

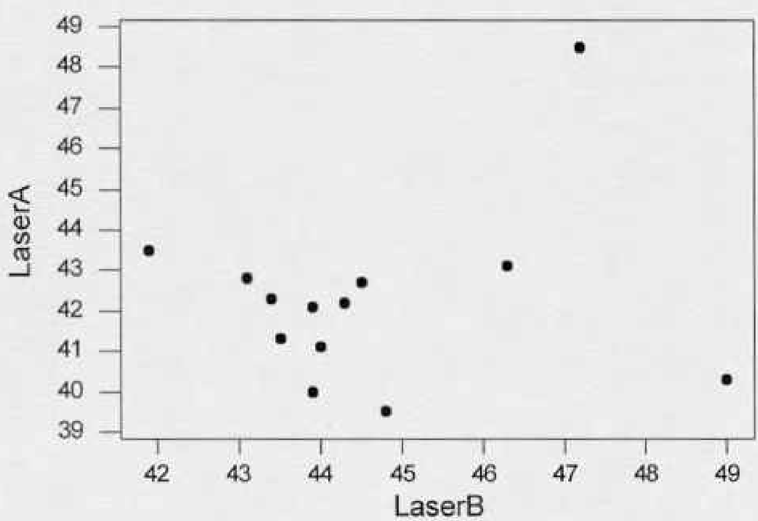
(e) For Laser A, Q1 = 40.9, median = 42.2, Q3 = 42.875 and the IQR = 1.975
For EDM, Q1 = .75(44.2 - 44) + 44 = 44.15, m = 44.6, Q3 = .25(45.3 -45.3) + 45.3 = 45.3 and IQR = 1.15.

(f) EDM produced the most consistent results and produced an average angle closest to the nominal value of 45 degrees.

(g) The two sets of Laser measurements are paired data.

(h) For most pieces of material, the Laser B hole has a larger angle than that corresponding to the Laser A hole.

Laser A Hole Angle vs Laser B Hole Angle

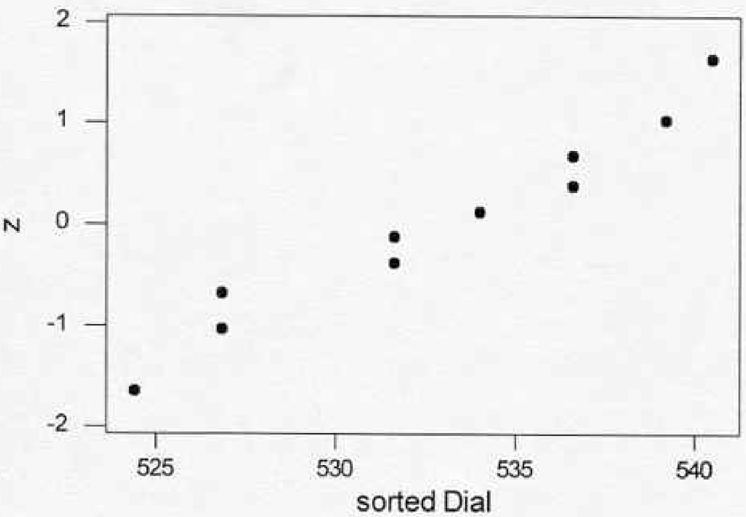


(i) $\bar{d} = -2.338$, s_d = 2.705 where d = Hole A angle - Hole B angle. $\bar{d}$ estimates the average angle difference (Laser A - Laser B) when both methods are applied to the same piece of material. s_d estimates the standard deviation of all angle differences (Laser A - Laser B) when both methods are applied to the same piece of material.

21 (a) median = .5(534 - 531.6) + 531.6 = 532.8, Q1 = 526.8 and Q3 = 536.6. Since p = .27, the 27th quantile = .2(531.6 - 526.8) + 526.8 = 527.76.

(b)

Normal Probability Plot for Dial Data



The Dial hardness data appears to have a normal distribution.

(c) $\bar{x} = 532.81$, s = 5.54 and R = 16.1

(d)

Dial		Digital
	50	1, 6
	51	4, 8, 9
6, 6, 4	52	2, 2, 2, 3
9, 6, 6, 4, 1,1	53	1
0	54	

(e) Dial Hardness: Q1 = 526.8, median = 532.8, Q3 = 536.6 and IQR = 9.8
Digital Hardness: Q1 = 514.2, median = 520.7, Q3 = 522 and IQR = 7.8
Brinnell Hardness: Q1 = 516, median = 534, Q3 = 580.4 and IQR =64.4

(f) The Dial Rockwell was most precise because the range was the minimum and the IQR was only slightly larger than the IQR for Digital.

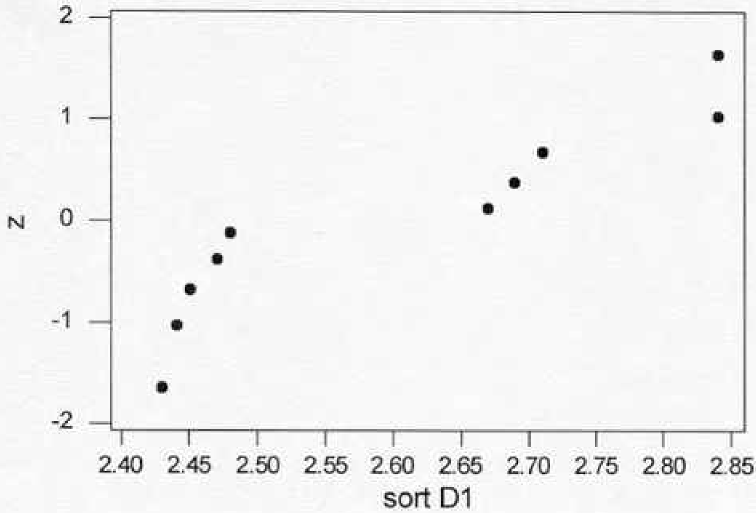
(g) No, the true hardness of the sample material is not known, so accuracy cannot be evaluated.

22(a) Q1 = 2.45 is the 3rd ranked data value because (i - .5)/10 = .25
m = .5(2.67 - 2.48) + 2.48 = 2.575
Q3 = 2.71 is the 8th ranked data value because (i - .5)/10 = .75

p = .62 implies the .62 quantile = .7(2.69 - 2.67) + 2.67 = 2.684

(b)

Normal Probability Plot for Design 1 Data



It is clear the data is not normally distributed, i.e., it is not bell-shaped. It appears the data come from two different populations. The 5 largest data values seem to come from a different population than the smallest 5 data values.

(c) $\bar{x} = 2.602$, s = .1662, R = 2.84 - 2.43 = .41 for Design 1 data.

(d)

D1		D2
8, 7, 5, 4, 3	24	
	25	
9, 7	26	7, 9
1	27	
4, 4	28	7
	29	
	30	
	31	
	32	9

33
34 0, 2, 7
35 0, 1, 3

(e) Design 1: Q1 = 2.45, m = 2.575, Q3 = 2.71, IQR = .26,
Range = 2.84 - 2.43 = .41

Design 2: Q1 = 2.87, m = 3.41, Q3 = 3.5, IQR = .63,
Range = 3.53 - 2.67 = .86.

(f) Design 1 has the most consistent results (minimum range, minimum IQR, minimum standard deviation). Design 2 has the longest flight times.

(g) If the object of the study was to identify a superior design, dropping 10 helicopters of each design once will identify a helicopter design that produces superior helicopters.

Chapter 4: Computationally Intensive Descriptive Statistics

Section 1

1. (a) The following table shows the necessary computations.

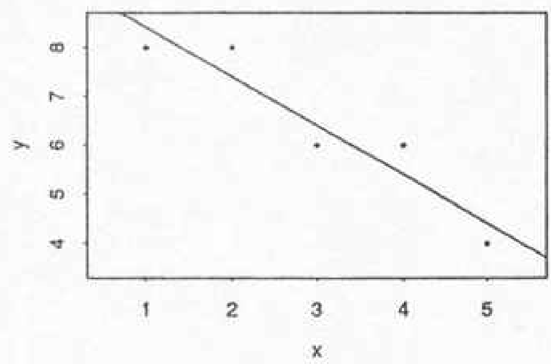
i	x_i	x_i^2	y_i	y_i^2	$x_i y_i$
1	1	1	8	64	8
2	2	4	8	64	16
3	3	9	6	36	18
4	4	16	6	36	24
5	5	25	4	16	20
	15	55	32	216	86

$$b_1 = \frac{\sum x_i y_i - \frac{(\sum x_i)(\sum y_i)}{n}}{\sum x_i^2 - \frac{(\sum x_i)^2}{n}} = \frac{86 - \frac{(15)(32)}{5}}{55 - \frac{(15)^2}{5}} = -1.0$$

$$b_0 = \bar{y} - b_1\bar{x} = \frac{32}{5} - (-1.0)\frac{15}{5} = 9.4$$

So the least squares equation is

$$\hat{y} = 9.4 - 1.0x.$$



(b)

$$\begin{aligned} r &= \frac{\sum x_i y_i - \frac{(\sum x_i)(\sum y_i)}{n}}{\sqrt{\left(\sum x_i^2 - \frac{(\sum x_i)^2}{n}\right)\left(\sum y_i^2 - \frac{(\sum y_i)^2}{n}\right)}} \\ &= \frac{86 - \frac{(15)(32)}{5}}{\sqrt{\left(55 - \frac{(15)^2}{5}\right)\left(216 - \frac{(32)^2}{5}\right)}} = -.945 \end{aligned}$$

(c) The necessary calculations are given in the table below.

i	x_i	y_i	y_i^2	$\hat{y}_i = 9.4 - 1.0x_i$	$\hat{y}_i^2$	$y_i\hat{y}_i$
1	1	8	64	8.4	70.56	67.2
2	2	8	64	7.4	54.76	59.2
3	3	6	36	6.4	40.96	38.4
4	4	6	36	5.4	29.16	32.4
5	5	4	16	4.4	19.36	17.6
		32	216	32	214.8	214.8

$$r = \frac{214.8 - \frac{(32)(32)}{5}}{\sqrt{\left(216 - \frac{(32)^2}{5}\right)\left(214.8 - \frac{(32)^2}{5}\right)}} = .945$$

This is the negative of the r in part (b). Since the $\hat{y}$'s are on the least squares line, they are perfectly negatively correlated with the x's. So the correlation between the $\hat{y}$'s and the y's is the same as the correlation between the x's and the y's, except for a difference in the sign.

(d) The calculations are in the table below.

i	y_i	$\bar{y}$	$\hat{y}_i$	$(y_i - \bar{y})$	$(y_i - \bar{y})^2$	$e_i = (y_i - \hat{y}_i)$	$(y_i - \hat{y}_i)^2$
1	8	6.4	8.4	1.6	2.56	−.4	.16
2	8	6.4	7.4	1.6	2.56	.6	.36
3	6	6.4	6.4	−.4	.16	−.4	.16
4	6	6.4	5.4	−.4	.16	.6	.36
5	4	6.4	4.4	−2.4	5.76	−.4	.16
					11.2		1.2

$$R^2 = \frac{\sum(y_i - \bar{y})^2 - \sum(y_i - \hat{y}_i)^2}{\sum(y_i - \bar{y})^2} = \frac{11.2 - 1.2}{11.2} = .893$$

This is equal to the square of sample correlation in both (b) and (c).

(e) The residuals e_i are given in the table in (d). These are the vertical distances from each data point to the least squares line.

2. The following printout was produced using Version 9.1 of Minitab.

```
MTB > info

Column   Name          Count
C1       y                 5
C2       x                 5

MTB > print c1-c2

 ROW    y    x

   1    8    1
   2    8    2
   3    6    3
   4    6    4
   5    4    5
```

```
MTB > gstd
* NOTE  * Standard Graphics are enabled.
          Professional Graphics are disabled.
          Use the GPRO command to enable Professional Graphics.
MTB > plot c1 c2

 y        -
          -   *             *
          -
      7.5+
          -
          -
          -
          -
      6.0+                               *            *
          -
          -
          -
          -
      4.5+
          -
          -                                                        *
          -
           +---------+---------+---------+---------+---------+------x
          0.80      1.60      2.40      3.20      4.00      4.80

 MTB > regress c1 on 1 x variable c2;
 SUBC> fits c3;
 SUBC> residuals c4.
                                              least squares line
 The regression equation is
 y = 9.40 - 1.00 x

 Predictor       Coef          Stdev     t-ratio         p
 Constant       9.4000  b0    0.6633       14.17     0.001
 x             -1.0000  b1    0.2000       -5.00     0.015

 s = 0.6325      R-sq = 89.3%      R-sq(adj) = 85.7%
                               R^2
 Analysis of Variance

 SOURCE        DF          SS          MS          F        p
 Regression     1      10.000      10.000      25.00    0.015
 Error          3       1.200       0.400
 Total          4      11.200

 MTB > name c3 'fits' c4 'resids'
 MTB > corr c1 c2

 Correlation of y and x = -0.945

 MTB > corr c1 c3

 Correlation of y and fits = 0.945
```

```
MTB > print c1-c4

 ROW    y    x   fits   resids

   1    8    1    8.4     -0.4
   2    8    2    7.4      0.6
   3    6    3    6.4     -0.4
   4    6    4    5.4      0.6
   5    4    5    4.4     -0.4
```

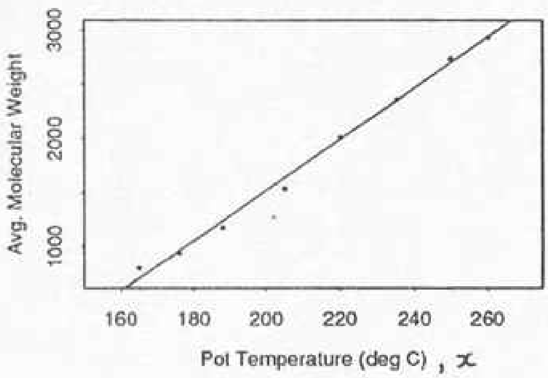
residuals

3. (a) $R^2 = .994$.

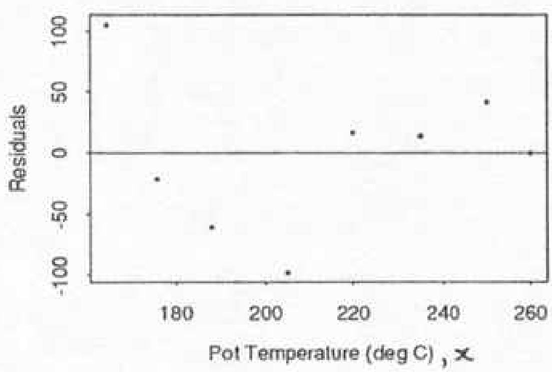
(b) The least squares equation is

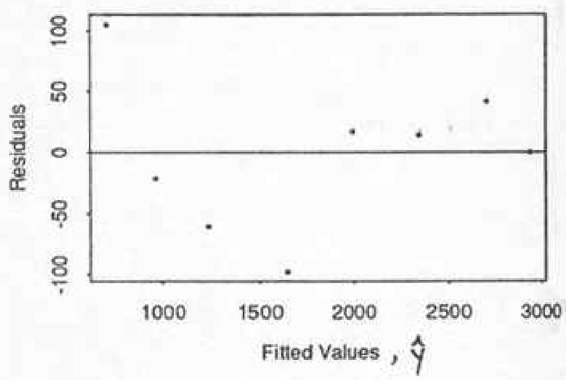
$$\hat{y} = -3174.6 + 23.5x.$$

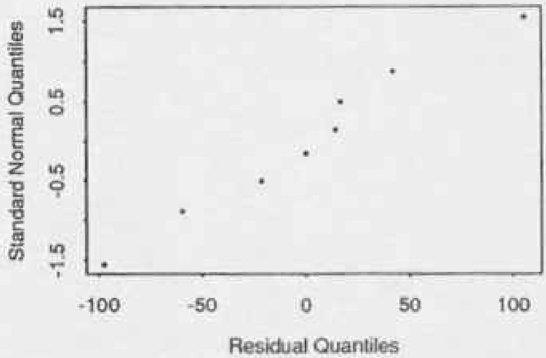
β_1 represents the "true" average change in molecular weight that accompanies a 1°C increase in pot temperature (assuming that a straight-line model is correct). $b_1 = 23.5$ is a data-based approximation of this value.



(c) The residuals are: 105.36, −21.13, −60.11, −97.58, 16.95, 14.48, 42.00, and .02.







It is difficult to evaluate the appropriateness of the fitted equation based on so little data. The plots of residuals versus x and residuals versus $\hat{y}$ do not contain any obvious patterns, and thus provide no evidence that the equation is inappropriate. The normal plot of residuals is fairly linear, providing no evidence that the residuals are not bell-shaped.

(d) There is no replication (multiple experimental runs at a particular pot temperature). Replication would validate any conclusions drawn from the experiment, and provide more information to confirm the appropriateness of the fitted equation.

(e) For $x = 188°$C,

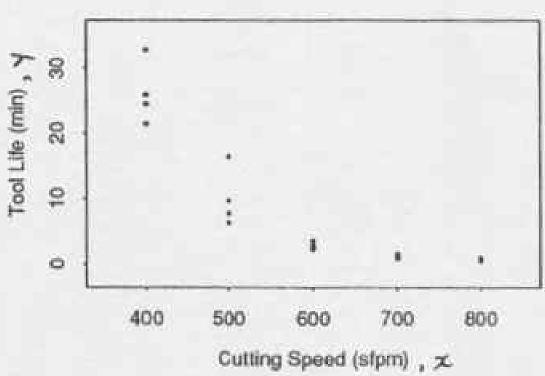
$$\hat{y} = -3174.6 + 23.5(188) = 1243.1.$$

For $x = 200°$C,

$$\hat{y} = -3174.6 + 23.5(200) = 1525.1.$$

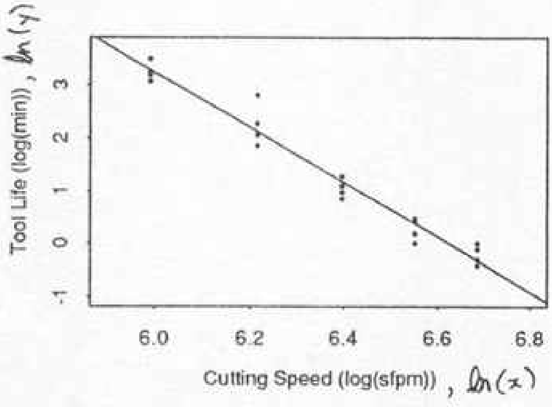
It would not be wise to make a similar prediction at $x = 70°$C because there is no evidence that the fitted relationship is correct for pot temperatures as low as $x = 70°$C. This would be an extrapolation. Some data should be obtained around $x = 70°$C before making such a prediction.

4. (a)



The scatterplot is not linear, so the given straight-line relationship does not seem appropriate. The least squares line is $\hat{y} = 44.075 - .059650x$. The corresponding R^2 is .723.

(b)



This scatterplot is much more linear, and a straight-line relationship seems appropriate for the transformed variables. The least squares line is $\widehat{\ln y} = 34.344 - 5.1857 \ln x$. The corresponding R^2 is .965.

(c) The least squares line is given in part (b). For $x = 550$,

$$\widehat{\ln y} = 34.344 - 5.1857 \ln(550) = 1.6229 \ln(\text{minutes}),$$

so $\hat{y} = e^{1.6229} = 5.07$ minutes. The implied relationship between x and y is

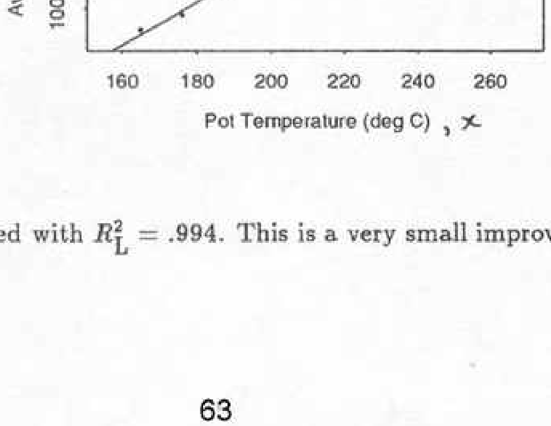
$$\begin{aligned} y &\approx e^{\beta_0 + \beta_1 \ln x} \\ y &\approx e^{\beta_0} e^{\ln x^{\beta_1}} \\ y &\approx e^{\beta_0} x^{\beta_1}. \end{aligned}$$

With slight rearrangement, this is the same as Taylor's equation for tool life.

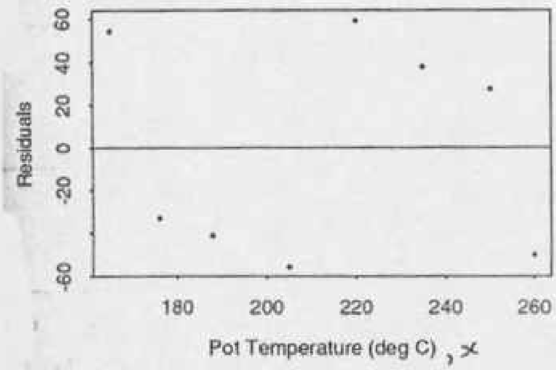
Section 2

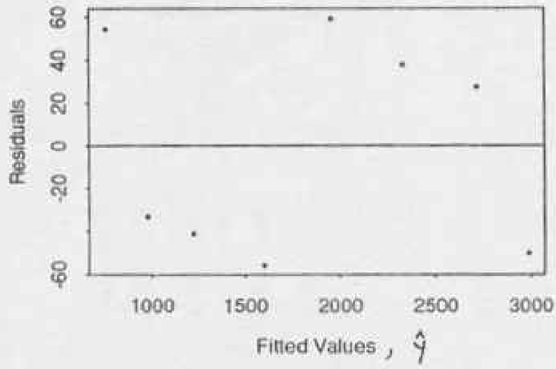
1. The least squares equation is

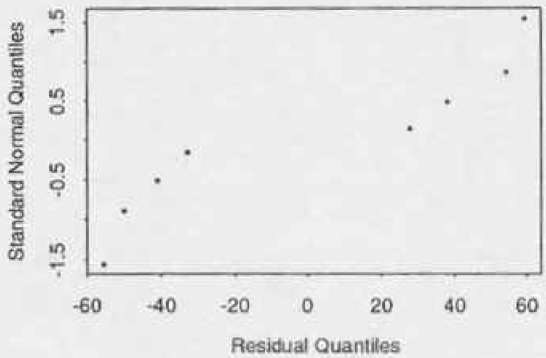
$$\hat{y} = -1315 + 5.59x + .04212x^2.$$



$R^2_{\text{Q}} = .996$, compared with $R^2_{\text{L}} = .994$. This is a very small improvement, at the cost of using a more complex equation.







The residuals here are smaller, as they will always be for a more complex model. There is no noticeable improvement in the residual plots, compared to those from the straight-line model. In fact, the residual plots for the quadratic model look more patterned. The scatterplot of y versus x also indicates that the quadratic model would be "overfitting" the data. The simpler straight-line relationship seems to be adequate.

For the quadratic model, at $x = 200°\text{C}$,

$$\hat{y} = -1315 + 5.59(200) + .04212(200)^2 = 1487.2,$$

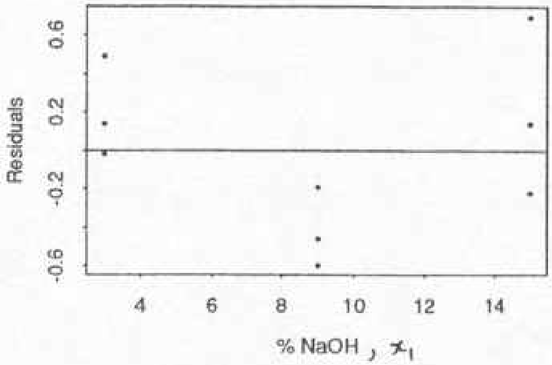
which is relatively close to 1525.1 from Ch. 4, Sec. 1, Ex. 3.

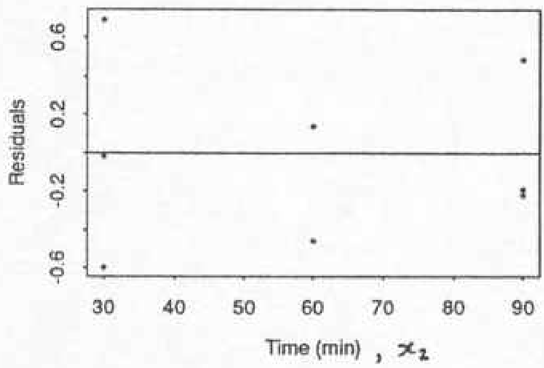
2. (a) The least squares equation is

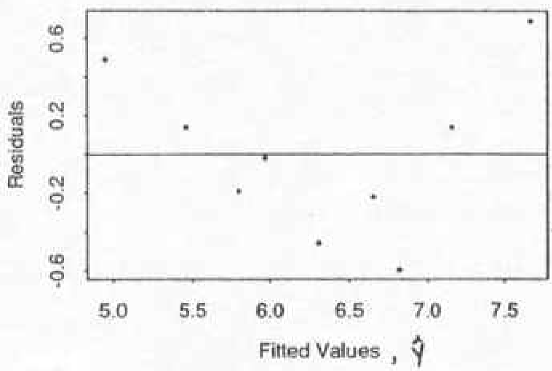
$$\hat{y} = 6.0483 + .14167x_1 - .016944x_2.$$

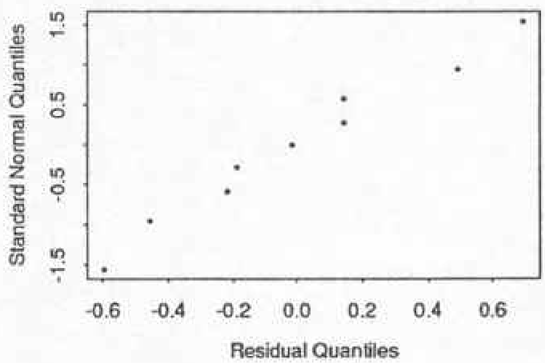
Assuming the fitted equation is appropriate, this means that as x_1 increases by 1% (holding x_2 constant), y increases by roughly .14167 cm^3/g. As x_2 increases by 1 minute (holding x_1 constant), y decreases by roughly .016944 cm^3/g. The R^2 corresponding to this equation is .807.

(b) The residuals are $-.015$, $.143$, $.492$, $-.595$, $-.457$, $-.188$, $.695$, $.143$, $-.218$.









Both the plots of residuals versus x_1 and residuals versus $\hat{y}$ show a positive-negative-positive pattern of residuals, indicating that the relationship between x_1 and y is not completely accounted for by the current model. These plots suggest adding an x_1^2 term. The plot of residuals versus x_2 is patternless; x_2 seems to be well represented. The normal plot of residuals is fairly linear, indicating that the residuals are bell-shaped.

(c) For $x_2 = 30$, the equation is

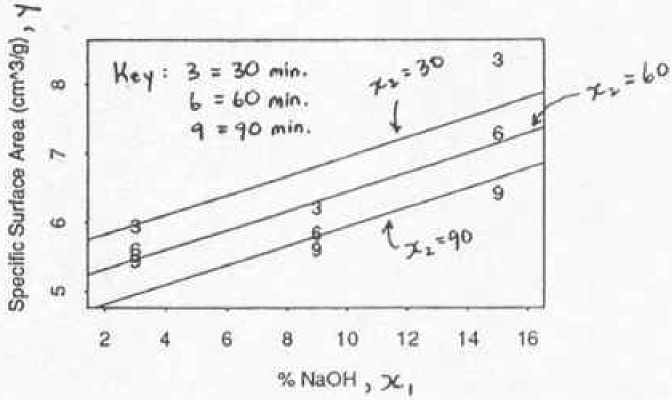
$$\begin{aligned}\hat{y} &= 6.0483 + .14167x_1 - .016944(30) \\ &= 5.53998 + .14167x_1.\end{aligned}$$

For $x_2 = 60$, the equation is

$$\begin{aligned}\hat{y} &= 6.0483 + .14167x_1 - .016944(60) \\ &= 5.03166 + .14167x_1.\end{aligned}$$

For $x_2 = 90$, the equation is

$$\begin{aligned}\hat{y} &= 6.0483 + .14167x_1 - .016944(90) \\ &= 4.52334 + .14167x_1.\end{aligned}$$



The fitted responses do not match up well, because the relationship between y and x_1 (%NaOH) is not linear for any of the x_2 values (Time).

(d) At $x_1 = 10\%$ and $x_2 = 70$ minutes,

$$\hat{y} = 6.0483 + .14167(10) - .016944(70) = 6.279 \text{ cm}^3/\text{g}.$$

It would not be wise to make a similar prediction at $x_1 = 10\%$ and $x_2 = 120$ minutes because there is no evidence that the fitted relationship is correct under these conditions. This would be extrapolating. Some data should be obtained around $x_1 = 10\%$ and $x_2 = 120$ minutes before making such a prediction.

(e) The least squares equation is

$$\hat{y} = 4.9833 + .260x_1 + .00081x_2 - .001972x_1x_2,$$

and the corresponding R^2 is .876. The increase in R^2 from .807 to .876 is not very large; using the more complicated equation may not be desirable (this is subjective). Residual plots for this more complicated equation should also be examined before evaluating its appropriateness.

(f) For $x_2 = 30$, the equation is

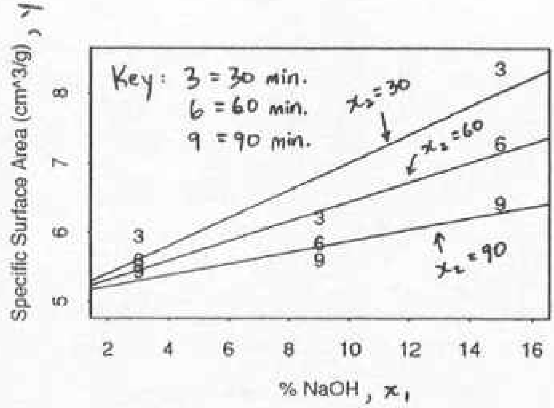
$$\begin{aligned}\hat{y} &= 4.9833 + .260x_1 + .00081(30) - .001972x_1(30)\\ &= 5.0076 + .20084x_1.\end{aligned}$$

For $x_2 = 60$, the equation is

$$\begin{aligned}\hat{y} &= 4.9833 + .260x_1 + .00081(60) - .001972x_1(60)\\ &= 5.0319 + .14168x_1.\end{aligned}$$

For $x_2 = 90$, the equation is

$$\begin{aligned}\hat{y} &= 4.9833 + .260x_1 + .00081(90) - .001972x_1(90)\\ &= 5.0562 + .08252x_1.\end{aligned}$$



The new model allows there to be a different slope for different values of x_2, so these lines fit the data better than the lines in part (c). But they still do not account for the non-linearity between x_1 and y. An equation with an x_1^2 term would fit much better.

(g) There is no replication (multiple experimental runs at a particular NaOH-Time combination). Replication would validate any conclusions drawn from the experiment, and it would allow for better comparisons among the different possible fitted equations.

(h) This data has a complete (full) factorial structure. The straight-line least squares equation for x_1 is

$$\hat{y} = 5.0317 + .14167x_1$$

with a corresponding R^2 of .594. The straight-line least squares equation for x_2 is

$$\hat{y} = 7.3233 - .01694x_2$$

with a corresponding R^2 of .212. The slopes in these one-variable linear equations are the same as the corresponding slopes in the two-variable equation from (a). The R^2 value in (a) is the sum of the R^2 values from the two one-variable linear equations.

Section 3 1. (a) The averages needed are given in the table below.

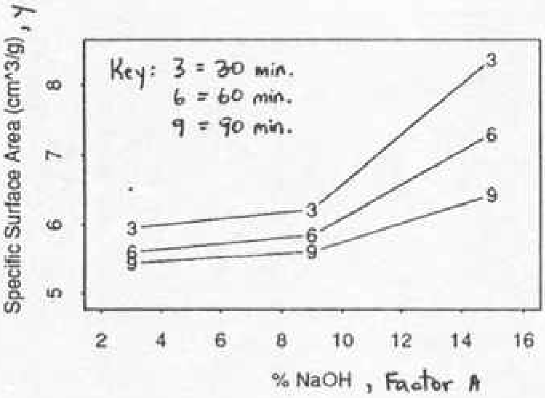
		TIME (Factor B) 30	60	90	
% NaOH (Factor A)	3.0	$\bar{y}_{11} = 5.95$	$\bar{y}_{12} = 5.60$	$\bar{y}_{13} = 5.44$	$\bar{y}_{1.} = 5.663$
	9.0	$\bar{y}_{21} = 6.22$	$\bar{y}_{22} = 5.85$	$\bar{y}_{23} = 5.61$	$\bar{y}_{2.} = 5.893$
	15.0	$\bar{y}_{31} = 8.36$	$\bar{y}_{32} = 7.30$	$\bar{y}_{33} = 6.43$	$\bar{y}_{3.} = 7.363$
		$\bar{y}_{.1} = 6.843$	$\bar{y}_{.2} = 6.250$	$\bar{y}_{.3} = 5.827$	$\bar{y}_{..} = 6.307$

The fitted main effects are

$$a_1 = \bar{y}_{1.} - \bar{y}_{..} = -.643$$
$$a_2 = \bar{y}_{2.} - \bar{y}_{..} = -.413$$
$$a_3 = \bar{y}_{3.} - \bar{y}_{..} = 1.057$$
$$b_1 = \bar{y}_{.1} - \bar{y}_{..} = .537$$
$$b_2 = \bar{y}_{.2} - \bar{y}_{..} = -.057$$
$$b_3 = \bar{y}_{.3} - \bar{y}_{..} = -.480$$

The fitted interactions are

$$
\begin{aligned}
ab_{11} &= \bar{y}_{11} - (\bar{y}_{..} + a_1 + b_1) = -.250 \\
ab_{12} &= \bar{y}_{12} - (\bar{y}_{..} + a_1 + b_2) = -.007 \\
ab_{13} &= \bar{y}_{13} - (\bar{y}_{..} + a_1 + b_3) = .257 \\
ab_{21} &= \bar{y}_{21} - (\bar{y}_{..} + a_2 + b_1) = -.210 \\
ab_{22} &= \bar{y}_{22} - (\bar{y}_{..} + a_2 + b_2) = .013 \\
ab_{23} &= \bar{y}_{23} - (\bar{y}_{..} + a_2 + b_3) = .197 \\
ab_{31} &= \bar{y}_{31} - (\bar{y}_{..} + a_3 + b_1) = .460 \\
ab_{32} &= \bar{y}_{32} - (\bar{y}_{..} + a_3 + b_2) = -.007 \\
ab_{33} &= \bar{y}_{33} - (\bar{y}_{..} + a_3 + b_3) = -.453.
\end{aligned}
$$



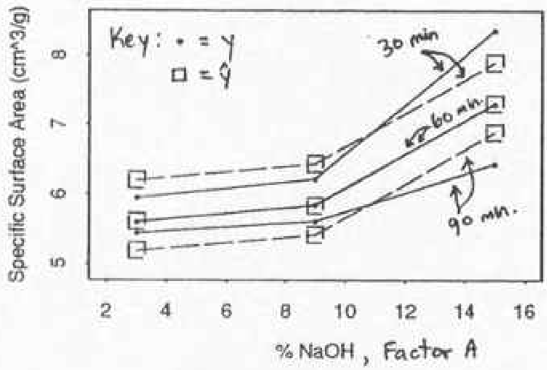
The fitted interactions ab_{31} and ab_{33} are large (relative to the fitted main effects) indicating that the effect on y of changing NaOH from 9% to 15% depends on the Time (non-parallelism in the plot). The a's are somewhat larger than the b's, indicating that Time has a slightly smaller overall effect than %NaOH. Overall, increasing Time decreases the specific surface area and increasing % NaOH increases the specific surface area. However, in each case the size of the change depends on the level of the other factor. It would not be wise to use the fitted main effects alone to summarize the data, since there may be an importantly large interaction between the two factors.

(b) For the factor-level combination with Factor A at level i and Factor B at level j, the fitted/predicted response for a "main effects only" model is computed as

$$\hat{y} = \bar{y}_{..} + a_i + b_j$$

The 9 fitted/predicted responses are given below.

$$
\begin{aligned}
\hat{y}_{11} &= \bar{y}_{..} + a_1 + b_1 = 6.20 \\
\hat{y}_{12} &= \bar{y}_{..} + a_1 + b_2 = 5.61 \\
\hat{y}_{13} &= \bar{y}_{..} + a_1 + b_3 = 5.18 \\
\hat{y}_{21} &= \bar{y}_{..} + a_2 + b_1 = 6.43 \\
\hat{y}_{22} &= \bar{y}_{..} + a_2 + b_2 = 5.84 \\
\hat{y}_{23} &= \bar{y}_{..} + a_2 + b_3 = 5.41 \\
\hat{y}_{31} &= \bar{y}_{..} + a_3 + b_1 = 7.90 \\
\hat{y}_{32} &= \bar{y}_{..} + a_3 + b_2 = 7.31 \\
\hat{y}_{33} &= \bar{y}_{..} + a_3 + b_3 = 6.88
\end{aligned}
$$



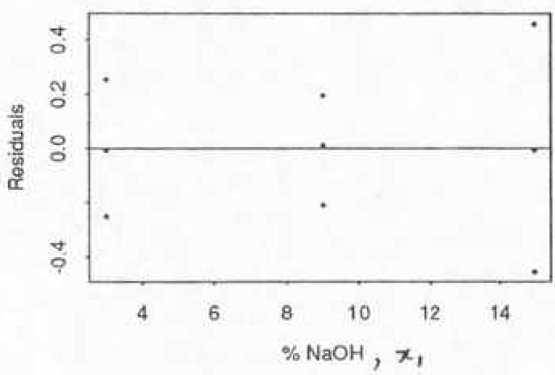
Like the plot in Chapter 4, section 2, problem 2 the fitted values for each level of B (x_2) must be parallel; no interactions are allowed. However, unlike Ch. 4, sec. 2, prob. 2, the current model allows these fitted values to be non-linear in x_1. Factorial models are generally more flexible than lines, curves, and surfaces.

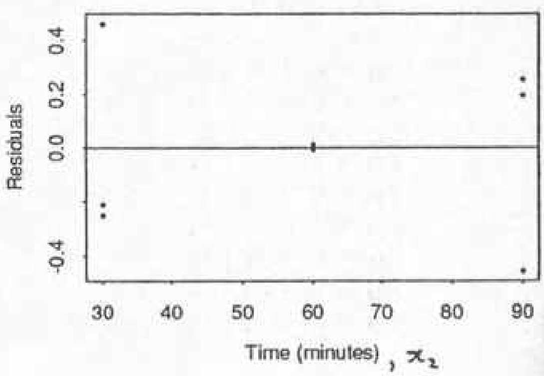
(c) The computations are given in the table below. (Note: $\bar{y}$ represents the average of all observations. It is equal to $\bar{y}_{..}$ only because all the sample sizes are equal.)

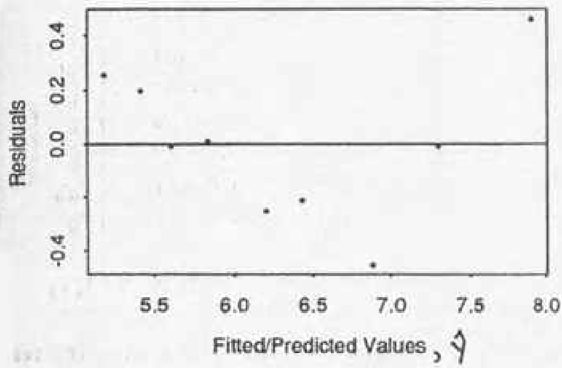
i	y_i	$\bar{y}$	$\hat{y}_i$	$(y_i - \bar{y})$	$(y_i - \bar{y})^2$	$e_i = (y_i - \hat{y}_i)$	$(y_i - \hat{y}_i)^2$
1	5.95	6.307	6.200	−.357	.127	−.250	.063
2	5.60	6.307	5.607	−.707	.499	−.007	.000
3	5.44	6.307	5.183	−.867	.751	.257	.066
4	6.22	6.307	6.430	−.087	.008	−.210	.044
5	5.85	6.307	5.837	−.457	.209	.013	.000
6	5.61	6.307	5.413	−.697	.485	.197	.039
7	8.36	6.307	7.900	2.053	4.216	.460	.212
8	7.30	6.307	7.307	.993	.987	−.007	.000
9	6.43	6.307	6.883	.123	.015	−.453	.206
					7.2972		.6285

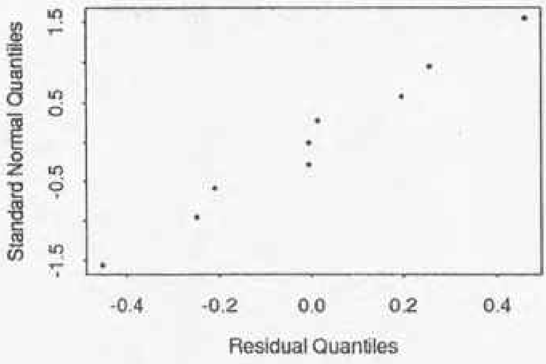
$$R^2 = \frac{\sum(y_i - \bar{y})^2 - \sum(y_i - \hat{y}_i)^2}{\sum(y_i - \bar{y})^2} = \frac{7.2972 - .6285}{7.2972} = .914.$$

The residuals $e_i = y_i - \hat{y}_i$ are given in the table above.









The plots of residuals versus Time and residuals versus $\hat{y}_i$ both have patterns; these show that the "main effects only" model is not accounting for the apparent interaction between the two factors. Even though R^2 is higher than both of the models in 4-8, this model does not seem to be adequate.

2. (a) Using the Yates algorithm:

Combination	$\bar{y}$	Cycle 1	Cycle 2	Cycle 3	Cycle 3 ÷ 8	
(1)	21.0100	42.8333	54.843	166.337	20.7921	$= \bar{y}_{...}$
a	21.8233	12.0100	111.493	.903	.1129	$= a_2$
b	6.0067	95.5633	.810	−110.457	−13.8071	$= b_2$
ab	6.0033	15.9300	.093	−.690	−.0863	$= ab_{22}$
c	47.7900	.8133	−30.823	56.650	7.0812	$= c_2$
ac	47.7733	−.0033	−79.633	−.717	−.0896	$= ac_{22}$
bc	7.9100	−.0167	−.817	−48.810	−6.1012	$= bc_{22}$
abc	8.0200	.1100	.127	.943	.1179	$= abc_{222}$

Other fitted effects can be obtained by appropriately changing the signs of the fitted effects in the last column. Since b_2, c_2, and bc_{22} are relatively large, the simplest possible interpretation is that Diameter, Fluid, and their interaction are the only effects on Time. Generally, the .314 diameter (B +) results in shorter times than the .188 diameter (B −), and this is reflected by the negative sign of b_2. Also, ethylene glycol (C +) results in longer times than water (C −), and this is reflected by the positive sign of c_2. The negative sign of bc_{22} indicates that the decrease in time due to changing the diameter from .188 to .314 is smaller for water than it is for ethylene glycol. All of these observations are consistent with simple graphical summaries of the sample means.

(b)

Combination	$\bar{y}$	Cycle 1	Cycle 2	Cycle 3	Cycle 3 ÷ 8	
(1)	3.04497	6.12795	9.7130	21.5936	2.69920	$= \bar{y}_{...}$
a	3.08298	3.58506	11.8806	.0507	.00634	$= a_2$
b	1.79282	7.73100	.0374	−6.1243	−.76554	$= b_2$
ab	1.79224	4.14960	.0133	−.0251	−.00314	$= ab_{22}$
c	3.86554	.03801	−2.5429	2.1676	.27095	$= c_2$
ac	3.86547	−.00058	−3.5814	−.0241	−.00301	$= ac_{22}$
bc	2.06811	−.00007	−.0386	−1.0385	−.12981	$= bc_{22}$
abc	2.08149	.01338	.0134	.0520	.00650	$= abc_{222}$

Yes, but the Diameter×Fluid interaction still seems to be important.

(c) Using the reverse Yates algorithm:

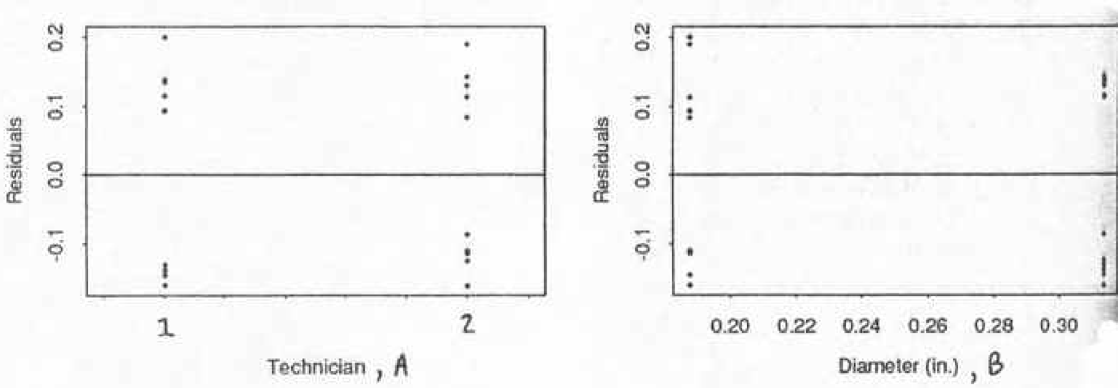
Fitted Effect	Value	Cycle 1	Cycle 2	Cycle 3 ($\hat{y}$)	
abc_{222}	0	0	.27095	2.20461	$= \hat{y}_{abc}$
bc_{22}	0	.27095	1.93366	2.20461	$= \hat{y}_{bc}$
ac_{22}	0	−.76554	.27095	3.73569	$= \hat{y}_{ac}$
c_2	.27095	2.69920	1.93366	3.73569	$= \hat{y}_{c}$
ab_{22}	0	0	.27095	1.66271	$= \hat{y}_{ab}$
b_2	−.76554	.27095	3.46474	1.66271	$= \hat{y}_{b}$
a_2	0	−.76554	.27095	3.19379	$= \hat{y}_{a}$
$\bar{y}_{...}$	2.69920	2.69920	3.46474	3.19379	$= \hat{y}_{(1)}$

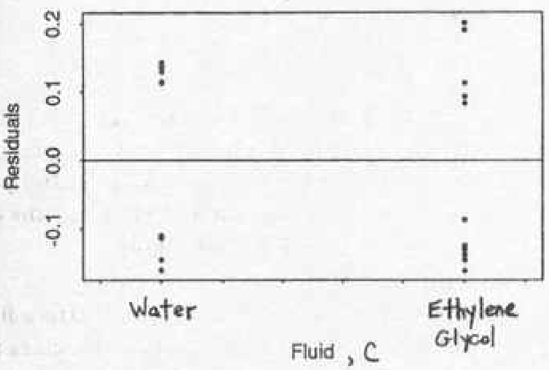
There will be a total of 24 residuals, one for each observation. To compute the residuals, take each (transformed) observation and subtract the $\hat{y}$ that corresponds to the factor-level combination from which the observation came. For example, $\hat{y}_{(1)} = 3.19379$ should be subtracted from the natural logs of each of the 3 observations from combination (1), 21.12, 21.11, and 20.80, producing the 3 residuals −.143569, −.110963, and .138995.

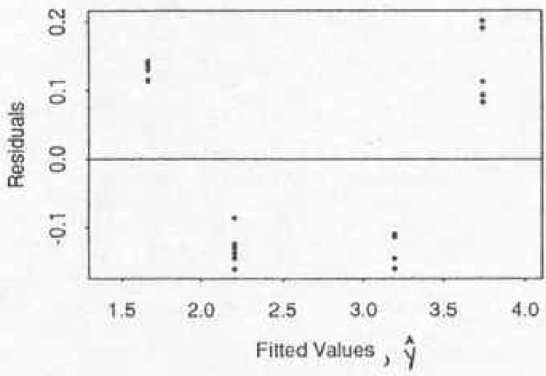
$\bar{y}$ = the average of all 24 observations = 2.69920. (This is equal to $\bar{y}_{...}$ in this case because the data are balanced—all sample sizes are equal.) Use this and the 24 residuals e_i to compute R^2:

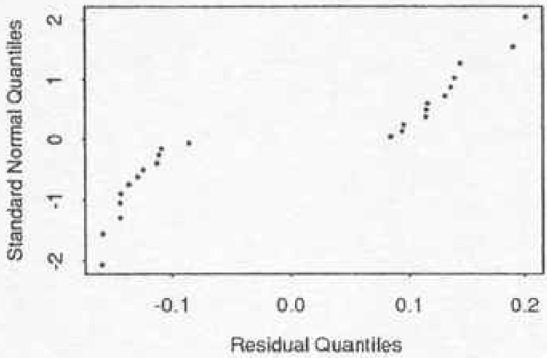
$$
\begin{aligned}
R^2 &= \frac{\sum(y_i - \bar{y})^2 - \sum(y_i - \hat{y}_i)^2}{\sum(y_i - \bar{y})^2} \\
&= \frac{\sum(y_i - \bar{y})^2 - \sum e_i^2}{\sum(y_i - \bar{y})^2} \\
&= \frac{16.25126 - .424197}{16.25126} = .974
\end{aligned}
$$

A model with all factorial effects is a "saturated" model. The fitted/predicted values for this model will exactly match the $\bar{y}$'s for each factor-level combination. The residuals for this model are just the differences between each observation and the sample mean from its combination. The resulting R^2 is .995.









The plots of Residuals versus Technician, Diameter, and Fluid all show that there is a gap in the residuals; there are no residuals near zero. The plot of residuals versus $\hat{y}$ shows a positive-negative-positive pattern. All of these plots show that the current model is inadequate (even though its R^2 is high). It does not account for the apparent interaction between Diameter and Fluid.

(d) If you believe that there are no interactions, there is an approximate $b_1 - b_2 = 1.532$ ln(sec) decrease in log drain time. The change in raw drain time is then a multiplicative change. You would need to divide the .188 raw drain time by $e^{1.532}$ to get the .314 raw drain time. This suggests that (.188 drain time/.314 drain time) $= e^{1.532} = 4.63$; the theory predicts this ratio to be

$$\frac{\frac{1}{(\frac{.188}{2})^4}}{\frac{1}{(\frac{.314}{2})^4}} = 7.78.$$

3. Interpolation, and possibly some cautious extrapolation, is only possible using surface-fitting methods. In many engineering situations, an "optimal" setting of quantitative factors is sought. This can be facilitated by interpolation (or extrapolation) using a surface-fitting model.

Section 4

1. Transforming data can sometimes make relationships among variables simpler. Sometimes non-linear relationships can be made linear, or factors and response can be transformed so that there are no interactions among the factors. Transformations can also potentially make the shape of a distribution simpler, allowing the use of statistical models that assume a particular distributional shape (such as the bell-shaped normal distribution).

2. In terms of the raw response, there will be interactions, since x_1 and x_2 are multiplied together in the power law. The suggested plot of raw y versus x_1 will have different slopes for different values of x_2. This means that the effect of changing x_1 *depends* on the setting of x_2, which is one way to define an interaction.

   In terms of the log of y, there will not be interactions, since x_1 and x_2 appear additively in the equation for $\ln y$. Therefore, the suggested plot of $\ln y$ versus x_1 will have the same slope for all values of x_2. This means that the effect of changing x_1 does not depend on the setting of x_2 (there are no interactions).

Section 5

1. A deterministic model is used to describe a situation where the outcome can be almost exactly predicted if certain variables are known. A stochastic/probabilistic model is used in situations where it is not possible to predict the exact outcome. This may happen when important variables are unknown, or when no known deterministic theory can describe the situation.

   An example of a deterministic model is the classical Economic Order Quantity (EOQ) model for inventory control. Given constant rate of demand R, order quantity X, ordering cost P,

   and per unit holding cost C, the total cost per time period is

$$Y = P\left(\frac{R}{X}\right) + C\left(\frac{X}{2}\right)$$

End Chapter Exercises

1. (a) The following table shows the necessary computations.

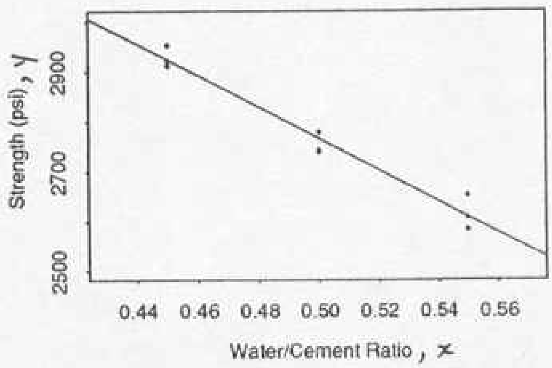
i	x_i	x_i^2	y_i	y_i^2	x_iy_i
1	.45	.2025	2954	8726116	1329.30
2	.45	.2025	2913	8485569	1310.85
3	.45	.2025	2923	8543929	1315.35
4	.50	.2500	2743	7524049	1371.50
5	.50	.2500	2779	7722841	1389.50
6	.50	.2500	2739	7502121	1369.50
7	.55	.3025	2652	7033104	1458.60
8	.55	.3025	2607	6796449	1433.85
9	.55	.3025	2583	6671889	1420.65
	4.5	2.265	24893	69006067	12399.1

$$b_1 = \frac{\sum x_iy_i - \frac{(\sum x_i)(\sum y_i)}{n}}{\sum x_i^2 - \frac{(\sum x_i)^2}{n}} = \frac{12399.1 - \frac{(4.5)(24893)}{9}}{2.265 - \frac{(4.5)^2}{9}} = -3160$$

$$b_0 = \bar{y} - b_1\bar{x} = \frac{24893}{9} - (-3160)\frac{4.5}{9} = 4345.889$$

So the least squares equation is

$$\hat{y} = 4345.889 - 3160x.$$



(b)

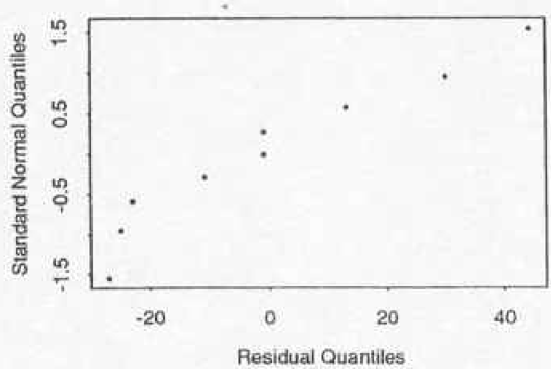
$$r = \frac{\sum x_iy_i - \frac{(\sum x_i)(\sum y_i)}{n}}{\sqrt{\left(\sum x_i^2 - \frac{(\sum x_i)^2}{n}\right)\left(\sum y_i^2 - \frac{(\sum y_i)^2}{n}\right)}}$$

$$= \frac{12399.1 - \frac{(4.5)(24893)}{9}}{\sqrt{\left(2.265 - \frac{(4.5)^2}{9}\right)\left(69006067 - \frac{(24893)^2}{9}\right)}} = -.984$$

Since r is negative and close to -1, there is a strong negative linear relationship between Water/Cement Ratio and 14-Day Compressive Strength.

(c) Since this is a straight-line model, $R^2 = r^2 = .968$.

(d) The residuals are 30.11, −10.89, −.089, −22.89, 13.11, −26.89, 44.11, −.89, and −24.90.



The normal plot of residuals is fairly linear; this implies that the residuals are roughly bell-shaped. There are no outliers.

(e) For $x = .48$,

$$\hat{y} = 4345.889 - 3160(.48) = 2829.09 \text{ psi.}$$

(f) The following printout was produced using Version 9.1 of Minitab.

```
MTB > print c1 c2

 ROW  Strength  Ratio

   1      2954   0.45
   2      2913   0.45
   3      2923   0.45
   4      2743   0.50
   5      2779   0.50
   6      2739   0.50
   7      2652   0.55
   8      2607   0.55
   9      2583   0.55

MTB > regress c1 on 1 x variable c2;
SUBC> fits c3;
SUBC> resids c4.
                                                  least squares line
The regression equation is
Strength = 4346 - 3160 Ratio

Predictor         Coef        Stdev     t-ratio        p
Constant        4345.9  b0    109.6       39.66    0.000
Ratio          -3160.0  b1    218.5      -14.47    0.000

s = 26.76         R-sq = 96.8%       R-sq(adj) = 96.3%
                           R^2
Analysis of Variance

SOURCE       DF        SS          MS          F          p
Regression    1    149784      149784     209.24      0.000
Error         7      5011         716
Total         8    154795
```

```
MTB > name c3 'fits' c4 'resids'
MTB > corr c1 c2

Correlation of Strength and Ratio = -0.984
```

r

```
MTB > print c1-c4
```

ROW	Strength	Ratio	fits	resids
1	2954	0.45	2923.89	30.1111
2	2913	0.45	2923.89	-10.8889
3	2923	0.45	2923.89	-0.8889
4	2743	0.50	2765.89	-22.8889
5	2779	0.50	2765.89	13.1111
6	2739	0.50	2765.89	-26.8889
7	2652	0.55	2607.89	44.1111
8	2607	0.55	2607.89	-0.8889
9	2583	0.55	2607.89	-24.8889

residuals

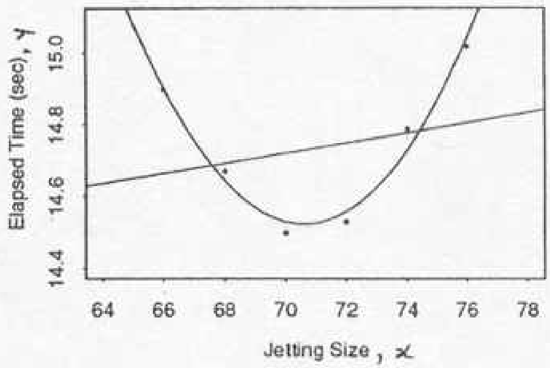
2. (a) There is no replication (multiple experimental runs at a particular jetting size). Replication would validate any conclusions drawn from the experiment, and provide more information to confirm the appropriateness of any particular fitted equation.

(b) The least squares line is

$$\hat{y}_{\mathrm{L}} = 13.731 + .01414x.$$

The least squares quadratic equation is

$$\hat{y}_{\mathrm{Q}} = 103.989 - 2.5343x + .017946x^2.$$



(c) $R^2_{\mathrm{L}} = .066$; $R^2_{\mathrm{Q}} = .969$.

(d) You want to *minimize* the least squares quadratic equation with respect to x. To do this, take the derivative with respect to x, set it equal to zero, and solve for x:

$$\begin{aligned} \frac{d\hat{y}_{\mathrm{Q}}}{dx} &= 0 \\ -2.5343 + 2(.017946)x &= 0 \end{aligned}$$

So $x_{\mathrm{opt}} = 70.61$. Note that the data point corresponding to $x = 70$ has a smaller actual time than the $\hat{y}$ corresponding to x_{opt}. (This can be seen in the plot.) More data should be obtained to validate the relationship.

3. (a) This data has a complete (full) factorial data structure. There is no replication (multiple experimental runs at a particular Temp-Time combination). Replication would validate any conclusions drawn from the experiment, and provide more information to confirm the appropriateness of any particular fitted equation.

(b) For the first equation,

$$\hat{y} = -515.15 + .35667x_1 + .01069x_2$$

with corresponding $R^2 = .886$ For the second equation,

$$\hat{y} = -528.46 + .35667x_1 + 3.711 \ln x_2$$

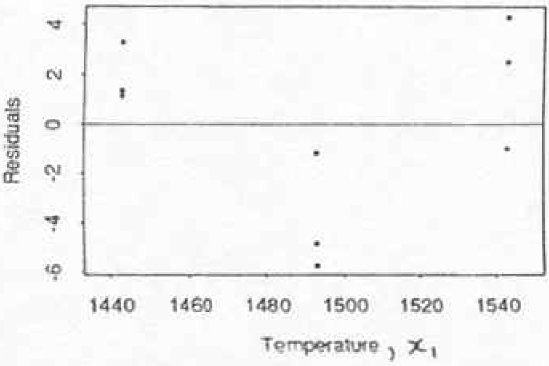
with corresponding $R^2 = .889$. For the third equation,

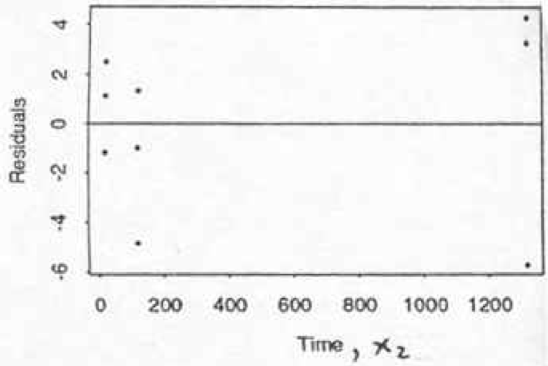
$$\hat{y} = -42.4 + .0311x_1 - 93.72 \ln x_2 + .06526x_1 \ln x_2$$

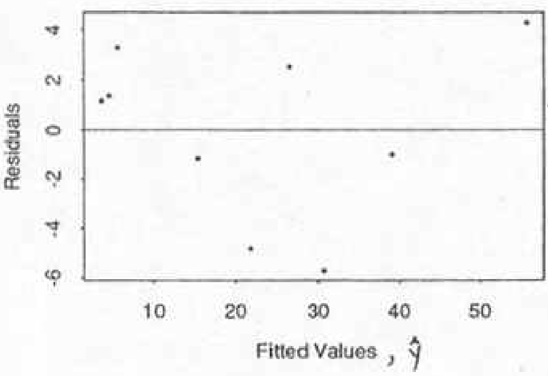
with corresponding $R^2 = .962$. The 1st equation is the least complex, followed by the 2nd and then the 3rd equation. The 3rd equation's R^2 value is much larger than the first two;

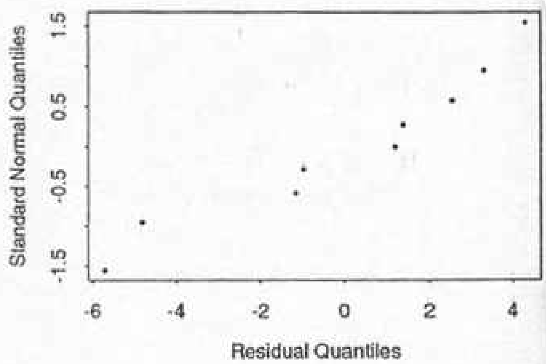
because of the large size of this increase, it seems that the 3rd equation would predict y better even though it is more complex.

(c) The residuals are 1.17078, 1.36828, 3.29428, −1.15724, −4.80582, −5.70360 2.51475, −.97992, and 4.29851









Both the plots of Residuals versus x_1 and Residuals versus $\hat{y}$ show a positive-negative-positive pattern, indicating that the relationship between x_1 and y is not completely accounted for by the third equation. These plots suggest adding an x_1^2 term. The plot of Residuals versus $\ln x_2$ is patternless; x_2 seems to be well-represented. The normal plot of residuals is fairly linear, indicating that the residuals are bell-shaped. There are no outliers.

(d) For $x_1 = 1443$, the equation is

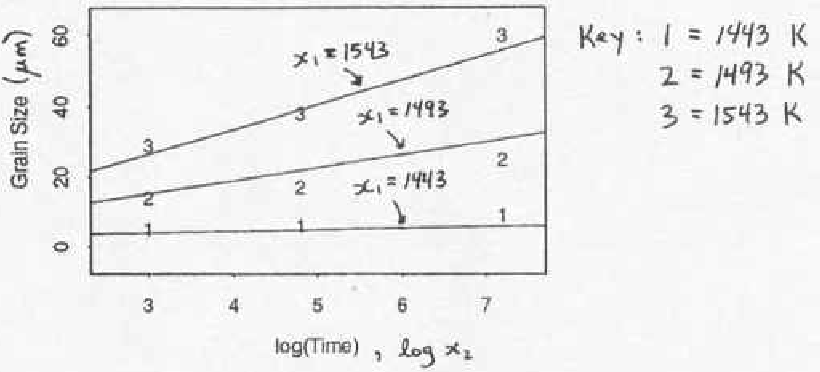
$$\hat{y} = 2.4773 + .45018 \ln x_2.$$

For $x_1 = 1493$, the equation is

$$\hat{y} = 4.0323 + 3.71318 \ln x_2.$$

For $x_1 = 1543$, the equation is

$$\hat{y} = 5.5873 + 6.97618 \ln x_2.$$

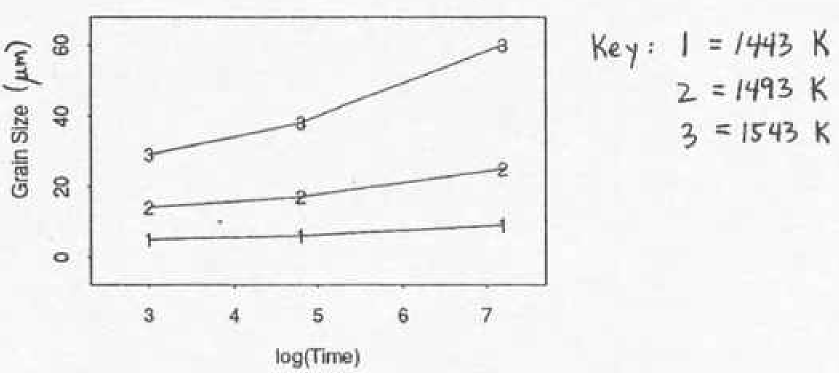


The 3rd equation allows there to be a different slope for different values of x_1; a similar plot for the 2nd equation would have parallel lines because the 2nd equation does not allow for different slopes.

(e) For $x_1 = 1500$ and $x_2 = 500$,

$$\hat{y} = -42.4 + .0311(1500) - 93.72\ln(500) + .06526(1500)(\ln(500)) = 30.16\ \mu\text{m}.$$

(f)



The non-parallelism in the plot indicates that there may be an interaction between Temperature and Time. The effect on y of changing Temperature seems to depend on the Time. It would not be wise to use the fitted main effects alone to summarize the data, since there may be an importantly large interaction.

The averages needed are given in the table below.

		TIME (Factor B) 20	120	1320	
TEMPERATURE (Factor A)	1443	$\bar{y}_{11} = 5$	$\bar{y}_{12} = 6$	$\bar{y}_{13} = 9$	$\bar{y}_{1.} = 6.67$
	1493	$\bar{y}_{21} = 14$	$\bar{y}_{22} = 17$	$\bar{y}_{23} = 25$	$\bar{y}_{2.} = 18.67$
	1543	$\bar{y}_{31} = 29$	$\bar{y}_{32} = 38$	$\bar{y}_{33} = 60$	$\bar{y}_{3.} = 42.33$
		$\bar{y}_{.1} = 16.0$	$\bar{y}_{.2} = 20.33$	$\bar{y}_{.3} = 31.33$	$\bar{y}_{..} = 22.56$

The fitted main effects are

$a_1 = \bar{y}_{1\cdot} - \bar{y}_{\cdot\cdot} = -15.89$
$a_2 = \bar{y}_{2\cdot} - \bar{y}_{\cdot\cdot} = -3.89$
$a_3 = \bar{y}_{3\cdot} - \bar{y}_{\cdot\cdot} = 19.78$
$b_1 = \bar{y}_{\cdot 1} - \bar{y}_{\cdot\cdot} = -6.56$
$b_2 = \bar{y}_{\cdot 2} - \bar{y}_{\cdot\cdot} = -2.22$
$b_3 = \bar{y}_{\cdot 3} - \bar{y}_{\cdot\cdot} = 8.78$

The fitted interactions are

$$
\begin{aligned}
ab_{11} &= \bar{y}_{11} - (\bar{y}_{\cdot\cdot} + a_1 + b_1) = 4.89\\
ab_{12} &= \bar{y}_{12} - (\bar{y}_{\cdot\cdot} + a_1 + b_2) = 1.56\\
ab_{13} &= \bar{y}_{13} - (\bar{y}_{\cdot\cdot} + a_1 + b_3) = -6.44\\
ab_{21} &= \bar{y}_{21} - (\bar{y}_{\cdot\cdot} + a_2 + b_1) = 1.89\\
ab_{22} &= \bar{y}_{22} - (\bar{y}_{\cdot\cdot} + a_2 + b_2) = .56\\
ab_{23} &= \bar{y}_{23} - (\bar{y}_{\cdot\cdot} + a_2 + b_3) = -2.44\\
ab_{31} &= \bar{y}_{31} - (\bar{y}_{\cdot\cdot} + a_3 + b_1) = -6.78\\
ab_{32} &= \bar{y}_{32} - (\bar{y}_{\cdot\cdot} + a_3 + b_2) = -2.11\\
ab_{33} &= \bar{y}_{33} - (\bar{y}_{\cdot\cdot} + a_3 + b_3) = 8.89.
\end{aligned}
$$

4. (a) There is no replication (multiple experimental runs at a particular factor-level combination). Replication would validate any conclusions drawn from the experiment, and provide more information to confirm the appropriateness of any particular model.

(b) Using the Yates algorithm:

Combination	$\bar{y}$	Cycle 1	Cycle 2	Cycle 3	Cycle 3 ÷ 8	
(1)	6.7	18.6	43.6	88.1	11.0125	$= \bar{y}_{\cdots}$
a	11.9	25.0	44.5	31.7	3.9625	$= a_2$
b	8.5	21.4	13.2	8.1	1.0125	$= b_2$
ab	16.5	23.1	18.5	3.7	.4625	$= ab_{22}$
c	6.3	5.2	6.4	.9	.1125	$= c_2$
ac	15.1	8.0	1.7	5.3	.6625	$= ac_{22}$
bc	6.7	8.8	2.8	−4.7	−.5875	$= bc_{22}$
abc	16.4	9.7	.9	−1.9	−.2375	$= abc_{222}$

The fitted main effects for A and B are much larger than the rest. The positive signs of a_2 and b_2 indicate that setting A and B at their (+) levels (1.2 and 40 mol % respectively) results in the largest impact strength. This confirms the pattern in the raw data.

(c) Using the reverse Yates algorithm:

Fitted Effect	Value	Cycle 1	Cycle 2	Cycle 3 ($\hat{y}$)	
abc_{222}	0	0	0	14.975	$= \hat{y}_{\text{abc}}$
bc_{22}	0	0	14.975	7.050	$= \hat{y}_{\text{bc}}$
ac_{22}	0	0	0	14.975	$= \hat{y}_{\text{ac}}$
c_2	0	14.975	7.050	7.050	$= \hat{y}_{\text{c}}$
ab_{22}	0	0	0	14.975	$= \hat{y}_{\text{ab}}$
b_2	0	0	14.975	7.050	$= \hat{y}_{\text{b}}$
a_2	3.9625	0	0	14.975	$= \hat{y}_{\text{a}}$
$\bar{y}_{\cdots}$	11.0125	7.050	7.050	7.050	$= \hat{y}_{(1)}$

Since the model only includes the main effect for A, only the level of A affects the fitted values.

The following table shows the computations necessary to compute R^2. (Note: $\bar{y}$ represents the average of all observations. It is equal to $\bar{y}_{...}$ only because all the sample sizes are equal.)

i	y_i	$\bar{y}$	$\hat{y}_i$	$(y_i - \bar{y})$	$(y_i - \bar{y})^2$	$e_i = (y_i - \hat{y}_i)$	$(y_i - \hat{y}_i)^2$
1	6.7	11.0125	7.050	−4.3125	18.5977	−.35000	.12250
2	11.9	11.0125	14.975	.8875	.7877	−3.07500	9.45564
3	8.5	11.0125	7.050	−2.5125	6.3127	1.45000	2.10250
4	16.5	11.0125	14.975	5.4875	30.1127	1.52500	2.32562
5	6.3	11.0125	7.050	−4.7125	22.2077	−.75000	.56250
6	15.1	11.0125	14.975	4.0875	16.7077	.12500	.01562
7	6.7	11.0125	7.050	−4.3125	18.5977	−.35000	.12250
8	16.4	11.0125	14.975	5.3875	29.0252	1.42500	2.03062
					142.35		16.738

$$R^2 = \frac{\sum(y_i - \bar{y})^2 - \sum(y_i - \hat{y}_i)^2}{\sum(y_i - \bar{y})^2} = \frac{142.35 - 16.738}{142.35} = .882.$$

(d) The least squares equation is

$$\hat{y} = 3.088 + 9.906x.$$

Since there are only 2 levels for A, the least squares line goes through the sample mean of the observations at $x = .4$ and the sample mean of the observations at $x = 1.2$. These sample means are precisely the fitted values for the model in (c), so the curve fitting model fits the same as the one in (c). (The fitted values and R^2 are the same as in (c).)

5. (a) Using the Yates algorithm,

← sample mean of natural logs, not natural log of sample mean

Combination	$\overline{\ln y}$	Cycle 1	Cycle 2	Cycle 3	Cycle 3 ÷ 8	
(1)	2.56495	6.88235	12.8817	26.4969	3.31211	$= \overline{\ln y_{...}}$
a	4.31740	5.99934	13.6152	6.5991	.82488	$= a_2$
b	2.24990	7.22331	3.2520	−1.7144	−.21430	$= b_2$
ab	3.74943	6.39190	3.3471	−.3912	−.04890	$= ab_{22}$
c	2.74032	1.75245	−.8830	.7335	.09169	$= c_2$
ac	4.48299	1.49953	−.8314	.0951	.01189	$= ac_{22}$
bc	2.39375	1.74267	−.2529	.0516	.00645	$= bc_{22}$
abc	3.99816	1.60441	−.1383	.1147	.01433	$= abc_{222}$

The fitted main effect for A is much larger than the rest. Also, the fitted B main effect is at least twice as large as the rest. a_2 is positive, which means that the .7 mm lead requires more clips to break than the .5 mm lead (this is obvious from the raw data). On average, it takes about 1.65 more log(clips) to break .7 mm lead than it does to break .5 mm lead. In other words, the number of clips needed to break .7 mm lead is $e^{1.65} = 5.21$ times the number of clips needed to break .5 mm lead. Lead hardness (Factor C) does not seem to play an important role in determining this kind of breaking strength.

(b) $\widehat{\overline{\ln y}_{abc}} = \overline{\ln y_{...}} + a_2 = 3.31211 + .82488 = 4.13699$ ln(clips). Exponentiating, $\hat{y} = e^{4.13699} = 62.6$. This is a bit higher than the data for this factor-level combination, because the main effect for B was not taken into account.

(c) For one thing, you would not want a model to predict a negative number of paper clips, and this would be prevented by analyzing the data on the log scale. There may also be a power law that governs the breaking strength of this material, in which case taking logs would eliminate interactions among the factors.

6. (a) Using the Yates algorithm,

Combination	y	Cycle 1	Cycle 2	Cycle 3	Cycle 3 $\div$ 8	
(1)	2.348	4.428	8.706	18.152	2.2690	$= \overline{y}_{...}$
a	2.080	4.278	9.446	−.656	−.0820	$= a_2$
b	2.298	4.668	−.586	−.040	−.0050	$= b_2$
ab	1.980	4.778	−.070	−.040	−.0050	$= ab_{22}$
c	2.354	−.268	−.150	.740	.0925	$= c_2$
ac	2.314	−.318	.110	.516	.0645	$= ac_{22}$
bc	2.404	−.040	−.050	.260	.0325	$= bc_{22}$
abc	2.374	−.030	.010	.060	.0075	$= abc_{222}$

(b) On average, 6 mesh particles result in greater densities, since a_2 is negative. Also, on average, vibrated cylinders result in greater densities, since c_2 is positive. However, since there is an interaction, the size of the change in density when going from unvibrated to vibrated cylinders is not the same for 6 mesh particles as it is for 60 mesh particles. Specifically, since ac_{22} is positive, the increase in going from unvibrated to vibrated cylinders is larger for 60 mesh particles than it is for 6 mesh particles. A careful look at the sample means confirms this.

(c) $\hat{y}_{\text{abc}} = \overline{y}_{...} + a_2 + c_2 + ac_{22} = 2.2690 - .0820 + .0925 + .0645 = 2.344$ g/cc.

(d) Since there seems to be an interaction between A and C, the answer to this question really depends on whether 6 or 60 mesh particles are used. If you believe that the AC interaction is unimportant, then the average change would be $a_1 - a_2 =\ .0820 + .0820 =\ .1640$ g/cc, regardless of the mesh size.

7. (a) The least squares equation is

$$\begin{aligned}\hat{y} \ =\ & 5423 + 80.51x_1 - 4046x_2 + 137.2x_3 + .09547x_1^2 + 275.7x_2^2 - 1.933x_3^2 \\ & -9.130x_1x_2 - .9427x_1x_3 + 28.60x_2x_3\end{aligned}$$

and the corresponding R^2 is .938. This equation is not very simple nor easy to interpret.

(b) The least squares equation is

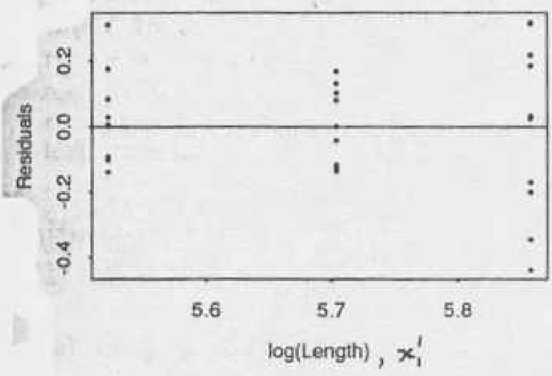
$$\hat{y}' = 3.865 + 4.9504x_1' - 5.6537x_2' - 3.5030x_3'$$

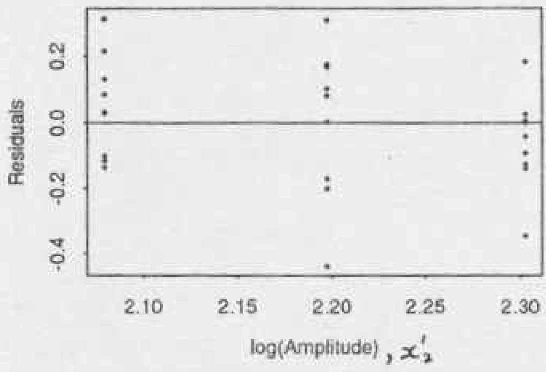
with corresponding $R^2 = .967$. A decrease of about 3.5030 log(cycle) accompanies a 1 ln(g) increase in x_3'.

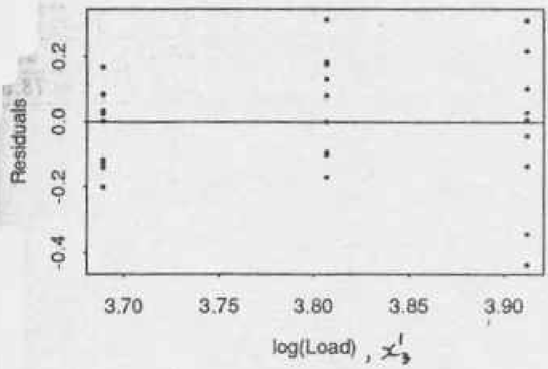
(c) The regression equation is

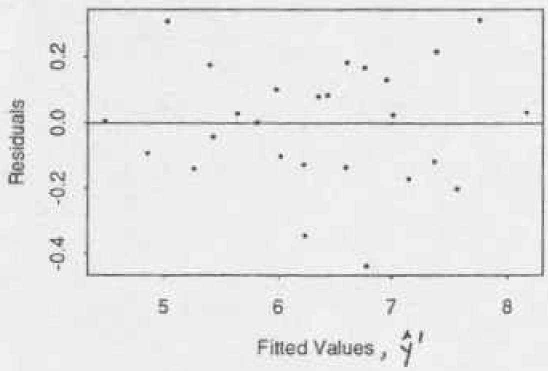
$$\hat{y}' = 1.571 + 5.1650(x_1' - x_2') - 3.5030x_3'$$

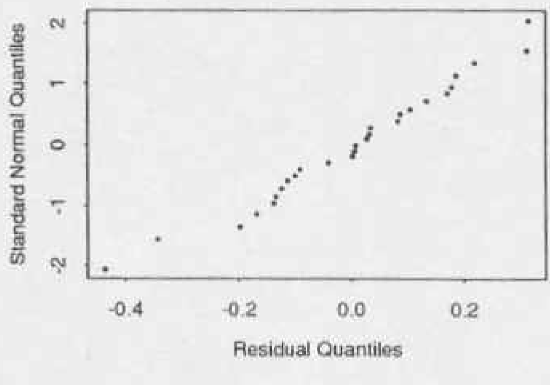
with an R^2 of .964.











There are no strong patterns in the residual plots, and the R^2 value is very high for such a simple model. The equation seems to summarize the data well, and is much simpler than the one from (a). The implied power law is

$$\begin{aligned} y &= e^{\beta_0} e^{\beta_1 \ln x_1 - \beta_1 \ln x_2} e^{\beta_2 \ln x_3} \\ &= e^{\beta_0} \left(\frac{x_1}{x_2} \right)^{\beta_1} x_3^{\beta_2} \end{aligned}$$

(d) For $x_1 = 300$, $x_2 = 9$, and $x_3 = 45$,

$$\hat{y}' = 1.571 + 5.1650(\ln(300) - \ln(9)) - 3.5030\ln(45) = 6.348,$$

so $\hat{y} = e^{6.348} = 571$. For $x_1 = 325$, $x_2 = 9.5$, and $x_3 = 47$,

$$\hat{y}' = 1.571 + 5.1650(\ln(325) - \ln(9.5)) - 3.5030\ln(47) = 6.329,$$

so $\hat{y} = e^{6.329} = 561$. It would not be wise to make a similar prediction at $x_1 = 375$, $x_2 = 10.5$, and $x_3 = 51$ because there is no evidence that the fitted relationship is correct under these conditions. This would be an extrapolation. Some data should be obtained under conditions like these before making such a prediction.

8. The averages needed are given in the table below.

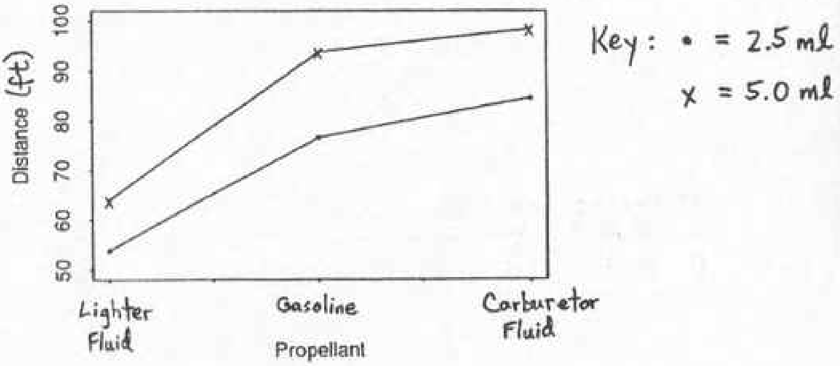
		PROPELLANT (Factor B)			
		Lighter Fluid	Gasoline	Carburetor Fluid	
CHARGE SIZE (Factor A)	2.5 ml	$\bar{y}_{11} = 53.8$	$\bar{y}_{12} = 76.6$	$\bar{y}_{13} = 84.6$	$\bar{y}_{1.} = 71.67$
	5.0 ml	$\bar{y}_{21} = 64.0$	$\bar{y}_{22} = 93.8$	$\bar{y}_{23} = 98.4$	$\bar{y}_{2.} = 85.4$
		$\bar{y}_{.1} = 58.9$	$\bar{y}_{.2} = 85.2$	$\bar{y}_{.3} = 91.5$	$\bar{y}_{..} = 78.53$

The fitted main effects are

$$a_1 = \bar{y}_{1.} - \bar{y}_{..} = -6.87$$
$$a_2 = \bar{y}_{2.} - \bar{y}_{..} = 6.87$$
$$b_1 = \bar{y}_{.1} - \bar{y}_{..} = -19.63$$
$$b_2 = \bar{y}_{.2} - \bar{y}_{..} = 6.67$$
$$b_3 = \bar{y}_{.3} - \bar{y}_{..} = 12.97$$

The fitted interactions are

$$ab_{11} = \bar{y}_{11} - (\bar{y}_{..} + a_1 + b_1) = 1.77$$
$$ab_{12} = \bar{y}_{12} - (\bar{y}_{..} + a_1 + b_2) = -1.73$$
$$ab_{13} = \bar{y}_{13} - (\bar{y}_{..} + a_1 + b_3) = -.03$$
$$ab_{21} = \bar{y}_{21} - (\bar{y}_{..} + a_2 + b_1) = -1.77$$
$$ab_{22} = \bar{y}_{22} - (\bar{y}_{..} + a_2 + b_2) = 1.73$$
$$ab_{23} = \bar{y}_{23} - (\bar{y}_{..} + a_2 + b_3) = .03$$



The plot shows near parallelism, indicating that interactions may be small and unimportant. This is also reflected in the fitted interactions, since they are small relative to the fitted main effects. If interactions are assumed to be unimportant, it then makes sense to summarize the results in terms of the main effects only. Both factors seem to have an effect, but Propellant appears to have a slightly larger effect than Charge Size. For each Charge Size, gasoline and carburetor fluid result in longer distances than lighter fluid. Carburetor fluid gives slightly longer distances than gasoline. Switching from lighter fluid to gasoline results in an increase of about $b_2 - b_1 = 26.3$ feet, and switching from gasoline to carburetor fluid increases the distance by about $b_3 - b_2 = 6.3$ feet. For each Propellant, the 5.0 ml charge results in a longer distance. The increase in distance due to changing Propellant from 2.5 to 5.0 ml is about $a_2 - a_1 = 13.73$ feet.

9. (a) Using the Yates algorithm:

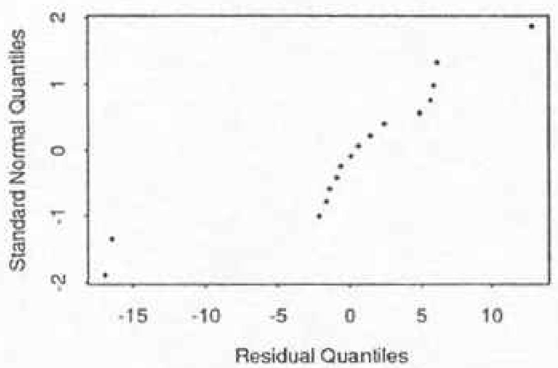
Combination	$\bar{y}$	Cycle 1	Cycle 2	Cycle 3	Cycle 4	Cycle 4 ÷ 16	
(1)	9	79	129	223	282	17.625	$= \bar{y}_{....}$
a	70	50	94	59	184	11.500	$= a_2$
b	8	68	17	149	−96	−6.000	$= b_2$
ab	42	26	42	35	−74	−4.625	$= ab_{22}$
c	13	9	95	−71	−10	−.625	$= c_2$
ac	55	8	54	−25	−24	−1.500	$= ac_{22}$
bc	7	33	9	−57	−36	−2.250	$= bc_{22}$
abc	19	9	26	−17	−26	−1.625	$= abc_{222}$
d	3	61	−29	−35	−164	−10.250	$= d_2$
ad	6	34	−42	25	−114	−7.125	$= ad_{22}$
bd	1	42	−1	−41	46	2.875	$= bd_{22}$
abd	7	12	−24	17	40	2.500	$= abd_{222}$
cd	5	3	−27	−13	60	3.750	$= cd_{22}$
acd	28	6	−30	−23	58	3.625	$= acd_{222}$
bcd	3	23	3	−3	−10	−.625	$= bcd_{222}$
abcd	6	3	−20	−23	−20	−1.250	$= abcd_{2222}$

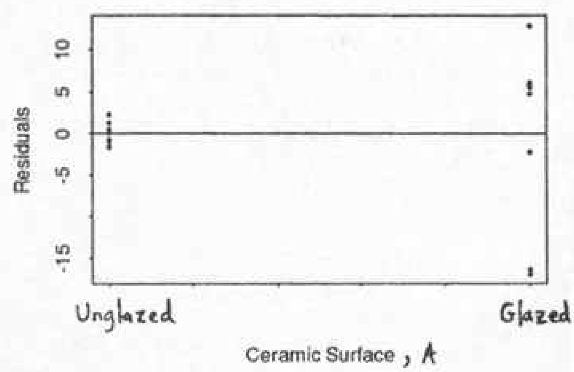
The dominant effects seem to be the A, B, and D main effects, and the A×D and A×B interactions.

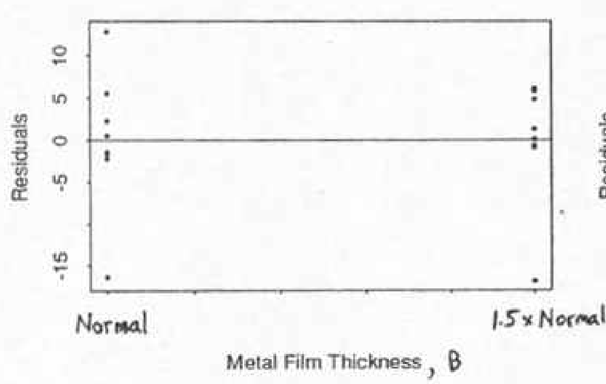
(b) Using the fitted main effects and interactions identified in (a), the reverse Yates algorithm gives

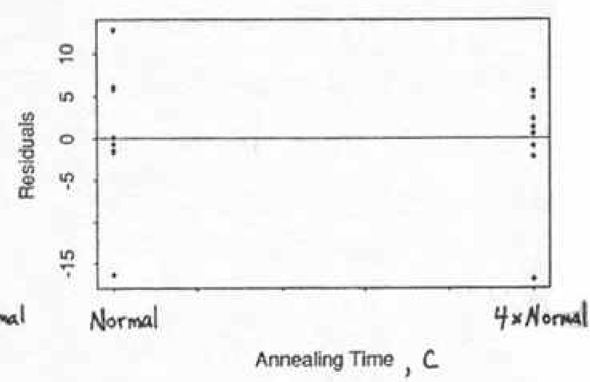
Fitted Effect	Value	Cycle 1	Cycle 2	Cycle 3	Cycle 4 ($\hat{y}$)	
$abcd_{2222}$	0	0	0	−17.375	1.125	$= \hat{y}_{\text{abcd}}$
bcd_{222}	0	0	−17.375	18.500	1.625	$= \hat{y}_{\text{bcd}}$
acd_{222}	0	0	0	−3.125	22.375	$= \hat{y}_{\text{acd}}$
cd_{22}	0	−17.375	18.500	4.750	4.375	$= \hat{y}_{\text{cd}}$
abd_{222}	0	0	0	−17.375	1.125	$= \hat{y}_{\text{abd}}$
bd_{22}	0	0	−3.125	39.750	1.625	$= \hat{y}_{\text{bd}}$
ad_{22}	−7.125	−10.625	0	−3.125	22.375	$= \hat{y}_{\text{ad}}$
d_2	−10.250	29.125	4.750	7.500	4.375	$= \hat{y}_{\text{d}}$
abc_{222}	0	0	0	−17.375	35.875	$= \hat{y}_{\text{abc}}$
bc_{22}	0	0	−17.375	18.500	7.875	$= \hat{y}_{\text{bc}}$
ac_{22}	0	0	0	−3.125	57.125	$= \hat{y}_{\text{ac}}$
c_2	0	−3.125	39.750	4.750	10.625	$= \hat{y}_{\text{c}}$
ab_{22}	−4.625	0	0	−17.375	35.875	$= \hat{y}_{\text{ab}}$
b_2	−6.000	0	−3.125	39.750	7.875	$= \hat{y}_{\text{b}}$
a_2	11.500	−1.375	0	−3.125	57.125	$= \hat{y}_{\text{a}}$
$\bar{y}_{....}$	17.625	6.125	7.500	7.500	10.625	$= \hat{y}_{(1)}$

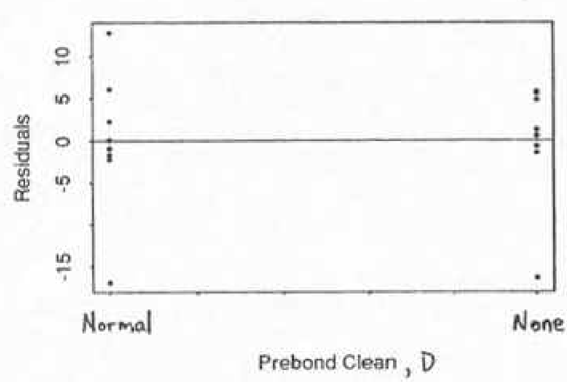
The residuals are computed as $y - \hat{y}$ for each observation. They are (in Yates standard order) -1.625, 12.875, $.125$, 6.125, 2.375, -2.125, $-.875$, -16.875, -1.375, -16.375, $-.625$, 5.875, $.625$, 5.625, $.375$, and 4.875.

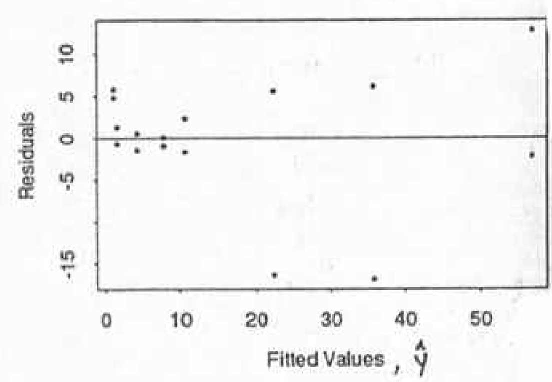












The plot of Residuals versus A indicates that there is more variation in the response for glazed surfaces than for unglazed surfaces, and the plot of Residuals versus Fitted Values suggests that variation tends to increase for larger values of the response. The normal plot identifies two unusually large (negative) residuals, and both of these correspond to glazed surface observations. The patternless plot of Residuals versus C indicates that C is not important (since it is not in the current fitted model).

(c) According to the model, C is not important, so set it to its cheapest level (probably normal, since time is money). Then to minimize the number of pull-outs (based on the smallest fitted responses), set A at glazed (+), B at 1.5 times normal (+), and D at no clean (+).

10. The fitted main effects are:

$$a_1 = .02814815 \quad a_2 = -.01685185 \quad a_3 = -.01129630$$
$$b_1 = .03203704 \quad b_2 = .04648148 \quad b_3 = -.07851852$$
$$c_1 = -36.65185 \quad c_2 = -21.68185 \quad c_3 = 58.33370$$

The ab_{ij}'s are:

		j = 1	2	3
i	1	.021851852	.009074074	−.03092593
	2	−.008148148	−.012592593	.02074074
	3	−.013703704	.003518519	.01018519

The ac_{ik}'s are:

		k = 1	2	3
i	1	−.025925926	−.0159259259	.041851852
	2	.007407407	−.0009259259	−.006481481
	3	.018518519	.0168518519	−.035370370

The bc_{jk}'s are:

		k = 1	2	3
j	1	−.0331481481	−.001481481	.03462963
	2	−.0009259259	−.020925926	.02185185
	3	.0340740741	.022407407	−.05648148

The abc_{ijk}'s are:

k		i	j = 1	2	3
1	i	1	−.024074074	−.002962963	.0270370370
		2	.012592593	−.004629630	−.0079629630
		3	.011481481	.007592593	−.0190740741
2	i	1	−.010740741	−.002962963	.0137037037
		2	.004259259	−.011296296	.0070370370
		3	.006481481	.014259259	−.0207407407
3	i	1	.034814815	.005925926	−.0407407407
		2	−.016851852	.015925926	.0009259259
		3	−.017962963	−.021851852	.0398148148

Refer to the solution to Ex. 5, Ch. 3, for a graphical summary. The b's show that Scale 3 reads lower than the other two scales on average, since b_3 is negative and b_1 and b_2 are positive. Student 1 may have a tendency to produce higher measurements than students 2 and 3, according to a_1. The fitted main effects for Factor C (Weight) are the largest, as would be expected.

The interactions are harder to interpret, especially the three-way interactions. The graph for 20 gram weighings given in the solution to Ex. 5, Ch. 3, shows a lack of parallelism, and this is measured by the ab's. The lack of consistency in all 3 graphs is measured by the abc's. The roles of the factors in the graphs would need to be switched to see the lack of parallelism that is measured by the ac's and the bc's.

11. If a situation can be accurately described without interactions, each factor may be set independently of the rest. This simplifies the task of improving and controlling the process. When interactions are present, the effect of changing one factor depends on the current settings of the other factors, so making changes to improve and control a system is more difficult and complex.

12. (a) The least squares equation is

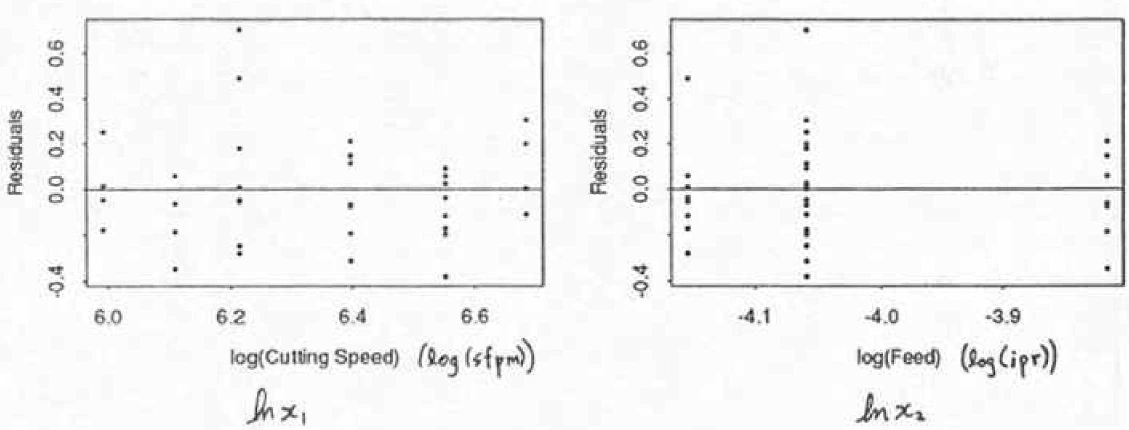
$$\widehat{\ln y} = 18.750 - 5.1209 \ln x_1 - 3.7379 \ln x_2,$$

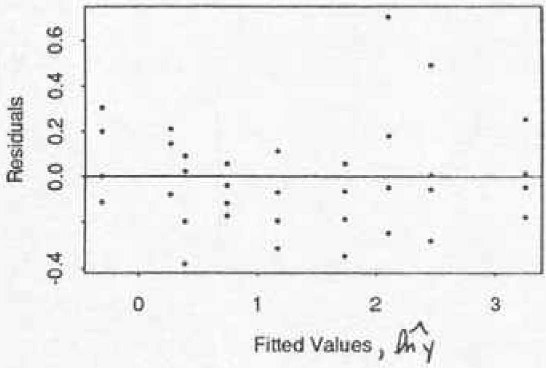
with an R^2 of .960. The relationship $y x_1^{\alpha_1} x_2^{\alpha_2} = C$ implies that

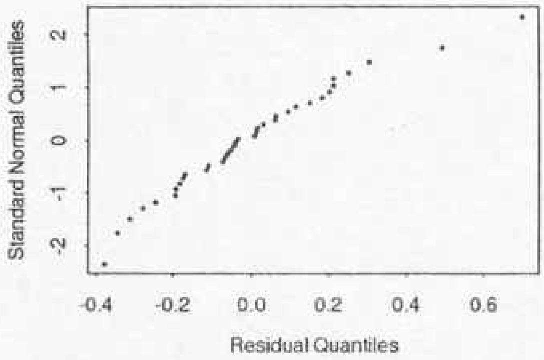
$$\ln y = \ln C - \alpha_1 \ln x_1 - \alpha_2 \ln x_2,$$

so $\hat{\alpha}_1 = -b_1 = 5.1209$, $\hat{\alpha}_2 = -b_2 = 3.7379$, and $\hat{C} = e^{b_0} = 1.39 \times 10^8$.

(b)







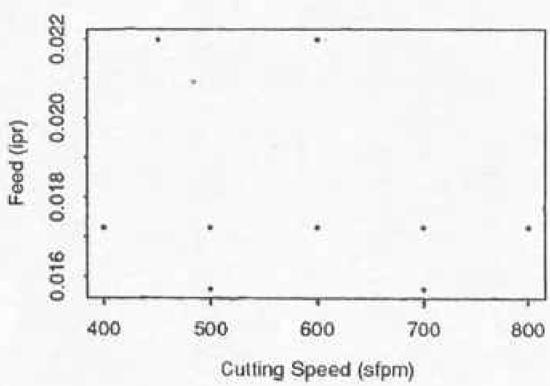
The plot of Residuals versus $\widehat{\ln y}$ shows a slight amount of curvature, but the pattern is not strong. The plot of Residuals versus $\ln x_1$ shows that there is more spread in the response when $\ln x_1 = 6.2146$ ($x_1 = 500$), and the plot of Residuals versus $\ln x_2$ shows that there is more spread in the response when $\ln x_2 = -4.05994$ ($x_2 = .01725$). The normal plot of residuals is fairly linear, indicating that the residuals are bell-shaped. Overall, the residual plots do not reveal any major problems with the fitted model.

(c) For $x_1 = 550$ and $x_2 = .01650$,

$$\widehat{\ln y} = \ln \hat{C} - \hat{\alpha}_1 \ln(550) - \hat{\alpha}_2 \ln(.01650) = 1.7789,$$

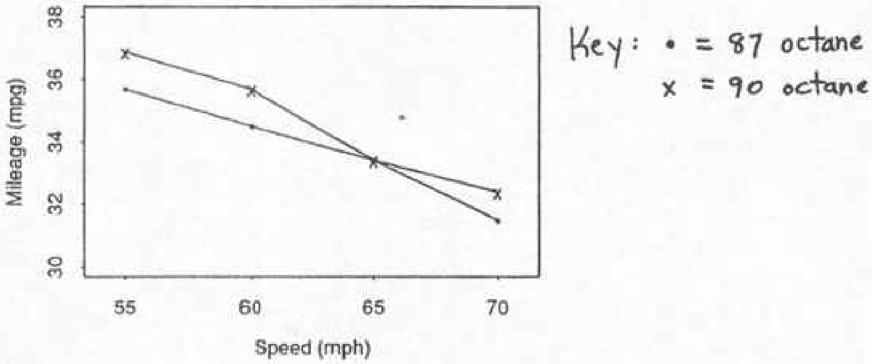
so $\hat{y} = e^{1.7789} = 5.92$ minutes.

(d)



This is subjective, and also more difficult with surface fitting than with line fitting.

13. (a)



The lines almost cross, but only because they are so close together. Relative to the effect of Speed the interactions appear to be small. They are large relative to any Octane main effect.

(b) The averages needed are given in the table below.

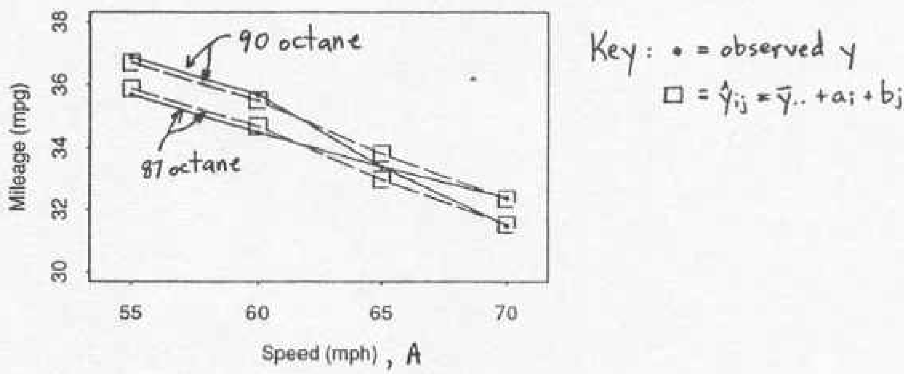
		SPEED (Factor B)				
		55	60	65	70	
OCTANE (Factor A)	87	$\bar{y}_{11} = 35.7$	$\bar{y}_{12} = 34.5$	$\bar{y}_{13} = 33.4$	$\bar{y}_{14} = 31.5$	$\bar{y}_{1\cdot} = 33.775$
	90	$\bar{y}_{21} = 36.9$	$\bar{y}_{22} = 35.7$	$\bar{y}_{23} = 33.4$	$\bar{y}_{24} = 32.4$	$\bar{y}_{2\cdot} = 34.6$
		$\bar{y}_{\cdot 1} = 36.3$	$\bar{y}_{\cdot 2} = 35.1$	$\bar{y}_{\cdot 3} = 33.4$	$\bar{y}_{\cdot 4} = 31.95$	$\bar{y}_{\cdot\cdot} = 34.1875$

The fitted main effects are

$$
\begin{aligned}
a_1 &= \bar{y}_{1\cdot} - \bar{y}_{\cdot\cdot} = -.4125 \\
a_2 &= \bar{y}_{2\cdot} - \bar{y}_{\cdot\cdot} = .4125 \\
b_1 &= \bar{y}_{\cdot 1} - \bar{y}_{\cdot\cdot} = 2.1125 \\
b_2 &= \bar{y}_{\cdot 2} - \bar{y}_{\cdot\cdot} = .9125 \\
b_3 &= \bar{y}_{\cdot 3} - \bar{y}_{\cdot\cdot} = -.7875 \\
b_4 &= \bar{y}_{\cdot 4} - \bar{y}_{\cdot\cdot} = -2.2375
\end{aligned}
$$

The fitted interactions are

$$
\begin{aligned}
ab_{11} &= \bar{y}_{11} - (\bar{y}_{\cdot\cdot} + a_1 + b_1) = -.1875 \\
ab_{12} &= \bar{y}_{12} - (\bar{y}_{\cdot\cdot} + a_1 + b_2) = -.1875 \\
ab_{13} &= \bar{y}_{13} - (\bar{y}_{\cdot\cdot} + a_1 + b_3) = .4125 \\
ab_{14} &= \bar{y}_{14} - (\bar{y}_{\cdot\cdot} + a_1 + b_4) = -.0375 \\
ab_{21} &= \bar{y}_{21} - (\bar{y}_{\cdot\cdot} + a_2 + b_1) = .1875 \\
ab_{22} &= \bar{y}_{22} - (\bar{y}_{\cdot\cdot} + a_2 + b_2) = .1875 \\
ab_{23} &= \bar{y}_{23} - (\bar{y}_{\cdot\cdot} + a_2 + b_3) = -.4125 \\
ab_{24} &= \bar{y}_{24} - (\bar{y}_{\cdot\cdot} + a_2 + b_4) = .0375
\end{aligned}
$$



The fitted values match up well with the observed data for 55 and 60 mph, but they are not able to cross and account for the possible interaction in the observed values.

(c) With Speed = x_1 and Octane = x_2, the least squares equation is

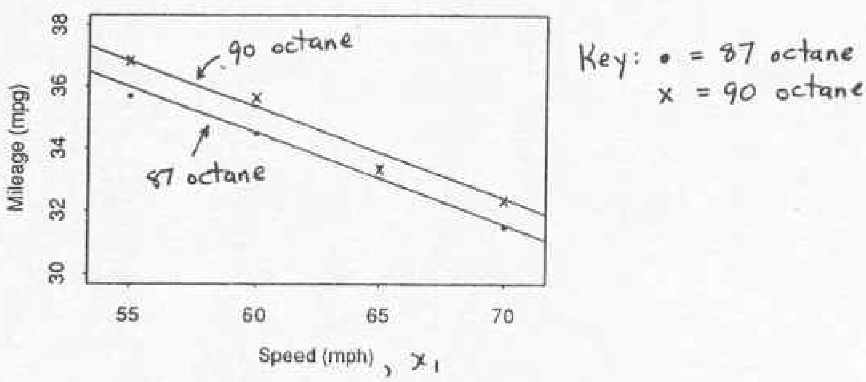
$$\hat{y} = 28.2875 - .295x_1 + .275x_2.$$

For 87 octane, the equation is

$$\hat{y} = 28.2875 - .295x_1 + .275(87) = 52.2125 - .295x_1.$$

For 90 octane, the equation is

$$\hat{y} = 28.2875 - .295x_1 + .275(90) = 53.0375 - .295x_1.$$

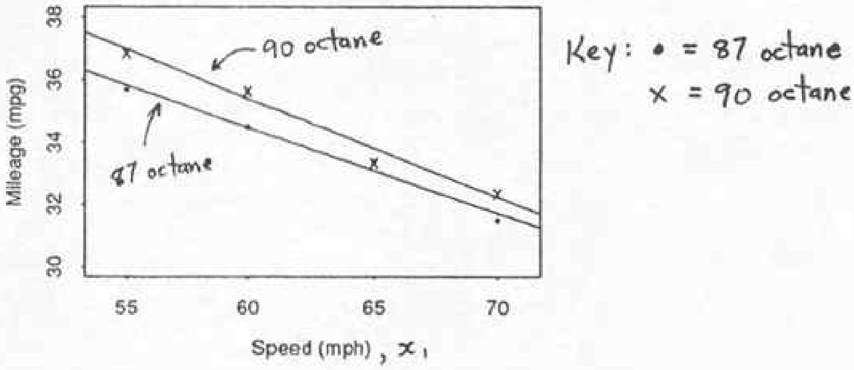


(d) For the 87 octane data, the least squares equation is

$$\hat{y} = 50.9 - .274x_1.$$

For the 90 octane data, the least squares equation is

$$\hat{y} = 54.35 - .316x_1.$$



(e) The plots in (b) and (c) are fitted models that do not account for interactions (non-parallelism). The plot in (b) is more flexible than the one in (c) because factorial models are (in general) more flexible than curve and surface models. The plot in (d) represents a model that allows interactions, as reflected by the non-parallelism of the lines. This was possible because the two lines were fit separately for each level of Octane. None of the models fit the data perfectly, and it is hard to say if any fits better than the rest.

(f) There is no replication (multiple experimental runs at a particular Speed-Octane combination). Replication would validate any conclusions drawn from the experiment, and it would allow for better comparisons among the different possible models. The second weakness is a potential problem because any apparent effect of changing the Octane could be attributed to differences in conditions between the early runs and the later runs.

14. (a) The least squares equation is

$$\widehat{\ln y} = 20.539 - 1.2060 \ln x_1 - 1.3988 \ln x_2.$$

with corresponding R^2 equal to .782.

The relationship $y x_1^{\alpha_1} x_2^{\alpha_2} = C$ implies that

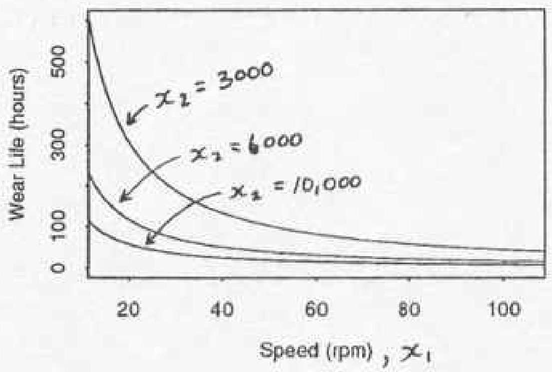
$$\ln y = \ln C - \alpha_1 \ln x_1 - \alpha_2 \ln x_2,$$

so $\hat{\alpha}_1 = -b_1 = 1.2060$, $\hat{\alpha}_2 = -b_2 = 1.3988$, and $\hat{C} = e^{b_0} = 8.317 \times 10^8$.

For $x_2 = 3000$, the equation is $y = 11381 x_1^{-1.2060}$

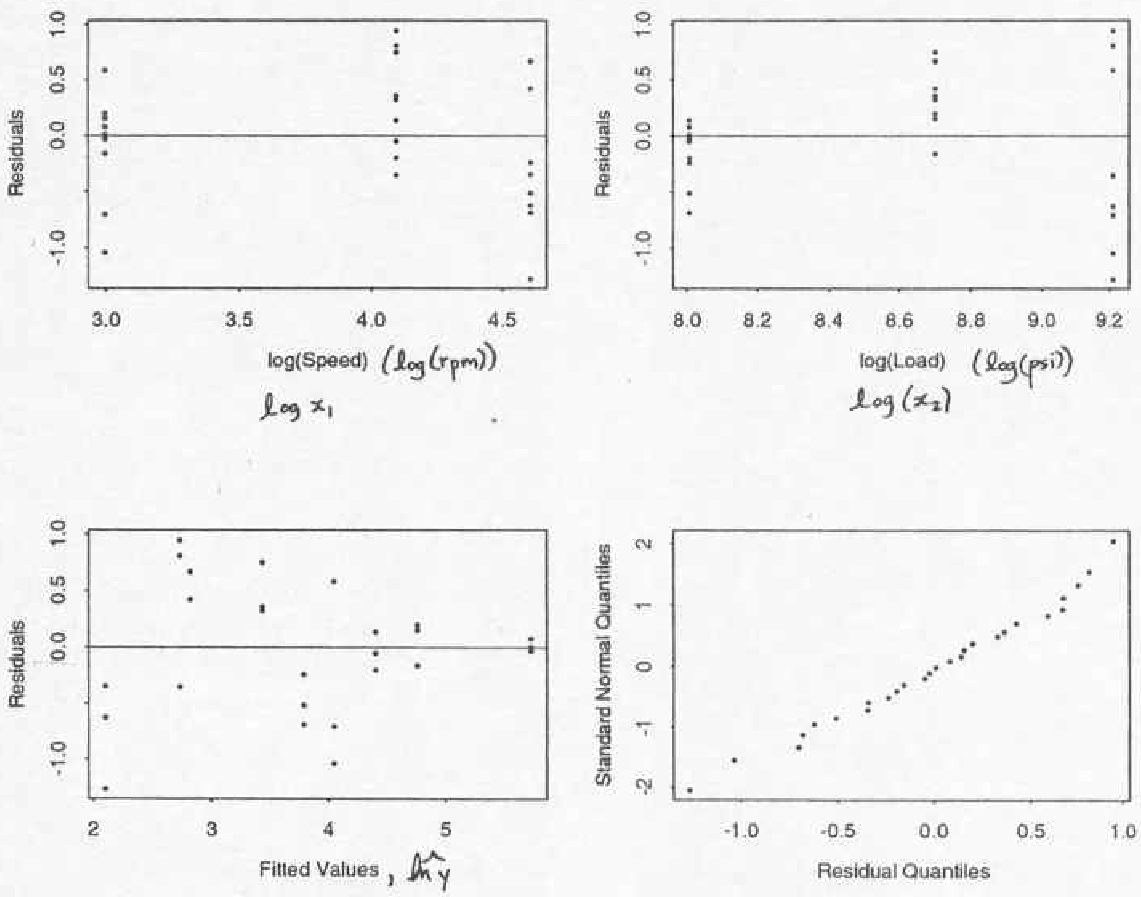
for $x_2 = 6000$, the equation is $y = 4316 x_1^{-1.2060}$

for $x_2 = 10,000$, the equation is $y = 2112 x_1^{-1.2060}$



For this equation, there are interactions between x_1 and x_2. This is reflected by the non-parallelism of the plots.

(b)



The plot of Residuals versus $\widehat{\ln y}$ is patternless, so it reveals no problems with the fitted model. The plots of Residuals versus $\ln x_1$ and $\ln x_2$ both have a slight negative-positive-negative pattern, indicating that there may be something about these variables that the model is not accounting for. In addition, the plot of Residuals versus $\ln x_2$ shows that there is more variation in y when $\ln x_2 = 9.21034$ (when $x_2 = 10,000$). The normal plot of residuals is fairly linear, indicating that the residuals are bell-shaped.

(c) For $x_1 = 20$ and $x_2 = 10,000$,

$$\widehat{\ln y} = 20.539 - 1.2060 \ln(20) - 1.3988 \ln(10,000) = 4.043,$$

so $\hat{y} = e^{4.043} = 57.0$ hours.

(d) For $x_1 = 15$ and $x_2 = 1500$,

$$\widehat{\ln y} = 20.539 - 1.2060 \ln(15) - 1.3988 \ln(1500) = 7.043,$$

so $\hat{y} = e^{7.043} = 1145$ hours.

15. (a) The least squares equations are

$$\hat{y}_1 = -.4750 + .020000x_1 + .140000x_2,$$

with corresponding $R^2 = .998$, and

$$\hat{y}_2 = .1200 - .020000x_1 + .024000x_2,$$

with corresponding $R^2 = .600$. Based on the R^2 values, it seems that y_1 is well described as a linear function of x_1 and x_2, but not y_2.

(b) Solving the first equation for x_1 yields

$$x_1 = \left(\frac{1}{.02}\right)(y_1 - .14x_2 + .475).$$

Plugging this into the second equation and solving for x_2 gives

$$\hat{x}_2 = \frac{y_2 + y_1 + .355}{.164}.$$

Predictions using this equation and the one from theory can be compared to the actual values for x_2:

x_2	$\hat{x}_2$(Theory)	$\hat{x}_2$(Eq)
25	23.2	24.7256
25	24.0	25.3354
30	29.6	29.6037
30	32.0	30.2134
35	36.0	35.7012
35	37.6	34.4817
40	41.6	39.9695
40	43.2	39.9695

The data-derived equation above does seem to do a better job than the one from chemical theory, but the data-derived equation was fit "especially" to this data set. The two equations should be compared in an additional experiment.

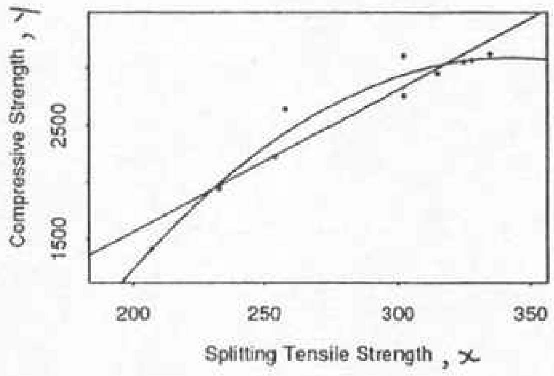
(c) The least squares equation is

$$\hat{x}_2 = 3.007 + 6.1396y_1 + 3.511y_2$$

with corresponding R^2=.996. This is not the same as the equation derived in (b).

(d) The equation in (c) is guaranteed to win, because the method of least squares minimizes the sum of squared differences between the predicted and observed values of x_2 in (c). The equation derived in (b) came from two least squares equations that minimized this quantity for y_1 and y_2 respectively, not x_2.

16. (a)



The relationship is not quite linear.

(b) The calculations are given below:

i	x_i	x_i^2	y_i	y_i^2	x_iy_i
1	207	42849	1420	2016400	293940
2	233	54289	1950	3802500	454350
3	254	64516	2230	4972900	566420
4	328	107584	3070	9424900	1006960
5	325	105625	3060	9363600	994500
6	302	91204	3110	9672100	939220
7	258	66564	2650	7022500	683700
8	335	112225	3130	9796900	1048550
9	315	99225	2960	8761600	932400
10	302	91204	2760	7617600	833520
	2859	835285	26340	72451000	7753560

$$r = \frac{\sum x_iy_i - \frac{(\sum x_i)(\sum y_i)}{n}}{\sqrt{\left(\sum x_i^2 - \frac{(\sum x_i)^2}{n}\right)\left(\sum y_i^2 - \frac{(\sum y_i)^2}{n}\right)}}$$

$$= \frac{7753560 - \frac{(2859)(26340)}{10}}{\sqrt{\left(835285 - \frac{(2859)^2}{10}\right)\left(72451000 - \frac{(26340)^2}{10}\right)}} = .951.$$

This is close to 1, so there is a fairly strong positive linear relationship between y and x.

(c)

$$b_1 = \frac{\sum x_iy_i - \frac{(\sum x_i)(\sum y_i)}{n}}{\sum x_i^2 - \frac{(\sum x_i)^2}{n}} = \frac{7753560 - \frac{(2859)(26340)}{10}}{835285 - \frac{(2859)^2}{10}} = 12.45769$$

$$b_0 = \bar{y} - b_1\bar{x} = \frac{26340}{10} - (12.45769)\frac{2859}{10} = -927.6531$$

So the least squares equation is

$$\hat{y} = -927.6531 + 12.45769x.$$

(d) There is an approximate $b_1 = 12.45769$ psi increase in compressive strength that accompanies a 1 psi increase in splitting tensile strength.

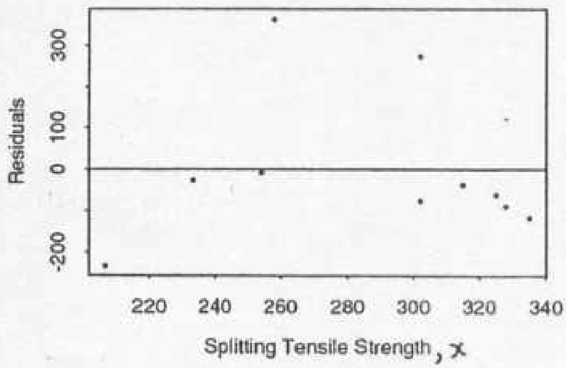
(e) $R^2 = r^2 = (.951)^2 = .904$.

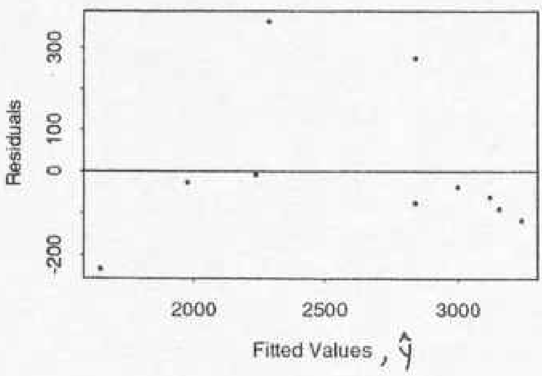
(f) For $x = 245$,

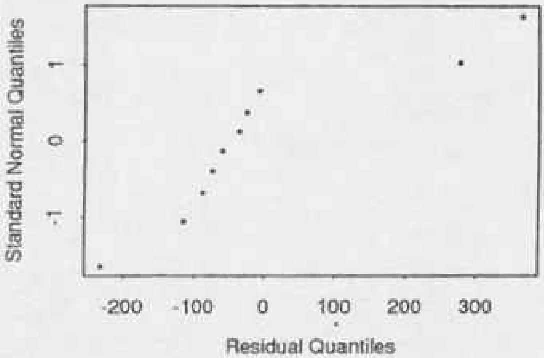
$$\hat{y} = -927.6531 + 12.45769(245) \approx 2124 \text{ psi}.$$

(g) The residuals e_i are computed below.

i	x_i	y_i	$\hat{y}_i = -927.6531 + 12.45769x_i$	$e_i = y_i - \hat{y}$
1	207	1420	1651.088	−231.088401
2	233	1950	1974.988	−24.988294
3	254	2230	2236.600	−6.599746
4	328	3070	3158.469	−88.468673
5	325	3060	3121.096	−61.095609
6	302	3110	2834.569	275.431220
7	258	2650	2286.430	363.569501
8	335	3130	3245.672	−115.672491
9	315	2960	2996.519	−36.518727
10	302	2760	2834.569	−74.568780







The plots of Residuals versus x and $\hat{y}$ show a negative-positive-negative pattern, indicating that the line does not account for the curvature in the relationship between y and x. The normal plot is not very linear, indicating that the residuals are not bell-shaped.

(h) The following printout was produced using Version 9.1 of Minitab.

```
MTB > print c1 c2

 ROW      y      x

   1   1420    207
   2   1950    233
   3   2230    254
   4   3070    328
   5   3060    325
   6   3110    302
   7   2650    258
   8   3130    335
   9   2960    315
  10   2760    302

MTB > regress c1 on 1 x variable c2;
SUBC> fits c3;
SUBC> residuals c4.

The regression equation is          least squares equation
y = - 928 + 12.5 x

Predictor       Coef         Stdev     t-ratio        p
Constant      -927.7  b0     414.1       -2.24    0.055
x             12.458  b1     1.433        8.69    0.000

s = 191.7       R-sq = 90.4%      R-sq(adj) = 89.2%
                           R^2
Analysis of Variance

SOURCE       DF          SS          MS        F        p
Regression    1     2777491     2777491    75.59    0.000
Error         8      293949       36744
Total         9     3071440
```

```
Unusual Observations
Obs.        x         y        Fit  Stdev.Fit   Residual    St.Resid
  7       258    2650.0     2286.4       72.6      363.6       2.05R

R denotes an obs. with a large st. resid.

MTB > name c3 'fits' c4 'resids'

MTB > corr c1 c2

Correlation of y and x = 0.951
```

← r

```
MTB > print c1-c4
```

ROW	y	x	fits	resids
1	1420	207	1651.09	-231.088
2	1950	233	1974.99	-24.988
3	2230	254	2236.60	-6.600
4	3070	328	3158.47	-88.469
5	3060	325	3121.10	-61.096
6	3110	302	2834.57	275.431
7	2650	258	2286.43	363.570
8	3130	335	3245.67	-115.673
9	2960	315	2996.52	-36.519
10	2760	302	2834.57	-74.569

residuals

(i) The least squares equation is

$$\hat{y} = -7634.571 + 62.59917x - .09132849x^2$$

with corresponding R^2 equal to .956. This equation does appear to be an important improvement in describing the relationship between y and x in the range of the data.

(j) The increase in R^2 is from .904 to .956. The quadratic equation is guaranteed to have a larger R^2, so you should look at the size of the increase in R^2. The increase here may be considered large enough to use the quadratic equation, especially after looking at the scatterplot.

(k) For $x = 245$,

$$\hat{y} = -7634.571 + 62.59917(245) - .09132849(245)^2 \approx 2220.2 \text{ psi.}$$

This is relatively close to the prediction in (f).

(l) For the linear equation at $x = 400$,

$$\hat{y} = -927.6531 + 12.45769(400) \approx 4055.4 \text{ psi.}$$

For the quadratic equation at $x = 400$,

$$\hat{y} = -7634.571 + 62.59917(400) - .09132849(400)^2 \approx 2792.5 \text{ psi.}$$

These are not as close as the predictions for $x = 245$. It would be unwise to use either of these predictions without collecting some data around $x = 400$, since there is no way of knowing what the relationship between y and x is like from the given data. (It is certainly unlikely that y decreases after x increases beyond a certain point, as is suggested by the quadratic equation.)

17. It is important to have *replication* (several y observations for at least some x's) so that you can be more sure of the true relationship between the variables. Replication validates any conclusions drawn from the experiment, and provides more information to confirm the appropriateness of any particular fitted equation.

18. (a) This is a complete (full) factorial data structure. There is no replication (multiple experimental runs at a particular (x_1, x_2) combination).

(b) The first least squares equation is

$$\hat{y}_1 = 96.621 - .01570x_1$$

with $R^2 = .537$. The second least squares equation is

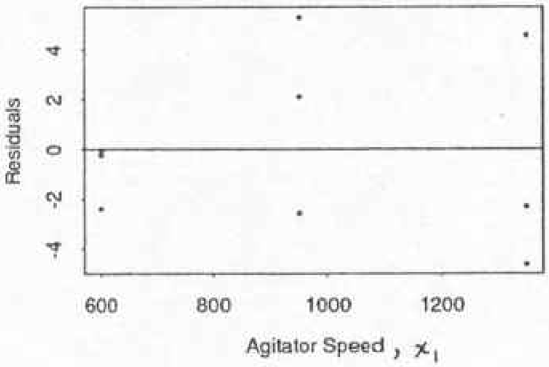
$$\hat{y}_1 = 100.61 - .19167x_2$$

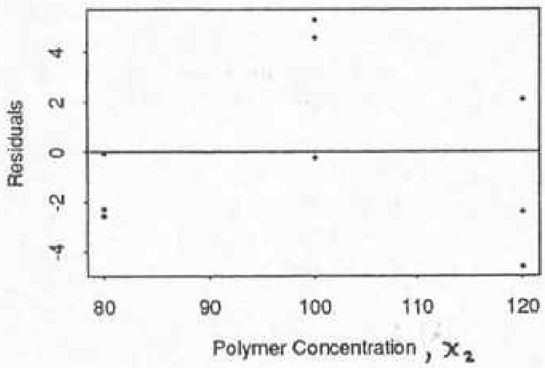
with $R^2 = .227$. The third least squares equation is

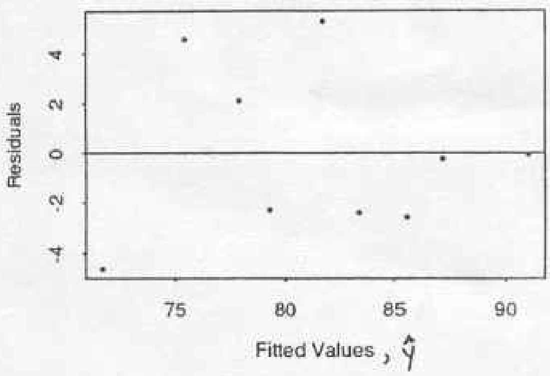
$$\hat{y}_1 = 115.788 - .01570x_1 - .19167x_2$$

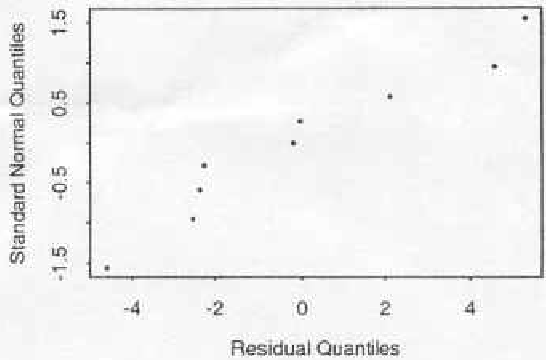
with $R^2 = .764$. The third equation is more complex than the first two, but seems to do a much better job of fitting the data (the increase in R^2 is large). It would probably do a better job of predicting y_1, even though it is more complex.

(c) The residuals are: -2.25937, -2.53944, $-.03452$, 4.57397, 5.29388, $-.20119$, -4.59270, 2.12722, and -2.36785.









The plots of Residuals versus x_1 and $\hat{y}_1$ are patternless, revealing no problems with the model. The plot of Residuals versus x_2 has a negative-positive-negative pattern, indicating that there may be some curvature in the relationship between x_2 and y_1 that the model is not accounting for. The normal plot of residuals is fairly linear, so the residuals are roughly bell-shaped.

(d) For $x_1 = 1350$, the equation is

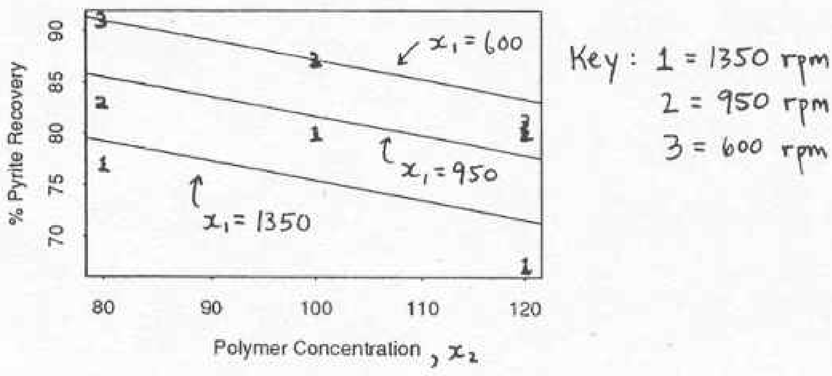
$$\begin{aligned}\hat{y}_1 &= 115.788 - .01570(1350) - .19167x_2 \\ &= 94.593 - .19167x_2.\end{aligned}$$

For $x_1 = 950$, the equation is

$$\begin{aligned}\hat{y}_1 &= 115.788 - .01570(950) - .19167x_2 \\ &= 100.873 - .19167x_2.\end{aligned}$$

For $x_1 = 600$, the equation is

$$\begin{aligned}\hat{y}_1 &= 115.788 - .01570(600) - .19167x_2 \\ &= 106.368 - .19167x_2.\end{aligned}$$



(e) For $x_1 = 1000$ and $x_2 = 110$,

$$\hat{y}_1 = 115.788 - .01570(1000) - .19167(110) = 79.00\%.$$

(f) The least squares equation is

$$\hat{y}_1 = -17.92 + .01952x_1 + 2.235x_2 - .00001746x_1^2 - .012083x_2^2 - .0000104x_1x_2$$

with $R^2 = .915$. This is a big increase from the third equation in part (b), but the quadratic equation is much more complex.

For $x_1 = 1350$, the equation is

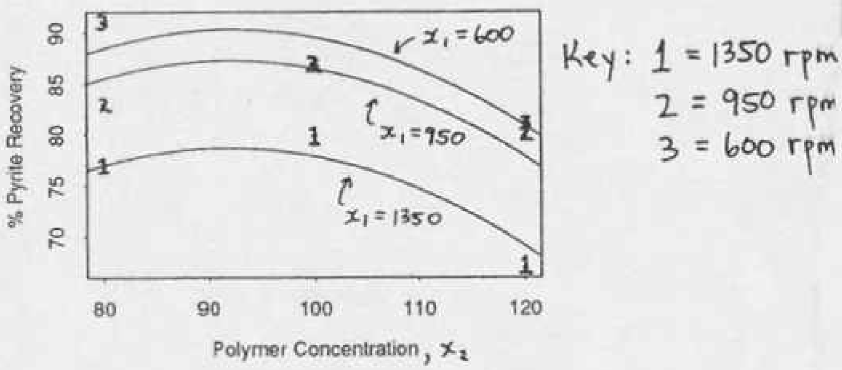
$$\begin{aligned}\hat{y}_1 &= -17.92 + .01952(1350) + 2.235x_2 - .00001746(1350)^2 \\ &\quad -.012083x_2^2 - .0000104(1350)x_2 \\ &= -23.37535 + 2.22096x_2 - .012083x_2^2.\end{aligned}$$

For $x_1 = 950$, the equation is

$$\begin{aligned}\hat{y}_1 &= -17.92 + .01952(950) + 2.235x_2 - .00001746(950)^2 \\ &\quad -.012083x_2^2 - .0000104(950)x_2 \\ &= -15.12415 + 2.22512x_2 - .012083x_2^2.\end{aligned}$$

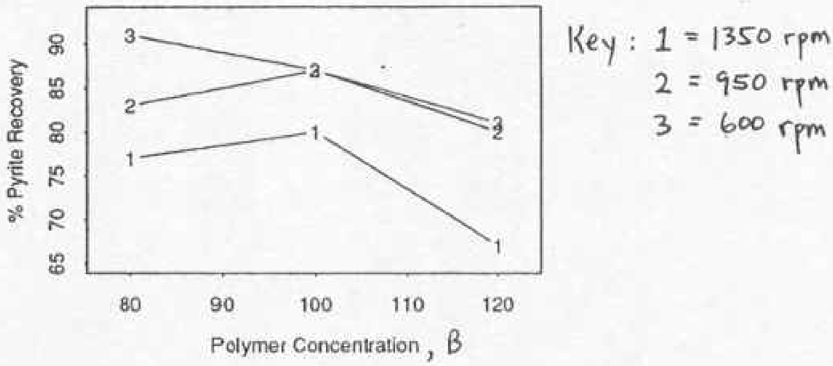
For $x_1 = 600$, the equation is

$$\begin{aligned}\hat{y}_1 &= -17.92 + .01952(600) + 2.235x_2 - .00001746(600)^2 \\ &\quad -.012083x_2^2 - .0000104(600)x_2 \\ &= -12.4876 + 2.22876x_2 - .012083x_2^2.\end{aligned}$$



It is hard to notice, but the three curves are not parallel because of the x_1x_2 term. This lack of parallelism represents an interaction (although it is very small for this fitted equation).

(g)



The plot shows that the effect on y_1 of changing Polymer Concentration depends on the Speed (there seems to be an interaction). It would not be wise to use the fitted main effects alone to summarize the data, since there may be an importantly large interaction. (The interaction seems to be almost as large as either main effect.)

The averages needed are given in the table below.

POLYMER CONC. (Factor B)

		80	100	120	
SPEED (Factor A)	1350	$\bar{y}_{11} = 77$	$\bar{y}_{12} = 80$	$\bar{y}_{13} = 67$	$\bar{y}_{1.} = 74.67$
	950	$\bar{y}_{21} = 83$	$\bar{y}_{22} = 87$	$\bar{y}_{23} = 80$	$\bar{y}_{2.} = 83.33$
	600	$\bar{y}_{31} = 91$	$\bar{y}_{32} = 87$	$\bar{y}_{33} = 81$	$\bar{y}_{3.} = 86.33$
		$\bar{y}_{.1} = 83.67$	$\bar{y}_{.2} = 84.67$	$\bar{y}_{.3} = 76.0$	$\bar{y}_{..} = 84.44$

The fitted main effects are

$$
\begin{aligned}
a_1 &= \bar{y}_{1.} - \bar{y}_{..} = -6.78 \\
a_2 &= \bar{y}_{2.} - \bar{y}_{..} = 1.89 \\
a_3 &= \bar{y}_{3.} - \bar{y}_{..} = 4.89 \\
b_1 &= \bar{y}_{.1} - \bar{y}_{..} = 2.22 \\
b_2 &= \bar{y}_{.2} - \bar{y}_{..} = 3.22 \\
b_3 &= \bar{y}_{.3} - \bar{y}_{..} = -5.44
\end{aligned}
$$

The fitted interactions are

$$
\begin{aligned}
ab_{11} &= \bar{y}_{11} - (\bar{y}_{..} + a_1 + b_1) = .11 \\
ab_{12} &= \bar{y}_{12} - (\bar{y}_{..} + a_1 + b_2) = 2.11 \\
ab_{13} &= \bar{y}_{13} - (\bar{y}_{..} + a_1 + b_3) = -2.22 \\
ab_{21} &= \bar{y}_{21} - (\bar{y}_{..} + a_2 + b_1) = -2.56 \\
ab_{22} &= \bar{y}_{22} - (\bar{y}_{..} + a_2 + b_2) = .44 \\
ab_{23} &= \bar{y}_{23} - (\bar{y}_{..} + a_2 + b_3) = 2.11 \\
ab_{31} &= \bar{y}_{31} - (\bar{y}_{..} + a_3 + b_1) = 2.44 \\
ab_{32} &= \bar{y}_{32} - (\bar{y}_{..} + a_3 + b_2) = -2.56 \\
ab_{33} &= \bar{y}_{33} - (\bar{y}_{..} + a_3 + b_3) = .11.
\end{aligned}
$$

(h) No, since x_1 and x_2 enter the equation additively. This can be seen graphically in part (d). The parallelism indicates that the effect on y_1 of changing x_2 does not depend on the setting of x_1. The equation in part (f) does imply an interaction because there is an x_1x_2 term. The plotted curves in (f) are (slightly) non-parallel.

19. (a) Exercise 18 on previous page

(b) The first least squares equation is

$$\hat{y}_2 = 60.08 - .002840x_1$$

with $R^2 = .012$. The second least squares equation is

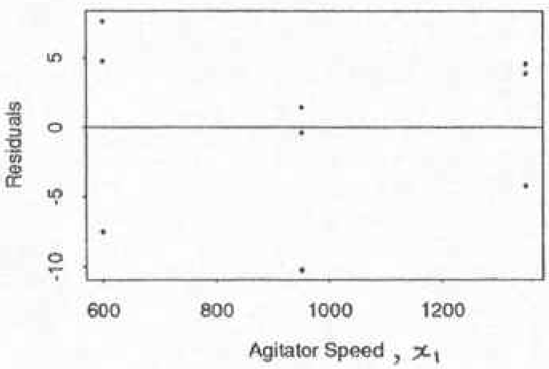
$$\hat{y}_2 = 91.50 - .3417x_2$$

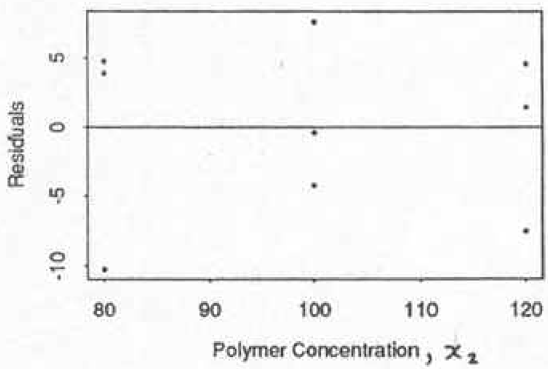
with $R^2 = .478$. The third least squares equation is

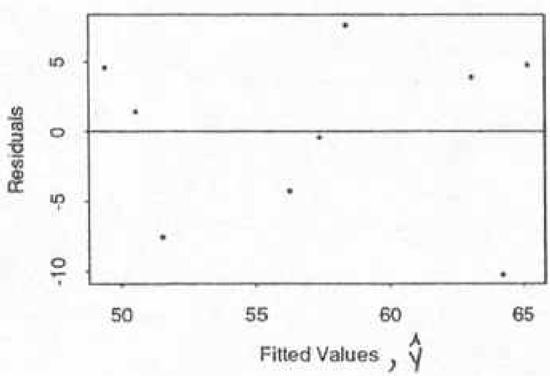
$$\hat{y}_2 = 94.25 - .002840x_1 - .3417x_2$$

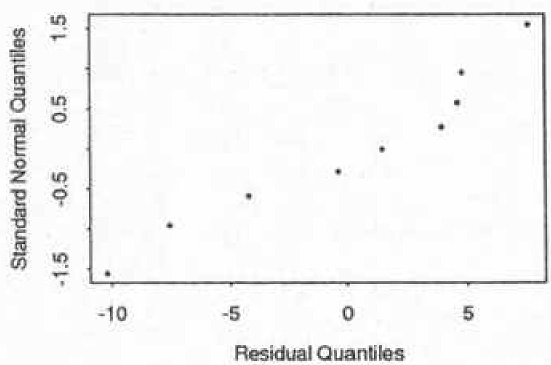
with $R^2 = .490$. The third equation is more complex than the first two, and does not do much better in fitting the data than the second equation (the increase in R^2 is small). The second equation would probably do a better job of predicting y_2, since it fits almost as well as the third and is simpler.

(c) The residuals are: 3.9221, -10.2140, 4.7919, -4.2446, $-.3807$, 7.6252, 4.5888, 1.4527, and -7.5414.









The plots of Residuals versus x_2 and $\hat{y}_2$ are patternless, revealing no problems with the model. The plot of Residuals versus x_1 has some slight hint of a positive-negative-positive pattern, indicating that there may be some curvature in the relationship between x_1 and y_2 that the model is not accounting for. The normal plot of residuals is fairly linear, so the residuals are roughly bell-shaped.

(d) For $x_1 = 1350$, the equation is

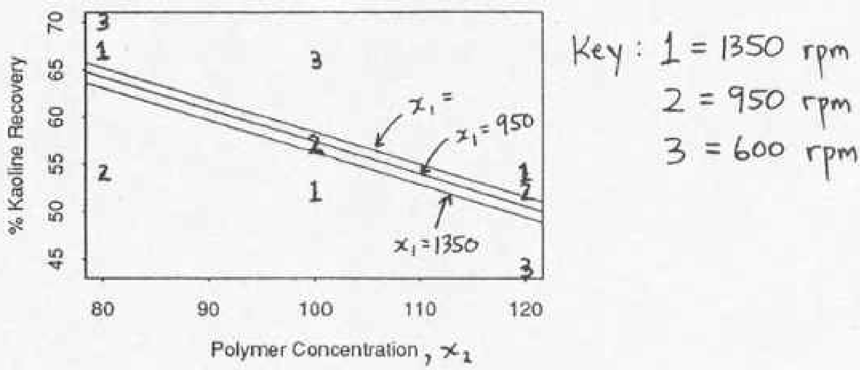
$$\begin{aligned} \hat{y}_2 &= 94.25 - .002840(1350) - .3417x_2 \\ &= 90.416 - .3417x_2 \end{aligned}$$

For $x_1 = 950$, the equation is

$$\begin{aligned} \hat{y}_2 &= 94.25 - .002840(950) - .3417x_2 \\ &= 91.552 - .3417x_2 \end{aligned}$$

For $x_1 = 600$, the equation is

$$\begin{aligned} \hat{y}_2 &= 94.25 - .002840(600) - .3417x_2 \\ &= 92.546 - .3417x_2 \end{aligned}$$



(e) For $x_1 = 1000$ and $x_2 = 110$,

$$\hat{y}_2 = 94.25 - .002840(1350) - .3417(110) = 53.82\%.$$

(f) The least squares equation is

$$\hat{y}_2 = 125.3 - .1076x_1 + .015x_2 + .00003270x_1^2 - .00375x_2^2 + .0004068x_1x_2$$

with $R^2 = .632$. This is a fairly large increase from the third equation in part (b), but the quadratic equation is much more complex.

For $x_1 = 1350$, the equation is

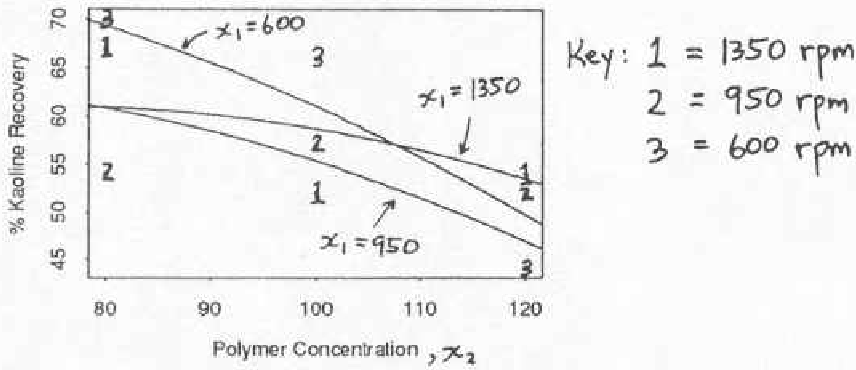
$$\begin{aligned} \hat{y}_2 &= 125.3 - .1076(1350) + .015x_2 + .00003270(1350)^2 - .00375x_2^2 + .0004068(1350)x_2 \\ &= 39.636 + .56418x_2 - .00375x_2^2. \end{aligned}$$

For $x_1 = 950$, the equation is

$$\begin{aligned} \hat{y}_2 &= 125.3 - .1076(950) + .015x_2 + .00003270(950)^2 - .00375x_2^2 + .0004068(950)x_2 \\ &= 52.592 + .40146x_2 - .00375x_2^2. \end{aligned}$$

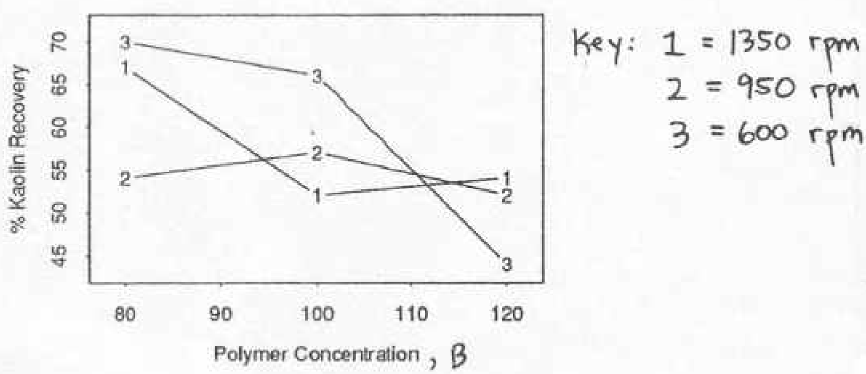
For $x_1 = 600$, the equation is

$$\begin{aligned} \hat{y}_2 &= 125.3 - .1076(600) + .015x_2 + .00003270(600)^2 - .00375x_2^2 + .0004068(600)x_2 \\ &= 72.512 + .25908x_2 - .00375x_2^2. \end{aligned}$$



The three curves are not parallel because of the x_1x_2 term. This lack of parallelism means that an interaction is implied by the equation.

(g)



The plot shows that the effect on y_2 of changing Polymer Concentration depends on the Speed (there seems to be an interaction). It would not be wise to use the fitted main effects alone to summarize the data, since there seems to be a large interaction. (The interaction seems to be as large as either main effect.)

The averages needed are given in the table below.

POLYMER CONC. (Factor B)

		80	100	120	
SPEED (Factor A)	1350	$\bar{y}_{11} = 67$	$\bar{y}_{12} = 52$	$\bar{y}_{13} = 54$	$\bar{y}_{1.} = 57.67$
	950	$\bar{y}_{21} = 54$	$\bar{y}_{22} = 57$	$\bar{y}_{23} = 52$	$\bar{y}_{2.} = 54.33$
	600	$\bar{y}_{31} = 70$	$\bar{y}_{32} = 66$	$\bar{y}_{33} = 44$	$\bar{y}_{3.} = 60.0$
		$\bar{y}_{.1} = 63.67$	$\bar{y}_{.2} = 58.33$	$\bar{y}_{.3} = 50.0$	$\bar{y}_{..} = 57.33$

The fitted main effects are

$a_1 = \bar{y}_{1.} - \bar{y}_{..} = .33$
$a_2 = \bar{y}_{2.} - \bar{y}_{..} = -3.0$
$a_3 = \bar{y}_{3.} - \bar{y}_{..} = 2.67$
$b_1 = \bar{y}_{.1} - \bar{y}_{..} = 6.33$
$b_2 = \bar{y}_{.2} - \bar{y}_{..} = 1.0$

$b_3 = \bar{y}_{.3} - \bar{y}_{..} = -7.33$

The fitted interactions are

$$\begin{aligned}
ab_{11} &= \bar{y}_{11} - (\bar{y}_{..} + a_1 + b_1) = 3.0\\
ab_{12} &= \bar{y}_{12} - (\bar{y}_{..} + a_1 + b_2) = -6.67\\
ab_{13} &= \bar{y}_{13} - (\bar{y}_{..} + a_1 + b_3) = 3.67\\
ab_{21} &= \bar{y}_{21} - (\bar{y}_{..} + a_2 + b_1) = -6.67\\
ab_{22} &= \bar{y}_{22} - (\bar{y}_{..} + a_2 + b_2) = 1.67\\
ab_{23} &= \bar{y}_{23} - (\bar{y}_{..} + a_2 + b_3) = 5.0\\
ab_{31} &= \bar{y}_{31} - (\bar{y}_{..} + a_3 + b_1) = 3.67\\
ab_{32} &= \bar{y}_{32} - (\bar{y}_{..} + a_3 + b_2) = 5.0\\
ab_{33} &= \bar{y}_{33} - (\bar{y}_{..} + a_3 + b_3) = -8.67.
\end{aligned}$$

(h) See 4-34.

20. (a) Using the Yates algorithm:

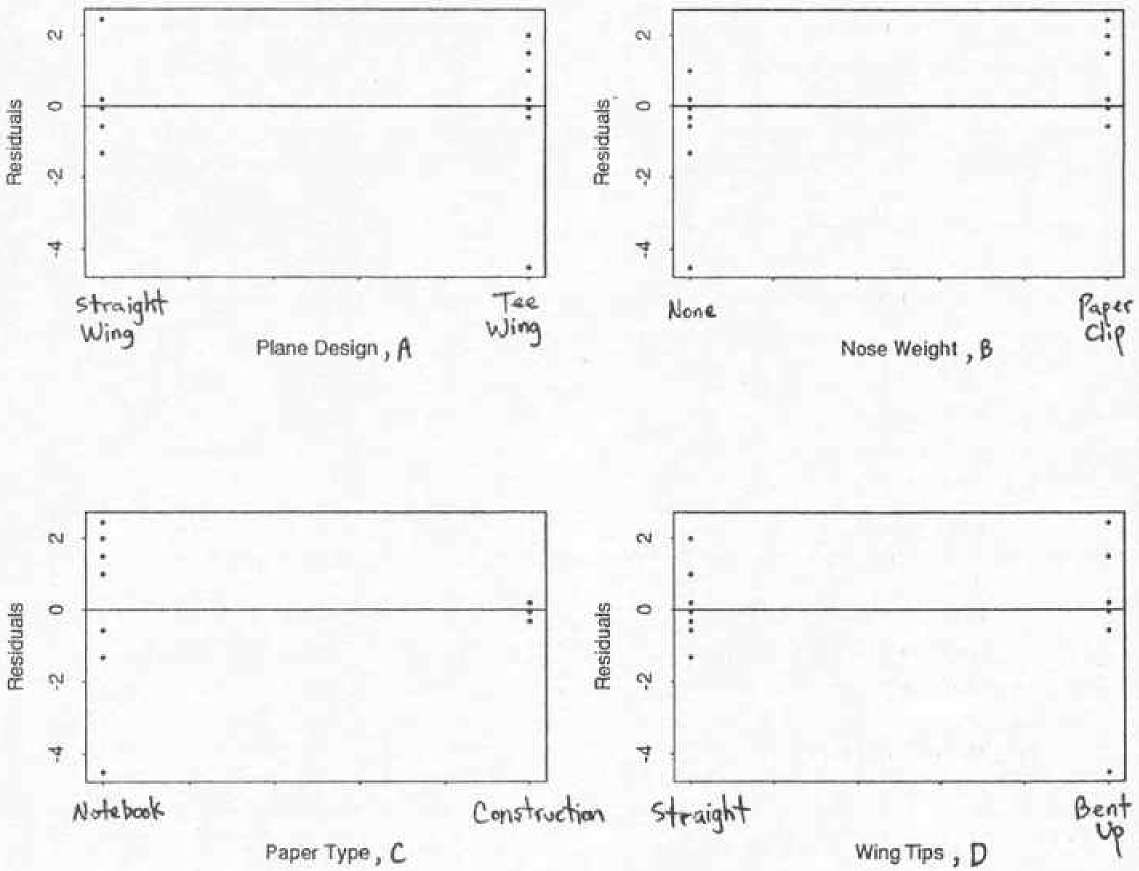
Comb	$\bar{y}$	Cycle 1	Cycle 2	Cycle 3	Cycle 4	Cycle 4 $\div$ 16	
(1)	6.25	21.75	45.25	66.00	129.75	8.10937	$= \bar{y}_{....}$
a	15.50	23.50	20.75	63.75	32.75	2.04687	$= a_2$
b	7.00	10.25	43.00	21.00	10.75	.67188	$= b_2$
ab	16.50	10.50	20.75	11.75	3.75	.23437	$= ab_{22}$
c	4.75	17.00	18.75	2.00	−46.75	−2.92188	$= c_2$
ac	5.50	26.00	2.25	8.75	−22.75	−1.42188	$= ac_{22}$
bc	4.50	10.50	9.00	1.00	−10.75	−.67188	$= bc_{22}$
abc	6.00	10.25	2.75	2.75	−2.75	−.17188	$= abc_{222}$
d	7.00	9.25	1.75	−24.50	−2.25	−.14063	$= d_2$
ad	10.00	9.50	.25	−22.25	−9.25	−.57813	$= ad_{22}$
bd	10.00	.75	9.00	−16.50	6.75	.42188	$= bd_{22}$
abd	16.00	1.50	−.25	−6.25	1.75	.10938	$= abd_{222}$
cd	4.50	3.00	.25	−1.50	2.25	.14063	$= cd_{22}$
acd	6.00	6.00	.75	−9.25	10.25	.64063	$= acd_{222}$
bcd	4.50	1.50	3.00	.50	−7.75	−.48437	$= bcd_{222}$
abcd	5.75	1.25	−.25	−3.25	−3.75	−.23437	$= abcd_{2222}$

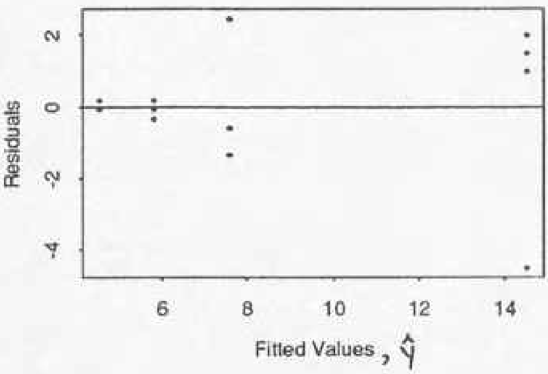
(b) The fitted effects that stand out are a_2, c_2, and ac_{22}. Since a_2 is positive, the tee wing planes generally flew farther. c_2 is negative, which means that overall, the notebook paper planes flew farther. The negative sign of ac_{22} indicates that the difference in flight distance between straight wing and tee wing planes is larger for notebook paper than it is for construction paper. (This can be seen by looking at the raw data.) Because of this interaction, the experiment should not be summarized in terms of main effects only; there is more to the story.

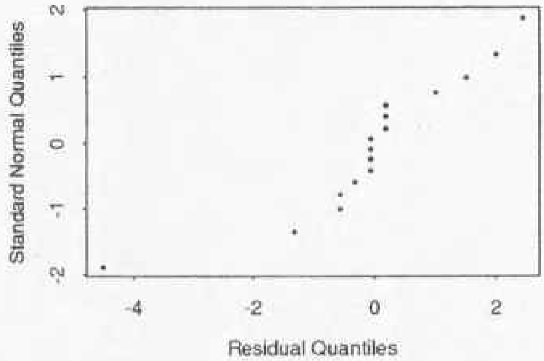
(c) Using the fitted effects and interactions for only factors A and C, the reverse Yates algorithm gives

Fitted Effect	Value	Cycle 1	Cycle 2	Cycle 3	Cycle 4 ($\hat{y}$)	
$abcd_{2222}$	0	0	0	0	5.8125	$= \hat{y}_{\text{abcd}}$
bcd_{222}	0	0	0	5.8125	4.5625	$= \hat{y}_{\text{bcd}}$
acd_{222}	0	0	-4.3437	0	5.8125	$= \hat{y}_{\text{acd}}$
cd_{22}	0	0	10.1563	4.5625	4.5625	$= \hat{y}_{\text{cd}}$
abd_{222}	0	0	0	0	14.5000	$= \hat{y}_{\text{abd}}$
bd_{22}	0	-4.3437	0	5.8125	7.5625	$= \hat{y}_{\text{bd}}$
ad_{22}	0	0	-1.5000	0	14.5000	$= \hat{y}_{\text{ad}}$
d_2	0	10.1563	6.0625	4.5625	7.5625	$= \hat{y}_{\text{d}}$
abc_{222}	0	0	0	0	5.8125	$= \hat{y}_{\text{abc}}$
bc_{22}	0	0	0	14.5000	4.5625	$= \hat{y}_{\text{bc}}$
ac_{22}	-1.42188	0	-4.3437	0	5.8125	$= \hat{y}_{\text{ac}}$
c_2	-2.92188	0	10.1563	7.5625	4.5625	$= \hat{y}_{\text{c}}$
ab_{22}	0	0	0	0	14.5000	$= \hat{y}_{\text{ab}}$
b_2	0	-1.5000	0	14.5000	7.5625	$= \hat{y}_{\text{b}}$
a_2	2.04687	0	-1.5000	0	14.5000	$= \hat{y}_{\text{a}}$
$\bar{y}_{....}$	8.10937	6.0625	6.0625	7.5625	7.5625	$= \hat{y}_{(1)}$

The residuals are computed as $y - \hat{y}$ for each observation. They are (in Yates standard order) -1.3125, 1.0000, $-.5625$, 2.0000, $.1875$, $-.3125$, $-.0625$, $.1875$, $-.5625$, -4.5000, 2.4375, 1.5000, $-.0625$, $.1875$, $-.0625$, and $-.0625$.







Each plot reveals an outlier; it is from the combination ad. There is a slight increasing pattern for the plot of Residuals versus Factor B, which indicates that B may have a small effect that the model is not accounting for. The plot of Residuals versus Paper Type shows that there is more variation in distance for notebook paper than there is for construction paper. This is also reflected in the plot of Residuals versus Fitted Values. Overall, the plots show no evidence that the fitted model is inadequate; it seems to summarize the data well.

(d)
$$R^2 = \frac{\sum(y_i-\bar{y})^2 - \sum(y_i-\hat{y}_i)^2}{\sum(y_i-\bar{y})^2} = \frac{271.9961 - 36.01563}{271.9961} = .868.$$

This is a fairly large R^2 value. Based on this and the work in part (c), this model seems to provide an effective summary of flight distance.

21. (a) The first least squares equation is

$$\hat{y} = -886 + 8.267x_1$$

with an R^2 of .115. The second least squares equation is

$$\hat{y} = 211.0 + 2.6388x_2$$

with an R^2 of .504. The third least squares equation is

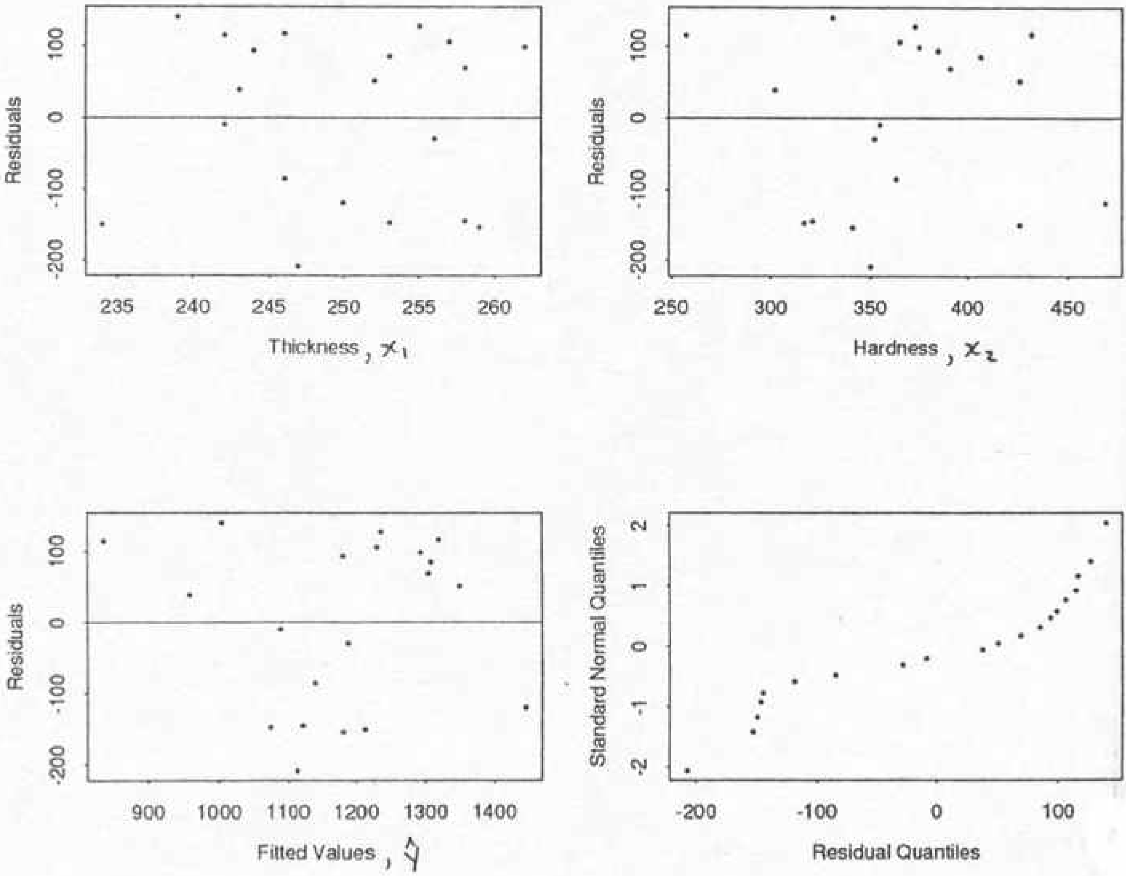
$$\hat{y} = -1674.1 + 7.612x_1 + 2.5939x_2$$

with an R^2 of .601. The third equation is more complex than the first two, but accounts for 10% more variability than the second equation. The addition of x_1 to the second equation seems to be important, so the third equation may do a better job of predicting y even though it is more complex.

(b) The correlation between x_1 and y is equal to the square root of R^2 for the first equation, .340. The correlation between x_2 and y is equal to the square root of R^2 for the second equation, .710.

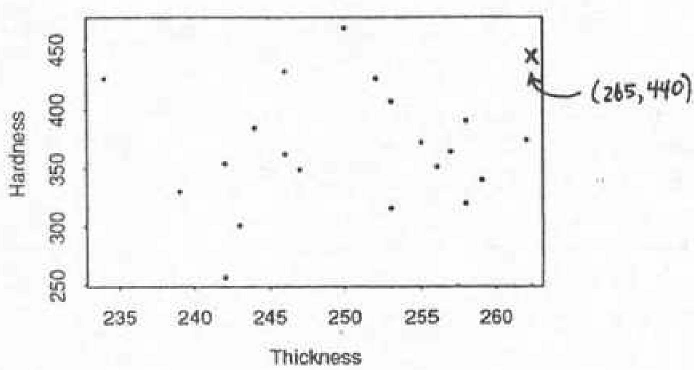
(c) Based on the R^2's and r's, Hardness seems to have a strong effect on Ballistic Limit, while Thickness seems to have a minor effect. One reason for this could be that the thicknesses used in the experiment vary over a small range. If a greater range of thicknesses were tested, the effect of thickness might be more important.

(d) The residuals are: -147.074, -144.511, -154.001, -207.999, -28.697, -85.107, 105.970, 98.970, 127.444, 69.917, 85.476, 51.804, 116.914, −118.509, 115.294, 38.957, 140.183, −8.907, 94.052, and −150.176.



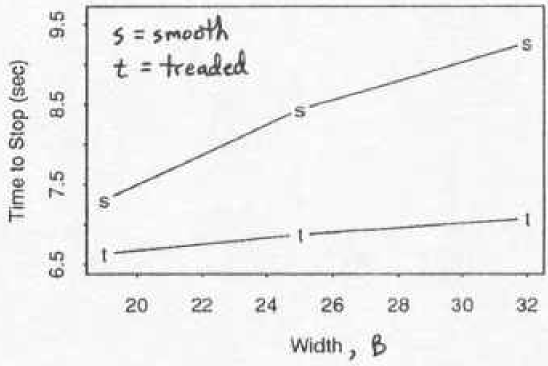
The first three plots look random, indicating no problems with the third fitted equation. The normal plot is not very linear, indicating that the residuals are not very bell-shaped

(e)



This (x_1, x_2) combination is not within the space of the data used in the study, so this would be extrapolating. There is no information from the data about the relationship between x_1, x_2, and y for these x's. More data should be collected for x's like these before making such a prediction.

22. (a)



The plot shows that, overall, Tread Type seems to have a slightly larger effect on Time to Stop than Tire Width. Smooth tires generally take longer to stop than treaded tires, and wide tires generally take longer to stop than narrow tires. The lines on the plot are slightly non-parallel, indicating the possibility of an interaction between Tread Type and Tire Width. Specifically, it looks like there might be a bigger difference between the two types of treads for larger tire widths than for smaller tire widths. The size of the interaction is not too large relative to the main effects.

(b) The averages needed are given in the table below.

		TIRE WIDTH (Factor B)			
		19c	25c	32c	
TREAD (Factor A)	Smooth	$\bar{y}_{11} = 7.30$	$\bar{y}_{12} = 8.44$	$\bar{y}_{13} = 9.27$	$\bar{y}_{1.} = 8.337$
	Treaded	$\bar{y}_{21} = 6.63$	$\bar{y}_{22} = 6.87$	$\bar{y}_{23} = 7.07$	$\bar{y}_{2.} = 6.857$
		$\bar{y}_{.1} = 6.965$	$\bar{y}_{.2} = 7.655$	$\bar{y}_{.3} = 8.17$	$\bar{y}_{..} = 7.597$

The fitted main effects are

$a_1 = \bar{y}_{1.} - \bar{y}_{..} = .74$
$a_2 = \bar{y}_{2.} - \bar{y}_{..} = -.74$
$b_1 = \bar{y}_{.1} - \bar{y}_{..} = -.632$
$b_2 = \bar{y}_{.2} - \bar{y}_{..} = .058$
$b_3 = \bar{y}_{.3} - \bar{y}_{..} = .573$

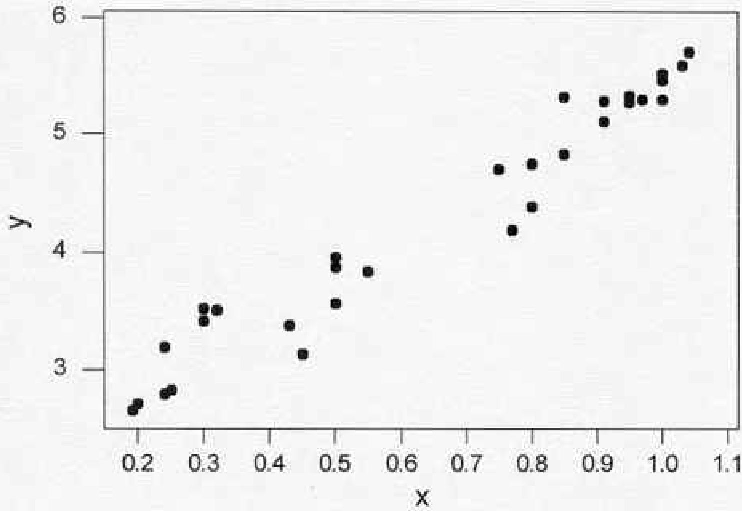
The fitted interactions are

$ab_{11} = \bar{y}_{11} - (\bar{y}_{..} + a_1 + b_1) = -.405$
$ab_{12} = \bar{y}_{12} - (\bar{y}_{..} + a_1 + b_2) = .045$
$ab_{13} = \bar{y}_{13} - (\bar{y}_{..} + a_1 + b_3) = .36$
$ab_{21} = \bar{y}_{21} - (\bar{y}_{..} + a_2 + b_1) = .405$
$ab_{22} = \bar{y}_{22} - (\bar{y}_{..} + a_2 + b_2) = -.045$
$ab_{23} = \bar{y}_{23} - (\bar{y}_{..} + a_2 + b_3) = -.36$

The a's quantify half of the average distance between the two lines on the plot. The b's quantify the average slope of the lines. The ab's quantify the lack of parallelism in the lines. The relatively large values of ab_{11} and ab_{21} reflect the lack of parallelism for the 700/19c tire width.

23 (a)

Scatterplot of Y vs X, Problem 23



(b) The sample correlation coefficient is .975. There is a strong linear relationship between y and x.

(c) $\sum(x-\ddot{x})(y-\ddot{y}) = 8.693$, $\sum(x-\ddot{x})^2 = 2.653$, $b_1 = 8.693/2.653 = 3.276$

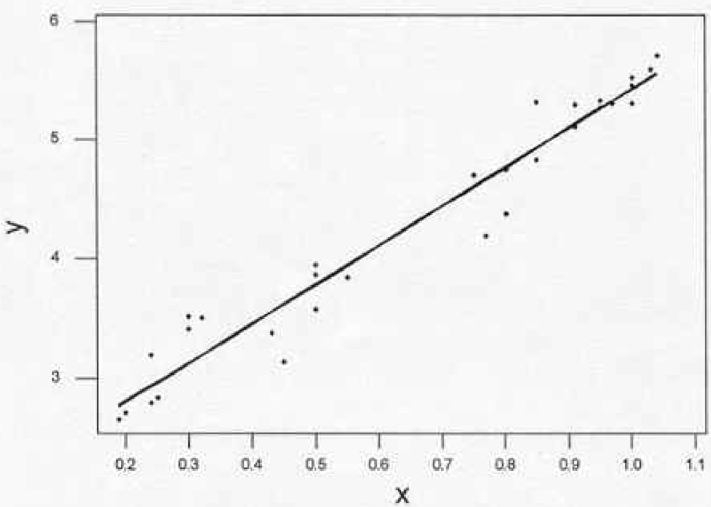
$b_o = 4.28 - 3.276(.6517) = 2.144$

$\hat{y}(x) = 2.144 + 3.276x$

Regression Plot

y = 2.14472 + 3.27664 x

S = 0.228372 R-Sq = 95.1 % R-Sq(adj) = 95.0 %



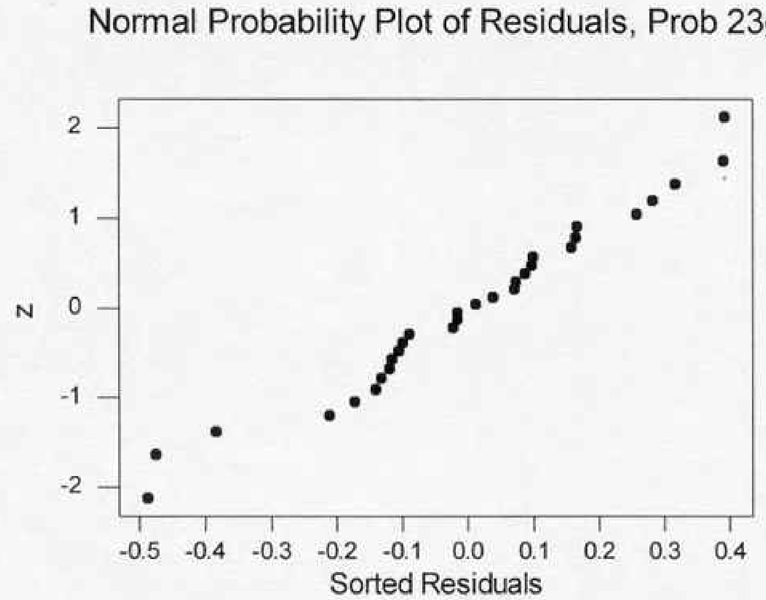
(d)3.276 increase in detonation velocity for 1g/cc increase in PETN density. A .3276 increase in detonation velocity for .1g/cc increase in PETN density.

(e) 95.1% = R^2

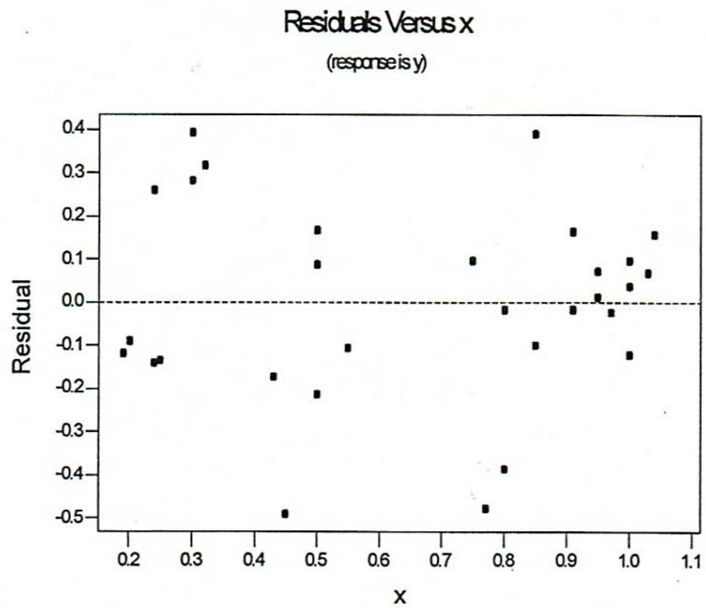
(f) $\hat{y}(x=.65) = 2.144 + 3.276(x=.65)$ = 4.2734 is the predicted detonation velocity for a PETN density of .65 g/cc.

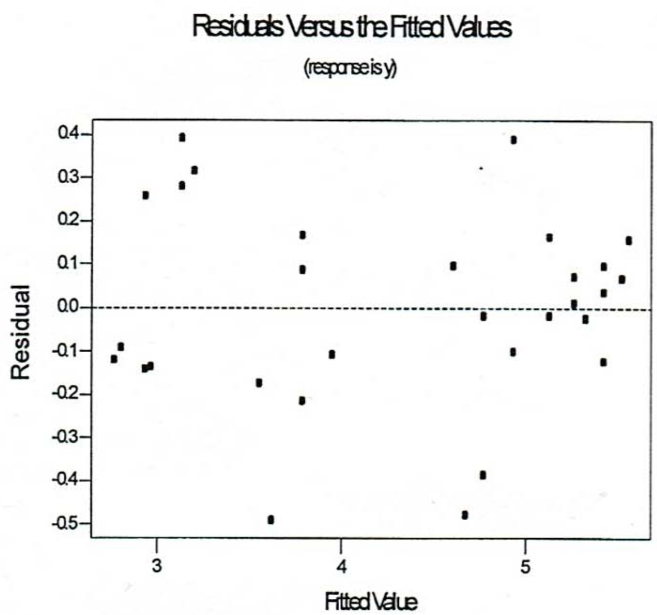
5 = 2.144 + 3.276x implies x = .872 g/cc is the "predicted" density that would produce a 5.00 km/sec detonation velocity.

(g)



Upon examination of the three plots, it seems the straight line model is appropriate and the usual assumptions (normal distribution and common variance) hold.


Residuals Versus x
(response is y)
Residual
0.4
0.3
0.2
0.1
0.0
-0.1
-0.2
-0.3
-0.4
-0.5
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.1
x


Residuals Versus the Fitted Values
(response is y)
Residual
0.4
0.3
0.2
0.1
0.0
-0.1
-0.2
-0.3
-0.4
-0.5
3
4
5
Fitted Value

Residuals.

-0.117283
-0.090050
-0.141116
0.258885
-0.133882
0.392286
0.282286
0.316753
-0.173678
-0.489210
0.166957
0.086957
-0.213043
-0.106875
0.097797
-0.477736
-0.016035
-0.386035
-0.099868
0.390133
0.163534
-0.016466
0.072468
0.012468
-0.023064
0.098636
0.038636
-0.121364
0.070337
0.157571

(h)

Regression Analysis: y versus x

```
The regression equation is
y = 2.14 + 3.28 x

30 cases used 1 cases contain missing values

Predictor        Coef     SE Coef          T        P
```

Constant	2.1447	0.1003	21.38	0.000
x	3.2766	0.1400	23.41	0.000

S = 0.2284 R-Sq = 95.1% R-Sq(adj) = 95.0%

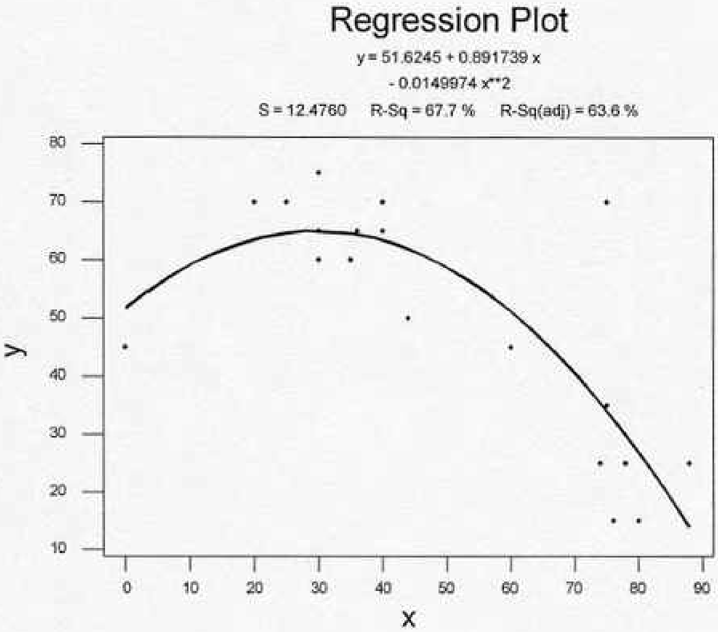
Analysis of Variance

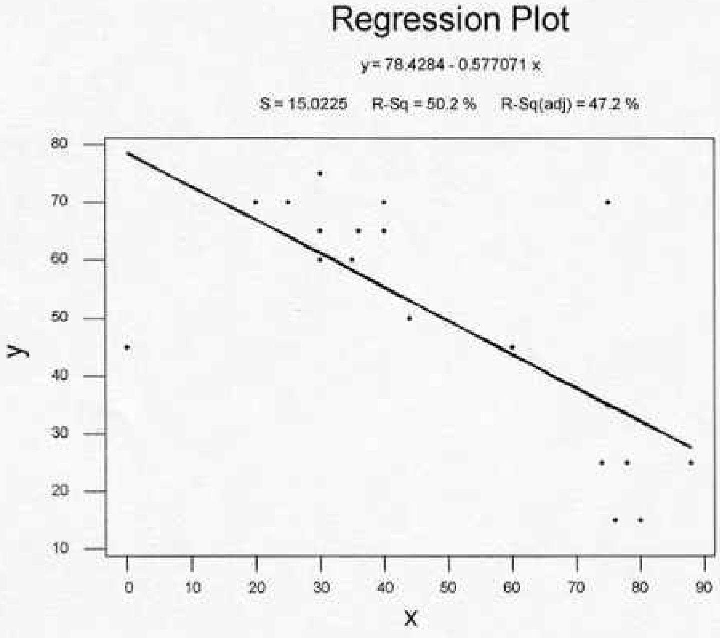
Source	DF	SS	MS	F	P
Regression	1	28.570	28.570	547.80	0.000
Residual Error	28	1.460	0.052		
Total	29	30.030			

Obs	x	y	Fit	SE Fit	Residual	St Resid
1	0.19	2.6500	2.7673	0.0769	-0.1173	-0.55
2	0.20	2.7100	2.8000	0.0757	-0.0900	-0.42
3	0.24	2.7900	2.9311	0.0711	-0.1411	-0.65
4	0.24	3.1900	2.9311	0.0711	0.2589	1.19
5	0.25	2.8300	2.9639	0.0700	-0.1339	-0.62
6	0.30	3.5200	3.1277	0.0645	0.3923	1.79
7	0.30	3.4100	3.1277	0.0645	0.2823	1.29
8	0.32	3.5100	3.1932	0.0624	0.3168	1.44
9	0.43	3.3800	3.5537	0.0520	-0.1737	-0.78
10	0.45	3.1300	3.6192	0.0504	-0.4892	-2.20R
11	0.50	3.9500	3.7830	0.0468	0.1670	0.75
12	0.50	3.8700	3.7830	0.0468	0.0870	0.39
13	0.50	3.5700	3.7830	0.0468	-0.2130	-0.95
14	0.55	3.8400	3.9469	0.0441	-0.1069	-0.48
15	0.75	4.7000	4.6022	0.0439	0.0978	0.44
16	0.77	4.1900	4.6677	0.0449	-0.4777	-2.13R
17	0.80	4.7500	4.7660	0.0466	-0.0160	-0.07
18	0.80	4.3800	4.7660	0.0466	-0.3860	-1.73
19	0.85	4.8300	4.9299	0.0501	-0.0999	-0.45
20	0.85	5.3200	4.9299	0.0501	0.3901	1.75
21	0.91	5.2900	5.1265	0.0552	0.1635	0.74
22	0.91	5.1100	5.1265	0.0552	-0.0165	-0.07
23	0.95	5.3300	5.2575	0.0590	0.0725	0.33
24	0.95	5.2700	5.2575	0.0590	0.0125	0.06
25	0.97	5.3000	5.3231	0.0610	-0.0231	-0.10
26	1.00	5.5200	5.4214	0.0642	0.0986	0.45
27	1.00	5.4600	5.4214	0.0642	0.0386	0.18
28	1.00	5.3000	5.4214	0.0642	-0.1214	-0.55
29	1.03	5.5900	5.5197	0.0674	0.0703	0.32
30	1.04	5.7100	5.5524	0.0685	0.1576	0.72
31	0.65	*	4.2745	0.0417	*	*

R denotes an observation with a large standardized residual

24(a)

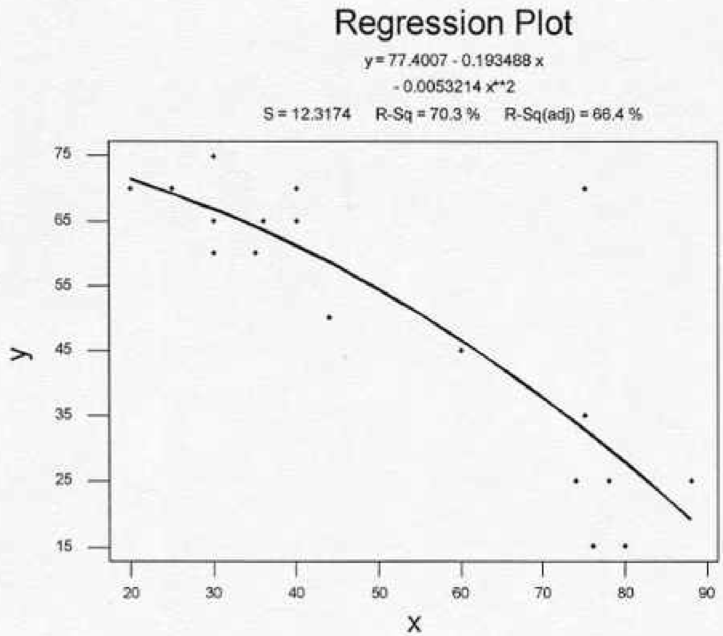




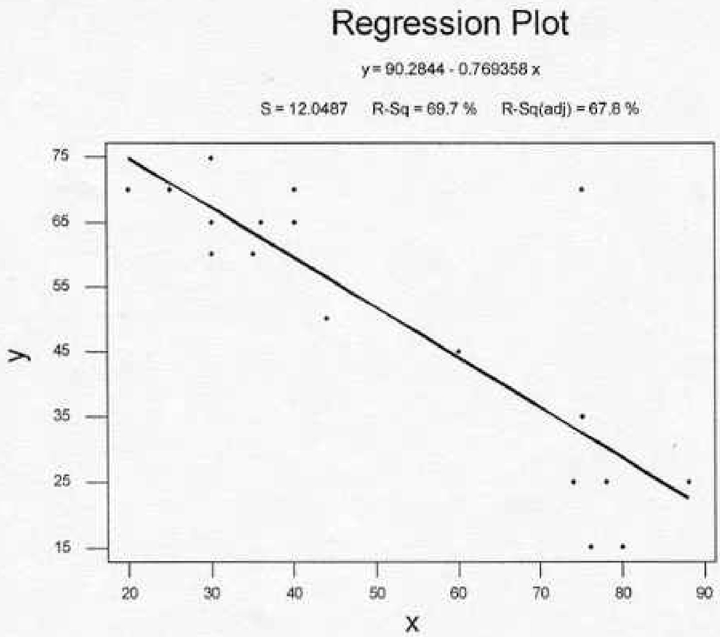
50.2% of raw variability is accounted for using the straight-line approach. 67.7% of the raw variability is accounted for using the quadratic approach.

(b)

Leaving out x = 0 and y = 45 gives:



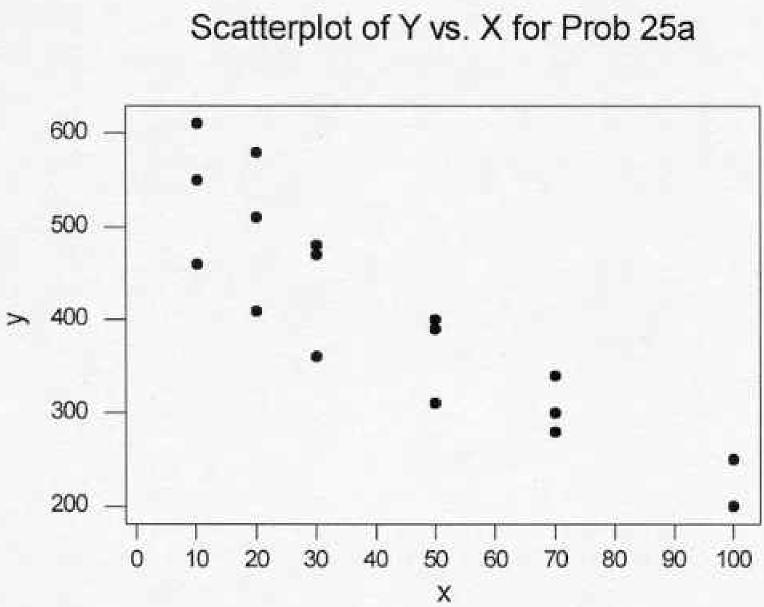
69.7% of the raw variability is accounted for using the straight-line model. 70.3% of the raw variability is accounted for using the quadratic model. A single point can



significantly affect how well a particular model fits the data (in this case, the straight-line model).

(c) Taking the derivative and setting it equal to zero and solving for x (from the fitted quadratic model in (a)) gives x = 29.73 (in 10^{-3} in. above .400). I would screw a few studs into the block at this depth and measure the torque required to strip the stud out. If the torque was close to the maximum given at x = 29.73, then recommend this level for the assembly process.

25 (a)

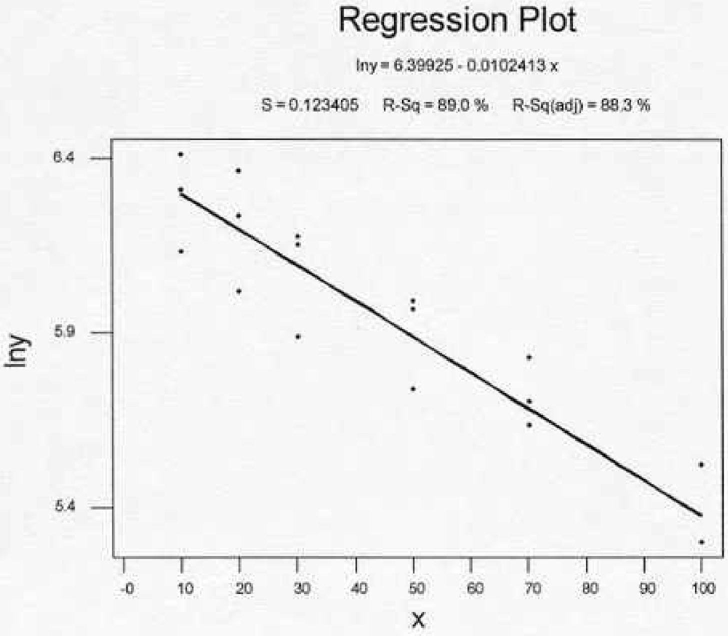


(b)

The graph relating lnY to X appears to be slightly more linear (see (b)).

(c) The sample correlation between lnY and X is r = - .943. A strong linear relationship exists between the natural log of Y = grip force and X= % drag. As X increases, lnY decreases in a linear fashion.

(d) $\sum(x-\bar{x})^2 = 19{,}857.59$, $\sum(y'-\bar{y}')(x-\bar{x}) = -203.367$

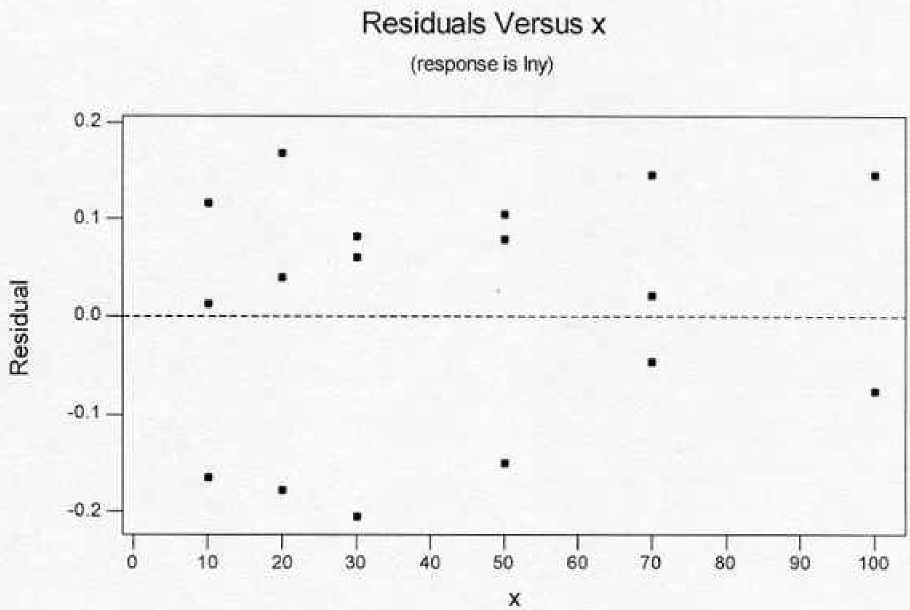


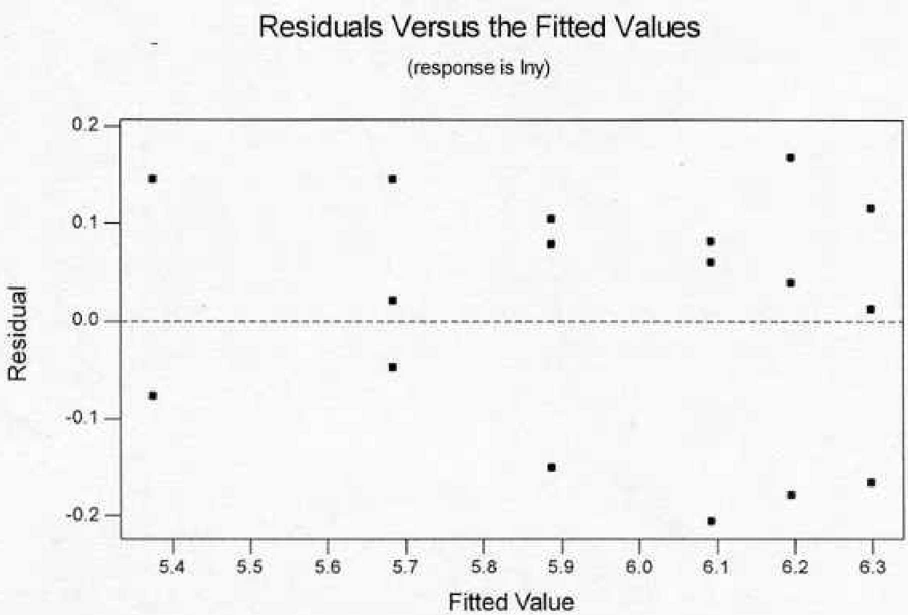
(e) 10(-.0102) = -.102 change in log grip force when drag is increased by 10% , i.e., from, say 20% to 30% or from 30% to 40%. The raw grip force after increasing drag by 10% is about 90.3% of the raw grip force before the drag was increased by 10%.

(f) 89% of the raw variability in log grip force is accounted for in the fitting of a line to the data in part (d).

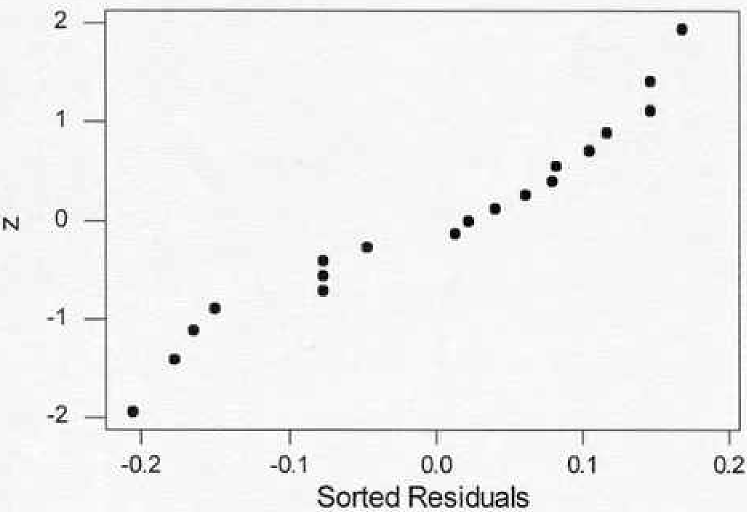
(g) $\hat{y}(x=40) = 6.399 - .0102(40) = 5.9896$. This is the log grip force (log lbs.) predicted with 40% drag. Exp(5.9896) = 399.25 lbs. is the predicted raw-grip force.

(h)


Residuals Versus x
(response is lny)
Residual
0.2
0.1
0.0
-0.1
-0.2
0
10
20
30
40
50
60
70
80
90
100
x


Residuals Versus the Fitted Values
(response is lny)
Residual
0.2
0.1
0.0
-0.1
-0.2
5.4
5.5
5.6
5.7
5.8
5.9
6.0
6.1
6.2
6.3
Fitted Value

Normal Probability Plot of Residuals Prob 25h



It seems linearity between y' and x is reasonable together with the usual assumptions of normal distribution and common variance.

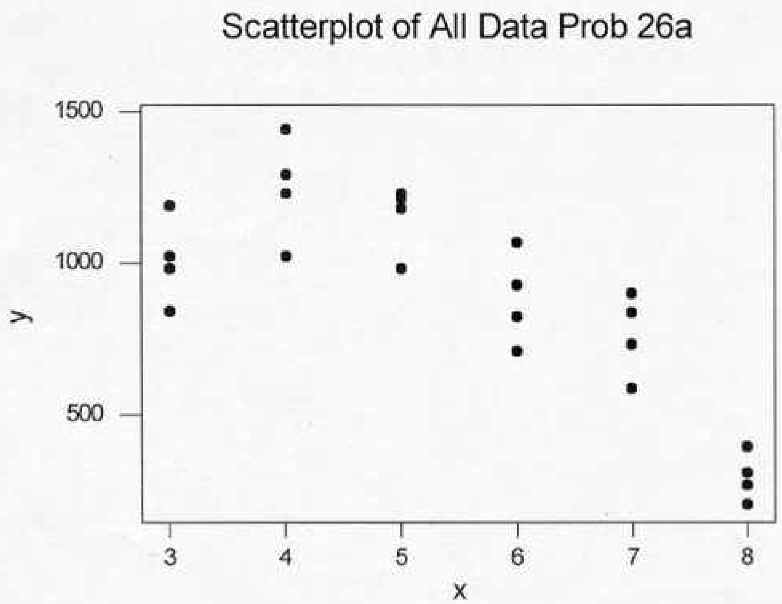
(i)

Regression Analysis: lny versus x

```
The regression equation is
lny = 6.40 - 0.0102 x

19 cases used 1 cases contain missing values

Predictor         Coef      SE Coef          T        P
Constant       6.39925      0.05172     123.72    0.000
x           -0.0102413    0.0008749     -11.71    0.000

S = 0.1234       R-Sq = 89.0%     R-Sq(adj) = 88.3%

Analysis of Variance

Source            DF          SS          MS          F        P
Regression         1      2.0866      2.0866     137.02    0.000
Residual Error    17      0.2589      0.0152
Total             18      2.3455

Obs          x        lny          Fit       SE Fit    Residual    St Resid
```

1	10	6.3099	6.2968	0.0447	0.0131	0.11
2	10	6.1312	6.2968	0.0447	-0.1656	-1.44
3	10	6.4135	6.2968	0.0447	0.1166	1.01
4	20	6.2344	6.1944	0.0383	0.0400	0.34
5	20	6.0162	6.1944	0.0383	-0.1783	-1.52
6	20	6.3630	6.1944	0.0383	0.1686	1.44
7	30	6.1527	6.0920	0.0330	0.0607	0.51
8	30	5.8861	6.0920	0.0330	-0.2059	-1.73
9	30	6.1738	6.0920	0.0330	0.0818	0.69
10	50	5.9661	5.8872	0.0283	0.0790	0.66
11	50	5.7366	5.8872	0.0283	-0.1506	-1.25
12	50	5.9915	5.8872	0.0283	0.1043	0.87
13	70	5.7038	5.6824	0.0335	0.0214	0.18
14	70	5.6348	5.6824	0.0335	-0.0476	-0.40
15	70	5.8289	5.6824	0.0335	0.1466	1.23
16	100	5.5215	5.3751	0.0525	0.1463	1.31
17	100	5.2983	5.3751	0.0525	-0.0768	-0.69
18	100	5.2983	5.3751	0.0525	-0.0768	-0.69
19	100	5.2983	5.3751	0.0525	-0.0768	-0.69
20	40	*	5.9896	0.0295	*	*

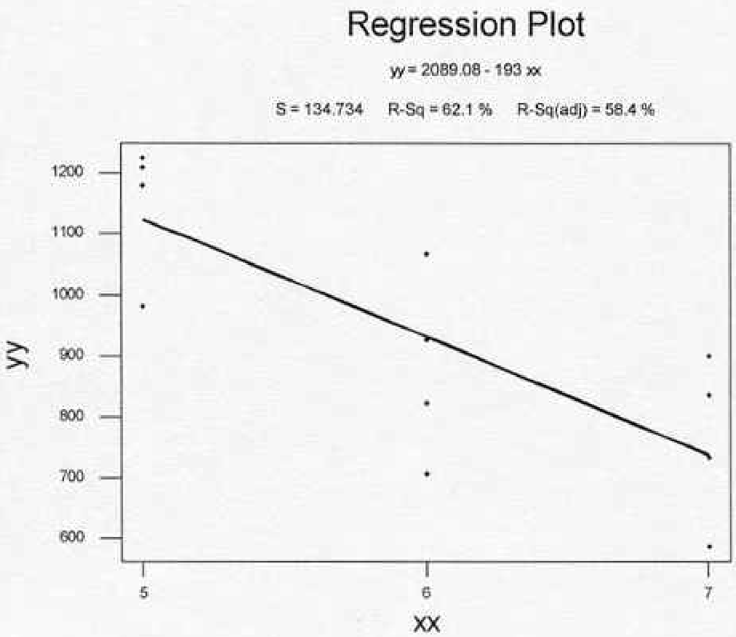
26 (a)



For X= 5 to 7 the graph looks linear. For all the data, the plot looks curvilinear (like a parabola).

(b) The correlation coefficient between X and Y for X = 5, 6, and 7 is r = -.778. It appears there is a strong linear relationship between Y and X for X from 5 to 7.

(c)



$\sum(x-\bar{x})^2$ = 7.998 , $\sum(x-\bar{x})(y-\bar{y})$ = -1543.697

b_1 = -193 , b_0 = $\bar{y}$ - $b_1\bar{x}$ = 2089.08

$\hat{y}(x) = 2089.08 - 193x$

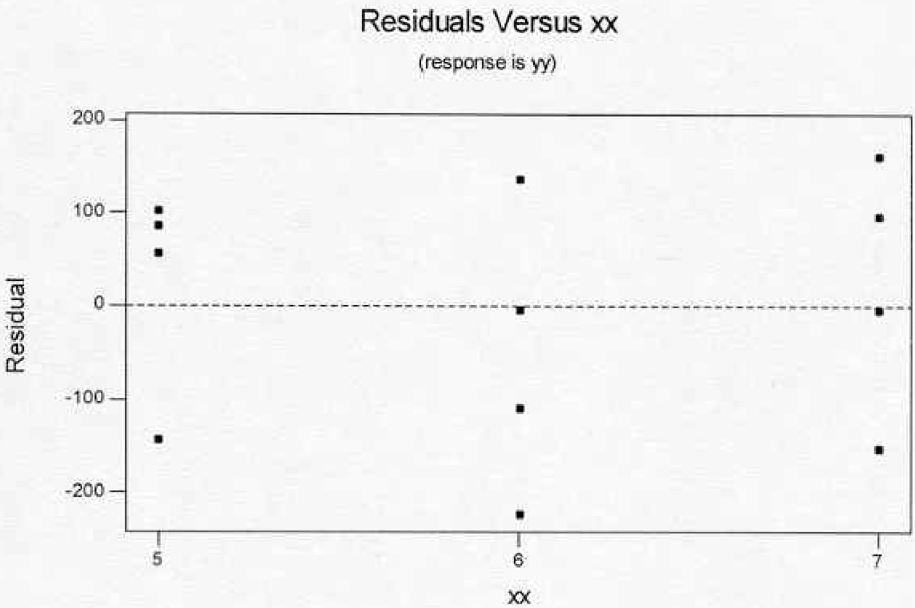
(d) b_1 = -193 , i.e., there is a decrease of 193 in permeability for 1% increase in asphalt content (change in x of 1).

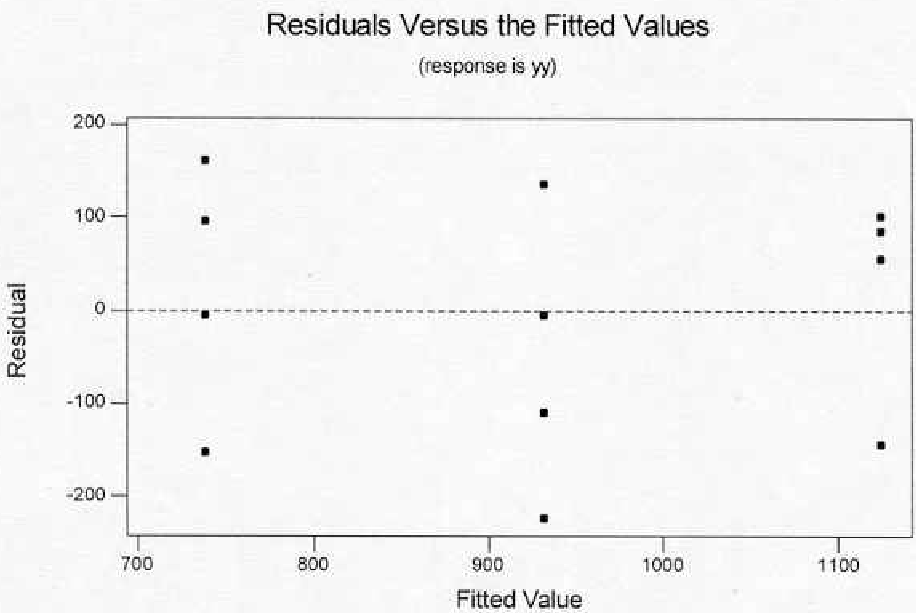
(e)R^2 = 62.1% of raw variability in permeability is accounted for by the straight-line fit.

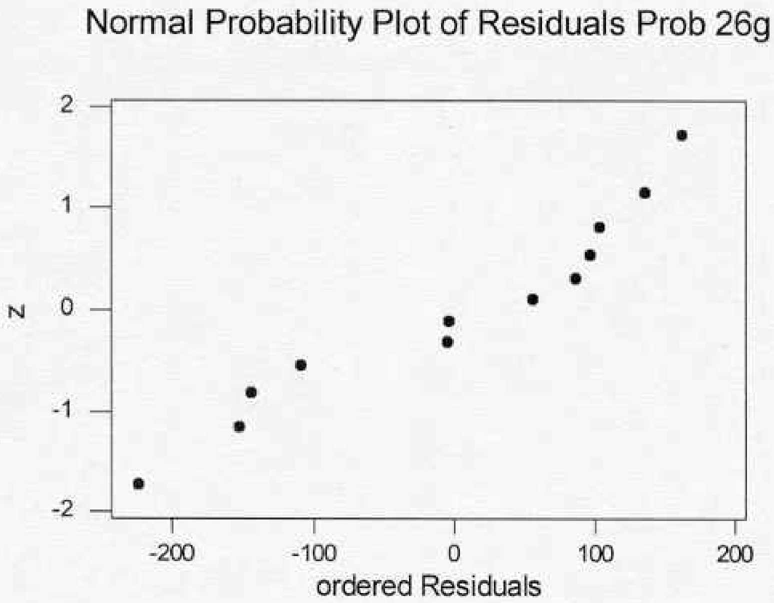
(f) $\hat{y}(x=5.5) = 2089.08 - 193(5.5) =$ 1027.6

(g) Three Plots

It seems the straight-line model fits well the data for X=5, 6, and 7. The common variance assumption and normal distribution seems reasonable.


Residuals Versus xx
(response is yy)
200
100
0
-100
-200
Residual
5
6
7
xx


Residuals Versus the Fitted Values
(response is yy)
200
100
0
-100
-200
Residual
700
800
900
1000
1100
Fitted Value



(h)

Results for: prob 26.mtw

Plot y * x

Correlations: xx, yy

```
Pearson correlation of xx and yy = -0.788
P-Value = 0.002
```

```
Macro is running ... please wait
```

Regression Analysis: yy versus xx

```
The regression equation is
yy = 2089.08 - 193 xx

S = 134.734       R-Sq = 62.1 %       R-Sq(adj) = 58.4 %
```

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	1	297992	297992	16.4153	0.002
Error	10	181533	18153		
Total	11	479525			

Fitted Line Plot: yy versus xx

Regression Analysis: yy versus xx

The regression equation is
yy = 2089 - 193 xx

12 cases used 1 cases contain missing values

Predictor	Coef	SE Coef	T	P
Constant	2089.1	288.4	7.24	0.000
xx	-193.00	47.64	-4.05	0.002

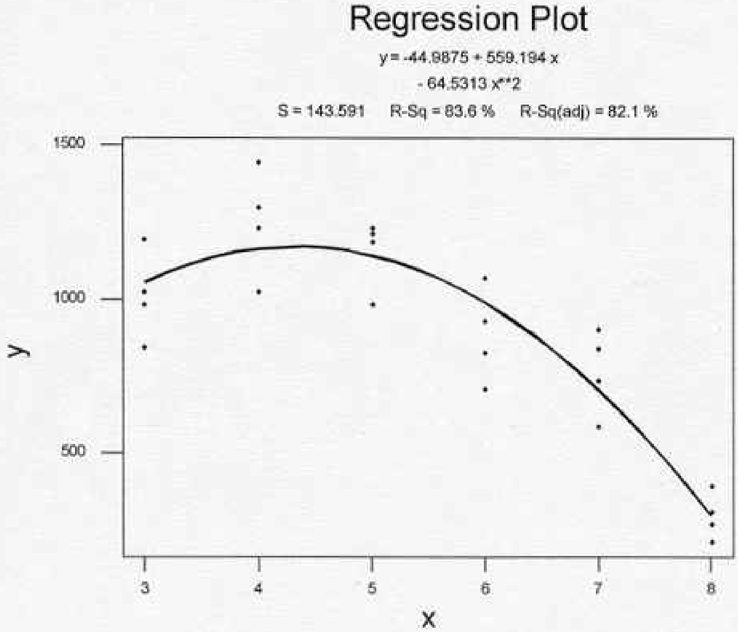
S = 134.7 R-Sq = 62.1% R-Sq(adj) = 58.4%

Analysis of Variance

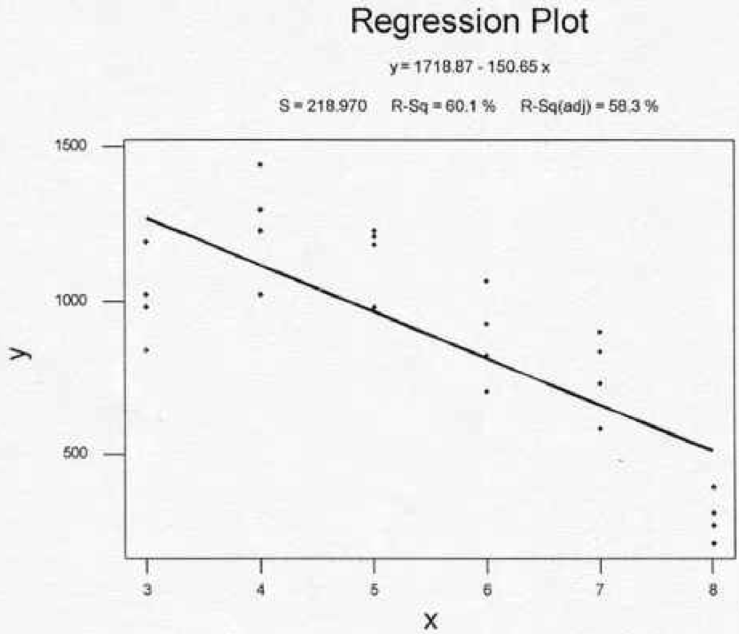
Source	DF	SS	MS	F	P
Regression	1	297992	297992	16.42	0.002
Residual Error	10	181533	18153		
Total	11	479525			

Obs	xx	yy	Fit	SE Fit	Residual	St Resid
1	5.00	1227.0	1124.1	61.5	102.9	0.86
2	5.00	1180.0	1124.1	61.5	55.9	0.47
3	5.00	980.0	1124.1	61.5	-144.1	-1.20
4	5.00	1210.0	1124.1	61.5	85.9	0.72
5	6.00	707.0	931.1	38.9	-224.1	-1.74
6	6.00	927.0	931.1	38.9	-4.1	-0.03
7	6.00	1067.0	931.1	38.9	135.9	1.05
8	6.00	822.0	931.1	38.9	-109.1	-0.85
9	7.00	835.0	738.1	61.5	96.9	0.81
10	7.00	900.0	738.1	61.5	161.9	1.35
11	7.00	733.0	738.1	61.5	-5.1	-0.04
12	7.00	585.0	738.1	61.5	-153.1	-1.28
13	5.50	*	1027.6	45.6	*	*

(i) Quadratic Fit to All data



(j) Linear Fit and Plot to All data

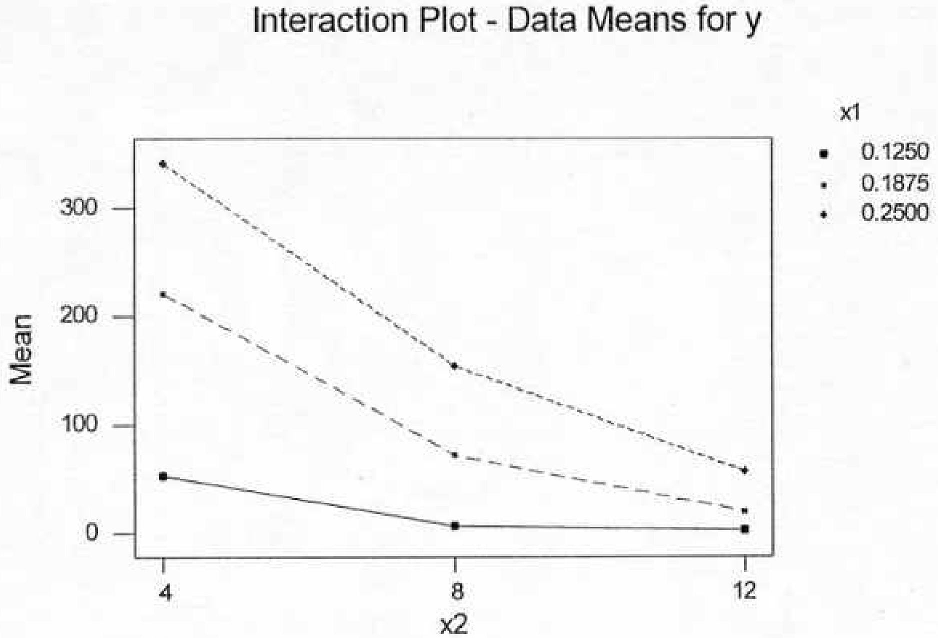


Since R^2 = 83.6% for the quadratic model and only 60.1% for the straight-line model, it seems the quadratic model is much better and should be implemented.

(k) For the quadratic model using the whole set of data, $\hat{y}(x = 5.5) = 1078.51$. In (f), the prediction was 1027.6. We see a large difference in the prediction. Since all the data was used in (k) and curvature was permitted, prediction based on the quadratic model is preferred, although the prediction based on the linear model with restricted data, is reasonable.

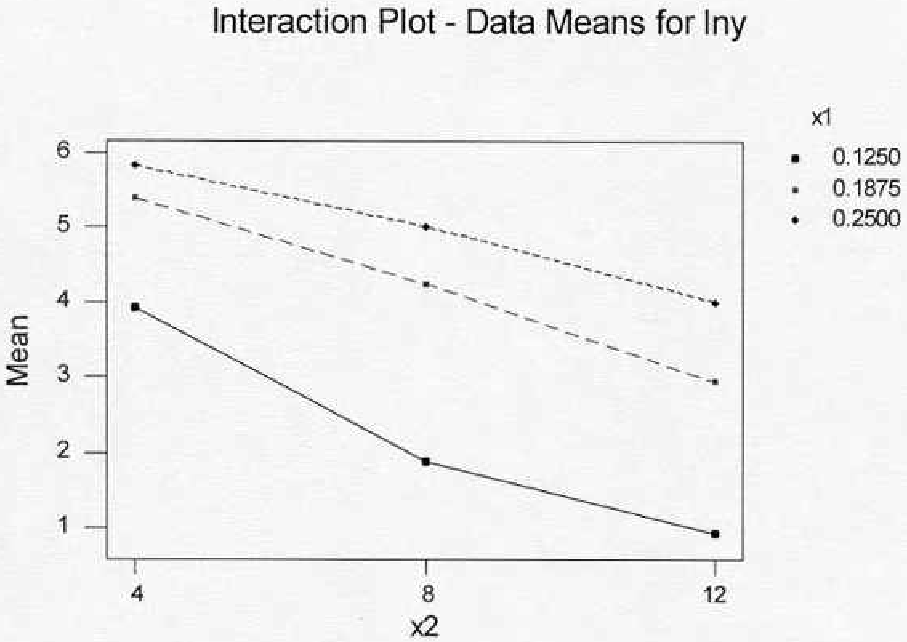
(l) For the quadratic model using the whole set of data, $\hat{y}(x = 2) = 815.28$.
For the straight-line model using the whole set of data, $\hat{y}(x = 2) = 1417.57$.
For the straight-line model using only the restricted set of data $\hat{y}(x = 2) = 1703.08$. Since x=2 is significantly outside the domain of the observed x values, extrapolating (extending) the fitted model is very unwise. It is unknown whether a straight-line or quadratic model is appropriate for x values outside the domain of the observed x values. In (f) and (k), x = 5.5 is inside the domain of the x values and therefore predictions made at x = 5.5 can be legitimately considered as reasonable.

27(a)



It appears that y is not linear in x2 for each fixed x1.

(b)



It seems lny is linear in x2 for each x1.

(c) $\ln y = -0.261 + 21.6\, x1$, $R^2 = .515$

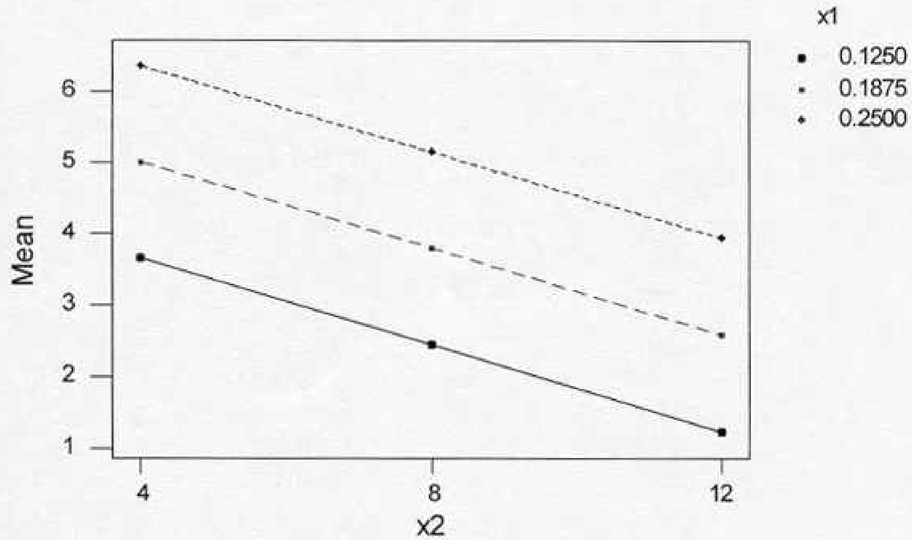
$\ln y = 6.22 - 0.303\, x2$, $R^2 = .414$

$\ln y = 2.17 + 21.6\, x1 - 0.303\, x2$, $R^2 = .929$

It seems the model relating lny to x1 and x2 as a "plane" fits best according to R^2.

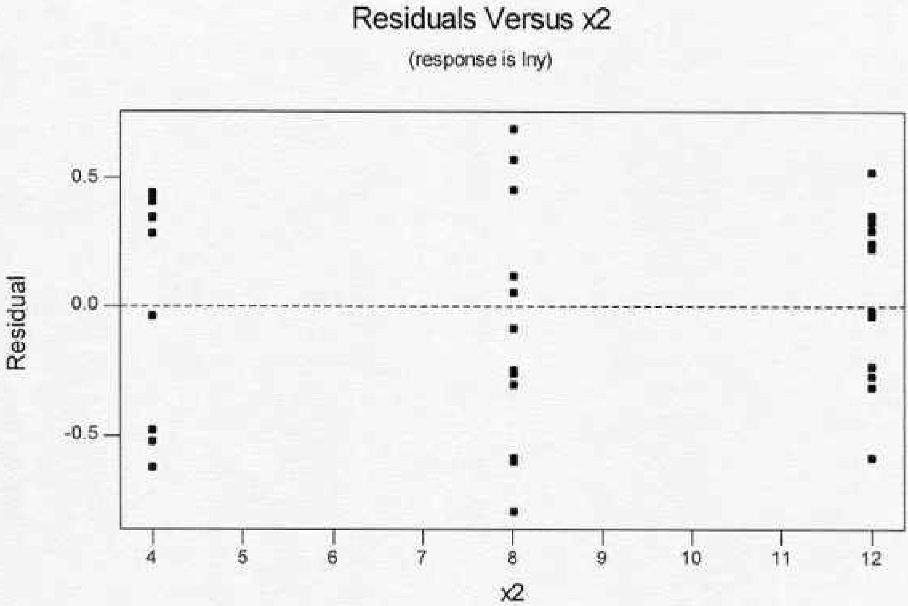
(d)

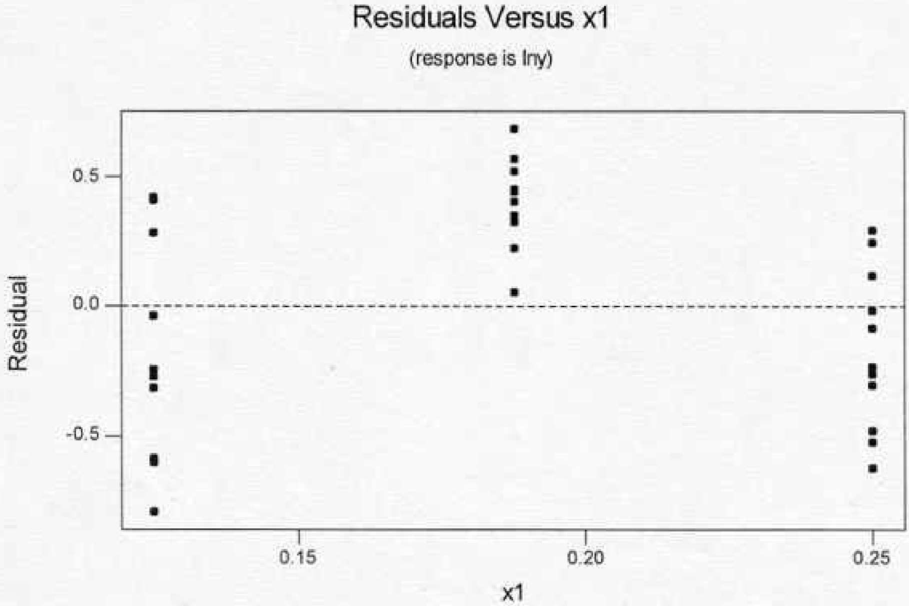
Interaction Plot - Data Means for FITS1

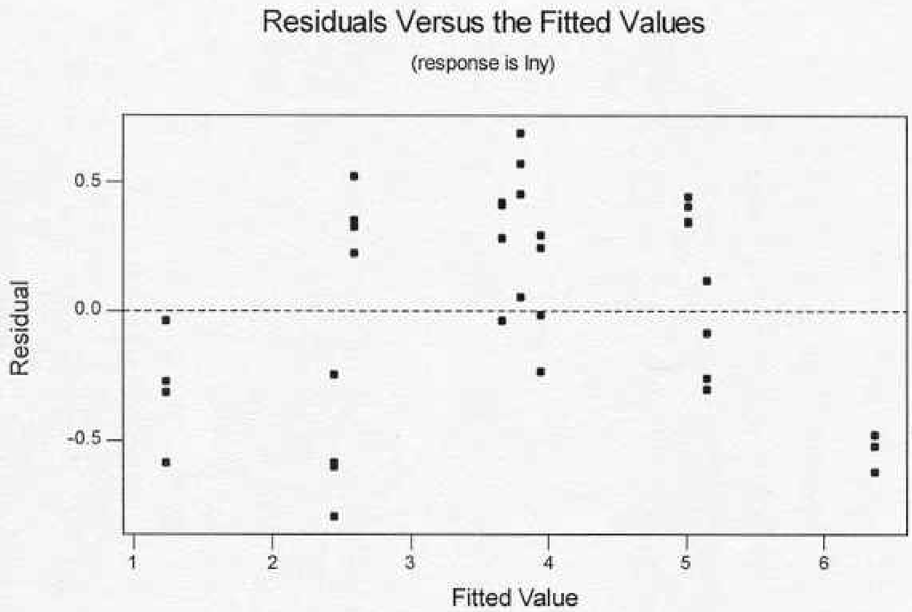


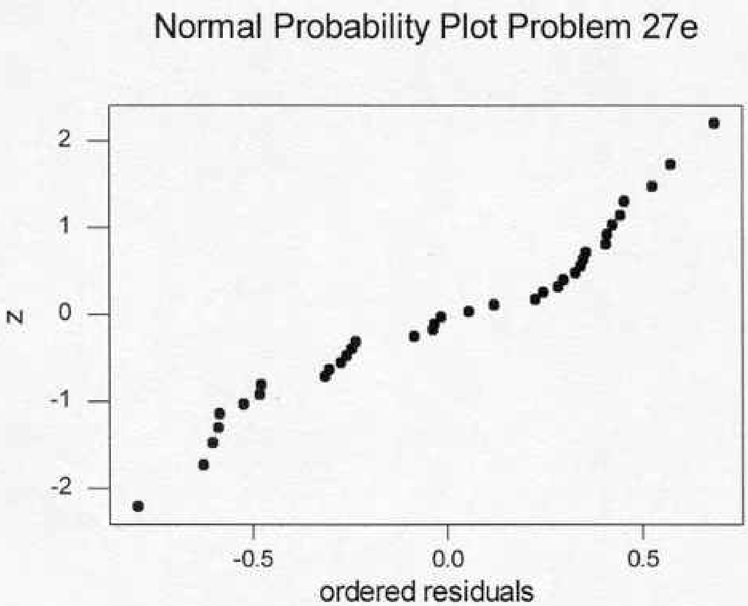
predicted lny(x1=.2, x2=10) = 3.46157. So the predicted "y" is exp(3.46157) = 31.8669. Since the ordered pair (x1 = .5, x2 = 24) is outside the domain of the (x1,x2) ordered pairs used in the study, it is not wise to extrapolate the fitted plane and predicted lny or for that matter y at (x1 = .5, x2 = 24).

(e)


Residuals Versus x2
(response is lny)
Residual
0.5
0.0
-0.5
4
5
6
7
8
9
10
11
12
x2


Residuals Versus x1
(response is lny)
Residual
0.5
0.0
-0.5
0.15
0.20
0.25
x1



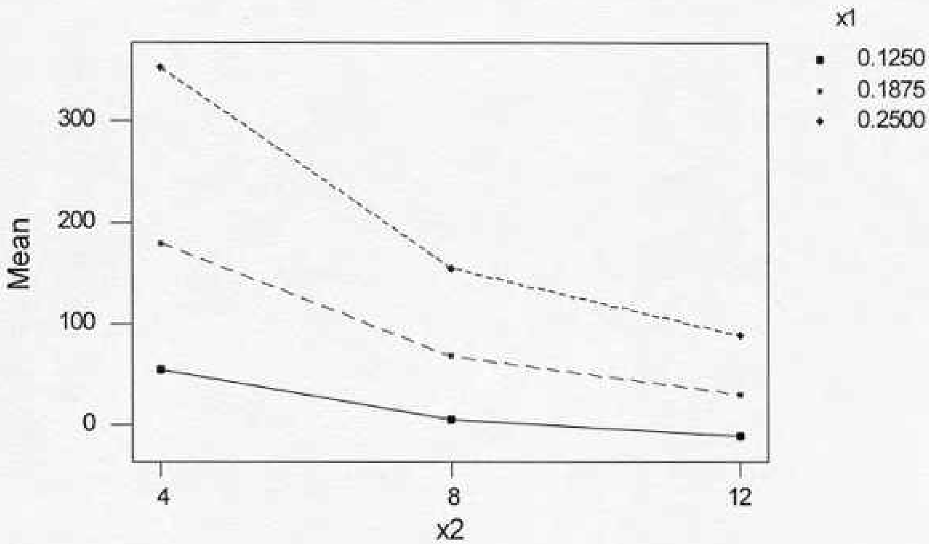


Yes, these plots suggest something concerning the x1 variable needs to be addressed (see the residual plot vs. X1). Also, the residual plot vs the fitted values suggests there needs to be some change in the model (possibly involving a transformed version of the x1 variable).

(f) The fitted equation is : $\ln y = 14.9 + 3.96 \ln X1 - 2.18 \ln X2$ and $R^2 = .948$.

The fitted equation y as a function of X3 is: $y = - 44.1 + 25443\ x3$ and $R^2 = .955$.

Interaction Plot - Data Means for FITS4



(g) low X1 = 20.3 - 102.6333 = -82.333
medX1 = 103.7-102.6333 = 1.0667
hi X1 = 183.9 - 102.6333 = 81.2667

lowX2 = 205 - 102.6333 = 102.3667
medX2 = 77 - 102.6333 = -25.6333
hiX2 = 26 -102.633 = -76.6333

LowX1/LowX2 = 51.675 - (102.6333 -82.33+102.3667) = -70.995

LowX1/medX2 = 6.725 -(102.6333 - 82.33 -25.6333) = 12.055

LowX1/hiX2 = 2.575 - (102.6333 - 82.33-76.6333) = 58.905

MedX1/LowX2 = 220.8 - (102.6333 + 1.0667 + 102.3667) = 14.733

MedX1/MedX2 = 71.275 - (102.6333 + 1.0667- 25.6333) = -6.7917

MedX1/hiX2 = 19.075 - (102.6333 + 1.0667 - 76.6333) = -7.9917

HiX1/LowX2 = 342.425 - (102.6333 + 81.2667 + 102.3667) = 56.1583

HiX1/MedX2 = 153.025 - (102.6333 + 81.2667 - 25.6333) = -5.2417

HiX1/HiX2 = 56.325 - (102.6333 + 81.2667 - 76.6333) = -50.9417.

. The Low and Hi levels of X1 have larger effects than the interaction terms. The Low and Hi levels of X2 also have larger effects than the interaction terms. It does seem that interaction effects are important using the "raw" data.

(h) Using the lnYs we have:

LowX1: 2.2471 - 3.7977 = -1.5506
MedX1: 4.1928 - 3.7977 = .3951
HiX1: 4.9532 - 3.7977 = 1.1555

Low X2: 5.0531 - 3.7977 = 1.2554
Med X2: 3.7139 - 3.7977 = - .0838
Hi X2: 2.6261 - 3.7977 = -1.1716

LowX1/LowX2: 3.92872 - (3.7977 - 1.5506 + 1.2554) = .4262
LowX1/MedX2: 1.88568 - (3.7977 - 1.5506 -.0838) = -.27762
LowX1/HiX2: .92689 - (3.7977 - 1.5506 - 1.1716) = - .14861

MedX1/LowX2: 5.39637 - (3.7977 + .3951 + 1.2554) = - .05183
MedX1/MedX2: 4.23968 -(3.7977 + .3951 - .0838) = .13068
MedX1/HiX2: 2.94249 -(3.7977 + .3951 -1.1716) = - .07871

HiX1/LowX2: 5.8343 - (3.7977 + 1.1555 + 1.2554) = - .3743
HiX1/MedX2: 5.01644 - (3.7977 + 1.1555 - .0838) = .14704
HiX1/HiX2: 4.00888 - (3.7977 + 1.1555 - 1.1716) = .22728

The same main effects are larger than the interactions as when the data were not transformed. The plot in (b) suggests that working with lnYs eliminates interaction effects.

28 (a)

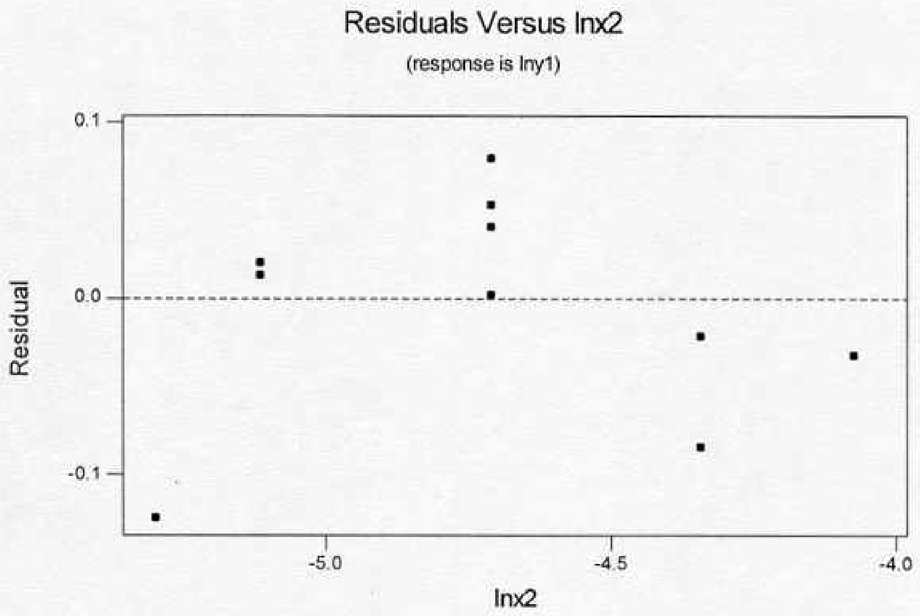
$\hat{y}_1' = 7.0752 + .9935x_1'$ $R^2 = .417$

$\hat{y}_1' = 9.1283 + .6770x_2'$ $R^2 = .549$

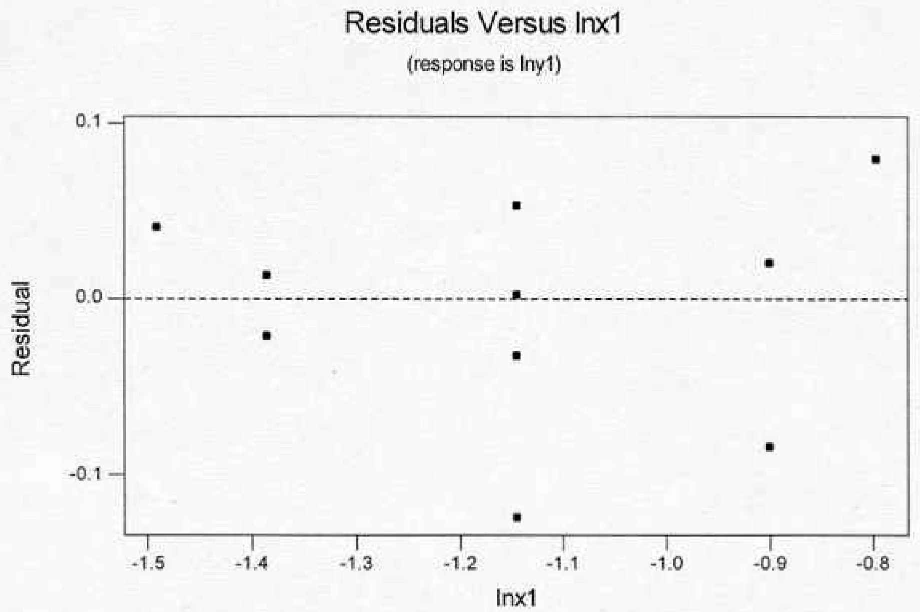
$\hat{y}_1' = 10.2665 + .99372x_1' + .6772x_2'$ $R^2 = .966$

It appears the 3rd equation fits well (without having looked at the residuals). Also, since the coefficients of x_1' and x_2' are almost the same when the other variable is not in the model, suggests there is little correlation between x_1' and x_2'.

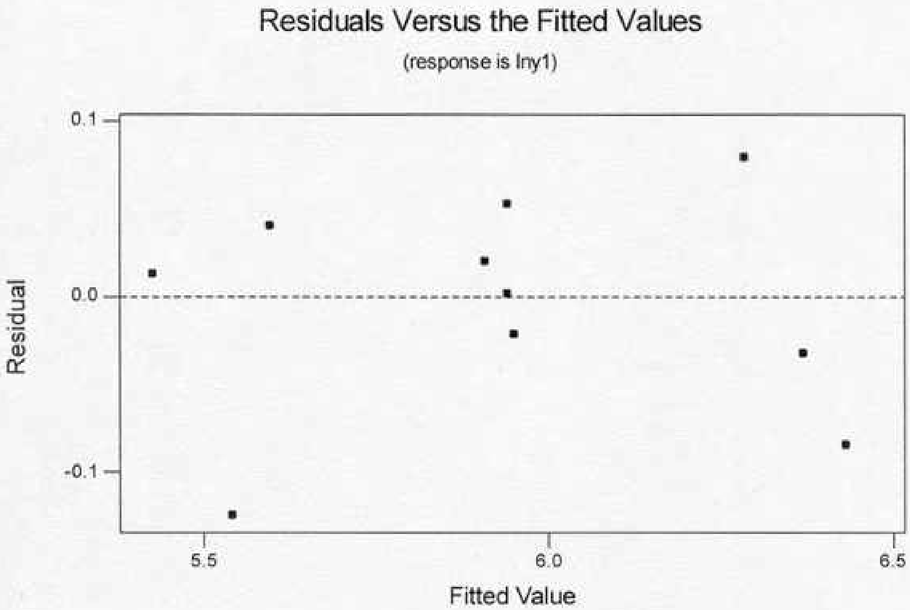
(b)

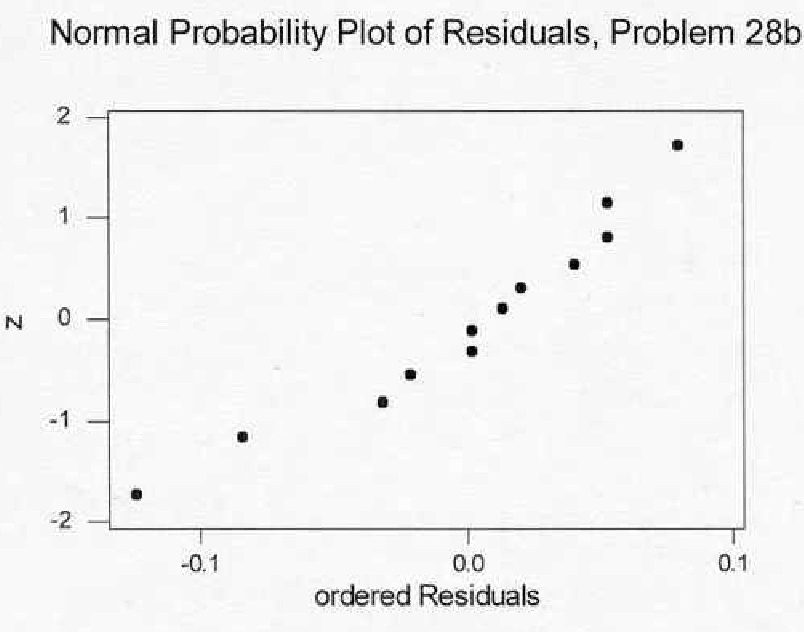


Residuals Versus lnx2
(response is lny1)
0.1
0.0
-0.1
Residual
-5.0
-4.5
-4.0
lnx2



Residuals Versus lnx1
(response is lny1)
0.1
0.0
-0.1
Residual
-1.5
-1.4
-1.3
-1.2
-1.1
-1.0
-0.9
-0.8
lnx1



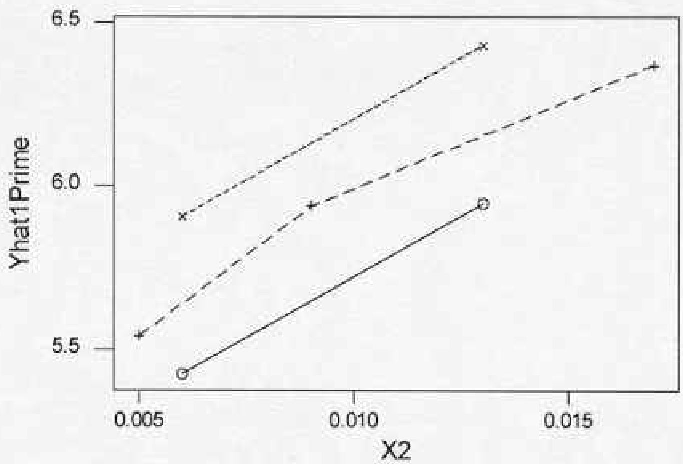


The Residuals vs lnx2 plot suggests perhaps a need for a cross-product term or "squared" term of lnx2 is needed. The other plots do not reveal any real evidence to say the chosen fitted model is in appropriate.

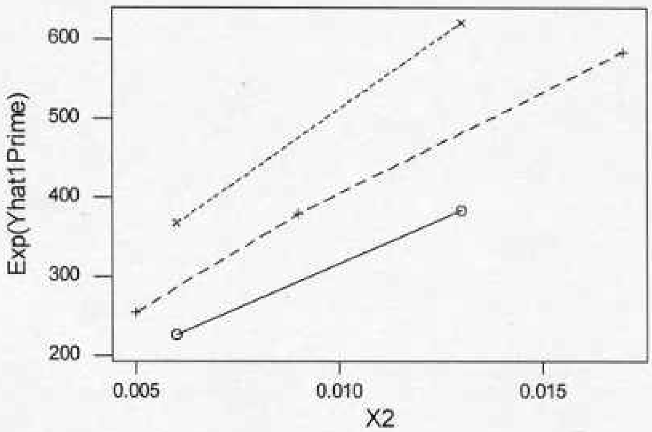
(c) Setting $x_1 = .36$ and $x_2 = .011$, the predicted $\ln y_1 = 10.2665 + .99372 \ln(.36) + .67712 \ln(.011) = 6.19760$. The predicted y_1 is thus, $\exp(6.19760) = 491.568$. Since $x_1 = .45$ and $x_2 = .017$ is outside the domain of the observed set of (x_1, x_2) pairs, it is not wise to use the fitted equation involving both lnx1 and lnx2 to predict y_1 or $\ln y_1$.

(d)

Plot of YhatPrime vs. X2 for a selected value of X1



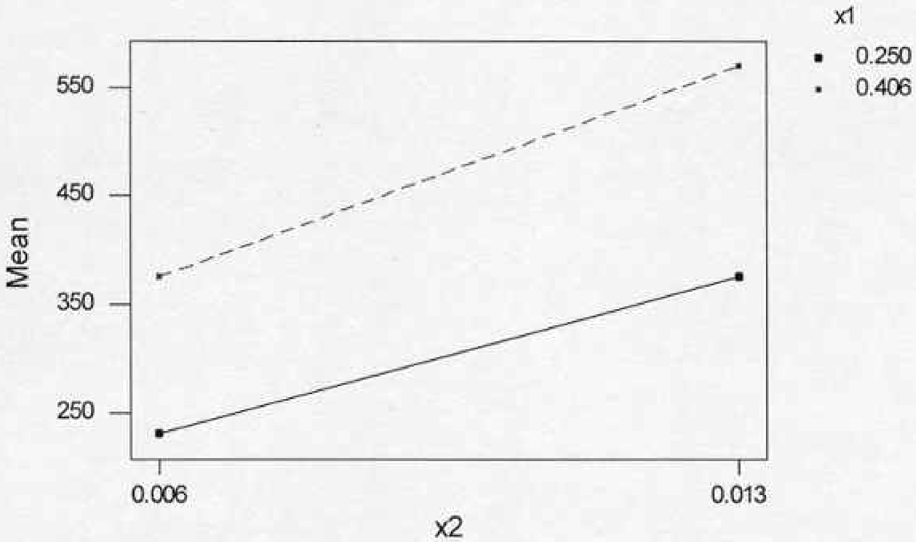
Plot of Exp(Yhat1Prime) vs. X2 for a selected value of X1



The "circle points" correspond to x1 = .25, the "plus" points correspond to x1 = .318 and the "cross" points correspond to x1 = .406. It seems there is some interaction in the "raw" y values, since these lines above are not parallel. It does seem, however, that on the log scale, there is no interaction (X1 by X2).

(e)

Interaction Plot - Data Means for y1



<u>Main Effects</u>

Lo X1 = 302.5 - 387.5 = -85
Hi X1 = 472.5 - 387.5 = 85

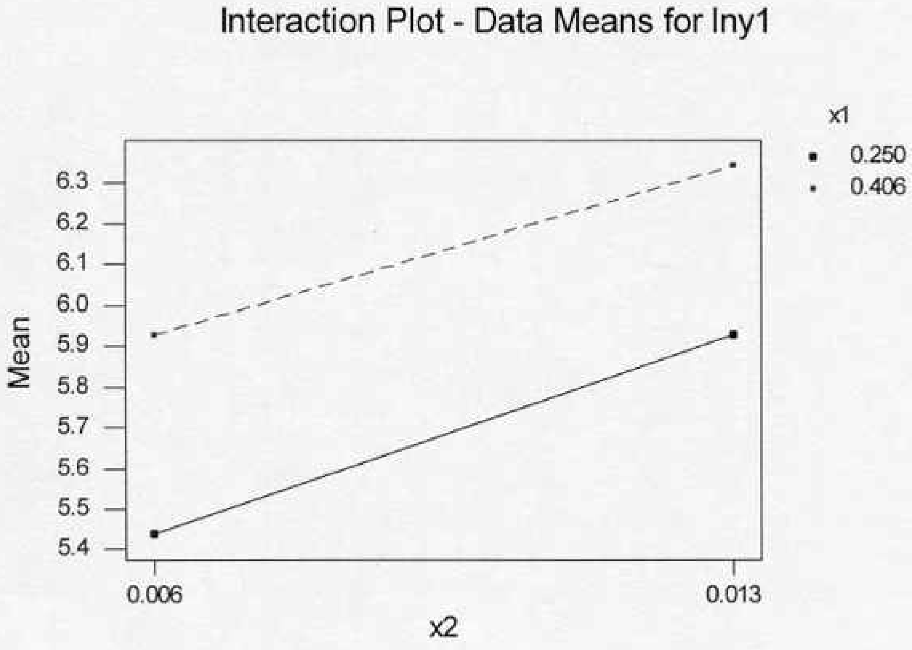
Lo X2 = 302.5 - 387.5 = -85
Hi X2 = 472.5 - 387.5 = 85

<u>Interactions:</u>

Low X1/Low X2 = 230 - (387.5 - 85 - 85) = 12.5
Low X1/Hi X2 = 375 - (387.5 - 85 + 85) = - 12.5
Hi X1/Low X2 = 375 - (387.5 + 85 - 85) = - 12.5
Hi X1/Hi X2 = 570 - (387.5 + 85 + 85) = 12.5

It seems the effect of Diameter on Thrust is smaller at the low (.006) feed rate compared to the Diameter effect at the hi (.013) Feed Rate. In both cases, the interaction plot reveals minimal interaction. The larger Diameter produces a larger Thrust for a given feed rate.

(f)



At the low level (.006) of Feed Rate (X2), the Diameter effect on Thrust is larger than the Diameter effect on Thrust at the hi level (.013) of Feed Rate (X2). For both diameter levels, as Feed Rate (X2) increases, Thrust (lnY1) increases about the same amount.

<u>Main Effects</u>

Low X1 = 5.682505 - 5.909395 = - .22689
Hi X1 = 6.136285 - 5.909395 = .22689
Low X2 = 5.682505 - 5.909395 = - .22689
Hi X2 = 6.136285 - 5.909395 = .22689

<u>Interaction Effects</u>

Low X1/ Low X2 = 5.43808 - (5.909395 - .22689 - .22689) = - .017535
Low X1/ Hi X2 = .017535
Hi X1/ Low X2 = .017535
Hi X1/ Hi X2 = - .017535

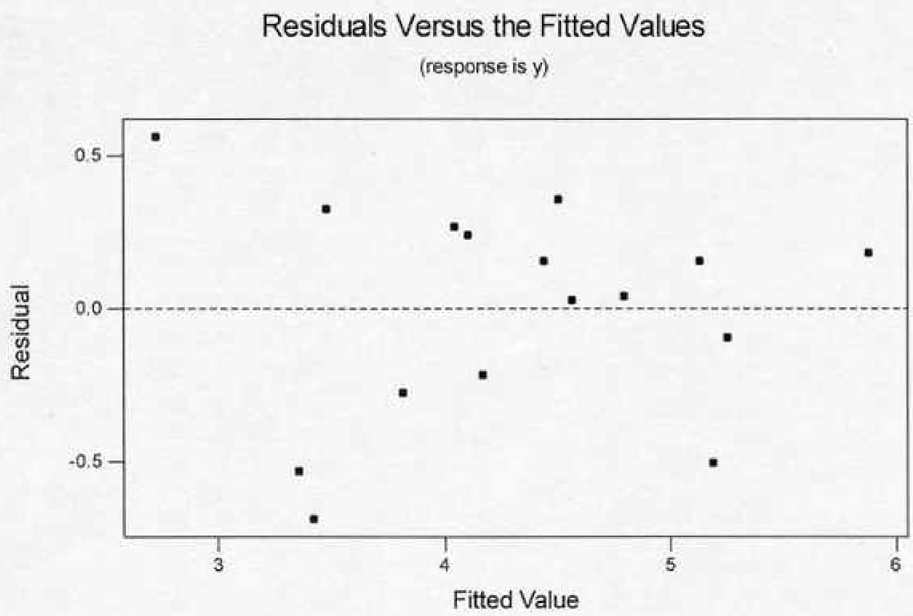
(g) Yes, the plots in (d) complement what is seen in parts (e) and (f).

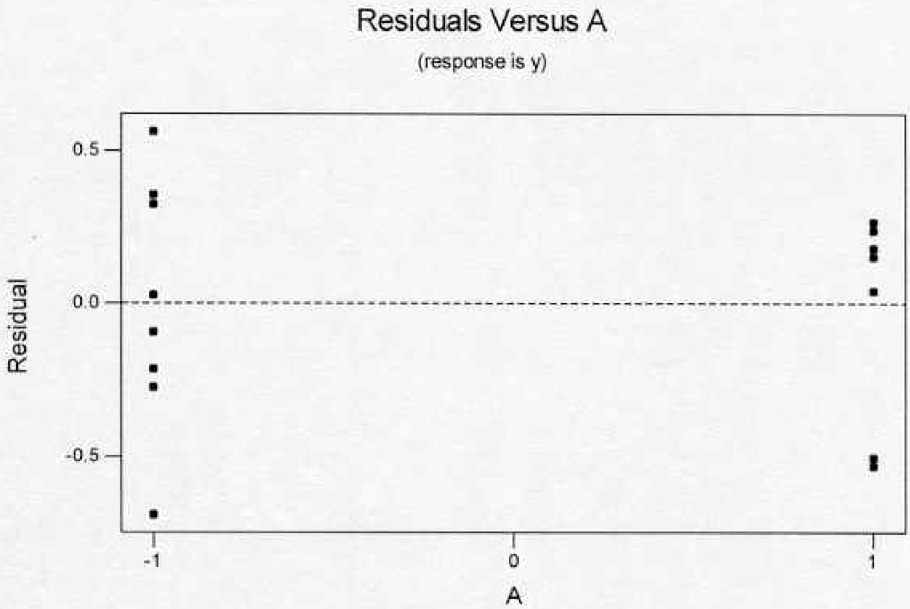
29.

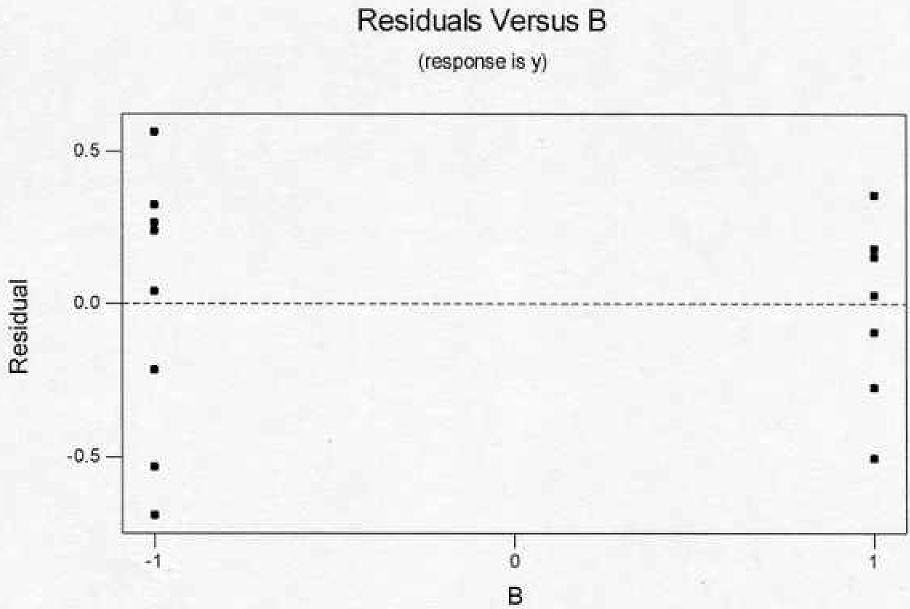
(a) $\bar{y} = 4.30188$, a_2 = .31187, b_2 = .54312, c_2 = .34563, d_2 = -.09688, ab_{22} = -.00438, ac_{22}= .16062, ad_{22} = -.04938, bc_{22} = .14937, bd_{22} = .37437, cd_{22} = .01437, abc_{222} = -.13813, abd_{222} = -.01063, acd_{222} = .19687, bcd_{222} = - .02187, $abcd_{2222}$ = -.01688.

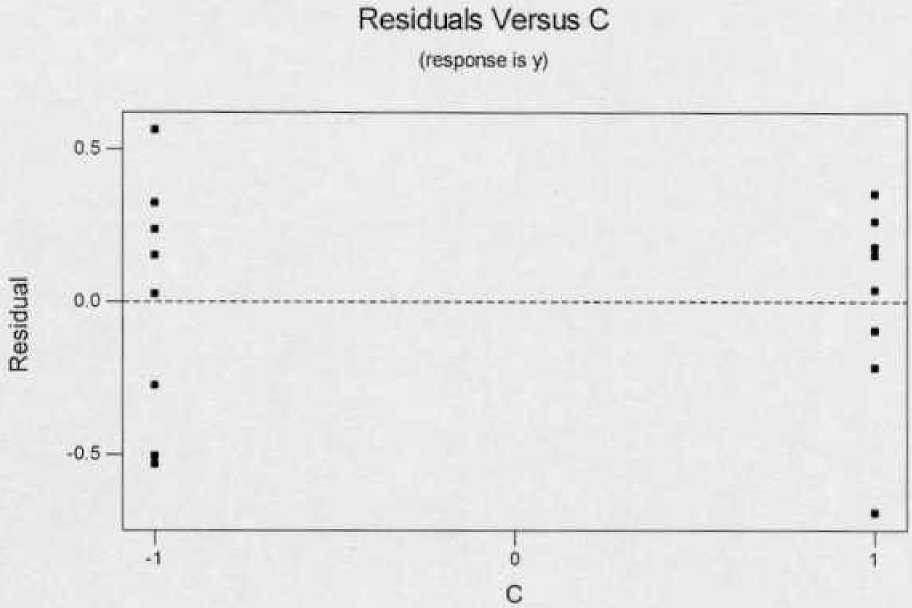
(b) It appears there is an important A, B, and C main effect together with an important BD interaction effect. The higher level of glue, the higher pre-drying temperature, the higher tunnel temperature and the higher level of pressure applied will maximize adhesive force.

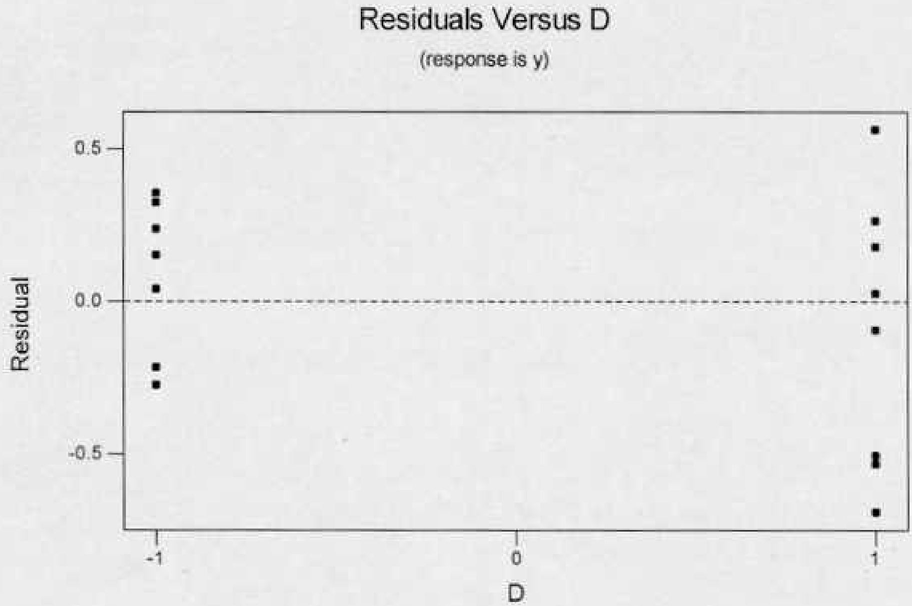
(c)



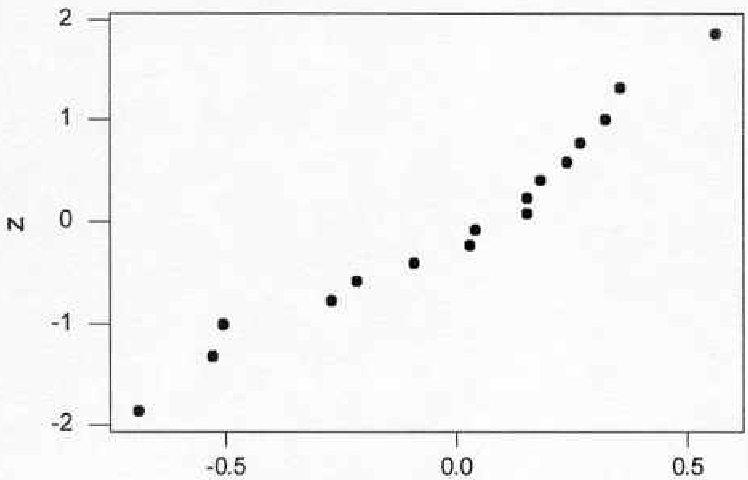

Residuals Versus A
(response is y)
Residual
0.5
0.0
-0.5
-1
0
1
A


Residuals Versus B
(response is y)
Residual
0.5
0.0
-0.5
-1
0
1
B


Residuals Versus C
(response is y)
0.5
0.0
-0.5
Residual
-1
0
1
C


Residuals Versus D
(response is y)
0.5
0.0
-0.5
Residual
-1
0
1
D

Normal Probability Plot of Residuals Prob 29c



There appears to be more variability at the high level of D, not so consistent results may occur when factor D is set at its high level. The other plots do not reveal anything noticeably unusual.

Residuals	Predicts	y
0.324375	3.47563	3.80
0.240625	4.09938	4.34
-0.273125	3.81313	3.54
0.153125	4.43688	4.59
-0.216875	4.16688	3.95
0.039375	4.79063	4.83
0.355625	4.50438	4.86
0.151875	5.12813	5.28
0.563125	2.72688	3.29
-0.530625	3.35063	2.82
0.028125	4.56188	4.59
-0.505625	5.18563	4.68
-0.688125	3.41813	2.73
0.268125	4.04188	4.31
-0.093125	5.25313	5.16
0.183125	5.87688	6.06

(d) $R^2 = .846$ using the model that includes factors A,B,C and BD interaction. When factors A,B,C and the interaction term BD are included in the linear model representing adhesive force, 84.6% of the overall variability of the responses about their grand mean is explained by the A,B and C main effects and the BD interaction effect.

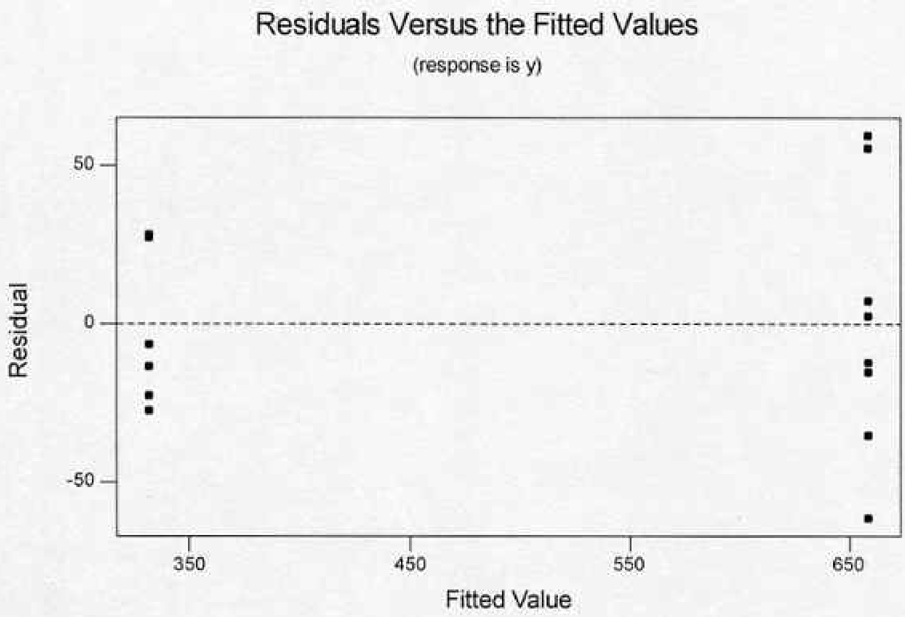
(30)

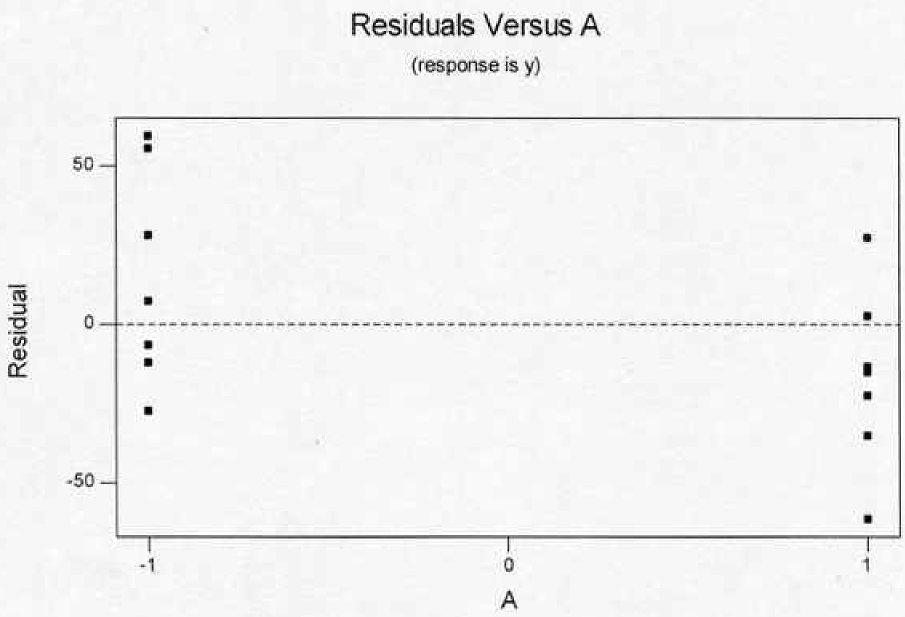
(a) $\bar{y} = 495.063$, $a_2 = -16.5625$, $b_2 = 12.0625$, $c_2 = -163.438$, $d_2 = -3.4375$, $ab_{22} = -5.5625$, $ac_{22} = 10.9375$, $ad_{22} = -3.8125$, $bc_{22} = -13.4375$, $bd_{22} = 10.5625$, $cd_{22} = -5.4375$, $abc_{222} = -1.0625$, $abd_{222} = 1.1875$, $acd_{222} = .1875$, $bcd_{222} = 7.0625$, $abcd_{2222} = -6.3125$.

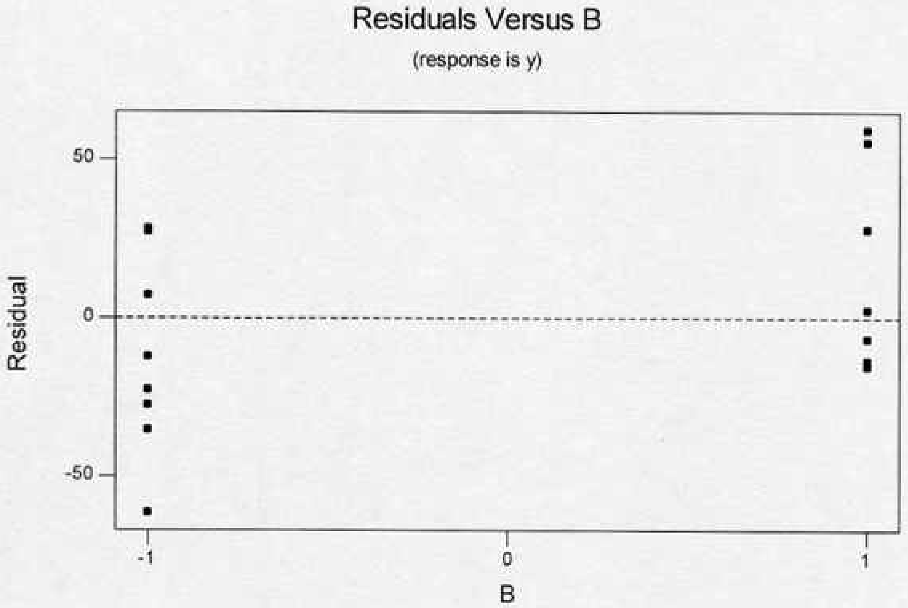
(b) It is clear the C effect is an important contributor to the variability of average sheet resistivity. In fact, at the high level of C, average sheet resistivity is significantly minimized. The A and B main effects and their interaction with C are somewhat influential on variability of the average sheet resistivity. The high level of A decreases average sheet resistivity, the high level of B increases the average sheet resistivity. The AC interaction (both at their high levels) somewhat increase average sheet resistivity and the BC interaction (both at their high levels) somewhat decrease average sheet resistivity.

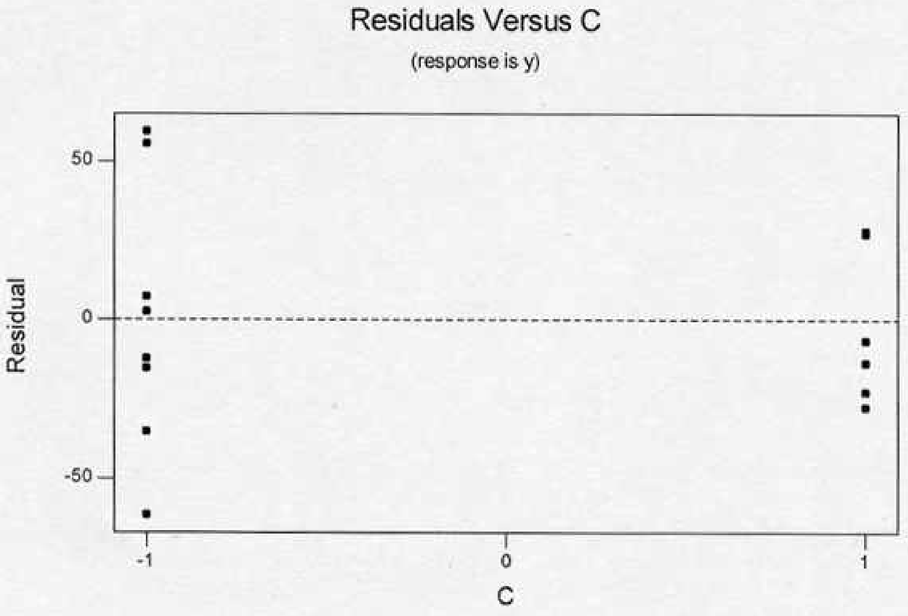
(c) $\bar{y} = 495.063$, $c_2 = -163.438$

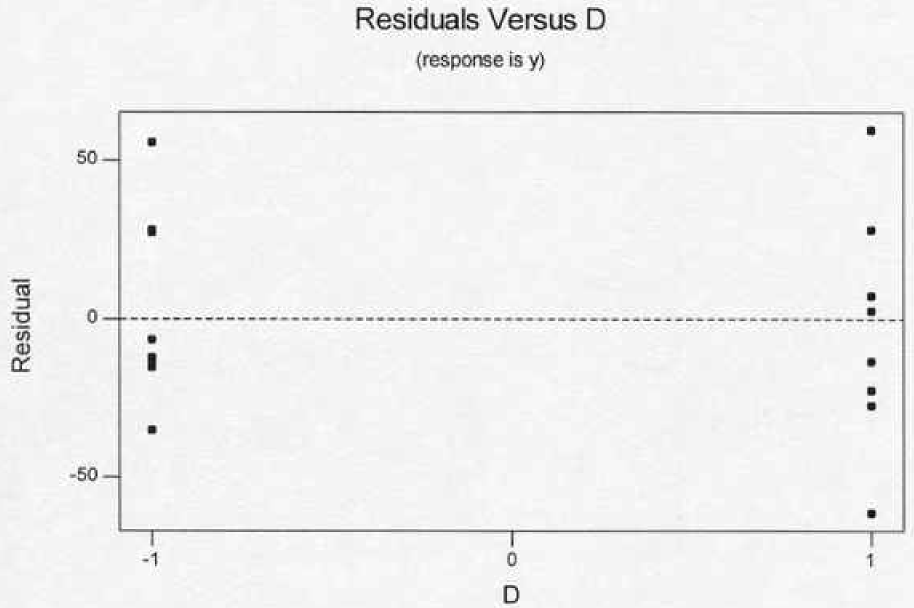
Predictions	Residuals	y
658.500	-12.500	646
658.500	-35.500	623
658.500	55.500	714
658.500	-15.500	643
331.625	28.375	360
331.625	27.375	359
331.625	-6.625	325
331.625	-13.625	318
658.500	7.500	666
658.500	-61.500	597
658.500	59.500	718
658.500	2.500	661
331.625	-27.625	304
331.625	-22.625	309
331.625	28.375	360
331.625	-13.625	318


Residuals Versus the Fitted Values
(response is y)
Residual
50
0
-50
350
450
550
650
Fitted Value


Residuals Versus A
(response is y)
Residual
50
0
-50
-1
0
1
A


Residuals Versus B
(response is y)
50
0
-50
Residual
-1
0
1
B


Residuals Versus C
(response is y)
50
0
-50
Residual
-1
0
1
C



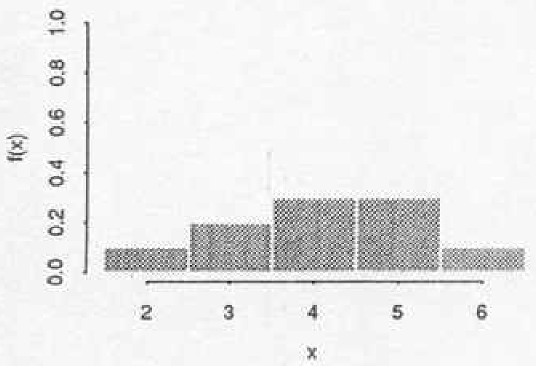
The residual plots vs A and B suggest perhaps A and B should be in the model and there seems to be different variability of average sheet resistivity at the high level of the factor of C.

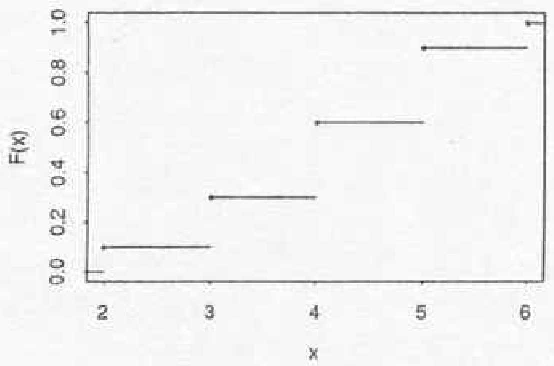
(d) The R^2 = .964 for the C main effects only model. It would appear from the residual plots there is a need for factors A and B and their interactions together with an interaction term AC. However, the C main effects only model does okay, especially with such a large R^2.

Chapter 5: Probability: The Mathematics of Randomness

Section 1

1. (a)





(b) Using equation (5-1),

$$EX = 2(.1) + 3(.2) + 4(.3) + 5(.3) + 6(.1) = 4.1.$$

Using equation (5-2),

$$\mathrm{Var}X = 2^2(.1) + 3^2(.2) + 4^2(.3) + 5^2(.3) + 6^2(.1) - (4.1)^2 = 1.29,$$

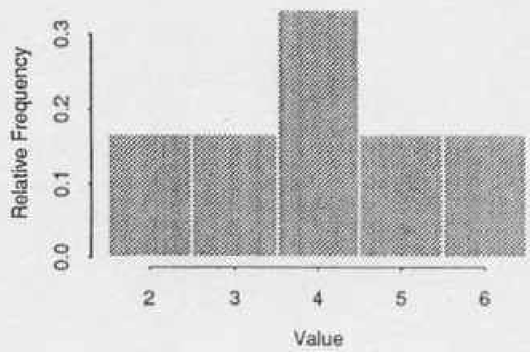
so the standard deviation of X is $\sqrt{1.29} = 1.136$.

2. (a) X has a binomial distribution with $n = 10$ and $p = \frac{1}{3}$. Use equation (5-3) with $n = 10$ and $p = \frac{1}{3}$.

x	$P(X = x)$
0	.0173
1	.0867
2	.1951
3	.2601
4	.2276
5	.1366
6	.0569
7	.0163
8	.0030
9	.0003
10	.0000

(b) Assuming that they are just guessing, how likely is it that 7 (or more) out of 10 subjects would be correct? This is $P(X \geq 7) = .0197$. Under the hypothesis that they are only guessing, this kind of extreme outcome would only happen about 1 in 50 times, so the outcome is strong evidence that they are not just guessing.

3. (a)



Using equations (3-4) and (3-5), $\mu = 4$, $\sigma^2 = \frac{5}{3}$, and $\sigma = 1.291$.

(b)

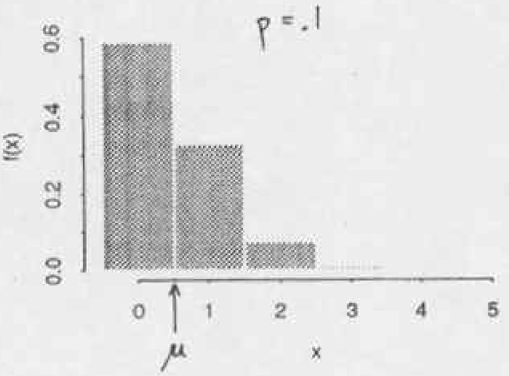
x	2	3	4	5	6
$P(X = x)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

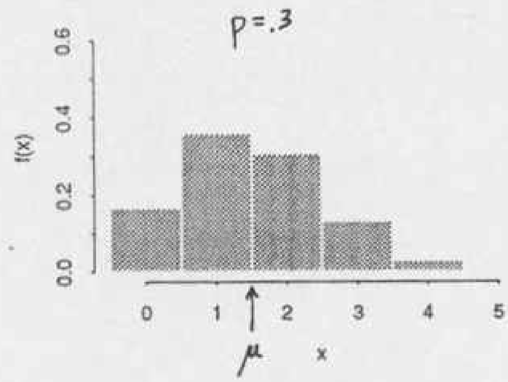
Since all members of the population are equally likely to be chosen, the probability histogram for X is the same as the population relative frequency distribution. Using equations (5-1) and (5-2), $EX = 4$ and Var$X = \frac{5}{3}$.

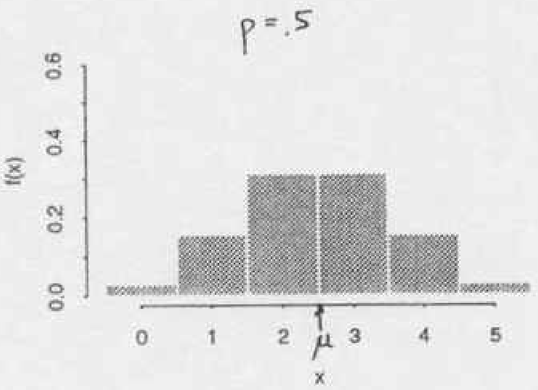
(c) Label the values $2, 3, 4_1, 4_2, 5, 6$.

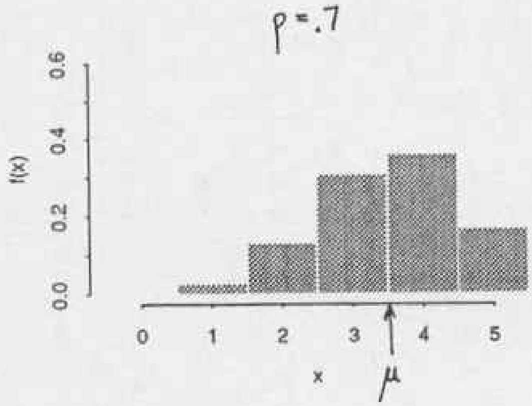
4. Use equation (5-3) with $n = 5$.

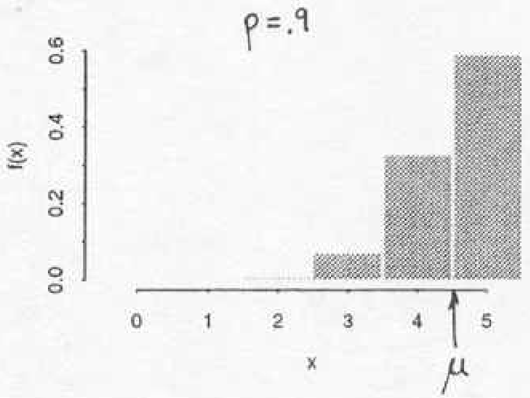
p	$f(0)$	$f(1)$	$f(2)$	$f(3)$	$f(4)$	$f(5)$	$EX = np$	Var$X =$ $np(1-p)$	St. Dev.
.1	.5905	.3280	.0729	.0081	.0005	.0000	.5	.45	.6708
.3	.1681	.3601	.3087	.1323	.0283	.0024	1.5	1.05	1.0247
.5	.0312	.1562	.3125	.3125	.1562	.0312	2.5	1.25	1.1180
.7	.0024	.0284	.1323	.3087	.3601	.1681	3.5	1.05	1.0247
.9	.0000	.0004	.0081	.0729	.3280	.5905	4.5	.45	.6708











5. Use the binomial distribution, equation (5-3), with $n = 8$ and $p = .20$.

(a) $P(W = 3) = .1468$.

(b) $P(W \leq 2) = P(W = 0) + P(W = 1) + P(W = 2) = .7969$.

(c) Using equation (5-4), $EW = np = 1.6$.

(d) Using equation (5-5), $\text{Var}W = np(1-p) = 1.28$.

(e) $\sqrt{1.28} = 1.1314$.

6. Use the geometric distribution, equation (5-6), with $p = .20$.

(a) $P(Y = 5) = .08192$.

(b) $P(Y \leq 4) = P(Y = 1) + P(Y = 2) + P(Y = 3) + P(Y = 4) = .5904$.

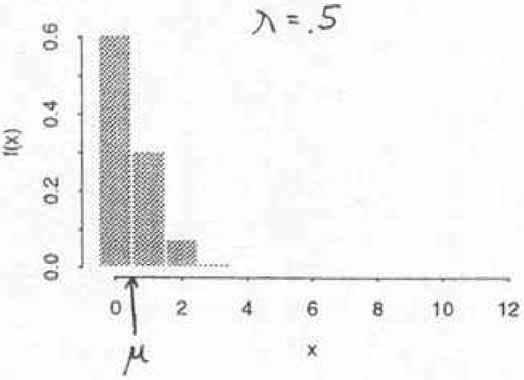
(c) Using equation (5-8), $EY = \frac{1}{p} = 5$.

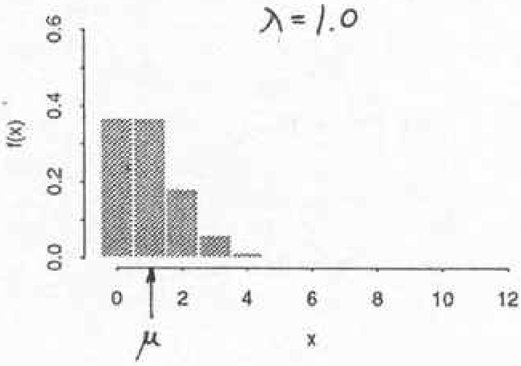
(d) Using equation (5-9), $\text{Var}Y = \frac{1-p}{p^2} = 20$.

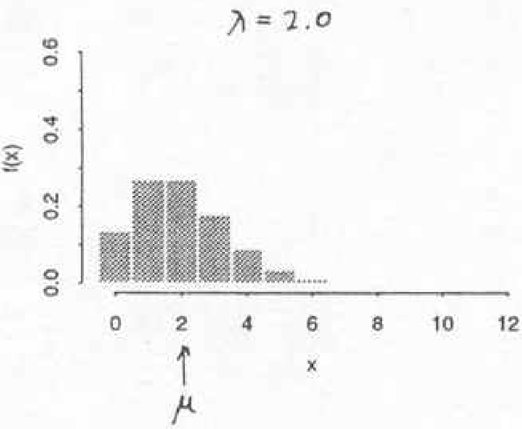
(e) $\sqrt{20} = 4.4721$.

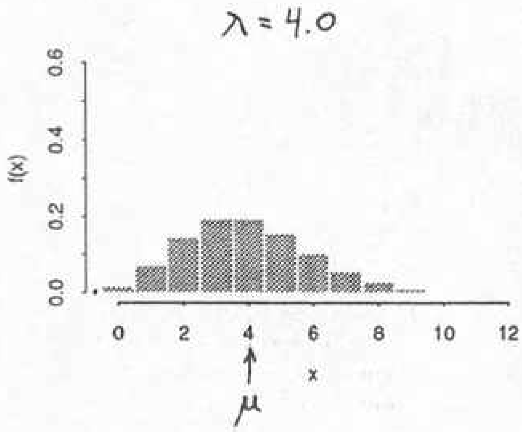
7. Use equation (5-10).

x	$P(X=x)$ $\lambda = .5$	$\lambda = 1.0$	$\lambda = 2.0$	$\lambda = 4.0$
0	.6065	.3679	.1353	.0183
1	.3033	.3679	.2707	.0733
2	.0758	.1839	.2707	.1465
3	.0126	.0613	.1804	.1954
4	.0016	.0153	.0902	.1954
5	.0002	.0031	.0361	.1563
6	.0000	.0005	.0120	.1042
7	.0000	.0001	.0034	.0595
8	.0000	.0000	.0009	.0298
9	.0000	.0000	.0002	.0132
10	.0000	.0000	.0000	.0053
11	.0000	.0000	.0000	.0019
12	.0000	.0000	.0000	.0006
	⋮	⋮	⋮	⋮
$EX = \lambda$	.5	1.0	2.0	4.0
Var$X = \lambda$	.5	1.0	2.0	4.0
Std. Dev.	.7071	1.0	1.4142	2.0









8. (a) Use the Poisson distribution, equation (5-10), with $\lambda = 2$.

$$P(X > 2) = 1 - P(X \leq 2) = 1 - .6767 = .3233.$$

(b) Use the Poisson distribution, equation (5-10), with $\lambda = 1$.

$$P(X = 0) = .3679.$$

9. (a) Use the Poisson distribution, equation (5-10), with $\lambda = 5$.

$$P(X = 0) = .0067.$$

(b) $Y \sim$ Binomial with $n = 4$ and $p = .0067$. Use equation (5-3) with $n = 4$ and $p = .0067$.

$$P(Y = 2) = .00027.$$

10. Probability is a mathematical system used to describe random phenomena. It is based on a set of axioms, and all the theory is deduced from the axioms. Once a model is specified, probability provides a deductive process that enables predictions to be made based on the theoretical model.

Statistics uses probability theory to describe the source of variation seen in data. Statistics tries to create realistic probability models that have (unknown) parameters with meaningful interpretations. Then, based on observed data, statistical methods try to estimate the unknown parameters as accurately and precisely as possible. This means that statistics is inductive, using data to draw conclusions about the process or population from which the data came.

Neither is a subfield of the other. Just as engineering uses calculus and differential equations to model physical systems, statistics uses probability to model variation in data. In each case the mathematics can stand alone as theory, so calculus is not a subfield of engineering and probability is not a subfield of statistics. Conversely, statistics is not a subfield of probability just as engineering is not a subfield of calculus; many simple statistical methods do not require the use of probability, and many engineering techniques do not require calculus.

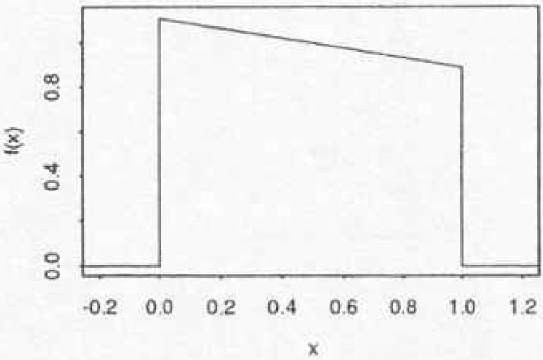
11. A relative frequency distribution is based on *data*. A probability distribution is based on a theoretical model for probabilities. Since probability can be interpreted as long-run relative frequency, a relative frequency distribution approximates the underlying probability distribution, with the approximation getting better as the amount of data increases.

Section 2

1. (a) Use equation (5-13) and solve for $k = \frac{2}{9}$.

(b)

$$f(x) = \begin{cases} \frac{2}{9}(5-x) & \text{for } 0 < x < 1 \\ 0 & \text{otherwise.} \end{cases}$$

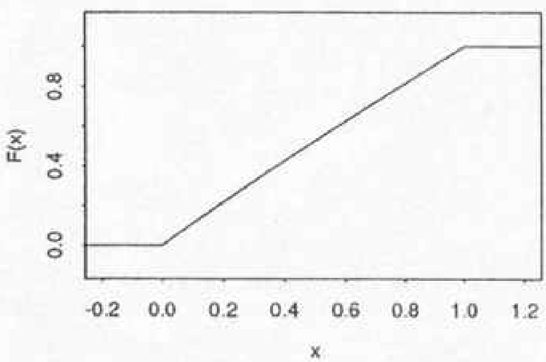


(c)

$$\begin{aligned} P(.25 < X < .75) &= \int_{.25}^{.75} f(x)\,dx \\ &= .5. \end{aligned}$$

(d) Using equation (5-16),

$$\begin{aligned} F(x) &= P(X \le x) \\ &= \int_{-\infty}^{x} f(x)\,dx \\ &= \begin{cases} 0 & \text{for } x \le 0 \\ \frac{2}{9}(5x - \frac{1}{2}x^2) & \text{for } 0 < x < 1 \\ 1 & \text{for } x \ge 1. \end{cases} \end{aligned}$$



(e) Using equation (5-18),

$$EX = \frac{13}{27}.$$

Using equation (5-19),

$$\text{Var}X = .08299,$$

so the standard deviation of X is .288.

2. The values of $\Phi(z) = P(Z \le z)$ are given in Table B-3. All of these probabilities correspond to areas under the standard normal curve.

(a) $P(Z < -.62) = \Phi(-.62) = .2676.$

(b) $P(Z > 1.06) = 1 - P(Z \le 1.06) = 1 - \Phi(1.06) = 1 - .8554 = .1446.$

(c) $P(-.37 < Z < .51) = P(Z < .51) - P(Z \le -.37) = .6950 - .3557 = .3393.$

(d) $P(|Z| \le .47) = P(-.47 \le Z \le .47) = P(Z \le .47) - P(Z < -.47) = .6808 - .3192 = .3616.$

(e) $P(|Z| > .93) = P(Z < -.93) + P(Z > .93) = 2(P(Z < -.93)) = 2(.1762) = .3524.$

(f) $P(-3.0 < Z < 3.0) = P(Z < 3.0) - P(Z \le -3.0) = .9987 - .0013 = .9974.$

(g) Looking up .90 in the body of the table, $\# \approx 1.28$.

(h) $P(|Z| < \#) = .90$ is equivalent to $P(Z < \#) = .95$ (by symmetry). Looking up .95 in the body of the table, $\# \approx 1.645$.

(i) $P(|Z| > \#) = .03$ is equivalent to $P(Z < \#) = .985$ (by symmetry). Looking up .985 in the body of the table, $\# \approx 2.17$.

3. Probabilities involving X are just areas under the normal curve with $\mu = 43.0$ and $\sigma = 3.6$. Each of these areas has an equal corresponding area under the standard normal curve.

Define $Z = \frac{X-43.0}{3.6}$. Then Z is a standard normal random variable. Re-express each of the problems below in terms of Z.

(a) $P(X < 45.2) = P(Z < .61) = .7291.$

(b) $P(X \le 41.7) = P(Z \le -.36) = .3594.$

(c) $P(43.8 < X < 47.0) = P(.22 < Z < 1.11) = P(Z < 1.11) - P(Z \le .22)$
$= .8665 - .5871 = .2794.$

(d) $P(|X - 43.0| \le 2.0) = P(41.0 \le X \le 45.0) = P(-.56 \le Z \le .56)$
$= P(Z \le .56) - P(Z < -.56) = .7123 - .2877 = .4246.$

(e) $P(|X - 43.0| > 1.7) = 1 - P(|X - 43.0| \le 1.7) = 1 - P(41.3 \le X \le 44.7)$
$= 1 - P(-.47 \le Z \le .47) = 1 - (P(Z \le .47) - P(Z < -.47))$
$= 1 - (.6808 - .3192) = .6384.$

(f) $P(X < \#) = .95$ is equivalent to $P(Z < \frac{\#-43.0}{3.6}) = .95$. Looking up .95 in the body of the table,

$$\frac{\# - 43.0}{3.6} \approx 1.645$$

so $\# \approx 48.922$.

(g) $P(X \geq \#) = .30$ is equivalent to $P(X < \#) = .70$, which is equivalent to $P(Z < \frac{\#-43.0}{3.6}) = .70$. Looking up .70 in the body of the table,

$$\frac{\# - 43.0}{3.6} \approx .52$$

so $\# \approx 44.872$.

(h) $P(|X - 43.0| > \#) = .05$ is equivalent to $P(|X - 43.0| \leq \#) = .95$, which is equivalent to $P(X - 43.0 \leq \#) = .975$ (by symmetry). This is equivalent to $P(Z \leq \frac{\#}{3.6}) = .975$. Looking up .975 in the body of the table,

$$\frac{\#}{3.6} \approx 1.96$$

so $\# \approx 7.056$.

4. (a) The probability that one journal is within specifications is the same as the long-run fraction of journals within specifications, if successive journal diameters can be thought of as repeated observations of the same random variable X. Since X is normal with $\mu = 2.0005$ and $\sigma = .0004$, $Z = \frac{X-2.0005}{.0004}$ is a standard normal random variable.

$$\begin{aligned} P(1.9995 \leq X \leq 2.0005) &= P(X \leq 2.0005) - P(X < 1.9995) \\ &= P(Z \leq 0) - P(Z < -2.50) \\ &= .5 - .0062 = .4938. \end{aligned}$$

(b) Because of the symmetry and shape of the normal distribution, setting μ to the midpoint of the specifications will increase the fraction of diameters in specifications. (This assumes that it is easy to make an adjustment that will change μ as desired.) With $\mu = 2.0000$,

$$\begin{aligned} P(1.9995 \leq X \leq 2.0005) &= P(X \leq 2.0005) - P(X < 1.9995) \\ &= P(Z \leq 1.25) - P(Z < -1.25) \\ &= .8944 - .1056 = .7888. \end{aligned}$$

(c) Want

$$P(1.9995 \leq X \leq 2.0005) \geq .95.$$

This is equivalent to

$$P(X \leq 2.0005) \geq .975$$

because of the symmetry of the normal distribution. Expressing this in terms of Z,

$$P(Z \leq \frac{.0005}{\sigma}) \geq .975.$$

Looking up .975 in the body of Table B-3,

$$\frac{.0005}{\sigma} \geq 1.96$$

so $\sigma \leq .0002551$ will do the trick.

5 (a)

E(X) = 1000 = $\alpha = \mu$, $\sigma^2 = \alpha^2$ = Var(X) = $(1000)^2 = 10^6$, $\sigma = 10^3$.

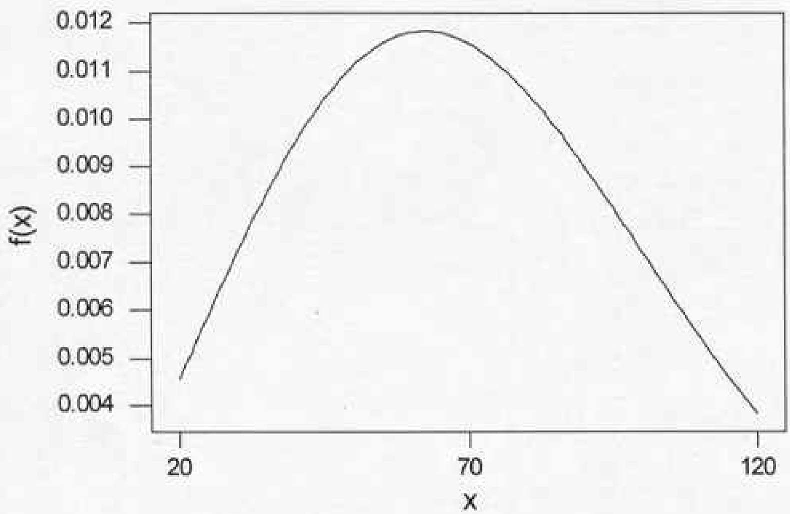
Consider equation (5.25). P(X < 500) = 1 - exp[- .001(500)] = .3934

P(X > 2000) = 1 - (1 - exp[-.001(2000)]) = exp[-2] = .1353.

(b) Consider equation (5.25). .05 = 1 - exp[- .001(x)] or .95 = exp[-.001(x)] or ln(.95) = -.001x which implies that x = 51.29. Thus, the .05 quantile is x = 51.29. The .90 quantile can be found in a similar fashion. Consider equation (5.25). .9 = 1 - exp[-.001(x)] or .1 = exp[-.001(x)]. Therefore, ln(.1) = -.001(x) and x = 2,302.58. The .90 quantile is x = 2,302.58.

6. (a)

Weibull Probability Density: Shape parameter = 2.3,
Scale parameter = 80



(b) Let x = median lifetime. Since X is Weibull ($\alpha = 80$, $\beta = 2.3$), equation (5-26) gives:

$.5 = 1 - \exp[-(x/80)^{2.3}]$

$.5 = \exp[-(x/80)^{2.3}]$

$\ln(.5) = -(x/80)^{2.3}$

$x = 68.2156 \times 10^6$ is the median lifetime.

(c) Continuing, using equation (5-26),

$.05 = 1 - \exp[-(x/80)^{2.3}]$

$.95 = \exp[-(x/80)^{2.3}]$

$-.051293 = -(x/80)^{2.3}$

$x = 21.99 \times 10^6$ is the .05 quantile.

Using equation (5-26), let x equal the .95 quantile.

$.95 = 1 - \exp[-(x/80)^{2.3}]$

$.05 = \exp[-(x/80)^{2.3}]$

$2.99573 = (x/80)^{2.3}$

$x = 128.903 \times 10^6$ is the .95 quantile.

Section 3

1. Data that are being generated from a particular distribution will have roughly the same shape as the density of the distribution, and this is more true for larger samples. Probability plotting provides a sensitive graphical way of deciding if the data have the same shape as a theoretical probability distribution. If a distribution can be found that accurately describes the data generating process, one can then estimate probabilities and quantiles and make predictions about future process behaviour based on the model.

2. Fit a line (by hand or some other method) through the points on the plot. The x-intercept is an approximate mean, and an approximate standard deviation is

$$\sigma \approx \frac{1}{\text{slope}} = \frac{\Delta x}{\Delta y} = \frac{\Delta \text{ data quantiles}}{\Delta \text{ std. normal quantiles}}.$$

3. a) For Minitab Version 9.1, with the data in C1, the commands are

```
MTB > nscores c1 c2
MTB > name c2 'SNQT'
MTB > gstd
* NOTE  * Standard Graphics are enabled.
          Professional Graphics are disabled.
          Use the GPRO command to enable Professional Graphics.
MTB > plot c2 c1
```

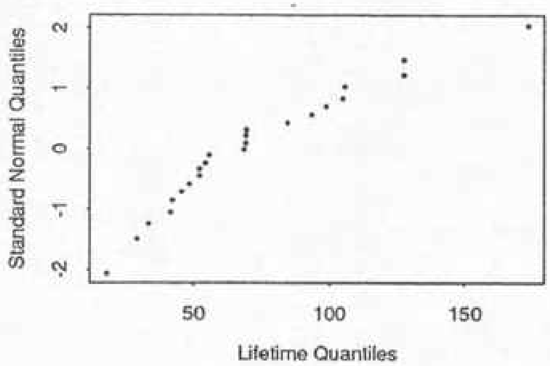
The plot given in section 2, prob. 3 was produced by a user-written function in S-Plus.

b) See the solution to section 2, prob. 3

$$\mu \approx \text{x-intercept} = 69.6;$$

$$\sigma \approx \frac{1}{\text{slope}} = 2.1.$$

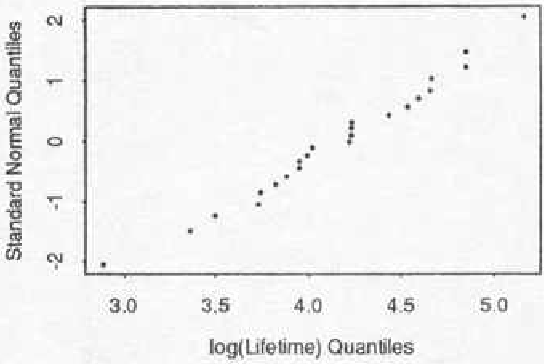
4. (a) The first 3 coordinates of the normal plot of the raw data are: (17.88, −2.05), (28.92, −1.48), (33.00, −1.23).



This normal plot is not linear, so a Gaussian (normal) distribution does not seem to fit the raw data.

The first 3 coordinates of the normal plot of the natural log of the data are: (2.884,

$-2.05)$, $(3.365, -1.48)$, $(3.497, -1.23)$.



This normal plot is fairly linear, indicating that a normal distribution fits the log-transformed data well (the lognormal distribution fits the raw data well). To find an approximate μ and σ for the lognormal distribution, fit a line to the normal plot of the log-transformed data.

$$\mu \approx \text{x-intercept} = 4.15;$$

$$\sigma \approx \frac{1}{\text{slope}} = .54.$$

(You could also use the sample mean and standard deviation of the log-transformed data.)

For these parameters, the .05 quantile of ln(life) is the # that satisfies the expression

$$P(\ln(\text{life}) \leq \#) = .05.$$

This is equivalent to

$$P(Z \leq \frac{\# - 4.15}{.54}) = .05,$$

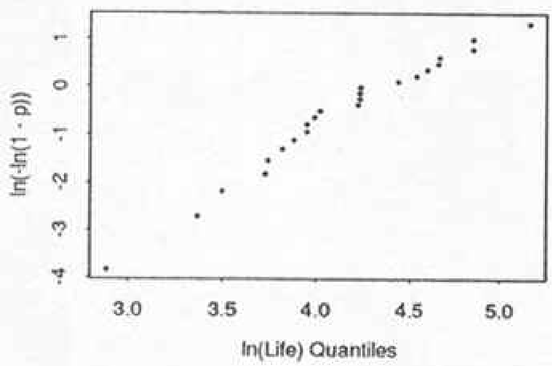
where Z is a standard normal random variable. Looking up .05 in the body of Table B-3,

$$\frac{\# - 4.15}{.54} \approx -1.645$$

so $\# \approx 3.26$. This corresponds to a .05 quantile of $e^{3.26} = 26.09$ for raw life.

(b) The computations necessary for the Weibull plot are given below.

i	ith Smallest Life	ln(ith Smallest Life)	$p = \frac{i-.5}{23}$	$\ln(-\ln(1-p))$
1	17.88	2.8837	.0217	-3.8177
2	28.92	3.3645	.0652	-2.6965
3	33.00	3.4965	.1087	-2.1622
4	41.52	3.7262	.1522	-1.8013
5	42.12	3.7405	.1957	-1.5245
6	45.60	3.8199	.2391	-1.2972
7	48.40	3.8795	.2826	-1.1022
8	51.84	3.9482	.3261	-.9297
9	51.96	3.9505	.3696	-.7736
10	54.12	3.9912	.4130	-.6296
11	55.56	4.0175	.4565	-.4947
12	67.80	4.2166	.5000	-.3665
13	68.64	4.2289	.5435	-.2432
14	68.64	4.2289	.5870	-.1231
15	68.88	4.2324	.6304	-.0046
16	84.12	4.4322	.6739	.1139
17	93.12	4.5339	.7174	.2340
18	98.64	4.5915	.7609	.3582
19	105.12	4.6551	.8043	.4894
20	105.84	4.6619	.8478	.6327
21	127.92	4.8514	.8913	.7971
22	128.04	4.8523	.9348	1.0043
23	173.40	5.1556	.9783	1.3425



The Weibull plot is fairly linear, indicating that a Weibull distribution might be used to describe bearing load life. From a line fit to the plot,

$$\ln \alpha \approx \text{x-intercept} = 4.396$$

so $\alpha \approx e^{4.396} = 81.12$.

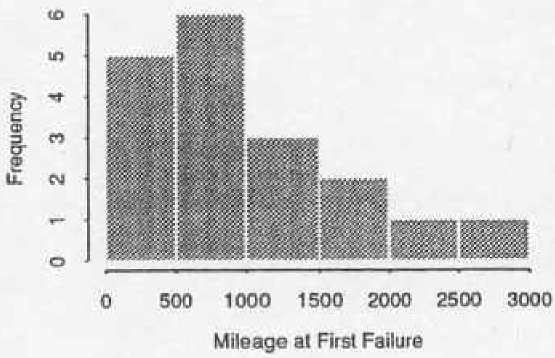
$$\beta \approx \text{slope} = 2.30.$$

To find the p quantile of the distribution, set the CDF equal to p and solve for x:

$$\begin{aligned} F(x) = 1 - e^{-\left(\frac{x}{\alpha}\right)^{\beta}} &= p \\ \ln(1-p) &= -\left(\frac{x}{\alpha}\right)^{\beta} \\ x &= \alpha(-\ln(1-p))^{\frac{1}{\beta}} \end{aligned}$$

Using $p = .05$ and the above approximations for α and β, the .05 quantile is approximately $x = 22.31$.

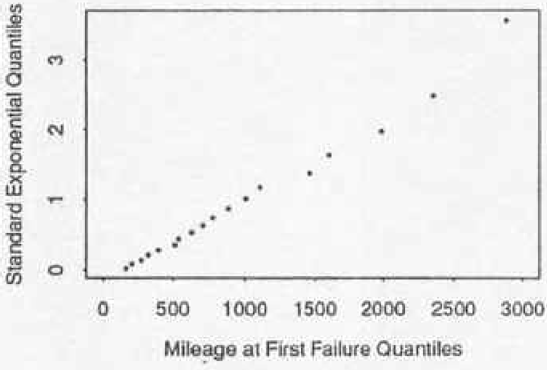
5. (a)



The distribution of the data is right-skewed.

(b) The plotting positions are given in the following table.

i	$\frac{i-.5}{18}$	$Q(\frac{i-.5}{18})$	$Q_{\text{Exp}}(\frac{i-.5}{18})$
1	.02777778	162	.02817088
2	.08333333	200	.08701138
3	.13888889	271	.14953173
4	.19444444	320	.21622311
5	.25000000	393	.28768207
6	.30555556	508	.36464311
7	.36111111	539	.44802472
8	.41666667	629	.53899650
9	.47222222	706	.63907996
10	.52777778	777	.75030559
11	.58333333	884	.87546874
12	.63888889	1008	1.01856958
13	.69444444	1101	1.18562367
14	.75000000	1462	1.38629436
15	.80555556	1603	1.63760879
16	.86111111	1984	1.97408103
17	.91666667	2355	2.48490665
18	.97222222	2880	3.58351894



The plot is very linear, indicating that the data have an exponential shape. It seems that the exponential distribution fits the data well. Since a line fit to the plot gives $Q(0) \approx 0$, no strong need for a threshold parameter greater than zero is indicated.

Section 4

1. If X and Y are independent, then they are uncorrelated—they do not influence each other in any way. Put differently, if the probability distribution of X and Y are both known completely, observing the actual value of X does not in any way change the probability distribution of the yet-to-be-observed Y, and vice-versa.

   One practical advantage of X and Y being independent is that the variance of a linear combination of the two can be easily computed using equation (5-59). Another advantage is that it is easy to describe the joint probability distribution of X and Y—it is just the product of the marginal distributions. In general, independence allows the probability of two events happening together to be computed as the product of the probabilities of each event.

2. (a) Compute $f_X(x)$ by summing down the columns and $f_Y(y)$ by summing across the rows.

x	0	1	2
$f_X(x)$	.5	.4	.1

y	0	1	2	3	4
$f_Y(y)$	.21	.19	.26	.21	.13

   (b) No, since $f(x,y) \neq f_X(x)f_Y(y)$. For example, $f(0,0) = .15$ and $f_X(0)f_Y(0) = .5(.21) = .105$.

   (c) Use equations (5-1) and (5-2), and the marginal distribution of X from part (a).

$$EX = (0)(.5) + (1)(.4) + (2)(.1) = .6$$

   and

$$\text{Var}X = (0)^2(.5) + (1)^2(.4) + (2)^2(.1) - (.6)^2 = .44.$$

   (d) Use equations (5-1) and (5-2), and the marginal distribution of Y from part (a).

$$EY = (0)(.21) + (1)(.19) + (2)(.26) + (3)(.21) + (4)(.13) = 1.86$$

   and

$$\text{Var}Y = (0)^2(.21) + (1)^2(.19) + (2)^2(.26) + (3)^2(.21) + (4)^2(.13) - (1.86)^2 = 1.7404.$$

   (e) Use equation (5-42). For example

$$f_{Y|X}(0|0) = \frac{f(0,0)}{f_X(0)} = \frac{.15}{.5} = .3$$

   The rest are given in the table below.

y	0	1	2	3	4
$f_{Y\|X}(y\|0)$	.3	.2	.2	.2	.1

   Using equation (5-1),

$$E(Y|X=0) = (0)(.3) + (1)(.2) + (2)(.2) + (3)(.2) + (4)(.1) = 1.6.$$

3. (a) If $X = 0$, then all of the specimens will be tested with probability 1.

y	$f_{Y\|X}(y\|0)$
1	0
2	0
3	0
4	1

If $X = 1$, and the contaminated specimen is equally likely to be tested first, second, third, or fourth, then all possible values of Y are equally likely.

y	$f_{Y\|X}(y\|1)$
1	.25
2	.25
3	.25
4	.25

The joint distribution can be obtained using equation (5-52): $f(x, y) = f_{Y|X}(y|x)f_X(x)$.

$f(x, y)$

		x	
		0	1
	1	0	$.25(1-p)$
y	2	0	$.25(1-p)$
	3	0	$.25(1-p)$
	4	p	$.25(1-p)$

(b) Use the marginal distribution for Y. This is obtained by summing across the rows of the table for $f(x, y)$:

y	$f_Y(y)$
1	$.25(1-p)$
2	$.25(1-p)$
3	$.25(1-p)$
4	$.25(1-p)+p$

Then applying equation (5-1),

$$\begin{aligned} EY &= (1)((.25)(1-p)) + (2)((.25)(1-p)) + (3)((.25)(1-p)) + (4)((.25)(1-p)+p) \\ &= 2.5 + 1.5p \end{aligned}$$

(c) For the second method, Y has two possible values, 1 and 3. $P(Y = 1) = P(X = 0) = p$ because if there is not a contaminated specimen, this will be known after the first composite test. If there is a contaminated specimen, then the lab *must* do 3 tests, and the probability of this is $P(Y = 3) = P(X = 1) = 1 - p$. Applying equation (5-1), $EY = (1)(p) + (3)(1 - p) = 3 - 2p$ for the second method. Based only on the criterion of minimizing EY, the second method will be better when

$$3 - 2p < 2.5 + 1.5p.$$

Solving this inequality for p results in $p > .143$. This makes sense, because if p is large it is likely that there are no contaminated specimens, in which case the second method is more efficient.

4. (a) Since X and Y are independent, $f(x,y) = f_X(x)f_Y(y)$ (definition (5-27)).

$$f(x,y) = \begin{cases} \frac{1}{.05}\frac{1}{.06} & \text{for } x \in (1.97, 2.02) \text{ and } y \in (2.00, 2.06) \\ 0 & \text{otherwise} \end{cases}$$

$$= \begin{cases} 333.33 & \text{for } x \in (1.97, 2.02) \text{ and } y \in (2.00, 2.06) \\ 0 & \text{otherwise} \end{cases}$$

(b) Integrate $f(x,y)$ over the region in which $y < x$.

$$\begin{aligned} P(Y - X < 0) &= P(Y < X) \\ &= \int_{2.00}^{2.02}\int_{2.00}^{x} 333.33\, dx\, dy \\ &= .0667. \end{aligned}$$

5(a) f(x) = 2x for 0 < X < 1 and 0 otherwise. f(y) = 2(1 - y) for 0 < Y < 1 and 0 otherwise. So, μ = E(X) = $\int_0^1 2x^2 dx = (1/3)(2x^3)$. Letting X = 1 gives 2/3. Letting X =0, gives 0. Thus, E(X) = 2/3.

(b) Yes, since $f(x,y) = f_X(x)f_Y(y)$.

(c) Integrate $f(x,y)$ over the region defined by $x + 2y \geq 1$. This is equivalent to the region defined by $y \geq \frac{1}{2}(1-x)$.

$$\begin{aligned} P(X + 2Y \geq 1) &= \int_0^1 \int_{\frac{1}{2}(1-x)}^{1} 4x(1-y)\, dy\, dx \\ &= .7083. \end{aligned}$$

(d) f(x| y) = f(x,y)/f(y) = [4x(1-y)] / [2(1 - y)]. For y = .5, f(x | y = .5) becomes 2x. Thus,

E[(X | y = .5)] = $\int_0^1 2x^2 dx = (1/3)(2x^3)$ = 2/3.

6. (a) Since X and Y are independent, $f(x,y) = f_X(x)f_Y(y)$ (definition (5-27)).

$$f(x,y) = \begin{cases} e^{-x}e^{-y} & \text{for } x > 0 \text{ and } y > 0 \\ 0 & \text{otherwise} \end{cases}$$

(b) Integrate $f(x,y)$ over the region in which both x and y are greater than t.

$$\begin{aligned} P(X > t \text{ and } Y > t) &= \int_t^\infty \int_t^\infty e^{-x}e^{-y}\,dx\,dy \\ &= e^{-2t}\,,\, t > 0 \end{aligned}$$

(c) Use equation (5-17).

$$f(t) = \begin{cases} 2e^{-2t} & \text{for } t > 0 \\ 0 & \text{otherwise.} \end{cases}$$

T has an exponential distribution with mean .5.

(d) Integrate $f(x,y)$ over the region in which both x and y are less than t.

$$\begin{aligned} P(X < t \text{ and } Y < t) &= \int_0^t \int_0^t e^{-x}e^{-y}\,dx\,dy \\ &= (1-e^{-t})^2\,,\, t > 0 \end{aligned}$$

(e) The answer to (d) is $F(t)$. Using equation (5-17),

$$f(t) = \begin{cases} 2(1-e^{-t})e^{-t} & \text{for } t > 0 \\ 0 & \text{otherwise} \end{cases}$$

Using equation (5-18),

$$\begin{aligned} ET &= \int_0^\infty 2t(1-e^{-t})e^{-t}\,dt \\ &= (-2e^{-t}(t+1) + 2(\tfrac{1}{4})e^{-2t}(2t+1))|_0^\infty \\ &= 1.5. \end{aligned}$$

Section 5

1. The total thickness is the sum of the thicknesses of the layers. Define $X_1, \ldots, X_5$ to be the thicknesses of the layers; then $U = X_1 + \cdots + X_5$ is the total thickness. Using equation (5-58),

$$EU = EX_1 + \cdots + EX_5 = .750 \text{ in.}$$

To use equation (5-59), you have to square the given standard deviations first.

$$\mathrm{Var}U = \mathrm{Var}X_1 + \cdots + \mathrm{Var}X_5 = .000014,$$

so the standard deviation of U is $\sqrt{.000014} = .00374$ in.

2. (a) Take the given measured values to be approximately equal to the means of the random variables. The following Minitab (Version 9.1) session shows a simulation of observed α's using a normal distribution for each input random variable.

```
MTB > random 1000 c1;
SUBC> normal 50 .1.
MTB > name c1 'T1'
MTB > random 1000 c2;
SUBC> normal 100 .1.
MTB > name c2 'T2'
MTB > random 1000 c3;
SUBC> normal 1.0 .00005.
MTB > name c3 'L1'
MTB > random 1000 c4;
SUBC> normal 1.00095 .00005.
MTB > name c4 'L2'
MTB > let c5 = (c4 - c3)/(c3*(c2 - c1))
MTB > name c5 'alpha'

MTB > gstd
* NOTE  * Standard Graphics are enabled.
          Professional Graphics are disabled.
          Use the GPRO command to enable Professional Graphics.
MTB > hist c5

Histogram of alpha   N = 1000
Each * represents 5 obs.

 Midpoint   Count
0.0000155       2  *
0.0000160      19  ****
```

```
0.0000165      21  *****
0.0000170      56  ************
0.0000175      81  *****************
0.0000180     135  ***************************
0.0000185     142  *****************************
0.0000190     155  *******************************
0.0000195     115  ***********************
0.0000200     109  **********************
0.0000205      59  ************
0.0000210      51  ***********
0.0000215      34  *******
0.0000220      15  ***
0.0000225       5  *
0.0000230       0
0.0000235       1  *

MTB > stdev c5
   ST.DEV. =0.0000013380
```

The sample standard deviation of the simulated values of α is .0000013380. This can be used as a rough approximation to the underlying standard deviation of the probability distribution of α.

To use the propagation of error formula (5-59), the partial derivatives need to be evaluated at the means of the input random variables:

$$\frac{\partial\alpha}{\partial L_1} = \frac{-L_1(T_2 - T_1) - (L_2 - L_1)(T_2 - T_1)}{(L_1(T_2 - T_1))^2} = -.020019$$

$$\frac{\partial\alpha}{\partial L_2} = \frac{1}{L_1(T_2 - T_1)} = .02$$

$$\frac{\partial\alpha}{\partial T_1} = \frac{L_1(L_2 - L_1)}{(L_1(T_2 - T_1))^2} = 3.8 \times 10^{-7}$$

$$\frac{\partial\alpha}{\partial T_2} = \frac{-L_1(L_2 - L_1)}{(L_1(T_2 - T_1))^2} = -3.8 \times 10^{-7}$$

Then applying equation (5-59),

$$\begin{aligned} \mathrm{Var}(\alpha) &\approx (-.020019)^2(.00005)^2 + (.02)^2(.00005)^2 + (3.8 \times 10^{-7})^2(.1)^2 \\ &\quad + (-3.8 \times 10^{-7})^2(.1)^2 \\ &= 1.0019 \times 10^{-12} + 10^{-12} + 1.444 \times 10^{-15} + 1.444 \times 10^{-15} \\ &= 2.0047889 \times 10^{-12}, \end{aligned}$$

so the approximate standard deviation of α is

$$\sqrt{2.0047889 \times 10^{-12}} = .000001415 \frac{1}{^\circ\mathrm{C}}$$

This is almost the same as the approximation from the simulation.

(b) The lengths, since their terms (variance $\times$ squared partial derivative) contribute much more to the propagation of error formula than the temperatures' terms.

(c) T_2 can only be made so large before the brass melts. It is difficult to measure the length of melting brass. Also, it may be difficult to find measuring instruments that will accurately and precisely measure extremely cold or hot temperatures.

3. From Exercise 1, Section 2, Chapter 5, $EX = \mu = \frac{13}{27}$ and $\text{Var}X = \sigma = .28808$.

(a) Using equation (5-55), $E\bar{X} = \mu = \frac{13}{27}$. Using equation (5-56),

$$\begin{aligned}\sqrt{\text{Var}\bar{X}} &= \frac{\sigma}{\sqrt{n}} \\ &= \frac{.28808}{\sqrt{25}} = .05762.\end{aligned}$$

(b) Because n is large and the individual X's are independent, the central limit theorem says that the distribution of $\bar{X}$ is approximately normal. From part (a), the distribution of $\bar{X}$ is approximately normal with mean $\frac{13}{27}$ and standard deviation .05762.

(c)

$$\begin{aligned}P(\bar{X} > .5) &= 1 - P(\bar{X} \le .5) \\ &= 1 - P\left(\frac{\bar{X} - \frac{13}{27}}{.05762} \le \frac{.5 - \frac{13}{27}}{.05762}\right) \\ &= 1 - P(Z \le .321) \text{ where Z is standard normal} \\ &= 1 - .6255 = .3745.\end{aligned}$$

(d)

$$\begin{aligned}P(.4615 \le \bar{X} \le .5015) &= P(\bar{X} \le .5015) - P(\bar{X} < .4615) \\ &= P(Z \le .35) - P(Z < -.35) \\ &= .6368 - .3632 = .2736.\end{aligned}$$

(e) $\bar{X}$ is approximately normal with mean $\frac{13}{27}$ and standard deviation $\frac{.28808}{\sqrt{100}} = .02881$.

$$\begin{aligned}P(\bar{X} > .5) &= 1 - P(\bar{X} \le .5) \\ &= 1 - P\left(\frac{\bar{X} - \frac{13}{27}}{.02881} \le \frac{.5 - \frac{13}{27}}{.02881}\right) \\ &= 1 - P(Z \le .64) \text{ where Z is standard normal} \\ &= 1 - .7389 = .2611.\end{aligned}$$

$$\begin{aligned}P(.4615 \le \bar{X} \le .5015) &= P(\bar{X} \le .5015) - P(\bar{X} < .4615) \\ &= P(Z \le .69) - P(Z < -.69) \\ &= .7549 - .2451 = .5098.\end{aligned}$$

4. All of these sample sizes are large. A simple random sample from a large population results in random variables that are approximately independent. The central limit theorem then says that $\bar{X}$ is approximately normal, even if the individual X's are not. Equation (5-55) says that the mean of $\bar{X}$ is equal to μ, the mean of the individual X's. Equation (5-56) says that the standard deviation of $\bar{X}$ is $\frac{\sigma}{\sqrt{n}}$, which is equal to $\frac{.0004}{\sqrt{n}}$.

For $n = 25$,

$$\begin{aligned}
P(\mu - .0001 \leq \bar{X} \leq \mu + .0001) &= P(\bar{X} \leq \mu + .0001) - P(\bar{X} < \mu - .0001) \\
&= P\left(Z \leq \frac{.0001}{\frac{.0004}{\sqrt{n}}}\right) - P\left(Z < \frac{-.0001}{\frac{.0004}{\sqrt{n}}}\right) \\
&= P(Z \leq 1.25) - P(Z < -1.25) \\
&= .8944 - .1056 = .7888.
\end{aligned}$$

For $n = 100$,

$$\begin{aligned}
P(\mu - .0001 \leq \bar{X} \leq \mu + .0001) &= P(\bar{X} \leq \mu + .0001) - P(\bar{X} < \mu - .0001) \\
&= P(Z \leq 2.5) - P(Z < -2.5) \\
&= .9938 - .0062 = .9876.
\end{aligned}$$

For $n = 400$,

$$\begin{aligned}
P(\mu - .0001 \leq \bar{X} \leq \mu + .0001) &= P(\bar{X} \leq \mu + .0001) - P(\bar{X} < \mu - .0001) \\
&= P(Z \leq 5) - P(Z < -5) \\
&\approx 1 - 0 = 1.
\end{aligned}$$

5. Rearrange the relationship in terms of g:

$$g = \frac{4\pi^2 L}{\tau^2}.$$

Take the given length and period to be approximately equal to the means of these input random variables. To use the propagation of error formula (5-59), the partial derivatives need to be evaluated at the means of the input random variables:

$$\frac{\partial g}{\partial L} = \frac{4\pi^2}{\tau^2} = 6.418837$$

$$\frac{\partial g}{\partial \tau} = \frac{-8\pi^2 L}{\tau^3} = -25.8824089$$

Then applying equation (5-59),

$$\begin{aligned}
\text{Var}(g) &\approx (6.418837)^2(.0208)^2 + (-25.8824089)^2(.1)^2 \\
&= .01783 + 6.699 \\
&= 6.7168 \text{ ft}^2/\text{sec}^4,
\end{aligned}$$

so the approximate standard deviation of g is

$$\sqrt{6.7168} = 2.592 \text{ ft/sec}^2.$$

The precision in the period measurement is the principal limitation on the precision of the derived g because its term (variance $\times$ squared partial derivative) contributes much more to the propagation of error formula than the length's term.

End Chapter Exercises

1. Use equation (5-3) with $n = 6$ and $p = .9$.

   (a) $P(X = 6) = .531$.

   (b) $P(X \geq 4) = .984$.

   (c) $P(X < 4) = 1 - P(X \geq 4) = .016$.

   (d) $EX = np = 5.4$.

   (e) $\text{Var}X = np(1-p) = .54$; std. dev. of $X = \sqrt{.54} = .735$.

2. Use the binomial distribution, equation (5-3), with $n = 10$ and $p = .15$.

   (a) $P(X = 2) = .276$.

   (b) $P(X \geq 1) = 1 - P(X < 1) = 1 - P(X = 0) = 1 - .197 = .803$.

   (c) $EX = np = 1.5$.

   (d) $\text{Var}X = np(1-p) = 1.275$.

   (e) $\sqrt{1.275} = 1.129$.

3. The Poisson distribution, equation (5-10), is often used in these kinds of situations.

   (a) It is given that the mean of Y is equal to 1.3, since Y is the number of defects on one bumper. λ is the mean of the Poisson distribution (see equation (5-11)), so use $\lambda = 1.3$. $P(Y = 2) = .230$.

   (b) $P(Y \geq 1) = 1 - P(Y < 1) = 1 - P(Y = 0) = 1 - .273 = .727$.

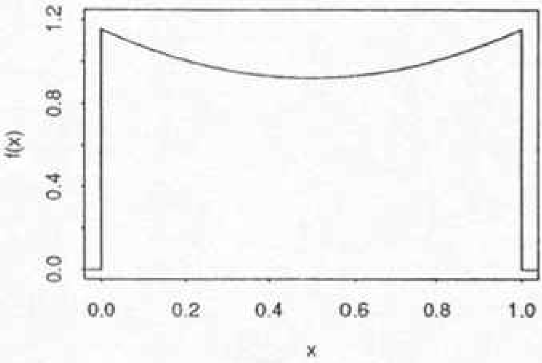
   (c) Using equation (5-12), $\sqrt{\text{Var}Y} = \sqrt{\lambda} = \sqrt{1.3} = 1.14$.

   (d) If the average number of defects per bumper is 1.3, then the average number of defects per 2 bumpers is 2.6. Use the Poisson distribution for $W = Y + Z$, with $\lambda = 2.6$.

$$\begin{aligned} P(W \geq 2) = 1 - P(W < 2) &= 1 - (P(W = 0) + P(W = 1)) \\ &= 1 - (.0743 + .1931) = .7326. \end{aligned}$$

4. (a) Use equation (5-13) and solve for $k = .9231$.

$$f(x) = \begin{cases} .9231((x-.5)^2+1) & \text{for } 0 < x < 1 \\ 0 & \text{otherwise.} \end{cases}$$



(b) All of the following probabilities correspond to areas under $f(x)$.

$$\begin{aligned} P(X \geq .5) &= \int_{.5}^{1} f(x)\,dx \\ &= .5. \end{aligned}$$

$$P(X > .5) = P(X \geq .5) = .5$$

$$\begin{aligned} P(.75 > X \geq .5) &= \int_{.5}^{.75} f(x)\,dx \\ &= .2356. \end{aligned}$$

$$\begin{aligned} P(|X-.5| \geq .2)) &= 1 - P(|X-.5| < .2)) \\ &= 1 - P(-.2 < X - .5 < .2)) \\ &= 1 - P(.3 < X < .7)) \\ &= 1 - \int_{.3}^{.7} f(x)\,dx \\ &= 1 - .3742 = .6258. \end{aligned}$$

(c) Using equation (5-18),

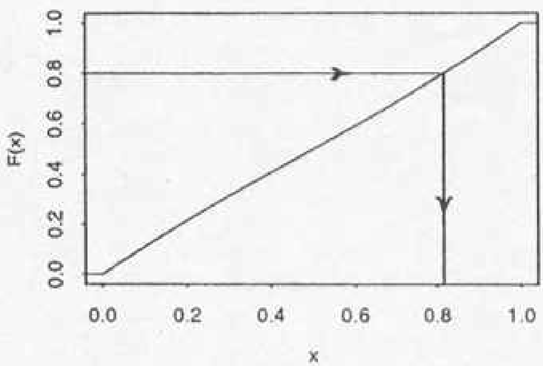
$$EX = .5.$$

Using equation (5-19),

$$\text{Var}X = .08846.$$

(d) Using equation (5-16),

$$\begin{aligned} F(x) &= P(X \le x) \\ &= \int_{-\infty}^{x} f(x)\,dx \\ &= \begin{cases} 0 & \text{for } x \le 0 \\ .9231(\frac{1}{3}x^3 - \frac{1}{2}x^2 + 1.25x) & \text{for } 0 < x < 1 \\ 1 & \text{for } x \ge 1. \end{cases} \end{aligned}$$



The .8 quantile of the distribution is the x such that $P(X \le x) = .8$. This is just $F^{-1}(.8)$, so you need to find .8 on the vertical axis, and find the x that produces this value for $F(x)$. (By trial and error, the exact value is $x = .81458$.)

5. The values of $\Phi(z) = P(Z \le z)$ are given in Table B-3. All of these probabilities correspond to areas under the standard normal curve.

(a) $P(Z \le 1.13) = \Phi(1.13) = .8708$.

(b) $P(Z > -.54) = 1 - P(Z \le -.54) = 1 - \Phi(-.54) = 1 - .2946 = .7054$.

(c) $P(-1.02 < Z < .06) = P(Z < .06) - P(Z \le -1.02) = .5239 - .1539 = .3700$.

(d) $P(|Z| \le .25) = P(-.25 \le Z \le .25) = P(Z \le .25) - P(Z < -.25) = .5987 - .4013 = .1974$.

(e) $P(|Z| > 1.51) = P(Z < -1.51) + P(Z > 1.51) = 2(P(Z < -1.51)) = 2(.0655) = .1310$.

(f) $P(-3.0 < Z < 3.0) = P(Z < 3.0) - P(Z \le -3.0) = .9987 - .0013 = .9974$.

(g) $P(|Z| < \#) = .80$ is equivalent to $P(Z < \#) = .90$ (by symmetry). Looking up .90 in the body of the table, $\# \approx 1.28$.

(h) Looking up .80 in the body of the table, $\# \approx .84$.

(i) $P(|Z| > \#) = .04$ is equivalent to $P(Z < \#) = .98$ (by symmetry). Looking up .98 in the body of the table, $\# \approx 2.05$.

6. Probabilities involving X are just areas under the normal curve with $\mu = 10.2$ and $\sigma = .7$.

Each of these areas has an equal corresponding area under the standard normal curve. Define $Z = \frac{X-10.2}{.7}$. Then Z is a standard normal random variable. Re-express each of the problems below in terms of Z.

(a) $P(X \leq 10.1) = P(Z < -.14) = .4443$.

(b) $P(X > 10.5) = 1 - P(X \leq 10.5) = 1 - P(Z \leq .43) = 1 - .6664 = .3336$.

(c) $P(9.0 < X < 10.3) = P(-1.71 < Z < .14) = P(Z < .14) - P(Z \leq -1.71) = .5557 - .0436 = .5121$.

(d) $P(|X - 10.2| \leq .25) = P(9.95 \leq X \leq 10.45) = P(-.36 \leq Z \leq .36) = P(Z \leq .36) - P(Z < -.36) = .6406 - .3594 = .2812$.

(e) $P(|X - 10.2| > 1.51) = 1 - P(|X - 10.2| \leq 1.51) = 1 - P(8.69 \leq X \leq 11.71) = 1 - P(-2.16 \leq Z \leq 2.16) = 1 - (P(Z \leq 2.16) - P(Z < -2.16)) = 1 - (.9846 - .0154) = .0308$.

(f) $P(|X - 10.2| < \#) = .80$ is equivalent to $P(X - 10.2 \leq \#) = .90$ (by symmetry). This is equivalent to $P(Z \leq \frac{\#}{.7}) = .90$. Looking up .90 in the body of the table,

$$\frac{\#}{.7} \approx 1.28$$

so $\# \approx .896$.

(g) $P(X < \#) = .80$ is equivalent to $P(Z < \frac{\#-10.2}{.7}) = .80$. Looking up .80 in the body of the table,

$$\frac{\# - 10.2}{.7} \approx .84$$

so $\# \approx 10.788$.

(h) $P(|X - 10.2| > \#) = .04$ is equivalent to $P(|X - 10.2| \leq \#) = .96$, which is equivalent to $P(X - 10.2 \leq \#) = .98$ (by symmetry). This is equivalent to $P(Z \leq \frac{\#}{.7}) = .98$. Looking up .98 in the body of the table,

$$\frac{\#}{.7} \approx 2.05$$

so $\# \approx 1.435$.

7. The probability of one part failing to meet inspection is equal to the long-run fraction of parts failing to meet inspection, if the part measurements are considered to be identical random variables. You need to make μ small enough so that

$$P(X > 3.150) \leq .03.$$

This is equivalent to $P(X \leq 3.150) > .97$, which is equivalent to $P\left(Z \leq \frac{3.150-\mu}{.002}\right) > .97$, where Z is a standard normal random variable. Looking up .97 in the body of Table B-3,

$$\frac{3.150 - \mu}{.002} > 1.88,$$

or $\mu < 3.14624$.

8. (a) Let $X_1, \ldots, X_{50}$ be the 50 individual strengths. The strength of the cable is then

$U = \sum X_i$. Using equation (5-55), the mean of the sum is the sum of the means:

$$EU = EX_1 + EX_2 + \cdots + EX_{50} = 50(45) = 2250 \text{ lbs.}$$

Assuming that the individual strengths are independent, you can use equation (5-56) to say that the variance of the sum is the sum of the variances:

$$\text{Var}U = \text{Var}X_1 + \text{Var}X_2 + \cdots + \text{Var}X_{50} = 50(4)^2 = 800 \text{ lbs.}^2,$$

so the standard deviation of U is $\sqrt{800} = 28.28$ lbs.

(b) Since $n = 50$ is large, the central limit theorem says that $\bar{X} = \frac{\sum X_i}{50}$ is approximately normal. The mean of $\bar{X}$ is 45 and the standard deviation of $\bar{X}$ is $\frac{4}{\sqrt{50}}$ (see equations (5-55) and (5-56). Since $\bar{X} = \frac{U}{50}$,

$$\begin{aligned} P(U \geq 2230) = P\left(\bar{X} \geq \frac{2230}{50}\right) &= 1 - P\left(\bar{X} < \frac{2230}{50}\right) \\ &= 1 - P\left(Z < \frac{\frac{2230}{50} - 45}{\frac{4}{\sqrt{50}}}\right) \\ &= 1 - P(Z < -.71) \\ &= 1 - .2389 = .7611. \end{aligned}$$

(Z is a standard normal random variable.)

9. Assume that the nominal values given are approximately equal to the means of these input random variables.

(a) To use the propagation of error formula (5-59), the partial derivatives need to be evaluated at the means of the input random variables:

$$\frac{\partial \rho}{\partial L} = -\frac{R\pi D^2}{4L^2} = -8.4823 \times 10^{-6}$$

$$\frac{\partial \rho}{\partial D} = \frac{2R\pi D}{4L} = 1.69646 \times 10^{-5}$$

$$\frac{\partial \rho}{\partial R} = \frac{\pi D^2}{4L} = \pi \times 10^{-6}$$

Then applying equation (5-59),

$$\begin{aligned} \text{Var}(\rho) &\approx (-8.4823 \times 10^{-6})^2(10^{-3})^2 + (1.69646 \times 10^{-5})^2(10^{-4})^2 \\ &\quad +(\pi \times 10^{-6})^2(5 \times 10^{-4})^2 \\ &= 7.1949 \times 10^{-17} + 2.87798 \times 10^{-18} + 2.467397 \times 10^{-18} \\ &= 7.72948 \times 10^{-17}\ (\Omega\text{m})^2, \end{aligned}$$

so the approximate standard deviation of ρ is

$$\sqrt{7.72948 \times 10^{-17}} = 8.792 \times 10^{-9}\ \Omega\text{m}.$$

(b) Length, since its term (variance × squared partial derivative) contributes more to the propagation of error formula than the other input measurements' terms.

(c) Over a period of years, there may be other sources of variation (besides measurement error for these 3 variables) that affect the relationship. These might include variation in the purity of the wires, the accuracy of temperature measurement, and the deviation from cylindricalness of the wires, just to name a few.

10. Let $X_1, \ldots, X_{370}$ be the 370 individual sheet thicknesses. The thickness of the text is then $U = \sum X_i$. Using equation (5-53), the mean of the sum is the sum of the means:

$$EU = EX_1 + EX_2 + \cdots + EX_{370} = 370(.1) = 37.0 \text{ mm}.$$

Assuming that the individual thicknesses are independent, you can use equation (5-54) to say that the variance of the sum is the sum of the variances:

$$\text{Var}U = \text{Var}X_1 + \text{Var}X_2 + \cdots + \text{Var}X_{370} = 370(.003)^2 = .00333 \text{ mm}^2,$$

so the standard deviation of U is $\sqrt{.00333} = .0577$ mm.

11. (a) To find an approximate mean for I, use formula (5-58):

$$EI = EV\left(\frac{1}{ER_1} + \frac{1}{ER_2}\right) = 1.8 \text{ A}.$$

To use the propagation of error formula (5-59), the partial derivatives need to be evaluated at the means of the input random variables:

$$\frac{\partial I}{\partial V} = \frac{1}{R_1} + \frac{1}{R_2} = .2$$

$$\frac{\partial I}{\partial R_1} = -\frac{V}{R_1^2} = -.09$$

$$\frac{\partial I}{\partial R_2} = -\frac{V}{R_2^2} = -.09$$

Then applying equation (5-59),

$$\begin{aligned}\text{Var}I &\approx (.2)^2(.2)^2 + (-.09)^2(.1)^2 + (-.09)^2(.1)^2 \\ &= .0016 + .000081 + .000081 \\ &= .001762\,,\end{aligned}$$

so the approximate standard deviation of I is

$$\sqrt{.001762} = .041976 \text{ A}.$$

(b) Voltage, since its term (variance × squared partial derivative) contributes much more to the propagation of error formula than the sum of the terms for the two resistances.

12. (a)

$$P(.48 \le X \le .52) = P(X \le .52) - P(X < .48) = P(Z \le .67) - P(Z < -.67)$$

where Z is a standard normal random variable. From Table B-3, this is equal to $.7486 - .2514 = .4972$.

(b) From equations (5-55) and (5-56), the mean of $\bar{X}$ is .5 and (assuming that students' measurements are independent), the standard deviation of $\bar{X}$ is $\frac{.03}{\sqrt{25}} = .006$.

$$P(.48 \le \bar{X} \le .52) = P(\bar{X} \le .52) - P(\bar{X} < .48) = P(Z \le 3.33) - P(Z < -3.33)$$

which from Table B-3 is equal to $.9996 - .0004 = .9992$.

(c) Use the binomial distribution, equation (5-3), with $n = 5$ and $p = .4972$.

$$P(Y \ge 2) = 1 - P(Y < 2) = 1 - (P(Y = 0) + P(Y = 1)) = 1 - (.03213 + .15888) = .80898.$$

13. (a) Let X be the strength of a particular individual cable. Using the normal distribution,

$$\begin{aligned} P(X < 400) &= P\left(\frac{X - 450}{50} < \frac{400 - 450}{50}\right) \\ &= P(Z < -1) \text{ where } Z \text{ is a standard normal random variable} \\ &= .1587 \end{aligned}$$

from Table B-3. Now let Y be the number of cables that fail. If the cable strengths are independent, Y has a binomial distribution, equation (5-3), with $n = 5$ and $p = .1587$.

$$P(Y \ge 1) = 1 - P(Y < 1) = 1 - P(Y = 0) = 1 - .4215 = .5785.$$

(b) Since the sample size is large, it is not necessary to assume that the individual cable strengths are normally distributed. The central limit theorem says that the sample mean $\bar{X}$ will be approximately normal, regardless of the distribution of the individual X's. From equations (5-55) and (5-56), the mean of $\bar{X}$ is 45 lb, and the standard deviation of $\bar{X}$ is $\frac{50}{\sqrt{100}}$ lb.

$$\begin{aligned} P(\bar{X} < 457) &= P\left(\frac{\bar{X} - 450}{\frac{50}{\sqrt{100}}} < \frac{457 - 450}{\frac{50}{\sqrt{100}}}\right) \\ &= P(Z < 1.4) \text{ where } Z \text{ is a standard normal random variable} \\ &= .9192 \end{aligned}$$

from Table B-3.

14. Using equation (5-18),

$$EX = \int_0^1 .3x\,dx + \int_1^2 .7x\,dx = 1.2.$$

using equation (5-19),

$$\text{Var}X = \int_0^1 .3x^2\,dx + \int_1^2 .7x^2\,dx - (1.2)^2 = .2933.$$

15. (a)

$$P(X > 2945) = 1 - P(X \leq 2945) = 1 - P(Z \leq .75).$$

(Z is a standard normal random variable.) From Table B-3, this is equal to $1 - .7734 = .2266$.

(b) Use the binomial distribution, equation (5-3), with $n = 4$ and $p = .2266$.

$$P(Y \geq 1) = 1 - P(Y < 1) = 1 - P(Y = 0) = 1 - .3578 = .6422.$$

(c) Using equations (5-55) and (5-56), the mean of $\bar{X}$ is 2930 psi and the standard deviation of $\bar{X}$ is $\frac{20}{\sqrt{25}}$.

$$\begin{aligned} P(2925 \leq \bar{X} \leq 2935) &= P(\bar{X} \leq 2935) - P(\bar{X} < 2925) \\ &= P(Z \leq 1.25) - P(Z < -1.25) \\ &= .8944 - .1056 = .7888. \end{aligned}$$

(Z is a standard normal random variable.)

(d) The Poisson distribution, equation (5-10), is often used in this type of situation to model the number of occurrences. λ is the mean. In this situation, since the average number of air pockets is 1 per 50 cubic yards, $\lambda = 3$.

$$\begin{aligned} &P(W \geq 2) = 1 - P(W < 2) = 1 - (P(W = 0) + P(W = 1)) \\ &= 1 - (.049787 + .1493612) = .80085. \end{aligned}$$

16.

$$P(X < 1.0) = \int_0^1 .5x\,dx = .25.$$

using equation (5-18),

$$EX = \int_0^2 .5x^2\,dx = \frac{4}{3}.$$

17. (a) To use the propagation of error formula (5-59), the partial derivatives need to be evaluated at the means of the input random variables:

$$\frac{\partial \eta}{\partial F} = \frac{L}{vA} = 1.3259 \times 10^{-5}$$

$$\frac{\partial \eta}{\partial A} = -\frac{FL}{vA^2} = -1.59278 \times 10^{-6}$$

$$\frac{\partial \eta}{\partial L} = \frac{F}{vA} = .004004$$

$$\frac{\partial \eta}{\partial v} = -\frac{FL}{v^2 A} = -6.6737 \times 10^{-5}$$

Then applying equation (5-59),

$$\begin{aligned} \mathrm{Var}(\eta) &\approx (1.3259 \times 10^{-5})^2(.05)^2 + (-1.59278 \times 10^{-6})^2(.2)^2 + (.004004)^2(.05)^2 \\ &\quad +(-6.6737 \times 10^{-5})^2(1)^2 \\ &= 4.395 \times 10^{-13} + 1.014776 \times 10^{-13} + 6.41358 \times 10^{-7} + 4.453878 \times 10^{-9} \\ &= = 4.4534 \times 10^{-8} \end{aligned}$$

so the approximate standard deviation of η is

$$\sqrt{4.4534 \times 10^{-8}} = 2.1103 \times 10^{-4}$$

(b) The approximation above only takes measurement error into account. Other effects that change over time and from place to place could also affect the relationship, causing more variability. These might include differences in the shapes of the container and the rotor, variability in oil quality, and differences in experimental techniques, just to name a few.

18. (a) Using equation (5-59),

$$\begin{aligned} \mathrm{Var}(\lambda) &\approx (-.249)^2(.1)^2 + (.199)^2(.1)^2 + (-.00199)^2(1)^2 + (.00199)^2(1)^2 \\ &\quad +(.000825)^2(10)^2 + (.000332)^2(1)^2 \\ &= .00062001 + .00039601 + 3.9601 \times 10^{-6} + 3.9601 \times 10^{-6} \\ &\quad +6.80625 \times 10^{-5} + 1.10224 \times 10^{-7} \\ &= .001092, \end{aligned}$$

so the approximate standard deviation of λ is

$$\sqrt{.001092} = .033047.$$

(b) D, followed by L. These two variables' terms (variance $\times$ squared partial derivative) contribute the most to the propagation of error formula.

19. Use the binomial distribution, equation (5-3), with $n = 5$ and $p = .15$.

$$P(X \leq 1) = P(X = 0) + P(X = 1) = .4437 + .3915 = .8352.$$

20. Since the sample size is large, the central limit theorem says that the distribution of $\bar{X}$ is approximately normal. Using equation (5-55), the mean of $\bar{X}$ is μ and the standard deviation of $\bar{X}$ is $\frac{.1}{\sqrt{25}}$.

$$\begin{aligned} P(\mu - .03 \leq \bar{X} \leq \mu + .03) &= P(\bar{X} \leq \mu + .03) - P(\bar{X} < \mu - .03) \\ &= P\left(\frac{\bar{X}-\mu}{\frac{.1}{\sqrt{25}}} \leq \frac{.03}{\frac{.1}{\sqrt{25}}}\right) - P\left(\frac{\bar{X}-\mu}{\frac{.1}{\sqrt{25}}} < \frac{-.03}{\frac{.1}{\sqrt{25}}}\right) \\ &= P(Z \leq 1.5) - P(Z < -1.5) \\ &= .9332 - .0668 = .8664. \end{aligned}$$

(Z is a standard normal random variable.)

21. (a) Using equation (5-59),

$$\begin{aligned} \mathrm{Var}Y &\approx (5.09 \times 10^4)^2(10)^2 + (-6.11 \times 10^8)^2(.001)^2 + (1.54 \times 10^8)^2(.01)^2 \\ &\quad +(-1.53 \times 10^8)^2(.01)^2 \\ &= 2.59081 \times 10^{11} + 3.733211 \times 10^{11} + 2.3716 \times 10^{12} + 2.3409 \times 10^{12} \\ &= 5.3449 \times 10^{12}, \end{aligned}$$

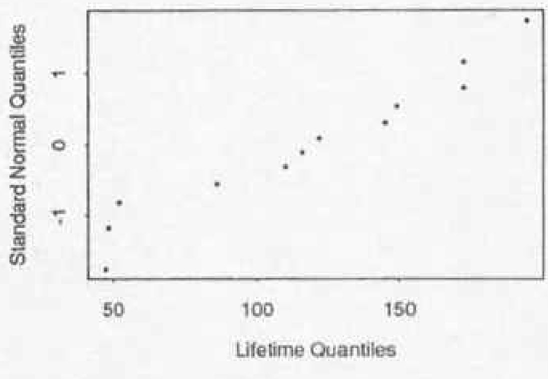
so the approximate standard deviation of Y is

$$\sqrt{5.3449 \times 10^{12}} = 2,311,904.41.$$

(b) L_0, followed closely by L_1. These two variables' terms (variance × squared partial derivative) contribute the most to the propagation of error formula.

(c) F and D will have important interactions. The equation says that the effect on ΔL of changing F depends on D, and vice-versa. The slope of a plot of ΔL versus F would be different for different values of D.

22. (a)



The smallest data point is too large—it would need to be made smaller to make the plot more linear, and thus make the data more bell-shaped. So the data have a shorter left tail than the normal shape. Overall, the plot is not all that non-linear.

(b)

$$\begin{aligned} P(Y < 40) &= P\left(\frac{Y - 117.75}{51.1} < \frac{40 - 117.75}{51.1}\right) \\ &= P(Z < -1.52) \text{ where } Z \text{ is a standard normal random variable} \\ &= .0643 \end{aligned}$$

from Table B-3.

(c) It would be high because the actual data have a shorter left tail than the normal distribution. If the underlying distribution has a shorter left tail than the normal distribution, then there will be less probability to the left of 40.

23. (a) Use equation (5-10) with $\lambda = .03$.

$$P(X \geq 1) = 1 - P(X < 1) = 1 - P(X = 0) = 1 - e^{-.03} = .0296$$

(b) Now use $\lambda = .3$.

$$P(Y \geq 1) = 1 - P(Y < 1) = 1 - P(Y = 0) = 1 - e^{-.3} = .2592.$$

(c) Use the binomial distribution, equation (5-3), with $n = 10$ and $p = .0296$.

$$P(W = 1) = .2256.$$

24. If the mean is ideal, it is equal to 2 cm. Want

$$P(1.998 \leq X \leq 2.002) = .95$$

This is equivalent to $P(X \leq 2.002) = .975$, by symmetry. This is equivalent to $P\left(Z \leq \frac{.002}{\sigma}\right) = .975$, where Z is a standard normal random variable. Looking up .975 in the body of Table B-3,

$$\frac{.002}{\sigma} \approx 1.96,$$

or $\sigma \approx .00102$. σ must be less than this number to ensure that 95% of the parts are within these specifications.

25.

z	$g_S(z)$	$g_N(z)$	$g_D(z)$	$\Phi(z)$
.5	.6950	.6874	.6914	.6915
1.0	.8400	.8336	.8413	.8413
1.5	.9350	.9187	.9332	.9332
2.0	.9800	.9717	.9772	.9773
2.5	.9900	.9922	.9938	.9938

The last column of the table is from Table B-3. For these z's, $g_D(z)$ is the best approximation, followed by $g_S(z)$. $g_N(z)$ is too low for moderate values of z.

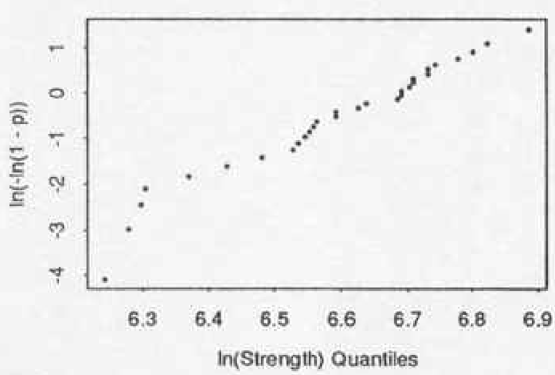
26.

p	$Q_{\text{approx}}(p)$	$Q_{\text{table}}(z)$
.01	-2.326	-2.33
.05	-1.645	-1.645
.1	-1.282	-1.28
.3	-.524	-.52
.7	.524	.52
.9	1.282	1.28
.95	1.645	1.645
.99	2.326	2.33

The approximation is excellent for these p's.

27. (a) The computations necessary for the Weibull plot are given below.

i	ith Smallest Strength	$\ln(i\text{th Smallest Strength})$	$p = \frac{i-.5}{30}$	$\ln(-\ln(1-p))$
1	514	6.2422	.0167	-4.0860
2	533	6.2785	.0500	-2.9702
3	543	6.2971	.0833	-2.4417
4	547	6.3044	.1167	-2.0870
5	584	6.3699	.1500	-1.8170
6	619	6.4281	.1833	-1.5969
7	653	6.4816	.2167	-1.4098
8	684	6.5280	.2500	-1.2459
9	689	6.5352	.2833	-1.0992
10	695	6.5439	.3167	-.9656
11	700	6.5511	.3500	-.8422
12	705	6.5582	.3833	-.7269
13	709	6.5639	.4167	-.6180
14	729	6.5917	.4500	-.5144
15	729	6.5917	.4833	-.4150
16	753	6.6241	.5167	-.3188
17	763	6.6373	.5500	-.2250
18	800	6.6846	.5833	-.1330
19	805	6.6908	.6167	-.0420
20	805	6.6908	.6500	.0486
21	814	6.7020	.6833	.1397
22	819	6.7081	.7167	.2320
23	819	6.7081	.7500	.3266
24	839	6.7322	.7833	.4249
25	839	6.7322	.8167	.5285
26	849	6.7441	.8500	.6403
27	879	6.7788	.8833	.7647
28	900	6.8024	.9167	.9102
29	919	6.8233	.9500	1.0972
30	979	6.8865	.9833	1.4096



The Weibull plot is fairly linear, except in the lower tail. There tends to be more variability in the tails of probability plots, so the Weibull model fits the data reasonably well.

(b) From a line fit to the plot,

$$\ln \alpha \approx \text{x-intercept} = 6.672515$$

so $\alpha \approx e^{4.396} = 790.4$.

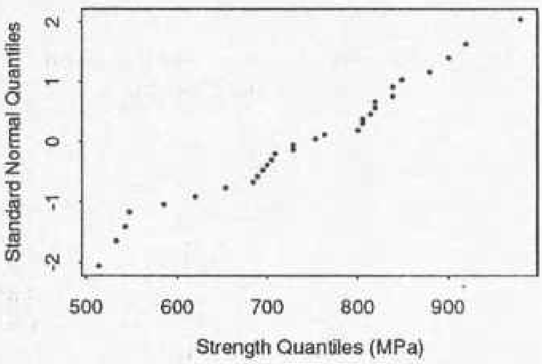
$$\beta \approx \text{slope} = 7.202.$$

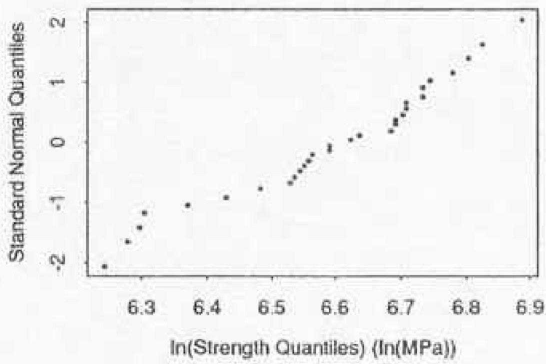
To find the p quantile of the distribution, set the CDF equal to p and solve for x:

$$\begin{aligned} F(x) = 1 - e^{-\left(\frac{x}{\alpha}\right)^\beta} &= p \\ \ln(1-p) &= -\left(\frac{x}{\alpha}\right)^\beta \\ x &= \alpha(-\ln(1-p))^{\frac{1}{\beta}} \end{aligned}$$

The median strength is the .5 quantile. Using $p = .5$ and the above approximations for α and β, the median strength is approximately $x = 751.2$. The strength exceeded by 80% of such specimens is the .2 quantile; for $p = .2$, $x = 641.8$. For $p = .1$, $x = 578.3$, and for $p = .01$, $x = 417.3$.

(c) The first 3 coordinates of the normal plot of the raw data are (514, −2.05), (533, −1.64), (543, −1.41). The first 3 coordinates of the normal plot of the logs of the raw data are (6.24, −2.05), (6.28, −1.64), (6.30, −1.41).





The normal plot of the logs of the data is not much more linear than the normal plot of the raw data, but both of these plots are more linear than the Weibull plot. This suggests a preference for the normal or lognormal model over the Weibull.

(d) For the raw data, the approximate mean and standard deviation are

$$\mu \approx \text{x-intercept} = 740.5;$$

$$\sigma \approx \frac{1}{\text{slope}} = 124.26.$$

The p quantile is the number x such that

$$P(X \leq x) = p.$$

This is equivalent to $P\left(Z \leq \frac{x-\mu}{\sigma}\right) = p$, where Z is a standard normal random variable. To find the p quantile, look up p in the body of Table B-3, and find corresponding z in the margin. Then set this z equal to $\frac{x-\mu}{\sigma}$ and solve for x. For example, the .01 quantile can be found by looking up .01 in the body of the table. This gives $z = -2.33$. Use the above approximations for μ and σ, and solve

$$\frac{x-\mu}{\sigma} = -2.33$$

for x. This gives $x = 451.0$.

For the logs of the data, the approximate mean and standard deviation are

$$\mu \approx \text{x-intercept} = 6.59369;$$

$$\sigma \approx \frac{1}{\text{slope}} = .1770572.$$

Quantiles are computed as shown above. In this case, the result needs to be exponentiated to get it back into the original strength units.

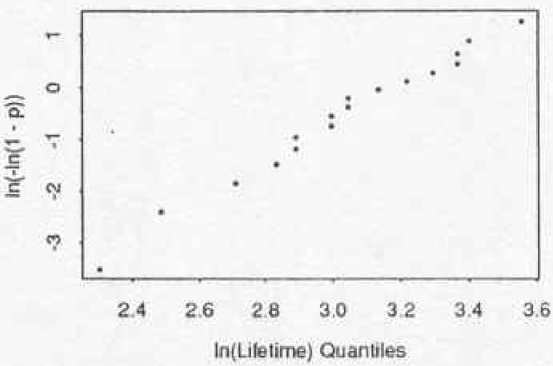
The following table summarizes the estimated quantiles using each of the 3 fitted distributions.

Quantile	Weibull	Normal	Lognormal
.01	417.3	451.0	483.5
.10	578.3	581.4	582.3
.20	641.8	636.1	629.5
.50	751.2	740.5	730.5

The estimated quantiles agree more for larger quantiles. Different fitted distributions can generally give very different results, especially in the tails of the distribution.

28. (a) The computations necessary for the Weibull plot are given below.

i	ith Smallest Lifetime	ln(ith Smallest Lifetime)	$p = \frac{i-.5}{17}$	$\ln(-\ln(1-p))$
1	10	2.3026	.0294	-3.5115
2	12	2.4849	.0882	-2.3819
3	15	2.7081	.1471	-1.8384
4	17	2.8332	.2059	-1.4674
5	18	2.8904	.2647	-1.1793
6	18	2.8904	.3235	-.9394
7	20	2.9957	.3824	-.7301
8	20	2.9957	.4412	-.5414
9	21	3.0445	.5000	-.3665
10	21	3.0445	.5588	-.2005
11	23	3.1355	.6176	-.0394
12	25	3.2189	.6765	.1209
13	27	3.2958	.7353	.2845
14	29	3.3673	.7941	.4577
15	29	3.3673	.8529	.6507
16	30	3.4012	.9118	.8870
17	35	3.5553	.9706	1.2603



The plot is very linear, indicating that the Weibull distribution fits the data well. There is no evidence that the Weibull distribution is an unreasonable description of roller life.

(b) From a line fit to the plot,

$$\ln \alpha \approx \text{x-intercept} = 3.183074$$

so $\alpha \approx e^{4.396} = 24.12$.

$$\beta \approx \text{slope} = 3.694.$$

(c) To find the p quantile of the distribution, set the CDF equal to p and solve for x:

$$\begin{aligned} F(x) = 1 - e^{-\left(\frac{x}{\alpha}\right)^{\beta}} &= p \\ \ln(1-p) &= -\left(\frac{x}{\alpha}\right)^{\beta} \\ x &= \alpha\left(-\ln(1-p)\right)^{\frac{1}{\beta}} \end{aligned}$$

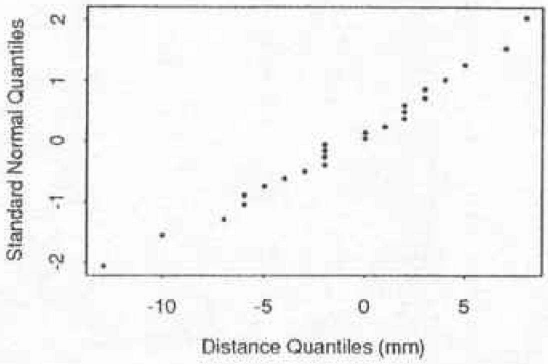
Using $p = .10$ and the above approximations for α and β, the .10 quantile is approximately 13.1 shifts.

29. (a)

```
                                             .
                                             :       .
      .        .        . :  . . . :      :  . :  : .  .    . .
-----+---------+---------+---------+---------+---------+-- Distance
   -12.0     -8.0      -4.0       0.0       4.0       8.0    (mm)
```

$\bar{x} = -1.04$ and $s = 5.17$ mm.

(b)



$$\mu \approx \text{x-intercept} = -1.04 \quad \text{(I fit the plot using least squares);}$$

$$\sigma \approx \frac{1}{\text{slope}} = 5.22.$$

The graphical estimates should be reasonably close to $\bar{x}$ and s.

(c)

$$\begin{aligned} P(X < -10 \text{ or } X > 10) &= 1 - P(-10 \leq X \leq 10) \\ &= 1 - (P(X \leq 10) - P(X < -10)) \\ &= 1 - (P\left(Z \leq \frac{10-(-1.04)}{5.17}\right) - P\left(Z < \frac{-10-(-1.04)}{5.17}\right)) \\ &= 1 - (P(Z \leq 2.14) - P(Z < -1.73)) \\ &= 1 - (.9838 - .0418) = .0580. \end{aligned}$$

Even if properly aimed ($\mu = 0$), the fraction falling outside of specifications is

$$\begin{aligned} P(X < -10 \text{ or } X > 10) &= 1 - P(-10 \leq X \leq 10) \\ &= 1 - (P(X \leq 10) - P(X < -10)) \\ &= 1 - (P\left(Z \leq \frac{10-0}{5.17}\right) - P\left(Z < \frac{-10-0}{5.17}\right)) \\ &= 1 - (P(Z \leq 1.93) - P(Z < -1.93)) \\ &= 1 - (.9732 - .0268) = .0536. \end{aligned}$$

The process is only capable of producing about 95% of the deviations in specifications, even if the process is properly aimed.

30. (a)

$$\begin{aligned} P(\text{Green}) &= P(1.178 \leq X \leq 1.182) \\ &= P(X \leq 1.182) - P(X < 1.178) \\ &= P(Z \leq .5) - P(Z < -1.5) \\ &= .6915 - .0668 = .6247. \end{aligned}$$

$$\begin{aligned}
P(\text{Red}) &= P(X \leq 1.176 \text{ or } X \geq 1.184) \\
&= 1 - P(1.176 \leq X \leq 1.184) \\
&= 1 - (P(X \leq 1.184) - P(X < 1.176)) \\
&= 1 - (P(Z \leq 1.5) - P(Z < -2.5)) \\
&= 1 - (.9332 - .0062) = .0730.
\end{aligned}$$

$$P(\text{Yellow}) = 1 - (P(\text{Green}) + P(\text{Red})) = .3023.$$

(b) Use the geometric distribution, equation (5-6), with $p = .0730$.

$$\begin{aligned}
P(Y > 10) &= 1 - P(Y \leq 10) \\
&= 1 - (P(Y = 1) + P(Y = 2) + \cdots + P(Y = 10)) \\
&= 1 - (.0730 + .0730(1 - .0730) + \cdots + .0730(1 - .0730)^9) \\
&= .4686.
\end{aligned}$$

From equation (5-8), $EY = \frac{1}{p} = 13.7$.

(c) Use the binomial distribution, equation (5-3), with $n = 8$ and $p = .1865$. $P(W = 2) = .2823$. $EW = np = 1.49$.

31. (a)

$$P(X \leq .32) = F(.32) = \sin(.32) = .3146 \text{ rad}$$

(b) Using equation (5-17),

$$f(x) = \begin{cases} 0 & \text{for } x \leq 0 \\ \cos(x) & \text{for } 0 < x \leq \pi/2 \\ 0 & \text{for } \pi/2 < x \end{cases}$$

(c) Using equation (5-18),

$$\begin{aligned}
EX &= \int_0^{\pi/2} x \cos(x)\, dx \\
&= (\cos(x) + x\sin(x)) \Big|_0^{\pi/2} \\
&= \pi/2 - 1 = .5708
\end{aligned}$$

Using equation (5-19),

$$\begin{aligned}
\text{Var}X &= \int_0^{\pi/2} x^2 \cos(x)\, dx - (EX)^2 \\
&= (x^2 \sin(x))\Big|_0^{\pi/2} - 2\int_0^{\pi/2} x \sin(x)\, dx - (EX)^2 \\
&= (x^2 \sin(x) - 2(\sin(x) - x\cos(x)))\Big|_0^{\pi/2} - (EX)^2 \\
&= (\pi/2)^2 - 2 - (.5708)^2 = .1416.
\end{aligned}$$

32. (a)

$$ED = 5EX + EY = 4.86.$$

(b) Use the joint distribution.

$$\begin{aligned} P(D \le 7) &= P(5X + Y \le 7) \\ &= P(Y \le 7 - 5X) \\ &= .15 + .05 + .10 + .08 + .10 + .14 + .10 + .05 = .77. \end{aligned}$$

(c) Use the geometric distribution with $p = .77$. From equation (5-8), the mean is $\frac{1}{.77} = 1.30$.

33. (a) $P(XY \ge \frac{1}{4})$ is equivalent to $P(Y \ge \frac{1}{4X})$. Integrate $f(x, y)$ over the region defined by this inequality.

$$\begin{aligned} P(Y \ge \frac{1}{4X}) &= \int_{\frac{1}{4}}^{1} \int_{\frac{1}{4x}}^{1} (x + y)\, dy\, dx \\ &= .5625. \end{aligned}$$

(b) Use definition (5-25).

$$\begin{aligned} f_X(x) &= \begin{cases} \int_0^1 (x + y)\, dy & \text{for } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases} \\ &= \begin{cases} (x + .5) & \text{for for } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases} \end{aligned}$$

Using equation (5-18),

$$\begin{aligned} EX &= \int_0^1 x(x + .5)\, dx \\ &= .5833. \end{aligned}$$

Using equation (5-19),

$$\begin{aligned} \text{Var}X &= \int_0^1 x^2(x + .5)\, dx - (.5833)^2 \\ &= .076389, \end{aligned}$$

so the standard deviation of X is $\sqrt{.076389} = .2764$.

(c) No, since $f(x, y) \ne f_X(x) f_Y(y)$.

(d) Using equation (5-53),

$$E(X + Y) = EX + EY = .5833 + .5833 = 1.167.$$

($EX = EY$ because the roles of x and y in $f(x, y)$ are exactly the same.) Formula (5-54) cannot be used because X and Y are not independent.

34. (a) For both X and Y, $\alpha = 1$ and $\beta = 1$. For X, $a = 1.97$ and $b = .05$. For Y, $a = 2.00$ and $b = .06$. Using formulas (5-28) and (5-29), $EX = 1.995$, $\text{Var}X = .00020833$, $EY = 2.03$, and $\text{Var}Y = .0003$.

(b)

$$E(Y - X) = EY - EX = .035$$

and

$$\text{Var}(Y - X) = \text{Var}Y + (-1)^2\text{Var}X = .00050833.$$

35. (a) Given that $X = 5$, use the binomial distribution (equation (5-3)) with $n = 5$ and $p = .80$.

$$P(Y = 3|X = 5) \doteq .2048.$$

(b)

$$f(x, 0) = .2^x \frac{e^{-3}3^x}{x!}.$$

Summing this over all possible values of X gives

$$\begin{aligned} P(Y = 0) &= \sum_{x=0}^{\infty} .2^x \frac{e^{-3}3^x}{x!} \\ &= e^{-3} \sum_{x=0}^{\infty} \frac{(.6)^x}{x!} \\ &= e^{-3} \frac{1}{e^{-.6}} \sum_{x=0}^{\infty} e^{-.6} \frac{(.6)^x}{x!} \\ &= \frac{e^{-3}}{e^{-.6}} = .0907. \end{aligned}$$

($\sum_{x=0}^{\infty} e^{-.6}\frac{(.6)^x}{x!} = 1$, because this is the sum of all probabilities for a Poisson random variable with $\lambda = .6$.)

(c) In general, $f_Y(y) = \frac{e^{-2.4}2.4^y}{y!}$. Y has a Poisson distribution with $\lambda = 3(.8) = 2.4$.

36. (a)

$$\begin{aligned} P(V > 15.07) &= 1 - P(V \leq 15.07) \\ &= 1 - P(Z \leq -.6) \text{ where } Z \text{ is a standard normal random variable} \\ &= 1 - .2743 = .7257. \end{aligned}$$

(b) Because this is a large sample, the central limit theorem says that $\bar{X}$ is approximately normal. Using equation (5-55), the mean of $\bar{X}$ is 15.10. Using equation (5-56), the standard deviation of $\bar{X}$ is $\frac{.05}{\sqrt{100}}$.

$$\begin{aligned}
P(\bar{X} > 15.105) &= 1 - P(\bar{X} \leq 15.105) \\
&= 1 - P(Z \leq 1.0) \text{ where } Z \text{ is a standard normal random variable} \\
&= 1 - .8413 = .1587.
\end{aligned}$$

(c) Using equation (5-58),

$$\mu_h \approx \frac{\mu_v}{\pi \mu_r^2} = 4.8 \text{ in.}$$

To use formula (5-59), the partial derivatives need to be evaluated at the means of the input random variables:

$$\frac{\partial h}{\partial r} = -\frac{v}{\pi r^3} = -4.80648$$

$$\frac{\partial h}{\partial v} = \frac{1}{\pi r^2} = \frac{1}{\pi}$$

Then applying equation (5-59),

$$\begin{aligned}
\mathrm{Var}(h) &\approx (-4.80648)^2(.02)^2 + (\frac{1}{\pi})^2(.05)^2 \\
&= .0092409 + .000253302 \\
&= .0094942,
\end{aligned}$$

so the approximate standard deviation of h is

$$\sqrt{.0094942} = .097438 \text{ in.}$$

(d) The variation in radius has the biggest impact, since its term (variance × squared partial derivative) contributes much more to the propagation of error formula than volume term.

37. (a) Integrate $f(x, y)$ over the region where $y \leq 1.5$.

$$\begin{aligned}
P(Y \leq 1.5) &= \int_1^{1.5}\int_0^1 e^x e^{-y}\,dx\,dy + \int_0^1\int_0^y e^x e^{-y}\,dx\,dy \\
&= .2487 + .3679 = .6166.
\end{aligned}$$

(b) Use definition (5-25).

$$\begin{aligned}
f_X(x) &= \begin{cases} \int_x^{\infty} e^x e^{-y}dy & \text{for } 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases} \\
&= \begin{cases} 1 & \text{for } 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}
\end{aligned}$$

$$f_Y(y) = \begin{cases} \int_0^y e^x e^{-y} dx \text{ for } 0 \le y \le 1 \\ \int_0^1 e^x e^{-y} dx \text{ for } y > 1 \\ 0 \quad \text{otherwise} \end{cases}$$

$$= \begin{cases} 1 - e^{-y} & \text{for } 0 \le y \le 1 \\ e^{1-y} - e^{-y} & \text{for } y > 1 \\ 0 & \text{otherwise} \end{cases}$$

(c) No. For example, given that $X = .5$, Y must be greater than or equal to .5, by the definition of the joint density. Unconditionally, there is a positive probability associated with the event $Y < 5$. However, given that $X = .5$, $P(Y < 5) = 0$. Since knowing the observed value of X changes the marginal probability of Y, X and Y are not independent. It can also be seen that $f(x, y) \neq f_X(x) f_Y(y)$.

(d) Use equation (5-48).

$$f_{Y|X}(y|.25) = \begin{cases} \frac{f(.25,y)}{f_X(.25)} & \text{for } y \ge .25 \\ 0 & \text{otherwise} \end{cases}$$

$$= \begin{cases} e^{.25} e^{-y} & \text{for } y \ge .25 \\ 0 & \text{otherwise} \end{cases}$$

Use $f_{Y|X}(y|.25)$ in equation (5-18) to find the conditional mean.

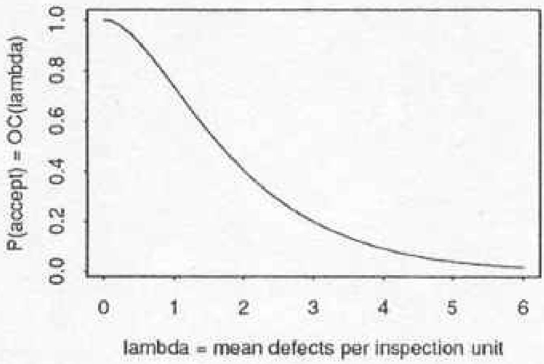
$$\begin{aligned} E(Y|X = .25) &= \int_{.25}^{\infty} y e^{.25} e^{-y}\, dy \\ &= e^{.25}\left(-e^{-y}(y+1)\right)\Big|_{.25}^{\infty} \\ &= 1.25. \end{aligned}$$

38. (a) Use equation (5-10) with various $\lambda > 0$. (Note: If $\lambda = 0$ then there will certainly be zero nonconformities, and so $P(\text{accept}) = 1$.)

$$\begin{aligned} \text{OC}(\lambda) = P(X \le 1) &= P(X = 0) + P(X = 1) \\ &= e^{-\lambda} + e^{-\lambda}\lambda. \end{aligned}$$

Some of the points of the OC function are given below.

λ	OC(λ)
.0	1.0000
.4	.9384
.8	.8088
1.2	.6626
1.6	.5249
2.0	.4060
2.4	.3084
2.8	.2311
3.2	.1712
3.6	.1257
4.0	.0916
4.4	.0663
4.8	.0477
5.2	.0342
5.6	.0244
6.0	.0174



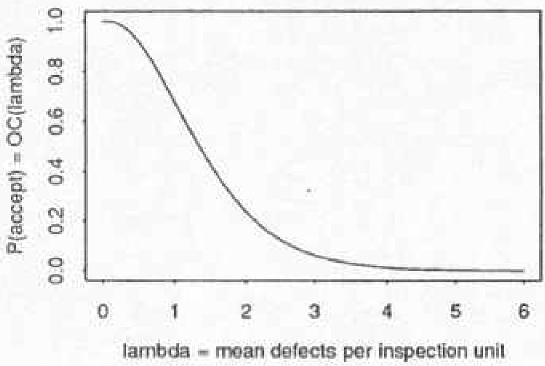
As might be expected, as the mean number of defects per unit (λ) increases, the probability of accepting the lot decreases.

(b)

$$\begin{aligned} \mathrm{OC}(\lambda) = P(X \leq 2) &= P(X=0) + P(X=1) + P(X=2) \\ &= e^{-2\lambda} + e^{-2\lambda}2\lambda + e^{-2\lambda}\frac{(2\lambda)^2}{2}. \end{aligned}$$

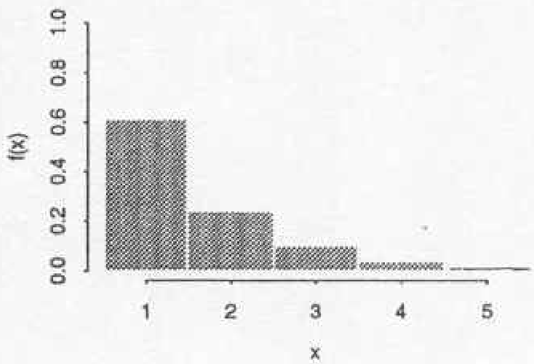
Some of the points of the OC function are given below.

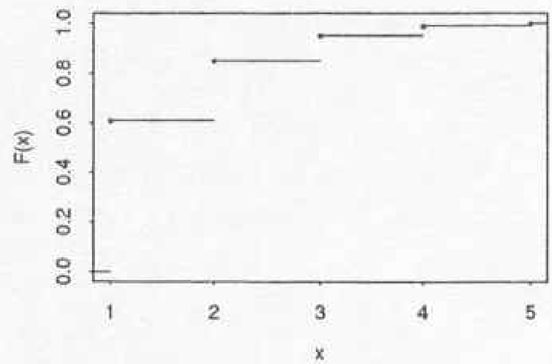
λ	OC(λ)
.0	1.0000
.4	.9526
.8	.7834
1.2	.5697
1.6	.3799
2.0	.2381
2.4	.1425
2.8	.0824
3.2	.0463
3.6	.0255
4.0	.0138
4.4	.0073
4.8	.0038
5.2	.0020
5.6	.0010
6.0	.0005



This OC curve is steeper than the one from part (a). This reflects the increase in information about λ, since here we are inspecting 2 units.

39. (a)





(b) Using equation (5-1),

$$EX = (1)(.61) + (2)(.24) + (3)(.10) + (4)(.04) + (5)(.01) = 1.6.$$

Using equation (5-2),

$$\text{Var}X = (1)^2(.61) + (2)^2(.24) + (3)^2(.10) + (4)^2(.04) + (5)^2(.01) - (1.6)^2 = .8,$$

so the standard deviation of X is $\sqrt{.8} = .8944$.

(c)

$$P(X \geq 3) = P(X = 3) + P(X = 4) + P(X = 5) = .15.$$

$$P(X < 3) = 1 - P(X \geq 3) = 1 - .15 = .85.$$

40. (a) Use equation (5-10), with $\lambda = 3.87$.

$$P(X \geq 2) = 1 - P(X < 2) = 1 - (P(X = 0) + P(X = 1)) = 1 - (.0209 + .0807) = .8984.$$

(b) Using equation (5-12), $\sqrt{\text{Var}X} = \sqrt{\lambda} = \sqrt{3.87} = 1.967$.

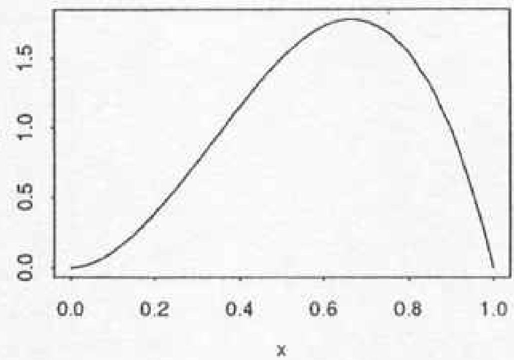
(c) Note that λ is the mean, or the average rate. If the average number of collisions per 8 minutes is 3.87, then the average number of collisions per 16 minutes is 7.74. Use the Poisson distribution for $W = X + Y$, with $\lambda = 7.74$. $P(W = 6) = .1299$.

(d)

$$\begin{aligned} P(W \geq 3) = 1 - P(W < 3) &= 1 - (P(W = 0) + P(W = 1) + P(W = 2)) \\ &= 1 - (.00044 + .0034 + .0130) = .9832. \end{aligned}$$

41. (a) Use equation (5-13) and solve for $k = .12$.

$$f(x) = \begin{cases} 12(x^2(1-x)) & \text{for } 0 < x < 1 \\ 0 & \text{otherwise.} \end{cases}$$



(b) All of the following probabilities correspond to areas under $f(x)$.

$$\begin{aligned} P(X \leq .25) &= \int_0^{.25} f(x)\,dx \\ &= .0508. \end{aligned}$$

$$\begin{aligned} P(X \leq .75) &= \int_0^{.75} f(x)\,dx \\ &= .7383. \end{aligned}$$

$$P(.25 < X \leq .75) = P(X \leq .75) - P(X < .25) = .7383 - .0508 = .6875$$

$$\begin{aligned} P(|X - .5| > .1)) &= 1 - P(|X - .5| \leq .1)) \\ &= 1 - P(-.1 < X - .5 < .1)) \\ &= 1 - P(.4 < X < .6)) \\ &= 1 - \int_{.4}^{.6} f(x)\,dx \\ &= 1 - .2960 = .7040. \end{aligned}$$

(c) Using equation (5-18),

$$EX = .6.$$

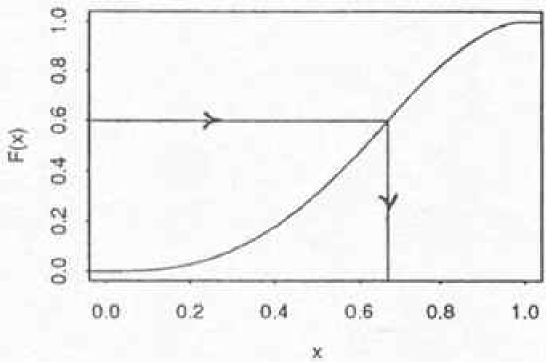
Using equation (5-19),

$$\sqrt{\mathrm{Var} X} = \sqrt{.04} = .2.$$

(d) Using equation (5-16),

$$\begin{aligned} F(x) &= P(X \leq x) \\ &= \int_{-\infty}^{x} f(x)\,dx \end{aligned}$$

$$= \begin{cases} 0 & \text{for } x \le 0 \\ 12\left(\frac{1}{3}x^3 - \frac{1}{4}x^4\right) & \text{for } 0 < x < 1 \\ 1 & \text{for } x \ge 1. \end{cases}$$



The .6 quantile of the distribution is the x such that $P(X \le x) = .6$. This is just $F^{-1}(.6)$, so you need to find .6 on the vertical axis, and find the x that produces this value for $F(x)$. (By trial and error, the exact value is $x = .67082$.)

42. (a) The probability of an individual depth being within specifications is the same as the long-run fraction of depths within specifications, if successive shelf depths can be considered identical random variables.

$$\begin{aligned} P(.0275 \le X \le .0278) &= P(X \le .0278) - P(X < .0275) \\ &= P(Z \le 2) - P(Z < -1) \\ &= .9773 - .1587 = .8186. \end{aligned}$$

(Z is a standard normal random variable.)

(b) Assuming that $\mu = .02765$, we want

$$P(.0275 \le X \le .0278) = .98.$$

By symmetry, this is equivalent to $P(X \le .0278) = .99$, which is equivalent to

$$P\left(Z \le \frac{.0278 - .02765}{\sigma}\right) = .99.$$

Looking up .99 in the body of Table B-3, this means that

$$\frac{.00015}{\sigma} \approx 2.33$$

or that $\sigma \approx .00006438$.

43. (a) Let $X_1, \ldots, X_{30}$ be the 30 individual resistances. The resistance of the assembly is then $U = \sum X_i$. Using equation (5-53), the mean of the sum is the sum of the means:

$$EU = EX_1 + EX_2 + \cdots + EX_{30} = 30(9.91) = 297.3\,\Omega.$$

Assuming that the individual resistances are independent, you can use equation (5-54) to say that the variance of the sum is the sum of the variances:

$$\text{Var}U = \text{Var}X_1 + \text{Var}X_2 + \cdots + \text{Var}X_{30} = 30(.08)^2 = .192\,\Omega^2,$$

so the standard deviation of U is $\sqrt{.192} = .438\Omega$.

(b) Since $n = 30$ is large, the central limit theorem says that $\bar{X} = \frac{\sum X_i}{30}$ is approximately normal. The mean of $\bar{X}$ is 9.91 and the standard deviation of $\bar{X}$ is $\frac{.438}{\sqrt{30}}$ (see equations (5-55) and (5-56). Since $\bar{X} = \frac{U}{30}$,

$$\begin{aligned} P(U > 298.2) = P\left(\bar{X} > \frac{298.2}{30}\right) &= 1 - P\left(\bar{X} \leq 9.94\right) \\ &= 1 - P\left(Z \leq \frac{9.94 - 45}{\frac{.438}{\sqrt{30}}}\right) \\ &= 1 - P(Z \leq 2.05) \\ &= 1 - .9798 = .0202. \end{aligned}$$

(Z is a standard normal random variable.)

44. (a) Set μ equal to the midpoint of the specifications to maximize the fraction of lengths in specifications. (This follows from the symmetry and shape of the normal distribution.) With $\mu = 33.69$,

$$\begin{aligned} P(33.68 \leq X \leq 33.70) &= P(X \leq 33.70) - P(X < 33.68) \\ &= P(Z \leq 2) - P(Z < -2) \\ &= .9773 - .0228 = .9545. \end{aligned}$$

(Z is a standard normal random variable.)

(b) All of these sample sizes are large. If individual lengths can be considered as independent, the central limit theorem says that $\bar{X}$ is approximately normal, even if the individual X's are not. Equation (5-55) says that the mean of $\bar{X}$ is equal to μ, the mean of the individual X's. Equation (5-56) says that the standard deviation of $\bar{X}$ is $\frac{\sigma}{\sqrt{n}}$, which is equal to $\frac{.005}{\sqrt{n}}$.

For $n = 25$,

$$\begin{aligned} P(\mu - .0005 \leq \bar{X} \leq \mu + .0005) &= P(\bar{X} \leq \mu + .0005) - P(\bar{X} < \mu - .0005) \\ &= P\left(Z \leq \frac{.0005}{\frac{.005}{\sqrt{n}}}\right) - P\left(Z < \frac{-.0005}{\frac{.005}{\sqrt{n}}}\right) \\ &= P(Z \leq .5) - P(Z < -.5) \\ &= .6915 - .3085 = .3830. \end{aligned}$$

For $n = 100$,

$$\begin{aligned} P(\mu - .0005 \leq \bar{X} \leq \mu + .0005) &= P(\bar{X} \leq \mu + .0005) - P(\bar{X} < \mu - .0005) \\ &= P(Z \leq 1) - P(Z < -1) \\ &= .8413 - .1587 = .6826. \end{aligned}$$

For $n = 400$,

$$\begin{aligned} P(\mu - .0005 \leq \bar{X} \leq \mu + .0005) &= P(\bar{X} \leq \mu + .0005) - P(\bar{X} < \mu - .0005) \\ &= P(Z \leq 2) - P(Z < -2) \\ &\approx .9773 - .0228 = .9545. \end{aligned}$$

45. (a)

$$\begin{aligned} P(4.65 \le X \le 4.70) &= P(X \le 4.70) - P(X < 4.65) \\ &= P(Z \le .67) - P(Z < -1) \\ &= .7486 - .1587 = .5899. \end{aligned}$$

(Z is a standard normal random variable.)

(b) Use the binomial distribution, equation (5-3), with $n = 5$ and $p = .5899$.
$P(Y = 4) = .2483$.

(c) Use the geometric distribution, equation (5-6), with $p = .5899$.

$$\begin{aligned} P(W = 2, 3, \text{or } 4) &= P(W = 2) + P(W = 3) + P(W = 4) \\ &= .24192 + .09921 + .04069 = .3818. \end{aligned}$$

(d) Equation (5-55) says that the mean of $\bar{X}$ is equal to μ, the mean of the individual X's. Equation (5-56) says that the standard deviation of $\bar{X}$ is $\frac{\sigma}{\sqrt{n}}$, which is equal to $\frac{.03}{\sqrt{25}}$.

$$\begin{aligned} P(\mu - .01 \le \bar{X} \le \mu + .01) &= P(\bar{X} \le \mu + .01) - P(\bar{X} < \mu - .01) \\ &= P\left(Z \le \frac{.01}{\frac{.03}{\sqrt{25}}}\right) - P\left(Z < \frac{-.01}{\frac{.03}{\sqrt{25}}}\right) \\ &= P(Z \le 1.67) - P(Z < -1.67) \\ &= .9525 - .0475 = .9050. \end{aligned}$$

(e) Want

$$P(\mu - .005 \le \bar{X} \le \mu + .005) = .90.$$

By symmetry, this is equivalent to $P(\bar{X} \le \mu + .005) = .95$, which is equivalent to

$$P\left(Z \le \frac{.005}{\frac{.03}{\sqrt{n}}}\right) = .95.$$

Looking up .95 in Table B-3, this means that

$$\frac{.005}{\frac{.03}{\sqrt{n}}} \approx 1.645$$

or $n = 97.42$. Bump this up to the next highest integer (98), to make the probability at least .90.

46 (a) Since $1 - F(x) = \exp(-x/\alpha)$ and $\alpha = 15{,}000$,

$1 - F(20{,}000) = \exp[-20{,}000/15{,}000] = .2636$.

(b) Let Y = # disk drives that fail after 20,000 hours from 10 randomly selected disk drives. Y ~ Binomial (n = 10, p = .2636).

$P(Y > 8) = P(Y = 9) + P(Y = 10) = .000045 + .000002 = .000047$.

47 (a) $\mu_X = 2.577$, $\sigma_X = .061$. For the "Box" $\mu_Y = 9.566$ and $\sigma_Y = .053$.

$E(U) = 9.566 - 4(2.577) = -.742$

$Var(U) = (.061)^2 + 4(.053)^2 = .014957$.

$\sigma_U = .1222$.

(b) $P(U < 0) = P(Z < (0 + .742)/.1222) = P(Z < 6.067) = 1.00$

(c) $P(U < 0) = P(Z < (0 - ?)/.1222) = .005$. Thus,

$(0 - ?)/.1222 = -2.575$ or $? = .314665 = \mu_Y - 10.308$ (note: $10.308 = 4(2.577)$)

$\mu_Y = 10.6226$.

Chapter 6: Introduction to Formal Statistical Inference

Section
1 1. [6.3, 7.9] ppm is a set of plausible values for the mean. The method used to construct this interval correctly contains means in 95% of repeated applications. This particular interval either contains the mean or it doesn't (there is no probability involved). However, because the *method* is correct 95% of the time, we might say that we have 95% confidence that it was correct this time.

2. (a) You can use equation (6-9), since this is a large sample. The appropriate z for 90% confidence is 1.645. The interval is

$$\begin{aligned} 142.7 \pm 1.645\left(\frac{98.2}{\sqrt{26}}\right) &= 142.7 \pm 31.68 \\ &= [111.02, 174.38]. \end{aligned}$$

(b) Now $z = 1.96$, and the interval is

$$\begin{aligned} 142.7 \pm 1.96\left(\frac{98.2}{\sqrt{26}}\right) &= 142.7 \pm 37.75 \\ &= [104.95, 180.45]. \end{aligned}$$

This interval is wider than the one from (a). In order to have more confidence of containing the mean, the interval must be wider.

(c) To make a 90% one-sided confidence interval, construct a 80% two-sided confidence interval, and use the upper endpoint. The appropriate z for a 80% two-sided confidence interval is 1.28, so the 90% one-sided confidence interval is

$$\begin{aligned} 142.7 + 1.28\left(\frac{98.2}{\sqrt{26}}\right) &= 142.7 + 24.65 \\ &= 167.35. \end{aligned}$$

This value is smaller than the upper endpoint from part (a). Setting the lower endpoint equal to $-\infty$ requires you to move the upper endpoint in so that the confidence remains at 90%.

(d) To make a 95% one-sided confidence interval, construct a 90% two-sided confidence interval, and use the upper endpoint. This was done in part (a), so the 90% one-sided confidence interval is 174.38. This is larger than the answer to (c); in order to achieve higher confidence, you must make the interval "wider".

(e) [111.02, 174.38] ppm is a set of plausible values for the mean aluminum content of samples of recycled PET plastic from the recycling pilot plant at Rutgers University. The method used to construct this interval correctly contains means in 90% of repeated applications. This particular interval either contains the mean or it doesn't (there is no probability involved). However, because the *method* is correct 90% of the time, we might say that we have 90% confidence that it was correct this time.

3. $n = [Z\, s/ B]^2 = [\,(1.645)(98.2)/ 20\,]^2 = 65.24$ or about 66.

4. (a) $\bar{x} = 4.6858$ and $s = .02900317$.

(b) Since this is a large sample, you can use equation (6-9), with $z = 2.33$ for 98% confidence. The two-sided confidence interval is

$$\begin{aligned} 4.6858 \pm 2.33\left(\frac{.02900317}{\sqrt{50}}\right) &= 4.6858 \pm .009556884 \\ &= [4.676, 4.695] \text{ mm.} \end{aligned}$$

(c) $z = 2.58$ for 98% confidence. The two-sided confidence interval is

$$\begin{aligned} 4.6858 \pm 2.58\left(\frac{.02900317}{\sqrt{50}}\right) &= 4.6858 \pm .0105823 \\ &= [4.675, 4.696] \text{ mm.} \end{aligned}$$

This interval is wider than the one in (b). To increase the confidence that μ is in the interval, you need to make the interval wider.

(d) To make a 98% one-sided interval, construct a 96% two-sided interval and use the lower endpoint. For a 96% two-sided interval, the appropriate z is $Q_{SN}(.98) = 2.05$. The resulting 98% one-sided interval is

$$\begin{aligned} 4.6858 - 2.05\left(\frac{.02900317}{\sqrt{50}}\right) &= 4.6858 - .008408418 \\ &= 4.677 \text{ mm.} \end{aligned}$$

This is larger than the lower endpoint of the interval in (b). Since the upper endpoint here is set to ∞, the lower endpoint must be increased to keep the confidence level the same.

(e) To make a 99% one-sided interval, construct a 98% two-sided interval and use the lower endpoint. This was done in part (a), and the resulting lower bound is 4.676. This is smaller than the value in (d); to increase the confidence, the interval must be made "wider".

(f) [4.676, 4.695] ppm is a set of plausible values for the mean diameter of this type of screw as measured by this student with these calipers. The method used to construct this interval correctly contains means in 98% of repeated applications. This particular interval either contains the mean or it doesn't (there is no probability involved). However, because the *method* is correct 98% of the time, we might say that we have 98% confidence that it was correct this time.

Section 2

1. Since the natural logarithms of the data are more bell-shaped than the raw data (see exercise 2 chapter 3) , it would be better to test the null hypothesis that the mean of the logs is equal to ln(200) versus the alternative that the mean of the logs is greater than ln(200). However, since this is a large sample, using the raw data poses no major problem.

1. H_0: $\mu = 200$ ppm.
2. H_a: $\mu > 200$ ppm.
3. The test statistic is

$$Z = \frac{\bar{x} - 200}{\frac{s}{\sqrt{26}}}$$

and the reference distribution is the standard normal distribution. Observed values of Z far above zero will be considered as evidence against H_0.

4. The sample gives

$$z = -2.98.$$

5. The observed level of significance is

$$\begin{aligned} & P(\text{a standard normal random variable} > -2.98) \\ = \; & P(\text{a standard normal random variable} < 2.98) \end{aligned}$$

which is equal to .9986, according to Table B-3. There is no evidence that the mean aluminum content for samples of recycled plastic is greater than 200 ppm.

2. (a) 1. H_0: $\mu = .500$ in.

2. H_a: $\mu \neq .500$ ppm.

3. The test statistic is

$$Z = \frac{\bar{x} - .500}{\frac{s}{\sqrt{405}}}$$

and the reference distribution is the standard normal distribution. Observed values of Z far above or below zero will be considered as evidence against H_0.

4. The sample gives

$$z = 1.55.$$

5. The observed level of significance is

$$\begin{aligned} & 2P(\text{a standard normal random variable} > 1.55) \\ = \; & 2P(\text{a standard normal random variable} < -1.55) \end{aligned}$$

which is equal to $2(.0606) = .1212$, according to Table B-3. There is some (weak) evidence that the mean height of the punches is not equal to .500 in.

It is interesting to note that the rounded $\bar{x}$ and s given produce a z that is quite a bit different from what the exact values produce. $\bar{x} = .005002395$ and $s = .002604151$, computed from the raw data given in exercise 9, ch. 3, produce $z = 1.85$, and a p-value of $2(.0322) = .0644$.

(b) You can use equation (6-9), since this is a large sample. The appropriate z for 98% confidence is 2.33. The interval is

$$\begin{aligned} .5002 \pm 2.33 \left(\frac{.0026}{\sqrt{405}} \right) &= .5002 \pm .000301 \\ &= [.49990, .50050]. \end{aligned}$$

3. The mean of the punch heights is almost certainly not exactly equal to .50000000 inches. Given enough data, a hypothesis test would detect this as a "statistically significant" difference (and produce a small p-value). What is practically important is whether the mean is "close enough" to .500 inches. The confidence interval in part (a) answers this more practical question.

4. 1. H_0: $\mu = 4.70$ mm.
2. H_a: $\mu \neq 4.70$ mm.
3. The test statistic is

$$Z = \frac{\bar{x} - 4.70}{\frac{s}{\sqrt{50}}}$$

and the reference distribution is the standard normal distribution. Observed values of Z far above or below zero will be considered as evidence against H_0.
4. The sample gives

$$z = -3.46.$$

5. The observed level of significance is

$$2P(\text{a standard normal random variable} < -3.46)$$

which is equal to $2(.0003) = .0006$, according to Table B-3. There is very strong evidence that the mean measured diameter differs from nominal.

5. Although there is evidence that the mean is not equal to nominal, the test does not say anything about how *far* the mean is from nominal. It may be "significantly" different from nominal, but the difference may be practically unimportant. A confidence interval is more practical for determining how far the mean is from nominal.

Section 3

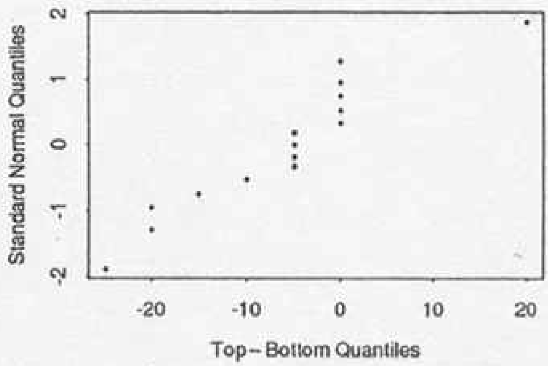
1. The normal distribution is bell-shaped and symmetric, with no outliers. The confidence interval methods depend on this regularity. If the distribution is skewed or prone to outliers, the normal-theory methods will not properly take this into account. The result is an interval whose real confidence level is lower than the nominal value associated with it. For example, if the data are skewed to the right (long right tail), a 90% normal-theory confidence interval for the mean will tend to underestimate the mean, and so the method will produce intervals that contain the mean *less* than 90% of the time.

2. **(a)** It is required that the top bolt torques for each piece are independent and approximately normally distributed. The normal probability plot suggests the torque values for the top bolt come from a normal distribution.

(b) Ho: $\mu = 100$ vs. Ha: $\mu \neq 100$ t = $(\bar{x} - 100)/(s/\sqrt{n})$ = $(111 - 100)/(9.6732/\sqrt{15})$ = 4.4. p-value = 2 P[$t_{14} > 4.4$] = .001. Conclude Ha: : $\mu \neq 100$.

(c) $\bar{x} \pm ts/\sqrt{n}$ becomes 111 $\pm (2.624)(9.6732)/\sqrt{15}$ or 111 ± 6.553. The interval [104.45, 117.55] is a 98% confidence interval for the mean torque for the top bolt.

(d) Since the data are paired, one should take the difference for each pair and analyze the differences. This is a small sample (small number of pairs), so the differences need to be iid normal to use the methods in Section 6-3. One way to check this assumption is to make a normal plot of the differences. (I have taken the differences as Top−Bottom.)



Given the number of ties in the data, this plot is fairly linear, indicating that the differences are roughly bell-shaped. Other than the discrete (chunky) nature of the data, there is no evidence against the assumption of a normal distribution for the differences.

(e) 1. H_0: $\mu_d = 0$.
2. H_a: $\mu_d < 0$.
3. The test statistic is given by equation (6-26), with $\# = 0$. The reference distribution is the t_{14} distribution. Observed values of T far below zero will be considered as evidence against H_0.
4. The sample gives

$$t = -2.10.$$

5. The observed level of significance is

$$\begin{aligned} &P(\text{a } t_{14} \text{ random variable} < -2.10) \\ &\quad = P(\text{a } t_{14} \text{ random variable} > 2.10) \end{aligned}$$

which is between .025 and .05, according to Table B-4. This is fairly strong evidence that there is a mean increase in required torques as one moves from the top to the bottom bolts.

(f) Use equation (6-25). For 98% confidence, the appropriate t is $t = Q_{14}(.99) = 2.624$, from Table B-4.

$$\begin{aligned} -6.0 \pm 2.624\left(\frac{11.0518}{\sqrt{15}}\right) &= -6.0 \pm 7.4878 \\ &= [-13.49, 1.49]. \end{aligned}$$

3. (a) Use equation (6-22). For 90% confidence, the appropriate z is 1.645. The interval is

$$.0004 \pm 1.645\left(\frac{.01159873}{\sqrt{50}}\right) = .0004 \pm .002698308$$

$$= [-0.0023, .0031] \text{ mm.}$$

(b) 1. H_0: $\mu_d = 0$.
2. H_a: $\mu_d \neq 0$.
3. The test statistic is given by equation (6-24), with $\# = 0$. The reference distribution is the standard normal distribution. Observed values of Z far above or below zero will be considered as evidence against H_0.
4. The sample gives

$$z = .24.$$

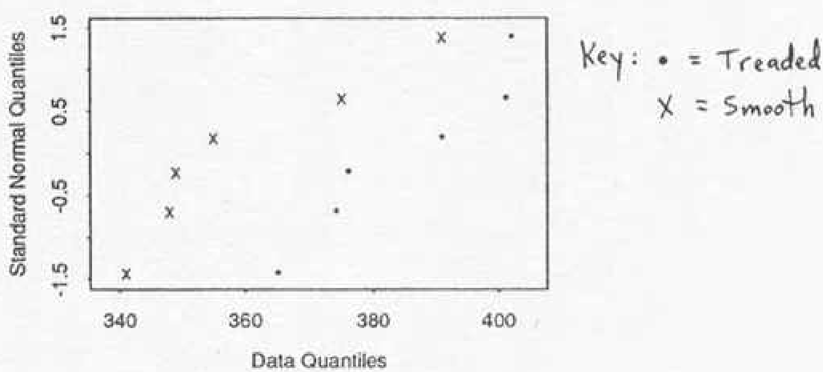
5. The observed level of significance is

$$\begin{aligned} & 2P(\text{a standard normal random variable} > .24) \\ = \; & 2P(\text{a standard normal random variable} < -.24) \end{aligned}$$

which is equal to $2(.4052) = .8104$, according to Table D-3. There is no evidence of a systematic difference in the readings produced by the two calipers.

(c) The confidence interval in part (a) contains zero; in fact, zero is near the middle of the interval. This means that zero is a very plausible value for the mean difference—there is no evidence that the mean is not equal to zero. This is reflected by the large p-value in part (b).

4. (a) The data within each sample must be iid normal, and the two distributions must have the same variance σ^2. One way to check these assumptions is to normal plot each data set on the same axes (see Figure 6-15).



For such small sample sizes, it is difficult to verify the assumptions. The plots are roughly linear with no outliers, indicating that the normal part of the assumption may be reasonable. The slopes are similar, indicating that the common variance assumption may be reasonable.

(b) Label the Treaded data Sample 1 and the Smooth data Sample 2.

1. H_0: $\mu_1 - \mu_2 = 0$.
2. H_a: $\mu_1 - \mu_2 \neq 0$.
3. The test statistic is given by equation (6-36) with $\# = 0$, and the reference distribution is the t_{10} distribution. Observed values of T far above or below zero will be considered as evidence against H_0.
4. The sample gives

$$t = 2.49$$

5. The observed level of significance is

$$2P(\text{a } t_{10} \text{ random variable} > 2.49) = 2(\text{something between .01 and .025})$$

which is between .02 and .05, according to Table B-4. This is strong evidence that there is a difference in mean skid lengths.

(c) Use equation (6-35). For 95% confidence, the appropriate t is $t = Q_{10}(.975) = 2.228$ from Table B-4, and the resulting interval is

$$384.83 - 359.83 \pm 2.228(17.377)\sqrt{\frac{1}{6}+\frac{1}{6}} = 25.0 \pm 22.3529 = [2.65, 47.35].$$

(d) $\hat{\upsilon} =$

$(s_1^2/n_1 + s_2^2/n_2)^2$ divided by $(s_1^4/(n_1-1)n_1^2 + s_2^4/(n_2-1)n_2^2)$

$= [236.567/6 + 367.367/6]^2$ divided by

$(236.567)^2/(5)(36) + (367.367)^2/(5)(36)$ gives

$\hat{\upsilon} = 10{,}131.56/1{,}060.68 = 9.55$

$(s_1^2/n_1 + s_2^2/n_2)^{1/2} = [(236.567)/6 + (367.367)/6]^{1/2} = 10.033$

Let the df be 10.

$(\bar{x}_1 - \bar{x}_2) \pm \hat{t}\sqrt{(s_1^2/n_1 + s_2^2/n_2)}$, $\quad \hat{t} = 2.228(10df)$. Thus,

$(384.833 - 359.833) \pm (2.228)(10.033)$

25 ± 22.3535 gives $[2.65, 47.35]$. Using 9 df., $\hat{t} = 2.262(9df)$ and the interval becomes $[2.3, 47.7]$.

Section 4

1. (a) Use equation (6-42) and Table B-5. For a 95% two-sided interval, $U = Q_5(.975) = 12.833$ and $L = Q_5(.025) = .831$. The resulting interval for σ^2 is $[92.17123, 1423.385]$; taking the square root of each endpoint, the interval for σ is $[9.60, 37.73]$ cm.

   (b) For a 99% one-sided interval, $L = Q_5(.01) = .554$ and the interval for σ^2 is $[-\infty, 3315.584]$; taking the square root, the interval for σ is $[-\infty, 57.58]$ cm.

   (c) 1. H_0: $\frac{\sigma_1^2}{\sigma_2^2} = 1$.
   2. H_a: $\frac{\sigma_1^2}{\sigma_2^2} \neq 1$.
   3. The test statistic is given by equation (6-49) with $\# = 1$, and the reference distribution is the $F_{5,5}$ distribution. Small or large observed values of F (relative to 1) will be considered as evidence against H_0.
   4. The sample gives

$$f = .644.$$

   5. The observed level of significance is

$$2P(\text{an } F_{5,5} \text{ random variable} < .644).$$

   It is necessary to switch the degrees of freedom, invert the observed f, and change the inequality to find the probability to the left of this small quantile using Tables B-6. (Switching the degrees of freedom has no effect here, since the degrees of freedom are the same.)

$$\begin{aligned} &= 2P(\text{an } F_{5,5} \text{ random variable} > \frac{1}{.644}) \\ &= 2P(\text{an } F_{5,5} \text{ random variable} > 1.55) \\ &= 2(\text{something greater than } .25), \end{aligned}$$

   so the p-value is greater than .5, according to Tables B-6. There is no evidence of a difference in variability between treaded and smooth stopping distances.

   (d) Use equation (6-47) and Tables B-6. For 90% confidence, $U = Q_{5,5}(.95) = 5.05$ and $L = Q_{5,5}(.05) = \frac{1}{Q_{5,5}(.95)} = \frac{1}{5.05}$. The resulting interval for $\frac{\sigma_1^2}{\sigma_2^2}$ is $[.1275153, 3.25196]$. Taking the square root of each endpoint, the interval for $\frac{\sigma_1}{\sigma_2}$ is $[.357, 1.803]$.

2. **(a)** $[\ \sqrt{(n-1)s^2/U}, +\infty\ ]$ which becomes $[\ \sqrt{14/23.685}(9.67323), +\infty\ ]$ or $[\ 7.437, +\infty\ ]$ is a lower one-sided 95% confidence interval for the standard deviation of the top bolt torques.

   (b) $[\ 6\sqrt{(n-1)s^2/U}, +\infty\ ]$ becomes $[\ 44.622, +\infty\ ]$, a lower one-sided 95% confidence interval for 6σ.

   (c) Torque of the top bolt is not independent of the torque on the bottom bolt for a given piece.

Section 5 1. (a) Using equation (6-57), the appropriate z for 95% confidence is 1.96. The resulting interval is

$$\begin{aligned} .66 \pm 1.96\frac{1}{2\sqrt{100}} &= .66 \pm .098 \\ &= [.562, .758]. \end{aligned}$$

For a 95% one-sided interval, construct a 90% two-sided interval and use the lower endpoint. The appropriate z for a 90% two-sided interval is 1.645, so the 95% one-sided interval is

$$\begin{aligned} .66 - 1.645\frac{1}{2\sqrt{100}} &= .66 - .08225 \\ &= .578. \end{aligned}$$

Using equation (6-59), the appropriate z for 95% confidence is 1.96. The resulting interval is

$$\begin{aligned} .66 \pm 1.96\sqrt{\frac{.66(1-.66)}{100}} &= .66 \pm .0928 \\ &= [.567, .753]. \end{aligned}$$

A 95% one-sided interval is is

$$\begin{aligned} .66 - 1.645\sqrt{\frac{.66(1-.66)}{100}} &= .66 - .077925 \\ &= .582. \end{aligned}$$

The two different methods give similar results, because $\hat{p} = .66$ is close to $\frac{1}{2}$.

(b) 1. H_0: $p = .55$.
2. H_a: $p > .55$.
3. The test statistic is given by equation (6-53) with $\# = .55$, and the reference distribution is the standard normal distribution. Observed values of Z far above zero will be considered as evidence against H_0.
4. The sample gives

$$z = 2.21.$$

5. The observed level of significance is

$$\begin{aligned} & P(\text{a standard normal random variable} > 2.21) \\ = \; & P(\text{a standard normal random variable} < -2.21) \\ = \; & .0136 \end{aligned}$$

using Table B-3. This is strong evidence of an improvement in yield.

(c) Label the small shot size Sample 1 and the large shot size Sample 2. Using equation (6-65), the appropriate z for 95% confidence is 1.96. The resulting interval is

$$\begin{aligned} .66 - .53 \pm 1.96\left(\frac{1}{2}\right)\sqrt{\frac{1}{100}+\frac{1}{100}} &= .13 \pm .13859 \\ &= [-.0086, .2686]. \end{aligned}$$

Using equation (6-67), the resulting interval is

$$\begin{aligned} .66 - .53 \pm 1.96\sqrt{\frac{.66(1-.66)}{100}+\frac{.53(1-.53)}{100}} &= .13 \pm .13487 \\ &= [-.00487, .2649]. \end{aligned}$$

Both methods show that there is some evidence that the fraction conforming is larger for the small shot size, but the evidence is not conclusive.

(d) 1. H_0: $p_1 - p_2 = 0$.
2. H_a: $p_1 - p_2 \neq 0$.
3. The test statistic is given by equation (6-72), and the reference distribution is the standard normal distribution. Observed values of Z far above or below zero will be considered as evidence against H_0.
4. The sample gives

$$z = 1.87$$

5. The observed level of significance is

$$\begin{aligned} & 2P(\text{a standard normal random variable} > 1.87) \\ = \; & 2P(\text{a standard normal random variable} < -1.87) \\ = \; & 2(.0307) = .0614. \end{aligned}$$

using Table B-3. This is moderate evidence that the shot size affects the fraction of pellets conforming.

2. To ensure that the sample size is large enough (no matter what p really is), assume that $p = .5$ and use the conservative interval given by equation (6-57). For 95% confidence, $z = 1.96$, so $\Delta = 1.96\frac{1}{2\sqrt{n}}$. We want this to be less than or equal to .01. Solving the inequality for n gives $n \geq 9604$. Pollsters use $\Delta = .03$, resulting in $n = 1068$, which is the minimum sample size that you will usually see when the "margin of error" is $\pm 3\%$.

3. Using equation (6-57), the appropriate z for 99% confidence is 2.58. The resulting interval is

$$\begin{aligned} \frac{405-290}{405} \pm 2.58\frac{1}{2\sqrt{405}} &= .28395 \pm .064101 \\ &= [.220, .348]. \end{aligned}$$

Using equation (6-59), the appropriate z for 99% confidence is 2.58. The resulting interval is

$$\begin{aligned} .28395 \pm 2.58\sqrt{\frac{.28395(1-.28395)}{405}} &= .28395 \pm .057808 \\ &= [.226, .342]. \end{aligned}$$

4. 1. H_0: $p_1 - p_2 = 0$.
2. H_a: $p_1 - p_2 \neq 0$.
3. The test statistic is given by equation (6-70), and the reference distribution is the standard normal distribution. Observed values of Z far above or below zero will be considered as evidence against H_0.
4. The sample gives

$$z = -.97$$

5. The observed level of significance is

$$\begin{aligned} &2P(\text{a standard normal random variable} < -.97) \\ &\qquad = 2(.1660) = .3320. \end{aligned}$$

using Table B-3. There is little or no evidence of a difference in machine nonconforming rates. This suggests that large sample sizes are needed to detect even moderate differences in underlying proportions. In general, large samples are needed to make definitive conclusions based on qualitative data.

Section 6

1. A consumer about to purchase a single auto would be most interested in a prediction bound, because the single auto that the consumer will purchase is likely to have mileage above the bound. This is not true for a confidence bound for the mean, so a confidence bound would be less practical for the consumer. It may be more useful for the EPA official, since this person wants to be sure that the manufacturer is producing cars that exceed some minimum average mileage. The design engineer would be most interested in a lower tolerance bound for most mileages, to be sure that a high percentage of the cars produced are able to cruise for at least 350 miles. A confidence or prediction bound does not answer this question.

2. **(a)** $\bar{x} \pm ts\sqrt{(1+1/n)}$ becomes 215.1 ± (1.833)(42.943)(1 + 1/10)$^{1/2}$ or (132.543, 297.656) is a two-sided 90% prediction interval for an additional spring lifetime under this stress.

(b) $\bar{x} \pm \tau_2 s$ or 215.1 ± (2.856)(42.943). Thus, (92.455, 337.745) includes 90% of the population with 95% confidence.

(c) The 95% tolerance interval for 90% of the population (interval in (b)) is much wider than the 90% prediction interval for the next observation (interval in (a)). The interval in (b) contains 90% of future observations with 95% confidence. In repeated applications, the interval constructed as in (a) will contain an average of 90% of future observations. Any one interval constructed like the one in (a) may contain less or more than 90% of all future observations.

(d) The 90% interval for the mean lifetime (900 N/m^2) is shorter than both intervals given respectively in (a) and (b). The 90% interval for the mean lifetime (at 900 N/m^2) is such that 90% of all intervals similarly constructed from samples of size n = 10 will cover or include the true average lifetime (at 900 N/m^2 stress).

(e) $\bar{x} - ts\sqrt{(1+1/n)}$ produces 215.1 - (1.383)(42.943)$\sqrt{1.1}$ = 152.811

Thus, [152.811, + ∞] is the 90% confidence lower one-sided prediction interval for an additional spring lifetime under this stress.

(f) $\bar{x}$ - τ_1s = 215.1 - (2.355)(42.943) = 215.1 - 101.13 = 113.969. Thus, [113.969, + ∞] is a 95% lower tolerance bound for 90% of all spring lifetimes under this stress.

3. (a) Use equation (6-83). With 99% confidence, $n \approx 25$, and $p = .90$, the appropriate value for τ_2 is 2.506 (see Table B-7-A). The interval is then

$$4.9 \pm 2.506(.59) = 4.9 \pm 1.47854 = [3.42146, 6.37854].$$

Exponentiating each endpoint, [30.61, 589.07] is á 99% tolerance interval for 90% of additional raw aluminum contents.

(b) Use equation (6-78). For 90% confidence, the appropriate t is $t = Q_{25}(.95) = 1.708$, from Table B-4. The resulting interval is

$$\begin{aligned} 4.9 \pm 1.708(.59)\sqrt{1+\frac{1}{26}} &= 4.9 \pm 1.026916 \\ &= [3.873084, 5.926916]. \end{aligned}$$

Exponentiating each endpoint, [48.09, 375.00] is a 90% prediction interval for a single additional raw aluminum content.

(c) The interval in (a) is wider than the interval in (b). This is usually true when applying tolerance intervals (with large p) and prediction intervals in the same situation, with similar confidences.

4. 30 is the minimum value in the data set, and 511 is the maximum. Using equation (6-93), the confidence in this interval as a prediction interval is

$$\frac{n-1}{n+1} = \frac{25}{27} = .926$$

or 92.6%. Using equation (6-95) with $p = .90$, the confidence in this interval as a tolerance interval for a fraction .90 of all future aluminum contents is

$$1 - p^n - n(1-p)p^{n-1} = 1 - (.90)^{26} - 26(1 - .90)(.90)^{25} = .749$$

or 74.9%.

End Chapter Exercises

1. (a) The two-sided 95% confidence interval is given by equation (6-20). The required t is $Q(.975)$ of the t_9 distribution, since (by symmetry) there must be probability .025 in each tail. From Table B-4, $t = Q_9(.975) = 2.262$. From the data, $n = 10$, $\bar{x} = 9082.2$, and $s = 841.87$, so the confidence interval is

$$\begin{aligned} 9082.2 \pm 2.262\left(\frac{841.87}{\sqrt{10}}\right) &= 9082.2 \pm 602.19 \\ &= [8480.0, 9684.4] \text{ g.} \end{aligned}$$

To make the 95% one-sided confidence interval, construct a 90% two-sided confidence interval and use the lower endpoint. The appropriate t for a 90% two-sided confidence interval is $t = Q_9(.95) = 1.833$, and so the 95% one sided interval is

$$\begin{aligned} 9082.2 - 1.833\left(\frac{841.87}{\sqrt{10}}\right) &= 9082.2 - 487.99 \\ &= 8594.2 \text{ g.} \end{aligned}$$

(b) Use equation (6-78). The two-sided 95% prediction interval is

$$\begin{aligned} 9082.2 \pm 2.262(841.87)\sqrt{1+\frac{1}{10}} &= 9082.2 \pm 1997.25 \\ &= [7084.9, 11079.5] \text{ g.} \end{aligned}$$

To make the 95% one-sided prediction interval, construct a 90% two-sided prediction interval and use the lower endpoint.

$$\begin{aligned} 9082.2 - 1.833(841.87)\sqrt{1+\frac{1}{10}} &= 9082.2 - 1618.5 \\ &= 7463.7 \text{ g.} \end{aligned}$$

(c) Use formulas (6-83) and (6-85). $p = .99$, and from Table B-7-A, for 95% confidence, with $n = 10$, $\tau_2 = 4.437$. The resulting two-sided tolerance interval is

$$\begin{aligned} 9082.2 \pm 4.437(841.87) &= 9082.2 \pm 3735.37 \\ &= [5346.8, 12817.6] \text{ g.} \end{aligned}$$

From Table B-7-B, for 95% confidence, with $n = 10$ and $p = .99$, $\tau_1 = 3.981$. The resulting one-sided tolerance interval is

$$\begin{aligned} 9082.2 - 3.981(841.87) &= 9082.2 - 3351.48 \\ &= 5730.7 \text{ g}. \end{aligned}$$

(d) Use equation (6-42) and Table B-5. For a 95% two-sided interval, $U = Q_9(.975) = 19.023$ and $L = Q_9(.025) = 2.700$. The resulting interval for σ^2 is $[335314, 2362476]$; taking the square root of each endpoint, the interval for σ is $[579.1, 1537.0]$ g.

For a 95% one-sided interval, $U = Q_9(.95) = 16.919$ and the interval for σ^2 is $[377013, \infty]$; taking the square root, the interval for σ is $[614.0, \infty]$ g.

(e) 1. H_0: $\mu = 9,500$ g.
2. H_a: $\mu < 9,500$ g.
3. The test statistic is

$$T = \frac{\bar{x} - 9,500}{\frac{s}{\sqrt{10}}}$$

and the reference distribution is the t_9 distribution. Observed values of T far below zero will be considered as evidence against H_0.
4. The sample gives

$$t = -1.57$$

5. The observed level of significance is

$$\begin{aligned} &P(\text{a } t_9 \text{ random variable} < -1.57) \\ &\qquad = P(\text{a } t_9 \text{ random variable} > 1.57) \end{aligned}$$

which is between .05 and .1, according to Table B-4. This is moderate evidence that the mean breaking strength of generic towels is less than 9,500 g.

(f) 1. H_0: $\sigma = 400$ g.
2. H_a: $\sigma > 400$ g.
3. The test statistic is

$$X^2 = \frac{(n-1)s^2}{(400)^2}$$

and the reference distribution is the χ^2_9 distribution. Large observed values of X^2 will be considered as evidence against H_0.
4. The sample gives

$$x^2 = 39.87$$

5. The observed level of significance is

$$P(\text{a } \chi^2_9 \text{ random variable} > 39.87)$$

which is less than .005, according to Table B-5. This is very strong evidence that the standard deviation of breaking strengths of generic towels is greater than 400 g.

2. (a) Label the laid gears Sample 1 and the hung gears Sample 2. Since both of these samples are large, use equation (6-31) as the test statistic.

1. H_0: $\mu_1 - \mu_2 = 0$.
2. H_a: $\mu_1 - \mu_2 < 0$.
3. The test statistic is

$$Z = \frac{\bar{x}_1 - \bar{x}_2 - 0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

and the reference distribution is the standard normal distribution. Observed values of Z far below zero will be considered as evidence against H_0.

4. The sample gives

$$z = -4.18$$

5. The observed level of significance is

$$P(\text{a standard normal random variable} < -4.18)$$

which is less than .0002, according to Table B-3. This is very strong evidence that the mean of the laying method is smaller than the mean of the hanging method.

(b) Use equation (6-30). For a 90% two-sided confidence interval, the appropriate z is 1.645 (from Table 6-1).

$$\begin{aligned} 17.949 - 12.632 \pm 1.645\sqrt{\frac{47.89}{39} + \frac{14.83}{38}} &= 5.317 \pm 2.0927 \\ &= [3.22, 7.41]. \end{aligned}$$

To make a one-sided 90% one-sided interval, make an 80% two-sided confidence interval, and use the lower endpoint. For a 80% two-sided confidence interval, the appropriate z is 1.28, so the 90% one-sided confidence interval is

$$\begin{aligned} 5.317 - 1.28\sqrt{\frac{47.89}{39} + \frac{14.83}{38}} &= 5.317 - 1.6283 \\ &= 3.69. \end{aligned}$$

(c) Use equation (6-9) with $z = 1.645$. The two-sided confidence interval is

$$\begin{aligned} 12.632 \pm 1.645\left(\frac{3.85}{\sqrt{38}}\right) &= 12.632 \pm 1.028 \\ &= [11.60, 13.66]. \end{aligned}$$

(d) The distribution of the laid gears is slightly skewed to the right (is not bell-shaped). More importantly, there is an outlier, which is not usual for a normal distribution. For the prediction interval confidence, use (6-92).

$$\frac{n}{n+1} = \frac{38}{39} = .974$$

so the confidence associated with 27 as an upper prediction bound is 97.4%. For the tolerance interval confidence, use equation (6-94) with $p = .95$.

$$1 - p^n = 1 - (.95)^{38} = .858$$

so the confidence associated with 27 as an upper tolerance bound for 95% of additional runouts is 85.8%.

3. (a) Label the 2,000 psi data as Sample 1 and the 4,000 psi data as Sample 2.

1. H_0: $\mu_1 - \mu_2 = 0$.
2. H_a: $\mu_1 - \mu_2 < 0$.
3. The test statistic is given by equation (6-36) with $\# = 0$, and the reference distribution is the t_4 distribution. Observed values of T far below zero will be considered as evidence against H_0.
4. The sample gives

$$t = -11.99$$

5. The observed level of significance is

$$\begin{aligned} &P(\text{a } t_4 \text{ random variable} < -11.99) \\ &\quad = P(\text{a } t_4 \text{ random variable} > 11.99) \end{aligned}$$

which is less than .0005, according to Table B-4. This is very strong evidence that increasing pressure increases the mean density of the resulting cylinders.

(b) Use equation (6-35). To make a 99% one-sided confidence interval, make a 98% two-sided confidence interval and use the lower endpoint. For a 98% two-sided confidence interval, the appropriate t is $Q_4(.99) = 3.747$ (from Table B-4), and so the 99% one-sided confidence interval is

$$\begin{aligned} 2.569 - 2.479 - 3.747(.0092286)\sqrt{\frac{1}{3}+\frac{1}{3}} &= .09033 - .007535 \\ &= .06210 \text{ g/cc.} \end{aligned}$$

(c) 1. H_0: $\frac{\sigma_1^2}{\sigma_2^2} = 1$.
2. H_a: $\frac{\sigma_1^2}{\sigma_2^2} \neq 1$.
3. The test statistic is given by equation (6-49) with $\# = 1$, and the reference distribution is the $F_{2,2}$ distribution. Small or large observed values of F (relative to 1) will be considered as evidence against H_0.
4. The sample gives

$$f = .404.$$

5. The observed level of significance is

$$2P(\text{an } F_{2,2} \text{ random variable} < .404).$$

It is necessary to switch the degrees of freedom, invert the observed f, and change the inequality to find the probability to the left of this small quantile using Tables B-6. (Switching the degrees of freedom has no effect here, since the degrees of freedom are the same.)

$$\begin{aligned} &= 2P(\text{an } F_{2,2} \text{ random variable} > \frac{1}{.404}) \\ &= 2P(\text{an } F_{2,2} \text{ random variable} > 2.48) \\ &= 2(\text{something greater than } .25), \end{aligned}$$

so the p-value is greater than .5, according to Tables B-6. There is no evidence of a difference in variability between the two conditions.

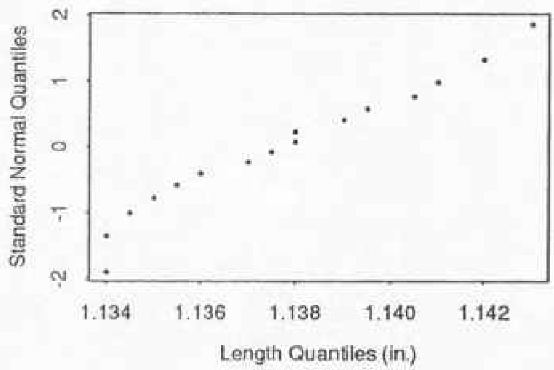
(d) Use equation (6-47) and Tables B-6. For 90% confidence, $U = Q_{2,2}(.95) = 19.00$ and $L = Q_{2,2}(.05) = \frac{1}{Q_{2,2}(.95)} = \frac{1}{19.00}$. The resulting interval for $\frac{\sigma_1^2}{\sigma_2^2}$ is [.021255, 7.673077]. Taking the square root of each endpoint, the interval for $\frac{\sigma_1}{\sigma_2}$ is [.14579, 2.77003].

(e) There must be independence within and between samples, and the individual data points from each sample must be normal. Also, the methods in (a) and (b) assume that $\sigma_1 = \sigma_2$.

(4) $\hat{p} = 18/26$ = sample proportion of PET samples that had aluminum contents above 100 ppm. An approximate 95% confidence interval for the true proportion of PET samples that have aluminum contents above 100 ppm is

$\hat{p} \pm z\sqrt{(\hat{p}(1-\hat{p})/n}$ becomes $.692 \pm 1.96\sqrt{(.692)(.308)/26}$ or $.692 \pm .177$. Thus, the 95% confidence interval is [.515, .869].

5. (a) Since this is a small sample, the data must be iid normal. One way to check this assumption is to make a normal plot of the data.



The normal plot is roughly linear with no outliers, giving no evidence that the normal assumption is unreasonable.

(b) The two-sided 90% confidence interval is given by equation (6-20). The required t is $Q(.95)$ of the t_{15} distribution, since (by symmetry) there must be probability .05 in each tail. From Table B-4, $t = Q_{15}(.95) = 1.753$. From the data, $n = 16$, $\bar{x} = 1.13778$, and $s = .002869$, so the confidence interval is

$$\begin{aligned} 1.13778 \pm 1.753\left(\frac{.002869}{\sqrt{16}}\right) &= 1.13778 \pm .0012574 \\ &= [1.13652, 1.13904] \text{ in.} \end{aligned}$$

(c) To make a 90% one-sided confidence interval, construct a 80% two-sided confidence interval and use the upper endpoint. The appropriate t for a 80% two-sided confidence interval is $t = Q_{15}(.90) = 1.341$, and so the 90% one sided interval is

$$\begin{aligned} 1.13778 + 1.341\left(\frac{.002869}{\sqrt{16}}\right) &= 1.13778 + .0009619 \\ &= 1.13874 \text{ in.} \end{aligned}$$

(d) Use equation (6-78). For 90% confidence, the appropriate t is the same as the one in (b). The resulting interval is

$$\begin{aligned} 1.13778 \pm 1.753(.002869)\sqrt{1+\frac{1}{16}} &= 1.13778 \pm .005184499 \\ &= [1.13260, 1.14297] \text{ in.} \end{aligned}$$

(e) Use equation (6-83). With 95% confidence, $n = 16$, and $p = .99$, the appropriate value for τ_2 is 3.819 (see Table B-7-A). The interval is then

$$1.13778 \pm 3.819(.002869) = 1.13778 \pm .0109575 = [1.12682, 1.14874] \text{ in.}$$

(f) Using equation (6-93), the confidence in this interval as a prediction interval is

$$\frac{n-1}{n+1} = \frac{15}{17} = .882$$

or 88.2%. Using equation (6-95) with $p = .99$, the confidence in this interval as a tolerance interval for a fraction .99 of all future bushing lengths is

$$1 - p^n - n(1-p)p^{n-1} = 1 - (.99)^{16} - 16(1-.99)(.99)^{15} = .011$$

or 1.1%. This interval is not very useful as a tolerance interval for a fraction .99 of all future bushing lengths. n needs to be increased substantially to make the confidence larger. Alternatively, you could be less ambitious and lower p to increase the confidence.

6. (a) The formulas are for comparing two means based on two independent samples. Because each bushing was measured twice by each student, there is one paired sample here, not two independent samples.

(b) Compute the differences between students A and B for each bushing, and use equation (6-25). (I took the differences as Student A–Student B.) For 95% confidence, the appropriate t is $t = Q_{15}(.975) = 2.131$, from Table B-4.

$$\begin{aligned} -00009375 \pm 2.131\left(\frac{.0004552929}{\sqrt{16}}\right) &= -00009375 \pm .0002414191 \\ &= [-0.0003352, .0001477]. \end{aligned}$$

Since zero is in this interval, there is no evidence of a mean difference between students.

7. (a) $Q_5(.90) = 1.476$, from Table B-4.

(b) $Q_5(.10) = -Q_5(.90)$ (by symmetry) $= -1.476$.

(c) $Q_7(.95) = 14.067$, from Table B-5.

(d) $Q_7(.05) = 2.167$, from Table B-5.

(e) $Q_{8,4}(.95) = 6.04$, from Table B-6-C.

(f) $Q_{8,4}(.05) = \frac{1}{Q_{4,8}(.95)} = \frac{1}{3.84} = .2604$.

8. (a) $Q_{13}(.99) = 2.650$, from Table B-4.

(b) $Q_{13}(.01) = -Q_{13}(.99)$ (by symmetry) $= -2.650$.

(c) $Q_3(.975) = 9.348$, from Table B-5.

(d) $Q_3(.025) = .216$, from Table B-5.

(e) $Q_{6,12}(.75) = 1.53$, from Table B-6-C.

(f) $Q_{6,12}(.25) = \frac{1}{Q_{12,6}(.75)} = \frac{1}{1.77} = .5650$.

9. (a) Use equation (6-9) with $z = 1.96$ for 95% confidence. The two-sided confidence interval is

$$\begin{aligned} .0287 \pm 1.96\left(\frac{.0119}{\sqrt{50}}\right) &= .0287 \pm .003299 \\ &= [.02540, .03200] \text{ in.} \end{aligned}$$

(b) To make a 95% one-sided confidence interval, construct a 90% two-sided confidence interval and use the lower endpoint. The appropriate z for a 90% two-sided confidence interval is 1.645, so the 95% one sided interval is

$$\begin{aligned} .0287 - 1.645\left(\frac{.0119}{\sqrt{50}}\right) &= .0287 - .0027684 \\ &= .02593 \text{ in.} \end{aligned}$$

(c) 1. H_0: $\mu = .025$ in.
2. H_a: $\mu > .025$ in.
3. The test statistic is

$$Z = \frac{\bar{x} - .025}{\frac{s}{\sqrt{50}}}$$

and the reference distribution is the standard normal distribution. Observed values of Z far above zero will be considered as evidence against H_0.
4. The sample gives

$$z = 2.20.$$

5. The observed level of significance is

$$\begin{aligned} & P(\text{a standard normal random variable} > 2.20) \\ = \; & P(\text{a standard normal random variable} < -2.20) \end{aligned}$$

which is equal to .0139, according to Table B-3. There is strong evidence that the mean wobble exceeds .025 in.

(d) No. The requirement in part (c) pertains to the mean. *Individual* wobbles have a distribution around the mean, with a spread that is approximated by $s = .0119$. The mean could be just below .025 in., but the lot could have many individual wobbles exceed .025 in. because of the spread in the wobbles.

(e) Using equation (6-57), the appropriate z for 90% confidence is 1.645. The resulting interval is

$$\begin{aligned} \frac{19}{50} \pm 1.645\frac{1}{2\sqrt{50}} &= .38 \pm .116 \\ &= [.264, .496]. \end{aligned}$$

Using equation (6-59), the appropriate z for 90% confidence is 1.645. The resulting interval is

$$\begin{aligned} .38 \pm 1.645\sqrt{\frac{.38(1-.38)}{50}} &= .38 \pm .113 \\ &= [.267, .493]. \end{aligned}$$

10. (a) Use equation (6-78). For a 90% one-sided interval, construct a 80% two-sided interval, and use the upper endpoint. For a 80% two-sided interval, the appropriate t is $t = Q_2(.90) = 1.886$, from Table B-4. The resulting 90% one-sided interval is

$$\begin{aligned} .8733 + 1.886(.011547)\sqrt{1+\frac{1}{3}} &= .8733 + .025147 \\ &= .898 \text{ cm.} \end{aligned}$$

(b) Use equation (6-84). With 95% confidence, $n = 3$, and $p = .90$, the appropriate value for τ_1 is 6.155 (see Table B-7-B). The interval is then

$$.8733 + 6.155(.011547) = .8733 + .07107 = .944 \text{ cm.}$$

(c) For the mean, use equation (6-20). For a 90% two-sided interval, the required t is $Q(.95)$ of the t_2 distribution, since (by symmetry) there must be probability .05 in each tail. From Table B-4, $t = Q_2(.95) = 2.920$, and the confidence interval is

$$\begin{aligned} .8733 \pm 2.920\left(\frac{.011547}{\sqrt{3}}\right) &= .8733 \pm .01947 \\ &= [.854, .893] \text{ cm.} \end{aligned}$$

For the standard deviation, use equation (6-42) and Table B-5. For a 90% two-sided interval, $U = Q_2(.95) = 5.991$ and $L = Q_2(.05) = .103$. The resulting interval for σ^2 is $[.00004451121, .002588997]$; taking the square root of each endpoint, the interval for σ is $[.00667, .05088]$ cm.

(d) Label Brand B as Sample 1 and Brand D as Sample 2. Use equation (6-47) and Tables B-6. For 90% confidence, $U = Q_{2,2}(.95) = 19.00$ and $L = Q_{2,2}(.05) = \frac{1}{Q_{2,2}(.95)} = \frac{1}{19.00}$. The resulting interval for $\frac{\sigma_1^2}{\sigma_2^2}$ is $[.01754386, 6.3333]$. Taking the square root of each endpoint, the interval for $\frac{\sigma_1}{\sigma_2}$ is $[.132, 2.517]$.

(e) Use equation (6-35). For 90% confidence, the appropriate t is $t = Q_4(.95) = 2.132$ from Table B-4, and the resulting interval is

$$\begin{aligned} .8733 \pm 2.132(.01633)\sqrt{\frac{1}{3}+\frac{1}{3}} &= -.1667 \pm .02843 \\ &= [-.195, .138] \text{ cm.} \end{aligned}$$

Since zero is not in this interval, there appears to be a difference between the mean stretch values of the two brands.

(f) 1. H_0: $\mu_1 - \mu_2 = 0$.
2. H_a: $\mu_1 - \mu_2 \neq 0$.
3. The test statistic is given by equation (6-36) with $\# = 0$, and the reference distribution is the t_4 distribution. Observed values of T far above or below zero will be considered as evidence against H_0.
4. The sample gives

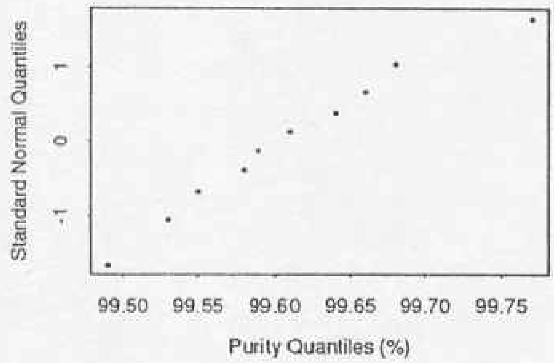
$$t = -12.5.$$

5. The observed level of significance is

$$2P(\text{a } t_4 \text{ random variable} < -12.5) = 2P(\text{a } t_4 \text{ random variable} > 12.5)$$

which is less than $2(.0005) = .001$, according to Table B-4. This is very strong evidence that there is a difference between the mean stretch values of the two brands. This conclusion agrees with the confidence interval in part (e).

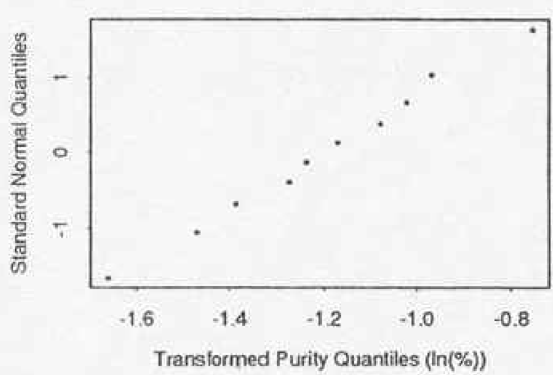
11. (a)



The normal plot is fairly linear, except for the largest point. It would have to be pushed in to make the plot more straight (and thus make the data more bell-shaped), so the data have a longer right tail than a bell-shaped distribution.

(b) The most common and easily applied small-sample methods assume that the data are generated from an underlying normal distribution. If the data are not bell-shaped, these small-sample methods are less valid.

(c)



(d) Use equation (6-78). The appropriate t is $t = Q_9(.975) = 2.262$, from Table B-4. The two-sided 95% prediction interval is

$$-1.203 \pm 2.262(.263)\sqrt{1 + \frac{1}{10}} = -1.203 \pm .6239427$$

$$= [-1.826943, -.5790573].$$

To get this in terms of raw purity, exponeniate the endpoints and then add 99.3. This results in [99.146, 99.860] %.

(e) Use equation (6-83). With 99% confidence, $n = 10$, and $p = .95$, the appropriate value for τ_2 is 4.294 (see Table B-7-A). The interval is then

$$-1.203 \pm 4.294(.263) = -1.203 \pm 1.129322 = [-2.332322, -.073678].$$

To get this in terms of raw purity, exponeniate the endpoints and then add 99.3. This results in [93.397, 94.229] %.

(f) 1. H_0: $\mu_{y'} = -1.61$.
2. H_a: $\mu_{y'} < -1.61$.
3. The test statistic is

$$T = \frac{\bar{y}' - (-1.61)}{\frac{s_{y'}}{\sqrt{10}}}$$

and the reference distribution is the t_9 distribution. Observed values of T far below zero will be considered as evidence against H_0.
4. The sample gives

$$t = 4.894.$$

5. The observed level of significance is

$$P(\text{a } t_9 \text{ random variable} < 4.894)$$

which is greater than .9995, according to Table B-4. There is absolutely no evidence that the mean purity is substandard.

12. (a) Use equation (6-20). To make the 90% one-sided confidence interval, construct an 80% two-sided confidence interval and use the lower endpoint. The appropriate t for an 80% two-sided confidence interval is $t = Q_{11}(.90) = 1.363$, and so the 90% one sided interval is

$$\begin{aligned} 117.75 - 1.363\left(\frac{51.1}{\sqrt{12}}\right) &= 117.75 - 20.10602 \\ &= 97.6 \text{ holes.} \end{aligned}$$

(b) The test would have a large p-value because there are plausible values of μ which are below 100. Therefore, there is not strong evidence that $\mu > 100$, which corresponds to a large p-value.

(c) Use equation (6-78). To make the 90% one-sided prediction interval, construct an 80% two-sided prediction interval and use the lower endpoint.

$$\begin{aligned} 117.75 - 1.363(51.1)\sqrt{1 + \frac{1}{12}} &= 117.75 - 72.49329 \\ &= 45.3 \text{ holes.} \end{aligned}$$

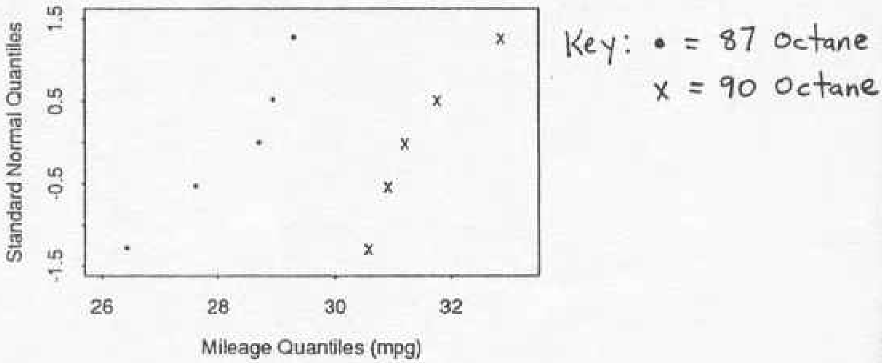
(d) Use equation (6-83). With 95% confidence, $n = 12$, and $p = .90$, the appropriate value for τ_2 is 2.670 (see Table B-7-A). The interval is then

$$117.75 \pm 2.670(51.1) = 117.75 \pm 136.437 = [-18.7, 254.2].$$

Embarrassingly, the lower bound is negative. Set it equal to zero, since lifetimes cannot be negative: [0, 254.2] holes. (It is common to analyze lifetime data on the log scale to avoid this problem. The exponentiation at the end will always result in a positive interval.)

(e) Use equation (6-40) and Table D-5. For a 90% two-sided interval, $U = Q_{11}(.95) = 19.675$ and $L = Q_{11}(.05) = 4.575$. The resulting interval for σ^2 is [1459.889, 6278.319]; taking the square root of each endpoint, the interval for σ is [38.21, 79.24] holes.

13. (a)



For such small sample sizes, it is difficult to verify the assumptions. The plots are roughly linear with no outliers, indicating that the normal part of the assumption may be reasonable. The slopes are similar, indicating that the common variance assumption may be reasonable.

(b) Use equation (6-32).

$$s_P^2 = \frac{(4)(1.37467) + (4)(.80328)}{8} = 1.088975$$

so $s_P = 1.089$. This measures the amount of baseline variation within either condition, assuming it is the same for each condition.

(c) Label the 87 Octane data Sample 1 and the 90 Octane data Sample 2. Use equation (6-35). For a 95% two-sided confidence interval, the appropriate t is $Q_8(.975) = 2.306$ (from Table B-4), and so interval is

$$\begin{aligned} 28.198 - 31.464 \pm 2.306(1.089)\sqrt{\frac{1}{5} + \frac{1}{5}} &= -3.266 \pm 1.521943 \\ &= [-4.788, -1.744] \text{ mpg.} \end{aligned}$$

The test will produce a small p-value because zero is not in the confidence interval; there is evidence against the null hypothesis.

(d) 1. $\mathrm{H_0}$: $\mu_1 - \mu_2 = 0$.
2. $\mathrm{H_a}$: $\mu_1 - \mu_2 < 0$.
3. The test statistic is given by equation (6-36) with $\# = 0$, and the reference distribution is the t_8 distribution. Observed values of T far below zero will be considered as evidence against $\mathrm{H_0}$.
4. The sample gives

$$t = -4.949.$$

5. The observed level of significance is

$$\begin{aligned} &P(\text{a } t_8 \text{ random variable} < -4.949) \\ &\qquad = P(\text{a } t_8 \text{ random variable} > 4.949) \end{aligned}$$

which is between .0005 and .001, according to Table B-4. This is very strong evidence that the higher-octane gasoline provides higher mean mileage.

(e) Use equation (6-78). To make the 95% one-sided prediction intervals, construct 90% two-sided prediction intervals and use the lower endpoints. For each 90% two-sided interval, the appropriate t is $t = Q_4(.95) = 2.132$ (from Table B-4). For the 87 octane fuel, the 95% one-sided prediction interval is

$$\begin{aligned} 28.198 - 2.306(1.172463)\sqrt{1+\frac{1}{5}} &= 28.198 - 2.961755 \\ &= 25.236 \text{ mpg.} \end{aligned}$$

For the 90 octane fuel, the 95% one-sided prediction interval is

$$\begin{aligned} 31.464 - 2.306(.8962589)\sqrt{1+\frac{1}{5}} &= 31.464 - 2.264036 \\ &= 29.200 \text{ mpg.} \end{aligned}$$

(f) Use formula (6-85). From Table B-7-B, for 95% confidence, with $n = 5$ and $p = .95$, $\tau_1 = 4.203$. The resulting one-sided tolerance interval for the 87 octane fuel is

$$\begin{aligned} 28.198 - 4.203(1.172463) &= 28.198 - 4.927862 \\ &= 23.270 \text{ mpg.} \end{aligned}$$

The resulting one-sided tolerance interval for the 90 octane fuel is

$$\begin{aligned} 31.464 - 4.203(.8962589) &= 31.464 - 3.766976 \\ &= 27.697 \text{ mpg.} \end{aligned}$$

14. (a) Using equation (6-57) for a 95% one-sided interval, construct a 90% two-sided interval and use the upper endpoint. The appropriate z for a 90% two-sided interval is 1.645, so the 95% one-sided interval is

$$\begin{aligned} \frac{147}{250} + 1.645\frac{1}{2\sqrt{250}} &= .588 + .05201947 \\ &= .640. \end{aligned}$$

Using equation (6-59), the appropriate z is the same. The resulting 95% one-sided interval is

$$\begin{aligned} .588 + 1.645\sqrt{\frac{.588(1-.588)}{250}} &= .588 + .05120745 \\ &= .639. \end{aligned}$$

(b) Using equation (6-57) for a 95% one-sided interval, construct a 90% two-sided interval and use the upper endpoint. The appropriate z for a 90% two-sided interval is 1.645, so the 95% one-sided interval is

$$\begin{aligned} \frac{12}{250} + 1.645\frac{1}{2\sqrt{250}} &= .048 + .05201947 \\ &= .100. \end{aligned}$$

Using equation (6-59), the appropriate z is the same. The resulting 95% one-sided interval is

$$\begin{aligned} .048 + 1.645\sqrt{\frac{.048(1-.048)}{250}} &= .048 + .02224001 \\ &= .070. \end{aligned}$$

(c) Using equation (6-65), the appropriate z for 95% confidence is 1.96. The resulting interval is

$$
\begin{aligned}
.588 - .048 \pm 1.96\left(\frac{1}{2}\right)\sqrt{\frac{1}{250}+\frac{1}{250}} &= .54 \pm .08765386 \\
&= [.452, .628].
\end{aligned}
$$

Using equation (6-67), the resulting interval is

$$
\begin{aligned}
.588 - .048 \pm 1.96\sqrt{\frac{.588(1-.588)}{250}+\frac{.048(1-.048)}{250}} &= .54 \pm .06651906 \\
&= [.473, .607].
\end{aligned}
$$

There is a clear difference in defective rates for the two machines because even the conservative interval does not contain zero. The test will have a small p-value, because there is strong evidence against the null hypothesis.

(d) 1. H_0: $p_H - p_L = 0$.
2. H_a: $p_H - p_L \neq 0$.
3. The test statistic is given by equation (6-70), and the reference distribution is the standard normal distribution. Observed values of Z far above or below zero will be considered as evidence against H_0.
4. The sample gives

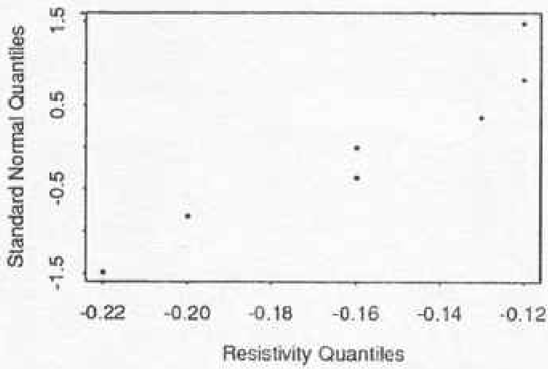
$$z = 12.96.$$

5. The observed level of significance is

$$
\begin{aligned}
& 2P(\text{a standard normal random variable} > 12.96) \\
= \; & 2P(\text{a standard normal random variable} < -12.96)
\end{aligned}
$$

which is less than $2(.0002) = .0004$, using Table B-3. This is overwhelming evidence that the defective rates are different. The high-speed operation has a higher defective rate.

15. (a) I computed the differences as 0.0°C − 21.8°C.



Given the amount of data, there is no evidence that the normal assumption is unreasonable. The plot is roughly linear with no outliers.

(b) Use equation (6-25). For 90% confidence, the appropriate t is $t = Q_6(.95) = 1.943$, from Table B-4.

$$\begin{aligned} -.15857 \pm 1.943\left(\frac{.03933979}{\sqrt{7}}\right) &= -.15857 \pm .02889055 \\ &= [-.1875, -.1297]\Omega\text{m}. \end{aligned}$$

(c) Use equation (6-78), with the same t that was used in part (b). The two-sided 90% prediction interval is

$$\begin{aligned} -.15857 \pm 1.943(.03933979)\sqrt{1+\frac{1}{7}} &= -.15857 \pm .08171482 \\ &= [-.2403, -.0769]\Omega\text{m}. \end{aligned}$$

16. (a) Use equation (6-20). For a 99% two-sided interval, the required t is $Q(.995)$ of the t_6 distribution, since (by symmetry) there must be probability .005 in each tail. From Table B-4, $t = Q_6(.995) = 3.707$, and the confidence interval is

$$\begin{aligned} 2.6814 \pm 3.707\left(\frac{.0908164}{\sqrt{7}}\right) &= 2.6814 \pm .1272442 \\ &= [2.554, 2.809]\Omega\text{m}. \end{aligned}$$

(b) Use equation (6-78). The appropriate t is $t = Q_6(.975) = 2.447$, from Table B-4. The two-sided 95% prediction interval is

$$\begin{aligned} 2.6814 \pm 2.447(.0908164)\sqrt{1+\frac{1}{7}} &= 2.6814 \pm .2375714 \\ &= [2.444, 2.919]\Omega\text{m}. \end{aligned}$$

(c) Use equation (6-83). With 95% confidence, $n = 7$, and $p = .99$, the appropriate value for τ_2 is 5.241 (see Table B-7-A). The interval is then

$$2.6814 \pm 5.241(.0908164) = 2.6814 \pm .4759688 = [2.2055, 3.1574]\Omega\text{m}.$$

(d) Use equation (6-42) and Table B-5. For a 95% two-sided interval, $U = Q_6(.975) = 14.449$ and $L = Q_6(.025) = 1.237$. The resulting interval for σ^2 is $[.003424854, .04000462]$; taking the square root of each endpoint, the interval for σ is $[.05852, .20001]\Omega$m.

(e) Label the copper data as Sample 1 and the aluminum data as Sample 2.

1. H_0: $\frac{\sigma_1^2}{\sigma_2^2} = 1$.
2. H_a: $\frac{\sigma_1^2}{\sigma_2^2} \neq 1$.
3. The test statistic is given by equation (6-47) with $\# = 1$, and the reference distribution is the $F_{6,6}$ distribution. Small or large observed values of F (relative to 1) will be considered as evidence against H_0.
4. The sample gives

$$f = .4036.$$

5. The observed level of significance is

$$2P(\text{an } F_{6,6} \text{ random variable} < .4036).$$

It is necessary to switch the degrees of freedom, invert the observed f, and change the inequality to find the probability to the left of this small quantile using Tables B-6. (Switching the degrees of freedom has no effect here, since the degrees of freedom are the same.)

$$\begin{aligned}
&= 2P(\text{an } F_{6,6} \text{ random variable} > \frac{1}{.4036}) \\
&= 2P(\text{an } F_{6,6} \text{ random variable} > 2.478) \\
&= 2(\text{something between .1 and .25}),
\end{aligned}$$

so the p-value is between .2 and .5, according to Tables B-6. There is no evidence of a difference in precisions between the measured copper and aluminum resistivities.

(f) Use equation (6-45) and Tables B-6. For 90% confidence, $U = Q_{6,6}(.95) = 4.28$ and $L = Q_{6,6}(.05) = \frac{1}{Q_{6,6}(.95)} = \frac{1}{4.28}$. The resulting interval for $\frac{\sigma_1^2}{\sigma_2^2}$ is $[.09429432, 1.727321]$. Taking the square root of each endpoint, the interval for $\frac{\sigma_1}{\sigma_2}$ is $[.3071, 1.3143]$.

17. (a) The appropriate t is $t = Q_4(.975) = 2.776$, from Table B-4. So you need to find the smallest integer n such that

$$n \geq \left(\frac{2.776(1.27)}{.5}\right)^2 = 49.72$$

which gives $n = 50$. This means that a total of $n = 50$ resistors must be sampled, so an additional $n_2 = 50 - 5 = 45$ resistors must be sampled.

(b) The same t and s_1 should be used. The resulting interval is

$$\begin{aligned}
102.8 \pm 2.776\left(\frac{1.27}{\sqrt{50}}\right) &= 102.8 \pm .49858 \\
&= [102.30, 103.30]\ \Omega.
\end{aligned}$$

18. (a) For prediction interval confidence, use (6-92).

$$\frac{n}{n+1} = \frac{65}{66} = .985$$

so the confidence associated with 87 seconds as an upper prediction bound for a single additional service time is 98.5%.

(b) For tolerance interval confidence, use equation (6-94) with $p = .95$.

$$1 - p^n = 1 - (.95)^{65} = .964$$

so the confidence associated with 87 seconds as an upper tolerance bound for 95% of additional service times is 96.4%.

19. (a) For Employee 1, using equation (6-57), the appropriate z for 95% confidence is 1.96. The resulting interval is

$$\begin{aligned}
\frac{5}{54} \pm 1.96\frac{1}{2\sqrt{54}} &= .09259 \pm .1333611 \\
&= [-.041, .226].
\end{aligned}$$

Since proportions must be between zero and one, set the lower endpoint equal to zero: $[0, .226]$. Still for Employee 1, using equation (6-59), the resulting interval is

$$\begin{aligned}
.09259 \pm 1.96\sqrt{\frac{.09259(1 - .09259)}{54}} &= .09259 \pm .07731228 \\
&= [.015, .170].
\end{aligned}$$

For Employee 2, using equation (6-57), the appropriate z for 95% confidence is 1.96. The resulting interval is

$$\begin{aligned}\frac{22}{73} \pm 1.96\frac{1}{2\sqrt{73}} &= .30137 \pm .1147003\\ &= [.187, .416].\end{aligned}$$

Still for Employee 2, using equation (6-59), the resulting interval is

$$\begin{aligned}.30137 \pm 1.96\sqrt{\frac{.30137(1-.30137)}{73}} &= .30137 \pm .1052612\\ &= [.196, .407].\end{aligned}$$

(b) Using equation (6-65), the appropriate z for 95% confidence is 1.96. The resulting interval is

$$\begin{aligned}.09259 - .30137 \pm 1.96\left(\frac{1}{2}\right)\sqrt{\frac{1}{54}+\frac{1}{73}} &= -.208778 \pm .1759015\\ &= [-.385, -.033].\end{aligned}$$

Using equation (6-67), the resulting interval is

$$\begin{aligned}&.09259 - .30137 \pm 1.96\sqrt{\frac{.09259(1-.09259)}{54}+\frac{.30137(1-.30137)}{73}}\\ =\ &-.208778 \pm .1306028\\ =\ &[-.339, -.078].\end{aligned}$$

(c) 1. $\mathrm{H_0}$: $p_1 - p_2 = 0$.
2. $\mathrm{H_a}$: $p_1 - p_2 \neq 0$.
3. The test statistic is given by equation (6-72), and the reference distribution is the standard normal distribution. Observed values of Z far above or below zero will be considered as evidence against $\mathrm{H_0}$.
4. The sample gives

$$z = -2.84.$$

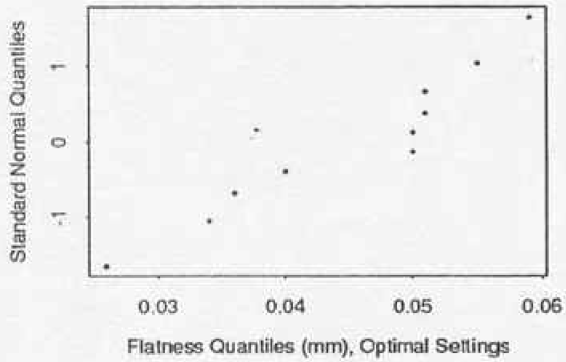
5. The observed level of significance is

$$\begin{aligned}&2P(\text{a standard normal random variable} < -2.84)\\ &\qquad = 2(.0023) = .0046.\end{aligned}$$

using Table B-3. There is strong evidence of a difference in rates of unacceptable keys produced by the two employees.

20. (a) The optimal settings y_1 (or y_2) data must be iid normal, because this is a small sample.

(b)



For such a small data set, this plot is roughly linear (with no outliers), indicating that it may be reasonable to treat the optimal settings' flatness distribution as normal.

(c) (i) The two-sided 90% confidence interval is given by equation (6-20). The required t is $Q(.95)$ of the t_9 distribution, since (by symmetry) there must be probability .05 in each tail. From Table B-4, $t = Q_9(.95) = 1.833$. From the data, $n = 10$, $\bar{x} = .0452$, and $s = .01057$, so the confidence interval is

$$\begin{aligned} .0452 \pm 1.833\left(\frac{.01057}{\sqrt{10}}\right) &= .0452 \pm .006127 \\ &= [.03907, .05133] \text{ mm.} \end{aligned}$$

(ii) Use equation (6-78). The t is the same as in part (i). The resulting interval is

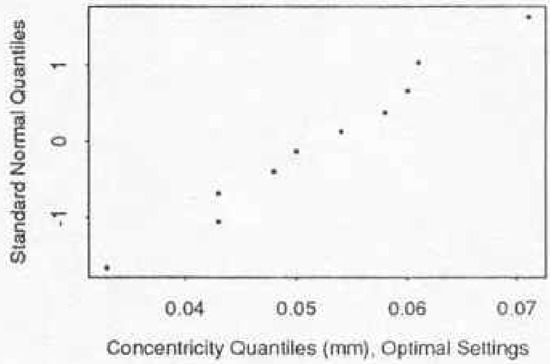
$$\begin{aligned} .0452 \pm 1.833(.01057)\sqrt{1+\frac{1}{10}} &= .0452 \pm .020321 \\ &= [.02488, .06552]. \end{aligned}$$

(iii) Use equation (6-83). With 95% confidence, $n = 10$, and $p = .90$, the appropriate value for τ_2 is 2.856 (see Table B-7-A). The interval is then

$$.0452 \pm 2.856(.01057) = .0452 \pm .030188 = [.01501, .07539].$$

(iv) Use equation (6-42) and Table B-5. For a 90% two-sided interval, $U = Q_9(.95) = 16.919$ and $L = Q_9(.05) = 3.325$. The resulting interval for σ^2 is [00005943614, .0003024361]; taking the square root of each endpoint, the interval for σ is [.007709, .017391] mm.

(d)



This plot is fairly linear, giving no indication that the normal distribution assumption for y_2 at the optimal settings is unreasonable.

(i) The two-sided 90% confidence interval is given by equation (6-20). The required t is $Q(.95)$ of the t_9 distribution, since (by symmetry) there must be probability .05 in each tail. From Table B-4, $t = Q_9(.95) = 1.833$. From the data, $n = 10$, $\bar{x} = .0521$, and $s = .01099949$, so the confidence interval is

$$\begin{aligned} .0521 \pm 1,833\left(\frac{.01099949}{\sqrt{10}}\right) &= .0521 \pm .0063758 \\ &= [.045724, .058476] \text{ mm.} \end{aligned}$$

(ii) Use equation (6-78). The t is the same as in part (i). The resulting interval is

$$\begin{aligned} .0521 \pm 1.833(.01099949)\sqrt{1+\frac{1}{10}} &= .0521 \pm .021146 \\ &= [.03095, .07325] \text{ mm.} \end{aligned}$$

(iii) Use equation (6-83). With 95% confidence, $n = 10$, and $p = .90$, the appropriate value for τ_2 is 2.856 (see Table B-7-A). The interval is then

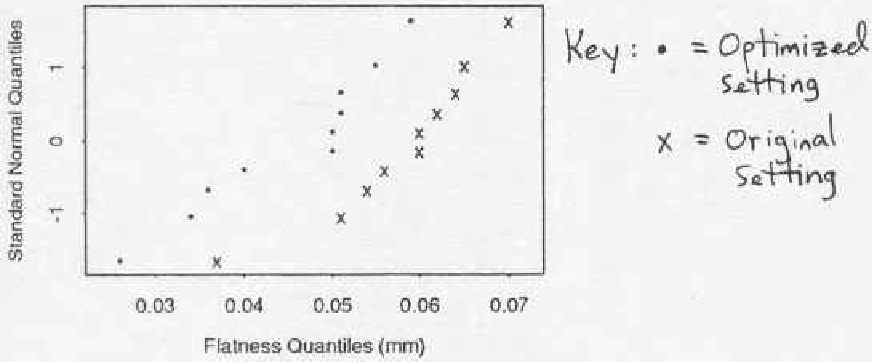
$$.0521 \pm 2.856(.01099949) = .0521 \pm .03141454 = [.02069, .08351].$$

(iv) Use equation (6-42) and Table B-5. For a 90% two-sided interval, $U = Q_9(.95) = 16.919$ and $L = Q_9(.05) = 3.325$. The resulting interval for σ^2 is $[.0000643596, .0003274887]$; taking the square root of each endpoint, the interval for σ is $[.008022443, .01809665]$ mm.

(e) Since both measurements are made on each of the 10 gears, the sample standard deviations would not be based on two independent samples. The two sample standard deviations are correlated; treating them as independent would result in underestimating the difference between the two standard deviations.

(f) The data within each y_1 (or y_2) sample must be iid normal. The two samples must be independent (and this seems to be true). Also, the standard deviations of the two distributions must be the same in order to compare the means.

(g)



Both plots are fairly linear, giving no evidence against the normal assumptions. The slopes of the plots are similar, giving no evidence that the equal variance assumption is unreasonable.

(h) (i) Label the Optimal Settings data Sample 1 and the Original Settings data Sample 2.

1. $\mathrm{H_0}$: $\mu_1 - \mu_2 = 0$.
2. $\mathrm{H_a}$: $\mu_1 - \mu_2 < 0$.
3. The test statistic is given by equation (6-36) with $\# = 0$, and the reference distribution is the t_{18} distribution. Observed values of T far below zero will be considered as evidence against $\mathrm{H_0}$.
4. The sample gives

$$t = -2.865$$

5. The observed level of significance is

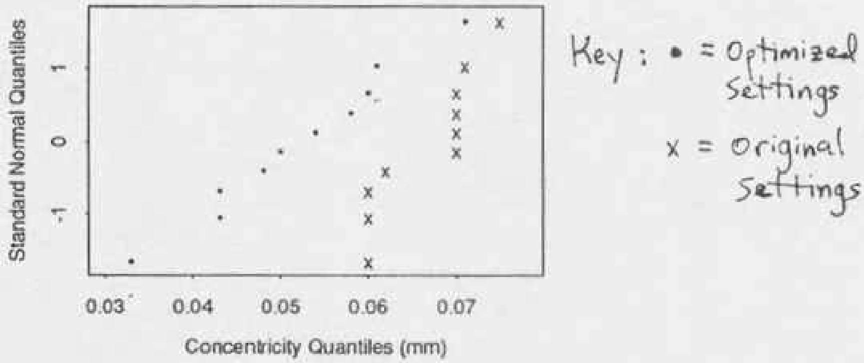
$$\begin{aligned} &P(\text{a } t_{18} \text{ random variable} < -2.865) \\ &\qquad = P(\text{a } t_{18} \text{ random variable} > 2.865) \\ &\qquad = (\text{something between .005 and .01}), \end{aligned}$$

according to Table B-4. There is strong evidence that the optimized settings produce a reduction in mean flatness distortion.

(ii) Use equation (6-35). To make a 90% one-sided confidence interval, make a 80% two-sided confidence interval and use the lower endpoint. For a 80% two-sided confidence interval, The appropriate t is $Q_{18}(.90) = 1.330$ (from Table B-4), and so the 90% one-sided confidence interval for $\mu_2 - \mu_1$ is

$$\begin{aligned} .0579 - .0452 - 1.330(.009912114)\sqrt{\frac{1}{10} + \frac{1}{10}} &= .0127 - .005895667 \\ &= .0068 \text{ mm.} \end{aligned}$$

(i)



Given the number of ties (the data are rather discrete), both of these plots are roughly linear, giving no strong indication that the normal distribution assumption is unreasonable. The slopes are not similar; there appears to be less spread in the original settings y_2 data. The common variance assumption may not be reasonable.

(i) Label the Optimal Settings data Sample 1 and the Original Settings data Sample 2.

1. $\mathrm{H_0}$: $\mu_1 - \mu_2 = 0$.
2. $\mathrm{H_a}$: $\mu_1 - \mu_2 < 0$.
3. The test statistic is given by equation (6-36) with $\# = 0$, and the reference distribution is the t_{18} distribution. Observed values of T far below zero will be considered as evidence against $\mathrm{H_0}$.
4. The sample gives

$$t = -3.759.$$

5. The observed level of significance is

$$\begin{aligned} &P(\text{a } t_{18} \text{ random variable} < -3.759) \\ &\quad = P(\text{a } t_{18} \text{ random variable} > 3.759) \\ &\quad = (\text{something between .0005 and .001}), \end{aligned}$$

according to Table B-4. There is very strong evidence that the optimized settings produce a reduction in mean concentricity distortion.

(ii) Use equation (6-35). To make a 90% one-sided confidence interval, make a 80% two-sided confidence interval and use the lower endpoint. For a 80% two-sided confidence interval, The appropriate t is $Q_{18}(.90) = 1.330$ (from Table B-4), and so the 90% one-sided confidence interval for $\mu_2 - \mu_1$ is

$$\begin{aligned} .0668 - .0521 - 1.330(.00874484)\sqrt{\frac{1}{10} + \frac{1}{10}} &= .0147 - .005201379 \\ &= .0095 \text{ mm}. \end{aligned}$$

21. (a) The sample may still be reasonably representative of the entire parking lot of cars. One might argue that the location of any car in the parking has little to do with whether the car's tires are over- or underinflated. If this is true, the convenience sample is still fairly

"random", and might be treated as a random sample.

(b) Using equation (6-57), the appropriate z for 90% confidence is 1.645. The resulting interval is

$$\begin{aligned}\frac{9}{25} \pm 1.645\frac{1}{2\sqrt{25}} &= .36 \pm .1645 \\ &= [.196, .525].\end{aligned}$$

Using equation (6-59), the resulting interval is

$$\begin{aligned}.36 \pm 1.645\sqrt{\frac{.36(1-.36)}{25}} &= .36 \pm .15792 \\ &= [.202, .518].\end{aligned}$$

(c) Using equation (6-57), the appropriate z for 90% confidence is 1.645. The resulting interval is

$$\begin{aligned}\frac{14}{25} \pm 1.645\frac{1}{2\sqrt{25}} &= .56 \pm .1645 \\ &= [.396, .725].\end{aligned}$$

Using equation (6-59), the resulting interval is

$$\begin{aligned}.56 \pm 1.645\sqrt{\frac{.56(1-.56)}{25}} &= .56 \pm .1633113 \\ &= [.397, .723].\end{aligned}$$

(d) Using equation (6-57) for a 90% one-sided interval, construct an 80% two-sided interval and use the lower endpoint. The appropriate z for a 80% two-sided interval is 1.28, so the 90% one-sided interval is

$$\begin{aligned}\frac{19}{25} - 1.28\frac{1}{2\sqrt{25}} &= .76 - .128 \\ &= .632.\end{aligned}$$

Using equation (6-59), the appropriate z is the same. The resulting 90% one-sided interval is

$$\begin{aligned}.76 - 1.28\sqrt{\frac{.76(1-.76)}{25}} &= .76 - .1093333 \\ &= .651.\end{aligned}$$

(e) There would not be two independent samples. The sample proportion of cars with at least one underinflated tire and the sample proportion of cars with at least one overinflated tire were both computed from the same 25 cars. They are not independent, and so formula (6-67) should not be used. There are other statistical methods (not covered in the text) that can be used to make such a confidence interval in this situation.

22. (a) Using equation (6-57), the appropriate z for 98% confidence is 2.33. The resulting interval is

$$\begin{aligned}.39 \pm 2.33\frac{1}{2\sqrt{442}} &= .39 \pm .0554134 \\ &= [.335, .445].\end{aligned}$$

Using equation (6-59), the resulting interval is

$$.39 \pm 2.33\sqrt{\frac{.39(1-.39)}{442}} = .39 \pm .05405576$$
$$= [.336, .444].$$

(b) Using equation (6-65), the appropriate z for 90% confidence is 1.645. The resulting interval is

$$.32 - .39 \pm 1.645\left(\frac{1}{2}\right)\sqrt{\frac{1}{442}+\frac{1}{100}} = -.07 \pm .09108029$$
$$= [-0.161, .021].$$

Using equation (6-67), the resulting interval is

$$-.07 \pm 1.645\sqrt{\frac{.32(1-.32)}{100}+\frac{.39(1-.39)}{442}} = -.07 \pm .08570174$$
$$= [-.156, .016].$$

Both of these intervals contain zero. This suggests that the data do not provide convincing evidence that a real process improvement has been accomplished. More data are needed to establish this.

23. Using equation (6-57), the appropriate z for 95% confidence is 1.96. The resulting interval is

$$\frac{16}{120} \pm 1.96\frac{1}{2\sqrt{120}} = .13333 \pm .08946135$$
$$= [.044, .223].$$

Using equation (6-59), the resulting interval is

$$.13333 \pm 1.96\sqrt{\frac{.13333(1-.13333)}{120}} = .13333 \pm .06082202$$
$$= [.073, .194].$$

24. (a) Label the rectangular bar data as Sample 1 and the circular bar data as Sample 2. Use equation (6-47) and Tables B-6. For 98% confidence, $U = Q_{4,4}(.99) = 15.98$ and $L = Q_{4,4}(.01) = \frac{1}{Q_{4,4}(.99)} = \frac{1}{15.98}$. The resulting interval for $\frac{\sigma_1^2}{\sigma_2^2}$ is $[.0378, 9.6465]$. Since 1 is contained in this interval, there is no evidence that the variabilities are different.

(b) Use equation (6-35). For a 95% two-sided confidence interval, the appropriate t is $Q_8(.975) = 2.306$ (from Table B-4), and so the interval is

$$82.6 - 87.4 \pm 2.306(5.128353)\sqrt{\frac{1}{5}+\frac{1}{5}} = -4.8 \pm 7.479407$$
$$= [-12.28, 2.68] \text{ psi.}$$

There is no evidence of a difference in means because zero is in this interval.

(c) Any conclusions made from this data can only be applied to the particular gripper in the study. To generalize conclusions to other grippers of this design, you would need to collect data on other grippers. This would result in more variation, since different grippers of the same design may have slightly different properties. It is this variation, however, that must be estimated in order to make generalizations beyond one particular gripper.

25. (a) Since this is a large sample, you can use equation (6-9), with $z = 1.96$ for 95% confidence. The two-sided confidence interval is

$$\begin{aligned} 10.1 \pm 1.96\left(\frac{3.2}{\sqrt{95}}\right) &= 10.1 \pm .6434936 \\ &= [9.46, 10.74] \times .001 \text{ inches above nominal.} \end{aligned}$$

Zero is not even close to being in this interval. There is strong evidence that the mean is not equal to zero, so the p-value for the test will be large.

(b) 1. H_0: $\mu = 0 \times .001$ inches above nominal.
2. H_a: $\mu > 0 \times .001$ inches above nominal.
3. The test statistic is

$$Z = \frac{\bar{x} - 0}{\frac{s}{\sqrt{95}}}$$

and the reference distribution is the standard normal distribution. Observed values of Z far above zero will be considered as evidence against H_0.
4. The sample gives

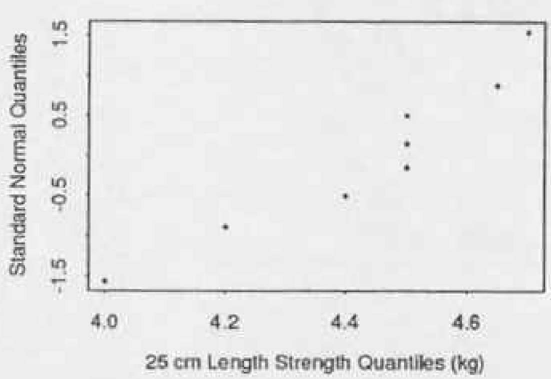
$$z = 30.76.$$

5. The observed level of significance is

$$\begin{aligned} & P(\text{a standard normal random variable} > 30.76) \\ = \; & P(\text{a standard normal random variable} < -30.76) \end{aligned}$$

which is less than .0002, according to Table B-3. There is overwhelming evidence that the mean thread length exceeds nominal.

26. (a) The strengths must be iid and normally distributed. A normal plot is given below.



For such a small data set, this plot is roughly linear (with no outliers), indicating that it may be reasonable to treat the strengths as normal.

(b) Use equation (6-20). For a 95% two-sided interval, the required t is $Q(.975)$ of the t_7 distribution, since (by symmetry) there must be probability .025 in each tail. From Table B-4, $t = Q_7(.975) = 2.365$, and the confidence interval is

$$\begin{aligned} 4.43125 \pm 2.365\left(\frac{.2313586}{\sqrt{8}}\right) &= 4.43125 \pm .1934513 \\ &= [4.238, 4.625] \text{ kg.} \end{aligned}$$

(c) To make a one-sided 95% confidence interval, use the lower endpoint of a 90% two-sided interval. For a 90% two-sided interval, the appropriate t is $t = Q_7(.95) = 1.895$, and the one-sided 95% confidence interval is

$$\begin{aligned} 4.43125 - 1.895\left(\frac{.2313586}{\sqrt{8}}\right) &= 4.43125 - .1550065 \\ &= 4.276 \text{ kg.} \end{aligned}$$

(d) Use equation (6-78). The t is the same as in part (b). The resulting interval is

$$\begin{aligned} 4.43125 \pm 2.365(.2313586)\sqrt{1+\frac{1}{8}} &= 4.43125 \pm .580354 \\ &= [3.851, 5.012] \text{ kg.} \end{aligned}$$

(e) Use equation (6-83). With 99% confidence, $n = 8$, and $p = .95$, the appropriate value for τ_2 is 4.968 (see Table B-7-A). The interval is then

$$4.43125 \pm 4.968(.2313586) = 4.43125 \pm 1.149389 = [3.282, 5.581] \text{ kg.}$$

(f) Using equation (6-93), the confidence in this interval as a prediction interval is

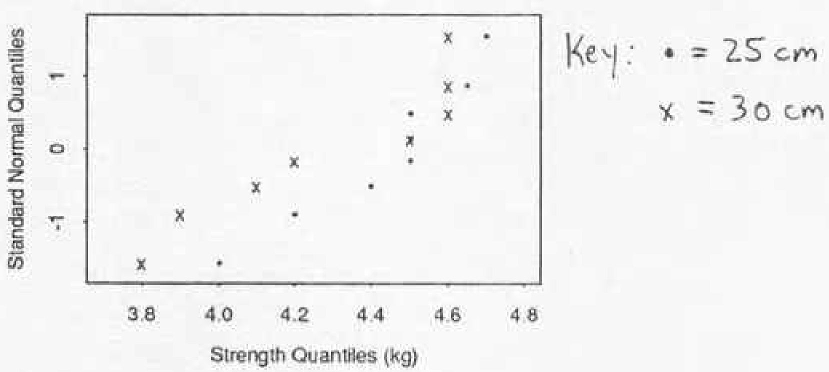
$$\frac{n-1}{n+1} = \frac{7}{9} = .778$$

or 77.8%. Using equation (6-95) with $p = .95$, the confidence in this interval as a tolerance interval for a fraction .95 of all future measured strengths is

$$1 - p^n - n(1-p)p^{n-1} = 1 - (.95)^8 - 8(1 - .95)(.95)^7 = .057$$

or 5.7%. This interval is not very useful as a tolerance interval for a fraction .95 of all future measured strengths. n needs to be increased substantially to make the confidence larger. Alternatively, you could be less ambitious and lower p to increase the confidence.

(g) The strengths within each sample must be iid and normally distributed, and the standard deviation of the two distributions must be the same. One way of checking these assumptions is to normal plot both sets of data on the same axes.



For such small sample sizes, it is difficult to verify the assumptions. The plots are roughly linear with no outliers, indicating that the normal part of the assumption may be reasonable. The slopes are similar, indicating that the common variance assumption may be reasonable.

(h) Label the 25 cm data Sample 1 and the 30 cm data Sample 2.

1. H_0: $\mu_1 - \mu_2 = 0$.
2. H_a: $\mu_1 - \mu_2 > 0$.
3. The test statistic is given by equation (6-36) with $\# = 0$, and the reference distribution is the t_{14} distribution. Observed values of T far above zero will be considered as evidence against H_0.
4. The sample gives

$$t = 1.006.$$

5. The observed level of significance is

$$P(\text{a } t_{14} \text{ random variable} > 1.006)$$

which is greater than .1, according to Table B-4. There is little or no evidence that an increase in specimen length produces a decrease in measured strength.

(i) Use equation (6-35). For a 98% two-sided confidence interval, the appropriate t is $Q_{14}(.99) = 2.624$ (from Table B-4), and so the interval is

$$\begin{aligned} 4.43125 - 4.2875 \pm 2.624(.2857868)\sqrt{\frac{1}{8}+\frac{1}{8}} &= .14375 \pm .3749523 \\ &= [-0.231, .519] \text{ kg.} \end{aligned}$$

27. (a) The researchers have essentially measured the same specimen 821 times. They learned a lot about this particular specimen, but they could not reasonably generalize their results to all such specimens. If the researchers want to apply their results more widely, they need to obtain replications which consist of particle diameter measurements from *different* specimens.

(b) Since this is a large sample, you can use equation (6-9), with $z = 2.33$ for 98% confidence. The two-sided confidence interval is

$$\begin{aligned} .055 \pm 2.33\left(\frac{.028}{\sqrt{821}}\right) &= .055 \pm .002276892 \\ &= [.05272, .057277]\ \mu\text{m}. \end{aligned}$$

(c) 1. $\mathrm{H_0}$: $\mu = .057$ μm.
2. $\mathrm{H_a}$: $\mu \neq .057$ μm.
3. The test statistic is

$$Z = \frac{\bar{x} - .057}{\frac{s}{\sqrt{821}}}$$

and the reference distribution is the standard normal distribution. Observed values of Z far above or below zero will be considered as evidence against $\mathrm{H_0}$.
4. The sample gives

$$z = -2.05.$$

5. The observed level of significance is

$$2P(\text{a standard normal random variable} < -2.05)$$

which is equal to $2(.0202) = .0404$, according to Table B-3. There is strong evidence that this specimen's mean particle diameter is different from the standard.

(d) The test in part (c) showed that there is strong evidence that $\mu \neq .057$, but it did not say anything about how far away μ is from .057. The difference between μ and .057 may be so small that, for all intents and purposes, $\mu = .057$. The confidence interval in (b) is more practical for determining how far μ is from .057.

28. (a) Use equation (6-42) and Table B-5. For a 98% two-sided interval, $U = Q_7(.99) = 18.475$ and $L = Q_7(.01) = 1.239$. The resulting interval for σ^2 is $[.02028078, .3024112]$; taking the square root of each endpoint, the interval for σ is $[.1424, .5499]$ kg.

(b) Use equation (6-42) and Table B-5. For a 95% upper confidence bound, $L = Q_7(.05) = 2.167$. The resulting bound for σ^2 is .3547531; taking the square root, the upper bound for σ is .5956.

(c) Label the 25 cm data as Sample 1 and the 30 cm data as Sample 2.

1. $\mathrm{H_0}$: $\frac{\sigma_1^2}{\sigma_2^2} = 1$.
2. $\mathrm{H_a}$: $\frac{\sigma_1^2}{\sigma_2^2} \neq 1$.
3. The test statistic is given by equation (6-47) with $\# = 1$, and the reference distribution is the $F_{7,7}$ distribution. Small or large observed values of F (relative to 1) will be considered as evidence against $\mathrm{H_0}$.
4. The sample gives

$$f = .4874.$$

5. The observed level of significance is

$$2P(\text{an } F_{7,7} \text{ random variable} < .4874).$$

It is necessary to switch the degrees of freedom, invert the observed f, and change the inequality to find the probability to the left of this small quantile using Tables B-6. (Switching the degrees of freedom has no effect here, since the degrees of freedom are the same.)

$$\begin{aligned} &= 2P(\text{an } F_{7,7} \text{ random variable} > \frac{1}{.4874}) \\ &= 2P(\text{an } F_{7,7} \text{ random variable} > 2.052) \\ &= 2(\text{something between } .1 \text{ and } .25), \end{aligned}$$

so the p-value is between .2 and .5, according to Tables B-6. There is no evidence of a difference in the wire lengths with respect to the variability in their measured tensile strengths.

(d) Use equation (6-47) and Tables B-6. For 98% confidence, $U = Q_{7,7}(.99) = 6.99$ and $L = Q_{7,7}(.01) = \frac{1}{Q_{7,7}(.99)} = \frac{1}{6.99}$. The resulting interval for $\frac{\sigma_1^2}{\sigma_2^2}$ is $[.06972795, 3.406915]$. Taking the square root of each endpoint, the interval for $\frac{\sigma_1}{\sigma_2}$ is $[.2641, 1.8458]$.

29. (a) $Q_4(.99) = 13.277$, from Table B-5.

(b) $Q_4(.025) = .484$, from Table B-5.

(c) $Q_{3,15}(.99) = 5.42$, from Table B-6-D.

(d) $Q_{3,15}(.25) = \frac{1}{Q_{15,3}(.75)} = \frac{1}{2.46} = .4065$, from Table B-6-A.

30. Using equation (6-57), the appropriate z for 95% confidence is 1.96. The resulting interval is

$$\begin{aligned} \frac{19}{50} \pm 1.96\frac{1}{2\sqrt{50}} &= .38 \pm .1385929 \\ &= [.241, .519]. \end{aligned}$$

Using equation (6-59), the appropriate z for 95% confidence is 1.96. The resulting interval is

$$\begin{aligned} .38 \pm 1.96\sqrt{\frac{.38(1-.38)}{50}} &= .38 \pm .1345423 \\ &= [.245, .515]. \end{aligned}$$

31. **(a)** $\bar{x} \pm z(s/\sqrt{n})$ becomes $.02046 \pm 1.96(.00178/\sqrt{50})$ or $.02046 \pm .00049$. [.01997, .02095] is the 95% two-sided confidence interval for the mean diameter of holes drilled by this process.

(b) $\bar{x} - z(s/\sqrt{n})$ = .02046 - (1.645)(.00178/$\sqrt{50}$) = .02046 - .00041 = .02004. Thus, (.02004, + ∞) is a 95% lower confidence interval (bound) for the mean diameter of holes drilled by this process. This lower bound (.02004) is larger than the lower bound (.01997) from part (a).

(c) $\bar{x} \pm z(s/\sqrt{n})$ becomes .02046 $\pm$ (1.645)(.00178/$\sqrt{50}$) or .02046 $\pm .00041$. (.02005, .02087) is the 90% two-sided confidence interval for the mean diameter of holes drilled by this process. This interval is not as wide as the one in (a).

(d) $\bar{x} - z(s/\sqrt{n})$ = .02046 - (1.28)(.00178/$\sqrt{50}$) = .02046 - .00032 = .02014. Thus, (.02014, + ∞) is a 90% lower confidence interval (bound) for the mean diameter of holes drilled by this process. This lower bound exceeds the lower bound found in (b).

(e) The structure of the interval in (a) will produce an interval that includes the true mean diameter in repeated application 95% of the time. The particular interval (.01997, .02095) may or may not include the true mean diameter.

(f) Yes, the p-value will be less than .05 because the 95% two-sided interval does not include μ = .0210.

(g) The p-value will be large because the 90% lower one-sided bound is .02014 which is less (not greater) than μ = .0210. The lower bound for the one-sided interval would need to exceed μ = .0210 to produce a small p-value.

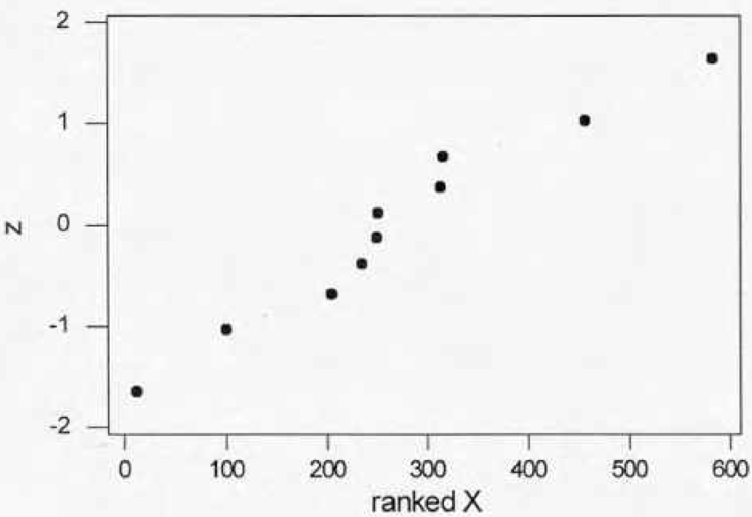
(h) No, this is not correct. We are 95% confident the interval (.01997, .02095) includes the average diameter of all holes drilled by the process under study.

(i)

1. Ho: μ = .0210 ,
2. Ha: $\mu \neq$ .0210,
3. Z = $(\bar{x} - .0210)/s/\sqrt{n}$,
4. (.02046 - .0210)/(.00178/$\sqrt{50}$) = - 2.16,
5. The observed significance level (p-value) is P(Z < -2.16) + P(Z > 2.16) = 2(.0154) = .0308.

(j) The average diameter could be between the specs (.0210 $\pm$ .0003) but half of the holes have diameters larger than the upper spec and half of the holes have diameters less than the lower spec. Thus, the process is poor, even though the average diameter is within specs. Section 4 in Chapter 6 presents inference methods to evaluate distribution spread (variance).

32. **(a)** We assume cycles till failure is normally distributed. The normal probability plot suggests the normal distribution is appropriate (straight-line plot). The normal probability plot of the data is below:



(b) $\bar{x} \pm t(s/\sqrt{n})$ becomes 271.8 $\pm$ 1.833 (163.235/$\sqrt{10}$) or 271.8 $\pm$ 94.618. The interval (177.18, 366.42) is a 90% interval estimate of the average cycles till failure from Heat 1. Units are 100s cycles.

(c) $\bar{x} - t(s/\sqrt{n})$ = 271.8 - (1.383) (163.2/$\sqrt{10}$) = 271.8 - 71.374 = 200.43. (200.43, + ∞) is a lower one-sided 90% confidence interval for the mean fatigue life of specimens from Heat 1. Units are 100s cycles.

(d) [$\sqrt{(n-1)s^2/U}, \sqrt{(n-1)s^2/L}$] becomes [$\sqrt{9(163.2)^2/19.023}, \sqrt{9(163.2)^2/2.7}$] or [112.25, 297.96] is a 95% two-sided confidence interval for σ.

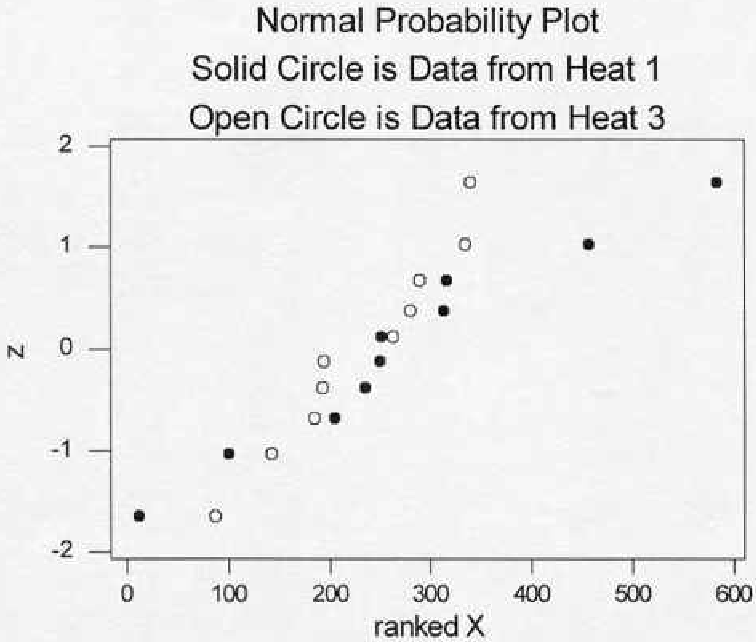
(e) $\bar{x} \pm ts\sqrt{(1+1/n)}$ becomes 271.8 $\pm$ (1.833)(1 + 1/10)$^{1/2}$(163.2) or

271.8 $\pm$ 313.74. The interval [-41.94, 585.54] or [0, 585.54] is a 90% two-sided prediction interval for a single additional fatigue life for a specimen from this heat.

(f) $\bar{x} \pm \tau s$ becomes 271.8 $\pm$ (2.856)(163.2) or 271.8 $\pm$ 466.09. The interval [-194.3, 737.9] or [0, 737.9] is a 95% two-sided tolerance interval for 90% of additional fatigue lives for specimens from this heat. This interval is much wider than the interval in (e).

(g) (n-1)/(n+1) = 9/11 = .82, i.e., 82% confidence associated with (11,548) as a prediction interval for a single additional fatigue life from this heat. Let p = .9. $1 - p^n - n(1-p)p^{n-1} = 1 - .9^{10} - 10(.1)(.9)^9$ = 1 - .3486 - .3874 = .264 or 26.4% confidence associated with (11, 548) as a tolerance interval for 90% of additional fatigue lives.

(h) To make formal inference about $\mu_1 - \mu_3$, we must assume the fatigue lives for both heats are approximately normally distributed and have the same variability. A normal probability plot (a) showed fatigue lives from heat 1 was approximately normal. The following plot is for heat 3.



This plot suggests data from heat3 is normally distributed but does not have the same variance (the slope of the line for data from heat 1) as the data from heat 1.

(i) Assuming $\sigma_1 = \sigma_3$ and data from both heats are normally distributed,
$s_p = \sqrt{s_p^2} = 129.6$. $s_p^2 = 16798.3$.

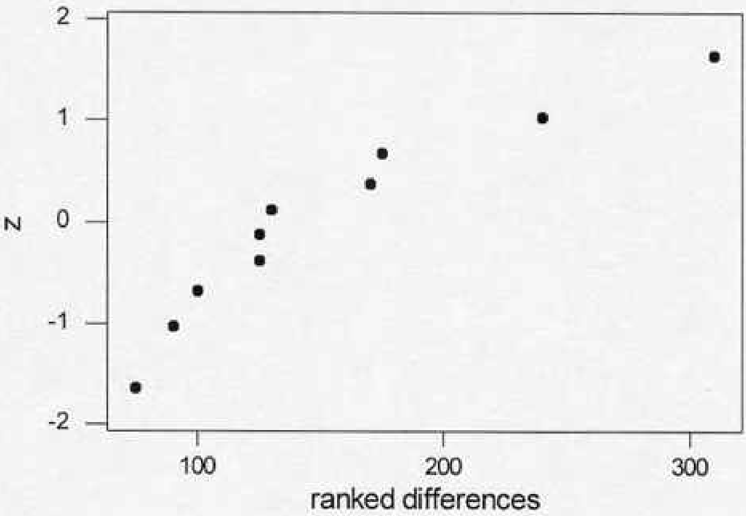
$(\bar{x}_1 - \bar{x}_3) \pm t s_p \sqrt{(1/n_1 + 1/n_3)}$ or
$(271.8 - 230.3) \pm (2.101)(129.6)\sqrt{(2/10)}$

41.5 ± 121.779 or [-80.28, 163.28] is a 95% two-sided confidence interval for $\mu_1 - \mu_3$.

33. **(a)** The methods of formulas (6.35), (6.36) and (6.38) are not appropriate because they assume the two groups of data are independent (among other things). In this example, the two groups of data (dial and air) are not independent. The measurement from dial on sleeve 1 is not independent of the measurement from air on sleeve 1.

(b) The differences (dial bore minus air spindler) for selected sleeves, respectively, must be approximately normal.

Normal Probability Plot for Differences (Dial minus Air)



It seems the differences are not normal. Two of the ten differences are skewed to the right (too large assuming a normal distribution).

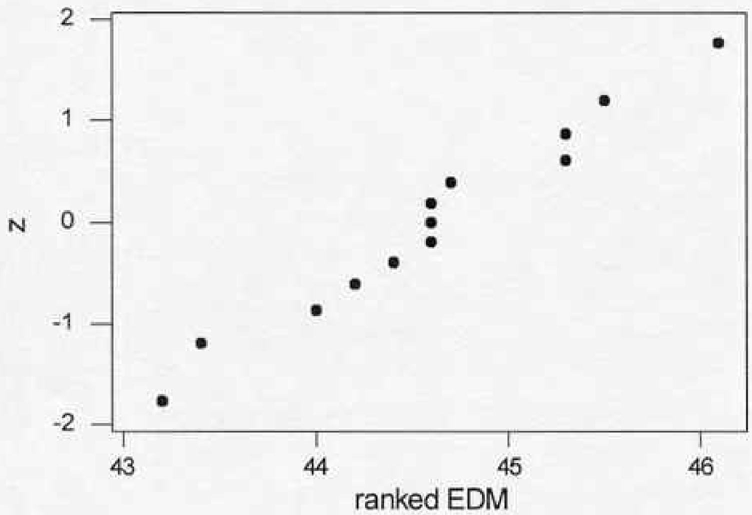
(c) Ho: $\mu_d = 0$ vs. Ha: $\mu_d \neq 0$. Let d = dial minus air. $\bar{d} = 154$ and $s_d = 72.984$. So, $t_9 = (\bar{d} - 0)/(s_d/\sqrt{n}) = (154 - 0)/(72.984/\sqrt{10}) = 6.67$ and the p-value is $2(t_9 > 6.67) < .0005$. Thus, conclude there is an important difference (dial minus air).

(d) $\bar{d} \pm t\,(s_d/\sqrt{n})$ becomes $154 \pm (2.262)(72.984)/\sqrt{10}$ or 154 ± 52.2. The interval (101.8, 206.2) is a 95% interval for the average difference (dial minus air) for any given sleeve.

(e) The interval in (d) does not include zero. Thus, the conclusion to (c) and (d) are consistent.

34. **(a)** The measured angles need to be approximately normally distributed. The measured angles need to be independent.

Normal Probability Plot for EDM Hole Angles



It seems the assumption of a normal distribution for hole angles using EDM is acceptable.

(b) $\bar{x} \pm t(s/\sqrt{n})$ becomes 44.61 ± 3.055(.8169)/$\sqrt{13}$ or 44.61 ± .692. The interval [43.92, 45.3] is the 99% two-sided confidence interval for the mean angle produced by the EDM drilling of this hole.

(c) $\bar{x} + t(s/\sqrt{n})$ = 44.61 + (2.681)(.8169)/$\sqrt{13}$ = 44.61 + .6074 = 45.22. Thus, the interval [- ∞ , 45.22] is a 99% upper one-sided confidence interval for the mean angle produced by the EDM drilling.

(d) $[\sqrt{(n-1)s^2/U}, \sqrt{(n-1)s^2/L}\,]$ becomes $[\sqrt{12(.6674)/23.337}, \sqrt{12(.6674)/4.404}\,]$ or [.5858, 1.3485] is a 95% two-sided confidence interval for the standard deviation of angles produced by the EDM drilling.

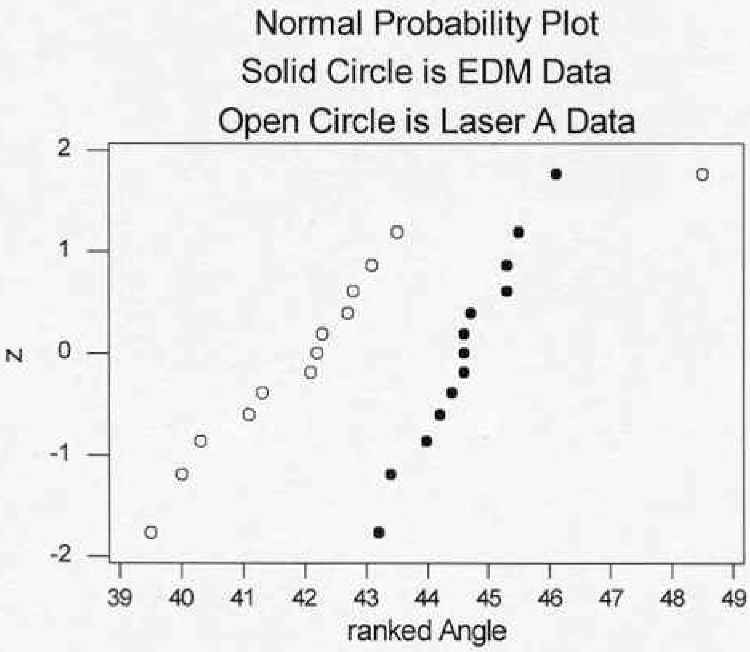
(e) $\bar{x} \pm ts\sqrt{(1+1/n)}$ becomes 44.61 ± (3.055)(.8169)$\sqrt{(1+1/13)}$ or

44.61 ± 2.5898. The interval [42.02, 47.19] is a 99% two-sided prediction interval for the next measured angle produced by EDM drilling.

(f) $\bar{x} \pm \tau_2 s$ becomes 44.61 ± (4.051)(.8169) or 44.61 ± 3.31. Thus, the interval [41.3, 47.92] is a 95% two-sided tolerance interval for 99% of angles produced by the EDM drilling.

(g) Equation 6.93 gives (n-1)/(n+1) = 12/14 = .857 as the confidence that should be associated with this interval as a prediction interval for a single additional measured angle. Equation 6.95 gives
$1 - p^n - n(1-p)p^{n-1}$ as the confidence level that should be associated with this interval when it is used as a tolerance interval for 99% (p = .99) of additional angles. Thus, $1 - (.99)^{13} - 13(.01)(.99)^{12} = 1 - .8775 - .1152 = .01$ or 1% confidence.

(h) Assume Laser (A) holes and EDM holes have angles that are normally distributed. Further, it is assumed angles from Laser A holes have the same variance as EDM holes have.



Except for one data value (Laser A, x = 48.5), both EDM and Laser A methods produce angles that are normally distributed (straight line normal probability plot) and have the same variance (parallel normal probability plots).

(i) Ho: $\mu_L - \mu_{EDM} = 0$ vs. Ha: $\mu_L - \mu_{EDM} \neq 0$.
$s_p^2 = [\ (12)(5.0442) + (12)(.6674)\ ]/24 = 2.8558$. $s_p = (2.8558)^{1/2} = 1.69$.
$t = [\ ((\bar{x}_L - \bar{x}_{EDM}) - 0]/s_p\sqrt{(2/n)} = (42.2615 - 44.6077)/1.69\sqrt{(2/13)} = -3.54$
p-value=2 $P[t_{24} < -3.54] = .002$. Thus, there is an important difference in the average angle for Laser compared to EDM drilling.

(j) $(\bar{x}_L - \bar{x}_{EDM}) \pm ts_p\sqrt{2/n} = (42.2615 - 44.6077) \pm (2.064)(1.69)\sqrt{(2/13)}$ or -2.3462 ± 1.3681. Thus [-3.7143, -.9781] is a 95% confidence interval for $\mu_L - \mu_{EDM}$.

(k) [$\sqrt{s_L^2/(Us_{EDM}^2)}, \sqrt{s_L^2/(Ls_{EDM}^2)}$] becomes [$\sqrt{5.0442/(2.69)(.6674)}, \sqrt{5.0442/(1/2.69)(.6674)}$] or [1.6762, 4.509] is a 95% confidence interval for $\sigma_L^2/\sigma_{EDM}^2$.

(l) The two angles (Laser A and B) on the same part are not independent.

(m) Let d = $X_{LA} - X_{LB}$. $\bar{d} \pm t\,(s_d/\sqrt{n})$ becomes -2.3384 ± (1.782)(2.7054)/$\sqrt{13}$ or -2.3384 ± 1.3371 gives [-3.68, -1.00] as the 90% two-sided confidence interval for Laser A minus Laser B average hole angle.

(n) Let d = $X_{LA} - X_{LB}$. Ho: $\mu_d = 0$ vs. Ha: $\mu_d \neq 0$.
So, $t_{12} = (\bar{d} - 0)/(s_d/\sqrt{n}) = (-2.3384 - 0)/(2.7054)/\sqrt{13}$) = -3.116.
p-value = 2P[t_{12} < -3.116] = .009. There is very strong evidence a difference exists between Laser A angles and Laser B angles.

(o) Since the 90% interval in (m) did not include 0, it is clear there is strong evidence the mean difference in hole angle is not 0. Thus, a small p-value was expected from part (n).

35. **(a)** The tilt table ratio must have an approximate normal distribution for Van1.

(b) $\bar{x}_1 \pm t(s/\sqrt{n})$ becomes 1.093 ± (3.182)(.0024495)(1/$\sqrt{4}$) or 1.093 ± .003897. Thus, the interval [1.0891, 1.0968] is a 95% two-sided confidence interval for the mean measured tilt table ratio for Van 1.

(c) $\bar{x}_1 - t(s/\sqrt{n})$ = 1.093 - (2.353)(.0024495)(1/$\sqrt{4}$) = 1.093 - .00288 or [1.09, + ∞] is a lower one-sided 95% confidence interval for the mean measured tilt table ratio for Van 1.

(d) [$\sqrt{(n-1)s^2/U}$,+∞] becomes [$\sqrt{(3)(.000006)/7.815}$,+∞] or [.00152, + ∞] is a 95% lower one-sided confidence interval for the standard deviation of tilt table ratios for Van 1.

(e) $\bar{x}_1 \pm ts\sqrt{(1+1/n)}$ becomes 1.093 ± (3.182)(.0024495)$\sqrt{1.25}$ or

1.093 ± .00871. The interval [1.084, 1.102] is a 95% two-sided prediction interval for a single additional measured tilt table ratio for Van 1 under conditions such as those experienced during testing.

(f) $\bar{x}_1 \pm \tau_2 s$ becomes 1.093 ± (11.118)(.0024495) or 1.093 ± .0272. The interval [1.066, 1.120] is a 99% two-sided tolerance interval for 95% of additional measured tilt table ratios for Van 1.

(g) (n-1)/(n+1) = 3/4 = 75% confidence the interval (1.09, 1.096) will include the next measured tilt table ratio for Van 1.
$1 - p^n - n(1-p)p^{n-1} = 1 - (.95)^4 - 4(.05)(.95)^3 = 1 - .8145 - .1715 = .014$ or 1.4% confidence the interval (1.09, 1.096) will include 95% of tilt table ratios for Van 1.

(h) Must assume similar variability of tilt-table ratios and normal distribution of tilt-table ratios for both vans.

(i) Ho: $\mu_1 - \mu_2 = 0$ vs. Ha: : $\mu_1 - \mu_2 \neq 0$.
s_p^2 = [(3)(.000006) + (3)(.0000109)]/6 = .00000845. $s_p = (.00000845)^{1/2}$ = .0029.

$t = [\,(\bar{x}_1 - \bar{x}_2) - 0]/s_p\sqrt{(2/n)} = (1.093 - .9663)/.0029\sqrt{(2/4)} = 61.787$

p-value=2 P[t_3 > 61.787] = 0. Thus, there is an important difference in the average angle for Laser compared to EDM drilling.

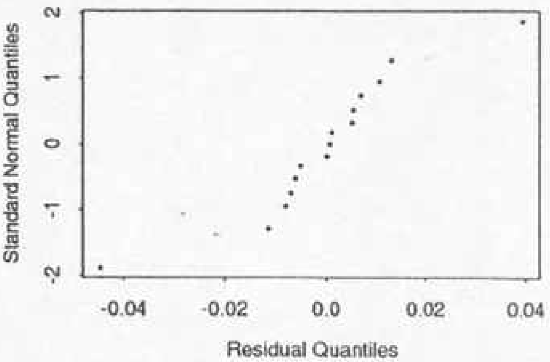
(j) $(\bar{x}_1 - \bar{x}_2) \pm ts_p\sqrt{2/n} = (1.093 - .9663) \pm (1.943)(.0029)\sqrt{(2/4)}$. The interval becomes .1267 ± .003984. Thus with 90% confidence [.1227, .1307] includes $\mu_1 - \mu_2$.

(k) [$\sqrt{s_1^2/(Us_2^2)}, \sqrt{s_1^2/(Ls_2^2)}$] becomes
[$\sqrt{(.000006)/[(9.28)(.0000109)]}, \sqrt{[(.000006)(9.28)]/(.0000109)}$] or
[.2436, 5.1083] is a 90% confidence interval for σ_1/σ_2.

Chapter 7: Inference for Unstructured Multisample Studies

1. (a) See equation (7-3). The necessary computations are given in the table below.

Pressure	y_{ij}	$\hat{y}_{ij} = \bar{y}_i$	e_{ij}
2000	2.486	2.4790	0.0070
2000	2.479	2.4790	0.0000
2000	2.472	2.4790	-0.0070
4000	2.558	2.5693	-0.0113
4000	2.570	2.5693	0.0007
4000	2.580	2.5693	0.0107
6000	2.646	2.6520	-0.0060
6000	2.657	2.6520	0.0050
6000	2.653	2.6520	0.0010
8000	2.724	2.7687	-0.0447
8000	2.774	2.7687	0.0053
8000	2.808	2.7687	0.0393
10000	2.861	2.8660	-0.0050
10000	2.879	2.8660	0.0130
10000	2.858	2.8660	-0.0080

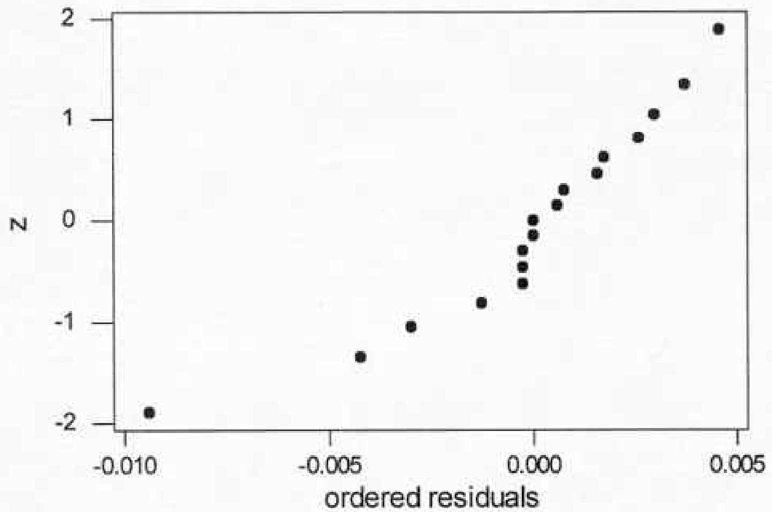


The plot reveals 2 outliers. The assumptions of the one-way normal model appear to be unreasonable for these data. Both of the outliers come from the 8000 psi condition. This is an indication that the common σ part of the one-way normal model assumptions is not reasonable. There seems to be a lot more spread in the 8000 psi sample than in the other samples.

(b) Using equation (7-7), $s_p = .02057$ g/cc, with $n - r = 15 - 5 = 10$ degrees of freedom associated with it. This measures the magnitude of baseline variation within any of the 5 conditions, assuming it is the same for all 5 conditions.

For the confidence interval, use equation (7-10) and Table B-5. For a 90% two-sided interval, $U = Q_{10}(.95) = 18.307$ and $L = Q_{10}(.05) = 3.940$. The resulting interval for σ^2 is $[.000231132, .001073942]$; taking the square root of each endpoint, the interval for σ is $[.01520, .03277]$ g/cc.

2. (a)



The plot reveals one test from Van 3 (y = 1.01) appears to deviate from a normal distribution assumption (residual = - .0094).

(b) Same as in (a), (standardized residual = -2.9148).

(c) s_{pooled} = .0036 measures the assumed common standard deviation of tilt angle for repeated tests on any of the four selected vans.
The interval defined as: [$s_p\sqrt{(13)/5.009}$, $s_p\sqrt{(13)/24.736}$] becomes (.0026, .0058) and is a 95% two-sided confidence interval for σ based on s_p.

Section 2

1. (a) Use equation (7-14). Δ is the same for all five intervals because all five sample sizes are the same. For 95% confidence, the appropriate t is $t = Q_{10}(.975) = 2.228$, from Table B-4. The resulting Δ is

$$2.228\frac{.02057}{\sqrt{3}} = .02646 \text{ g/cc.}$$

Using equation (7-26), the minimum overall (simultaneous) confidence is

$$1 - (.05 + .05 + .05 + .05 + .05) = .75$$

or 75%.

(b) Use equation (7-15). Δ is the same for all 10 intervals because all five sample sizes are the same. t is the same as in part (c). The resulting Δ is

$$2.228(.02057)\sqrt{\frac{1}{3}+\frac{1}{3}} = .03742 \text{ g/cc.}$$

(c) The estimate of μ_{6000} is 2.652 = $\bar{y}_{6000}$, the estimate of μ_{4000} is 2.5693 = $\bar{y}_{4000}$ and the estimate of μ_{2000} is $\bar{y}_{2000}$ = 2.479. The estimated L is 2.652 - 2(2.5693) + 2.479 = -.0076. The 95% confidence interval for L is:

-.0076 $\pm t_{10}$ (.02057)$\sqrt{(1/3+4/3+1/3)}$ or -.0076 $\pm$ .0648. The interval (-.0724, .0572) indicates mean density is a linear function of pressure from 2000 to 6000 psi because the interval for L includes 0.

2. **(a)** The 99% confidence interval for each of the Vans is of the form $\bar{x} \pm t_{13} s_p / \sqrt{4}$, where the $\bar{x}$ is the sample average for the selected van.

Van 1: 1.093 $\pm$ 3.012 (.0036/2) or 1.093 $\pm$ .00542 becomes (1.0875, 1.0984).

Van 2: .96625 $\pm$ (3.012)(.0036/2) or .96625 $\pm$ .00542 becomes (.9608, .9716).

Van 3: 1.0194 $\pm$ (3.012)(.0036/$\sqrt{5}$) or 1.0194 $\pm$ .004849 becomes (1.0145, 1.0242).

Van 4: 1.00225 $\pm$ .00542 becomes (.9968, 1.0076).

The Bonferroni Inequality guarantees $\gamma \geq 1-[.01+.01+.01+.01]$, i.e., at least 96% joint confidence.

(b) Let $n_1 = n_2 = 4$. $\Delta = t_{13}\, s_p\, (2/4)^{1/2} = 3.012(.0036)(1/2)^{1/2} = .0076$.
Let $n_1 = 4$, $n_2 = 5$, $\Delta = t_{13}\, s_p\, (1/4 + 1/5)^{1/2} = 3.012(.45)^{1/2}(.0036) = .0072$.

(c) The estimate of $1/2(\mu_1 + \mu_2) - 1/2(\mu_3 + \mu_4)$ is 1/2(1.093 + .96625) - 1/2(1.0194 + 1.00225) = .0188. The 99% two-sided confidence limit for $1/2(\mu_1 + \mu_2) - 1/2(\mu_3 + \mu_4)$ is:

$.0188 \pm t_{13}s_p(1/2)(1/4 + 1/4 + 1/5 + 1/4)^{1/2}$ or
.0188 $\pm$ .005284.

Thus, the 99% two-sided confidence limits are [.013516, .02408].

3. Before the data are collected, the probability is .05 that an individual 95% confidence interval will be in error—that it will not contain the quantity that it is supposed to contain. If several of these types of individual intervals are made, then the probability that *at least* one of the intervals is in error is greater than .05. (If each interval has a .05 chance of failing, then the overall chance of at least one failure is greater than .05.) When making several intervals, most people would like the overall or simultaneous error probability to be small. In order make sure, for example, that the overall error probability is .05, the error probability associated with the individual intervals must be made smaller than .05. This is equivalent to increasing the individual confidences (above 95%), which makes the intervals wider.

Section 3 **(1) (a)** Use equation 7-28. $r = 5$ and $\nu = n\text{-}r = 10$, $k_2{}^* = 3.10$. The resulting Δ becomes

$3.1(.02057)/\sqrt{3} = .03682$g/cc.
This $\Delta = .03682$ is larger than the $\Delta = .02646$ in Section 2, problem 1(a).

(b) Use equation 7-36. For 95% confidence, with Number of Means to be Compared = 5 and $\nu = n - r = 10$, $q^* = 4.65$. The resulting Δ is

$4.65(1/\sqrt{2})(.02057)\sqrt{(1/3+1/3)} = .05522$ g/cc.

This Δ is larger than the $\Delta = .03742$ in Section 2 problem 1b.

(2) **(a)** Use equation 7-28. $r = 4$ and $\nu = n\text{-}r = 13$, $k_2{}^* = 2.88$. Simultaneous 95% PR confidence limits for each van are:
Van1: $1.093 \pm (2.88)(.0036)/(4)^{1/2}$ or $1.093 \pm .0052$ gives (1.0878, 1.0982).

Van 2: $.96625 \pm .0052$ or (.9610, .9714).

Van 3: $1.0194 \pm (2.88)(.0036)/(5)^{1/2}$ becomes (1.0147, 1.024)

Van 4: $1.00225 \pm .0052$ or (.9970, 1.0074).

(b) For $n_1 = n_2 = 4$, $q^* = 5.4$, df = 13, r = 4,
$\Delta = [(5.4)/\sqrt{2}](s_p)(1/2)^{1/2} = [(5.4)/\sqrt{2}](.0036)(1/2)^{1/2} = .00972$

For $n_1 = 4$ and $n_2 = 5$, $q^* = 5.4$, df = 13, r = 4,
$\Delta = [(5.4)/\sqrt{2}](s_p)(.45)^{1/2} = [(5.4)/\sqrt{2}](.0036)(1/2)^{1/2} = .0092$.

The Δs here are larger. The confidence level for these Δs are applied to the complete set of confidence intervals. The Δs in Section 7-2, exercise 2(b) are for just a single interval.

Section 4 **(1)** **(a)** A small p-value is expected. The smallest difference among any two pressure means is 2.5693 - 2.479 = .09. This value of .09 exceeds the Δ in both problems 1(b) in sections 2 and 3.

(b) Using the general form given in Table 7-12, the calculations yield the following table.

Source	SS	df	MS	F
Treatments	.285135	4	.071284	168.47
Error	.004231	10	.000423	
Total	.289366	14		

Using equation (7-53),

$$R^2 = \frac{.285135}{.289366} = .985.$$

The p-value for an F test of the null hypothesis given in part (1) is

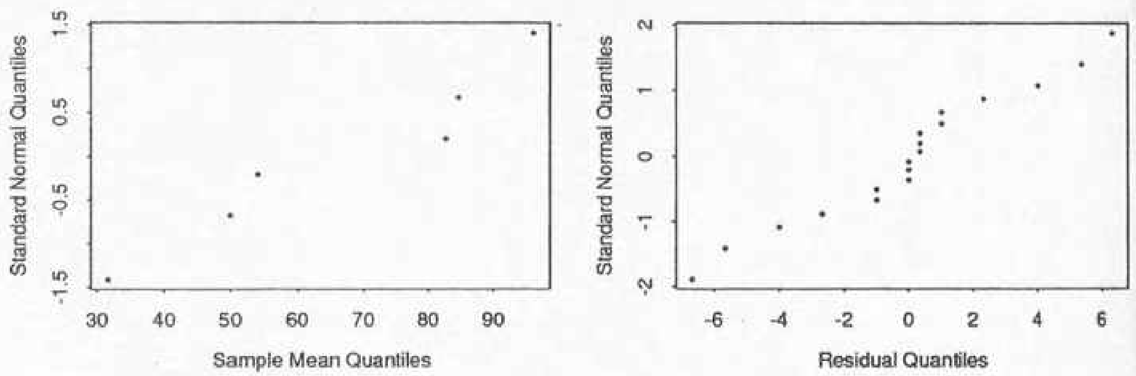
$$P(\text{an } F_{4,10} \text{ random variable} > 168.47).$$

Using Tables B-6, this is less than .001.

(2) **(a)** A small p-value is expected. In part (b) of Section 7-3, the difference between any two Vans exceeds the Δs calculated there (.00972 or .0092).

(b) SSTr = .034134, MSTr = .011378, df = 3; SSE = .000175, MSE = .000013, df = 13; SSTot = .034308, df = 16; f =846.67 on 3,13 df; p-value < .001. ; R^2=.995

3. (a) To check that the μ_i's are normal, make a normal plot of the $\bar{y}_i$'s. To check that the ϵ_{ij}'s are normal, make a normal plot of the residuals. (Normal plotting each sample individually will not be very helpful because the sample sizes are so small.)



Both plots are roughly linear, giving no evidence that the one-way random effects model assumptions are unreasonable.

(b) Using the general form given in Table 7-12, the calculations yield the following table.

Source	SS	df	MS	EMS	F
Rails	9310.5	5	1862.1	$\sigma^2 + 3\sigma_\tau^2$	115.18
Error	194.0	12	16.2	σ^2	
Total	9504.5	17			

An estimate of σ^2 is $MSE = s_p^2 = 16.2$, and so an estimate of σ is $\sqrt{16.2} = s_p = 4.025$ μsec (see equation (7-58)). Using equation (7-62), an estimate of σ_τ^2 is

$$\frac{1}{3}(1862.1 - 16.2) = 615.3,$$

so an estimate of σ_τ is $\sqrt{615.3} = 24.805$ μsec. The estimate of σ measures variation in the response from repeated measurements of the same rail; the estimate of σ_τ measures the variation in the response from differences among rails. It seems that most of the variation comes from differences among rails.

(c) Use equation (7-63). For a two-sided 90% confidence interval, $U = Q_{5,12}(.95) = 3.11$ and

$$L = Q_{5,12}(.05) = \frac{1}{Q_{12,5}(.95)} = \frac{1}{4.68} = .2136752$$

using Table B-6-C. The resulting interval for σ_τ^2/σ^2 is

$$\begin{aligned} &\left[\frac{1}{3}\left(\frac{1862.1}{(3.11)(16.2)} - 1\right), \frac{1}{3}\left(\frac{1862.1}{(.2136752)(16.2)} - 1\right)\right] \\ = \;& [11.98654, 178.98]. \end{aligned}$$

Taking the square roots of the endpoints, the interval for σ_τ/σ is $[3.46, 13.38]$. σ_τ/σ is a comparison of the size of variation among rails to the size of within-rail variation. The interval implies that, with 90% confidence, the variation among rails in travel time is between 3.5 to 13.4 times larger than the within-rail variation in travel time.

4. (a) Unstructured multisample data could also be thought of as data from one factor with r levels. In many situations, the specific levels of the factor included in the study are the levels of interest. For example, in comparing three drugs, the factor might be called "Treatment". It might have 4 levels: Drug 1, Drug 2, Drug 3, and Control. The experimenter is interested in comparing the specific drugs used in the study to each other and to the control.

Sometimes the specific levels of the factor are not of interest in and of themselves, but only because they may represent (perhaps they are a random sample of) many different possible levels that could have been used in the study. A random effects analysis is appropriate in this situation. For an example, see part (b).

(b) If there are many technicians, and 5 of these were randomly chosen to be in the study, then interest is in the variation among all technicians, not just the 5 chosen for the study. In this case, a random effects model is appropriate. If these are the only 5 technicians that will ever use the gage, than a fixed effects model should be used (the one-way normal model, in which the μ_i's are fixed).

(c) The standard deviation associated with repeat diameter measurements for a given technician is σ. The standard deviation of long-run mean measurements for various technicians (the μ_i's) is, by definition, σ_τ. An estimate of σ^2 is $MSE = .0000024$ (see equation (7-58)), and so an estimate of σ is $\sqrt{.0000024} = .001549193$ in. An estimate of σ_τ^2 is

$$\frac{1}{m}(MSTr - MSE) = \frac{1}{2}(.0000034 - .0000024) = .0000005,$$

so an estimate of σ_τ is

$$\sqrt{.0000005} = .0007071068 \text{ in.}$$

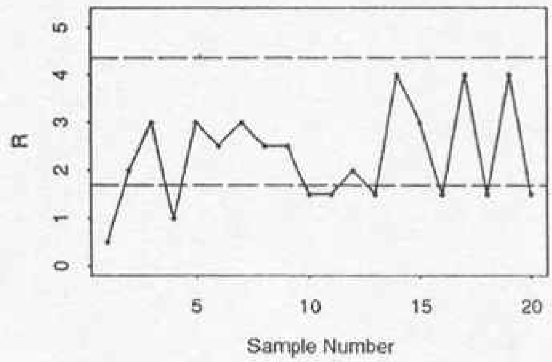
Section 5

1. (a) For the R chart, use the limits given in equation (7- 86). In Table B-2, for $m = 3$, D_1 is not given, so there is no lower control limit. $D_2 = 4.358$ and $d_2 = 1.693$, so

$$\text{Center Line}_R = d_2\sigma = 1.693(1.0) = 1.693 \text{ oz}$$

and

$$UCL_R = 4.358(1.0) = 4.358 \text{ oz.}$$



There is little evidence that the σ associated with the process was above 1.0 oz.

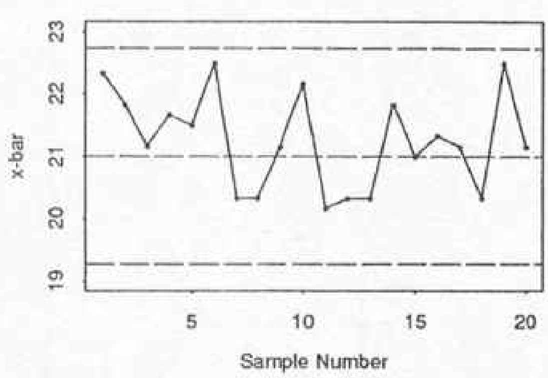
For the $\bar{x}$ chart, use μ as a center line, and use the limits given in equation (7-70).

$$\text{Center Line}_{\bar{x}} = 21.0 \text{ oz},$$

$$LCL_{\bar{x}} = 21.0 - 3\frac{1.0}{\sqrt{3}} = 19.26795 \text{ oz},$$

and

$$UCL_{\bar{x}} = 21.0 + 3\frac{1.0}{\sqrt{3}} = 22.73205 \text{ oz.}$$



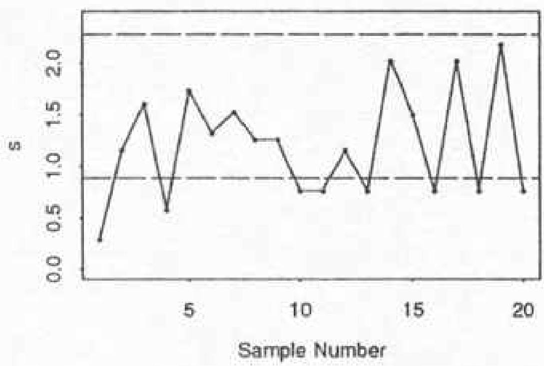
There is no evidence that the process mean μ was not equal to 21.0 oz.

(b) Use $c_4\sigma$ as a center line, and use the limits given in equation (7-90). From Table B-2 with $m = 3$, $c_4 = .8862$, B_5 is not given (so there will be no lower control limit), and $B_6 = 2.276$.

$$\text{Center Line}_s = .8862(1.0) = .8862 \text{ oz},$$

and

$$UCL_s = 2.276(1.0) = 2.276 \text{ oz}.$$



This chart is very similar in appearance to the R chart.

(c) Use equation (7-74) for the estimate based on $\bar{R}$.

$$\frac{\bar{R}}{d_2} = \frac{2.3}{1.693} = 1.358535 \text{ oz}.$$

Use equation (7-78) for the estimate based on $\bar{s}$.

$$\frac{\bar{s}}{c_4} = \frac{1.209262}{.8862} = 1.364548 \text{ oz}.$$

Using equation (7-7),

$$s_p^2 = 1.729167,$$

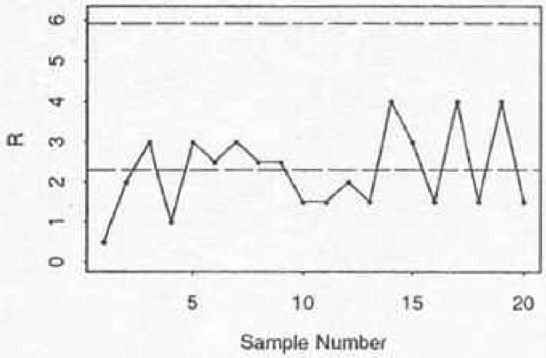
so $s_p = \sqrt{1.729167} = 1.314978$ oz. The two estimates above are slightly larger than s_p.

(d) For the R chart, use the limits given in equation (7-88). In Table B-2, for $m = 3$, D_3 is not given, so there is no lower control limit. $D_4 = 2.574$, so

$$\text{Center Line}_R = \bar{R} = 2.3 \text{ oz}$$

and

$$UCL_R = 2.574(2.3) = 5.9202 \text{ oz}.$$



There is little evidence that the σ associated with the process was unstable. The short-term variability seems to have been in control.

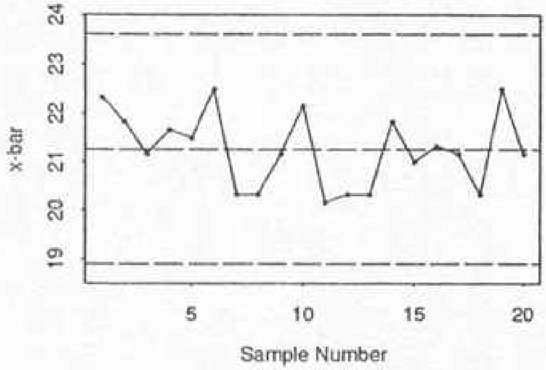
For the $\bar{x}$ chart, use $\bar{\bar{x}}$ as a center line, and use the limits given in equation (7-70), substituting $\bar{\bar{x}}$ for μ and $\frac{\bar{R}}{d_2}$ for σ.

$$\text{Center Line}_{\bar{x}} = 21.25833 \text{ oz},$$

$$LCL_{\bar{x}} = 21.25833 - 3\frac{1.358535}{\sqrt{3}} = 18.90528 \text{ oz},$$

and

$$UCL_{\bar{x}} = 21.25833 + 3\frac{1.358535}{\sqrt{3}} = 23.61138 \text{ oz}.$$



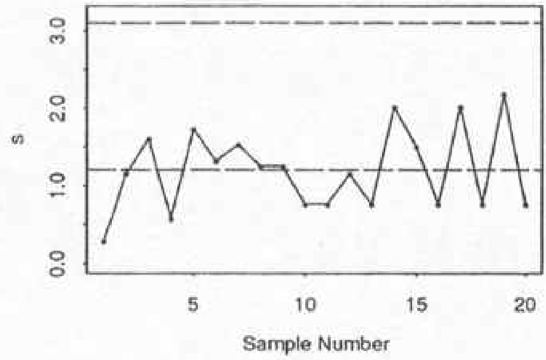
There is no evidence that the process mean μ was unstable. The process mean seems to have been in control.

(e) For the s chart, use the limits given in equation (7-92). In Table B-2, for $m = 3$, B_3 is not given, so there is no lower control limit. $B_4 = 2.568$, so

$$\text{Center Line}_s = \bar{s} = 1.209262 \text{ oz}$$

and

$$UCL_s = 2.568(1.209262) = 3.105385 \text{ oz}.$$



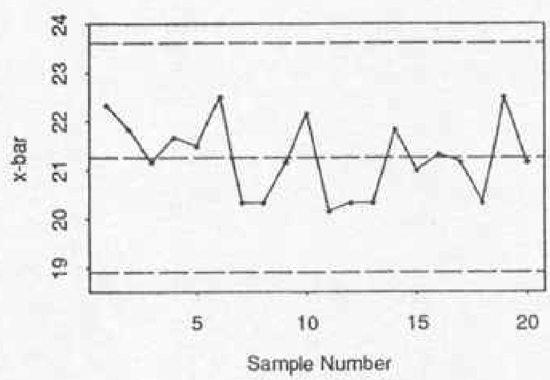
For the $\bar{x}$ chart, use $\bar{\bar{x}}$ as a center line, and use the limits given in equation (7-70), substituting $\bar{\bar{x}}$ for μ and $\frac{\bar{s}}{c_4}$ for σ.

$$\text{Center Line}_{\bar{x}} = 21.25833 \text{ oz},$$

$$LCL_{\bar{x}} = 21.25833 - 3\frac{1.364548}{\sqrt{3}} = 18.89487 \text{ oz},$$

and

$$UCL_{\bar{x}} = 21.25833 + 3\frac{1.364548}{\sqrt{3}} = 23.6218 \text{ oz}.$$



These charts are very similar to the ones made in part (d).

2. (a) Use equation (7-74) for the estimate based on $\bar{R}$. From Table B-2, for $m = 5$, $d_2 = 2.326$.

$$\frac{\bar{R}}{d_2} = \frac{4.052632}{2.326} = 1.742318 \times .001 \text{ in.}$$

Use equation (7-78) for the estimate based on $\bar{s}$. From Table B-2, for $m = 5$, $c_4 = .9400$.

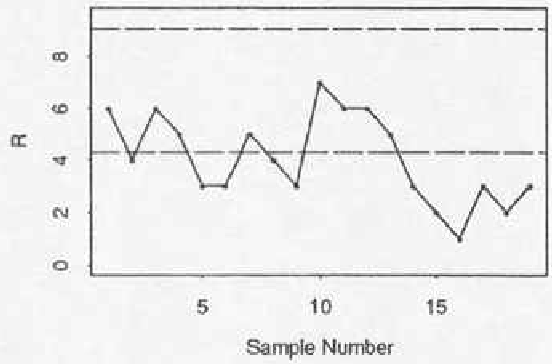
$$\frac{\bar{s}}{c_4} = \frac{1.732632}{.9400} = 1.843226 \times .001 \text{ in.}$$

(b) For the R chart, use the limits given in equation (7-86), with $\frac{\bar{s}}{c_4}$ substituted for σ. The center line will be at $d_2\frac{\bar{s}}{c_4}$. In Table B-2, for $m = 5$, D_1 is not given, so there is no lower control limit. $D_2 = 4.918$, so

$$\text{Center Line}_R = 2.326(1.843226) = 4.287344 \times .001 \text{ in.}$$

and

$$UCL_R = 4.918(1.843226) = 9.064985 \times .001 \text{ in.}$$

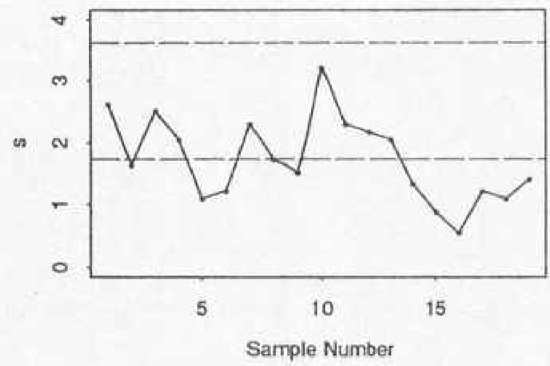


For the s chart, use the limits given in equation (7-92). The center line will be at $\bar{s}$. In Table B-2, for $m = 5$, B_3 is not given, so there is no lower control limit. $B_4 = 2.089$, so

$$\text{Center Line}_s = 1.732632 \times .001 \text{ in.}$$

and

$$UCL_s = 2.089(1.732632) = 3.619468 \times .001 \text{ in.}$$



Neither chart indicates that the short-term variability of the process (as measured by σ) was unstable.

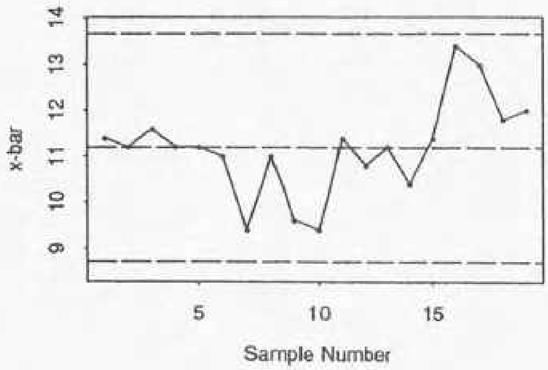
(c) Use $\bar{\bar{x}}$ as a center line, and use the limits given in equation (7-70), substituting $\bar{\bar{x}}$ for μ and $\frac{\bar{s}}{c_4}$ for σ.

$$\text{Center Line}_{\bar{x}} = 11.17895 \times .001 \text{ in. above nominal}$$

$$LCL_{\bar{x}} = 11.17895 - 3\frac{1.843226}{\sqrt{5}} = 8.706 \times .001 \text{ in. above nominal,}$$

and

$$UCL_{\bar{x}} = 11.17895 + 3\frac{1.843226}{\sqrt{5}} = 13.65189 \times .001 \text{ in. above nominal.}$$



The $\bar{x}$ from sample 16 comes close to the upper control limit, but overall the process mean seems to have been stable over the time period.

(d) The $\bar{x}$'s from samples 9 and 16 seem to have "jumped" from the previous $\bar{x}$. The coil change may be causing this jump, but it could also be explained by common cause variation. It may be something worth investigating.

(e) Assuming that the mean could be adjusted (down), you need to look at one of the estimates of σ to answer this question about individual thread lengths. (You should not use control limits to answer this question!) If μ could be made equal to zero, then (assuming normally distributed thread lengths), almost all of the thread lengths would fall in the interval $\pm 3\sigma$. Using the estimate of σ based on $\bar{s}$ from part (a), this can be approximated by $3(1.843226) = 5.53\times$ .001 in. It does seem that the equipment is capable of producing thread lengths within .01 in. of nominal.

If the equipment were not capable of meeting the given requirements, the company could invest in better equipment. This would "permanently" solve the problem, but it might not be feasible from a financial standpoint. A second option is to inspect the bolts and remove the ones that are not within .01 in. of nominal. This might be cheaper than investing in new equipment, but it will do nothing to improve the quality of the process in the long run. A third option is to study the process (through experimentation) to see if there might be some way of reducing the variability without making a large capital investment.

3. Control charting is used to monitor a process, and detect changes (lack of stability) in a process. The focus is on detecting changes in a meaningful parameter such as μ, σ, p, or λ. Points that plot out of control are a signal that the process is not stable at the standard parameter value (for a standards given chart) or was not stable at any parameter value (for a retrospective chart). The overall goal is to reduce process variability by identifying assignable causes and taking action to eliminate them. Reducing variability increases the quality of the process output.

4. Shewhart control charts do not physically control a process. They only monitor the process, trying to detect process instability. There is an entirely different field dedicated to "engineering control"; this field uses feedback techniques that manipulate process variables to control some response. Shewhart control charts simply monitor a response, and should not be used to make "real time" adjustments.

5. Out-of-control points should be investigated. If the causes of such points can be determined and eliminated, this will reduce long term variation from the process. There must be an active effort among those involved with the process to improve the quality; otherwise, control charts will do nothing to improve the process.

6. Control limits for an $\bar{x}$ chart are set so that, under the assumption that the process is stable, it would be very unusual for an $\bar{x}$ to plot outside the control limits. The chart recognizes that there will be *some* variation in the $\bar{x}$'s even if the process is in control, and prevents overadjustment by allowing the $\bar{x}$'s to vary "randomly" within the control limits. If the process mean or standard deviation changes, $\bar{x}$'s will be more likely to plot outside of the control limits, and sooner or later the alarm will sound. This provides an opportunity to investigate the cause of the change, and hopefully take steps to prevent it from happening again. In the long run, such troubleshooting may improve the process by making it less variable.

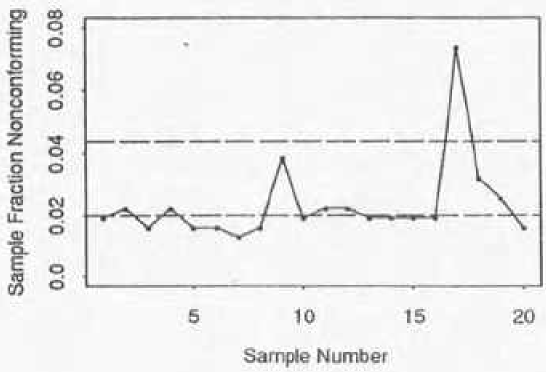
Section 6 1. (a) Use equations (7-95) and (7-96).

$$\text{Center Line}_{\hat{p}_i} = .02,$$

$$LCL_{\hat{p}_i} = .02 - 3\sqrt{\frac{.02(1-.02)}{312}} = -.003777818,$$

which is negative, so there is no lower control limit.

$$UCL_{\hat{p}_i} = .02 + 3\sqrt{\frac{.02(1-.02)}{312}} = .04377782.$$



The chart shows evidence (from sample 17) that the process fraction defective was not stable at $p = .02$ over the entire period.

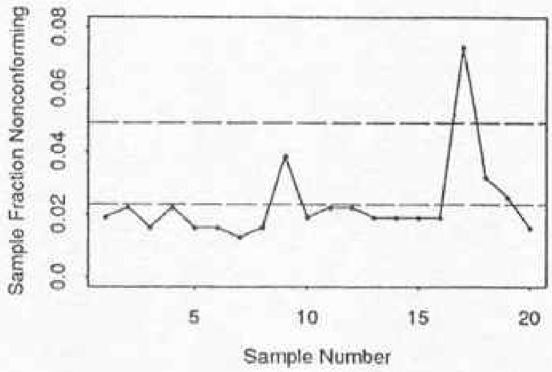
(b) Use equations (7-98) and (7-99). $\hat{p} = \frac{146}{6240} = .02339744$.

$$\text{Center Line}_{\hat{p}_i} = .02339744,$$

$$LCL_{\hat{p}_i} = .02339744 - 3\sqrt{\frac{.02339744(1-.02339744)}{312}} = -0.002276179,$$

which is negative, so there is no lower control limit.

$$UCL_{\hat{p}_i} = .02339744 + 3\sqrt{\frac{.02339744(1-.02339744)}{312}} = .04907105$$



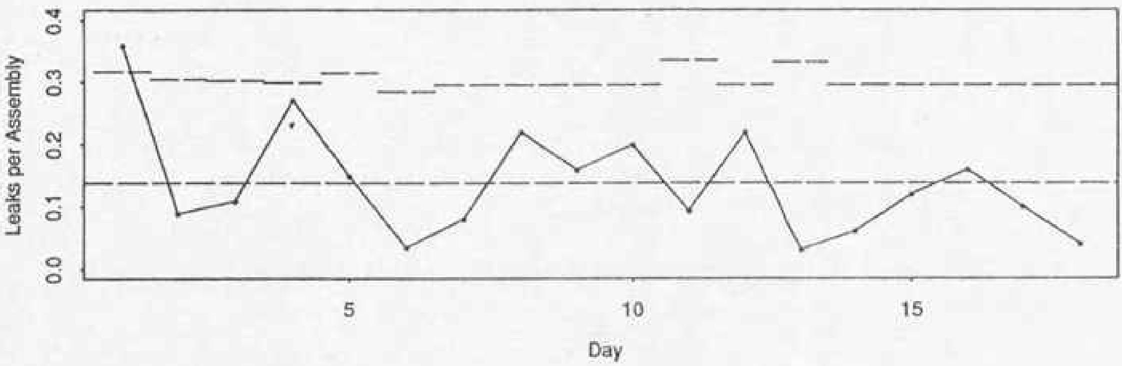
The chart shows evidence (from sample 17) that the process fraction defective was not stable at any value over the entire period.

2. Use equations (7-105) and (7-106). $\hat{u} = \frac{116}{841} = .137931$.

$$\text{Center Line}_{\hat{u}_i} = .137931.$$

The control limits depend on the number of units tested k_i. The following table gives upper control limits for each of the k_i's in the data set. (The lower control limits are all negative, so there are no lower control limits for any of these k_i's.)

k_i	$UCL_{\hat{u}_i}$
32	.3348907
33	.3318835
39	.3163413
40	.3140971
45	.304022
46	.3022067
48	.2987479
50	.2954988
58	.284229



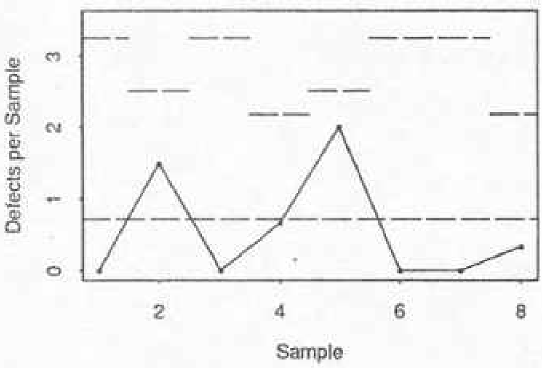
There is some evidence that the underlying leak rate was not stable over the time period studied. However, only the first $\hat{u}_i$ plotted outside its control limit, and there seems to be a general downward trend in the leak rate. The process may have stabilized during the period.

3. (a) Use equations (7-105) and (7-106). $\hat{u} = \frac{10}{14} = .7142857.$

$$\text{Center Line}_{\hat{u}_i} = .7142857.$$

The control limits depend on the number of units inspected k_i. The following table gives upper control limits for each of the k_i's in the data set. (The lower control limits are all negative, so there are no lower control limits for any of these k_i's.)

k_i	$UCL_{\hat{u}_i}$
1	3.249748
2	2.507129
3	2.178136



None of the $\hat{u}_i$'s plot outside the control limits, so there is no evidence that the underlying defect rate of the process was unstable over the time period studied.

(b) (i) If $k_i = 1$, then using equations (7-102) and (7-103), the center line will be at $\lambda = 1.2$ (standard),

$$LCL_{\hat{u}_i} = 1.2 - 3\sqrt{\frac{1.2}{1}} = -2.086335,$$

so there is no LCL, and

$$UCL_{\hat{u}_i} = 1.2 + 3\sqrt{\frac{1.2}{1}} = 4.486335.$$

Also, $X_i = k_i \hat{u}_i = \hat{u}_i$, so

$$\begin{aligned} P(\text{a sample produces an out of control signal}) &= P(\hat{u}_i \geq 4.486335) \\ &= P(X_i \geq 4.486335) \\ &= P(X_i \geq 5) \end{aligned}$$

where X_i is a Poisson random variable with $\lambda = 1.2$ (since the actual defect rate is standard here and $k_i = 1$). Using equation (5-10) with $\lambda = 1.2$,

$$\begin{aligned} P(X_i \geq 5) = 1 - P(X_i < 5) &= 1 - (P(X_i = 0) + \cdots + P(X_i = 4)) \\ &= 1 - .9922542 = .007745788. \end{aligned}$$

If $k_i = 2$, then using equations (7-102) and (7-103), the center line will be at $\lambda = 1.2$ (standard),

$$LCL_{\hat{u}_i} = 1.2 - 3\sqrt{\frac{1.2}{2}} = -1.12379,$$

so there is no LCL, and

$$UCL_{\hat{u}_i} = 1.2 + 3\sqrt{\frac{1.2}{2}} = 3.52379.$$

Also, $X_i = k_i \hat{u}_i = 2\hat{u}_i$, so

$$\begin{aligned} P(\text{a sample produces an out of control signal}) &= P(\hat{u}_i \geq 3.52379) \\ &= P(X_i \geq 7.04758) \\ &= P(X_i \geq 8) \end{aligned}$$

where X_i is a Poisson random variable with $\lambda = 2.4$ (since the actual defect rate is standard here and $k_i = 2$). Using equation (5-10) with $\lambda = 2.4$,

$$\begin{aligned} P(X_i \geq 8) = 1 - P(X_i < 8) &= 1 - (P(X_i = 0) + \cdots + P(X_i = 7)) \\ &= 1 - .9966614 = .003338617. \end{aligned}$$

This is slightly smaller than for $k_i = 1$, but both are very small. This is by design. For any k_i, the probability of "false alarm" would be small. As $k_i \to \infty$, the probability converges to

$$P(X_i < \mu - 3\sigma) + P(X_i > \mu + 3\sigma) = .0026$$

from the normal distribution. This is because the Poisson distribution converges to the normal distribution as $n \to \infty$.

(ii) The center line and control limits will be the same as in part (ii), because the standard hasn't changed.

If $k_i = 1$, then $X_i = k_i\hat{u}_i = \hat{u}_i$, so

$$\begin{aligned} P(\text{a sample produces an out of control signal}) &= P(\hat{u}_i \geq 4.486335) \\ &= P(X_i \geq 4.486335) \\ &= P(X_i \geq 5) \end{aligned}$$

where X_i is a Poisson random variable with $\lambda = 2.4$ (since the actual defect rate is twice the standard here and $k_i = 1$). Using equation (5-10) with $\lambda = 2.4$,

$$\begin{aligned} P(X_i \geq 5) = 1 - P(X_i < 5) &= 1 - (P(X_i = 0) + \cdots + P(X_i = 4)) \\ &= 1 - .9041314 = .09586859. \end{aligned}$$

If $k_i = 2$, then $X_i = k_i\hat{u}_i = 2\hat{u}_i$, so

$$\begin{aligned} P(\text{a sample produces an out of control signal}) &= P(\hat{u}_i \geq 3.52379) \\ &= P(X_i \geq 7.04758) \\ &= P(X_i \geq 8) \end{aligned}$$

where X_i is a Poisson random variable with $\lambda = 4.8$ (since the actual defect rate is twice standard here and $k_i = 2$). Using equation (5-10) with $\lambda = 4.8$,

$$\begin{aligned} P(X_i \geq 8) = 1 - P(X_i < 8) &= 1 - (P(X_i = 0) + \cdots + P(X_i = 7)) \\ &= 1 - .8866662 = .1133338. \end{aligned}$$

This is larger than for $k_i = 1$ because more information about the defect rate is obtained when 2 units are inspected. If the defect rate is actually twice the standard, this will be easier to detect by inspecting 2 units than by inspecting 1.

4. Use equations (7-98) and (7-99). $\hat{p} = \frac{18}{250} = .072$.

$$\text{Center Line}_{\hat{p}_i} = .072,$$

The control limits depend on the sample size n_i. For $n_i = 20$,

$$LCL_{\hat{p}_i} = .072 - 3\sqrt{\frac{.072(1-.072)}{20}} = -.101399,$$

which is negative, so there is no lower control limit.

$$UCL_{\hat{p}_i} = .072 + 3\sqrt{\frac{.072(1-.072)}{20}} = .245399.$$

For $n_i = 30$,

$$LCL_{\hat{p}_i} = .072 - 3\sqrt{\frac{.072(1-.072)}{30}} = -.06957966,$$

which is negative, so there is no lower control limit.

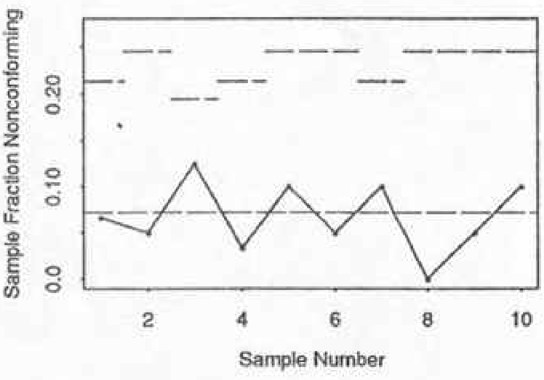
$$UCL_{\hat{p}_i} = .072 + 3\sqrt{\frac{.072(1-.072)}{30}} = .2135797.$$

For $n_i = 40$,

$$LCL_{\hat{p}_i} = .072 - 3\sqrt{\frac{.072(1-.072)}{40}} = -.05061158,$$

which is negative, so there is no lower control limit.

$$UCL_{\hat{p}_i} = .072 + 3\sqrt{\frac{.072(1-.072)}{40}} = .1946116.$$



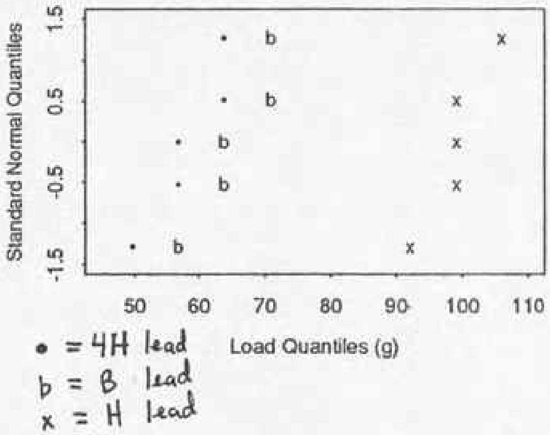
There is no evidence that the process fraction nonconforming was unstable (changing) over the time period studied.

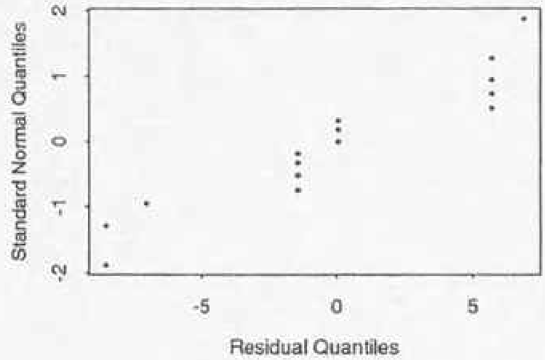
5. If different data collectors have different ideas of exactly what a "nonconformance" is, then the data collected will not be consistent. A stable process may look unstable (according to the c chart) because of these inconsistencies.

6. It may indicate that the chart was not applied properly. For example, if hourly samples of size $m = 4$ are collected, it may or may not be reasonable to use a retrospective $\bar{x}$ chart with $m = 4$. If each of the 4 items sampled are from 4 different machines, 3 of which are stable at some mean and the 4th stable at a different mean, then the sample ranges and standard deviations will be inflated. This will make the control limits on the $\bar{x}$ chart too wide. Also, the $\bar{x}$'s will show very little variation about a center line somewhere between the two means. This is all a result of the fact that each sample is really coming from 4 different processes. 4 different control charts should be used.

End Chapter Exercises

1. (a) Data within each sample must be iid normal, the samples must be independent, and the three distributions must have the same standard deviation.





The three normal plots of the data are roughly linear with no outliers, providing no evidence against the normal part of the assumption. The slopes are also similar, providing no evidence against the common standard deviation assumption. The normal plot of the residuals is roughly linear (considering the number of ties). This also provides no evidence against the normal part of the assumption.

(b) Using equation (7-7), $s_p^2 = 31.57733$, and so $s_p = \sqrt{31.57733} = 5.619$ g, with $n - r = 15 - 3 = 12$ degrees of freedom associated with it. This measures the magnitude of baseline variation within any of the 3 conditions, assuming it is the same for all 3 conditions.

(c) Use equation (7-14). The $\pm$ part is the same for all three intervals because all three sample sizes are the same. For 95% confidence, the appropriate t is $t = Q_{12}(.975) = 2.179$, from Table B-4. The resulting $\pm$ part is

$$2.179\frac{5.619}{\sqrt{5}} = 5.475956 \text{ g}.$$

The resulting confidence intervals for $\mu_{4\text{H}}$, μ_{H}, and μ_{B} are $[52.64, 63.60]$ g, $[93.66, 104.62]$ g, and $[59.74, 70.70]$ g respectively.

(d) Use equation (7-15). The $\pm$ part is the same for all 3 intervals because all three sample sizes are the same. t is the same as in part (c). The resulting $\pm$ part is

$$2.179(5.619)\sqrt{\frac{1}{5}+\frac{1}{5}} = 7.744171 \text{ g}.$$

The resulting confidence intervals for $\mu_{4\text{H}} - \mu_{\text{H}}$, $\mu_{4\text{H}} - \mu_{\text{B}}$, and $\mu_{\text{H}} - \mu_{\text{B}}$ are $[-48.76, -33.28]$ g, $[-14.84, .64]$ g, and $[26.18, 41.66]$ g respectively.

(e) Label the 4H lead Sample 1, the H lead Sample 2, and the B lead Sample 3. Use equation (7-17) with

$$c_1 = c_2 = \frac{1}{2}$$

and $c_3 = -1$. t is the same as in part (c). The resulting interval is

$$\begin{aligned} & \frac{58.12 + 99.14}{2} - 65.2213.41 \pm 2.179(5.619)\sqrt{.3} \\ = & \ 13.41 \pm 6.706649 \\ = & \ [6.70, 20.12] \text{ g}. \end{aligned}$$

(f) Use equation (7-28). With $r = 3$ and $\nu = n - r = 12$, Table B-8-A gives $k_2^* \approx 2.766$. The resulting $\pm$ part is

$$2.766 \frac{5.619}{\sqrt{5}} = 6.951122 \text{ g}.$$

The resulting confidence intervals for $\mu_{4\text{H}}$, μ_{H}, and μ_{B} are [51.17, 65.07] g, [92.19, 106.09] g, and [58.27, 72.17] g respectively. These intervals are wider than the ones in part (c). In order to ensure an overall (simultaneous) confidence of 95%, you need to make the individual 95% confidence intervals wider. Taken together, the intervals in part (c) have simultaneous confidence less than 95%.

(g) Use equation (7-36). For 95% confidence, with Number of Means to be Compared = 3 and $\nu = n - r = 12$, Table B-9-A gives $q^* = 3.77$. The resulting $\pm$ part is

$$\frac{3.77}{\sqrt{2}}(5.619)\sqrt{\frac{1}{5} + \frac{1}{5}} = 9.474233 \text{ g}.$$

The resulting confidence intervals for $\mu_{4\text{H}} - \mu_{\text{H}}$, $\mu_{4\text{H}} - \mu_{\text{B}}$, and $\mu_{\text{H}} - \mu_{\text{B}}$ are [−50.49, −31.55] g, [−16.57, 2.37] g, and [24.45, 43.39] g respectively. These intervals are wider than the ones in part (d), for the same reasons given in part (f).

(h) 1. H_0: $\mu_1 = \mu_2 = \mu_3$.
2. H_a: All 3 means are not the same.
3. The test statistic is given by equation (7-48). The reference distribution is the $F_{2,12}$ distribution. Large observed values of F will be considered as evidence against H_0.
4. The samples give

$$f = \frac{\frac{1}{2}(4806.0)}{(5.619)^2} = 76.10.$$

5. The observed level of significance is

$$P(\text{an } F_{2,12} \text{ random variable} > 76.10)$$

Using Tables B-6, the observed value of f is greater than $Q(.999) = 12.97$, and so the p-value is less than .001. This is overwhelming evidence that all 3 leads do not produce the same mean strength.

(i) Using the general form given in Table 7-12, the calculations yield the following table.

Source	SS	df	MS	F
Treatments	4806.0	2	2403.0	76.10
Error	378.9	12	31.6	
Total	5185.0	14		

(j) The following printout was produced using Minitab Version 9.1.

```
MTB > info

Column    Name            Count
C1        Load               15
C2        Lead               15

MTB > print c1 c2

 ROW    Load   Lead

   1    56.7      1
   2    63.8      1
   3    56.7      1
   4    63.8      1
   5    49.6      1
   6    99.2      2
   7    99.2      2
   8    92.1      2
   9   106.0      2
  10    99.2      2
  11    56.7      3
  12    63.8      3
  13    70.9      3
  14    63.8      3
  15    70.9      3

MTB > oneway c1 c2
```

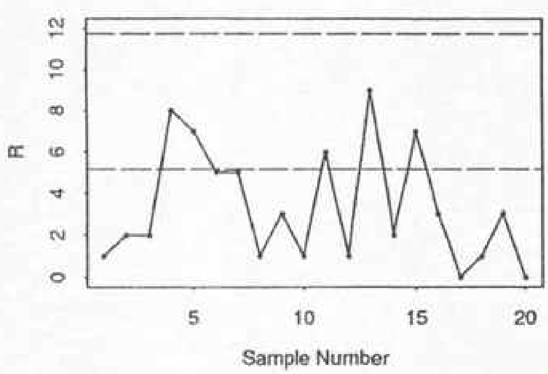
```
ANALYSIS OF VARIANCE ON Load
SOURCE      DF        SS        MS         F        p
Lead         2    4806.0    2403.0     76.10    0.000
ERROR       12     378.9      31.6
TOTAL       14    5185.0
                                     INDIVIDUAL 95 PCT CI'S FOR MEAN
                                     BASED ON POOLED STDEV
 LEVEL       N      MEAN     STDEV   -----+---------+---------+---------+-
     1       5     58.12      5.94   (---*--)
     2       5     99.14      4.92                                (---*---)
     3       5     65.22      5.94        (--*---)
                                     -----+---------+---------+---------+-
POOLED STDEV =      5.62                 60        75        90       105
```

2. (a) Use $d_2\sigma$ as a center line, and use the limits given in equation (7-86). From Table B-2 with $m = 4$, $d_2 = 2.059$, D_1 is not given (so there will be no lower control limit), and $D_2 = 4.698$. (I am continuing to work in the same units as the given data.)

$$\text{Center Line}_R = 2.059(2.5) = 5.1475 \times 10^{-4} \text{ in.},$$

and

$$UCL_R = 4.698(2.5) = 11.745 \times 10^{-4} \text{ in.}$$



This chart shows that there is no evidence that the process short-term variability (as measured by σ) was above 2.5×10^{-4} in. during the time period studied.

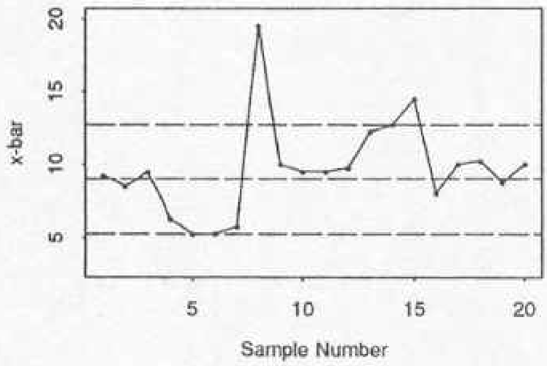
For the $\bar{x}$ chart, use $\mu = 9$ as a center line, and use the limits given in equation (7-70).

$$\text{Center Line}_{\bar{x}} = 9 \times 10^{-4} \text{ in. above } 1.1800,$$

$$LCL_{\bar{x}} = 9 - 3\frac{2.5}{\sqrt{4}} = 5.25 \times 10^{-4} \text{ in. above } 1.1800,$$

and

$$UCL_{\bar{x}} = 9 + 3\frac{2.5}{\sqrt{4}} = 12.75 \times 10^{-4} \text{ in. above } 1.1800.$$



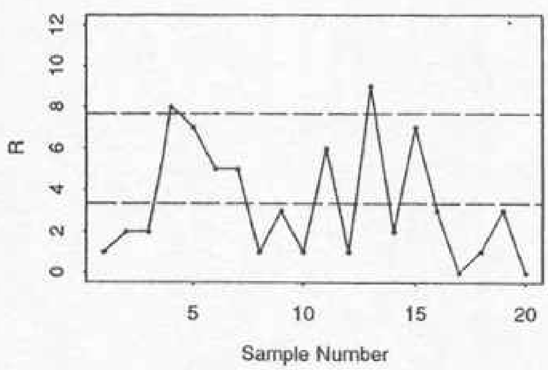
Several $\bar{x}$'s plot outside control limits, indicating that the process mean was not stable at 1.1809 in. during the period studied.

(b) For the R chart, use the limits given in equation (7-88). In Table B-2, for $m = 4$, D_3 is not given, so there is no lower control limit. $D_4 = 2.282$, so

$$\text{Center Line}_R = \bar{R} = 3.35 \times 10^{-4} \text{ in.},$$

and

$$UCL_R = 2.282(3.35) = 7.6447 \times 10^{-4} \text{ in.}$$



Two of the R's plot above the upper control limit. This indicates that the process short-term variability was not stable over the time period studied.

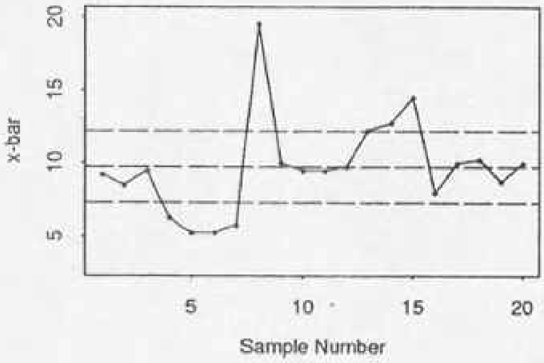
For the $\bar{x}$ chart, use $\bar{\bar{x}}$ as a center line, and use the limits given in equation (7-70), substituting $\bar{\bar{x}}$ for μ and $\frac{\bar{R}}{d_2}$ for σ.

$$\text{Center Line}_{\bar{x}} = 9.725 \times 10^{-4} \text{ in. above } 1.1800,$$

$$LCL_{\bar{x}} = 9.725 - 3\frac{1.627003}{\sqrt{4}} = 7.284496 \times 10^{-4} \text{ in. above } 1.1800,$$

and

$$UCL_{\bar{x}} = 9.725 + 3\frac{1.627003}{\sqrt{4}} = 12.1655 \times 10^{-4} \text{ in. above } 1.1800.$$



Many of the $\bar{x}$'s plot outside control limits, indicating that the process mean was not stable over the time period studied. (Long-term variability was not in control.)

(c) This estimate was used to make the $\bar{x}$ chart in part (b). See equation (7-74).

$$\frac{\bar{R}}{d_2} = \frac{3.35}{2.059} = 1.627003 \times 10^{-4} \text{ in.}$$

(d) (You should not use control limits to answer this question!) Assuming that the individual diameters are somewhat normally distributed, almost all of the diameters would fall in the interval $9 \pm 3\sigma$. Using the estimate of σ from part (c), this can be approximated by $9 \pm 3(1.627003) = 9 \pm 4.88$. It does seem that the process is capable of producing most diameters within the specifications. (The tails of the distribution of the individual diameters are quite close to the specifications, however. If the process cannot be kept at target, many diameters would fall outside the specifications.)

3. (a) Data within each sample must be iid normal, the samples must be independent, and the three distributions must have the same standard deviation.

(b) Using equation (7-7), $s_p = 64.274$ g, with $n - r = 9 - 3 = 6$ degrees of freedom associated with it.

(c) This measures the magnitude of baseline variation within any of the 3 conditions, assuming it is the same for all 3 conditions.

(d) Use equation (7-14). For a 90% one-sided confidence interval, make an 80% two-sided confidence interval and use the lower endpoint. For an 80% two-sided confidence interval, the appropriate t is $t = Q_6(.90) = 1.440$, from Table B-4. The resulting 90% lower confidence bound is

$$85.0 - 1.440\frac{64.274}{\sqrt{3}} = 31.56 \text{ g.}$$

(e) Label the Cotton/Polyester data Sample 1 and the Cotton/Acrylic data Sample 2. Use equation (7-15). t is the same as in part (e). The resulting interval is

$$\begin{aligned} 348.3 - 258.3 \pm 1.943(64.274)\sqrt{\frac{1}{3}+\frac{1}{3}} & \\ &= 90.0 \pm 101.9677 \\ &= [-11.97, 191.97]\text{ g.} \end{aligned}$$

Since zero is in this interval, there is not convincing evidence of a difference between the two mean weight losses.

(f) Use equation (7-36). For 95% confidence, with Number of Means to be Compared = 3 and $\nu = n - r = 6$, Table B-9 -A gives $q^* = 4.34$. The resulting Δ is

$$\frac{4.34}{\sqrt{2}}(64.274)\sqrt{\frac{1}{3}+\frac{1}{3}} = 161.05 \text{ g}.$$

(g) Using the one-way ANOVA identity (Proposition 7-1),

$$SSTr = SSTot - SSE = 132,247 - 24,787 = 107,460.$$

Using the general form given in Table 7-12, the degrees of freedom for Treatment are $r - 1 = 2$, the degrees of freedom for Error are $n - r = 6$, and the total degrees of freedom are $23 + 96 = 8 = n - 1$. $MSTr$ and MSE are obtained by dividing the sums of squares by the corresponding degrees of freedom, and F is obtained from $MSTr/MSE$.

Source	SS	df	MS	F
Treatment	107,460	2	53,730	13.01
Error	24,787	6	4131.167	
Total	132,247	8		

The p-value for the test is $P(\text{an } F_{2,6} \text{ random variable} > 13.01)$.

Using Tables B-6, the observed value of f is greater than $Q(.99) = 10.92$ and less than $Q(.999) = 27.00$, so the p-value is between .001 and .01. This is strong evidence that all 3 mean weight losses are not the same.

4. (a) Use formula (6-85). From Table B-7-B, for 95% confidence, with $n = 8$ and $p = .99$, $\tau_1 = 4.354$. The resulting one-sided tolerance interval for glue 1 is

$$\begin{aligned} 1821 - 4.353(214) &= 1821 - 931.542 \\ &= 889.46 \text{ kN}. \end{aligned}$$

(b) The two-sided 95% confidence interval is given by equation (6-20). To make a 95% one-sided confidence interval, construct a 90% two-sided confidence interval and use the lower endpoint. The appropriate t for a 90% two-sided confidence interval is $t = Q_7(.95) = 1.895$, and so the 95% one sided interval is

$$\begin{aligned} 1821 - 1.895\left(\frac{214}{\sqrt{8}}\right) &= 1821 - 143.3765 \\ &= 1677.62 \text{ kN}. \end{aligned}$$

(c) 1. H_0: $\mu_1 - \mu_2 = 0$.
2. H_a: $\mu_1 - \mu_2 \neq 0$.
3. The test statistic is given by equation (6-36) with $\# = 0$, and the reference distribution is the t_{14} distribution. Observed values of T far above or below zero will be considered as evidence against H_0.
4. The sample gives

$$t = -.858$$

5. The observed level of significance is

$$\begin{aligned} &2P(\text{a } t_{14} \text{ random variable} < -.858) \\ &\qquad = 2P(\text{a } t_{14} \text{ random variable} > .858) \end{aligned}$$

which is greater than $2(.1) = .2$, according to Table B-4. There is no evidence that the mean strengths differ for the two glues.

(d) For parts (a) and (b), the data must be iid normal because the sample size is small. For part (c), the data from each sample must be iid normal, and the standard deviations of the two distributions must be the same.

(e) Using equation (7-7), $s_p = 305.17$ kN, with $n - r = 64 - 8 = 56$ degrees of freedom associated with it.

(f) Assume common variance s = 305.17, avg. = 1821

1821 - 4.353(305.17) = 492.59kN

For part (b), use equation (7-14). To make a 95% one-sided confidence interval, construct a 90% two-sided confidence interval and use the lower endpoint. The appropriate t for a 90% two-sided confidence interval is $t = Q_{56}(.95) \approx 1.6725$, and so the 95% one sided interval is

$$\begin{aligned} 1821 - 1.6725\left(\frac{305.17}{\sqrt{8}}\right) &= 1821 - 180.4522 \\ &= 1640.55 \text{ kN}. \end{aligned}$$

The extra model assumptions are that the data within each of the 8 samples are iid normal, and that the standard deviations of all 8 distributions are the same.

(g) The statistic is given in equation (7-48).

$$f = \frac{\frac{1}{7}(8)(1200503)}{93128.37} = 14.73.$$

The reference distribution is the $F_{7,56}$ distribution.

(h) Use equation (7-28). With $r = 8$ and $\nu = n - r = 56$, Table B-8-A gives $k_2^* \approx 2.83$. The resulting Δ is

$$2.83\frac{305.169}{\sqrt{8}} = 305.34 \text{ kN}.$$

(i) Use equation (7-36). For 95% confidence, with Number of Means to be Compared = 8 and $\nu = n - r = 56$, Table B-9 -A gives $q^* \approx 4.456$. The resulting Δ is

$$\frac{4.456}{\sqrt{2}}(305.169)\sqrt{\frac{1}{8} + \frac{1}{8}} = 480.77 \text{ kN}.$$

5. (a) The two-sided 95% confidence interval is given by equation (6-20). The required t is $Q(.975)$ of the t_1 distribution, since (by symmetry) there must be probability .025 in each tail. From Table B-4, $t = Q_1(.975) = 12.706$. From the data, $n = 2$, $\bar{y} = 712.5$, and $s = 164.8$, so the confidence interval is

$$\begin{aligned} 712.5 \pm 12.706\left(\frac{164.8}{\sqrt{2}}\right) &= 712.5 \pm 1480.645 \\ &= [-768.15, 2193.15] \text{ psi.} \end{aligned}$$

Since pressures cannot be negative, set the lower endpoint equal to zero. The resulting interval is $[0, 2193.15]$ psi.

(b) Label the Pine/Lap data as Sample 1 and the Oak/Lap data as Sample 2.

1. $\mathrm{H_0}$: $\mu_1 - \mu_2 = 0$.
2. $\mathrm{H_a}$: $\mu_1 - \mu_2 \neq 0$.
3. The test statistic is given by equation (6-36) with $\# = 0$, and the reference distribution is the t_2 distribution. Observed values of T far above or below zero will be considered as evidence against $\mathrm{H_0}$.
4. The sample gives

$$t = -3.31$$

5. The observed level of significance is

$$\begin{aligned} &2P(\text{a } t_2 \text{ random variable} < -3.31) \\ &\qquad = P(\text{a } t_2 \text{ random variable} > 3.31) \end{aligned}$$

which is between $2(.025) = .05$ and $2(.05) = .1$, according to Table B-4. There is some evidence that the mean joint strength for lap joints is larger for oak wood than it is for pine.

(c) Use equation (6-47) and Tables B-6. For 90% confidence, $U = Q_{1,1}(.95) = 161.44$ and $L = Q_{1,1}(.05) = \frac{1}{Q_{1,1}(.95)} = \frac{1}{161.44}$. The resulting interval for $\frac{\sigma_1^2}{\sigma_2^2}$ is $[.00174021, 45.35486]$. Taking the square root of each endpoint, the interval for $\frac{\sigma_1}{\sigma_2}$ is $[.04, 6.73]$.

(d) Using equation (7-7), $s_p = 155.44$ psi, with $n - r = 7 - 4 = 3$ degrees of freedom associated with it.

(e) Use equation (7-30). With $r = 4$ and $\nu = n - r = 3$, Table B-8-B gives $k_1^* = 3.85$. The resulting interval for the Pine/Butt condition is

$$712.5 - 3.85\frac{155.44}{\sqrt{2}} = 289.35 \text{ psi.}$$

The resulting interval for the Oak/Butt condition is

$$1169 - 3.85\frac{155.44}{\sqrt{1}} = 570.57 \text{ psi.}$$

The resulting interval for the Pine/Lap condition is

$$929.5 - 3.85\frac{155.44}{\sqrt{2}} = 506.35 \text{ psi.}$$

The resulting interval for the Oak/Lap condition is

$$1428.0 - 3.85\frac{155.44}{\sqrt{2}} = 1004.85 \text{ psi.}$$

(f) Use equation (7-20) with

$$c_1 = c_2 = -\frac{1}{2}$$

and

$$c_3 = c_4 = \frac{1}{2}.$$

For a 95% two-sided interval, the appropriate t is $t = Q_3(.975) = 3.182$. The resulting interval is

$$\begin{aligned} & \frac{712.5 + 1169}{2} - \frac{929.5 + 1428.0}{2} \pm 3.182(155.44)\sqrt{.625} \\ = \; & -238.0 \pm 391.0127 \\ = \; & [-629.01, 153.01] \text{ psi.} \end{aligned}$$

6. (a) For the R chart, use the limits given in equation (7-88). In Table B-2, for $m = 2$, D_3 is not given, so there is no lower control limit. $D_4 = 3.267$, so

$$\text{Center Line}_R = \bar{R} = .00019 \text{ in.,}$$

and

$$UCL_R = 3.267(.00019) = .00062073 \text{ in.}$$

For the $\bar{x}$ chart, use $\bar{\bar{x}}$ as a center line, and use the limits given in equation (7-70), substituting $\bar{\bar{x}}$ for μ and $\frac{\bar{R}}{d_2}$ for σ. From Table B-2, for $m = 2$, $d_2 = 1.128$.

$$\text{Center Line}_{\bar{x}} = .35080 \text{ in.,}$$

$$LCL_{\bar{x}} = .35080 - 3\frac{.0001684397}{\sqrt{2}} = .3504427 \text{ in.,}$$

and

$$UCL_{\bar{x}} = .35080 + 3\frac{.0001684397}{\sqrt{2}} = .3511573 \text{ in.}$$

(b) Specifications apply to individual measurements; control limits apply to $\bar{x}$'s. $\bar{x}$'s have less variability than individual measurements, so control limits are generally much narrower than specifications for individuals.

Specifications are *external* standards used to judge quality; retrospective control limits are based on process history and are used to monitor process stability.

7. (a) Use equations (7-74), (7-78), and (7-7) respectively. From Table B-2, for $m = 4$, $d_2 = 2.059$ and $c_4 = .9213$.

$$\frac{\bar{R}}{d_2} = \frac{2.2}{2.059} = 1.06848 \times .001 \text{ in.}$$

$$\frac{\bar{s}}{c_4} = \frac{.948}{.9213} = 1.028981 \times .001 \text{ in.}$$

$$s_p = 1.087529 \times .001 \text{ in.}$$

(b) For $\frac{\bar{R}}{d_2}$ and the R chart, use the limits given in equation (7-88). In Table B-2, for $m = 4$, D_3 is not given, so there is no lower control limit. $D_4 = 2.282$, so

$$\text{Center Line}_R = \bar{R} = 2.2 \times .001 \text{ in.}$$

and

$$UCL_R = 2.282(2.2) = 5.0204 \text{ in.}$$

For $\frac{\bar{s}}{c_4}$ and the R chart, use the limits given in equation (7-86), with $\frac{\bar{s}}{c_4}$ substituted for σ. The center line will be at $d_2\frac{\bar{s}}{c_4}$. In Table B-2, for $m = 4$, D_1 is not given, so there is no lower control limit. $D_2 = 4.698$, so

$$\text{Center Line}_R = 2.059(1.028981) = 2.118672 \times .001 \text{ in.}$$

and

$$UCL_R = 4.698(1.028981) = 4.834153 \times .001 \text{ in.}$$

For s_p and the R chart, use the limits given in equation (7-86), with s_p substituted for σ. The center line will be at $d_2 s_p$.

$$\text{Center Line}_R = 2.059(1.087529) = 2.239222 \times .001 \text{ in.}$$

and

$$UCL_R = 4.698(1.087529) = 5.109211 \times .001 \text{ in.}$$

For $\frac{\bar{R}}{d_2}$ and the $\bar{x}$ chart, use $\bar{\bar{x}}$ as a center line, and use the limits given in equation (7-70), substituting $\bar{\bar{x}}$ for μ and $\frac{\bar{R}}{d_2}$ for σ.

$$\text{Center Line}_{\bar{x}} = 31.35 \times .001 \text{ in.}$$

$$LCL_{\bar{x}} = 31.35 - 3\frac{1.06848}{\sqrt{4}} = 29.74728 \times .001 \text{ in.}$$

and

$$UCL_{\bar{x}} = 31.35 + 3\frac{1.06848}{\sqrt{4}} = 32.95272 \times .001 \text{ in.}$$

For $\frac{\bar{s}}{c_4}$ and the $\bar{x}$ chart, use $\bar{\bar{x}}$ as a center line, and use the limits given in equation (7-70), substituting $\bar{\bar{x}}$ for μ and $\frac{\bar{s}}{c_4}$ for σ.

$$\text{Center Line}_{\bar{x}} = 31.35 \times .001 \text{ in.}$$

$$LCL_{\bar{x}} = 31.35 - 3\frac{1.028981}{\sqrt{4}} = 29.80653 \times .001 \text{ in.}$$

and

$$UCL_{\bar{x}} = 31.35 + 3\frac{1.028981}{\sqrt{4}} = 32.89347 \times .001 \text{ in.}$$

For s_p and the $\bar{x}$ chart, use $\bar{\bar{x}}$ as a center line, and use the limits given in equation (7-70), substituting $\bar{\bar{x}}$ for μ and s_p for σ.

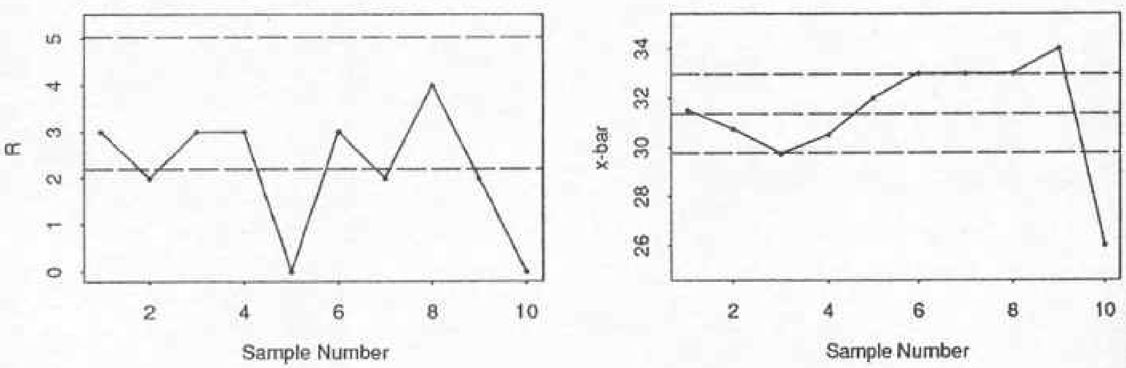
$$\text{Center Line}_{\bar{x}} = 31.35 \times .001 \text{ in.}$$

$$LCL_{\bar{x}} = 31.35 - 3\frac{1.087529}{\sqrt{4}} = 29.71871 \times .001 \text{ in.}$$

and

$$UCL_{\bar{x}} = 31.35 + 3\frac{1.087529}{\sqrt{4}} = 32.98129 \times .001 \text{ in.}$$

(c)



There are points that plot outside control limits on the $\bar{x}$ chart.

(d) The process is not stable. Short-term variability is reasonably stable, but the process mean is not constant over time. This is causing long-term variability that could possibly be avoided. To reduce variation, the company should first focus on stabilizing the process mean.

8. (a) Using equation (7-7), $s_p = .1430$ sec, with $n - r = 40 - 4 = 36$ degrees of freedom associated with it.

(b) Use equation (7-28). With $r = 4$ and $\nu = n - r = 36$, Table B-9-A gives $k_2^* \approx 2.62$. The resulting Δ is

$$2.62\frac{.1430}{\sqrt{10}} = .11849 \text{ sec.}$$

(c) Use equation (7-36). For 95% confidence, with Number of Means to be Compared = 4 and $\nu = n - r = 36$, Table B-9 -A gives $q^* \approx 3.814$. The resulting Δ is

$$\frac{3.814}{\sqrt{2}}(.1430)\sqrt{\frac{1}{10} + \frac{1}{10}} = .17248 \text{ sec.}$$

(d) 1. H_0: $\mu_1 = \mu_2 = \mu_3 = \mu_4$.

2. H_a: All 4 means are not the same.

3. The test statistic is given by equation (7-48). The reference distribution is the $F_{3,36}$ distribution. Large observed values of F will be considered as evidence against H_0.

4. The samples give

$$f = \frac{\frac{1}{3}(65.01539)}{(.1430)^2} = 1060.$$

5. The observed level of significance is

$$P(\text{an } F_{3,36} \text{ random variable} > 1060)$$

Using Tables B-6, the observed value of f is greater than $Q(.999) \approx 6.774$, and so the p-value is less than .001. This is overwhelming evidence that all 4 mean viscosities are not the same.

9. (a) Use equations (7-95) and (7-96).

$$\text{Center Line}_{\hat{p}_i} = 10^{-4},$$

$$LCL_{\hat{p}_i} = 10^{-4} - 3\sqrt{\frac{10^{-4}(1-10^{-4})}{100}} = -0.00289985,$$

which is negative, so there is no lower control limit.

$$UCL_{\hat{p}_i} = 10^{-4} + 3\sqrt{\frac{10^{-4}(1-10^{-4})}{100}} = .00309985.$$

(b) Using equation(5-3), with $X = 100\hat{p}$,

$$P(\hat{p}_i > .00309985) = P(X > .309985) = P(X > 1) = 1 - P(X = 0) = 1 - .9802 = .0198.$$

(c) Even with large sample sizes, attributes data will not provide enough information to detect even large changes in small p. It would be better to collect quantitative measurement data, if this is possible.

10. (a) Use equations (7-102) and (7-103). For a piece of .5 ft $\times$ .5 ft material (.25 sq ft),

$$UCL_{\hat{u}_i} = .04 + 3\sqrt{\frac{.04}{.25}} = 1.24.$$

For a piece of 5 ft $\times$ 5 ft material (25 sq ft),

$$UCL_{\hat{u}_i} = .04 + 3\sqrt{\frac{.04}{25}} = .16$$

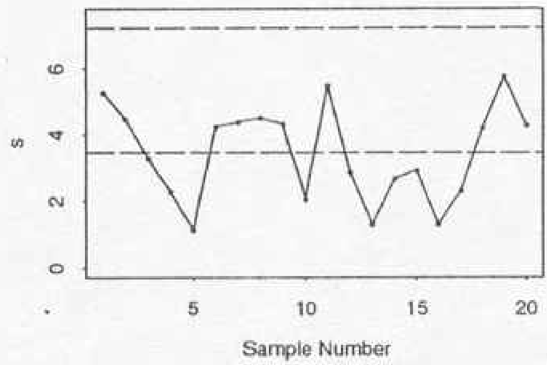
(b) A 1 ft $\times$ 1 ft specimen does not provide as much information about the underlying imperfection rate as a 10 ft $\times$ 10 ft specimen. Multiplying by 100 the number of imperfections on a 1 ft $\times$ 1 ft specimen provides a much less precise (more variable) estimate of the number of imperfections per 100 sq ft than counting the total number of imperfections on a 10 ft $\times$ 10 ft specimen.

11. (a) For an s chart, use the limits given in equation (7-92). The center line will be at $\bar{s}$. In Table B-2, for $m = 5$, B_3 is not given, so there is no lower control limit. $B_4 = 2.089$, so

$$\text{Center Line}_s = 3.4535 \times .001 \text{ in.}$$

and

$$UCL_s = 2.089(3.4535) = 7.214361 \times .001 \text{ in.}$$



This chart shows no evidence that the process short-term variation was unstable over the time period studied.

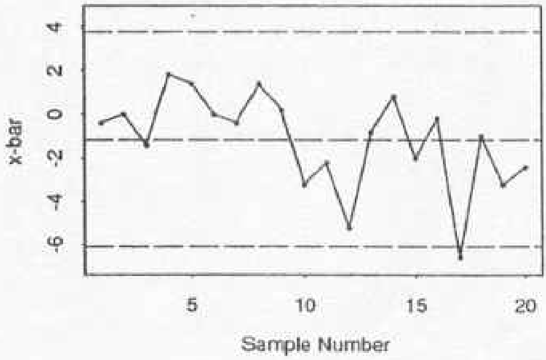
Use $\bar{\bar{x}}$ as a center line, and use the limits given in equation (7-70), substituting $\bar{\bar{x}}$ for μ and $\frac{\bar{s}}{c_4}$ for σ. In Table B-2, for $m = 5$, $c_4 = .9400$.

$$\text{Center Line}_{\bar{x}} = -1.17 \times .001 \text{ in. above target}$$

$$LCL_{\bar{x}} = -1.17 - 3\frac{3.673936}{\sqrt{5}} = -6.099103 \times .001 \text{ in. above target,}$$

and

$$UCL_{\bar{x}} = -1.17 + 3\frac{3.673936}{\sqrt{5}} = 3.759103 \times .001 \text{ in. above target.}$$



The $\bar{x}$ from sample 17 plots below the lower control limit, so there is evidence that the process mean was not constant over the time period. There seems to be a downward trend over the period.

(b) This estimate was used to make the $\bar{x}$ chart in part (a). See equation (7-74).

$$\frac{\bar{s}}{c_4} = \frac{3.4535}{.9400} = 3.673936 \times .001 \text{ in.}$$

(c) If the grinder is perfectly adjusted (u = 0), and stays that way, the best possible fraction of skives in specifications is roughly

$$\begin{aligned} P(-\ 6 \le X \le 6) &= P(X \le 6) - P(X < -6) \\ &= P(Z \le 1.63) - P(Z < -1.63) \\ &= .9484 - .0516 = .8968. \end{aligned}$$

(Z is a standard normal random variable.)

(d) These are standards given control limits. The standard for μ is $0 \times .001$ in. above target, and using the answer to part (b) to approximate σ, $3.673936 \times .001$ in. is the standard for σ.

For the R chart, use the limits given in equation (7-86). In Table B-2, for $m = 3$, D_1 is not given, so there is no lower control limit. $D_2 = 4.358$ and $d_2 = 1.693$, so

$$\text{Center Line}_R = d_2\sigma = 1.693(3.673936) = 6.219974 \times .001 \text{ in.}$$

and

$$UCL_R = 4.358(3.673936) = 16.01101 \times .001 \text{ in.}$$

For the $\bar{x}$ chart, use μ as a center line, and use the limits given in equation (7- 70).

$$\text{Center Line}_{\bar{x}} = 0 \times .001 \text{ in. above target,}$$

$$LCL_{\bar{x}} = 0 - 3\frac{3.673936}{\sqrt{3}} = -6.363444 \times .001 \text{ in. above target,}$$

and

$$UCL_{\bar{x}} = 0 + 3\frac{3.673936}{\sqrt{3}} = 6.363444 \times .001 \text{ in. above target.}$$

(e) The answer to part (b) is an estimate of the short-term standard deviation of skive lengths from that particular grinder. Different grinders may have different process means. The standard deviation mentioned in this part of the problem is measuring the sum of short-term variability for each grinder plus the variation from any differences in the means. It is bound to be at least as large as an estimate like the one in part (b).

(f) (i) The first method will allow you to get a good estimate of short-term variability for each grinder. This would be needed to estimate the σ used in control charts, and to statistically monitor each grinder over the long term. The disadvantage of the first method is that it only checks each grinder once an hour, and so is less likely to detect major problems quickly. The second method would be better for detecting major problems as quickly as possible.

(ii) From the description of the problem, it seems that hoses are being ground at a fast rate. If this is true, then sampling 5 hoses over the course of an hour would not give you good information about "short-term" variability. But this is the kind of information that is needed to see if the mean is in statistical control! The mean may change between 12 minute periods, and this would cause the estimate of short-term variation to be inflated. The resulting control limits on the $\bar{x}$ chart would then be widened, allowing the process to look more stable than it really is.

A compromise plan that incorporates the advantages of each of the two sampling methods would be ideal (but it may be too expensive).

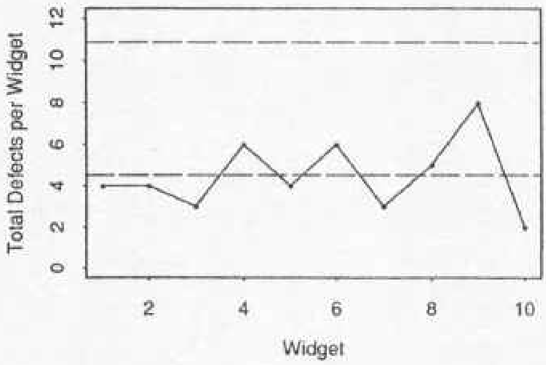
12. (a) Use equations (7-105) and (7-106) with all k_i's equal to 1. $\hat{u} = \frac{45}{10} = 4.5$.

$$\text{Center Line}_{\hat{u}_i} = 4.5.$$

$$LCL_{\hat{u}_i} = 4.5 - 3\sqrt{\frac{4.5}{1}} = -1.863961,$$

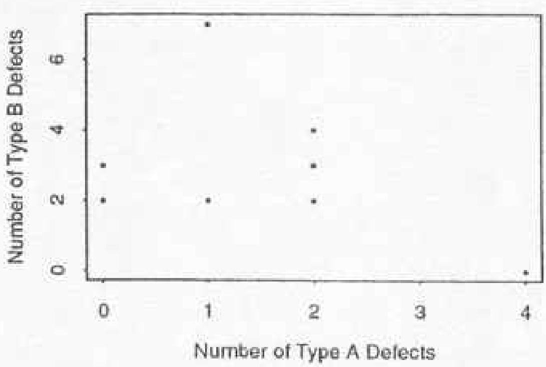
which is negative, so there is no LCL.

$$UCL_{\hat{u}_i} = 4.5 + 3\sqrt{\frac{4.5}{1}} = 10.86396.$$



There is no evidence of process instability, because none of the $\hat{u}_i$'s plotted outside the control limit.

(b) You might expect the number of type A defects to be positively correlated with the number of type B defects.



This does not seem to be the case here.

(c) (i) See equations (5-11), (5-12), (5-53), and (5-54). $EX = 2\lambda_1 + \lambda_2$, and $\text{Var}X = 4\lambda_1 + \lambda_2$. The center line would be at $2\lambda_1 + \lambda_2$, and the control limits would be

$$LCL_X = 2\lambda_1 + \lambda_2 - 3\sqrt{4\lambda_1 + \lambda_2}$$

and

$$UCL_X = 2\lambda_1 + \lambda_2 + 3\sqrt{4\lambda_1 + \lambda_2}$$

(ii) Estimate λ_1 with $\hat{u} = \frac{16}{10} = 1.6$ and λ_2 with $\hat{u} = \frac{29}{10} = 2.9$. The resulting center line is at $2(1.6) + 2.9 = 6.1$, and

$$LCL_X = 6.1 - 3\sqrt{4(1.6) + 2.9} = -3.04877,$$

which is negative, so there should be no LCL.

$$UCL_X = 6.1 + 3\sqrt{4(1.6) + 2.9} = 15.24877.$$

None of the 10 X's plotted above the UCL.

13. (a) With $\mu = 1.000$ and $\sigma = .005$,

$$\begin{aligned} P(X < .9902 \text{ or } X > 1.0098) &= 1 - P(.9902 \le X \le 1.0098) \\ &= 1 - (P(X \le 1.0098) - P(X < .9902)) \\ &= 1 - (P(Z \le 1.96) - P(Z < -1.96)) \\ &= 1 - (.9750 - .0250) = .0500. \end{aligned}$$

(Z is a standard normal random variable.) It is not possible for the fraction nonconforming to ever be less than this, unless σ can be made smaller.

(b) With $p = .05$ and $n_i = m = 10$, use equations (7-95) and (7-96).

$$\text{Center Line}_{\hat{p}_i} = .05,$$

$$LCL_{\hat{p}_i} = .05 - 3\sqrt{\frac{.05(1 - .05)}{10}} = -.1567607,$$

which is negative, so there is no lower control limit.

$$UCL_{\hat{p}_i} = .05 + 3\sqrt{\frac{.05(1 - .05)}{10}} = .2567607.$$

(c) For the p chart, define Y as the number of nonconforming items in the particular sample of 10. Then $Y = 10\hat{p}$, and

$$\begin{aligned} P(\text{out of control signal}) &= P(\hat{p} > .2567607) \\ &= P(Y > 2.567607) \\ &= P(Y \ge 3) \\ &= 1 - P(Y < 3). \end{aligned}$$

Y is a binomial random variable with $n = 10$. If $\mu = 1.000$, then $p = .05$ (from the calculation in part (a). Using equation (5-3),

$$1 - P(Y < 3) = 1 - .9884964 = .01150356.$$

For the x-bar chart, the control limits are given by equation (7-70).

$$\text{Center Line}_{\bar{x}} = 1.000,$$

$$LCL_{\bar{x}} = 1.000 - 3\frac{.005}{\sqrt{10}} = .9952566,$$

and

$$UCL_{\bar{x}} = 1.000 + 3\frac{.005}{\sqrt{10}} = 1.004743.$$

Based on samples of size $m = 10$, the mean of $\overline{X}$ is 1.000 and the standard deviation of $\overline{X}$ is $\frac{.005}{\sqrt{10}}$.

$$\begin{aligned} P(\text{out of control signal}) &= P(\overline{X} < .9952566 \text{ or } \overline{X} > 1.004743)) \\ &= 1 - P(.9952566 \le \overline{X} \le 1.004743) \\ &= 1 - (P(\overline{X} \le 1.004743) - P(\overline{X} < .9952566)) \\ &= 1 - (P(Z \le 3) - P(Z < -3)) \\ &= 1 - (.9987 - .0013) = .0026. \end{aligned}$$

(Z is a standard normal random variable.) The p chart is more than 4 times as likely to produce an out of control point as the x-bar chart, assuming that $\mu = 1.000$ (which implies that $p = .05$). However, this probability is still quite small for both charts.

(d) The control limits for both charts will be the same because the standards have not changed. For the p chart,

$$\begin{aligned} P(\text{out of control signal}) &= P(\hat{p} > .2567607) \\ &= P(Y > 2.567607) \\ &= P(Y \ge 3) \\ &= 1 - P(Y < 3). \end{aligned}$$

Y is a binomial random variable with $n = 10$. If $\mu = 1.005$, then the appropriate p is

$$\begin{aligned} P(X < .9902 \text{ or } X > 1.0098) &= 1 - P(.9902 \le X \le 1.0098) \\ &= 1 - (P(X \le 1.0098) - P(X < .9902)) \\ &= 1 - (P(Z \le .96) - P(Z < -2.96)) \\ &= 1 - (.8315 - .0015) = .1700. \end{aligned}$$

(Z is a standard normal random variable.) Using equation (5-3),

$$1 - P(Y < 3) = 1 - .7658695 = .2341305.$$

For the x-bar chart, the mean of $\overline{X}$ is now 1.005.

$$\begin{aligned} P(\text{out of control signal}) &= P(\overline{X} < .9952566 \text{ or } \overline{X} > 1.004743)) \\ &= 1 - P(.9952566 \le \overline{X} \le 1.004743) \\ &= 1 - (P(\overline{X} \le 1.004743) - P(\overline{X} < .9952566)) \\ &= 1 - (P(Z \le -.16) - P(Z < -6.16)) \\ &= 1 - (.4364 - .0000) = .5636. \end{aligned}$$

(Z is a standard normal random variable.) The $\bar{x}$ chart is much more likely to produce an out of control point than the p chart, assuming that $\mu = 1.005$ (which implies that $p = .17$). The story here is that the x-bar chart is much more sensitive to changes in μ than the p chart is to corresponding changes in p. In other words, the x-bar chart will generally detect changes in μ faster (on average) than the corresponding p chart.

14. (a) Using the one-way ANOVA identity (Proposition 7-1),

$$SSE = SSTot - SSTr = 1405.59 - 1052.39 = 353.20.$$

Using the general form given in Table 7-12, the degrees of freedom for Order(setup) are $r - 1 = 23$, the degrees of freedom for Error are $n - r = 120 - 24 = 96$, and the total degrees of freedom are $23 + 96 = 119 = n - 1$. MSOrder and MSE are obtained by dividing the appropriate sums of squares by the corresponding degrees of freedom, and F is obtained from MSOrder$/MSE$.

Source	SS	df	MS	F
Order (setup)	1052.39	23	45.75609	12.43654
Error	353.20	96	3.679167	
Total	1405.59	119		

(b) The focus is on variability among the many orders that are run over time, not just the 24 orders used in the study. The 24 orders used in the study can be thought of as a random sample of the many different orders that are run over time, and so the μ_i's can be thought of as random as well.

(c) An estimate of σ^2 is $MSE = s_p^2 = 3.679167$, and so an estimate of σ is $\sqrt{3.679167} = s_p = 1.918115 \times \frac{1}{32}$ in. (see equation (7-58)). Using equation (7-62), an estimate of σ_τ^2 is

$$\frac{1}{5}(45.75609 - 3.679167) = 8.415385,$$

so an estimate of σ_τ is $\sqrt{8.415385} = 2.900928 \times \frac{1}{32}$ in. The estimate of σ measures variability in skewness of boxes which all come from the same order (assuming this is the same for any order). The estimate of σ_τ measures the variation in the response from differences among orders. It seems that most of the variation comes from differences among orders.

(d) Use equation (7-63). For a two-sided 90% confidence interval, $U = Q_{23,96}(.95) \approx 1.64$ and

$$L = Q_{23,96}(.05) = \frac{1}{Q_{96,23}(.95)} \approx \frac{1}{1.83} = .5464481$$

using Table B-6-C. The resulting interval for σ_τ^2/σ^2 is

$$\begin{aligned} & \left[\frac{1}{5}\left(\frac{45.75609}{(1.64)(3.679167)} - 1\right), \frac{1}{5}\left(\frac{45.75609}{(.5464481)(3.679167)} - 1\right)\right] \\ = \; & [1.316651, 4.351772]. \end{aligned}$$

Taking the square roots of the endpoints, the interval for σ_τ/σ is $[1.147, 2.086]$. σ_τ/σ is a comparison of the size of variation from different setups to the size of within-setup variation in skewness. The interval implies that, with 90% confidence, the variation in skewness from different setups is between 1.3 to 4.4 times larger than the within-setup variation.

(e) Most of the variability seems to be coming from differences among orders (setups). The manufacturer should try to reduce variability from changing setups before buying new high-precision equipment. The new equipment would possibly only reduce within-setup variability.

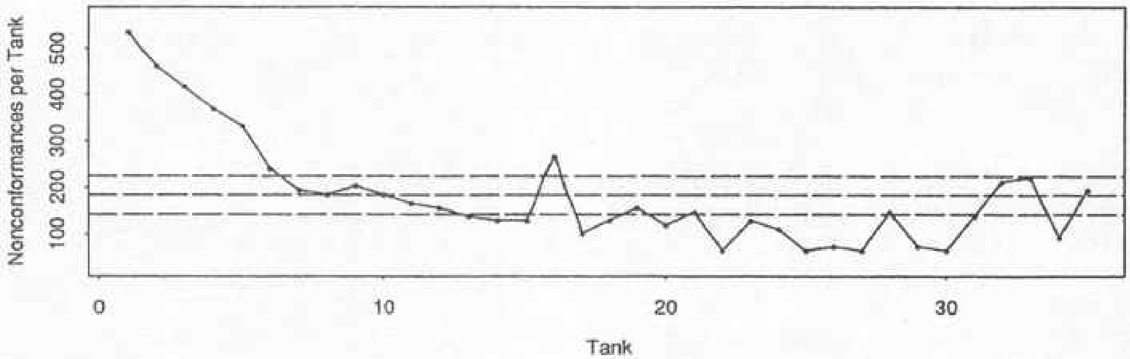
15. (a) Use equations (7-105) and (7-106) with all k_i's equal to 1. $\hat{u} = \frac{6434}{35} = 183.8286$.

$$\text{Center Line}_{\hat{u}_i} = 183.8286,$$

$$LCL_{\hat{u}_i} = 183.8286 - 3\sqrt{\frac{183.8286}{1}} = 143.1536,$$

and

$$UCL_{\hat{u}_i} = 183.8286 + 3\sqrt{\frac{183.8286}{1}} = 224.5036.$$



Many $\hat{u}_i$'s plot outside control limits. The rate of nonconformances could not have possibly been constant for all 35 tanks; there is strong evidence of a decrease in the rate over the time period studied.

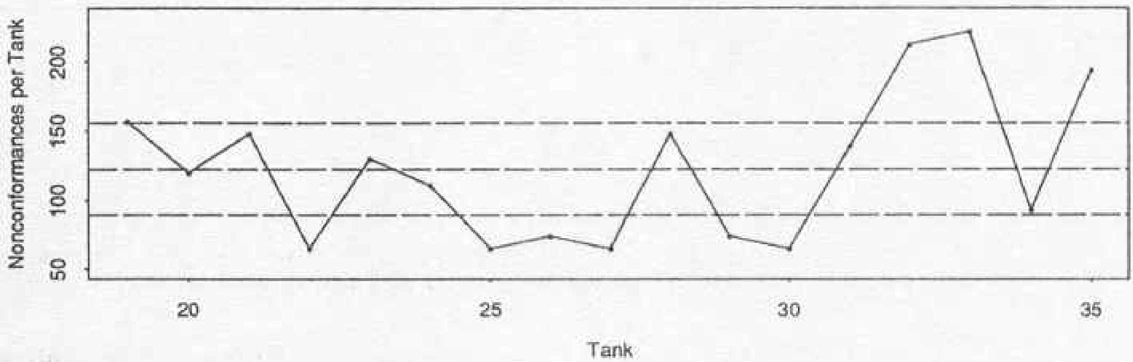
(b) $\hat{u} = \frac{2083}{17} = 122.5294$.

$$\text{Center Line}_{\hat{u}_i} = 122.5294,$$

$$LCL_{\hat{u}_i} = 122.5294 - 3\sqrt{\frac{122.5294}{1}} = 89.32151,$$

and

$$UCL_{\hat{u}_i} = 122.5294 + 3\sqrt{\frac{122.5294}{1}} = 155.7373.$$



This chart also has many $\hat{u}_i$'s outside control limits. It seems that quality (in terms of rate of nonconformances per tank) was not stable over the period represented by these tanks.

(c) No. Many $\hat{u}_i$'s were more than 15 nonconformances outside the control limits, so this measurement error cannot explain the lack of stability in the charts.

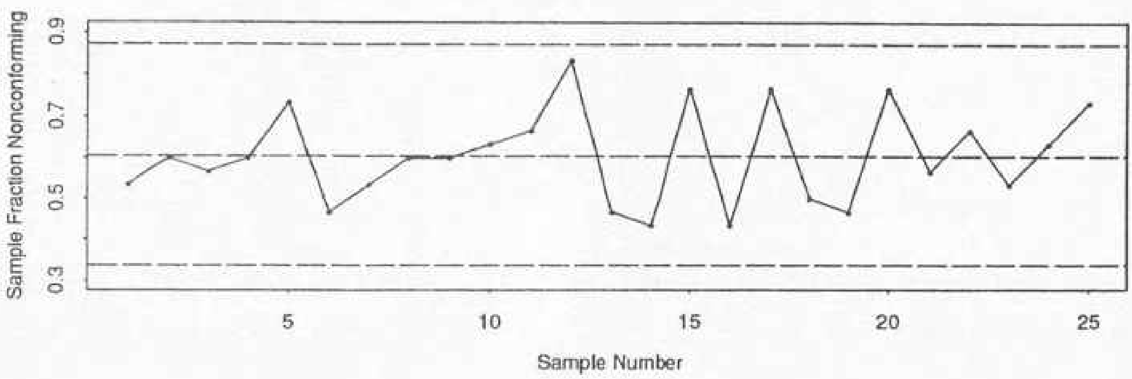
16. (a) Use equations (7-98) and (7-99). $\hat{p} = \frac{453}{750} = .604$.

$$\text{Center Line}_{\hat{p}_i} = .604,$$

$$LCL_{\hat{p}_i} = .604 - 3\sqrt{\frac{.604(1-.604)}{30}} = .3361284,$$

and

$$UCL_{\hat{p}_i} = .604 + 3\sqrt{\frac{.604(1-.604)}{30}} = .8718716.$$



There is no evidence from the chart that the process fraction nonconforming was unstable during Day 1, since none of the $\hat{p}_i$'s plot outside the control limits.

(b) Using equation (6-57), the appropriate z for 90% confidence is 1.645. The resulting interval is

$$\begin{aligned} .604 \pm 1.645\frac{1}{2\sqrt{750}} &= .604 \pm .03003345 \\ &= [.5740, .6340]. \end{aligned}$$

Using equation (6-57), the appropriate z for 90% confidence is 1.645. The resulting interval is

$$\begin{aligned} .604 \pm 1.645\sqrt{\frac{.604(1-.604)}{750}} &= .604 \pm .02937659 \\ &= [.5746, .6334]. \end{aligned}$$

(c) A process can be stable at an unacceptably large p, and therefore can consistently produce too much nonconforming product.

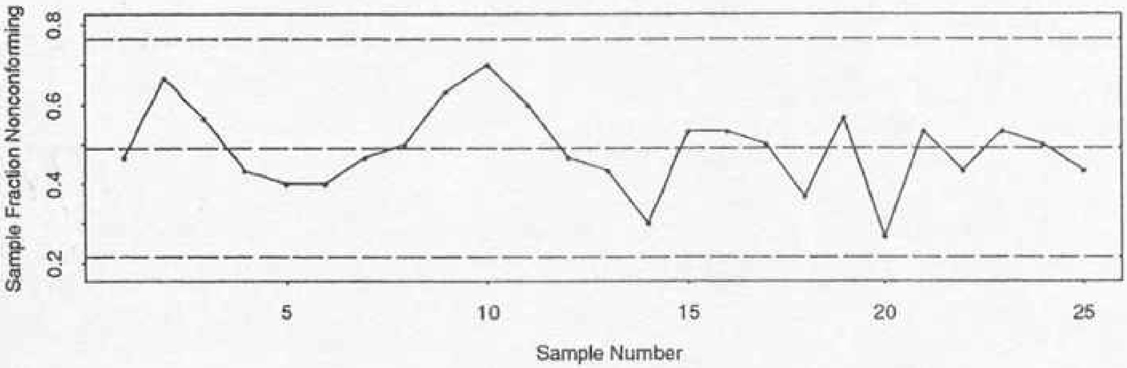
(d) Use equations (7-98) and (7-99). $\hat{p} = \frac{367}{750} = .48933$.

$$\text{Center Line}_{\hat{p}_i} = .48933,$$

$$LCL_{\hat{p}_i} = .48933 - 3\sqrt{\frac{.48933(1-.48933)}{30}} = .2155344,$$

and

$$UCL_{\hat{p}_i} = .48933 + 3\sqrt{\frac{.48933(1-.48933)}{30}} = .7631323.$$



There is no evidence from the chart that the process fraction nonconforming was unstable during Day 2, since none of the $\hat{p}_i$'s plot outside the control limits.

(e) Using equation (6-57), the appropriate z for 90% confidence is 1.645. The resulting interval is

$$\begin{aligned} .48933 \pm 1.645\frac{1}{2\sqrt{750}} &= .48933 \pm .03003345 \\ &= [.45930, .51937]. \end{aligned}$$

Using equation (6-59), the appropriate z for 90% confidence is 1.645. The resulting interval is

$$\begin{aligned} .48933 \pm 1.645\sqrt{\frac{.48933(1-.48933)}{750}} &= .48933 \pm .03002662 \\ &= [.45931, .51936]. \end{aligned}$$

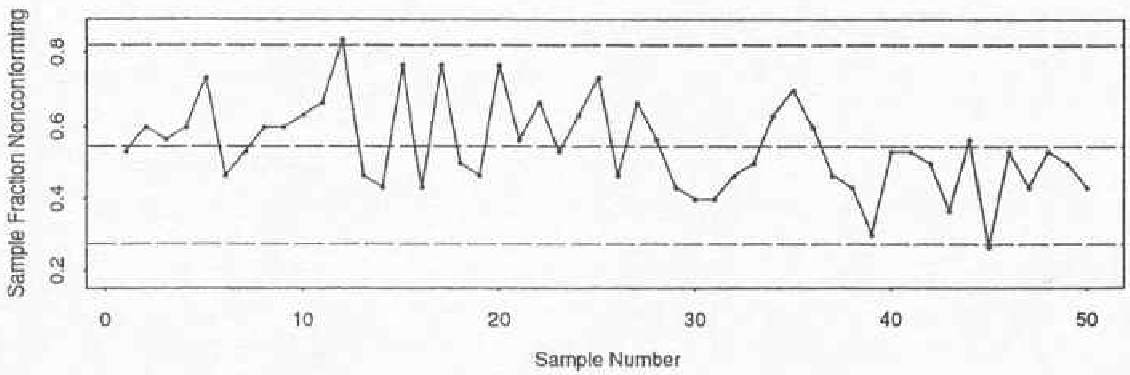
(f) Use equations (7-98) and (7-99). $\hat{p} = \frac{820}{1500} = .54667$.

$$\text{Center Line}_{\hat{p}_i} = .54667,$$

$$LCL_{\hat{p}_i} = .54667 - 3\sqrt{\frac{.54667(1-.54667)}{30}} = .2740008,$$

and

$$UCL_{\hat{p}_i} = .54667 + 3\sqrt{\frac{.54667(1-.54667)}{30}} = .8193325.$$



There is evidence from the chart that the process fraction nonconforming was unstable over the two days, since the $\hat{p}_i$'s for the 12th sample from Day 1 and the 20th sample from Day 2 were outside the control limits. This does not contradict previous results, because it is quite possible that a change in the underlying p happened between the two days. It is possible that, within each day, the process was stable. From the chart, it seems like p decreased slightly between Days 1 and 2.

(g) Using equation (6-65), the appropriate z for 98% confidence is 2.33. The resulting interval is

$$.604 - .48933 \pm 2.33\left(\frac{1}{2}\right)\sqrt{\frac{1}{750}+\frac{1}{750}} = .1146667 \pm .06016034 = [.0545, .1748].$$

Using equation (6-67), the resulting interval is

$$.604 - .48933 \pm 2.33\sqrt{\frac{.604(1-.604)}{750}+\frac{.48933(1-.48933)}{750}} = .1146667 \pm .05949917 = [.0552, .1742].$$

Since this interval does not contain zero, there is evidence of a difference in fraction nonconforming pellets between the two days.

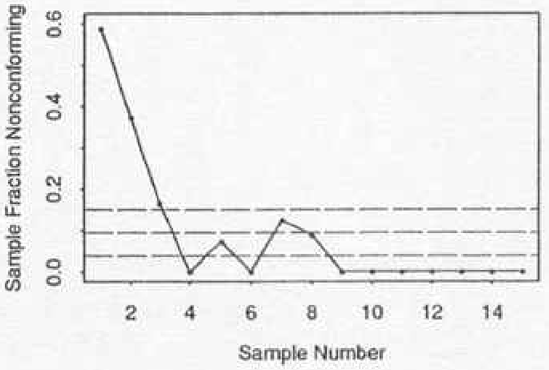
17. Use equations (7-98) and (7-99). $\hat{p} = \frac{352}{3750} = .093867$.

$$\text{Center Line}_{\hat{p}_i} = .093867,$$

$$LCL_{\hat{p}_i} = .093867 - 3\sqrt{\frac{.093867(1-.093867)}{250}} = .03853127,$$

and

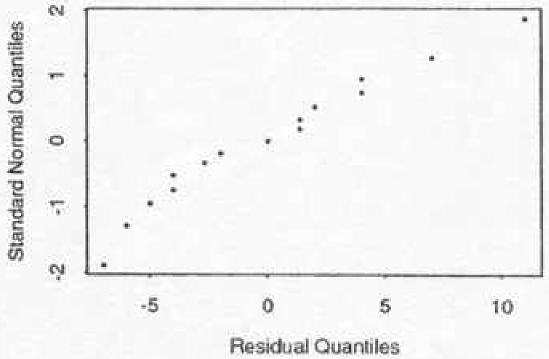
$$UCL_{\hat{p}_i} = .093867 + 3\sqrt{\frac{.093867(1-.093867)}{250}} = .1492021$$



The chart shows strong evidence that the fraction nonconforming was not stable at some value over the entire period. There must be a startup problem.

18. (a) Label brands C, H, W, Q, and P as 1, 2, 3, 4, and 5 respectively. See equation (7-3). The necessary computations are given in the table below.

Brand	y_{ij}	$\hat{y}_{ij} = \bar{y}_i$	e_{ij}
1	378	384.00	-6.00
1	386	384.00	2.00
1	388	384.00	4.00
2	357	361.00	-4.00
2	365	361.00	4.00
2	361	361.00	0.00
3	321	310.00	11.00
3	303	310.00	-7.00
3	306	310.00	-4.00
4	353	351.67	1.33
4	349	351.67	-2.67
4	353	351.67	1.33
5	390	383.00	7.00
5	378	383.00	-5.00
5	381	383.00	-2.00



The plot is fairly linear, giving no evidence against the normal part of the one-way normal model assumptions.

(b) Using equation (7-7), $s_p = 6.022°\text{F}$, with $n - r = 15 - 5 = 10$ degrees of freedom associated with it. This measures the magnitude of baseline variation within any of the 5 brands, assuming it is the same for all 5 brands.

For the confidence interval, use equation (7-10) and Table B-5. For a 90% two-sided interval, $U = Q_{10}(.95) = 18.307$ and $L = Q_{10}(.05) = 3.940$. The resulting interval for σ^2 is $[19.81027, 92.04738]$; taking the square root of each endpoint, the interval for σ is $[4.45, 9.59]°\text{F}$.

(c) Use equation (7-14). Δ is the same for all five intervals because all five sample sizes are the same. For 90% confidence, the appropriate t is $t = Q_{10}(.95) = 1.812$, from Table B-4. The resulting Δ is

$$1.812 \frac{6.022}{\sqrt{3}} = 6.30°\text{F}.$$

(d) Use equation (7-15). Δ is the same for all 10 intervals because all five sample sizes are the same. t is the same as in part (c). The resulting Δ is

$$1.812(6.022)\sqrt{\frac{1}{3} + \frac{1}{3}} = 8.91°\text{F}.$$

(e) Use equation (7-28). With $r = 5$ and $\nu = n - r = 10$, Table B-9-A gives $k_2^* = 3.10$. The resulting Δ is

$$3.01 \frac{6.022}{\sqrt{3}} = 10.47°\text{F}.$$

(f) Use equation (7-36). For 99% confidence, with Number of Means to be Compared = 5 and $\nu = n - r = 10$, Table B-9 -B gives $q^* = 6.14$. The resulting Δ is

$$\frac{6.14}{\sqrt{2}}(6.022)\sqrt{\frac{1}{3} + \frac{1}{3}} = 21.35°\text{F}.$$

(g) Using the general form given in Table 7-12, the calculations yield the following table.

Source	SS	df	MS	F
Treatments	10962.3	4	2740.6	75.57
Error	362.7	10	36.3	
Total	11324.9	14		

Using equation (7-53),

$$R^2 = \frac{10962.3}{11324.9} = .968.$$

The p-value for an F test of the null hypothesis given in part (i) is

$$P(\text{an } F_{4,10} \text{ random variable} > 75.57).$$

Using Tables B-6, this is less than .001. The conclusion is the same as in part (i).

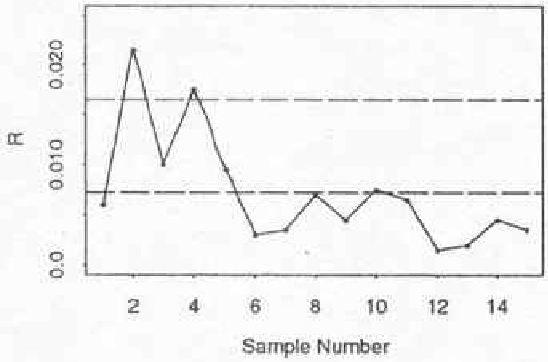
(h) If only one can of each brand is represented in the study, then (with respect to generalizing to all oil made by each brand), there is only one true replication for each brand. There is no guarantee that the 5 cans used are representative of all cans of oil made by the 5 companies. For the second scenario, there is true replication for each brand. Variability from differences in shipping lots is then represented in the data, and conclusions based on the experiment might safely be applied to the brands in general.

19. (a) Use the limits given in equation (7-88). In Table B-2, for $m = 4$, D_3 is not given, so there is no lower control limit. $D_4 = 2.282$, so

$$\text{Center Line}_R = \bar{R} = .0072 \text{ in}$$

and

$$UCL_R = 2.282(.0072) = .0164304 \text{ in.}$$



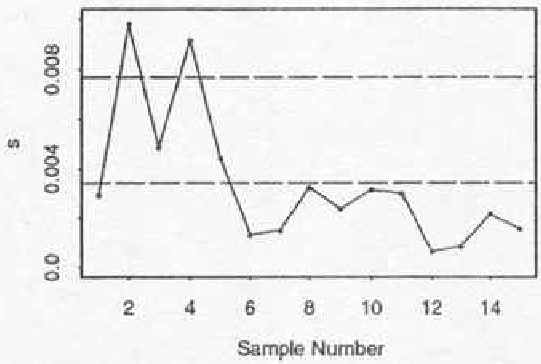
There are two R's that plot above the upper control limit. This indicates that the process short-term variability was not stable over the two days. It looks like there was more short-term variability on the first day than on the second day.

(b) Using the limits given in equation (7-92), the center line will be at $\bar{s}$. In Table B-2, for $m = 4$, B_3 is not given, so there is no lower control limit. $B_4 = 2.266$, so

$$\text{Center Line}_s = .00339 \text{ in}$$

and

$$UCL_s = 2.266(.00339) = .00768174 \text{ in.}$$



The plot, and conclusions from it, are similar to part (a).

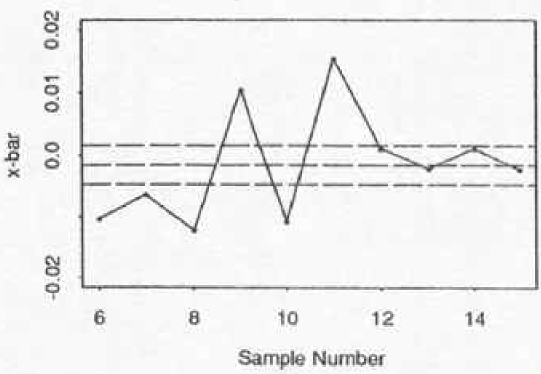
(c) Use $\bar{\bar{x}}$ as a center line, and use the limits given in equation (7-70), substituting $\bar{\bar{x}}$ for μ and $\frac{\bar{s}}{c_4}$ (or $\frac{\bar{R}}{d_2}$) for σ. From Table B-2, for $m = 4$, $d_2 = 2.059$ and $c_4 = .9213$. For the estimate of σ based on $\bar{R}$,

$$\text{Center Line}_{\bar{x}} = -.00159 \text{ in. above } 33.69,$$

$$LCL_{\bar{x}} = -.00159 - 3\frac{.002112676}{\sqrt{4}} = -.004759014 \text{ in. above } 33.69,$$

and

$$UCL_{\bar{x}} = -.00159 + 3\frac{.002112676}{\sqrt{4}} = .001579014 \text{ in. above } 33.69.$$



The $\bar{x}$'s from samples 6 through 11 are outside control limits, indicating that the process mean is changing between samples. (Long-term variation is not stable.) For the estimate of σ based on $\bar{s}$,

$$\text{Center Line}_{\bar{x}} = -.00159 \text{ in. above } 33.69,$$

$$LCL_{\bar{x}} = -.00159 - 3\frac{.00213177}{\sqrt{4}} = -0.004787655 \text{ in. above } 33.69,$$

and

$$UCL_{\bar{x}} = -.00159 + 3\frac{.00213177}{\sqrt{4}} = 001607655 \text{ in. above } 33.69.$$

This chart is virtually the same as the one based on $\bar{R}$.

(d) This procedure caused the within-bundle variability to be much smaller than the between-bundle variability. The within-sample variability is bound to be small, since all rods within a sample come from the same bundle and are all cut at once. The estimates of σ used above were "too small" because they only measured within-bundle variability. This caused the control limits for the $\bar{x}$ chart to be too narrow, resulting in many $\bar{x}$'s outside the limits.

For an $\bar{x}$ chart, σ is supposed to measure short-term variability from *different* units. There is really no way to accurately measure this, because not many bundles are cut in a short period of time.

(e) Based on $\bar{R}$, using equation (7-74),

$$\frac{\bar{R}}{d_2} = \frac{.00435}{2.059} = .002112676 \text{ in.}$$

Based on $\bar{s}$, using equation (7-92),

$$\frac{\bar{s}}{c_4} = \frac{.001964}{.9213} = .00213177 \text{ in.}$$

Both of these are measuring within-bundle variability in rod lengths (assuming this is the same for each of the last 10 bundles). They do not measure short-term process variability from different "units", because all rods in each sample come from the same unit (bundle).

(f) Using $\frac{\bar{R}}{d_2}$ to estimate σ,

$$\begin{aligned} P(33.66 \le X \le 33.72) &= P(X \le 33.66) - P(X < 33.66) \\ &= P(Z \le 14.2) - P(Z < -14.2) \approx 1 - 0 = 1. \end{aligned}$$

(Z is a standard normal random variable.)

(g) Due to bundling, the within-sample variability (as measured in (e)) is much smaller than the overall variability within and among bundles (as measured by s).

20. (a) Using the general form given in Table 7-12, the calculations yield the following table.

Source	SS	df	MS	EMS	F
Bundles	.0030053	9	.0003339	$\sigma^2 + 4\sigma_\tau^2$	71.71
Error	.0001397	30	.0000047	σ^2	
Total	.0031449	39			

An estimate of σ^2 is $MSE = s_p^2 = .0000047$, and so an estimate of σ is $\sqrt{.0000047} = s_p = .00216$ in. (see equation (7-58)). Using equation (7-62), an estimate of σ_τ^2 is

$$\frac{1}{4}(.0003339 - .0000047) = .0000823,$$

so an estimate of σ_τ is $\sqrt{.0000823} = .00907$ in. It seems that most of the variation comes from differences among bundles.

(b) Use equation (7-63). For a two-sided 90% confidence interval, $U = Q_{9,30}(.95) = 2.21$ and

$$L = Q_{9,30}(.05) = \frac{1}{Q_{30,9}(.95)} = \frac{1}{2.86} = .3496503$$

using Table B-6-C. The resulting interval for σ_τ^2/σ^2 is

$$\begin{aligned}&\left[\frac{1}{4}\left(\frac{.0003339}{(2.21)(.0000047)} - 1\right), \frac{1}{4}\left(\frac{.0003339}{(.3496503)(.0000047)} - 1\right)\right]\\ = \;&[7.786488, 50.54543].\end{aligned}$$

Taking the square roots of the endpoints, the interval for σ_τ/σ is $[2.79, 7.11]$. σ_τ/σ is a comparison of the size of variation in rod lengths among bundles to the size of within-bundle variation. The interval implies that, with 90% confidence, the variation in rod lengths among bundles is between 2.8 to 7.1 times larger than the within-bundle variation.

(c) Based on part (b), the principal origin of variability is from differences among bundles.

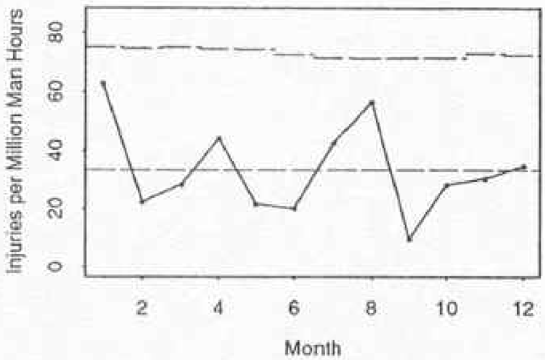
21. (a) Use equation (7-104). $\hat{u} = \frac{78}{2.327} = 33.51955$.

(b) Use equations (7-105) and (7-106).

$$\text{Center Line}_{\hat{u}_i} = 33.51955.$$

The control limits depend on the number of man hours k_i. The following table gives upper control limits for each of the k_i's in the data set. (The lower control limits are all negative, so there are no lower control limits for any of these k_i's.)

k_i	$UCL_{\hat{u}_i}$
.175	75.03898
.178	74.68761
.180	74.45826
.183	74.12131
.195	72.85219
.198	72.55308
.200	72.35742
.210	71.42143
.211	71.33151
.212	71.24222

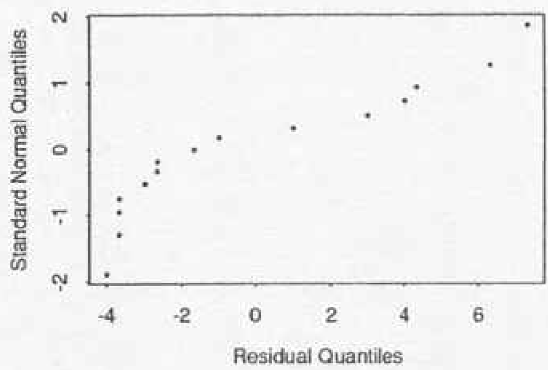


None of the $\hat{u}_i$'s plotted outside its control limit, so there is no evidence that the underlying accident rate was changing during the 12 months.

22. (a) Data within each sample must be iid normal, the samples must be independent, and the five distributions must have the same standard deviation.

Label the No Paper, Newsprint, Construction, Computer, and Magazine conditions Samples 1, 2, 3, 4, and 5 respectively. See equation (7-3). The necessary computations are given in the table below.

Condition	y_{ij}	$\hat{y}_{ij} = \bar{y}_i$	e_{ij}
1	24	26.67	-2.67
1	25	26.67	-1.67
1	31	26.67	4.33
2	61	54.67	6.33
2	51	54.67	-3.67
2	52	54.67	-2.67
3	72	73.00	-1.00
3	70	73.00	-3.00
3	77	73.00	4.00
4	59	62.67	-3.67
4	59	62.67	-3.67
4	70	62.67	7.33
5	54	58.00	-4.00
5	59	58.00	1.00
5	61	58.00	3.00



The plot is somewhat non-linear, giving some evidence that the normal part of the one-way normal model assumptions may not be true.

(b) Using equation (7-7), $s_p = 4.712$ oz., with $n - r = 15 - 5 = 10$ degrees of freedom associated with it. This measures the magnitude of baseline variation within any of the 5 conditions, assuming it is the same for all 5 conditions.

(c) Use equation (7-14). The $\pm$ is the same for all five intervals because all five sample sizes are the same. For 95% confidence, the appropriate t is $t = Q_{10}(.975) = 2.228$, from Table B-4. The resulting $\pm$ part is

$$2.228\frac{4.712}{\sqrt{3}} = 6.061 \text{ oz.}$$

For μ_1 the interval is $[20.61, 32.73]$ oz. For μ_2 the interval is $[48.61, 60.73]$ oz. For μ_3 the interval is $[66.94, 79.06]$ oz. For μ_4 the interval is $[56.61, 68.73]$ oz. For μ_5 the interval is $[51.94, 64.06]$ oz.

(d) Use equation (7-15). Δ is the same for all 10 intervals because all five sample sizes are the same. t is the same as in part (c). The resulting Δ is

$$2.228(4.712)\sqrt{\frac{1}{3} + \frac{1}{3}} = 8.57 \text{ oz.}$$

(e) Use equation (7-20) with

$$c_1 = -1$$

and

$$c_2 = c_3 = c_4 = c_5 = -\frac{1}{4}.$$

t is the same as in part (c). The resulting interval is

$$\begin{aligned} & 26.667 - \frac{54.667 + 73.0 + 62.667 + 58.0}{4} - 35.41667 \pm 2.228(4.712)\sqrt{.4166667} \\ = \quad & -35.41667 \pm 6.776197 \\ = \quad & [-42.19, -28.64] \text{ oz.} \end{aligned}$$

(f) Use equation (7-28). With $r = 5$ and $\nu = n - r = 10$, Table B-8-A gives $k_2^* = 3.10$. The resulting $\pm$ part is

$$3.10\frac{4.712}{\sqrt{3}} = 8.433 \text{ oz.}$$

For μ_1 the interval is $[18.23, 35.10]$ oz. For μ_2 the interval is $[46.23, 63.10]$ oz. For μ_3 the interval is $[64.57, 81.43]$ oz. For μ_4 the interval is $[54.23, 71.10]$ oz. For μ_5 the interval is $[49.57, 66.43]$ oz. These intervals are wider than the ones in part (c). In order to ensure an overall (simultaneous) confidence of 95%, you need to make the individual 95% confidence intervals wider. Taken together, the intervals in part (c) have simultaneous confidence less than 95%.

(g) Use equation (7-36). For 95% confidence, with Number of Means to be Compared = 5 and $\nu = n - r = 10$, Table B-10-A gives $q^* = 4.65$. The resulting Δ is

$$\frac{4.65}{\sqrt{2}}(4.712)\sqrt{\frac{1}{3}+\frac{1}{3}} = 10.33 \text{ oz.}$$

This value is larger than the one in part (d), for the same reasons given in part (f).

(h) 1. H_0: $\mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5$.
2. H_a: All 5 means are not the same.
3. The test statistic is given by equation (7-48). The reference distribution is the $F_{4,10}$ distribution. Large observed values of F will be considered as evidence against H_0.
4. The samples give

$$f = \frac{\frac{1}{4}(3584)}{(4.712)^2} = 40.36.$$

5. The observed level of significance is

$$P(\text{an } F_{4,10} \text{ random variable} > 40.36)$$

Using Tables B-6, the observed value of F is greater than $Q(.999) = 11.28$, and so the p-value is less than .001. This is overwhelming evidence that all 5 conditions do not produce the same mean cutting force.

(i) Using the general form given in Table 7-12, the calculations yield the following table.

Source	SS	df	MS	F
Treatments	3584.0	4	896.0	40.36
Error	222.0	10	22.2	
Total	3806.0	14		

23. (a) Using equation (7-7), $s_p = .1917 \times 10^3$ psi, with $n - r = 12 - 3 = 9$ degrees of freedom associated with it. This measures the magnitude of baseline variation within any of the 3 conditions, assuming it is the same for all 3 conditions.

(b) Use equation (7-14). For a 99% one-sided confidence interval, first construct a 98% two-sided confidence interval and use the lower endpoint. For a 98% two-sided confidence interval, the appropriate t is $t = Q_9(.99) = 2.821$ from Table B-4. The resulting 99% one-sided confidence interval is

$$5.3675 - 2.821\frac{.1917}{\sqrt{4}} = 5.09709 \times 10^3 \text{ psi.}$$

(c) Use equation (7-15). For a 99% two-sided confidence interval, the appropriate t is $t = Q_9(.995) = 3.250$ from Table B-4. The resulting interval is

$$\begin{aligned} 5.3675 - 4.9900 \pm 3.250(.1917)\sqrt{\frac{1}{4}+\frac{1}{4}} & \\ &= .3775 \pm .4405744 \\ &= [-.06307, .81807] \times 10^3 \text{ psi.} \end{aligned}$$

Since zero is in this interval, there is not convincing evidence of a difference between the two mean strengths. But zero is near the edge of the interval, so there is some evidence that the 3% air specimens have a larger mean strength than the 6% air specimens.

(d) Use equation (7-36). For 95% confidence, with Number of Means to be Compared = 3 and $\nu = n - r = 9$, Table B-9-A gives $q^* = 3.95$. The resulting Δ is

$$\frac{3.95}{\sqrt{2}}(.1917)\sqrt{\frac{1}{4}+\frac{1}{4}} = .378633 \times 10^3 \text{ psi.}$$

(e) Using proposition (7-1) and the definitions following it,

$$SSE = (n-r)s_p^2 = (9)(.1917)^2 = .33078,$$

and $SSTr = SSTot - SSE = 13.83002$. Using the general form given in Table 7-12, the degrees of freedom for Treatment are $r - 1 = 2$, the degrees of freedom for Error are $n - r = 9$, and the total degrees of freedom are $2 + 9 = 11 = n - 1$. $MSTr$ and MSE are obtained by dividing the appropriate sums of squares by the corresponding degrees of freedom, and F is obtained from $MSTr/MSE$.

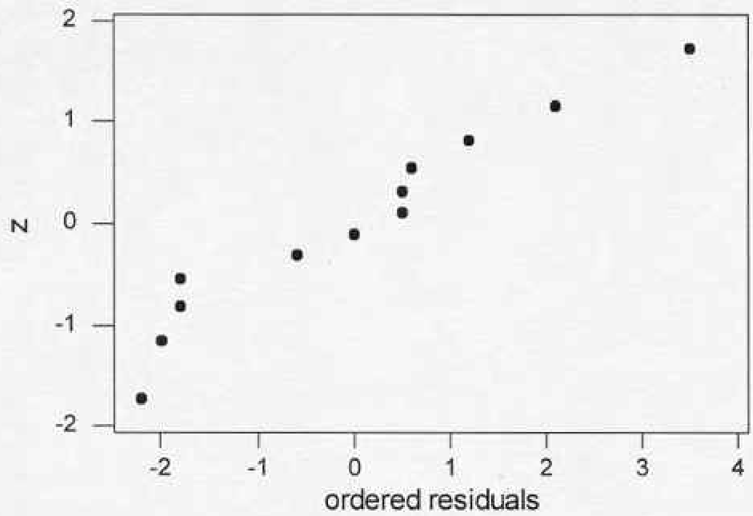
Source	SS	df	MS	F
Treatment	13.83002	2	6.91501	188.15
Error	.33078	9	.03675333	
Total	14.1608	11		

The p-value for the test is

$$P(\text{an } F_{2,9} \text{ random variable} > 188.14).$$

Using Tables B-6, the observed value of F is greater than $Q(.999) = 16.39$, so the p-value is less than .001. This is overwhelming evidence that all 3 mean strengths are not the same.

(24) **(a)** To analyze the data in this problem with methods of chapter 7 it is necessary that the depth data for each Pulse level has a common variability and the depth data is normally distributed for each Pulse level. It seems the data does deviate somewhat from a normal distribution as seen by the non-linear normal probability plot below.



(b) $s_{pooled} = 1.962$. This quantity estimates the common standard deviation of "depth" levels at a given level of Pulse.

(c) The form of each interval is: $\bar{x}_i \pm t(s_p/\sqrt{n})$

For 100 pulse: $7.4 \pm t_9(1.962/2)$ or 7.4 ± 2.21902. The 95% confidence interval for the mean depth at 100 Pulse is (5.181, 9.619).

For 500 pulse: 26 ± 2.21902. The 95% confidence interval for the mean depth of 500 Pulse level is (23.781, 28.219).

For 1000 pulse: 35.4 ± 2.21902. The 95% confidence interval for the mean depth of 1000 Pulse level is (33.18, 37.62).

(d) $\Delta = t_9\, s_p\, (1/4 + 1/4)^{1/2} = 2.262(1.962)(1/2)^{1/2} = 3.138$.

(e) $(1/400)(26 - 7.4) - (1/500)(35.4 - 26) = .0465 - .0188 = .0277$ estimates $\mu_{500}\ (1/400 + 1/500) - \mu_{100}\,(1/400) - \mu_{1000}\,(1/500)$.

$\Delta = t_9\,[s_p/2]\,(\,[.0045]^2 + [.0025]^2 + [.002]^2\,)^{1/2} = 2.262(1.962/2)(.0055227)$
$\Delta = .012255$. Thus, the 95% confidence interval for

$\mu_{500}\,(1/400 + 1/500) - \mu_{100}\,(1/400) - \mu_{1000}\,(1/500)$ is

$.0277 \pm .012255$ or (.015445, .03995). No, it does not seem reasonable that there is a linear increase in depth as the number of pulses increases. This interval does not include zero which implies with high confidence the "slope" is not the same from 100 pulses to 500 pulses as it is from 500 pulses to 1000 pulses.

(f) The PR "lengths" are longer. It is sensible because the PR confidence intervals have a level of confidence that applies to the "correctness" of the whole set. The intervals in (c) have a level of confidence that applies to a single interval, not the whole set. Thus, these intervals (in (c)) are shorter.

For the PR method, using equation 7-28, $D = k_2^*\, s_p/\sqrt{n}$. From Table B-8-A, $r = 3$ and $\nu = 9$. Thus, $k_2^* = 2.885$. Hence,

$\Delta = (2.885)(1.962)(1/2) = 2.83$

Pulse level 100: 7.4 ± 2.83 or (4.57, 10.23) is a PR 95% confidence interval for the mean depth at Pulse level of 100.

Pulse level 500: 26 ± 2.83 or (23.17, 28.83) is a PR 95% confidence interval for the mean depth at Pulse level of 500.

Pulse level 1000: 35.4 ± 2.83 or (32.57, 38.23) is a PR 95% confidence interval for the mean depth at Pulse level of 1000.

(g) Using equation 7-36 and Table B-9-A, $\nu = 9$, # of means = 3, $q^* = 3.95$,
$\Delta = [q^*/\sqrt{2}\,]s_p(1/4 + 1/4)^{1/2} = (3.95/\sqrt{2}\,)(1.962)(1/2)^{1/2} = 3.875$. The $\Delta = 3.138$ in (d) is smaller than the $\Delta = 3.875$ here.

(h) 1. Ho: $\mu_1 = \mu_2 = \mu_3$. 2. Ha: at least one inequality ,
3. F = MStreatment/Mserror, 4. F = 812.21/3.85 = 211.03 based on 2,9 df.
5. p-value = P(F > 211.03) = 0. Thus, conclude Ha: at least one difference exists amongst the three means.

(i)

Source	df	SS	MS	F	p
Pulse	2	1624.43	812.21	211.03	0
Error	9	34.64	3.85		
Total	11	1659.07			

(25)(a)

$s_p^2 = 3[(.096)^2 + (.426)^2 + (.174)^2 + (.168)^2]/12 = .0623$. The quantity $s_p^2 = .0623$ estimates the common variance of flight times for helicopters from a given design.

(b) $\bar{y}_1 \pm t_{12}(s_p/\sqrt{n})$ becomes 1.64 ± (2.179)(.24959/2) or 1.64 ± .27193. The interval (1.368, 1.912) is a 95% two-sided confidence interval for the mean flight time of helicopters of Design #1.

(c) Using equation 7-28, r = 4, $\nu = 12$, $k_2^* = 2.892$.
$\Delta = k_2^* s_p/(4)^{1/2} = (2.892)(.24959)/2 = .3609$.

(d) $\bar{y}_1 - \bar{y}_2 \pm t_{12} s_p \sqrt{(1/4 + 1/4)}$ becomes

(1.64 - 2.545) ± (2.179)(.24959)($1/\sqrt{2}$) or -.905 ± .38456. The interval (-1.2896, - .5204) is a 95% two-sided confidence interval for the difference (Design#1 - Design#2) in average time.

(e) Using equation 7-36, $\Delta = (q^* s_p)(1/\sqrt{2})(1/4 + 1/4)^{1/2}$
$\Delta = (4.2)(.24959)(.7071)(.7071) = .524$.

(f) Yes, some differences in average flight times exceed .524.

(g) 1. Ho: $\mu_1 = \mu_2 = \mu_3 = \mu_4$, 2. Ha: At least 1 inequality,
3. F = Mstreatment/ Mserror with 3,12 df. 4. F = .33498/.0623 =5.376.
5. P(F > 5.376) = p-value and thus, .01 < p < .05. Conclude Ha: at least 1 inequality.

(h) $1/2(\bar{y}_1 + \bar{y}_3) - 1/2(\bar{y}_2 + \bar{y}_4)$ = 1/2(1.64 + 1.51) - 1/2 (2.545 + 2.6) = -.9975.
$\Delta = t_{12}s_p(1/2)(1/4 + 1/4 + 1/4 + 1/4)^{1/2} = 2.179(.24959)(1/2)(1) = .271928$.

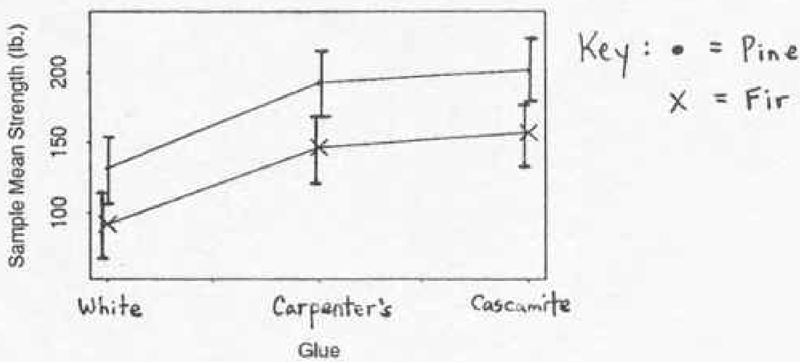
-.9975 ± .271928 or (-1.26943, -.72557) is a 95% interval estimate of $1/2(\mu_1 + \mu_3) - (1/2)(\mu_2 + \mu_4)$.

Chapter 8: Inference for Multisample Studies with Full Factorial Structure

Section 1

1. (a) Using equation (7-7), $s_p = 13.113$ lb., with $n - r = 18 - 6 = 12$ degrees of freedom associated with it. For the error bars, use equation (7-28). With $r = 6$ and $\nu = n - r = 18 - 6 = 12$, Table B-8-A gives $k_2^* \approx 3.11$. The six intervals will all be the same size, because the six sample sizes are all the same. The resulting $\pm$ part is

$$3.11 \frac{13.113}{\sqrt{3}} = 23.54 \text{ lb.}$$



(b) The averages needed are given in the table below.

		GLUE (Factor B) White	Carpenter's	Cascamite	
WOOD (Factor A)	Pine	$\bar{y}_{11} = 131.667$	$\bar{y}_{12} = 191.278$	$\bar{y}_{13} = 201.333$	$\bar{y}_{1.} = 175.222$
	Fir	$\bar{y}_{21} = 92.000$	$\bar{y}_{22} = 146.333$	$\bar{y}_{23} = 156.667$	$\bar{y}_{2.} = 131.667$
		$\bar{y}_{.1} = 111.833$	$\bar{y}_{.2} = 169.500$	$\bar{y}_{.3} = 179.000$	$\bar{y}_{..} = 153.444$

The fitted main effects are

$$a_1 = \bar{y}_{1.} - \bar{y}_{..} = 21.778$$
$$a_2 = \bar{y}_{2.} - \bar{y}_{..} = -21.778$$
$$b_1 = \bar{y}_{.1} - \bar{y}_{..} = -41.611$$
$$b_2 = \bar{y}_{.2} - \bar{y}_{..} = 16.056$$
$$b_3 = \bar{y}_{.3} - \bar{y}_{..} = 25.556$$

The fitted interactions are

$$ab_{11} = \bar{y}_{11} - (\bar{y}_{..} + a_1 + b_1) = -1.944$$
$$ab_{12} = \bar{y}_{12} - (\bar{y}_{..} + a_1 + b_2) = 1.389$$
$$ab_{13} = \bar{y}_{13} - (\bar{y}_{..} + a_1 + b_3) = .556$$
$$ab_{21} = \bar{y}_{21} - (\bar{y}_{..} + a_2 + b_1) = 1.944$$
$$ab_{22} = \bar{y}_{22} - (\bar{y}_{..} + a_2 + b_2) = -1.389$$
$$ab_{23} = \bar{y}_{23} - (\bar{y}_{..} + a_2 + b_3) = -.556$$

Use equation (8- 6) and Table 8.3 to construct the confidence intervals for the interactions. For 95% two-sided intervals, the appropriate t is $t = Q_{12}(.975) = 2.179$ from Table B-4. The confidence intervals are

$$\begin{aligned} & ab_{ij} \pm 2.179(13.113)\sqrt{\frac{(2-1)(3-1)}{(3)(2)(3)}} \\ = \; & ab_{ij} \pm 9.524 \text{ lb.} \end{aligned}$$

Looking at the ab_{ij}'s computed above, all of the confidence intervals for the underlying interactions contain zero. This means that the interactions are not detectable.

For the Wood main effects, use equation (8- 6) and Table 8.3. The confidence intervals are

$$\begin{aligned} & a_i \pm 2.179(13.113)\sqrt{\frac{2-1}{(3)(2)(3)}} \\ = \; & a_i \pm 6.735 \text{ lb.} \end{aligned}$$

Both of these confidence intervals do not contain zero, indicating that the main effects for Wood are detectable.

For the Glue main effects, use equation (8-16) and equation (8-11). The confidence intervals are

$$\begin{aligned} & b_j \pm 2.179(13.113)\sqrt{\frac{3-1}{(3)(2)(3)}} \\ = \; & b_j \pm 9.524 \text{ lb.} \end{aligned}$$

All of these confidence intervals do not contain zero, indicating that the main effects for Glue are detectable.

(c) Use equation (8- 10). For 95% confidence, with $\nu = n - IJ = 12$ and $J = 3$ means to be compared, Table B- 9 -A gives $q^* = 3.77$. The resulting $\pm$ part is

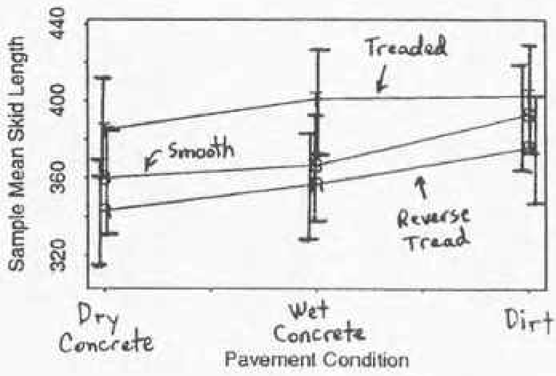
$$\frac{(3.77)(13.113)}{\sqrt{(2)(3)}} = 20.1818 \text{ lb.}$$

The resulting interval for the mean difference between white and carpenter's glue is $[-77.85, -37.48]$ lb. The resulting interval for the mean difference between white and cascamite glue is $[-87.35, -46.98]$ lb. The resulting interval for the mean difference between carpenter's and cascamite glue is $[-29.68, 10.68]$ lb.

2. (a) Using equation (7-7), $s_p = 33.252$ cm, with $n - r = 54 - 9 = 45$ degrees of freedom associated with it. This measures the magnitude of baseline variation within any of the 9 conditions, assuming it is the same for all 9 conditions.

(b) Use equation (7-14) for the error bars. The $\pm$ part is the same for all nine intervals because all nine sample sizes are the same. For 95% confidence, the appropriate t is $t = Q_{45}(.975) \approx 2.016$, from Table B-4. The resulting $\pm$ part is

$$2.016\frac{33.252}{\sqrt{6}} = 27.368 \text{ cm.}$$



(c) Considering the interactions first, the non-parallelism in the plot does not seem to be large relative to the size of the error bars. The error bars seem large enough that one could imagine that the true, underlying μ_{ij}'s might be parallel. The slopes of the lines seem small relative to the size of the error bars, suggesting that the main effects for Pavement Condition may not be distinguishable from background noise. The distance between the lines for treaded and smooth (or treaded and reverse) tires seen to be large enough to be distinguishable from background noise. These distances represent the main effects for Tire Type.

(d) The averages needed are given in the table below.

		PAVEMENT CONDITION (Factor B)			
		Dry Concrete	Wet Concrete	Dirt	
TIRE	Smooth	$\bar{y}_{11} = 359.8$	$\bar{y}_{12} = 366.5$	$\bar{y}_{13} = 393.0$	$\bar{y}_{1\cdot} = 373.100$
TYPE	Reverse	$\bar{y}_{21} = 343.0$	$\bar{y}_{22} = 356.7$	$\bar{y}_{23} = 375.7$	$\bar{y}_{2\cdot} = 358.467$
(Factor A)	Treaded	$\bar{y}_{31} = 384.8$	$\bar{y}_{32} = 400.8$	$\bar{y}_{33} = 402.5$	$\bar{y}_{3\cdot} = 396.033$
		$\bar{y}_{\cdot 1} = 362.533$	$\bar{y}_{\cdot 2} = 374.667$	$\bar{y}_{\cdot 3} = 390.400$	$\bar{y}_{\cdot\cdot} = 375.867$

The fitted main effects are

$$
\begin{aligned}
a_1 &= \bar{y}_{1\cdot} - \bar{y}_{\cdot\cdot} = -2.767 \\
a_2 &= \bar{y}_{2\cdot} - \bar{y}_{\cdot\cdot} = -17.400 \\
a_3 &= \bar{y}_{3\cdot} - \bar{y}_{\cdot\cdot} = 20.167 \\
b_1 &= \bar{y}_{\cdot 1} - \bar{y}_{\cdot\cdot} = -13.333 \\
b_2 &= \bar{y}_{\cdot 2} - \bar{y}_{\cdot\cdot} = -1.200 \\
b_3 &= \bar{y}_{\cdot 3} - \bar{y}_{\cdot\cdot} = 14.533
\end{aligned}
$$

The fitted interactions are

$$
\begin{aligned}
ab_{11} &= \bar{y}_{11} - (\bar{y}_{\cdot\cdot} + a_1 + b_1) = .033 \\
ab_{12} &= \bar{y}_{12} - (\bar{y}_{\cdot\cdot} + a_1 + b_2) = -5.400 \\
ab_{13} &= \bar{y}_{13} - (\bar{y}_{\cdot\cdot} + a_1 + b_3) = 5.367 \\
ab_{21} &= \bar{y}_{21} - (\bar{y}_{\cdot\cdot} + a_2 + b_1) = -2.133 \\
ab_{22} &= \bar{y}_{22} - (\bar{y}_{\cdot\cdot} + a_2 + b_2) = -.567 \\
ab_{23} &= \bar{y}_{23} - (\bar{y}_{\cdot\cdot} + a_2 + b_3) = 2.700 \\
ab_{31} &= \bar{y}_{31} - (\bar{y}_{\cdot\cdot} + a_3 + b_1) = 2.100 \\
ab_{32} &= \bar{y}_{32} - (\bar{y}_{\cdot\cdot} + a_3 + b_2) = 5.967 \\
ab_{33} &= \bar{y}_{33} - (\bar{y}_{\cdot\cdot} + a_3 + b_3) = -8.067
\end{aligned}
$$

(e) Use equation (8-6) and Table 8.3 to construct the confidence intervals. For 95% two-sided intervals, the appropriate t is the same as for part (b). The confidence intervals for the interactions are

$$
\begin{aligned}
& ab_{ij} \pm 2.016(33.252)\sqrt{\frac{(3-1)(3-1)}{(6)(3)(3)}} \\
= \; & ab_{ij} \pm 18.245 \text{ cm.}
\end{aligned}
$$

Looking at the ab_{ij}'s computed above, all of the confidence intervals for the underlying interactions contain zero. This means that the interactions are not detectable. This confirms the tentative conclusions made in part (c).

(f) Use equation (8- 6) and Table 8.3. For 95% two-sided intervals, the appropriate t is the same as for part (b). The resulting intervals are

$$
\begin{aligned}
& a_i - a_{i'} \pm 2.016(33.252)\sqrt{\frac{2}{(6)(3)}} \\
= \; & a_i - a_{i'} \pm 22.346 \text{ cm.}
\end{aligned}
$$

The resulting interval for the mean difference between smooth and reverse tread tires is $[-7.71, 36.98]$ cm. The resulting interval for the mean difference between smooth and treaded tires is $[-45.28, -.59]$ cm. The resulting interval for the mean difference between reverse tread and treaded tires is $[-59.91, -15.22]$ cm. There is not a detectable difference between smooth and reverse tread main effects, because the first of these confidence intervals contains zero. The last two intervals do not contain zero, indicating that there is a detectable difference between the main effects of both smooth and treaded and reverse and treaded tires. This agrees with the tentative conclusion reached in part (c).

(g) Use equation (8- 8). For 95% confidence, with $\nu = n - IJ = 45$ and $I = 3$ means to be compared, Table B-9 -A gives $q^* \approx 3.43$. The resulting $\pm$ part is

$$
\frac{(3.43)(33.252)}{\sqrt{(3)(6)}} = 26.883 \text{ cm.}
$$

The resulting interval for the mean difference between smooth and reverse tread tires is $[-12.25, 41.52]$ cm. The resulting interval for the mean difference between smooth and treaded tires is $[-49.82, 3.95]$ cm. The resulting interval for the mean difference between reverse tread and treaded tires is $[-64.45, -10.68]$ cm. The only detectable difference using 95% simultaneous confidence is the difference between the reverse tread main effect and the treaded tire main effect.

Section 2

1. (a) Using equation (7-7), $s_p = .03290416$, with $n - r = 24 - 8 = 16$ degrees of freedom associated with it. Use equation (8-13) for the confidence intervals. For 95% two-sided confidence intervals, the appropriate t is $t = Q_{16}(.975) = 2.120$ from Table B-4. The resulting $\pm$ part is

$$
2.120\frac{.03290416}{\sqrt{(3)(8)}} = .01423905.
$$

The BC interaction is detectable, as well as the B and C main effects, since their corresponding confidence intervals do not contain zero.

(b) This was done in Exercise 2, Section 3, Chapter 4, for a "B and C main effects only" model. However, the BC interaction seems to be important, so I will include it here. Using the reverse Yates algorithm:

Fitted Effect	Value	Cycle 1	Cycle 2	Cycle 3 ($\hat{y}$)	
abc_{222}	0	-.12981	.14113	2.07480	$= \hat{y}_{\text{abc}}$
bc_{22}	-.12981	.27095	1.93366	2.07480	$= \hat{y}_{\text{bc}}$
ac_{22}	0	-.76554	.14113	3.86550	$= \hat{y}_{\text{ac}}$
c_2	.27095	2.69920	1.93366	3.86550	$= \hat{y}_{\text{c}}$
ab_{22}	0	-.12981	.40076	1.79253	$= \hat{y}_{\text{ab}}$
b_2	-.76554	.27095	3.46474	1.79253	$= \hat{y}_{\text{b}}$
a_2	0	-.76554	.40076	3.06398	$= \hat{y}_{\text{a}}$
$\bar{y}_{...}$	2.69920	2.69920	3.46474	3.06398	$= \hat{y}_{(1)}$

There will be a total of 24 residuals, one for each observation. To compute the residuals, take each (transformed) observation and subtract the $\hat{y}$ that corresponds to the factor-level combination from which the observation came. For example, $\hat{y}_{(1)} = 3.06398$ should be subtracted from the natural logs of each of the 3 observations from combination (1), 21.12, 21.11, and 20.80, producing the 3 residuals $-.01375954$ $-.01423314$ $-.02902701$.

Using equation (8-16),

$$s^2_{\text{FE}} = \frac{1}{(3)(8)-4}(.01975849) = .0009879243,$$

so $s_{\text{FE}} = \sqrt{.0009879243} = .03143126$, with 20 degrees of freedom associated with it. This is relatively close to $s_p = .03290416$.

(c) Using equation (8-18) for a 95% two-sided interval, the appropriate t is $t = Q_{20}(.975) = 2.086$. The resulting interval is

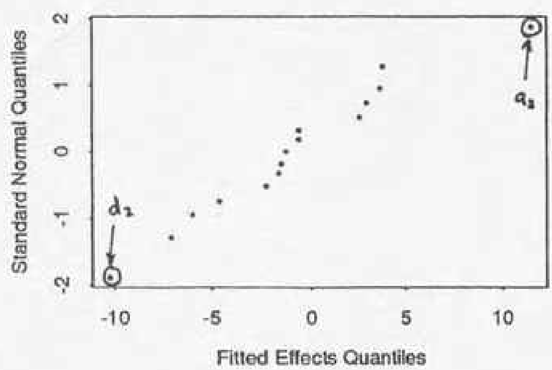
$$\begin{aligned} & 3.06398 \pm 2.086(.03143126)\sqrt{\frac{4}{(3)(8)}} \\ = \;& 3.06398 \pm .02676705 \\ = \;& [3.037213, 3.090747]. \end{aligned}$$

Using equation (7-14), for 95% confidence, the appropriate t is the same as the one used in part (a). The resulting interval is

$$\begin{aligned} & 3.044973 \pm 2.120\frac{.03290416}{\sqrt{3}} \\ = \;& 3.044973 \pm .04027412 \\ = \;& [3.004699, 3.085248]. \end{aligned}$$

The interval made using the few-effects model is shorter. This is expected, since the few-effects model is more specialized.

2. (a)



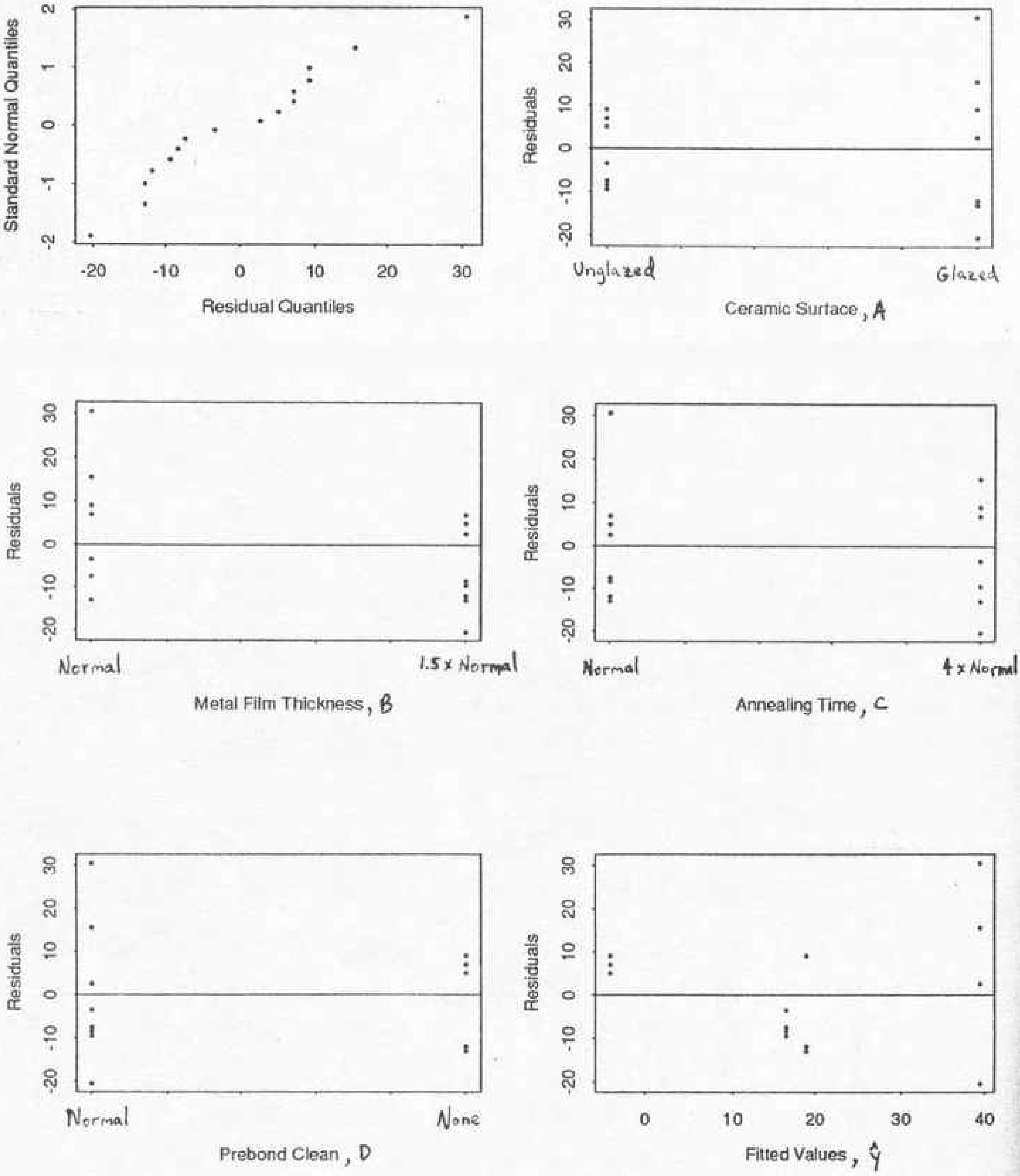
Only the point a_2 plots "off the line", suggesting that the main effect for factor A may be detectably larger than the other effects. It does not plot far away from the other points, so it is difficult to be very confident about this conclusion. d_2 is almost as big as a_2 in absolute value, so if the A main effect is judged detectable, so should the D main effect.

(b) One possibility is to include only the A and D main effects. Using the reverse Yates algorithm:

Fitted Effect	Value	Cycle 1	Cycle 2	Cycle 3	Cycle 4 ($\hat{y}$)	
$abcd_{2222}$	0	0	0	-10.250	18.875	$= \hat{y}_{\text{abcd}}$
bcd_{222}	0	0	-10.250	29.125	-4.125	$= \hat{y}_{\text{bcd}}$
acd_{222}	0	0	0	-10.250	18.875	$= \hat{y}_{\text{acd}}$
cd_{22}	0	-10.250	29.125	6.125	-4.125	$= \hat{y}_{\text{cd}}$
abd_{222}	0	0	0	-10.250	18.875	$= \hat{y}_{\text{abd}}$
bd_{22}	0	0	-10.250	29.125	-4.125	$= \hat{y}_{\text{bd}}$
ad_{22}	0	0	0	-10.250	18.875	$= \hat{y}_{\text{ad}}$
d_2	-10.250	29.125	6.125	6.125	-4.125	$= \hat{y}_{\text{d}}$
abc_{222}	0	0	0	-10.250	39.375	$= \hat{y}_{\text{abc}}$
bc_{22}	0	0	-10.250	29.125	16.375	$= \hat{y}_{\text{bc}}$
ac_{22}	0	0	0	-10.250	39.375	$= \hat{y}_{\text{ac}}$
c_2	0	-10.250	29.125	6.125	16.375	$= \hat{y}_{\text{c}}$
ab_{22}	0	0	0	-10.250	39.375	$= \hat{y}_{\text{ab}}$
b_2	0	0	-10.250	29.125	16.375	$= \hat{y}_{\text{b}}$
a_2	11.500	0	0	-10.250	39.375	$= \hat{y}_{\text{a}}$
$\bar{y}_{....}$	17.625	6.125	6.125	6.125	16.375	$= \hat{y}_{(1)}$

As you may notice, some of the $\hat{y}$'s for this model are negative! It would probably be better to analyze the natural logarithms of these data, but for rest of this problem, imagine that there can be a negative number of pull-outs.

There will be a total of 16 residuals, one for each observation. To compute the residuals, take each observation and subtract the $\hat{y}$ that corresponds to the factor-level combination from which the observation came. For example, $\hat{y}_{(1)} = 16.375$ should be subtracted from the observation from combination (1), 9, producing the residual 7.375.



The plot of residuals versus levels of B shows a fairly strong pattern, indicating that there is some effect due to factor B that the present model is not accounting for. The plot of Residuals versus A indicates that there is more variation in the response for glazed surfaces than for unglazed surfaces, and the plot of Residuals versus Fitted Values suggests that variation tends to increase for larger values of the response. The normal plot is roughly linear, and so provides no evidence that the current model is inappropriate. The patternless plot of residuals versus C indicates that C does not seem to affect the response.

(c) According to the smallest fitted values from part (b), Factor D (Prebond Clean) should be set at its high level (no clean), and Factor A (Ceramic Surface) should be set at its low level (unglazed).

Using equation (8-16),

$$s^2_{\mathrm{FE}} = \frac{1}{(1)(16)-3}(2594.75) = 199.5962,$$

so $s_{\mathrm{FE}} = \sqrt{199.5962} = 14.12785$, with 13 degrees of freedom associated with it.

Using equation (8-18) for a 95% two-sided interval, the appropriate t is $t = Q_{13}(.975) = 2.160$. The resulting interval is

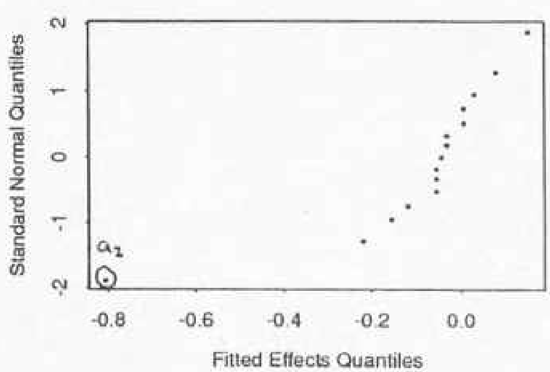
$$\begin{aligned} & -4.125 \pm 2.160(14.12785)\sqrt{\frac{3}{(1)(16)}} \\ = \; & -4.125 \pm 13.21388 \\ = \; & [-17.34, 9.09]. \end{aligned}$$

Since there cannot be a negative number of pull-outs, set the lower endpoint equal to zero: $[0, 9.09]$.

3. (a) Using the Yates algorithm:

Comb	$\bar{y}$	Cycle 1	Cycle 2	Cycle 3	Cycle 4	Cycle 4 ÷ 16	
(1)	4.2	7.3	14.7	29.2	57.5	3.59375	$= \bar{y}_{....}$
a	3.1	7.4	14.5	28.3	-12.9	-0.80625	$= a_2$
b	4.5	6.7	14.5	-5.2	2.5	0.15625	$= b_2$
ab	2.9	7.8	13.8	-7.7	-3.5	-0.21875	$= ab_{22}$
c	3.9	7.0	-2.7	1.2	-0.9	-0.05625	$= c_2$
ac	2.8	7.5	-2.5	1.3	-0.5	-0.03125	$= ac_{22}$
bc	4.6	6.5	-3.5	-0.8	1.3	0.08125	$= bc_{22}$
abc	3.2	7.3	-4.2	-2.7	0.5	0.03125	$= abc_{222}$
d	4.0	-1.1	0.1	-0.2	-0.9	-0.05625	$= d_2$
ad	3.0	-1.6	1.1	-0.7	-2.5	-0.15625	$= ad_{22}$
bd	5.0	-1.1	0.5	0.2	0.1	0.00625	$= bd_{22}$
abd	2.5	-1.4	0.8	-0.7	-1.9	-0.11875	$= abd_{222}$
cd	4.0	-1.0	-0.5	1.0	-0.5	-0.03125	$= cd_{22}$
acd	2.5	-2.5	-0.3	0.3	-0.9	-0.05625	$= acd_{222}$
bcd	5.0	-1.5	-1.5	0.2	-0.7	-0.04375	$= bcd_{222}$
abcd	2.3	-2.7	-1.2	0.3	0.1	0.00625	$= abcd_{2222}$

(b)



It appears that only the main effect for A is detectably larger than the rest of the effects, since the point for a_2 is far away from the rest of the fitted effects.

(c) Because a_2 is large and negative, using monk's cloth results in less inches burned for either treatment. However (because ab_{22} is negative), the difference between monk's cloth and sateen is greater for treatment Y than for treatment X. To minimize the number of inches burned, use monk's cloth and treatment Y.

Section 3

1. Multiplying the defining relation through by A, A↔BCDE. This means that only the sum of the A main effect and the BCDE interaction can be estimated. If these effects are large but opposite in sign, their sum will be small. Since only their sum can be estimated, each of these effects could be large and still go undetected.

2. (a) Using the Yates algorithm:

Use defining relation

Comb	$\bar{y}$	C1	C2	C3	C4	C4 ÷ 16	Sum Estimated
e	8.5	16.4	32.8	69.1000	131.7	8.23125	$\mu_{.....} + \alpha\beta\gamma\delta\epsilon_{22222}$
a	7.9	16.4	36.3	62.6000	5.9	0.36875	$\alpha_2 + \beta\gamma\delta\epsilon_{2222}$
b	7.7	18.2	30.3	1.5000	4.1	0.25625	$\beta_2 + \alpha\gamma\delta\epsilon_{2222}$
abe	8.7	18.1	32.3	4.4000	-0.9	-0.05625	$\alpha\beta_{22} + \gamma\delta\epsilon_{222}$
c	9.0	13.8	0.4	-0.1000	5.5	0.34375	$\gamma_2 + \alpha\beta\delta\epsilon_{2222}$
ace	9.2	16.5	1.1	4.2000	-1.1	-0.06875	$\alpha\gamma_{22} + \beta\delta\epsilon_{222}$
bce	8.6	15.4	3.1	2.3000	-1.3	-0.08125	$\beta\gamma_{22} + \alpha\delta\epsilon_{222}$
abc	9.5	16.9	1.3	-3.2000	-1.5	-0.09375	$\alpha\beta\gamma_{222} + \delta\epsilon_{22}$
d	5.8	-0.6	0.0	3.5000	-6.5	-0.40625	$\delta_2 + \alpha\beta\gamma\epsilon_{2222}$
ade	8.0	1.0	-0.1	2.0000	2.9	0.18125	$\alpha\delta_{22} + \beta\gamma\epsilon_{222}$
bde	7.8	0.2	2.7	0.7000	4.3	0.26875	$\beta\delta_{22} + \alpha\gamma\epsilon_{222}$
abd	8.7	0.9	1.5	-1.8000	-5.5	-0.34375	$\alpha\beta\delta_{222} + \gamma\epsilon_{22}$
cde	6.9	2.2	1.6	-0.1000	-1.5	-0.09375	$\gamma\delta_{22} + \alpha\beta\epsilon_{222}$
acd	8.5	0.9	0.7	-1.2000	-2.5	-0.15625	$\alpha\gamma\delta_{222} + \beta\epsilon_{22}$
bcd	8.6	1.6	-1.3	-0.9000	-1.1	-0.06875	$\beta\gamma\delta_{222} + \alpha\epsilon_{22}$
abcde	8.3	-0.3	-1.9	-0.6000	0.3	0.01875	$\alpha\beta\gamma\delta_{2222} + \epsilon_2$

(b) The appropriate t is $t = Q_2(.975) = 4.303$. The ± part is

$$(4.303)\frac{.29}{\sqrt{(1)(16)}} = .312.$$

The sums $\alpha_2 + \beta\gamma\delta\epsilon_{2222}$, $\gamma_2 + \alpha\beta\delta\epsilon_{2222}$, $\delta_2 + \alpha\beta\gamma\epsilon_{2222}$, and $\alpha\beta\delta_{222} + \gamma\epsilon_{22}$ are detectable. The simplest explanation is that the A, C, and D main effects and the CE interaction are responsible for these large sums. If you "knew" that the BCDE 4-factor interaction, the ABCE 4-factor interaction, and the ABD 3-factor interaction were small, then the above interpretation could be made confidently. If there is no reason to believe that these interactions are small, then the above interpretation must be tentative (because of the aliasing).

(c) Based on the signs of the estimates, to maximize bond strength, set A at its high level (1.2 in./sec), C at its high level (120 g), and D at its low level (120 mW). The estimate of $\alpha\beta\delta_{222} + \gamma\epsilon_{22}$ is negative, so if C is set at its high level, E should be set at its low level (10 ms) to maximize bond strength. The abc combination actually had these levels of A, C, D, and E. This combination did have the largest observed bond strength.

3. (a) See Exercise 3, Section 2, Chapter 8.

(b) Use the generator and multiply the appropriate columns.

A	B	C	D↔ABC	Combination
−	−	−	−	(1)
+	−	−	+	ad
−	+	−	+	bd
+	+	−	−	ab
−	−	+	+	cd
+	−	+	−	ac
−	+	+	−	bc
+	+	+	+	abcd

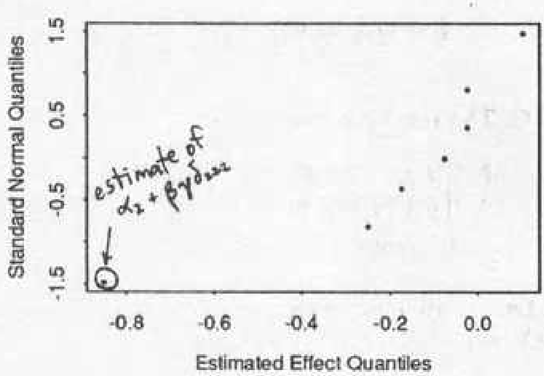
Using the Yates algorithm,

Combination	$\bar{y}$	Cycle 1	Cycle 2	Cycle 3	Cycle 3 ÷ 8	Sum Estimated
(1)	4.2	7.2	15.1	28.8	3.600	$\mu_{....} + \alpha\beta\gamma\delta_{2222}$
ad	3.0	7.9	13.7	-6.8	-.850	$\alpha_2 + \beta\gamma\delta_{222}$
bd	5.0	6.8	-3.3	.8	.100	$\beta_2 + \alpha\gamma\delta_{222}$
ab	2.9	6.9	-3.5	-2.0	-.250	$\alpha\beta_{22} + \gamma\delta_{22}$
cd	4.0	-1.2	.7	-1.4	-.175	$\gamma_2 + \alpha\beta\delta_{222}$
ac	2.8	-2.1	.1	-.2	-.025	$\alpha\gamma_{22} + \beta\delta_{22}$
bc	4.6	-1.2	-.9	-.6	-.075	$\beta\gamma_{22} + \alpha\delta_{22}$
abcd	2.3	-2.3	-1.1	-.2	-.025	$\alpha\beta\gamma_{222} + \delta_2$

The following table shows that these are sums of the appropriate estimates from Exercise 3, Section 2, Chapter 8.

$$
\begin{aligned}
3.600 &= \bar{y}_{....} + abcd_{2222} = 3.59375 + .00635 \\
-.850 &= a_2 + bcd_{222} = -.80625 + (-.04375) \\
.100 &= b_2 + acd_{222} = .15625 + (-.05625) \\
-.250 &= ab_{22} + cd_{22} = -.21875 + (-.03125) \\
-.175 &= c_2 + abd_{222} = -.05625 + (-.11875) \\
-.025 &= ac_{22} + bd_{22} = -.03125 + .00625 \\
-.075 &= bc_{22} + ad_{22} = .08125 + (-.15625) \\
-.025 &= abc_{222} + d_2 = .03125 + (-.05625)
\end{aligned}
$$

(c)



The estimate of $\alpha_2 + \beta\gamma\delta_{222}$ plots off the line. You still might conclude that this is due to the main effect for A, but the conclusion here would be a little more tentative.

Section 4

1. The advantage of fractional factorial experiments is that the same number of factors can be studied using less experimental runs. This is important when there are a large number of factors, and/or experimental runs are expensive. The disadvantage is that there will be ambiguity in the results; only sums of effects can be estimated. The advantage of using a complete factorial experiment is that all means can be estimated, so all effects can be estimated.

2. It will be impossible to separate main effects from 2-factor interactions. You would hope that any interactions are small compared to main effects; the results of the experiment can then be (tentatively) summarized in terms of main effects. (If all interactions are really zero, then it is possible to estimate all of the main effects.) Looking at Table 8-35 , the best possible resolution is 3 (at most).

3. Those effects (or sums of effects) which are nearly zero will have corresponding estimates which are "randomly" scattered about zero. If all of the effects are nearly zero, then one might expect the estimates from the Yates algorithm (excluding the one that includes the grand mean) to be bell-shaped around zero. A normal plot of these estimates would then be roughly linear. However, if there are effects (or sums of effects) which are relatively far from zero, the corresponding estimates will plot away from the rest (off the line), and may be considered more than just random noise. The principle of "sparsity of effects" says that in most situations, only a few of the many effects in a factorial experiment are dominant, and their estimates will then plot off the line on a normal plot.

4. There are a total of $2^7 = 128$ possible factor-level combinations. Since only 32 of these are to be included, this is a 2^{7-2} fractional factorial plan (a quarter fraction).

(a) Start with the generators. Multiply through by F on the first generator to get I↔ABCDF. Multiply through by G on the second generator to get I↔ABCEG. Now multiply these two "I" relationships to get a third:

$$I \leftrightarrow (ABCDF)(ABCEG)$$
$$I \leftrightarrow DEFG$$

So the entire defining relationship is

I↔ABCDF↔ABCEG↔DEFG.

(b) To answer this question, multiply through by C on the defining relation:

C↔ABDF↔ABEG↔CDEFG.

This means that the C main effect is aliased with the ABDF 4-factor interaction, the ABEG 4-factor interaction, and the CDEFG 5-factor interaction. This means that none of these effects can be estimated alone; only the sum of these 4 effects can be estimated based on this plan.

(c) Use the generators and multiply the appropriate columns.

A	B	C	D	E	F↔ABCD	G↔ABCE	Combination
−	−	−	−	−	+	+	fg
+	+	+	−	−	−	−	abc

(d) Using the defining relation,

A↔BCDF↔BCEG↔ADEFG,

ABCD↔F↔DEG↔ABCEFG,

and

BCD↔AF↔ADEG↔BCEFG.

The simplest interpretation is that the effects that are making these estimates large are the A and F main effects and the AF 2-factor interaction. This may not be correct, because other effects in the alias structure could be large; there is no way of knowing based on the information from this fractional factorial. If there is any reason to believe that 3- and higher-factor interactions are unimportant, then this would be the correct interpretation.

5. Use equation (8-12) with $p = 5 - 2 = 3$ in that formula. The appropriate t is $t = Q_4(.95) = 2.132$ from Table B-4. The resulting $\pm$ part is

$$(2.132)(5)\frac{1}{8}\sqrt{4 + \frac{4}{2}} = 3.264.$$

6. (a) Start with the generators. Multiply through by E on the first generator to get I↔ABCE. Multiply through by F on the second generator to get I↔BCDF. Now multiply these two "I" relationships to get a third:

$$\begin{aligned} \mathrm{I} &\leftrightarrow (\mathrm{ABCE})(\mathrm{BCDF}) \\ \mathrm{I} &\leftrightarrow \mathrm{ADEF} \end{aligned}$$

So the entire defining relationship is

I↔ABCE↔BCDF↔ADEF.

(b) Use the generators and multiply the appropriate columns.

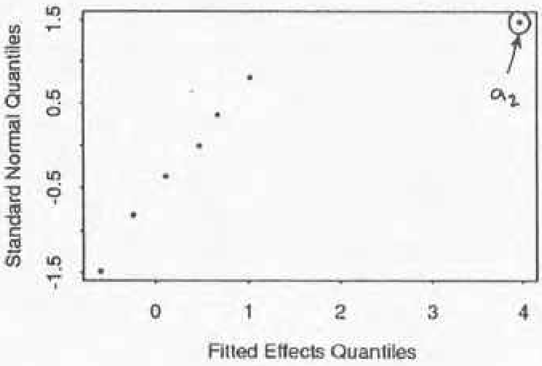
A	B	C	D	E↔ABC	F↔BCD	Combination
−	−	−	−	−	−	ef
+	−	−	−	+	−	ae

(c) Use equation (8-13) with $p = 6 - 2 = 4$ in that formula. The appropriate t is $t = Q_{32}(.95) \approx 1.6939$ from Table B-4. The resulting $\pm$ part is

$$(1.6939)\frac{2.00}{\sqrt{(3)(16)}} = .489.$$

End Chapter Exercises

1. See Ex. 4, Chap. 4 for the computation of the fitted factorial effects.



The point for a_2 plots far away from the rest of the points, indicating that the main effect for factor A is detectably larger than the other effects.

2. See Ex. 5, Chap. 4 for the fitted effects.

(a) Using equation (7-7), $s_p = .05603373$, with $n - r = 16 - 8 = 8$ degrees of freedom associated with it. Use equation (8-13) for the confidence intervals. For 95% two-sided confidence intervals, the appropriate t is $t = Q_8(.975) = 2.306$ from Table B-4. The resulting Δ is

$$2.306\frac{.05603373}{\sqrt{(2)(8)}} = .03230344.$$

(b) The AB interaction is detectable, as well as the A, B, and C main effects, since their corresponding confidence intervals do not contain zero. However, the fitted main effects of A and B are each at least twice as large as any other fitted effect. It seems that the main effects for A and B are the only effects large enough to have an important impact on the breaking strength.

$\alpha_2 - \alpha_1 = 2\alpha_2$ represents the average increase in log(clips) going from .3 mm lead to .7 mm lead. A confidence interval for α_2 can be made using the above Δ and a_2. Multiplying each endpoint by 2, the resulting confidence interval for $\alpha_2 - \alpha_1$ is [1.585153, 1.714367]. This means that the average number of clips needed to break .7 mm lead is between $e^{1.585153} = 4.88$ and $e^{1.714367} = 5.55$ times larger than the number of clips needed to break .3 mm lead (with 95% confidence).

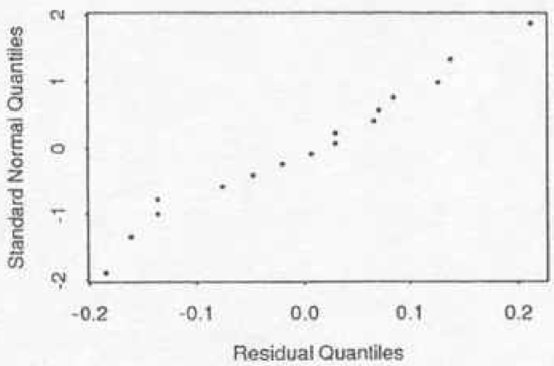
$\beta_1 - \beta_2 = -2\beta_2$ represents the average increase in log(clips) going from 4.5 cm to 3 cm length protruding. A confidence interval for β_2 can be made using the above Δ and b_2. Multiplying each endpoint by -2, the resulting confidence interval for $\beta_1 - \beta_2$ is [.3639931, .4932069]. This means that the average number of clips needed to break lead protruding 3 cm is between $e^{.3639931} = 1.44$ and $e^{.4932069} = 1.64$ times larger than the number of clips needed to break lead protruding 4.5 cm (with 95% confidence).

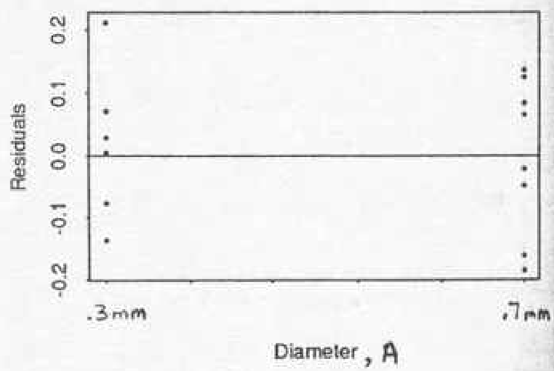
These confidence intervals provide some measure of the precision of the experiment. Reporting only single numbers may be misleading because they do not reflect the amount of data collected or the amount of variation in the data.

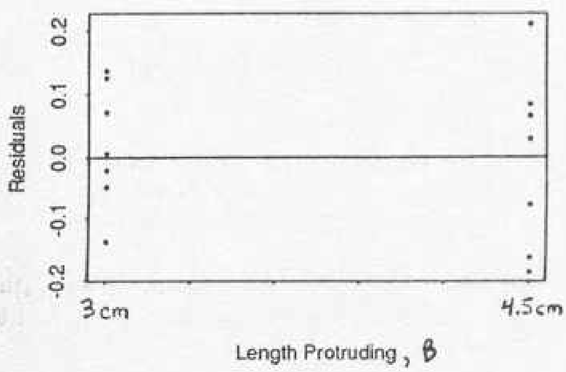
(c) I used a model with only A and B main effects. Using the reverse Yates algorithm:

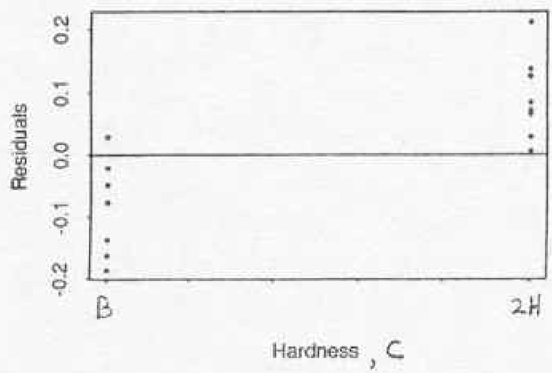
Fitted Effect	Value	Cycle 1	Cycle 2	Cycle 3 ($\hat{y}$)	
abc_{222}	0	0	0	3.92269	$= \hat{y}_{\text{abc}}$
bc_{22}	0	0	3.92269	2.27293	$= \hat{y}_{\text{bc}}$
ac_{22}	0	-0.21430	0	4.35130	$= \hat{y}_{\text{ac}}$
c_2	0	4.13700	2.27293	2.70153	$= \hat{y}_{\text{c}}$
ab_{22}	0	0	0	3.92269	$= \hat{y}_{\text{ab}}$
b_2	-0.21430	0	4.35130	2.27293	$= \hat{y}_{\text{b}}$
a_2	0.82488	-0.21430	0	4.35130	$= \hat{y}_{\text{a}}$
$\bar{y}_{...}$	3.31211	2.48723	2.70153	2.70153	$= \hat{y}_{(1)}$

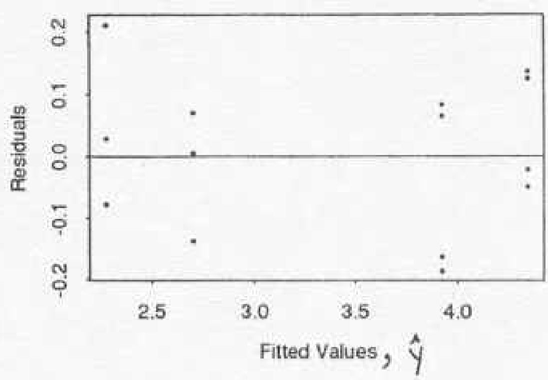
There will be a total of 16 residuals, one for each observation. To compute the residuals, take each (transformed) observation and subtract the $\hat{y}$ that corresponds to the factor-level combination from which the observation came. For example, $\hat{y}_{(1)} = 2.70153$ should be subtracted from the natural logs of both observations from combination (1), 13 and 13, producing the 2 residuals $-.136582$ and $-.136582$.











The normal plot is fairly linear, and so does not indicate any problems with the few-effects model. The plots of Residuals versus Diameter and Length Protruding are patternless, revealing no problems with the model. The plot of Residuals versus Hardness shows mostly negative residuals for B hardness and mostly positive residuals for 2H hardness. This is a sign that Hardness is having some effect on the response, and the model is not accounting for it. Finally the plot of Residuals versus Fitted Values is fairly random, revealing no major problems. Overall, if the effect of C is judged to be small and unimportant, the model fits fairly well.

(d) Using equation (8-16),

$$s^2_{\mathrm{FE}} = \frac{1}{(2)(8)-3}(.2041) = .0157,$$

so $s_{\mathrm{FE}} = \sqrt{.0157} = .1252996$, with 13 degrees of freedom associated with it.

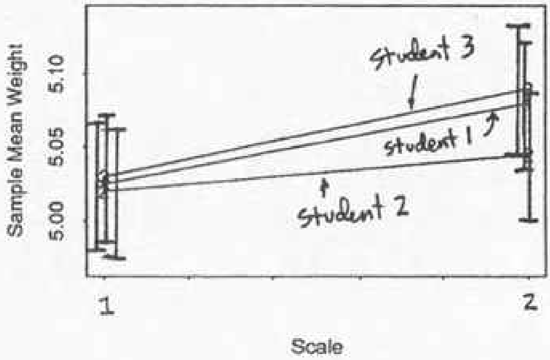
Using equation (8-18) for a 95% two-sided interval, the appropriate t is $t = Q_{13}(.975) = 2.160$. The resulting interval is

$$\begin{aligned} & 3.92269 \pm 2.160(.1252996)\sqrt{1+\frac{3}{(2)(8)}} \\ = \ & 3.92269 \pm .11719 \\ = \ & [3.8055\ ,\ 4.03988] \end{aligned}$$

Exponentiating each endpoint, the prediction interval for the median no. of clips is [44.947 , 56.8195] clips.

3. (a) $s_p = .02483$ g has $n - r = 12 - 6 = 6$ degrees of freedom associated with it. Use equation (7-14) for the error bars. The $\pm$ part is the same for all six intervals because all six sample sizes are the same. For 95% confidence, the appropriate t is $t = Q_6(.975) = 2.447$, from Table B-4. The resulting $\pm$ part is

$$2.447\frac{.02483}{\sqrt{2}} = .04297 \text{ g}.$$



The error bars seem to be too large for the interactions to be distinguishable from background noise. Given the size of the error bars, one could imagine that the underlying μ_{ij}'s could be parallel. The lack of parallelism in the plot could be due to random variation.

(b) Use equation (8-6) and Table 8.3 to construct the confidence intervals. For 95% two-sided intervals, the appropriate t is the same as for part (a). The confidence intervals for the interactions are

$$\begin{aligned} & ab_{ij} \pm 2.447(.02483)\sqrt{\frac{(3-1)(2-1)}{(2)(3)(2)}} \\ = \; & ab_{ij} \pm .024808 \text{ g}. \end{aligned}$$

Looking at the ab_{ij}'s, all of the confidence intervals for the underlying interactions contain zero. This means that the interactions are not detectable. This confirms the tentative conclusions made in part (a).

(c) Use equation (8-6) and Table 8.3 . For 95% two-sided intervals, the appropriate t is the same as for part (b). The resulting intervals are

$$\begin{aligned} & a_i - a_{i'} \pm 2.447(.02483)\sqrt{\frac{2}{(2)(2)}} \\ = \; & a_i - a_{i'} \pm .042968 \text{ g}. \end{aligned}$$

The resulting interval for the mean difference between student 1 and student 2 is $[-.022968, .062968]$ g. The resulting interval for the mean difference between student 1 and student 3 is $[-0.050468, .035468]$ g. The resulting interval for the mean difference between student 2 and student 3 is $[-0.070468, .015468]$ g. Since all of these intervals contain zero, the Student main effects are not detectable.

(d) Use equation (8-8). For 95% confidence, with $\nu = n - IJ = 6$ and $I = 3$ means to be compared, Table B-9 -A gives $q^* = 4.34$. The resulting $\pm$ part is

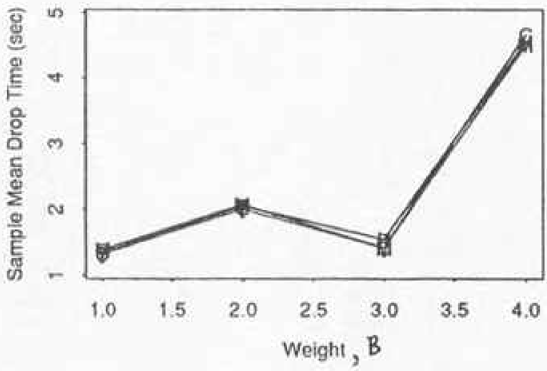
$$\frac{(4.34)(.02483)}{\sqrt{(2)(2)}} = .053887 \text{ g}.$$

The three resulting intervals are wider, as expected. All still contain zero, so Student main effects are not detectable with 95% simultaneous confidence.

4. (a) Using equation (7-7), $s_p = .12235$ sec, with $n - r = 120 - 12 = 108$ degrees of freedom associated with it.

(b) Use equation (7-14) for the error bars. The $\pm$ part is the same for all 12 intervals because all 12 sample sizes are the same. For 99% confidence, the appropriate t is $t = Q_{108}(.995) \approx 2.6256$, from Table B-4. The resulting $\pm$ part is

$$2.6256 \frac{.12235}{\sqrt{10}} = .1016 \text{ sec}.$$



The error bars are very small. It seems that the main effects for Weight dominates any interactions, even if they are detectable. The main effects for Weight also dominate any main effects for Brand, even if they are detectable.

(c) Use equation (8-8). For 95% confidence, with $\nu = n - IJ = 108$ and $I = 3$ means to be compared, Table B-9 -A gives $q^* \approx 3.368$. The resulting Δ is

$$\frac{(3.368)(.12235)}{\sqrt{(4)(10)}} = .0652 \text{ sec}.$$

(d) Use equation (8-10). For 95% confidence, with $\nu = n - IJ = 108$ and $J = 4$ means to be compared, Table B-9 -A gives $q^* \approx 3.692$. The resulting $\pm$ part is

$$\frac{(3.692)(.12235)}{\sqrt{(3)(10)}} = .0825 \text{ sec}.$$

(e) The averages needed are given in the table below.

		OIL WEIGHT (Factor B)				
		10W30	SAE 30	10W40	20W50	
BRAND (Factor A)	M	$\bar{y}_{11} = 1.385$	$\bar{y}_{12} = 2.066$	$\bar{y}_{13} = 1.414$	$\bar{y}_{14} = 4.498$	$\bar{y}_{1.} = 2.34075$
	C	$\bar{y}_{21} = 1.319$	$\bar{y}_{22} = 2.002$	$\bar{y}_{23} = 1.415$	$\bar{y}_{24} = 4.662$	$\bar{y}_{2.} = 2.34950$
	H	$\bar{y}_{31} = 1.344$	$\bar{y}_{32} = 2.049$	$\bar{y}_{33} = 1.544$	$\bar{y}_{34} = 4.549$	$\bar{y}_{3.} = 2.37150$
		$\bar{y}_{.1} = 1.34933$	$\bar{y}_{.2} = 2.03900$	$\bar{y}_{.3} = 1.45767$	$\bar{y}_{.4} = 4.56967$	$\bar{y}_{..} = 2.35392$

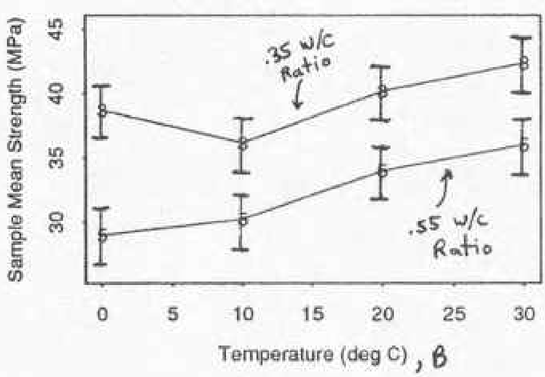
Attaching the Δ from part (c) to each of the 3 differences $\bar{y}_{i.} - \bar{y}_{i'.}$, all of these confidence intervals contain zero, so the Brand main effects are not detectable. Attaching the Δ from part (d) to each of the 6 differences $\bar{y}_{.j} - \bar{y}_{.j'}$, all of these confidence intervals do not contain zero, so the Weight main effects are detectable.

(f) If only one can of each Brand/Weight combination is represented in the study, then (with respect to generalizing to all oil made by each brand, or all oil of a certain weight), there is only one true replication for each combination. There is no guarantee that the 12 cans used are representative of all cans of oil for each of these factor-level combinations. If the students used different quarts of oil for each replication (for a total of 120 quarts of oil), then s_p would capture quart-to-quart variability, and conclusions based on the experiment might safely be applied to the brands and/or weights in general.

5. (a) Using equation (7-7), $s_p = 1.6196$ MPa, with $n - r = 40 - 8 = 32$ degrees of freedom associated with it.

(b) For the error bars, use equation (7-28). With $r = 8$ and $\nu = n - r = 32$, Table B-8-A gives $k_2^* \approx 2.90$. The 8 intervals will all be the same size, because the 8 sample sizes are all the same. The resulting $\pm$ part is

$$2.90\frac{1.6196}{\sqrt{5}} = 2.100 \text{ MPa.}$$



Relative to the size of the error bars, the lack of parallelism for the 0°C temperature may be large enough to be distinguishable from background noise. Tentatively, this interaction seems to be detectable. A cold climate engineer would need to know if the effect of changing the water/cement ratio is different at lower temperatures than it is at higher temperatures.

(c) The averages needed are given in the table below.

		TEMPERATURE (Factor B)				
		0°C	10°C	20°C	30°C	
W/C RATIO (Factor A)	.55	$\bar{y}_{11} = 28.99$	$\bar{y}_{12} = 30.24$	$\bar{y}_{13} = 33.99$	$\bar{y}_{14} = 36.02$	$\bar{y}_{1.} = 32.310$
	.35	$\bar{y}_{21} = 38.70$	$\bar{y}_{22} = 36.16$	$\bar{y}_{23} = 40.18$	$\bar{y}_{24} = 42.36$	$\bar{y}_{2.} = 39.350$
		$\bar{y}_{.1} = 33.845$	$\bar{y}_{.2} = 33.200$	$\bar{y}_{.3} = 37.085$	$\bar{y}_{.4} = 39.190$	$\bar{y}_{..} = 35.830$

The fitted main effects are

$$
\begin{aligned}
a_1 &= \bar{y}_{1.} - \bar{y}_{..} = -3.520\\
a_2 &= \bar{y}_{2.} - \bar{y}_{..} = 3.520\\
b_1 &= \bar{y}_{.1} - \bar{y}_{..} = -1.985\\
b_2 &= \bar{y}_{.2} - \bar{y}_{..} = -2.630\\
b_3 &= \bar{y}_{.3} - \bar{y}_{..} = 1.255\\
b_4 &= \bar{y}_{.4} - \bar{y}_{..} = 3.360
\end{aligned}
$$

The fitted interactions are

$$
\begin{aligned}
ab_{11} &= \bar{y}_{11} - (\bar{y}_{..} + a_1 + b_1) = -1.335\\
ab_{12} &= \bar{y}_{12} - (\bar{y}_{..} + a_1 + b_2) = .560\\
ab_{13} &= \bar{y}_{13} - (\bar{y}_{..} + a_1 + b_3) = .425\\
ab_{14} &= \bar{y}_{14} - (\bar{y}_{..} + a_1 + b_4) = .350\\
ab_{21} &= \bar{y}_{21} - (\bar{y}_{..} + a_2 + b_1) = 1.335\\
ab_{22} &= \bar{y}_{22} - (\bar{y}_{..} + a_2 + b_2) = -.56\\
ab_{23} &= \bar{y}_{23} - (\bar{y}_{..} + a_2 + b_3) = -.425\\
ab_{24} &= \bar{y}_{24} - (\bar{y}_{..} + a_2 + b_4) = -.350
\end{aligned}
$$

(d) Use equation (8-6) and Table 8.3 to construct the confidence intervals. For 95% two-sided intervals, the appropriate t is $t = Q_{32}(.975) \approx 2.0378$ using Table B-4. The confidence intervals for the interactions are

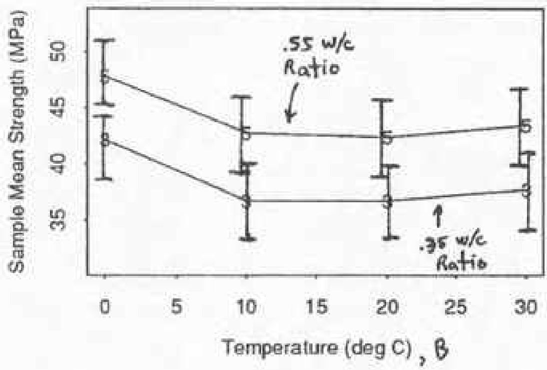
$$
\begin{aligned}
& ab_{ij} \pm 2.0378(1.6196)\sqrt{\frac{(2-1)(4-1)}{(5)(2)(4)}}\\
= \quad & ab_{ij} \pm .9039 \text{ MPa.}
\end{aligned}
$$

Looking at the ab_{ij}'s, the confidence intervals for $\alpha\beta_{11}$ and $\alpha\beta_{21}$ do not contain zero, so these interactions are statistically detectable. These correspond to the 0°C temperature, which agrees with the tentative conclusions made in part (b).

6. (a) Using equation (7-7), $s_p = 2.6060$ MPa, with $n - r = 40 - 8 = 32$ degrees of freedom associated with it.

(b) For the error bars, use equation (7-28). With $r = 8$ and $\nu = n - r = 32$, Table B-8-A gives $k_2^* \approx 2.90$. The 8 intervals will all be the same size, because the 8 sample sizes are all the same. The resulting $\pm$ part is

$$2.90\frac{2.6060}{\sqrt{5}} = 3.380 \text{ MPa.}$$



Relative to the size of the error bars, any lack of parallelism in the plot does not seem to be distinguishable from background noise. Tentatively, the interactions do not seem to be detectable. A cold climate engineer would need to know if the effect of changing the water/cement ratio is the same at lower temperatures as it is at higher temperatures.

(c) The averages needed are given in the table below.

		TEMPERATURE (Factor B)				
		0°C	10°C	20°C	30°C	
W/C RATIO	.55	$\bar{y}_{11} = 47.82$	$\bar{y}_{12} = 42.75$	$\bar{y}_{13} = 42.38$	$\bar{y}_{14} = 43.45$	$\bar{y}_{1.} = 44.100$
(Factor A)	.35	$\bar{y}_{21} = 42.14$	$\bar{y}_{22} = 36.72$	$\bar{y}_{23} = 36.72$	$\bar{y}_{24} = 37.70$	$\bar{y}_{2.} = 38.320$
		$\bar{y}_{.1} = 44.980$	$\bar{y}_{.2} = 39.735$	$\bar{y}_{.3} = 39.550$	$\bar{y}_{.4} = 40.575$	$\bar{y}_{..} = 41.210$

The fitted main effects are

$$\begin{aligned}
a_1 &= \bar{y}_{1.} - \bar{y}_{..} = 2.890 \\
a_2 &= \bar{y}_{2.} - \bar{y}_{..} = -2.890 \\
b_1 &= \bar{y}_{.1} - \bar{y}_{..} = 3.770 \\
b_2 &= \bar{y}_{.2} - \bar{y}_{..} = -1.475 \\
b_3 &= \bar{y}_{.3} - \bar{y}_{..} = -1.660 \\
b_4 &= \bar{y}_{.4} - \bar{y}_{..} = -.635
\end{aligned}$$

The fitted interactions are

$$\begin{aligned}
ab_{11} &= \bar{y}_{11} - (\bar{y}_{..} + a_1 + b_1) = -.050 \\
ab_{12} &= \bar{y}_{12} - (\bar{y}_{..} + a_1 + b_2) = .125 \\
ab_{13} &= \bar{y}_{13} - (\bar{y}_{..} + a_1 + b_3) = -.060 \\
ab_{14} &= \bar{y}_{14} - (\bar{y}_{..} + a_1 + b_4) = -.015 \\
ab_{21} &= \bar{y}_{21} - (\bar{y}_{..} + a_2 + b_1) = .050 \\
ab_{22} &= \bar{y}_{22} - (\bar{y}_{..} + a_2 + b_2) = -.125 \\
ab_{23} &= \bar{y}_{23} - (\bar{y}_{..} + a_2 + b_3) = .060 \\
ab_{24} &= \bar{y}_{24} - (\bar{y}_{..} + a_2 + b_4) = .015
\end{aligned}$$

(d) Use equation (8-6) and Table 8.3 to construct the confidence intervals. For 95% two-sided intervals, the appropriate t is $t = Q_{32}(.975) \approx 2.0378$ using Table B-4. The confidence intervals for the interactions are

$$\begin{aligned}
& ab_{ij} \pm 2.0378(2.6060)\sqrt{\frac{(2-1)(4-1)}{(5)(2)(4)}} \\
= \; & ab_{ij} \pm 1.4543 \text{ MPa.}
\end{aligned}$$

Looking at the ab_{ij}'s, all of the confidence intervals for the underlying interactions contain zero. This means that the interactions are not detectable. This confirms the tentative conclusions made in part (b).

(e) Use equation (8-6) and Table 8.3 . For a 95% two-sided interval, the appropriate t is the same as for part (d). The resulting interval is

$$\begin{aligned} & a_2 - a_1 \pm 2.0378(2.6060)\sqrt{\frac{2}{(5)(4)}} \\ = & -5.78 \pm 1.679325 \\ = & [-7.46, -4.10] \text{ MPa.} \end{aligned}$$

This would give an engineer in any climate an idea of the average difference between the two water/cement ratios, because the difference does not seem to depend on the temperature (interactions seem negligible).

7. Yes, since the nature of the Ratio × Temperature interaction depends on the type of concrete. (One way to describe a 3-factor interaction is to describe how a 2-factor interaction depends on the third factor.)

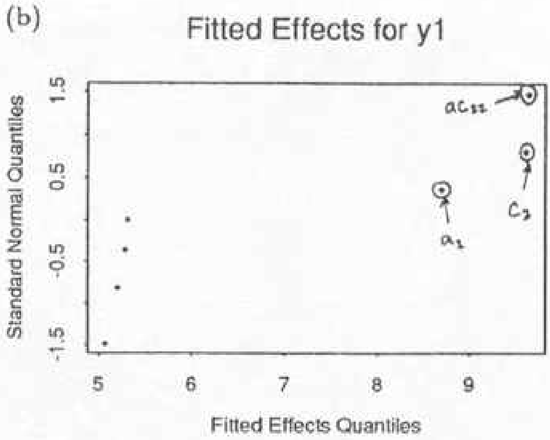
8. (a) Using the Yates algorithm for y_1:

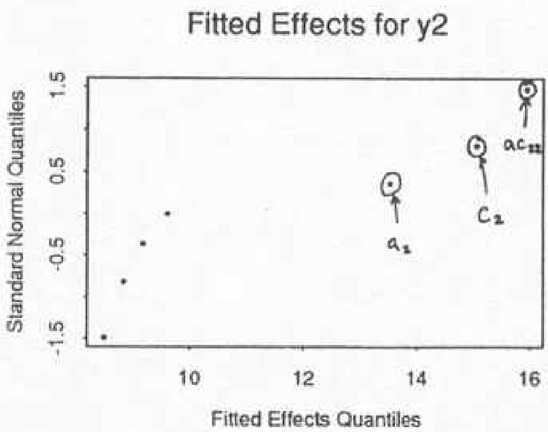
Combination	$\bar{y}$	Cycle 1	Cycle 2	Cycle 3	Cycle 3 ÷ 8	
(1)	2.7	3.6	7.7	92.2	11.525	$= \bar{y}_{...}$
a	0.9	4.1	84.5	69.6	8.700	$= a_2$
b	3.0	21.7	-3.7	41.6	5.200	$= b_2$
ab	1.1	62.8	73.3	42.2	5.275	$= ab_{22}$
c	3.1	-1.8	.5	76.8	9.600	$= c_2$
ac	18.6	-1.9	41.1	77.0	9.625	$= ac_{22}$
bc	2.5	15.5	-0.1	40.6	5.075	$= bc_{22}$
abc	60.3	57.8	42.3	42.4	5.300	$= abc_{222}$

Using the Yates algorithm for y_2:

Combination	$\bar{y}$	Cycle 1	Cycle 2	Cycle 3	Cycle 3 ÷ 8	
(1)	5.6	7.0	15.7	151.9	18.9875	$= \bar{y}_{...}$
a	1.4	8.7	136.2	108.1	13.5125	$= a_2$
b	7.1	30.5	-9.7	76.9	9.6125	$= b_2$
ab	1.6	105.7	117.8	68.3	8.5375	$= ab_{22}$
c	3.2	-4.2	1.7	120.5	15.0625	$= c_2$
ac	27.3	-5.5	75.2	127.5	15.9375	$= ac_{22}$
bc	6.0	24.1	-1.3	73.5	9.1875	$= bc_{22}$
abc	99.7	93.7	69.6	70.9	8.8625	$= abc_{222}$

(b)

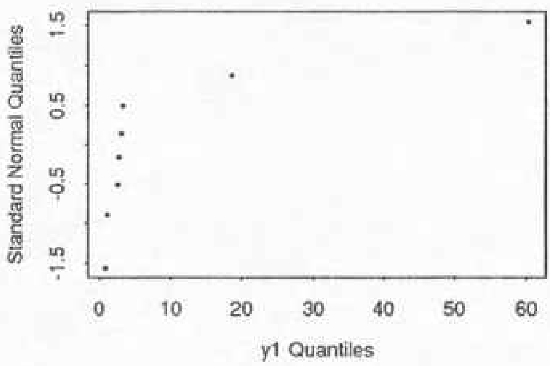


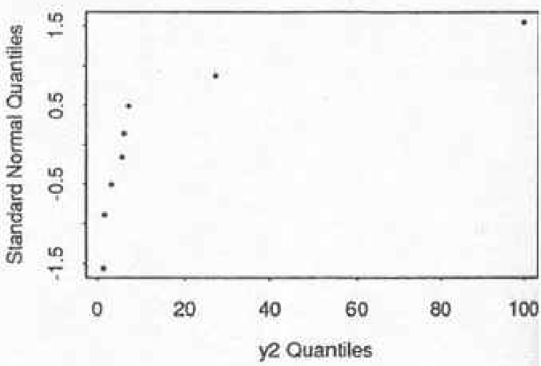


In both plots, a_2, c_2, and ac_{22} plot away from the rest of the effects. There is no obvious simple description of the effects of the factors on either response. It seems that any description would need to include at least the A main effect, the C main effect, and the AC interaction.

(c) The gap occurs because of the fact that the data for combinations ac and abc are much larger than the rest of the data, for each response. The response for combination abc is positive in the formulas for all of the fitted effects. The response for combination ac is positive in the formulas for $\bar{y}_{...}$, a_2, c_2, and ac_{22}, and is negative in the formulas for b_2, ab_{22}, bc_{22}, and abc_{222}.

(d)

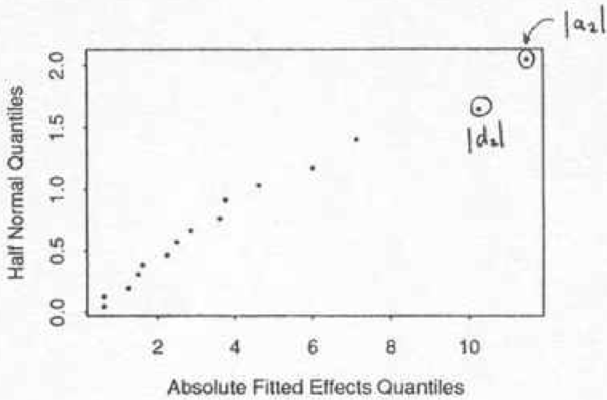




Unless the distributions of these responses are prone to extreme outliers, it is very unlikely that the data are just random variation (especially if the responses have a normal distribution). The factors are in some way having an effect here.

9. (a) The plot coordinates are given below.

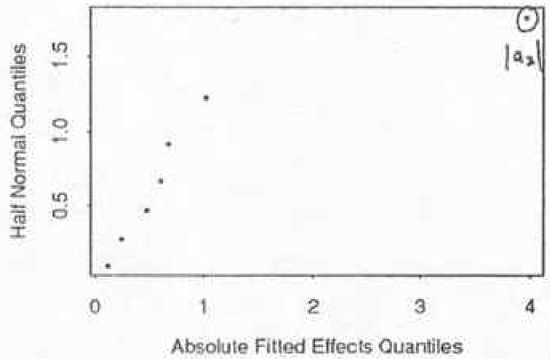
i	$p = \frac{i-.5}{15}$	$\frac{1+p}{2}$	$Q_{\lvert\text{Eff}\rvert}(p)$	$Q_{\text{HN}}(p) = Q_Z(\frac{1+p}{2})$
1	.0333	.5167	.625	.05
2	.1000	.5500	.625	.13
3	.1667	.5833	1.250	.20
4	.2333	.6167	1.500	.31
5	.3000	.6500	1.625	.39
6	.3667	.6833	2.250	.47
7	.4333	.7167	2.500	.58
8	.5000	.7500	2.875	.67
9	.5667	.7833	3.625	.77
10	.6333	.8167	3.750	.92
11	.7000	.8500	4.625	1.04
12	.7667	.8833	6.000	1.18
13	.8333	.9167	7.125	1.41
14	.9000	.9500	10.250	1.65
15	.9667	.9833	11.500	2.05



The absolute values of the fitted main effects for A and D plot slightly off the line; in ex. 2, sec. 2, ch. 8, , it appeared that only A was off the line. Neither of these absolute fitted effects plots far away from the rest, so it is hard to say if they are really detectable.

(b) The plot coordinates are given below.

i	$p = \frac{i-.5}{7}$	$\frac{1+p}{2}$	$Q_{\lvert\text{Eff}\rvert}(p)$	$Q_{\text{HN}}(p) = Q_Z(\frac{1+p}{2})$
1	.0714	.5357	.1125	.10
2	.2143	.6071	.2375	.28
3	.3571	.6786	.4625	.47
4	.5000	.7500	.5875	.67
5	.6429	.8214	.6625	.92
6	.7857	.8929	1.0125	1.23
7	.9286	.9643	3.9625	1.75



Here, the conclusions are the same as Ex. 1, Chapter 8. Only the absolute fitted main effect for A plots off the line, so the A main effect is detectably larger than the rest.

10. (a) Using the Yates algorithm:

Combination	$\bar{y}$	Cycle 1	Cycle 2	Cycle 3	Cycle 3 ÷ 8	
(1)	1.00	2.10	2.80	5.45	.68125	$= \bar{y}_{...}$
a	1.10	.70	2.65	.95	.11875	$= a_2$
b	.20	1.90	-.40	-2.55	-.31875	$= b_2$
ab	.50	.75	.55	.15	.01875	$= ab_{22}$
c	.80	.10	-1.40	-.15	-.01875	$= c_2$
ac	1.10	.30	-1.15	.15	.01875	$= ac_{22}$
bc	.25	.30	.20	.25	.03125	$= bc_{22}$
abc	.50	.25	-.05	-.25	-.03125	$= abc_{222}$

(b) Using equation (7-7), $s_p = .1225$ %, with $n - r = 11 - 8 = 3$ degrees of freedom associated with it.

(c) Use equation (8-12). For 90% two-sided confidence intervals, the appropriate t is $t = Q_3(.95) = 2.353$ from Table B-4. The resulting $\pm$ part is

$$2.353(.1225)\frac{1}{8}\sqrt{6.5} = .09184 \text{ \%}.$$

(d) The only effects whose confidence intervals do not contain zero are the main effect for A (α_2) and the main effect for B (β_2).

(e) It seems that Polymer Concentration has the biggest impact on impurity, followed by Polymer Type. The fact that b_2 is negative reflects that the higher polymer concentration results in lower impurity. The fact that a_2 is positive reflects that the standard polymer type results in lower impurity. To minimize impurity, this study suggests using the standard polymer at high concentration. Since Factor C (amount of an additive) does not seem to have a noticeable effect, it should be set at its least expensive level (probably 2 lbs.).

11. (a) Using equation (7-7), $s_p = 7.443957$, with $n - r = 64 - 16 = 48$ degrees of freedom associated with it. This measures the baseline variability in hour-to-hour missed lead counts for any of the 16 conditions, assuming this variability is the same for all 16 conditions.

(b) Using the Yates algorithm:

Comb	$\bar{y}$	Cycle 1	Cycle 2	Cycle 3	Cycle 4	Cycle 4 ÷ 16	
(1)	28.4	50.3	84.8	191.3	381.2	23.8250	$= \bar{y}_{....}$
a	21.9	34.5	106.5	189.9	-61.6	-3.8500	$= a_2$
b	20.2	55.5	98.4	-43.1	-36.4	-2.2750	$= b_2$
ab	14.3	51.0	91.5	-18.5	5.2	0.3250	$= ab_{22}$
c	30.4	56.0	-12.4	-20.3	14.8	0.9250	$= c_2$
ac	25.1	42.4	-30.7	-16.1	-6.8	-0.4250	$= ac_{22}$
bc	38.2	47.0	-15.0	-19.5	22.4	1.4000	$= bc_{22}$
abc	12.8	44.5	-3.5	24.7	-36.4	-2.2750	$= abc_{222}$
d	36.8	-6.5	-15.8	21.7	-1.4	-0.0875	$= d_2$
ad	19.2	-5.9	-4.5	-6.9	24.6	1.5375	$= ad_{22}$
bd	19.9	-5.3	-13.6	-18.3	4.2	0.2625	$= bd_{22}$
abd	22.5	-25.4	-2.5	11.5	44.2	2.7625	$= abd_{222}$
cd	25.5	-17.6	0.6	11.3	-28.6	-1.7875	$= cd_{22}$
acd	21.5	2.6	-20.1	11.1	29.8	1.8625	$= acd_{222}$
bcd	22.0	-4.0	20.2	-20.7	-0.2	-0.0125	$= bcd_{222}$
abcd	22.5	0.5	4.5	-15.7	5.0	0.3125	$= abcd_{2222}$

(c) Use equation (8-13). For 98% two-sided confidence intervals, the appropriate t is $t = Q_{48}(.99) \approx 2.4098$ from Table B-4. The resulting $\pm$ part is

$$2.4098 \frac{7.443957}{\sqrt{(4)(16)}} = 2.242306$$

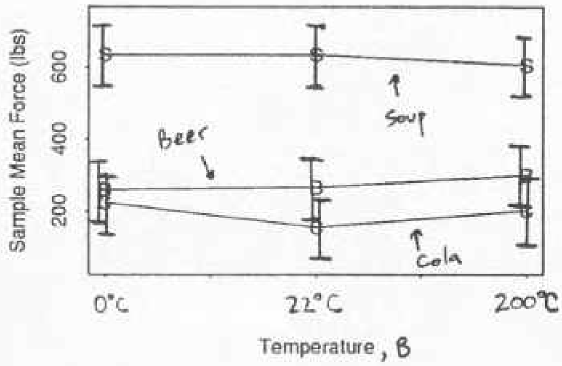
The ABD and ABC interactions are detectable, as well as the A and B main effects. There is really no simple interpretation of this set of effects. You could say that, overall, factors A and B have the greatest effect, but their effects seem to depend on how factors C and D are set.

(d) Yes, if you average over all the conditions. The negative sign of a_2 indicates that, averaging over levels of all the other factors, the new lead type results in a smaller number of missed leads than the standard.

(e) No, because the detectable interactions are large relative to the main effect for A. For example, the signs of abc_{222} and abd_{222} indicate that for plant 2, standard machine type, and shift 2, switching to the new lead type tends to increase the number of missed leads.

12. (a) $s_p = 57.50$ lb. has $n - r = 27 - 9 = 18$ degrees of freedom associated with it. Use equation (7-14) for the error bars. The $\pm$ part is the same for all nine intervals because all nine sample sizes are the same. For 98% confidence, the appropriate t is $t = Q_{18}(.99) = 2.552$, from Table B-4. The resulting $\pm$ part is

$$2.552 \frac{57.50}{\sqrt{3}} = 84.72287 \text{ lb.}$$



The error bars seem to be too large for the interactions to be distinguishable from background noise. Given the size of the error bars, one could imagine that the underlying μ_{ij}'s could be parallel. The lack of parallelism in the plot could be due to random variation.

(b) The averages needed are given in the table below.

		TEMPERATURE (Factor B) 0°C	22°C	200°C	
CAN TYPE (Factor A)	Cola	$\bar{y}_{11} = 224.00$	$\bar{y}_{12} = 154.33$	$\bar{y}_{13} = 200.67$	$\bar{y}_{1.} = 193.00$
	Beer	$\bar{y}_{21} = 260.00$	$\bar{y}_{22} = 265.67$	$\bar{y}_{23} = 300.00$	$\bar{y}_{2.} = 275.22$
	Soup	$\bar{y}_{31} = 635.33$	$\bar{y}_{32} = 635.67$	$\bar{y}_{33} = 605.33$	$\bar{y}_{3.} = 625.44$
		$\bar{y}_{.1} = 373.11$	$\bar{y}_{.2} = 351.89$	$\bar{y}_{.3} = 368.67$	$\bar{y}_{..} = 364.56$

The fitted main effects are

$a_1 = \bar{y}_{1.} - \bar{y}_{..} = -171.56$
$a_2 = \bar{y}_{2.} - \bar{y}_{..} = -89.33$
$a_3 = \bar{y}_{3.} - \bar{y}_{..} = 260.89$
$b_1 = \bar{y}_{.1} - \bar{y}_{..} = 8.56$
$b_2 = \bar{y}_{.2} - \bar{y}_{..} = -12.67$
$b_3 = \bar{y}_{.3} - \bar{y}_{..} = 4.11$

The fitted interactions are

$$
\begin{aligned}
ab_{11} &= \bar{y}_{11} - (\bar{y}_{..} + a_1 + b_1) = 22.44 \\
ab_{12} &= \bar{y}_{12} - (\bar{y}_{..} + a_1 + b_2) = -26.00 \\
ab_{13} &= \bar{y}_{13} - (\bar{y}_{..} + a_1 + b_3) = 3.56 \\
ab_{21} &= \bar{y}_{21} - (\bar{y}_{..} + a_2 + b_1) = -23.78 \\
ab_{22} &= \bar{y}_{22} - (\bar{y}_{..} + a_2 + b_2) = 3.11 \\
ab_{23} &= \bar{y}_{23} - (\bar{y}_{..} + a_2 + b_3) = 20.67 \\
ab_{31} &= \bar{y}_{31} - (\bar{y}_{..} + a_3 + b_1) = 1.33 \\
ab_{32} &= \bar{y}_{32} - (\bar{y}_{..} + a_3 + b_2) = 22.89 \\
ab_{33} &= \bar{y}_{33} - (\bar{y}_{..} + a_3 + b_3) = -24.22
\end{aligned}
$$

Use equation (8-6) and Table 8.3 to construct the confidence intervals for the interactions. For 98% two-sided intervals, the appropriate t is the same as the one used in part (a). The confidence intervals are

$$
\begin{aligned}
& ab_{ij} \pm 2.552(57.50)\sqrt{\frac{(3-1)(3-1)}{(3)(3)(3)}} \\
= \quad & ab_{ij} \pm 56.482 \text{ lbs.}
\end{aligned}
$$

Looking at the ab_{ij}'s computed above, all of the confidence intervals for the underlying interactions contain zero. This means that the interactions are not detectable.

For the Can Type main effects, use equation (8-6) and Table 8.3 . The confidence intervals are

$$
\begin{aligned}
& a_i \pm 2.552(57.50)\sqrt{\frac{3-1}{(3)(3)(3)}} \\
= \quad & a_i \pm 39.939 \text{ lbs.}
\end{aligned}
$$

All 3 of these confidence intervals do not contain zero, indicating that the main effects for Can Type are detectable.

For the Temperature main effects, use equation (8-6) and Table 8.3 . The confidence intervals are

$$
\begin{aligned}
& b_j \pm 2.552(57.50)\sqrt{\frac{3-1}{(3)(3)(3)}} \\
= \quad & b_j \pm 39.939 \text{ lb.}
\end{aligned}
$$

All of these confidence intervals contain zero, indicating that the main effects for Temperature are not detectable.

(c) For the Can Type main effects, Use equation (8- 8). For 99% confidence, with $\nu = n - IJ = 18$ and $I = 3$ means to be compared, Table B - 9 -B gives $q^* = 4.70$. The resulting $\pm$ part is

$$
\frac{(4.70)(57.50)}{\sqrt{(3)(3)}} = 90.086 \text{ lbs.}
$$

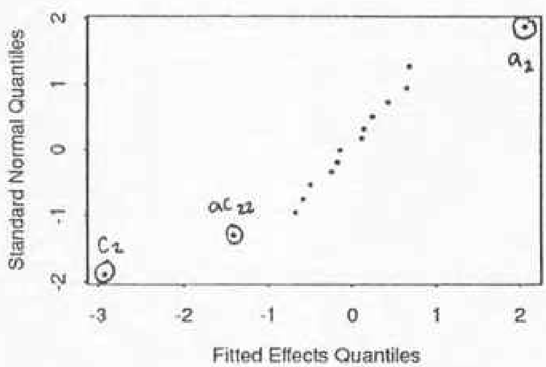
The resulting interval for the difference between cola and beer can main effects is $[-172.3, 7.9]$ lbs. The resulting interval for the difference between cola and soup can main effects is $[-522.5, -342.4]$ lbs. The resulting interval for the difference between beer and soup can main effects is $[-440.3, -260.1]$ lbs. The first interval contains zero, so there is not a detectable difference between cola and beer can main effects. The other two intervals do not contain zero, indicating that there is a detectable difference between both cola and soup and beer and soup cans.

For the Temperature main effects, Use equation (8-10). For 99% confidence, with $\nu = n - IJ = 18$ and $J = 3$ means to be compared, Table B-9-A gives $q^* = 4.70$. The resulting $\pm$ part is

$$\frac{(4.70)(57.50)}{\sqrt{(3)(3)}} = 90.086 \text{ lbs.}$$

The resulting interval for the difference between 0°C and 22°C temperature main effects is $[-68.9, 111.3]$ lbs. The resulting interval for the difference between 0°C and 200°C temperature main effects is $[-85.6, 94.5]$ lbs. The resulting interval for the difference between 22°C and 200°C temperature main effects is $[-106.9, 73.3]$ lbs. All of these intervals contain zero, so none of the main effects for Temperature are not detectable.

13.

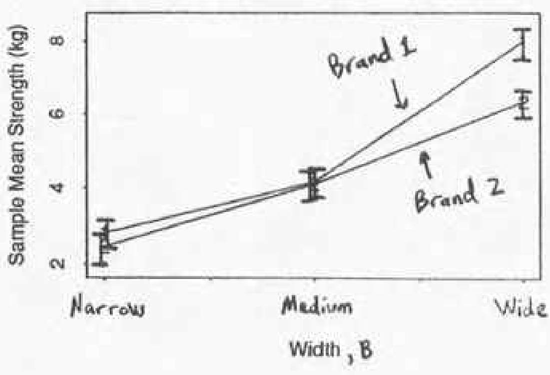


The points that plot away from the rest are c_2, a_2, and possibly ac_{22}. This means that the main effects for A and C, and possibly the AC interaction, are detectably larger than the rest. It seems that on average, tee wing planes fly farther than straight wing and notebook paper planes fly farther than construction paper planes. Also, the average increase in distance due to changing from straight wing to tee wing may be larger for notebook paper than for construction paper.

14. (a) Using equation (7-7), $s_p = .42118$ kg, with $n - r = 30 - 6 = 24$ degrees of freedom associated with it. This measures the magnitude of baseline variation within any of the 6 conditions, assuming it is the same for all 6 conditions.

(b) Use equation (7-14) for the error bars. The $\pm$ part is the same for all six intervals because all six sample sizes are the same. For 95% confidence, the appropriate t is $t = Q_{24}(.975) = 2.064$, from Table B-4. The resulting $\pm$ part is

$$2.064 \frac{.42118}{\sqrt{5}} = .38877 \text{ kg.}$$



(c) The error bars seem to be small enough so that the lack of parallelism in the plot is distinguishable from background noise. Main effects alone should not be used to summarize the situation. Overall, however, Width has a bigger effect on strength than Brand. The effect of Brand is only distinguishable for the wide width.

(d) The averages needed are given in the table below.

		WIDTH (Factor B)			
		Narrow	Medium	Wide	
BRAND	1	$\bar{y}_{11} = 2.811$	$\bar{y}_{12} = 4.164$	$\bar{y}_{13} = 8.001$	$\bar{y}_{1.} = 4.9920$
(Factor A)	2	$\bar{y}_{21} = 2.459$	$\bar{y}_{22} = 4.111$	$\bar{y}_{23} = 6.346$	$\bar{y}_{2.} = 4.3053$
		$\bar{y}_{.1} = 2.6350$	$\bar{y}_{.2} = 4.1375$	$\bar{y}_{.3} = 7.1735$	$\bar{y}_{..} = 4.6487$

The fitted main effects are

$$
\begin{aligned}
a_1 &= \bar{y}_{1.} - \bar{y}_{..} = .34333 \\
a_2 &= \bar{y}_{2.} - \bar{y}_{..} = -.34333 \\
b_1 &= \bar{y}_{.1} - \bar{y}_{..} = -2.01367 \\
b_2 &= \bar{y}_{.2} - \bar{y}_{..} = -.51117 \\
b_3 &= \bar{y}_{.3} - \bar{y}_{..} = 2.52483
\end{aligned}
$$

The fitted interactions are

$$
\begin{aligned}
ab_{11} &= \bar{y}_{11} - (\bar{y}_{..} + a_1 + b_1) = -.16733 \\
ab_{12} &= \bar{y}_{12} - (\bar{y}_{..} + a_1 + b_2) = -.31683 \\
ab_{13} &= \bar{y}_{13} - (\bar{y}_{..} + a_1 + b_3) = .48417 \\
ab_{21} &= \bar{y}_{21} - (\bar{y}_{..} + a_2 + b_1) = .16733 \\
ab_{22} &= \bar{y}_{22} - (\bar{y}_{..} + a_2 + b_2) = .31683 \\
ab_{23} &= \bar{y}_{23} - (\bar{y}_{..} + a_2 + b_3) = -.48417
\end{aligned}
$$

(e) Use equation (8-6) and Table 8.3 to construct the confidence intervals for the interactions. For 95% two-sided intervals, the appropriate t is the same as the one from part (b). The confidence intervals are

$$
\begin{aligned}
& ab_{ij} \pm 2.064(.42118)\sqrt{\frac{(2-1)(3-1)}{(5)(2)(3)}} \\
= \;& ab_{ij} \pm .22446 \text{ kg.}
\end{aligned}
$$

Looking at the ab_{ij}'s computed above, four of the six confidence intervals for the underlying interactions do not contain zero. This means that the interactions are detectable. This agrees with the tentative conclusion made in part (c).

(f) Use equation (8-6) and Table 8.3 . For 95% two-sided intervals, the appropriate t is the same as in part (b). The resulting intervals are

$$
\begin{aligned}
& b_j - b_{j'} \pm 2.064(.42118)\sqrt{\frac{2}{(5)(2)}} \\
= \;& b_i - b_{j'} \pm .38877 \text{ kg.}
\end{aligned}
$$

The resulting interval for the mean difference between narrow and medium widths is $[-1.89, -1.11]$ kg. The resulting interval for the mean difference between narrow and wide widths is $[-4.93, -4.15]$ kg. The resulting interval for the mean difference between medium and wide widths is $[-3.42, -2.65]$ kg. None of these intervals contain zero, so all of the differences between Width main effects are detectable. This agrees with the tentative conclusion made in part (c). It should be noted that the difference between medium and wide widths seems to depend on the brand (there is an interaction), and the above intervals do not include this information.

(g) Use equation (8-10). For 95% confidence, with $\nu = n - IJ = 24$ and $J = 3$ means to be compared, Table B-9-A gives $q^* = 3.53$. The resulting $\pm$ part is

$$\frac{(3.53)(.42118)}{\sqrt{(2)(5)}} = .47016 \text{ kg}.$$

None of the resulting intervals contain zero, so this does not change the conclusion made in part (f).

15. (a) Use the generator and multiply the appropriate columns.

A	B	C	D↔AB	Combination
−	−	−	+	d
+	−	−	−	a
−	+	−	−	b
+	+	−	+	abd
−	−	+	+	cd
+	−	+	−	ac
−	+	+	−	bc
+	+	+	+	abcd

(b) Multiply the generator through by D to get the defining relation: I↔ABD. Now multiply the defining relation through by various effects to get the alias structure:

I↔ABD
A↔BD
B↔AD
AB↔D
C↔ABCD
AC↔BCD
BC↔ACD
ABC↔CD

This means that the following sums can be estimated: $\mu_{....} + \alpha\beta\delta_{222}$, $\alpha_2 + \beta\delta_{22}$, $\beta_2 + \alpha\delta_{22}$, $\alpha\beta_{22} + \delta_2$, $\gamma_2 + \alpha\beta\gamma\delta_{2222}$, $\alpha\gamma_{22} + \beta\gamma\delta_{222}$, $\beta\gamma_{22} + \alpha\gamma\delta_{222}$, and $\alpha\beta\gamma_{222} + \gamma\delta_{22}$. None of the effects can be isolated individually.

(c) The first 4 lines of the Yates algorithm are estimating $\mu_{....} + \alpha\beta\delta_{222}$, $\alpha_2 + \beta\delta_{22}$, $\beta_2 + \alpha\delta_{22}$, and $\alpha\beta_{22} + \delta_2$ respectively. For each line, either of the effects (or both) may be causing the estimate to be large. It is very likely that $\mu_{....}$ is the primary cause of large estimate on the first line, and not $\alpha\beta\delta_{222}$. The other 3 lines are harder to interpret. Using the alias structure, 4 possible interpretations are:

1. The A, B, and D main effects are dominant;
2. The A and B main effects and the AB interaction are dominant;
3. The A and D main effects and the AD interaction are dominant;
4. The B and D main effects and the BD interaction are dominant.

(d) For D↔ABC, main effects are aliased with 3-factor interactions, which are usually smaller than 2-factor interactions. This allows the main effects to be detected less ambiguously (the ambiguity encountered in part (c) would not be as severe). For D↔AB, the main effects for A, B, and D are aliased with 2-factor interactions, which usually cannot be assumed small.

16. (a) Use the generators and multiply the appropriate columns.

A	B	C	$D \leftrightarrow AB$	$E \leftrightarrow AC$	Combination
−	−	−	+	+	de
+	−	−	−	−	a
−	+	−	−	+	be
+	+	−	+	−	abd
−	−	+	+	−	cd
+	−	+	−	+	ace
−	+	+	−	−	bc
+	+	+	+	+	abcd

(b) First find the defining relation. Start with the generators. Multiply through by D on the first generator to get $I \leftrightarrow ABD$. Multiply through by E on the second generator to get $I \leftrightarrow ACE$. Now multiply these two "I" relationships to get a third:

$$I \leftrightarrow (ABD)(ACE)$$
$$I \leftrightarrow BCDE$$

So the entire defining relationship is

$I \leftrightarrow ABD \leftrightarrow ACE \leftrightarrow BCDE$.

Now multiply through by A on the defining relation:

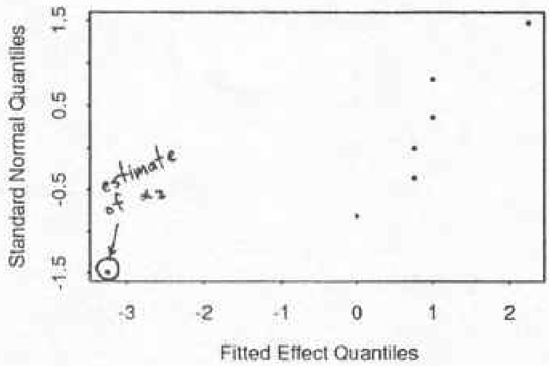
$$A \leftrightarrow BD \leftrightarrow CE \leftrightarrow ABCDE.$$

This means that the A main effect is aliased with the BD 2-factor interaction, the CE 2-factor interaction, and the ABCDE 5-factor interaction.

17. (a) Using the Yates algorithm:

Combination	$\bar{y}$	Cycle 1	Cycle 2	Cycle 3	Cycle 3 ÷ 8	Effect Estimated
(1)	70	131	262	532	66.50	$\mu_{...}$
a	61	131	270	-26	-3.25	α_2
b	72	132	-22	6	0.75	β_2
ab	59	138	-4	0	0.00	$\alpha\beta_{22}$
c	68	-9	0	8	1.00	γ_2
ac	64	-13	6	18	2.25	$\alpha\gamma_{22}$
bc	69	-4	-4	6	0.75	$\beta\gamma_{22}$
abc	69	0	4	8	1.00	$\alpha\beta\gamma_{222}$

(b) Take the last 7 estimates from the Yates algorithm and normal plot them.



The point that plots far from the others is the estimate of the main effect for A. Based on this plot, only the estimate of the A main effect is representing something more than background noise.

(c) Use equation (8-13). The appropriate t is $t = Q_8(.975) = 2.306$ from Table B-4. The resulting Δ is

$$(2.306)\frac{.9}{\sqrt{(2)(8)}} = .519.$$

The confidence intervals for α_2, β_2, γ_2, $\alpha\gamma_{22}$, $\beta\gamma_{22}$, and $\alpha\beta\gamma_{222}$ all do not contain zero, so all of these effects are detectable (their corresponding estimates are representing more than just background noise).

(d) First find the defining relation. Multiply through the generator by D to get I↔ABCD. Now multiply through the defining relation by various effects to get the alias structure:

I↔ABCD
A↔BCD
B↔ACD
AB↔CD
C↔ABD
AC↔BD
BC↔AD
ABC↔D

The "new" table of estimates can then be written using this alias structure.

Combination	$\bar{y}$	Cycle 1	Cycle 2	Cycle 3	Cycle 3 ÷ 8	Sum Estimated
(1)	70	131	262	532	66.50	$\mu.... + \alpha\beta\gamma\delta_{2222}$
ad	61	131	270	-26	-3.25	$\alpha_2 + \beta\gamma\delta_{222}$
bd	72	132	-22	6	0.75	$\beta_2 + \alpha\gamma\delta_{222}$
ab	59	138	-4	0	0.00	$\alpha\beta_{22} + \gamma\delta_{22}$
cd	68	-9	0	8	1.00	$\gamma_2 + \alpha\beta\delta_{222}$
ac	64	-13	6	18	2.25	$\alpha\gamma_{22} + \beta\delta_{22}$
bc	69	-4	-4	6	0.75	$\beta\gamma_{22} + \alpha\delta_{22}$
abcd	69	0	4	8	1.00	$\alpha\beta\gamma_{222} + \delta_2$

Based on the estimated sums, it is very likely that the first estimate is large primarily because of μ..... It is probably the A main effect that is making the second estimate large, unless there is some reason to believe that an ACD interaction may exist. For the 6th line, it could be either the AC interaction or the BD interaction (or both). But since the A main effect is important, it is more likely that it will be involved in an interaction. This conclusion is very tentative, though.

(e) First find the defining relation. Multiply through the first generator by D to get I↔ABCD. Multiply through the second generator by E to get I↔ACE. Now multiply these two "I" relationships to get a third:

$$I \leftrightarrow (ABCD)(ACE)$$
$$I \leftrightarrow BDE$$

So the entire defining relationship is

I↔ABCD↔ACE↔BDE.

Now multiply through the defining relation by various effects to get the alias structure. Focusing on the main effects,

I↔ABCD↔ACE↔BDE
A↔BCD↔CE↔ABDE
B↔ACD↔ABCE↔DE
C↔ABD↔AE↔BCDE
D↔ABC↔ACDE↔BE
E↔ABCDE↔AC↔BD

The "new" table of estimates can then be written using this alias structure.

Combination	$\bar{y}$	Cycle 1	Cycle 2	Cycle 3	Cycle 3 ÷ 8	Sum Estimated
e	70	131	262	532	66.50	μ..... + aliases
ad	61	131	270	-26	-3.25	$\alpha_2 + \gamma\epsilon_{22}$ + aliases
bde	72	132	-22	6	0.75	$\beta_2 + \delta\epsilon_{22}$ + aliases
ab	59	138	-4	0	0.00	$\alpha\beta_{22}$ + aliases
cd	68	-9	0	8	1.00	$\gamma_2 + \alpha\epsilon_{22}$ + aliases
ace	64	-13	6	18	2.25	$\alpha\gamma_{22} + \epsilon_2$ + aliases
bc	69	-4	-4	6	0.75	$\beta\gamma_{22}$ + aliases
abcde	69	0	4	8	1.00	$\alpha\beta\gamma_{222} + \delta_2$ + aliases

Based on the estimated sums, it is very likely that the first estimate is large primarily because of μ...... It is probably the A main effect that is making the second estimate large, unless there is some reason to believe that a DE interaction may exist. For the 6th line, probably the E main effect is the cause of this estimate being large, unless there is some reason to believe that an AC interaction may exist. It seems that the A and E main effects are dominant. This conclusion is tentative though, because the DE or AC interactions may really be causing these estimates to be large. There is no way of knowing for sure based on this fractional factorial data set.

18. (a) Use the generators and multiply the appropriate columns.

A	B	C	$D \leftrightarrow AB$	$E \leftrightarrow AC$	$F \leftrightarrow BC$	Combination
−	−	−	+	+	+	def
+	−	−	−	−	+	af
−	+	−	−	+	−	be
+	+	−	+	−	−	abd
−	−	+	+	−	−	cd
+	−	+	−	+	−	ace
−	+	+	−	−	+	bcf
+	+	+	+	+	+	abcdef

(b) First find the defining relation. Start with the generators. Multiply through by D on the first generator to get $I \leftrightarrow ABD$. Multiply through by E on the second generator to get $I \leftrightarrow ACE$. Multiply through by F on the third generator to get $I \leftrightarrow BCF$. Now multiply these three "I" relationships in pairs and all together to get 4 more:

$$I \leftrightarrow (ABD)(ACE)$$
$$I \leftrightarrow BCDE$$

$$I \leftrightarrow (ABD)(BCF)$$
$$I \leftrightarrow ACDF$$

$$I \leftrightarrow (ACE)(BCF)$$
$$I \leftrightarrow ABEF$$

$$I \leftrightarrow (ABD)(ACE)(BCF)$$
$$I \leftrightarrow DEF$$

So the entire defining relationship is

$I \leftrightarrow ABD \leftrightarrow ACE \leftrightarrow BCF \leftrightarrow BCDE \leftrightarrow ACDF \leftrightarrow ABEF \leftrightarrow DEF$.

Now, multiply through the defining relationship by A to get all the effects aliased with the A main effect:

$A \leftrightarrow BD \leftrightarrow CE \leftrightarrow ABCF \leftrightarrow ABCDE \leftrightarrow CDF \leftrightarrow BEF \leftrightarrow ADEF$.

19. (a) Use the generator and multiply the appropriate columns.

A	B	C	D	E$\leftrightarrow -$CD	Combination
$-$	$-$	$-$	$-$	$-$	(1)
$+$	$-$	$-$	$-$	$-$	a
$-$	$+$	$-$	$-$	$-$	b
$+$	$+$	$-$	$-$	$-$	ab
$-$	$-$	$+$	$-$	$+$	ce
$+$	$-$	$+$	$-$	$+$	ace
$-$	$+$	$+$	$-$	$+$	bce
$+$	$+$	$+$	$-$	$+$	abce
$-$	$-$	$-$	$+$	$+$	de
$+$	$-$	$-$	$+$	$+$	ade
$-$	$+$	$-$	$+$	$+$	bde
$+$	$+$	$-$	$+$	$+$	abde
$-$	$-$	$+$	$+$	$-$	cd
$+$	$-$	$+$	$+$	$-$	acd
$-$	$+$	$+$	$+$	$-$	bcd
$+$	$+$	$+$	$+$	$-$	abcd

(b) Multiply through the generator by E to get I$\leftrightarrow -$CDE. This design has resolution 3. The standard choice of the half fraction has defining relation I$\leftrightarrow$ABCDE, so it is resolution 5. The authors' choice seems to be unwise because the C, D, and E main effects are all aliased with 2-factor interactions. The standard choice has main effects aliased only with 4- and 5-factor interactions. It may be hard to clearly see main effects using the authors' design, especially if there are any 2-factor interactions involving C, D, and E. The standard design only requires that 4- and 5-factor interactions be small in order to be able to estimate any of the main effects.

(c) Multiply through the defining relation by the various effects to get the alias structure.

I$\leftrightarrow -$CDE
A$\leftrightarrow -$ACDE
B$\leftrightarrow -$BCDE
AB$\leftrightarrow -$ABCDE
C$\leftrightarrow -$DE
AC$\leftrightarrow -$ADE
BC$\leftrightarrow -$BDE
ABC$\leftrightarrow -$ABDE
D$\leftrightarrow -$CE
AD$\leftrightarrow -$ACE
BD$\leftrightarrow -$BCE
ABD$\leftrightarrow -$ABCE
CD$\leftrightarrow -$E
ACD$\leftrightarrow -$AE
BCD$\leftrightarrow -$BE
ABCD$\leftrightarrow -$ABE

Using this structure, the following differences of effects can be estimated:

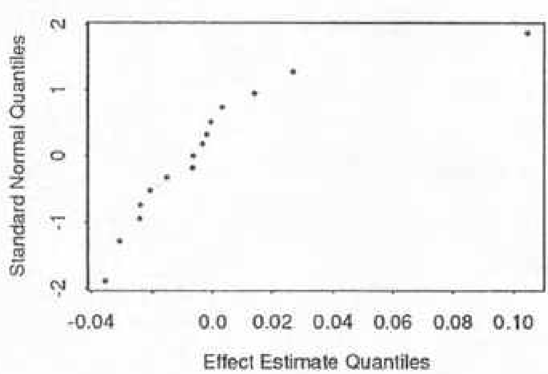
$\mu_{.....} - \gamma\delta\epsilon_{222}$
$\alpha_2 - \alpha\gamma\delta\epsilon_{2222}$
$\beta_2 - \beta\gamma\delta\epsilon_{2222}$
$\alpha\beta_{22} - \alpha\beta\gamma\delta\epsilon_{22222}$
$\gamma_2 - \delta\epsilon_{22}$
$\alpha\gamma_{22} - \alpha\delta\epsilon_{222}$
$\beta\gamma_{22} - \beta\delta\epsilon_{222}$

$\alpha\beta\gamma_{222} - \alpha\beta\delta\epsilon_{2222}$
$\delta_2 - \gamma\epsilon_{22}$
$\alpha\delta_{22} - \alpha\gamma\epsilon_{222}$
$\beta\delta_{22} - \beta\gamma\epsilon_{222}$
$\alpha\beta\delta_{222} - \alpha\beta\gamma\epsilon_{2222}$
$\gamma\delta_{22} - \epsilon_2$
$\alpha\gamma\delta_{222} - \alpha\epsilon_{22}$
$\beta\gamma\delta_{222} - \beta\epsilon_{22}$
$\alpha\beta\gamma\delta_{2222} - \alpha\beta\epsilon_{222}$

(d) Using the Yates algorithm:

Comb	$\bar{y}$	C1	C2	C3	C4	C4 ÷ 16	Diff. Estimated
(1)	.037	.077	.133	.389	2.447	.152938	$\mu_{.....} - \gamma\delta\epsilon_{222}$
a	.040	.056	.256	2.058	-.335	-.020938	$\alpha_2 - \alpha\gamma\delta\epsilon_{2222}$
b	.014	.163	1.213	.027	-.567	-.035438	$\beta_2 - \beta\gamma\delta\epsilon_{2222}$
ab	.042	.093	.845	-.362	-.057	-.003563	$\alpha\beta_{22} - \alpha\beta\gamma\delta\epsilon_{22222}$
ce	.063	.711	.031	-.091	-.245	-.015313	$\gamma_2 - \delta\epsilon_{22}$
ace	.100	.502	-.004	-.476	-.103	-.006438	$\alpha\gamma_{22} - \alpha\delta\epsilon_{222}$
bce	.067	.556	-.147	-.053	-.107	-.006687	$\beta\gamma_{22} - \beta\delta\epsilon_{222}$
abce	.026	.289	-.215	-.004	.223	.013938	$\alpha\beta\gamma_{222} - \alpha\beta\delta\epsilon_{2222}$
de	.351	.003	-.021	.123	1.669	.104313	$\delta_2 - \gamma\epsilon_{22}$
ade	.360	.028	-.070	-.368	-.389	-.024313	$\alpha\delta_{22} - \alpha\gamma\epsilon_{222}$
bde	.329	.037	-.209	-.035	-.385	-.024063	$\beta\delta_{22} - \beta\gamma\epsilon_{222}$
abde	.173	-.041	-.267	-.068	.049	.003062	$\alpha\beta\delta_{222} - \alpha\beta\gamma\epsilon_{2222}$
cd	.372	.009	.025	-.049	-.491	-.030688	$\gamma\delta_{22} - \epsilon_2$
acd	.184	-.156	-.078	-.058	-.033	-.002063	$\alpha\gamma\delta_{222} - \alpha\epsilon_{22}$
bcd	.158	-.188	-.165	-.103	-.009	-.000562	$\beta\gamma\delta_{222} - \beta\epsilon_{22}$
abcd	.131	-.027	.161	.326	.429	.026813	$\alpha\beta\gamma\delta_{2222} - \alpha\beta\epsilon_{222}$

The normal plot excludes the first estimate (.152938).



The only point that plots far away from the others is the estimate of $\delta_2 - \gamma\epsilon_{22}$. This means that this difference is statistically detectable. There is no way of knowing if this is due to the D main effect or the CE 2-factor interaction(or both).

20. (a) Start with the generators. Multiply through by E on the first generator to get I↔BCDE. Multiply through by F on the second generator to get I↔ACDF. Multiply through by G on the third generator to get I↔ABDG. Multiply through by H on the fourth generator to get I↔ABCH. Now multiply these four "I" relationships in pairs, triplets, and all together

to get 11 more:

$$I \leftrightarrow (BCDE)(ACDF)$$
$$I \leftrightarrow ABEF$$

$$I \leftrightarrow (BCDE)(ABDG)$$
$$I \leftrightarrow ACEG$$

$$I \leftrightarrow (BCDE)(ABCH)$$
$$I \leftrightarrow ADEH$$

$$I \leftrightarrow (ACDF)(ABDG)$$
$$I \leftrightarrow BCFG$$

$$I \leftrightarrow (ACDF)(ABCH)$$
$$I \leftrightarrow BDFH$$

$$I \leftrightarrow (ABDG)(ABCH)$$
$$I \leftrightarrow CDGH$$

$$I \leftrightarrow (BCDE)(ACDF)(ABDG)$$
$$I \leftrightarrow DEFG$$

$$I \leftrightarrow (BCDE)(ACDF)(ABCH)$$
$$I \leftrightarrow CEFH$$

$$I \leftrightarrow (BCDE)(ABDG)(ABCH)$$
$$I \leftrightarrow BEGH$$

$$I \leftrightarrow (ACDF)(ABDG)(ABCH)$$
$$I \leftrightarrow AFGH$$

$$I \leftrightarrow (BCDE)(ACDF)(ABDG)(ABCH)$$
$$I \leftrightarrow ABCDEFGH$$

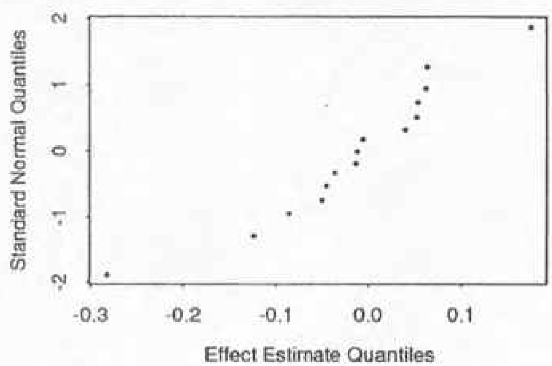
So the entire defining relationship is

$I \leftrightarrow BCDE \leftrightarrow ACDF \leftrightarrow ABDG \leftrightarrow ABCH \leftrightarrow ABEF \leftrightarrow ACEG \leftrightarrow ADEH \leftrightarrow BCFG \leftrightarrow BDFH \leftrightarrow CDGH \leftrightarrow DEFG \leftrightarrow CEFH \leftrightarrow BEGH \leftrightarrow AFGH \leftrightarrow ABCDEFGH$.

(b) Using the Yates algorithm:

Comb	$\bar{y}$	C1	C2	C3	C4	C4 ÷ 16	Sum Estimated
(1)	-.4425	-1.6414	-3.7226	-6.5962	-10.3753	-.648456	μ........ + aliases
afgh	-1.1989	-2.0812	-2.8736	-3.7791	-.0935	-.005844	α_2 + aliases
begh	-1.4307	-1.9199	-1.9530	.6496	.8425	.052656	β_2 + aliases
abef	-.6505	-.9537	-1.8261	-.7431	-.1861	-.011631	$\alpha\beta_{22}$ + aliases
cefh	-1.4230	-1.2030	.0238	.5264	.9759	.060994	γ_2 + aliases
aceg	-.4969	-.7500	.6258	.3161	.6305	.039406	$\alpha\gamma_{22}$ + aliases
bcfg	-.3267	-.8446	-.3858	.3102	.8161	.051006	$\beta\gamma_{22}$ + aliases
abch	-.6270	-.9815	-.3573	-.4963	-4.5261	-.282881	$\alpha\beta\gamma_{222} + \theta_2$ + aliases
defg	-.3467	-.7564	-.4398	.8490	2.8171	.176069	δ_2 + aliases
adeh	-.8563	.7802	.9662	.1269	-1.3927	-.087044	$\alpha\delta_{22}$ + aliases
bdfh	-.4369	.9261	.4530	.6020	-.2103	-.013144	$\beta\delta_{22}$ + aliases
abdg	-.3131	-.3003	-.1369	.0285	-.8065	-.050406	$\alpha\beta\delta_{222} + \eta_2$ + aliases
cdgh	-.6154	-.5096	1.5366	1.4060	-.7221	-.045131	$\gamma\delta_{22}$ + aliases
acdf	-.2292	.1238	-1.2264	-.5899	-.5735	-.035844	$\alpha\gamma\delta_{222} + \zeta_2$ + aliases
bcde	-.1190	.3862	.6334	-2.7630	-1.9959	-.124744	$\beta\gamma\delta_{222} + \epsilon_2$ + aliases
abcdefgh	-.8625	-.7435	-1.1297	-1.7631	.9999	.062494	$\alpha\beta\gamma\delta_{2222}$ + aliases

Note: ϵ_2 represents the main effect of factor E, ζ_2 represents the main effect of factor F, η_2 represents the main effect of factor G, and θ_2 represents the main effect of factor H. The normal plot excludes the first estimate (-.648456).

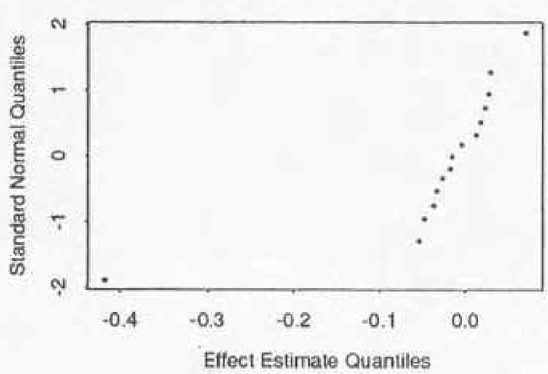


None of the points are obviously larger than the others, but the two that stand out most are the estimate of $\alpha\beta\gamma_{222} + \theta_2$ + aliases and the estimate of δ_2 + aliases. This means that these sums are marginally detectable. A simple tentative conclusion is that the main effects of factors D and H are causing these estimates to be large. In the context of the experiment, it seems that Susceptor Rotation and Nozzle Position are the primary factors that determine uniformity. Based on the signs of these estimates, to minimize y_2, set D at its low level (continuous) and H at its high level (6). It is not easy to tell from the description whether "6" is more expensive than "2" for the Nozzle Position. It may be more expensive to have continuous rotation than oscillating rotation, but this is also unclear from the problem description.

(c) Using the Yates algorithm:

Comb	$\bar{y}$	C1	C2	C3	C4	C4 ÷ 16	Sum Estimated
(1)	14.821	29.709	57.626	115.329	230.222	14.3889	μ........ + aliases
afgh	14.888	27.917	57.703	114.893	-.538	-.0336	α_2 + aliases
begh	14.037	28.025	57.262	-.231	-.874	-.0546	β_2 + aliases
abef	13.880	29.678	57.631	-.307	-.290	-.0181	$\alpha\beta_{22}$ + aliases
cefh	14.165	28.004	-.090	-.139	.446	.0279	γ_2 + aliases
aceg	13.860	29.258	-.141	-.735	.378	.0236	$\alpha\gamma_{22}$ + aliases
bcfg	14.757	29.810	-.368	.245	.202	.0126	$\beta\gamma_{22}$ + aliases
abch	14.921	27.821	.061	-.535	1.134	.0709	$\alpha\beta\gamma_{222} + \theta_2$ + aliases
defg	13.972	.067	-1.792	.077	-.436	-.0272	δ_2 + aliases
adeh	14.032	-.157	1.653	.369	-.076	-.0048	$\alpha\delta_{22}$ + aliases
bdfh	14.843	-.305	1.254	-.051	-.596	-.0373	$\beta\delta_{22}$ + aliases
abdg	14.415	.164	-1.989	.429	-.780	-.0488	$\alpha\beta\delta_{222} + \eta_2$ + aliases
cdgh	14.878	.060	-.224	3.445	.292	.0182	$\gamma\delta_{22}$ + aliases
acdf	14.932	-.428	.469	-3.243	.480	.0300	$\alpha\gamma\delta_{222} + \zeta_2$ + aliases
bcde	13.907	.054	-.488	.693	-6.688	-.4180	$\beta\gamma\delta_{222} + \epsilon_2$ + aliases
abcdefgh	13.914	.007	-.047	.441	-.252	-.0157	$\alpha\beta\gamma\delta_{2222}$ + aliases

Note: ϵ_2 represents the main effect of factor E, ζ_2 represents the main effect of factor F, η_2 represents the main effect of factor G, and θ_2 represents the main effect of factor H. The normal plot excludes the first estimate (14.3889).



The only point that plots far away from the others is the estimate of $\beta\gamma\delta_{222} + \epsilon_2$ + aliases. This means that this sum is statistically detectable. A simple tentative conclusion is that the main effect of factor E is causing this estimate to be large. In the context of the experiment, it seems that Deposition Time is the primary factor that determines average thickness. Based on the raw data, it seems the high Deposition Time (E $(-)$) results in thicknesses above target, and the low Deposition Time (E $(+)$) results in thicknesses below target. There may be some intermediate time that will result in a mean thickness close to target.

21. (a) According to the table, the best possible resolution is 4. To figure out the resolution of the students' design, first determine the defining relation. Multiply through the first generator by E to get I↔ABE. Multiply through the second generator by F to get I↔ACF. Now multiply these two "I" relationships to get a third:

$$\mathrm{I} \leftrightarrow (\mathrm{ABE})(\mathrm{ACF})$$
$$\mathrm{I} \leftrightarrow \mathrm{BCEF}$$

So the entire defining relationship is

I↔ABE↔ACF↔BCEF.

Since the shortest product in the defining relation has 3 letters, the students' plan has resolution 3, with some main effects aliased with 2-factor interactions. A resolution 4 design would have all main effects aliased with 3- or higher-factor interactions. In this case, all main effects could be estimated if all 3-factor interactions were small. In the students' plan, several 2-factor interactions would need to be small (as well as all 3-factor interactions) in order to estimate all of the main effects.

(b) One possibility is E↔BCD and F↔ABC (there are others). The defining relationship for these generators is

I↔BCDE↔ABCF↔ADEF.

Since the shortest product in this defining relation has 4 letters, this plan has resolution 4.

(c) Using the Yates algorithm:

Use defining relation

Comb	$\bar{y}$	C1	C2	C3	C4	C4 ÷ 16	Sum Estimated
ef	13.990	20.750	52.580	131.370	342.655	21.4159	μ...... + aliases
a	6.760	31.830	78.790	211.285	-22.835	-1.4272	α_2 + aliases
bf	20.710	35.340	98.730	-24.150	32.585	2.0366	β_2 + aliases
abe	11.120	43.450	112.555	1.315	1.555	.0972	$\alpha\beta_{22} + \epsilon_2$ + aliases
ce	19.610	49.470	-16.820	19.190	40.035	2.5022	γ_2 + aliases
acf	15.730	49.260	-7.330	13.395	12.985	.8116	$\alpha\gamma_{22} + \zeta_2$ + aliases
bc	23.450	49.475	-1.090	-1.930	10.845	.6778	$\beta\gamma_{22}$ + aliases
abcef	20.000	63.080	2.405	3.485	6.815	.4259	$\alpha\beta\gamma_{222}$ + aliases
def	24.940	-7.230	11.080	26.210	79.915	4.9947	δ_2 + aliases
ad	24.530	-9.590	8.110	13.825	25.465	1.5916	$\alpha\delta_{22}$ + aliases
bdf	24.970	-3.880	-.210	9.490	-5.795	-.3622	$\beta\delta_{22}$ + aliases
abde	24.290	-3.450	13.605	3.495	5.415	.3384	$\alpha\beta\delta_{222}$ + aliases
cde	25.075	-.410	-2.360	-2.970	-12.385	-.7741	$\gamma\delta_{22}$ + aliases
acdf	24.400	-.680	.430	13.815	-5.995	-.3747	$\alpha\gamma\delta_{222}$ + aliases
bcd	30.000	-.675	-.270	2.790	16.785	1.0491	$\beta\gamma\delta_{222}$ + aliases
abcdef	33.080	3.080	3.755	4.025	1.235	.0772	$\alpha\beta\gamma\delta_{2222}$ + aliases

Note: ϵ_2 represents the main effect of factor E, and ζ_2 represents the main effect of factor F.

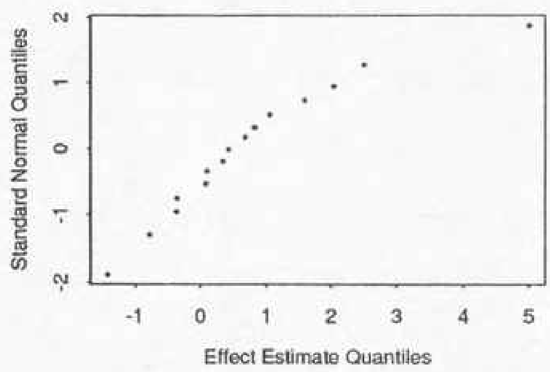
(d) Realistically, you should try to catch the variability that might be caused by changing the setups. One way to see this is to imagine that none of the factors have any effect on the response. If this is the case, then the variability between conditions should be the same as the variability within a condition. This is probably not the case here, since "replications" consisted of consecutive runs, with no change in setup. It is likely that any estimate of variability based on these replications will underestimate the true amount of variability that would be experienced over time. Basically, since the setup was changed between conditions, the estimate of baseline variation should be based on replications that have had a setup change between them. This type of estimate would be appropriate for use in confidence intervals. Using the given data, s_p will be too small, and many effects will (misleadingly) appear to be detectable.

(e) Using equation (7-7), $s_p = .3706$. The appropriate t is $t = Q_4(.95) = 2.132$ from Table B-4. The resulting ± part is

$$(2.132)(.3706)\frac{1}{16}\sqrt{12 + \frac{4}{2}} = .1848.$$

Based on this standard, all but two of the sums of effects are detectable. Since s_p is underestimating the true baseline variability, these intervals should not be used to judge detectability of effects; it gives "too many" detectable sums of effects.

(f) The normal plot excludes the first estimate (21.4159).



The only point that plots far away from the others is the estimate of δ_2 + aliases. This means that this sum is statistically detectable. A simple tentative conclusion is that the main effect of factor D is causing this estimate to be large. In the context of the experiment, it seems that Extruder Screw Speed is the primary factor that determines the amount of useful output. Based on the sign of this estimate, it seems that the "high" level of Extruder Screw Speed should be used to maximize amount of useful output.

(g) Any further data collection should focus closely on the effect of factor D, since the data from this fractional factorial experiment suggest that factor D seems to be the most important overall. Extruder Screw Speed seems to be a continuous variable, so line or curve fitting methods might be applied to data collected from several levels of Extruder Screw Speed. These methods could be used to predict an optimum Extruder Screw Speed. "True" replications should be conducted to avoid the same problem as in part (d) of this exercise. These replications would allow for the appropriate use of confidence, prediction, and tolerance intervals.

22. (a) Use the generators and multiply the appropriate columns.

A	B	C	D	E	F↔ −CD	G↔ −AD	H↔ −ABCD	Combination
−	−	−	−	−	−	−	−	(1)
+	−	−	−	−	−	+	+	agh
−	+	−	−	−	−	−	+	bh
+	+	−	−	−	−	+	−	abg
−	−	+	−	−	+	−	+	cfh
+	−	+	−	−	+	+	−	acfg
−	+	+	−	−	+	−	−	bcf
+	+	+	−	−	+	+	+	abcfgh
−	−	−	+	−	+	+	+	dfgh
+	−	−	+	−	+	−	−	adf
−	+	−	+	−	+	+	−	bdfg
+	+	−	+	−	+	−	+	abdfh
−	−	+	+	−	−	+	−	cdg
+	−	+	+	−	−	−	+	acdh
−	+	+	+	−	−	+	+	bcdgh
+	+	+	+	−	−	−	−	abcd
−	−	−	−	+	−	−	−	e
+	−	−	−	+	−	+	+	aegh
−	+	−	−	+	−	−	+	beh
+	+	−	−	+	−	+	−	abeg
−	−	+	−	+	+	−	+	cefh
+	−	+	−	+	+	+	−	acefg
−	+	+	−	+	+	−	−	bcef
+	+	+	−	+	+	+	+	abcefgh
−	−	−	+	+	+	+	+	defgh
+	−	−	+	+	+	−	−	adef
−	+	−	+	+	+	+	−	bdefg
+	+	−	+	+	+	−	+	abdefh
−	−	+	+	+	−	+	−	cdeg
+	−	+	+	+	−	−	+	acdeh
−	+	+	+	+	−	+	+	bcdegh
+	+	+	+	+	−	−	−	abcde

(b) Start with the generators. Multiply through by F on the first generator to get $I \leftrightarrow -CDF$. Multiply through by G on the second generator to get $I \leftrightarrow -ADG$. Multiply through by H on the third generator to get $I \leftrightarrow -ABCDH$. Now multiply these three "I" relationships in pairs and all together to get 4 more:

$$I \leftrightarrow (-CDF)(-ADG)$$
$$I \leftrightarrow ACFG$$

$$I \leftrightarrow (-CDF)(-ABCDH)$$
$$I \leftrightarrow ABFH$$

$$I \leftrightarrow (-ADG)(-ABCDH)$$
$$I \leftrightarrow BCGH$$

$$I \leftrightarrow (-CDF)(-ADG)(-ABCDH)$$
$$I \leftrightarrow -BDFGH$$

So the entire defining relationship is

$I \leftrightarrow -CDF \leftrightarrow -ADG \leftrightarrow -ABCDH \leftrightarrow ACFG \leftrightarrow ABFH \leftrightarrow BCGH \leftrightarrow -BDFGH$.

Since the number of letters in the shortest product is 3, the resolution of this design is 3. The best possible resolution is 4. If the engineers had used a resolution 4 design, they would have done better at minimizing the ambiguity. The resolution 3 design has some main effects aliased with 2-factor interactions; a resolution 4 design would have no main effects aliased with 2-factor interactions.

(c) The linear combinations of effects that appear to be detectable are the ones whose estimates are on lines 13, 17, and 29 of the Yates algorithm. Using the defining relation, the estimate on line 13 is for

CD interaction − F main effect + other aliases.

The estimate on line 17 is for

E main effect + other aliases.

The estimate on line 29 is for

CDE interaction − EF interaction + other aliases.

A tentative simple interpretation is that the dominant effects are the E and F main effects and the EF 2-factor interaction.

(d) The linear combinations of effects that appear to be detectable are the ones whose estimates are on lines 17 and 25 of the Yates algorithm. Using the defining relation, the estimate on line 17 is for

E main effect + other aliases.

The estimate on line 25 is for

DE interaction + other aliases.

A tentative simple interpretation is that the dominant effects are the E main effect and the DE 2-factor interaction.

(e) Based on the signs of the estimates, set E at its low level (Flux Type A857), F at its high level (90 Direction), and D at its high level (6.1 Conveyor Angle). Note: the sign of the estimate on line 29 of the Yates algorithm for y_1 suggests setting E and F both at their high or both at their low levels. However, the main effects for E and F are much larger, so they should be used primarily in trying to minimize the response.

The combinations dfgh, adf, dbfg, and abdfh all have the above combination of factor levels. These 4 conditions do seem to produce smaller y_1 and y_2 values than the rest. A follow-up study of these conditions should be done to validate the results of the experiment, before any permanent changes are made.

23. (a) The effect of shift is confounded with the effect of factor E. All runs with E low were made on the first shift, while all runs with E high were made on the second shift. Differences between the first 16 runs and the last 16 runs may be due to changing the level of E, or changing shifts (or both), so the engineers did run the risk of clouding their information about factor E's effect.

(b) Label Shift as factor J, with the "low" level corresponding to the first shift and the "high" level corresponding to the second shift. One possible generator for J is J↔ABE. The resulting assignment of combinations is given below.

Shift 1	Shift 2
(1), abg, cfh, abcfgh	agh, bh, acfg, bcf
dfgh, abdfh, cdg, abcd	adf, bdfg, acdh, bcdgh
aegh, beh, acefg, bcef	e, abeg, cefh, abcefgh
adef, bdefg, acdeh, acdegh	defgh, abdefh, cdeg, abcde

For example, the (1) combination has all of A, B, and E at their low levels, so J will also be at its low level (1st shift).

(c) Treat this as a 2^{9-4} fractional factorial. Find the defining relation, and use the resulting alias structure to interpret the estimates from the Yates algorithm. It may be possible to assume that there are no interactions involving J (Shift). If this is the case, all interactions in the defining relation involving J can be dropped. Hopefully, Shift will not have a large effect on the responses; if it does, the main effect of shift at least will not be confounded with any other main effects or 2-factor interactions.

24. (a) $s_p = 5.477$ with 3 degrees of freedom associated with it. Using the Yates algorithm:

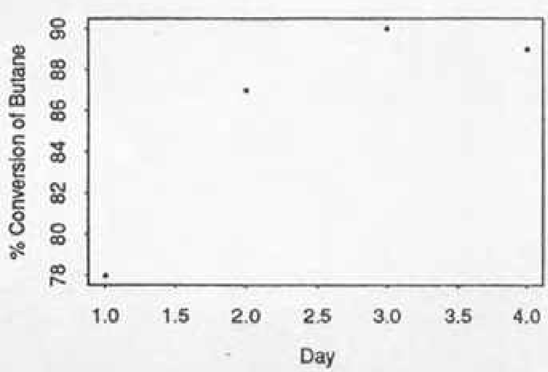
Combination	$\bar{y}$	Cycle 1	Cycle 2	Cycle 3	Cycle 3 ÷ 8	Effect Estimated
(1)	99	158	348	624	78.0	$\mu_{...}$
a	59	190	276	-112	-14.0	α_2
b	98	94	-46	120	15.0	β_2
ab	92	182	-66	84	10.5	$\alpha\beta_{22}$
c	76	-40	32	-72	-9.0	γ_2
ac	18	-6	88	-20	-2.5	$\alpha\gamma_{22}$
bc	95	-58	34	56	7.0	$\beta\gamma_{22}$
abc	87	-8	50	16	2.0	$\alpha\beta\gamma_{222}$

Using equation (8-13), the appropriate t is $t = Q_3(.975) = 3.182$ from Table B-4. Here, $m = 1$ and $p = 3$, so the $\pm$ part of the confidence intervals is

$$(3.182)\frac{5.477}{\sqrt{(1)(8)}} = 6.162.$$

Based on this $\pm$ part, the intervals for α_2, β_2, $\alpha\beta_{22}$, γ_2, and $\beta\gamma_{22}$ do not contain zero, so all of these effects are detectable. With two detectable interactions, no simple interpretation of the results is obvious.

(b)



The plot shows that y generally increased over the 4 days. It is difficult to tell if this is really a trend, or just random variation, because there is only one observation per day for only 4 days. If $\sigma \approx 5\%$ is expected, then the increase over the 4 days is relatively large for random variation. It is still possible that the plot represents random variation, though. (All of these points could have been reasonably generated from a stable mean at 84%.) If $\sigma \approx 1\%$ is expected, then the increase over the 4 days is extremely large for random variation. It would be unlikely that the plot represents random variation. In this case, the increase might reasonably be considered a real trend.

(c) Use the generators and multiply the appropriate columns.

A	B	C	D↔AB	E↔BC	Combination	Day
−	−	−	+	+	de	1
+	−	−	−	+	ac	3
−	+	−	−	−	b	2
+	+	−	+	−	abd	4
−	−	+	+	−	cd	4
+	−	+	−	−	ac	2
−	+	+	−	+	bce	3
+	+	+	+	+	abcde	1

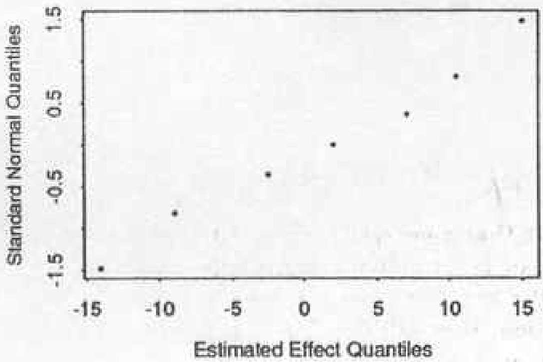
(d) Multiply through the defining relation by the various effects.

I↔ABD↔BCE↔ACDE
A↔BD↔ABCE↔CDE
B↔AD↔CE↔ABCDE
AB↔D↔ACE↔BCDE
C↔ABCD↔BE↔ADE
AC↔BCD↔ABE↔DE
BC↔ACD↔E↔ABDE
ABC↔CD↔AE↔BDE

If there are no interactions with days, then the effects of the factors A, B, and C (main effects and 2- and 3-factor interactions) are the same for each day. None of these effects change from day to day (although the mean may change due to a Day main effect). If there are no interactions with days, then any interaction in the above alias structure that involves D or E and another factor can be dropped. The simplified alias structure is then

I
A
B
AB↔D
C
AC↔DE
BC↔E
ABC

(e) The normal plot excludes the first estimate (78.0).

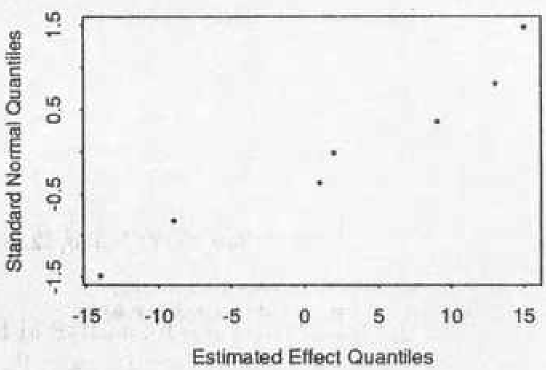


None of the estimates stand out on this plot. There is no indication that any of the sums of effects are detectable.

(f) Using the Yates algorithm:

Combination	$\bar{y}_d$	Cycle 1	Cycle 2	Cycle 3	Cycle 3 $\div$ 8	Effect Estimated
(1)	21	-10	4	-64	-8	$\mu_{...}$
a	-31	14	-68	-112	-14	α_2
b	11	-82	-60	120	15	β_2
ab	3	14	-52	104	13	$\alpha\beta_{22}$
c	-13	-52	24	-72	-9	γ_2
ac	-69	-8	96	8	1	$\alpha\gamma_{22}$
bc	5	-56	44	72	9	$\beta\gamma_{22}$
abc	9	4	60	16	2	$\alpha\beta\gamma_{222}$

The normal plot excludes the first estimate (-8).



None of these estimates stand out either. There is no indication that any of the effects are detectable. The result of this analysis is not substantially different from the analysis in part (e).

(25) (a) $s^2_{pooled} = [\,0^2 + (7.8)^2 + (4.9)^2 + (4.9)^2 + (6.4)^2 + (7.1)^2 + (6.4)^2 + (.7)^2]/8 = 30.21$. So, $s_{pooled} = 5.496$. s_{pooled} estimates the standard deviation (variability) of surface roughness for parts made at any one of the 8 combinations of Speed, Feed and Tool Condition.

(b) $\Delta = t_8 s_{pooled}/\sqrt{2} = [(2.306)/(1.4142)]5.49636 = 8.96$

(c) $\bar{y}_a - \bar{y}_1 = 45.5 - 33 = 12.5$, $\Delta = t_8 s_{pooled}\sqrt{1} = (2.306)(5.49636)(1) = 12.6746$ for a 95% confidence interval estimate of $\mu_a - \mu_1$. Since $\bar{y}_a - \bar{y}_1 = 12.5 < 12.675$, we do not have strong evidence to conclude μ_a is different from μ_1.

(d) $\Delta = t_8 s_{pooled}/\sqrt{2(2)^3} = (2.306)(5.49636)/4 = 3.16865$.

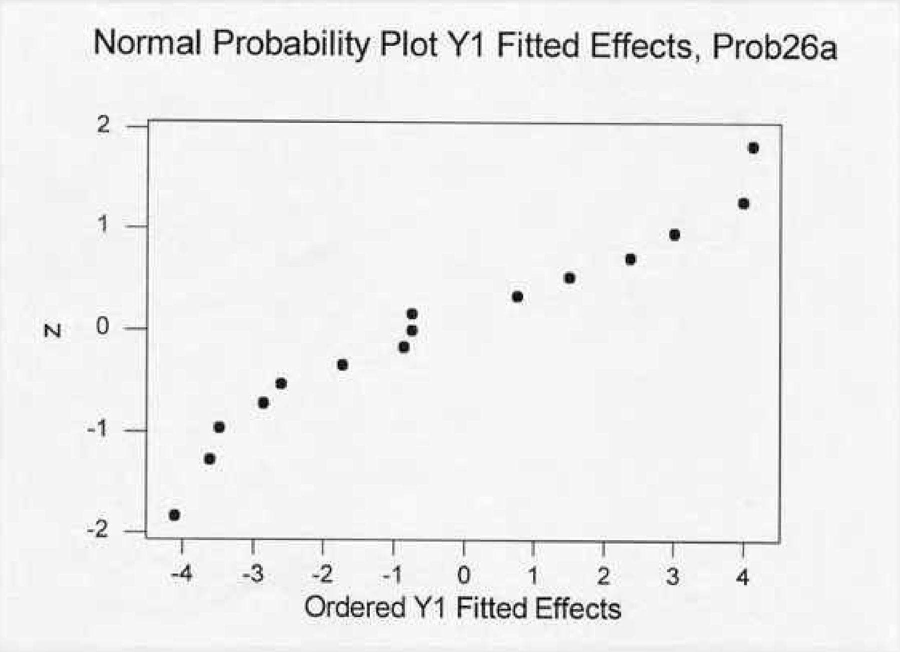
(e) $\hat{\mu} = 131.875$, $a_2 = 5$, $b_2 = 92.375$, $c_2 = -3.75$, $ab_{22} = -.25$, $ac_{22} = -2.875$, $bc_{22} = -4$, $abc_{222} = -1.875$.

(f) A, B, C and BC interactions are judged to be statistically detectable. The absolute values of a_2, b_2, c_2 and bc_{22} each exceed $\Delta = 3.168$ from (d).

(g) Initially, it seems the setting should be lo A (2500 rpm), lo B (.003 in/rev.), and hi C (Used). If only main effects are considered, this setting will minimize y and minimize cost. However, when considering the A, B and C main effects with the BC interaction, lo A (2500 rpm), lo B (.003 in/rev) and lo C (New) setting minimizes surface roughness but cost is higher than for lo A, lo B, hi C and lo B/hi C interaction.

(h) $\hat{\mu} + b_2 = 131.875 + 92.375 = 224.25$.

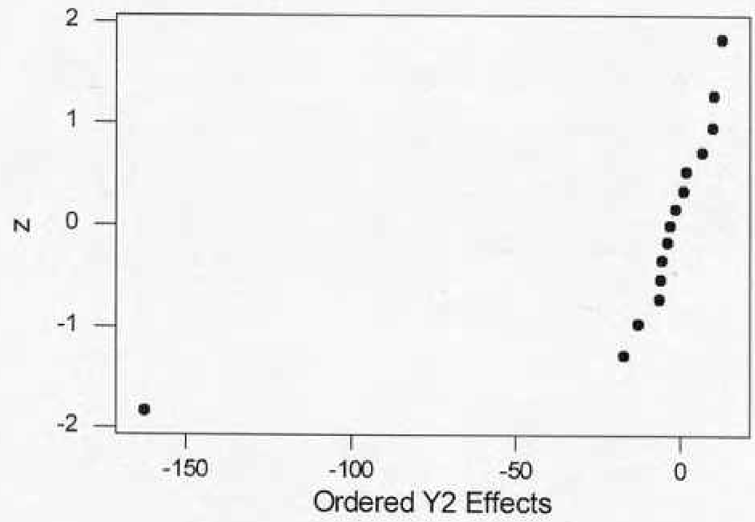
(26)(a) $\hat{\mu} = 80.125$, $a_2 = -.75$, $b_2 = 4.125$, $c_2 = -.875$, $d_2 = 2.375$, $ab_{22} = -1.75$, $ac_{22} = 1.5$, $ad_{22} = .75$, $bc_{22} = -2.875$, $bd_{22} = -4.125$, $cd_{22} = -3.625$, $abc_{222} = -3.5$, $abd_{222} = -.75$, $acd_{222} = 3$, $bcd_{222} = -2.625$, $abcd_{2222} = 4$.



It seems the B main effect, far most right point (b_2 = 4.125), is perhaps somewhat "different" and the BD interaction, far most left point (bd_{22} = -4.125), is somewhat "different". At high H_2 Flow (1000) (B main effect), there seems to be a smaller "percent step coverage" than would be expected if there was no B effect.

(b) $\hat{\mu}$ = 495.688, a_2= -17.188, b_2 = 12.687, c_2 = -162.813, d_2= -4.063, ab_{22} = -6.188, ac_{22}= 10.313, ad_{22} = -3.188, bc_{22} = -12.812, bd_{22} = 9.937, cd_{22}= -6.063, abc_{222} = -1.688, abd_{222}= 1.813, acd_{222}= .813, bcd_{222}= 6.438,

Normal Probability Plot Y2 Fitted Effects, Prob26b



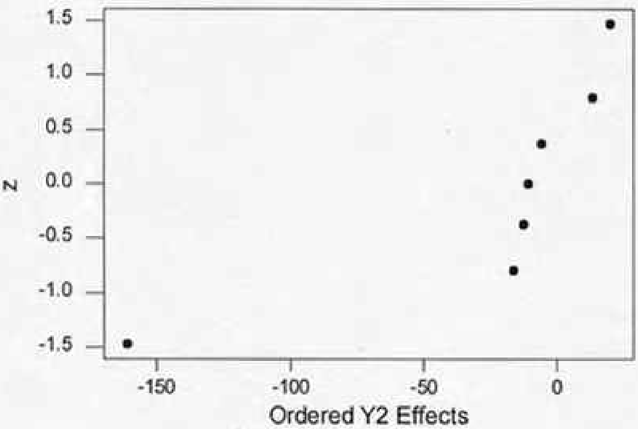
$abcd_{2222}$= -5.687. It seems there is one effect that is clearly different. This one effect is the C main effect c_2 = -162.813. At the high level of C (25), there is a significantly smaller "average" sheet resistance.

(c) The 8 treatment combinations that would have been run using the generator D $\leftrightarrow$ ABC or $I \leftrightarrow ABCD$ are:
1, ad, bd, ab, cd, ac, bc, abcd.

$\hat{\mu}$ + $abcd_{2222}$= 490, a_2 + bcd_{222}= -10.75, b_2 + acd_{222}= 13.5, ab_{22} + cd_{22}= -12.25, c_2 + abd_{222}= -161, ac_{22} + bd_{22} = 20.25, bc_{22} + ad_{22} = -16, d_2 + abc_{222} = -5.75. Yes, these estimates equal the sum of the estimates from (b) above for y_2 data.

(d) It seems the same conclusion would result with this analysis as was determined in part (b) above based on the full 2^4 factorial. . At the high level of C (25), there is a significantly smaller "average" sheet resistance. Assuming no 3-factor interaction.

Normal Probability Plot of Fitted Effects Prob 26d



(27)(a) The following 16 combinations of levels of the 7 factors will be run: e, af, bfg, abeg, cfg, aceg, bce, abcf, dg, adefg, bdef, abd, cdef, acd, bcdg, abcdefg.

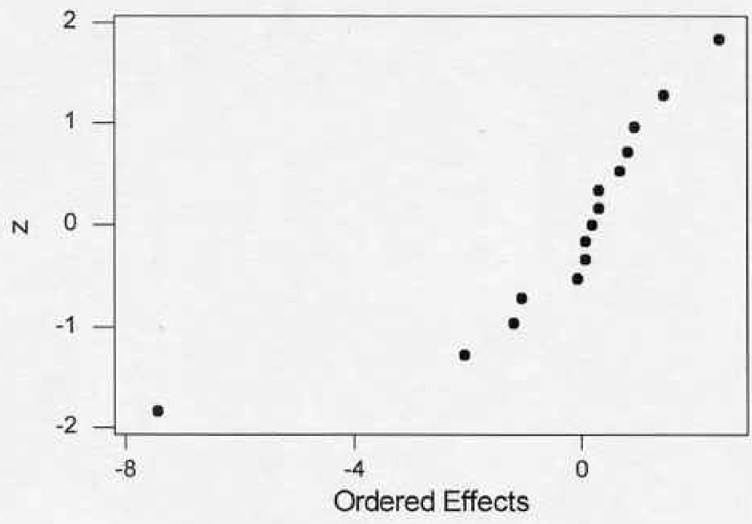
(b) I = ABCDE=ABCF=BCDG=DEF=AEG=ADFG=BCEFG.

(c) The defining relation given in this part (c) is better because each main effect is aliased with the sum of 7 three-factor interactions. The defining relation in (b) has main effects aliased with the sum of 7 effects, some of which are two-factor interactions. Thus, using the defining relation in (b) makes it difficult to estimate the main effects whereas using the defining relation in (c) permits one to estimate the main effects assuming the three factor interactions are negligible (assuming two factor interactions are negligible is very risky).

(28)(a) Using the defining relation $I \leftrightarrow ABCDE$, each main effect is aliased with a four-factor interaction. Thus, it is not unreasonable to think of the estimated sum of a main effect and four-factor interaction as just estimating the main effect (assuming the four-factor interaction is zero). Using the experimental plan (defining relation $I \leftrightarrow ABCE$), each main effect is aliased with only a three-factor interaction. It is not so obvious to assume every three-factor interaction is negligible. Thus, estimating main effects with this defining relation is less attractive (assuming three-factor interactions to be zero is less appealing than assuming four-factor interactions are zero).

(b)

Normal Probability Plot of 15 Fitted Sums, Prob 28b



(c) From largest to smallest, left to right: D, B, C, A, E. This order is based on the absolute value of the main effect plus an alias.

(d) Based on the normal probability plot above, it seems there is really only one clear significantly different "sum of effects". In this problem, it is $d_2 + abcd_{2222} = -7.437$.

(e) Set the Aerosol (D) at the high level (.6%), the Method of Preparation(A) at the low level (Usual), the Sugar Content at the low level (50%), the antiobiotic level (C) at the high level (16%). Since it seems only D plus alias is significantly different, the predicted value is [mean plus alias + D plus alias] which equals 37.563 - 7.437 = 30.126.

(f) No, it doesn't seem there is a "C effect", so either antibiotic level would have about the same effect.

(29)(a) df = 8(3)=24, $s_p = .118$, $\Delta = t_{24}\, s_p / \sqrt{4} = (2.064)(.118)/2 = .12154$.

(b) mean + aliases = 1.42375
a_2 + aliases = - .02125
b_2 + aliases = .20125
c_2 + aliases = .06625
ab_{22} + aliases = - .00875
ac_{22} + aliases = - .32375
bc_{22} + aliases = .12375
abc_{222} + aliases = -.01125

(c) Using equation 8-13, $\Delta = t_{24}\, s_p/\sqrt{(4)2^3}$ = (2.064)(.118)/(5.65685) = .04305.

(d) b_2 + aliases = .20125, c_2 + aliases = .06625, ac_{22} + aliases = - .32375, ac_{22} + aliases = .12375 all exceed (in absolute value) .04305.

(e) de, aef, bdf, ab, cf, acd, bce, abcdef are the combinations of the experimental factors A,B,C,D,E and F that were included in the experiment.

(f) $I \leftrightarrow ACD \leftrightarrow BCE \leftrightarrow ABCF \leftrightarrow ABDE \leftrightarrow BDF \leftrightarrow AEF \leftrightarrow CDEF$

(g) It seems the simplest explanation would be that there are important B,C, D and E main effects (if we assume the all 2-factor and above aliases with these main effects). The D and E main effects come from the fact that ac_{22} + aliases and ac_{22} + aliases are significant and the alias structure produces D and E main effects aliased with other 2-factor and above interactions that are important.

Chapter 9: Inference for Curve- and Surface-Fitting Analysis of Multisample Studies

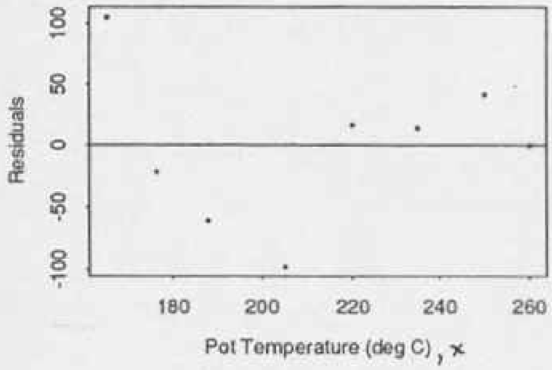
1. (a) See Exercise 3, Section 1, Chap. 4 for computations. Using equation (9-10),

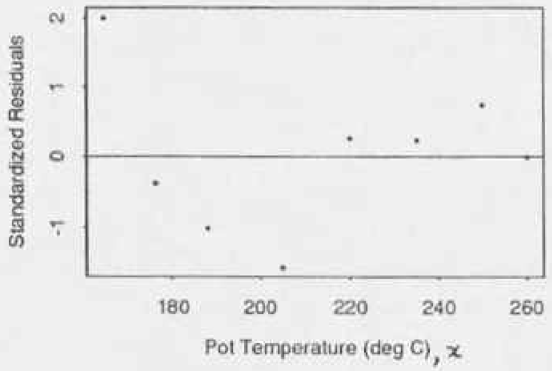
$$s_{\mathrm{LF}}^2 = \frac{1}{8-2}(26940.69) = 4490.116,$$

so $s_{\mathrm{LF}} = \sqrt{4490.116} = 67.01$, with 6 degrees of freedom associated with it. This measures the baseline variation in molecular weight that would be observed for any fixed pot temperature, assuming that model (9-4) is appropriate.

(b) The residuals were computed in Ex. 3, Sec. 1, Ch. 4. Use equation (9-12) to compute the standardized residuals. $\bar{x} = 212.375$, and $\sum(x-\bar{x})^2 = 8469.875$. The rest of the calculations are summarized below.

x	$\sqrt{1-\frac{1}{8}-\frac{(x-212.375)^2}{8469.875}}$	e	e^*
165	.78103	105.35535	2.01306
176	.84781	-21.12558	-.37186
188	.89714	-60.10477	-.99982
205	.93198	-97.57529	-1.56245
220	.93174	16.95072	.27150
235	.90253	14.47673	.23938
250	.84135	42.00275	.74503
260	.77924	.02009	.00038





The plots look almost exactly the same.

(c) This is β_1. Use equation (9-17). For 90% confidence, the appropriate t is $t = Q_6(.95) = 1.943$ from Table B-4. The resulting interval is

$$\begin{aligned} & 23.49827 \pm 1.943 \frac{67.01}{\sqrt{8469.875}} \\ = \ & 23.49827 \pm 1.414696 \\ = \ & [22.08, 24.91]. \end{aligned}$$

(d) Use equation (9-24). The appropriate t is the same as the one in part (c). The resulting interval for the mean at $x = 212$ is

$$\begin{aligned}
& 1807.063 \pm 1.943(67.01)\sqrt{\frac{1}{8} + \frac{.140625}{8469.875}} \\
= \ & 1807.063 \pm 46.03471 \\
= \ & [1761.03, 1853.10].
\end{aligned}$$

The resulting interval for the mean at $x = 250$ is

$$\begin{aligned}
& 2699.997 \pm 1.943(67.01)\sqrt{\frac{1}{8} + \frac{1415.641}{8469.875}} \\
= \ & 2699.997 \pm 70.37134 \\
= \ & [2629.63, 2770.37].
\end{aligned}$$

(e) Use equation (9-25). The appropriate f is $f = Q_{2,6}(.90) = 3.46$ from Table B-6-B. The resulting interval for the mean at $x = 212$ is

$$\begin{aligned}
& 1807.063 \pm \sqrt{2(3.46)}(67.01)\sqrt{\frac{1}{8} + \frac{.140625}{8469.875}} \\
= \ & 1807.063 \pm 62.32548 \\
= \ & [1744.74, 1869.39].
\end{aligned}$$

The resulting interval for the mean at $x = 250$ is

$$\begin{aligned}
& 2699.997 \pm \sqrt{2(3.46)}(67.01)\sqrt{\frac{1}{8} + \frac{1415.641}{8469.875}} \\
= \ & 2699.997 \pm \\
= \ & [2604.72, 2795.27].
\end{aligned}$$

(f) Use equation (9-26). For a 90% one-sided interval, appropriate t is $t = Q_6(.90) = 1.440$ from Table B-4. The resulting lower prediction bound at $x = 212$ is

$$\begin{aligned}
& 1807.063 - 1.440(67.01)\sqrt{1 + \frac{1}{8} + \frac{.140625}{8469.875}} \\
= \ & 1807.063 - 102.346 \\
= \ & 1704.72.
\end{aligned}$$

The resulting bound for the mean at $x = 250$ is

$$\begin{aligned}
& 2699.997 - 1.440(67.01)\sqrt{1 + \frac{1}{8} + \frac{1415.641}{8469.875}} \\
= \ & 2699.997 - 109.6846 \\
= \ & 2590.31.
\end{aligned}$$

(g) Use equation (9-27). For $x = 212$, first using equation (9-29) ,

$$A = \sqrt{\frac{1}{8} + \frac{.140625}{8469.875}} = .3535769.$$

Finally, using equation (9-30),

$$\tau = \frac{1.28 + (.3535769)(\ 1.645\ )\sqrt{1 + \frac{1}{2(6)}\left(\frac{(1.28)^2}{(.3535769)^2} - (\ 1.645\ )^2\right)}}{1 - \frac{(\ 1.645\ )^2}{2(6)}} = 2.6787$$

The resulting bound for $x = 212$ is

$$1807.063 - 2.6787\quad(67.01) = 1627.6.$$

For $x = 250$, first using equation (9-30),

$$A = \sqrt{\frac{1}{8} + \frac{1415.641}{8469.875}} = .5404982.$$

Finally, using equation (9-30),

$$\tau = \frac{1.28 + (.5404982)(\ 1.645\ )\sqrt{1 + \frac{1}{2(6)}\left(\frac{(1.28)^2}{(.5404982)^2} - (\ 1.645\ )^2\right)}}{1 - \frac{(\ 1.645\ )^2}{2(6)}} = 2.932$$

The resulting bound for $x = 250$ is

$$2699.997 - 2.932\quad(67.01) = 2503.5$$

(h) Using the general form given in Table 9-6.

Source	SS	df	MS	F
Regression	4676798	1	4676798	1041.58
Error	26941	6	4490	
Total	4703739	7		

The p-value is

$$P(\text{an } F_{1,6} \text{ random variable} > 1041.58),$$

which is less than .001, according to Tables B-6 ($Q_{1,6}(.999) = 35.51$). This is overwhelming evidence that the mean average molecular weight is related to the pot temperature. (The model used is an improvement over the model $y = \beta_0 + \epsilon$.)

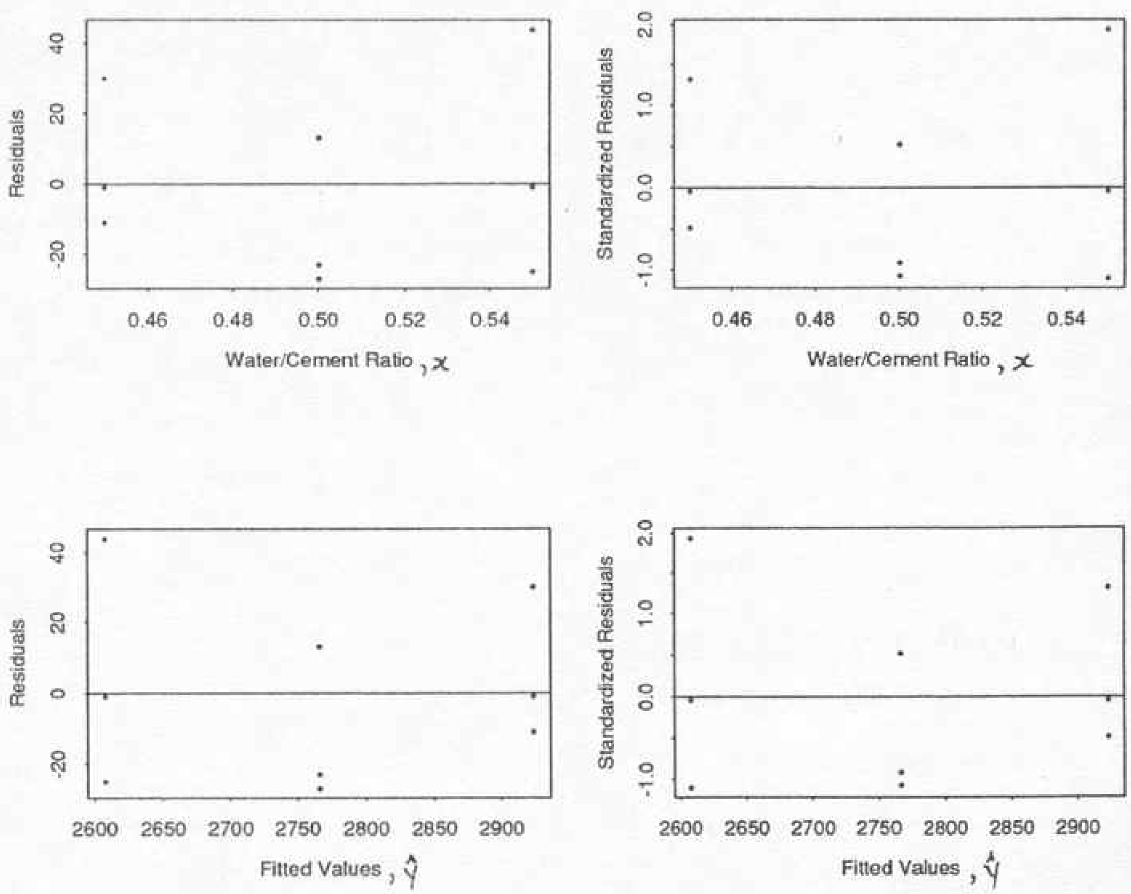
2. (a) In Ex. 3, Sec. 1, Ch. 4, , $b_1 = -3160$ and $b_0 = 4345.889$. The necessary computations for s_{LF} (the residuals) are also given there. Using equation (9-10),

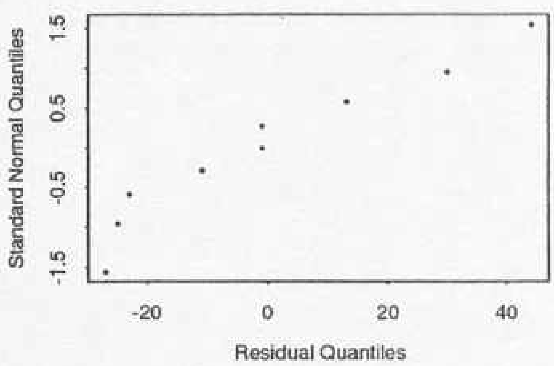
$$s^2_{\text{LF}} = \frac{1}{9-2}(5010.889) = 715.84,$$

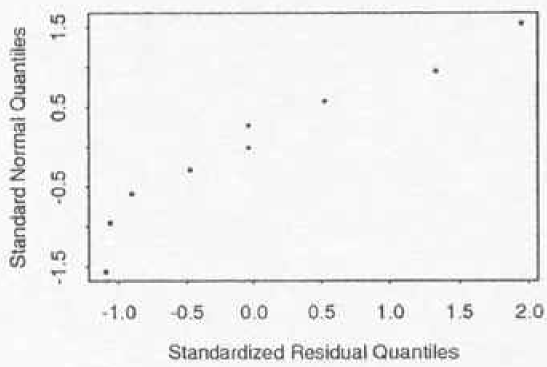
so $s_{\text{LF}} = \sqrt{715.84} = 26.755$ psi, with 7 degrees of freedom associated with it. This measures the baseline variation in 14-day compressive strength that would be observed for any fixed water/cement ratio, assuming that model (9-4) is appropriate. Using equation (7-7), $s_p = 26.890$ psi. These two estimates are very close, giving no indication that the model is inappropriate.

(b) The residuals were computed in Ex. 3, Sec. 1, Ch. 4. Use equation (9-12) to compute the standardized residuals. $\bar{x} = .50$, and $\sum(x - \bar{x})^2 = .015$. The rest of the calculations are summarized below.

x	$\sqrt{1 - \frac{1}{9} - \frac{(x-.50)^2}{.015}}$	e	e^*
.45	.8498	30.1111	1.3243
.45	.8498	-10.8889	-.4789
.45	.8498	-.8889	-.0391
.50	.9428	-22.8889	-.9074
.50	.9428	13.1111	.5198
.50	.9428	-26.8889	-1.0660
.55	.8498	44.1111	1.9400
.55	.8498	-.8889	-.0391
.55	.8498	-24.8889	-1.0946







For each of the three types of plots, the residuals and standardized residuals look almost exactly the same.

(c) First make a confidence interval for β_1, and then multiply the endpoints by .1. Use equation (9-17). For 90% confidence, the appropriate t is $t = Q_7(.95) = 1.895$ from Table B-4. The resulting interval for β_1 is

$$\begin{aligned} & -3160.0 \pm 1.895\frac{26.755}{\sqrt{.015}} \\ = & -3160.0 \pm 413.9729 \\ = & [-3573.973, -2746.027] \text{ psi.} \end{aligned}$$

Multiplying each endpoint by .1, the resulting interval for $.1\beta_1$ is $[-357.4, -274.6]$ psi.

(d) This can be done using the test statistic (9-16) or with (9-34). Using (9-16),

1. H_0: $\beta_1 = 0$.
2. H_a: $\beta_1 \neq 0$.
3. The test statistic is given by equation (9-16), with $\# = 0$. The reference distribution is the t_7 distribution. Observed values of t far above or below zero will be considered as evidence against H_0.
4. The samples give

$$t = \frac{-3160.0}{\frac{26.755}{\sqrt{.015}}} = -14.47.$$

5. The observed level of significance is

$$\begin{aligned} & 2P(\text{a } t_7 \text{ random variable} < -14.47) \\ = & 2P(\text{a } t_7 \text{ random variable} > 14.47) \\ = & 2(\text{less than } .0005) \end{aligned}$$

which is less than .001, according to Table B-4 (14.47 is greater than $Q(.9995) = 5.408$). This is overwhelming evidence that the mean compressive strength is related to the water/cement ratio. (The given model is an improvement over the model $y = \beta_0 + \epsilon$.)

Using (9-34),

1. H_0: $\beta_1 = 0$.

2. H_a: $\beta_1 \neq 0$.
3. The test statistic is given by equation (9-34). The reference distribution is the $F_{1,7}$ distribution. Large observed values of F will be considered as evidence against H_0.
4. The samples give $SSE = (n-2)(s_{LF}) = 5010.889$ and $SSTot = 154794.9$, so $SSR = SSTot - SSE = 149784$.

$$f = \frac{149784}{715.84} = 209.24.$$

5. The observed level of significance is

$$P(\text{an } F_{1,7} \text{ random variable} > 209.24)$$

which is less than .001, according to Tables B-6 (209.24 is greater than $Q(.999) = 29.24$). This is overwhelming evidence that the mean compressive strength is related to the water/cement ratio. (The given model is an improvement over the model $y = \beta_0 + \epsilon$.)

Note that the F statistic is equal to the square of the t statistic.

(e) Use equation (9-24). For 95% confidence, the appropriate t is $t = Q_7(.975) = 2.365$ from Table B-4. The resulting interval for the mean at $x = .5$ is

$$\begin{aligned} & 2765.889 \pm 2.365(26.755)\sqrt{\frac{1}{9} + \frac{0}{.015}} \\ = \; & 2765.889 \pm 21.09202 \\ = \; & [2744.8, 2787.0] \text{ psi.} \end{aligned}$$

(f) Use equation (9-26). The appropriate t is the same one used in part (e). The resulting prediction interval at $x = .5$ is

$$\begin{aligned} & 2765.889 \pm 2.365(26.755)\sqrt{1 + \frac{1}{9} + \frac{0}{.015}} \\ = \; & 2765.889 \pm 66.69884 \\ = \; & [2699.2, 2832.6] \text{ psi.} \end{aligned}$$

(g) Use equation (9-27). For $x = .5$, first using equation (9-29),

$$A = \sqrt{\frac{1}{9} + \frac{0}{.015}} = .3333.$$

Finally, using equation (9-30),

$$\tau = \frac{1.28 + (.3333)(1.645)\sqrt{1 + \frac{1}{2(7)}\left(\frac{(1.28)^2}{(.3333)^2} - (1.645)^2\right)}}{1 - \frac{(1.645)^2}{2(7)}} = 2.5136$$

The resulting bound for $x = .5$ is

$$2765.889 - 2.5136 \quad (26.755) = 2698.63 \text{ psi.}$$

Section 1.
2

(a) Using equation (9-40),

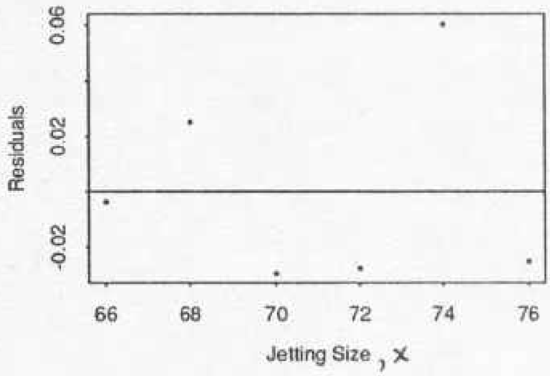
$$s^2_{\mathrm{SF}} = \frac{1}{6-2-1}(.006562857) = .002187619,$$

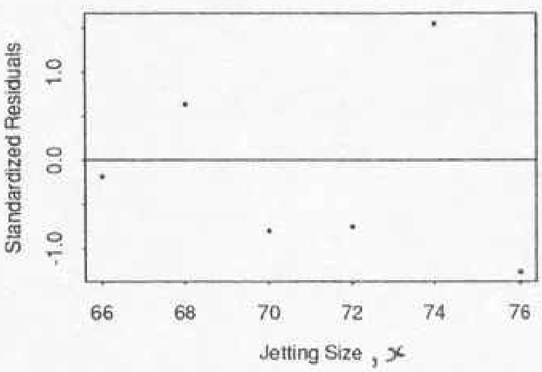
and so $s_{\mathrm{SF}} = \sqrt{.002187619} = .0468$ sec. This can also be read from the following Minitab Version 9.1 output. s_{SF} measures the variation in elapsed time for any fixed jetting size, assuming that the given model is appropriate.

```
MTB > brief = 3
MTB > print c1-c3

 ROW    Time  JetSize    xsq

   1   14.90       66   4356
   2   14.67       68   4624
   3   14.50       70   4900
   4   14.53       72   5184
   5   14.79       74   5476
   6   15.02       76   5776

MTB > regress c1 2 c2 c3;
SUBC> fits c4;
SUBC> residuals c5;
SUBC> sresiduals c6.
* NOTE *  JetSize is highly correlated with other  predictor variables
* NOTE *      xsq is highly correlated with other  predictor variables

The regression equation is
Time = 104 - 2.53 JetSize + 0.0179 xsq

Predictor        Coef        Stdev     t-ratio        p
Constant      103.989        9.633       10.80    0.002
JetSize       -2.5343       0.2718       -9.32    0.003
xsq          0.017946     0.001914        9.38    0.003

s = 0.04677      R-sq = 96.9%      R-sq(adj) = 94.9%
    S_SF
Analysis of Variance

SOURCE       DF          SS          MS         F        p
Regression    2     0.20639     0.10319     47.17    0.005
Error         3     0.00656     0.00219
Total         5     0.21295

SOURCE       DF      SEQ SS
JetSize       1     0.01400
xsq           1     0.19239

Obs.  JetSize       Time        Fit  Stdev.Fit   Residual    St.Resid
  1      66.0    14.9000    14.9036     0.0424    -0.0036       -0.18
  2      68.0    14.6700    14.6447     0.0259     0.0253        0.65
  3      70.0    14.5000    14.5294     0.0285    -0.0294       -0.79
  4      72.0    14.5300    14.5577     0.0285    -0.0277       -0.75
  5      74.0    14.7900    14.7296     0.0259     0.0604        1.55
  6      76.0    15.0200    15.0450     0.0424    -0.0250       -1.26
```

```
MTB > name c4 'fits' c5 'resids' c6 'stresids'
MTB > print c1-c6

ROW    Time  JetSize   xsq      fits       resids   stresids

  1   14.90       66  4356   14.9036  -0.0035715  -0.18070
  2   14.67       68  4624   14.6447   0.0252857   0.64948
  3   14.50       70  4900   14.5294  -0.0294285  -0.79361
  4   14.53       72  5184   14.5577  -0.0277147  -0.74739
  5   14.79       74  5476   14.7296   0.0604286   1.55216
  6   15.02       76  5776   15.0450  -0.0249996  -1.26486
```

(b)





There is a slight difference. The large positive residual is less extreme after it has been standardized. One of the negative residuals is more extreme after it has been standardized.

(c) Use equation (9-47) and the Minitab printout. The appropriate t is $t = Q_3(.95) = 2.353$ from Table B-4. The interval for β_0 is

$$\begin{aligned} & 103.989 \pm 2.353(9.633) \\ = \; & 103.989 \pm 22.66645 \\ = \; & [81.3, 126.7]. \end{aligned}$$

The interval for β_1 is

$$\begin{aligned} & -2.5343 \pm 2.353(.2718) \\ = \; & -2.5343 \pm .6395454 \\ = \; & [-3.17, -1.89]. \end{aligned}$$

The interval for β_2 is

$$\begin{aligned} & .017946 \pm 2.353(.001914) \\ = \; & .017946 \pm .004503642 \\ = \; & [.0134, .0225]. \end{aligned}$$

(d) Use equation (9-53) and the Minitab printout. The appropriate t is the same one used in part (c). For $x = 70$, the interval is

$$\begin{aligned} & 14.5294 \pm 2.353(.0285) \\ = \; & 14.5294 \pm .0670605 \end{aligned}$$

$$= [14.46, 14.60] \text{ sec.}$$

For $x = 76$, the interval is

$$\begin{aligned} & 15.0450 \pm 2.353(.0424) \\ = \; & 15.0450 \pm .0997672 \\ = \; & [14.95, 15.14] \text{ sec.} \end{aligned}$$

(e) Use equation (9-54) and the Minitab printout. The appropriate f is $f = Q_{3,3}(.90) = 5.39$ from Table B-6-B. For $x = 70$, the interval is

$$\begin{aligned} & 14.5294 \pm \sqrt{(3)(5.39)}(.0285) \\ = \; & 14.5294 \pm .114604 \\ = \; & [14.41, 14.64] \text{ sec.} \end{aligned}$$

For $x = 76$, the interval is

$$\begin{aligned} & 15.0450 \pm \sqrt{(3)(5.39)}(.0424) \\ = \; & 15.0450 \pm .170499 \\ = \; & [14.87, 15.22] \text{ sec.} \end{aligned}$$

(f) Use equation (9-56). For a 90% one-sided interval, construct an 80% two-sided interval and use the lower endpoint. For an 80% two-sided interval, the appropriate t is $t = Q_3(.90) = 1.638$ from Table B-4. The resulting lower prediction bound at $x = 70$ is

$$\begin{aligned} & 14.5294 - 1.638\sqrt{(.04677)^2 + (.0285)^2} \\ = \; & 14.5294 - .08971221 \\ = \; & 14.44 \text{ sec.} \end{aligned}$$

For $x = 76$, the interval is

$$\begin{aligned} & 15.0450 - 1.638\sqrt{(.04677)^2 + (.0424)^2} \\ = \; & 15.0450 - .1034043 \\ = \; & 14.94 \text{ sec.} \end{aligned}$$

(g) Use equation (9-58). For $x = 70$, $A = \frac{.0285}{.04677} = .609365$.
$p = .90$,

Finally, using equation (9-57)

$$\tau = \frac{1.28 + (.609365)(1.645\ )\sqrt{1 + \frac{1}{2(3)}\left(\frac{(1.28)^2}{(.609365)^2} - (1.645\ )^2\right)}}{1 - \frac{(1.645\ )^2}{2(3)}} = 4.4011$$

The resulting bound for $x = 70$ is

$$14.5294 - 4.4011 \quad (.04677) = 14.32 \text{ sec.}$$

For $x = 76$, $A = \frac{.0424}{.04677} = .906564$.

Finally, using equation (9-57)

$$\tau = \frac{1.28 + (.906564)(1.645)\sqrt{1 + \frac{1}{2(3)}\left(\frac{(1.28)^2}{(.906564)^2} - (1.645)^2\right)}}{1 - \frac{(1.645)^2}{2(3)}} = 4.88$$

The resulting bound for $x = 76$ is

$$15.0450 - 4.88 \quad (.04677) = 14.82 \text{ sec.}$$

(h) The ANOVA table is in the Minitab printout. This hypothesis means that the mean Elapsed Time is not related to Jetting Size. The p-value for the test is .005. There is very strong evidence that the quadratic model is an improvement over a model in which the mean of Elapsed Time does not depend on Jetting Size ($y = \beta_0 + \epsilon$).

(i) Use equation (9-46) as the test statistic. The observed value of T is $t = 9.38$, and the p-value is .003 (both numbers can be found on the Minitab printout). This hypothesis means that Elapsed Time depends only linearly on Jetting Size (no curvature). There is strong evidence against this hypothesis; the addition of the quadratic term is an improvement. (The model $y = \beta_0 + \beta_1 x + \beta_2 x^2 + \epsilon$ is an improvement over $y = \beta_0 + \beta_1 x + \epsilon$.)

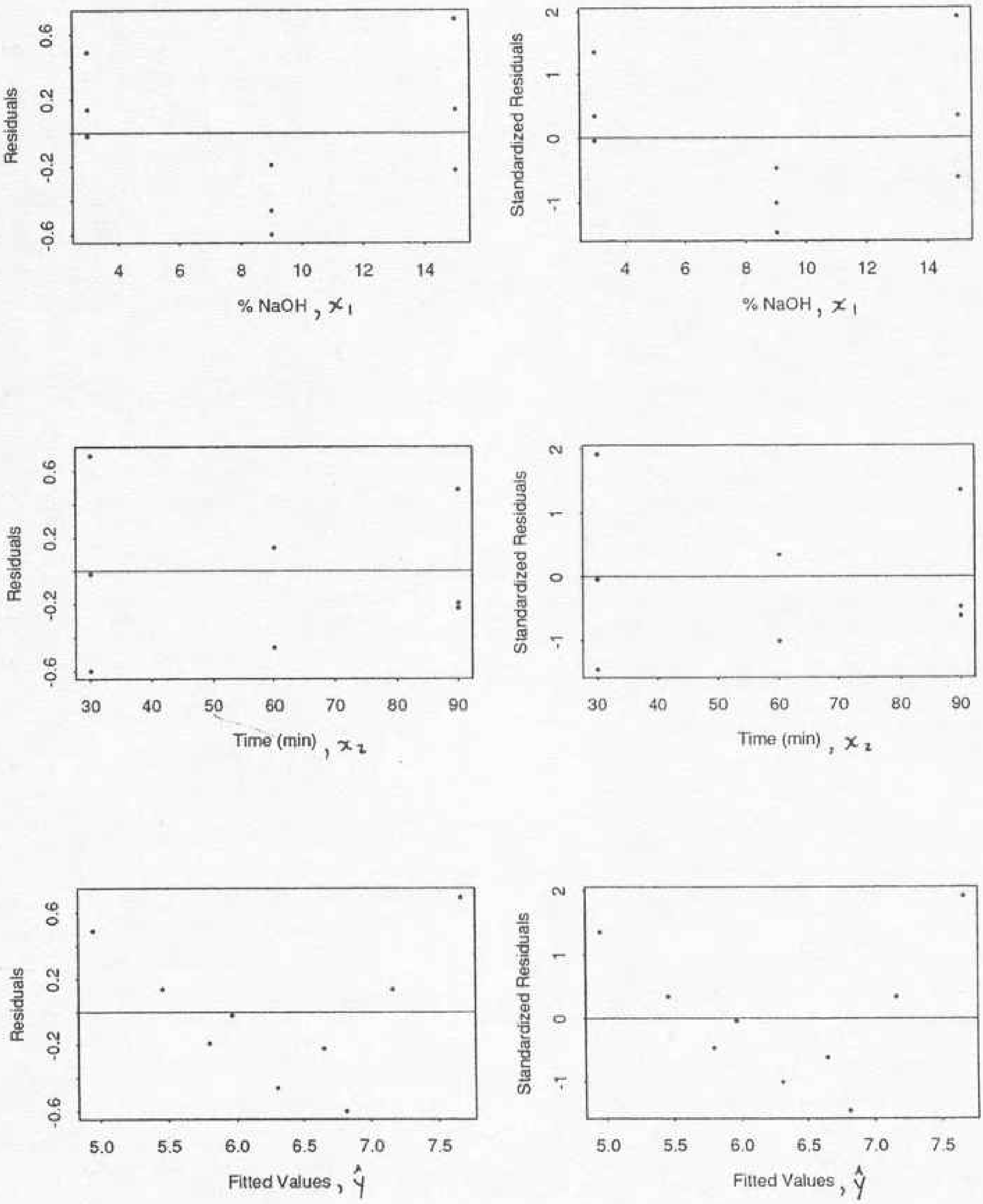
2. (a) The following output is from Minitab Version 9.1.

```
MTB > brief = 3
MTB > print c1-c3

 ROW  SurfArea  %NaOH   Time

   1      5.95      3     30
   2      5.60      3     60
   3      5.44      3     90
   4      6.22      9     30
   5      5.85      9     60
   6      5.61      9     90
   7      8.36     15     30
   8      7.30     15     60
   9      6.43     15     90
```

```
MTB > regress c1 2 c2 c3;
SUBC> fits c4;
SUBC> residuals c5;
SUBC> sresiduals c6;
SUBC> predict 10 70.

The regression equation is
SurfArea = 6.05 + 0.142 %NaOH - 0.0169 Time

Predictor       Coef        Stdev     t-ratio       p
Constant      6.0483       0.5208       11.61   0.000
%NaOH        0.14167      0.03301        4.29   0.005
Time       -0.016944     0.006601       -2.57   0.043

s = 0.4851        R-sq = 80.7%      R-sq(adj) = 74.2%
       Sse
Analysis of Variance

SOURCE         DF          SS          MS         F        p
Regression      2      5.8854      2.9427     12.51    0.007
Error           6      1.4118      0.2353
Total           8      7.2972

SOURCE         DF      SEQ SS
%NaOH           1      4.3350
Time            1      1.5504

Obs.    %NaOH   SurfArea        Fit  Stdev.Fit   Residual    St.Resid
  1       3.0      5.950      5.965      0.323     -0.015       -0.04
  2       3.0      5.600      5.457      0.256      0.143        0.35
  3       3.0      5.440      4.948      0.323      0.492        1.36
  4       9.0      6.220      6.815      0.256     -0.595       -1.44
  5       9.0      5.850      6.307      0.162     -0.457       -1.00
  6       9.0      5.610      5.798      0.256     -0.188       -0.46
  7      15.0      8.360      7.665      0.323      0.695        1.92
  8      15.0      7.300      7.157      0.256      0.143        0.35
  9      15.0      6.430      6.648      0.323     -0.218       -0.60

     Fit  Stdev.Fit         95% C.I.             95% P.I.
   6.279      0.178   (   5.844,   6.714) (   5.014,    7.543)

MTB > name c4 'fits' c5 'resids' c6 'stdres'
MTB > print c1-c6

 ROW  SurfArea  %NaOH   Time       fits      resids      stdres

   1      5.95      3     30    5.96500   -0.015000    -0.04149
   2      5.60      3     60    5.45667    0.143333     0.34770
   3      5.44      3     90    4.94833    0.491667     1.35987
   4      6.22      9     30    6.81500   -0.595000    -1.44335
   5      5.85      9     60    6.30667   -0.456666    -0.99854
   6      5.61      9     90    5.79833   -0.188333    -0.45686
   7      8.36     15     30    7.66500    0.695000     1.92226
   8      7.30     15     60    7.15667    0.143333     0.34770
   9      6.43     15     90    6.64833   -0.218334    -0.60388
```

$s_{SF} = .4851$ cm^3/g. Assuming that the model is appropriate, this measures the variation in Specific Surface Areas for a fixed NaOH/Time condition.

(b)



For each of the three types of plots, the residuals and standardized residuals look almost exactly the same.

(c) Use equation (9-47) and the Minitab printout. The appropriate t is $t = Q_6(.95) = 1.943$

from Table B-4. The interval for β_0 is

$$\begin{aligned} & 6.0483 \pm 1.943(.5208) \\ = \; & 6.0483 \pm 1.011914 \\ = \; & [5.04, 7.06]. \end{aligned}$$

The interval for β_1 is

$$\begin{aligned} & .14167 \pm 1.943(.03301) \\ = \; & .14167 \pm .06413843 \\ = \; & [.078, .206]. \end{aligned}$$

The interval for β_2 is

$$\begin{aligned} & -.016944 \pm 1.943(.006601) \\ = \; & -.016944 \pm .01282574 \\ = \; & [-.0298, -.0041]. \end{aligned}$$

(d) Use equation (9-53) and the Minitab printout. The appropriate t is the same one used in part (c). For $x_1 = 9.0$ and $x_2 = 60$, the interval is

$$\begin{aligned} & 6.30667 \pm 1.943(.162) \\ = \; & 6.30667 \pm .314766 \\ = \; & [5.99, 6.62] \text{ cm}^3/\text{g}. \end{aligned}$$

For $x_1 = 10.0$ and $x_2 = 70$, the interval is

$$\begin{aligned} & 6.279 \pm 1.943(.178) \\ = \; & 6.279 \pm .345854 \\ = \; & [5.93, 6.62] \text{ cm}^3/\text{g}. \end{aligned}$$

(e) Use equation (9-54) and the Minitab printout. The appropriate f is $f = Q_{3,6}(.90) = 3.29$ from Table B-6-B. For $x_1 = 9.0$ and $x_2 = 60$, the interval is

$$\begin{aligned} & 6.30667 \pm \sqrt{3(3.29)}(.162) \\ = \; & 6.30667 \pm .5089482 \\ = \; & [5.80, 6.82] \text{ cm}^3/\text{g}. \end{aligned}$$

For $x_1 = 10.0$ and $x_2 = 70$, the interval is

$$\begin{aligned} & 6.279 \pm \sqrt{3(3.29)}(.178) \\ = \; & 6.279 \pm .5592147 \\ = \; & [5.72, 6.84] \text{ cm}^3/\text{g}. \end{aligned}$$

(f) Use equation (9-56). For a 90% one-sided interval, construct an 80% two-sided interval and use the lower endpoint. For an 80% two-sided interval, the appropriate t is $t = Q_6(.90) = 1.440$ from Table B-4. For $x_1 = 9.0$ and $x_2 = 60$, the bound is

$$\begin{aligned} & 6.30667 - 1.440\sqrt{(.4851)^2 + (.162)^2} \\ = \; & 6.30667 - .7364668 \\ = \; & 5.57 \text{ cm}^3/\text{g}. \end{aligned}$$

For $x_1 = 10.0$ and $x_2 = 70$, the bound is

$$\begin{aligned} & 6.279 - 1.440\sqrt{(.4851)^2 + (.178)^2} \\ = \ & 6.279 - .7440858 \\ = \ & 5.53 \text{ cm}^3/\text{g}. \end{aligned}$$

(g) Use equation (9-58). For $x_1 = 9.0$ and $x_2 = 60$, $A = \frac{.162}{.4851} = .3339518$.

Finally, using equation (9-57)

$$\tau = \frac{1.28 + (.3339518)(1.645)\sqrt{1 + \frac{1}{2(6)}\left(\frac{(1.28)^2}{(.3339518)^2} - (1.645)^2\right)}}{1 - \frac{(1.645)^2}{2(6)}} = 2.65547$$

The resulting bound for $x_1 = 9.0$ and $x_2 = 60$ is

$$6.30667 - 2.65547\ (.4851) = 5.018 \text{cm}^3/\text{g}.$$

For $x_1 = 10.0$ and $x_2 = 70$, $A = \frac{.178}{.4851} = .3669347$.

Finally,
using equation (9-57)

$$\tau = \frac{1.28 + (.3669347)(1.645)\sqrt{1 + \frac{1}{2(6)}\left(\frac{(1.28)^2}{(.3669347)^2} - (1.645)^2\right)}}{1 - \frac{(1.645)^2}{2(6)}} = 2.69495$$

The resulting bound for $x_1 = 10.0$ and $x_2 = 70$ is

$$6.279 - 2.69495\ (.4851) = 4.97 \text{ cm}^3/\text{g}.$$

(h) The ANOVA table is in the Minitab output. The p-value is

$$P(\text{an } F_{2,6} \text{ random variable} > 12.51),$$

which is equal to .007, from the Minitab printout. This is very strong evidence that the model used is an improvement over a model which does not depend at all on NaOH and Time ($y = \beta_0 + \epsilon$).

Section 3

1. (a) The following printout comes from Minitab Version 9.1.

```
MTB > info c1-c7

Column     Name          Count
C1         Increase         48
C2         Time             48
C3         Temp             48
C4         lnTime           48
C5         lnTimeSq         48
C6         TempSq           48
C7         x2*lnx1          48
```

```
MTB > brief = 3
MTB > regress c1 5 c4 c3 c5 c6 c7;
SUBC> fits c9.
* NOTE *   lnTime is highly correlated with other  predictor variables
* NOTE *     Temp is highly correlated with other  predictor variables
* NOTE *   TempSq is highly correlated with other  predictor variables

The regression equation is
Increase = 31.4 + 7.43 lnTime - 0.0810 Temp - 0.276 lnTimeSq
                 + 0.000048 TempSq - 0.00660 x2*lnx1

Predictor        Coef        Stdev     t-ratio        p
Constant        31.40        25.85        1.21    0.231
lnTime          7.430        1.683        4.42    0.000
Temp         -0.08101      0.05380       -1.51    0.140
lnTimeSq      -0.2760       0.1185       -2.33    0.025
TempSq     0.00004792   0.00002810        1.71    0.095
x2*lnx1     -0.006596     0.001481       -4.45    0.000

s = 1.947        R-sq = 72.4%      R-sq(adj) = 69.1%
      S_SF                 R^2
Analysis of Variance

SOURCE        DF          SS          MS         F        p
Regression     5     416.845      83.369     22.00    0.000
Error         42     159.134       3.789
Total         47     575.979

SOURCE        DF      SEQ SS
lnTime         1     124.649
Temp           1     185.504
lnTimeSq       1      20.557
TempSq         1      11.021
x2*lnx1        1      75.114

Obs.   lnTime   Increase        Fit  Stdev.Fit   Residual    St.Resid
                                  .
                                  .
 13      3.91      3.000      1.454      0.657      1.546        0.84
                                  .
                                  .
```

```
MTB > name c9 'fits(a)'
MTB > delete 1:12 c9
MTB > info c11-c17

Column    Name          Count
C11       dIncr            36
C12       dTime            36
C13       dTemp            36
C14       dlnTime          36
C15       dlnTimsq         36
C16       dTempSq          36
C17       dx2*lnx1         36

MTB > regress c11 5 c14 c13 c15 c16 c17;
SUBC> fits c19.
                                          .
                                          .
                                          .
MTB > name c19 'fits(b)'

MTB > print c9 c19

 ROW   fits(a)   fits(b)
   1   1.45433   2.22387
   2   1.45433   2.22387
   3   1.45433   2.22387
   4  -1.08117  -0.75871
   5  -1.08117  -0.75871
   6  -1.08117  -0.75871
   7  -2.65833  -2.74129
   8  -2.65833  -2.74129
   9  -2.65833  -2.74129
  10  -3.27717  -3.72387
  11  -3.27717  -3.72387
  12  -3.27717  -3.72387
  13   1.11395   1.08060
  14   1.11395   1.08060
  15   1.11395   1.08060
  16  -2.14622  -2.30647
  17  -2.14622  -2.30647
  18  -2.14622  -2.30647
  19  -4.44806  -4.69353
  20  -4.44806  -4.69353
  21  -4.44806  -4.69353
  22  -5.79157  -6.08060
  23  -5.79157  -6.08060
  24  -5.79157  -6.08060
  25  -0.02430  -0.33781
  26  -0.02430  -0.33781
  27  -0.02430  -0.33781
  28  -4.07864  -4.16816
  29  -4.07864  -4.16816
  30  -4.07864  -4.16816
  31  -7.17465  -6.99851
  32  -7.17465  -6.99851
  33  -7.17465  -6.99851
  34  -9.31232  -8.82886
  35  -9.31232  -8.82886
  36  -9.31232  -8.82886
```

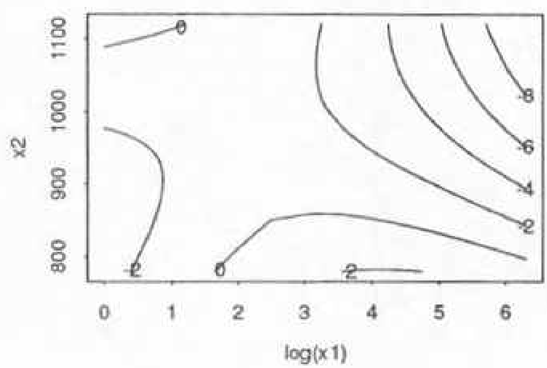
$R^2 = .724$, $s_{\text{SF}} = 1.947$, and $s_p = 2.136$. (The first two values were read from the Minitab printout, and the third value was computed using equation (7-7).) Since $s_{\text{SF}} = 1.947 < s_p = 2.136$, there is no indication that the model is inappropriate.

(b) The fitted values from each model are given in the Minitab printout. Factor-level combinations have fitted values which differ by as much as .77. This is a relatively large difference.

(c) Using Minitab,

```
MTB > grid c31=780:1120, c30=0,6.31
MTB > name c30 'lnx1' c31 'x2'
MTB > let c32 = 31.40 + 7.430*c30 - .08101*c31 - .2760*c30**2 + &
CONT> .00004792*c31**2 - .006596*c30*c31

MTB > contour c32 c31 c30

A =****,-9.6      B =-8.4,-7.2      C =-6.0,-4.8
D =-3.6,-2.4      E =-1.2, 0.0      F = 1.2, 2.4
         - ,,,,,,,,,,,,,,EEEEEEEE......DDDDD,,,,,CCCC...BBB,,,A
         - ,,,,,,,,,,EEEEEEEEEEEE.......DDDDDD,,,,CCCC....BBB,,,
     1080+ EEEEEEEEEEEEEEEEEEEEE........DDDDDD,,,,,CCCC....BBB,,
         - EEEEEEEEEEEEEEEEEEEE..........DDDDDD,,,,,CCCC....BBBB
x2       - ...EEEEEEEEEEEEEEEE...........DDDDDDD,,,,,CCCCC....BB
         - ........EEEEEEEEEEE.............DDDDDDD,,,,,CCCCC....
         - ............EEEEEEEE...............DDDDDDD,,,,,CCCCC..
      960+ .............EEEEEEEEEEE.............DDDDDD,,,,,,CCCC
         - D...........EEEEEEEEEEEEEEEEE...........DDDDDDD,,,,,C
         - DD..........EEEEEEEEEEEEEEEEEEEEEEE..........DDDDDD,,,
         - DDD........EEEEEEEEEEEEEEEEEEEEEEEEEEEEE........DDDDD
         - DDD.......EEEEEEEEEEEEE,,,,,,,,,,,,EEEEEEEEEEEE.......
      840+ DD......EEEEEEEEE,,,,,,,,,,,,,,,,,,,,,,,,,EEEEEEEEE.
         - DD.....EEEEEEE,,,,,,,,,,,,,FFFFFFFFFFFF,,,,,,,,,,,,,EE
         - D.....EEEEEE,,,,,,,,FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF,,,
           +---------+---------+---------+---------+---------+--lnx1
         0.00      1.20      2.40      3.60      4.80      6.00
```

Here's another plot using the S-Plus function "contour":



The region where tempering seems to provide an increase in hardness is the region in which $\hat{y}$ is positive. This is the lower-right part of each of the contour plots.

(d) (i) Use equation (9-53) and the Minitab printout. The appropriate t is $t = Q_{42}(.975) \approx 2.018$ from Table B-4. For $x_1 = 50$ and $x_2 = 800$, the interval is

$$\begin{aligned} & 1.454 \pm 2.018(.657) \\ = \ & 1.454 \pm 1.325826 \\ = \ & [.13, 2.78]. \end{aligned}$$

(ii) Use equation (9-56) and the Minitab printout. The t is the same as the one used in part (i). The resulting prediction interval at $x_1 = 50$ and $x_2 = 800$ is

$$\begin{aligned} & 1.454 \pm 2.018\sqrt{(1.947)^2 + (.657)^2} \\ = \ & 1.454 \pm 4.146712 \\ = \ & [-2.69, 5.60]. \end{aligned}$$

(iii) Use equation (9-58). For $x_1 = 50$ and $x_2 = 800$, $A = \frac{.657}{1.947} = .3374422$.

Finally,
using equation (9-57)

$$\tau = \frac{1.28 + (.3374422)(1.645)\sqrt{1 + \frac{1}{2(42)}\left(\frac{(1.28)^2}{(.3374422)^2} - (1.645)^2\right)}}{1 - \frac{(1.645)^2}{2(42)}} = 1.93476$$

The resulting bound for $x_1 = 50$ and $x_2 = 800$ is

$$1.454 - 1.93476\ (1.947) = -2.313$$

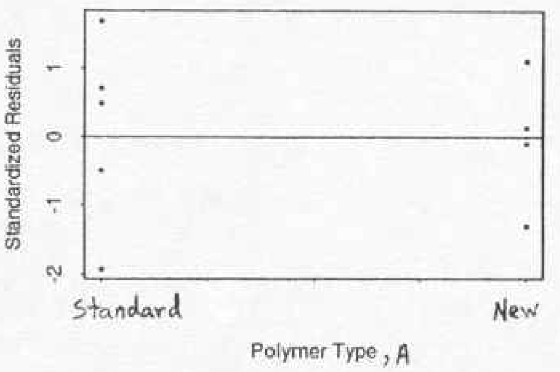
2. (a) The following output was obtained from Minitab Version 9.1.

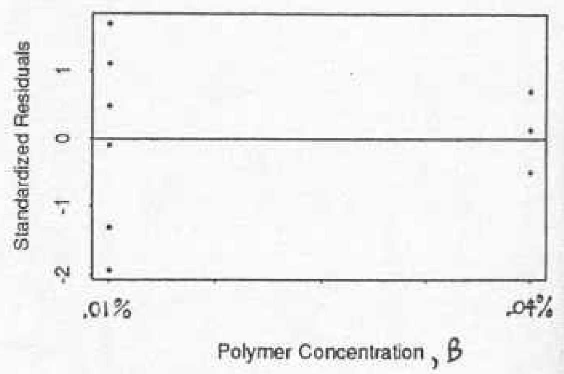
```
MTB > print c3-c6

 ROW      y   xa2   xb2   xc2

   1    1.0    -1    -1    -1
   2    1.0     1    -1    -1
   3    1.2     1    -1    -1
   4    0.2    -1     1    -1
   5    0.5     1     1    -1
   6    0.9    -1    -1     1
   7    0.7    -1    -1     1
   8    1.1     1    -1     1
   9    0.2    -1     1     1
  10    0.3    -1     1     1
  11    0.5     1     1     1
```

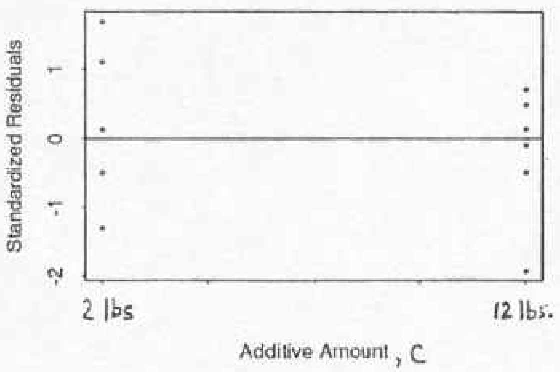
```
MTB > regress c3 2 c4 c5;
SUBC> fits c7;
SUBC> residuals c8;
SUBC> sresiduals c9.

The regression equation is
y = 0.674 + 0.124 xa2 - 0.309 xb2

Predictor       Coef       Stdev    t-ratio       p
Constant     0.67407     0.02928      23.02   0.000
xa2          0.12407     0.02928       4.24   0.003
xb2         -0.30926     0.02928     -10.56   0.000

s = 0.09623     R-sq = 94.6%     R-sq(adj) = 93.2%

Analysis of Variance

SOURCE       DF        SS          MS        F       p
Regression    2    1.29502     0.64751    69.93   0.000
Error         8    0.07407     0.00926
Total        10    1.36909

SOURCE       DF     SEQ SS
xa2           1    0.26209
xb2           1    1.03293

MTB > name c7 'fits' c8 'resids' c9 'stresids'
MTB > print c3-c9

 ROW     y   xa2   xb2   xc2      fits     resids  stresids

   1   1.0    -1    -1    -1   0.85926   0.140741   1.69941
   2   1.0     1    -1    -1   1.10741  -0.107407  -1.29692
   3   1.2     1    -1    -1   1.10741   0.092593   1.11803
   4   0.2    -1     1    -1   0.24074  -0.040741  -0.49193
   5   0.5     1     1    -1   0.48889   0.011111   0.14142
   6   0.9    -1    -1     1   0.85926   0.040741   0.49193
   7   0.7    -1    -1     1   0.85926  -0.159259  -1.92302
   8   1.1     1    -1     1   1.10741  -0.007407  -0.08944
   9   0.2    -1     1     1   0.24074  -0.040741  -0.49193
  10   0.3    -1     1     1   0.24074   0.059259   0.71554
  11   0.5     1     1     1   0.48889   0.011111   0.14142
```

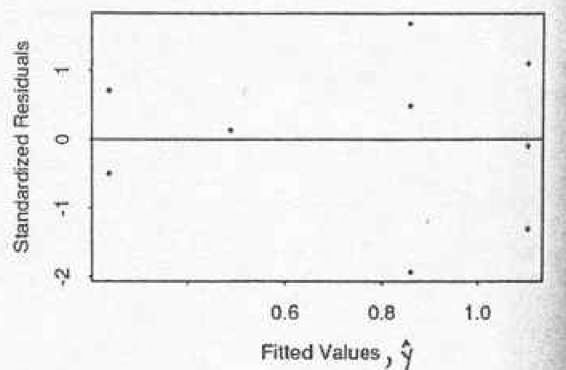
The estimate of $\mu_{...}$ is .67407; the estimate of α_2 is .12407; the estimate of β_2 is -.30926. See Ex. 10, Ch. 8 for the fitted effects obtained using the Yates algorithm. The estimates based on the few-effects model are slightly different than the estimates based on the full model.

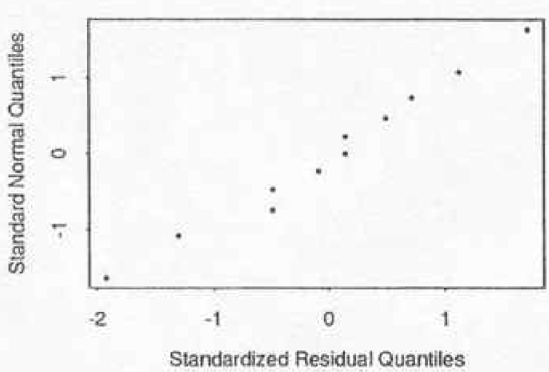
(b) The following plots use the standardized residuals.











There is some hint of a pattern in the plot of Standardized Residuals versus levels of C, indicating that the amount of additive may be having a small effect that the model is not accounting for. Otherwise, the residuals do not provide any evidence that the model is inadequate.

(c) $s_{\mathrm{FE}} = .09623$ %, compared to $s_p = .12247$ %. Since $s_{\mathrm{FE}} < s_p$, there is no evidence of lack of fit.

End Chapter Exercises

1. (a) The following output is from Minitab Version 9.1.

```
MTB > brief = 3
MTB > print c1-c5

 ROW  GrainSz   Temp   Time   logTime    x1*lnx2

   1        5   1443     20   2.99573     4322.8
   2        6   1443    120   4.78749     6908.4
   3        9   1443   1320   7.18539    10368.5
   4       14   1493     20   2.99573     4472.6
   5       17   1493    120   4.78749     7147.7
   6       25   1493   1320   7.18539    10727.8
   7       29   1543     20   2.99573     4622.4
   8       38   1543    120   4.78749     7387.1
   9       60   1543   1320   7.18539    11087.1

MTB > regress c1 3 c2 c4 c5;
SUBC> predict 1500 6.214608098 9321.912148.
* NOTE *  logTime is highly correlated with other  predictor variables
* NOTE *  x1*lnx2 is highly correlated with other  predictor variables

The regression equation is
GrainSz = - 42 + 0.031 Temp - 93.7 logTime + 0.0653 x1*lnx2

Predictor        Coef       Stdev    t-ratio        p
Constant        -42.4       165.0      -0.26    0.808
Temp           0.0311      0.1105       0.28    0.790
logTime        -93.72       31.27      -3.00    0.030
x1*lnx2       0.06526     0.02093       3.12    0.026

s = 4.401          R-sq = 96.2%     R-sq(adj) = 93.9%
      S_SF
Analysis of Variance

SOURCE       DF        SS         MS         F        p
Regression    3   2461.40     820.47     42.37    0.001
Error         5     96.82      19.36
Total         8   2558.22

SOURCE       DF    SEQ SS
Temp          1   1908.17
logTime       1    365.06
x1*lnx2       1    188.17
```

Obs.	Temp	GrainSz	Fit	Stdev.Fit	Residual	St.Resid
1	1443	5.00	3.83	3.55	1.17	0.45
2	1443	6.00	4.63	2.34	1.37	0.37
3	1443	9.00	5.71	3.77	3.29	1.45
4	1493	14.00	15.16	2.25	-1.16	-0.31
5	1493	17.00	21.81	1.48	-4.81	-1.16
6	1493	25.00	30.70	2.38	-5.70	-1.54
7	1543	29.00	26.49	3.55	2.51	0.97
8	1543	38.00	38.98	2.34	-0.98	-0.26
9	1543	60.00	55.70	3.77	4.30	1.89

Fit	Stdev.Fit	95% C.I.	95% P.I.
30.16	1.83	(25.46, 34.86)	(17.90, 42.41)

$s_{SF} = 4.401\ \mu$m.

(b) For $x_1 = 1443$, the equation is

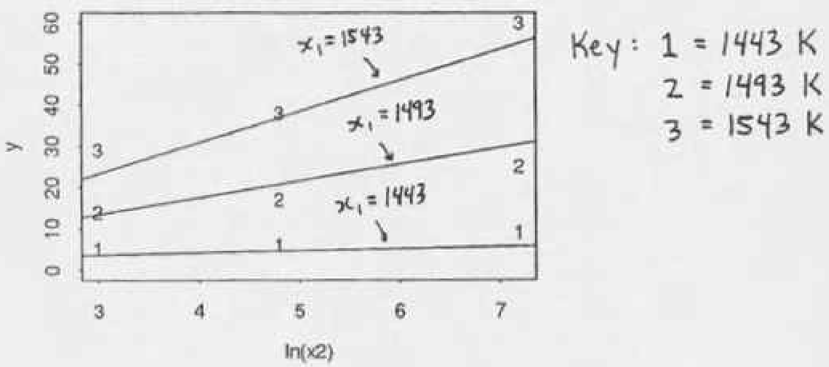
$$\begin{aligned}\hat{y} &= -42.4 + .0311(1443) - 93.72 \ln x_2 + .06526 \ln x_2(1443) \\ &= 2.4773 + .45018 \ln x_2.\end{aligned}$$

For $x_1 = 1493$, the equation is

$$\begin{aligned}\hat{y} &= -42.4 + .0311(1493) - 93.72 \ln x_2 + .06526 \ln x_2(1493) \\ &= 4.0323 + 3.71318 \ln x_2.\end{aligned}$$

For $x_1 = 1543$, the equation is

$$\begin{aligned}\hat{y} &= -42.4 + .0311(1543) - 93.72 \ln x_2 + .06526 \ln x_2(1543) \\ &= 5.5873 + 6.97618 \ln x_2.\end{aligned}$$



Complex models will always fit the given data well, but may not be good predictors of future process performance, especially for x's which were not used to fit the model. With so little data, it is difficult to tell if the fitted model is appropriate, so one should be very cautious about interpolating or extrapolating. The largest source of error in predictions (which statistical intervals cannot account for) is bound to be from an inappropriate model, not from variability (which statistical intervals do take into account).

(c) Use equation (9-53) and the Minitab printout. The appropriate t is $t = Q_5(.975) = 2.571$

from Table B-4. For $x_1 = 1493$ and $x_2 = 120$, the interval is

$$\begin{aligned} & 21.81 \pm 2.571(1.48) \\ = \; & 21.81 \pm 3.80508 \\ = \; & [18.0, 25.6]\ \mu\text{m}. \end{aligned}$$

(d) Use equation (9-56) and the Minitab printout. The t is the same as the one used in part (c). The resulting prediction interval at $x_1 = 1493$ and $x_2 = 120$ is

$$\begin{aligned} & 21.81 \pm 2.571\sqrt{(4.401)^2 + (1.48)^2} \\ = \; & 21.81 \pm 11.93764 \\ = \; & [9.9, 33.7]\ \mu\text{m}. \end{aligned}$$

(e) This can be read directly from the printout: [25.46, 34.86] μm. To "check" this answer, use equation (9-53) with the same t used in part (c):

$$\begin{aligned} & 30.16 \pm 2.571(1.83) \\ = \; & 30.16 \pm 4.70493 \\ = \; & [25.46, 34.86]\ \mu\text{m}. \end{aligned}$$

(f) This hypothesis means that Grain Size does not depend on Temperature or Time. (In other words, the given model is no improvement over a model which does not depend on Temperature or Time, $y = \beta_0 + \epsilon$.) The observed value of F and the p-value for this test can be read from the printout: $f = 42.37$ and p-value $= .001$. This is very strong evidence that Grain Size is related to Temperature or Time.

(g) The $x_1 \ln(x_2)$ term allows there to be an interaction between Temperature and Time (measured on the log scale). $\beta_3 = 0$ implies that the true relationship does not involve an interaction, $y = \beta_0 + \beta_1 x_1 + \beta_2 \ln(x_2) + \epsilon$. The observed value of T and the p-value for this test can be read from the printout: $t = 3.12$ and p-value $= .026$. This is strong evidence that there is an interaction between Temperature and Time (measured on the log scale).

2. (a) It appears that the order of the 11 runs was not randomized. A randomized order would have helped guard against time trends. The effect of x_1 is particularly bound to be confused with any time trends, because its levels generally increase throughout the experiment.

(b) With y_1 as the response, $R^2 = .974$, $s_{\text{SF}} = 49.85$, and $s_p = 19.14$. Although the R^2 is high, $s_{\text{SF}} > s_p$, indicating that the model may not be appropriate. With y_2 as the response, $R^2 = .793$, $s_{\text{SF}} = .7374$, and $s_p = .551$. Here, s_{SF} is still a bit larger than s_p, and the R^2 is smaller. With y_3 as the response, $R^2 = .964$, $s_{\text{SF}} = .05007$, and $s_p = .05196$. s_{SF} is similar to s_p, and the R^2 is close to 1.

(c) There is not a unique choice of the parameters that minimizes the sum of squared residuals. This is true because there are not enough data to find a unique solution.

(d) The following printout was made using Minitab Version 9.1.

```
MTB > info

Column    Name          Count
C1        y1               11
C2        y2               11
C3        y3               11
C4        x1               11
C5        x2               11
C6        x3               11
C7        x4               11
C8        x1sq             11
C9        x2sq             11
C10       x3sq             11
C11       x4sq             11
C12       x1*x2            11
C13       x1*x3            11
C14       x1*x4            11
C15       x2*x3            11
C16       x2*x4            11
C17       x3*x4            11

MTB > regress c2 8 c4 c5 c6 c7 c8 c9 c10 c11;
SUBC> fits c20;
SUBC> predict 325 550 1.2 200 105625 302500 1.44 40000.
* NOTE *      x1 is highly correlated with other  predictor variables
* NOTE *      x2 is highly correlated with other  predictor variables
* NOTE *      x3 is highly correlated with other  predictor variables
* NOTE *      x4 is highly correlated with other  predictor variables
* NOTE *    x1sq is highly correlated with other  predictor variables
* NOTE *    x2sq is highly correlated with other  predictor variables
* NOTE *    x3sq is highly correlated with other  predictor variables
* NOTE *    x4sq is highly correlated with other  predictor variables

The regression equation is
y2 = - 24.9 - 0.010 x1 - 0.035 x2 + 35.0 x3 + 0.204 x4 +0.000036 x1sq
         +0.000049 x2sq - 15.7 x3sq -0.000650 x4sq

Predictor        Coef        Stdev    t-ratio        p
Constant       -24.86        64.79      -0.38    0.738
x1            -0.0102       0.3664      -0.03    0.980
x2            -0.0353       0.1526      -0.23    0.838
x3              35.03        19.06       1.84    0.208
x4            0.20390      0.08855       2.30    0.148
x1sq        0.0000356    0.0006115       0.06    0.959
x2sq        0.0000489    0.0001529       0.32    0.779
x3sq          -15.694        9.554      -1.64    0.242
x4sq       -0.0006497    0.0002725      -2.38    0.140

s = 0.5508       R-sq = 96.2%     R-sq(adj) = 80.8%
```

Analysis of Variance

SOURCE	DF	SS	MS	F	p
Regression	8	15.1752	1.8969	6.25	0.145
Error	2	0.6067	0.3033		
Total	10	15.7818			

SOURCE	DF	SEQ SS
x1	1	2.9490
x2	1	5.1552
x3	1	4.0504
x4	1	0.3646
x1sq	1	0.0033
x2sq	1	0.0070
x3sq	1	0.9207
x4sq	1	1.7250

Unusual Observations

Obs.	x1	y2	Fit	Stdev.Fit	Residual	St.Resid
2	275	4.900	4.900	0.551	0.000	* X
3	275	4.600	4.600	0.551	0.000	* X
4	300	3.400	3.400	0.551	0.000	* X
5	300	4.600	4.600	0.551	0.000	* X
7	300	4.600	4.600	0.551	0.000	* X
8	325	5.000	5.000	0.551	0.000	* X
9	325	2.900	2.900	0.551	0.000	* X
10	325	5.600	5.600	0.551	0.000	* X

X denotes an obs. whose X value gives it large influence.

```
     Fit  Stdev.Fit          95% C.I.             95% P.I.
   5.156      0.530   (   2.875,   7.436)  (   1.867,   8.444)
```

```
MTB > regress c2 8 c4 c5 c7 c8 c9 c11 c12 c16;
SUBC> fits c21;
SUBC> predict 325 550 200 105625 302500 40000 178750 110000.
* NOTE *       x1 is highly correlated with other  predictor variables
* NOTE *       x2 is highly correlated with other  predictor variables
* NOTE *       x4 is highly correlated with other  predictor variables
* NOTE *     x1sq is highly correlated with other  predictor variables
* NOTE *     x2sq is highly correlated with other  predictor variables
* NOTE *     x4sq is highly correlated with other  predictor variables
* NOTE *    x1*x2 is highly correlated with other  predictor variables
* NOTE *    x2*x4 is highly correlated with other  predictor variables

The regression equation is
y2 = - 116 - 0.140 x1 + 0.448 x2 + 0.366 x4 + 0.00122 x1sq +0.000049 x2sq
          -0.000072 x4sq - 0.00121 x1*x2 -0.000739 x2*x4
```

```
Predictor          Coef         Stdev     t-ratio        p
Constant        -115.67         73.89       -1.57    0.258
x1              -0.1403        0.4271       -0.33    0.774
x2               0.4483        0.1896        2.36    0.142
x4               0.3655        0.1321        2.77    0.110
x1sq          0.0012186     0.0007363        1.65    0.240
x2sq          0.0000489     0.0001529        0.32    0.779
x4sq         -0.0000718     0.0003290       -0.22    0.848
x1*x2        -0.0012137     0.0003462       -3.51    0.073
x2*x4        -0.0007394     0.0002313       -3.20    0.085

s = 0.5508       R-sq = 96.2%     R-sq(adj) = 80.8%

Analysis of Variance

SOURCE        DF         SS          MS         F        p
Regression     8    15.1752      1.8969      6.25    0.145
Error          2     0.6067      0.3033
Total         10    15.7818

SOURCE        DF     SEQ SS
x1             1     2.9490
x2             1     5.1552
x4             1     0.0908
x1sq           1     0.0492
x2sq           1     0.0054
x4sq           1     2.2722
x1*x2          1     1.5525
x2*x4          1     3.1008

Unusual Observations
Obs.        x1         y2         Fit  Stdev.Fit   Residual    St.Resid
  2        275      4.900       4.900      0.551      0.000         * X
  3        275      4.600       4.600      0.551      0.000         * X
  4        300      3.400       3.400      0.551      0.000         * X
  5        300      4.600       4.600      0.551      0.000         * X
  7        300      4.600       4.600      0.551      0.000         * X
  8        325      5.000       5.000      0.551      0.000         * X
  9        325      2.900       2.900      0.551      0.000         * X
 10        325      5.600       5.600      0.551      0.000         * X

X denotes an obs. whose X value gives it large influence.

    Fit  Stdev.Fit          95% C.I.             95% P.I.
  0.766      1.136   (  -4.124,   5.655)  (  -4.668,   6.199) XX

X  denotes a row with X values away from the center
XX denotes a row with very extreme X values

MTB > name c20 'fits(i)' c21 'fits(ii)'
MTB > print c20 c21
```

```
ROW   fits(i)  fits(ii)

  1   2.33333   2.33333
  2   4.90000   4.90000
  3   4.60000   4.60000
  4   3.40000   3.40000
  5   4.60000   4.60000
  6   2.33333   2.33333
  7   4.60000   4.60000
  8   5.00000   5.00000
  9   2.90000   2.90000
 10   5.60000   5.60000
 11   2.33333   2.33333
```

All of the $\hat{y}_2$ values are exactly the same for each equation for the x's in the data set. The $\hat{y}_2$ values at $x_1 = 325$, $x_2 = 550$, $x_3 = 1.2$, and $x_4 = 200$ are very different. This shows that it is very difficult to trust any particular model when interpolating or extrapolating. Since the "optimum" set of x's are usually found through interpolation, this means that it is very difficult to confidently predict what the optimum will be based only on these data.

(e) Use equation (9-47) and following Minitab printout.

```
MTB > brief = 3
MTB > regress c3 4 c4 c5 c6 c7;
SUBC> predict 325 450 .8 125.

The regression equation is
y3 = 2.29 + 0.00327 x1 - 0.00133 x2 - 1.04 x3 -0.000532 x4

Predictor        Coef       Stdev    t-ratio        p
Constant       2.2896      0.2544       9.00    0.000
x1          0.0032704   0.0007621       4.29    0.005
x2         -0.0013315   0.0003811      -3.49    0.013
x3           -1.04120     0.09526     -10.93    0.000
x4         -0.0005321   0.0005094      -1.04    0.337

s = 0.05007     R-sq = 96.4%     R-sq(adj) = 94.0%

Analysis of Variance

SOURCE       DF        SS          MS          F        p
Regression    4   0.40321     0.10080      40.21    0.000
Error         6   0.01504     0.00251
Total        10   0.41825

SOURCE       DF    SEQ SS
x1            1   0.00267
x2            1   0.08305
x3            1   0.31476
x4            1   0.00273
```

```
Obs.     x1       y3       Fit   Stdev.Fit   Residual    St.Resid
  1     275   1.6300    1.6903     0.0276    -0.0603       -1.44
  2     275   1.3700    1.3968     0.0232    -0.0268       -0.60
  3     275   1.1000    1.1007     0.0433    -0.0007       -0.03
  4     300   1.5800    1.5239     0.0333     0.0561        1.50
  5     300   1.2600    1.2890     0.0328    -0.0290       -0.77
  6     275   1.7200    1.6903     0.0276     0.0297        0.71
  7     300   1.6500    1.6203     0.0330     0.0297        0.79
  8     325   1.4200    1.4187     0.0391     0.0013        0.04
  9     325   1.6900    1.7473     0.0393    -0.0573       -1.85
 10     325   1.5400    1.5124     0.0388     0.0276        0.87
 11     275   1.7200    1.6903     0.0276     0.0297        0.71

     Fit  Stdev.Fit         95% C.I.             95% P.I.
  1.8538     0.0394   ( 1.7573,  1.9503)  (  1.6978,  2.0098)
```

The appropriate t is $t = Q_6(.95) = 1.943$ from Table B-4. The interval for β_1 is

$$\begin{aligned} & .0032704 \pm 1.943(.0007621) \\ = \; & .0032704 \pm .00148076 \\ = \; & [.001790, .004751]. \end{aligned}$$

(f) Using the signs of the coefficients, to maximize y_3, set $x_1 = 325$, $x_2 = 450$, $x_3 = .8$, and $x_4 = 125$.

(g) Use equation (9-53) and the Minitab printout. The appropriate t is the same as the one used in part (e). The resulting interval is

$$\begin{aligned} & 1.8538 \pm 1.943(.0394) \\ = \; & 1.8538 \pm .0765542 \\ = \; & [1.78, 1.93]. \end{aligned}$$

The "optimal" conditions given in part (f) represent an extrapolation. The confidence interval is only meaningful if the fitted equation is appropriate for these conditions. Data should be collected under these conditions to confirm the validity of the equation and the confidence interval.

3. (a) The following (abbreviated) printout was made using Minitab Version 9.1.

```
MTB > brief = 3
MTB > regress c1 14 c2-c15

The regression equation is
y = - 1 - 2.30 x1 - 0.08 x2 + 0.836 x3 - 3.99 x4 + 0.0152 x1sq
         + 0.00130 x2sq - 0.00011 x3sq - 0.0078 x4sq + 0.0240 x1*x2
         - 0.0093 x1*x3 + 0.0755 x1*x4 - 0.00467 x2*x3
         + 0.0237 x2*c4 + 0.0007 x3*x4

s = 5.335        R-sq = 81.6%     R-sq(adj) = 64.5%
```

```
Analysis of Variance

SOURCE        DF        SS          MS        F        p
Regression    14     1898.33      135.59    4.76    0.002
Error         15      426.93       28.46
Total         29     2325.26

MTB > regress c1 4 c2-c5

The regression equation is
y = - 37.5 + 0.212 x1 + 0.498 x2 + 0.130 x3 + 0.258 x4

Predictor        Coef        Stdev      t-ratio        p
Constant       -37.48        13.10        -2.86    0.008
x1             0.2117       0.2106         1.01    0.324
x2            0.49833      0.07019         7.10    0.000
x3            0.12967      0.04211         3.08    0.005
x4             0.2583       0.2106         1.23    0.231

s = 5.158       R-sq = 71.4%     R-sq(adj) = 66.8%

Analysis of Variance

SOURCE        DF        SS          MS        F        p
Regression     4     1660.14      415.04   15.60    0.000
Error         25      665.12       26.60
Total         29     2325.26

Obs.       x1         y        Fit   Stdev.Fit   Residual    St.Resid
                                .
                                .
                                .
 24      35.0     41.800    40.990       2.307      0.810        0.18
```

The increase in R^2 going from the simpler to more complex model seems to be small compared to the increase in complexity. Based on the center points, $s_p = 6.016$, so the s_{SF} values for both equations are smaller than s_p. This indicates no problems with either model, but the linear response surface model is simpler.

(b) The "full" model is the quadratic response surface model and the "reduced" model is the hypothesis given.

1. H_0: $\beta_5 = \cdots = \beta_{14} = 0$.
2. H_a: not H_0.
3. The test statistic is given by equation (9-66). The reference distribution is the $F_{10,15}$ distribution. Large observed values of F will be considered as evidence against H_0.
4. The data give

$$f = \frac{\frac{1898.33-1660.14}{10}}{\frac{426.93}{15}} = .837.$$

5. The observed level of significance is

$$P(\text{an } F_{10,15} \text{ random variable} > .837).$$

From Table B -6-A, $Q_{10,15}(.75) = 1.45$, so the p-value is greater than .25. There is no evidence to reject the null hypothesis that the linear model is adequate. (There is no evidence that the full model is an improvement over the reduced model.)

(c) Use equation (9-47) and the Minitab printout. The appropriate t is $t = Q_{25}(.95) = 1.708$ from Table B -4. The interval for β_2 is

$$\begin{aligned} & .49833 \pm 1.708(.07019) \\ = \ & .49833 \pm .1198845 \\ = \ & [.378, .618] \text{ gm.} \end{aligned}$$

Assuming the model is accurate, β_2 represents the mean increase in the final ball bond shear strength that accompanies a 1 mw increase in Power, holding all of the other factors fixed. Since this interval does not contain zero, the p-value for this test would be small.

(d) Use equation (9-58). For $x_1 = 35$, $x_2 = 75$, $x_3 = 200$, and $x_4 = 20$, $A = \frac{2.307}{5.158} = .4472664$.

Finally, using equation (9-57)

$$\tau = \frac{2.05 + (.4472664)(\ 1.645\ )\sqrt{1 + \frac{1}{2(25)}\left(\frac{(2.05)^2}{(.4472664)^2} - (\ 1.645\ )^2\right)}}{1 - \frac{(1.645\ )^2}{2(25)}} = 3.0764259$$

The resulting bound for $x_1 = 35$, $x_2 = 75$, $x_3 = 200$, and $x_4 = 20$ is

$$40.990 - 3.0764259(5.158) = 25.12 \text{ gm.}$$

4. (a) There is no replication in these data sets, so you cannot compute s_p.

(b) 1. H_0: $\mu_{y|x} = \beta_0 + \beta_1 x$.
2. H_a: not H_0.
3. The test statistic is given in the Exercise. The reference distribution is the $F_{1,6}$ distribution. Large observed values of F will be considered as evidence against H_0.
4. The data give

$$f = \frac{\frac{(7)(26.76)^2-(6)(26.75521)^2}{1}}{(26.75521)^2} = 1.00$$

5. The observed level of significance is

$$P(\text{an } F_{1,6} \text{ random variable} > 1.00).$$

From Table B -6-A, $Q_{1,6}(.75) = 1.62$, so the p-value is greater than .25. There is no evidence to reject the null hypothesis that the model is correct.

(c) 1. H_0: $\mu_{y|x_1,x_2} = \beta_0 + \beta_1 \ln(x_1) + \beta_2 x_2 + \beta_3(\ln(x_1))^2 + \beta_4 x_2^2 + \beta_5 x_2 \ln(x_1)$.
2. H_a: not H_0.
3. The test statistic is given in the Exercise. The reference distribution is the $F_{10,32}$ distribution. Large observed values of F will be considered as evidence against H_0.
4. The data give

$$f = \frac{\frac{(42)(1.947)^2-(32)(2.136001)^2}{10}}{(2.136001)^2} = .288.$$

5. The observed level of significance is

$$P(\text{an } F_{10,32} \text{ random variable} > .288).$$

From Table B-6-A, $Q_{10,32}(.75) \approx 1.35$, so the p-value is greater than .25. There is no evidence to reject the null hypothesis that the model is correct.

(d) 1. H_0: $\mu_{y|x_1,x_2,x_3,x_4} = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4$.
2. H_a: not H_0.
3. The test statistic is given in the Exercise. The reference distribution is the $F_{20,5}$ distribution. Large observed values of F will be considered as evidence against H_0.
4. The data give

$$f = \frac{\frac{(25)(5.158)^2-(5)(6.01645)^2}{20}}{(6.01645)^2} = .669.$$

5. The observed level of significance is

$$P(\text{an } F_{20,5} \text{ random variable} > .669).$$

From Table B-6-A, $Q_{20,5}(.75) = 1.88$, so the p-value is greater than .25. There is no evidence to reject the null hypothesis that the model is correct.

5. (a) Some of the relevant Minitab output is given below.

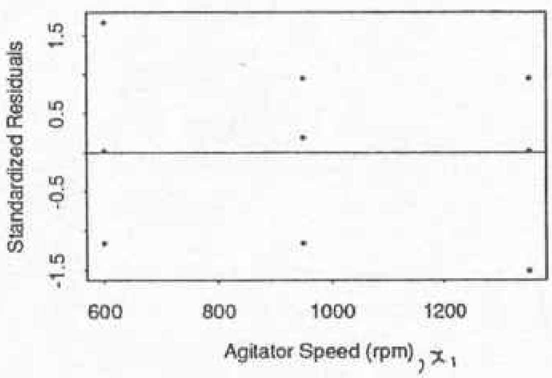
```
MTB > info c1-c7

Column   Name          Count
C1       x1                9
C2       x2                9
C3       y1                9
C4       y2                9
C5       x1sq              9
C6       x2sq              9
C7       x1*x2             9

MTB > regress c3 5 c1 c2 c5 c6 c7;
SUBC> fits c8;
SUBC> residuals c9;
SUBC> sresiduals c10.
* NOTE *       x1 is highly correlated with other  predictor variables
* NOTE *       x2 is highly correlated with other  predictor variables
* NOTE *     x2sq is highly correlated with other  predictor variables

The regression equation is
y1 = - 17.9 + 0.0195 x1 + 2.24 x2 -0.000017 x1sq - 0.0121 x2sq
           - 0.000010 x1*x2

Predictor        Coef         Stdev    t-ratio        p
Constant       -17.92         63.19      -0.28    0.795
x1            0.01953       0.03979       0.49    0.657
x2              2.235         1.195       1.87    0.158
x1sq      -0.00001746    0.00001679      -1.04    0.375
x2sq        -0.012083      0.005871      -2.06    0.132
x1*x2      -0.0000104     0.0002212      -0.05    0.966

s = 3.321        R-sq = 91.5%      R-sq(adj) = 77.3%

MTB > name c8 'fits1' c9 'resids1' c10 'stresid1'
MTB > regress c4 5 c1 c2 c5 c6 c7;
SUBC> fits c11;
SUBC> residuals c12;
SUBC> sresiduals c13.
* NOTE *       x1 is highly correlated with other  predictor variables
* NOTE *       x2 is highly correlated with other  predictor variables
* NOTE *     x2sq is highly correlated with other  predictor variables

The regression equation is
y2 = 125 - 0.108 x1 + 0.02 x2 +0.000033 x1sq - 0.0037 x2sq +0.000407 x1*x2

Predictor        Coef         Stdev    t-ratio        p
Constant        125.3         161.2       0.78    0.494
x1            -0.1076        0.1015      -1.06    0.367
x2              0.015         3.050       0.00    0.996
x1sq       0.00003270    0.00004283       0.76    0.501
x2sq         -0.00375       0.01498      -0.25    0.818
```

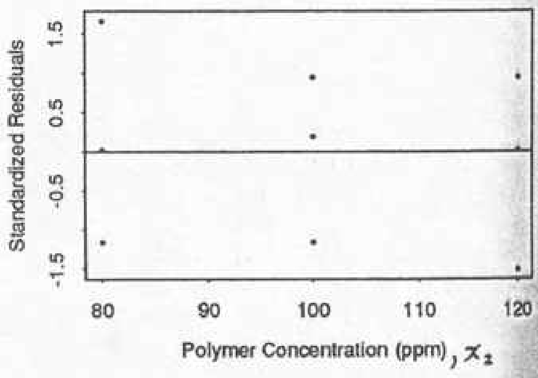
```
x1*x2        0.0004068    0.0005645         0.72    0.523

s = 8.473          R-sq = 63.2%     R-sq(adj) = 2.0%

MTB > name c11 'fits2' c12 'resids2' c13 'stresid2'
MTB > print c1-c4 c8-c13
```

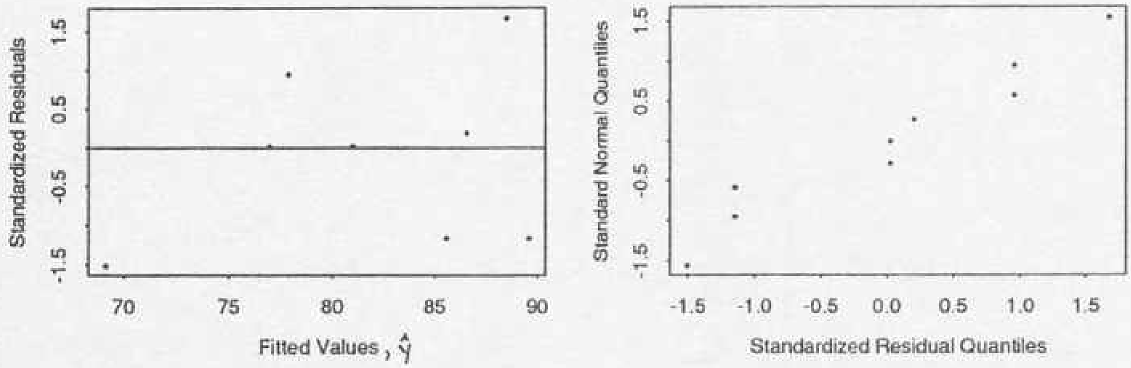
ROW	x1	x2	y1	y2	fits1	resids1	stresid1	fits2
1	1350	80	77	67	76.9683	0.03172	0.02229	60.8812
2	950	80	83	54	85.5521	-2.55210	-1.15335	60.8023
3	600	80	91	70	88.4796	2.52038	1.67298	69.3166
4	1350	100	80	52	77.8889	2.11111	0.95353	58.6667
5	950	100	87	57	86.5556	0.44444	0.20074	55.3333
6	600	100	87	66	89.5556	-2.55556	-1.15427	61.0000
7	1350	120	67	54	69.1428	-2.14283	-1.50587	53.4522
8	950	120	80	52	77.8923	2.10766	0.95250	46.8644
9	600	120	81	44	80.9648	0.03517	0.02335	49.6834

ROW	resids2	stresid2
1	6.11884	1.68538
2	-6.80227	-1.20489
3	0.68343	0.17781
4	-6.66667	-1.18022
5	1.66667	0.29505
6	5.00000	0.88516
7	0.54783	0.15089
8	5.13560	0.90967
9	-5.68343	-1.47865

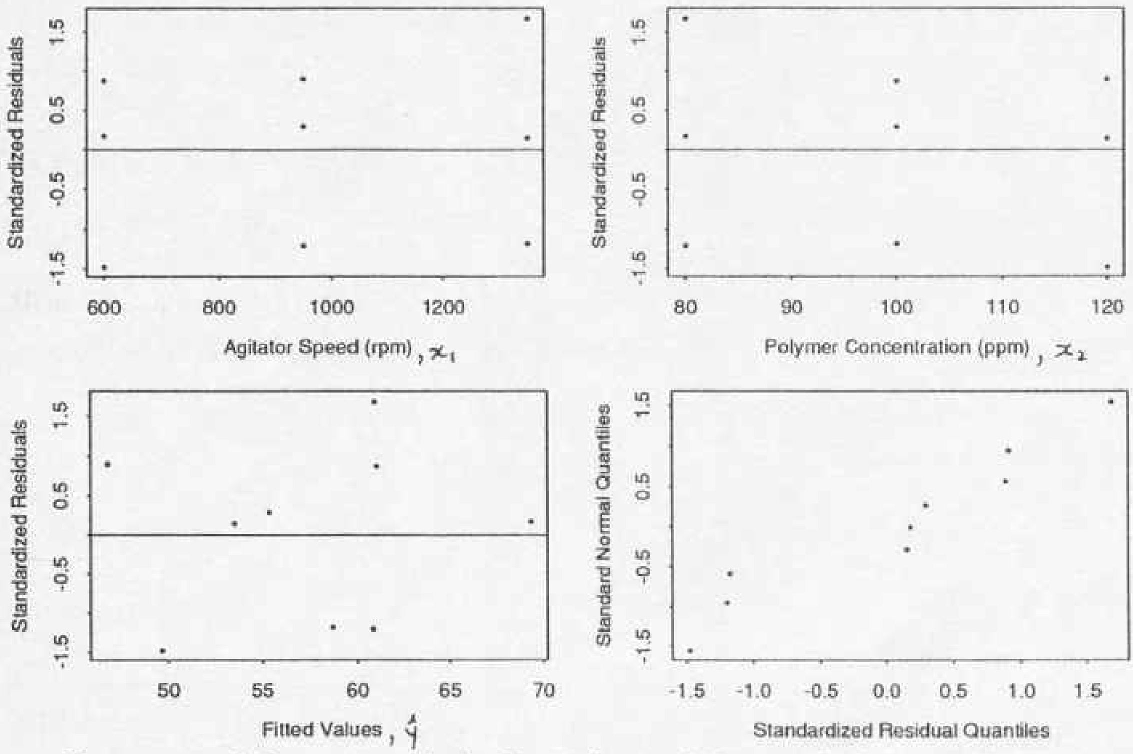
For the y_1 equation:







The first 3 plots are patternless, and the normal plot is roughly linear, so there is no indication that the model is inadequate. For the y_2 equation:



There are no obvious patterns in the first 3 plots, and the normal plot is roughly linear, so there is no indication that the model is inadequate.

(b) $\max \hat{y}_1 - \min \hat{y}_1 = 89.5556 - 69.1428 = 20.4128,$

and

$$4\sqrt{\frac{(6)(3.321)^2}{9}} = 10.84634.$$

The criterion is satisfied for the equation for y_1, so this response surface seems to be well enough determined by the data.

$$\max \hat{y}_2 - \min \hat{y}_2 = 69.3166 - 46.8644 = 22.4522,$$

and

$$4\sqrt{\frac{(6)(8.473)^2}{9}} = 27.6727.$$

The criterion is not satisfied for the equation for y_2, so this response surface is not well determined enough to justify the use of the surface for further analysis.

(c) Using the notation of Section 9-3, the fitted equation for y_1 has

$$b_0 = -17.92 \quad b_1 = .01953 \quad b_{11} = -.00001746 \quad b_{12} = -.0000104$$
$$b_2 = 2.235 \quad b_{22} = -.012083$$

The nature of the surface can be determined by finding the eigenvalues. For y_1,

$$\begin{aligned}\det(B - \lambda I) &= \det\begin{bmatrix} -.00001746 - \lambda & \frac{1}{2}(-.0000104) \\ \frac{1}{2}(-.0000104) & -.012083 - \lambda \end{bmatrix} \\ &= (-.00001746 - \lambda)(-.012083 - \lambda) - \frac{1}{4}(-.0000104)^2 \\ &= \lambda^2 + .01210046\lambda + .00000021.\end{aligned}$$

Setting this equal to zero, and using the quadratic formula, the roots are $\lambda = -.0000174$ and $\lambda = -.0120830$. Since both of the eigenvalues are negative, the surface is concave (inverted bowl-shape). The stationary point (maximum) is at

$$-\frac{1}{2}B^{-1}b = \begin{pmatrix} 531.80220 \\ 92.25645 \end{pmatrix},$$

or at $x_1 = 531.8$ rpm and $x_2 = 92.3$ ppm.

Using the notation of Section 9-3, the fitted equation for y_2 has

$$b_0 = 125.3 \quad b_1 = -.1076 \quad b_{11} = .00003270 \quad b_{12} = .0004068$$
$$b_2 = .015 \quad b_{22} = -.00375$$

The nature of the surface can be determined by finding the eigenvalues. For y_2,

$$\begin{aligned}\det(B - \lambda I) &= \det\begin{bmatrix} .00003270 - \lambda & \frac{1}{2}(.0004068) \\ \frac{1}{2}(.0004068) & -.00375 - \lambda \end{bmatrix} \\ &= (.00003270 - \lambda)(-.00375 - \lambda) - \frac{1}{4}(.0004068)^2 \\ &= \lambda^2 + .0037173\lambda - .00000016399656.\end{aligned}$$

Setting this equal to zero, and using the quadratic formula, the roots are $\lambda = .0000436056$ and $\lambda = -.003760906$. Since one of these eigenvalues is positive, and the other is negative, the surface is a saddle. The stationary point is at

$$-\frac{1}{2}B^{-1}b = \begin{pmatrix} 1220.90671 \\ 68.22198 \end{pmatrix},$$

or at $x_1 = 1220.91$ rpm and $x_2 = 68.22$ ppm.

(d) Using Minitab, the contour plot for the y_1 surface can be constructed as below.

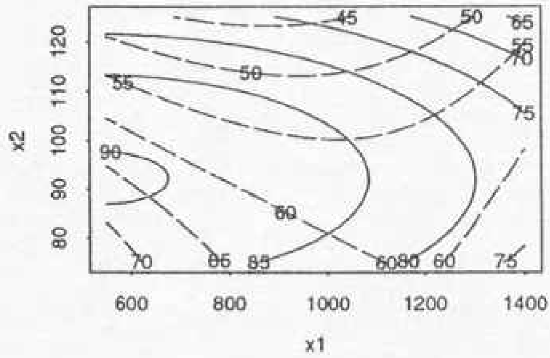
```
MTB > grid c32=75:125, c31=550:1400
MTB > name c32 'x2grid' c31 'x1grid'
MTB > let c33 = -17.92 + .01953*c31 + 2.235*c32 - .00001746*c31**2 &
CONT> - .012083*c32**2 - .0000104*c31*c32
MTB > name c33 'yhat1grd'
```

```
MTB > contour c33 c32 c31

A = 65.0, 67.5      B = 70.0, 72.5      C = 75.0, 77.5
D = 80.0, 82.5      E = 85.0, 87.5      F = 90.0, 92.5
          - ,,,,,,,,,,,,,,CCCCCCCCCCCC.........BBBBBBB,,,,,AAAAA
x2grid    - DDDDDDDDDDDDDD,,,,,,,,,,,,CCCCCCCCC.......BBBBBB,,,,,
          - ..........DDDDDDDDDDDDDDD,,,,,,,,,CCCCCCC......BBBBB,
          - .....................DDDDDDDDDD,,,,,,,,CCCCCC......BB
   112.0+ EEEEEEEEEEEEEEEE............DDDDDDD,,,,,,,CCCCCC....
          - ,,,,,EEEEEEEEEEEEEEEEE.........DDDDDDD,,,,,,CCCCCC.
          - ,,,,,,,,,,,,,,,EEEEEEEEEEEE.........DDDDDDD,,,,,,CCCC
          - ,,,,,,,,,,,,,,,,,,,,EEEEEEEEEE........DDDDDDD,,,,,CCC
          - F,,,,,,,,,,,,,,,,,,,,EEEEEEEEEE........DDDDDD,,,,,,C
    96.0+ FFFFFFF,,,,,,,,,,,,,,,,,EEEEEEEEE........DDDDDD,,,,,C
          - FFFFFFFF,,,,,,,,,,,,,,,,EEEEEEEEE........DDDDDD,,,,,C
          - FFFFF,,,,,,,,,,,,,,,,,,EEEEEEEEEE.......DDDDDD,,,,,,C
          - ,,,,,,,,,,,,,,,,,,,,,,EEEEEEEEEE........DDDDDD,,,,,,CC
          - ,,,,,,,,,,,,,,,,,,EEEEEEEEEEEE........DDDDDDD,,,,,,CCC
    80.0+ ,,,,,,,,,,,EEEEEEEEEEEEEEE.........DDDDDDD,,,,,,,CCCCC
          - EEEEEEEEEEEEEEEEEEE...........DDDDDDDD,,,,,,,CCCCCC..
            -----+---------+---------+---------+---------+-------x1grid
               640       800       960      1120      1280
```

The contour plot for the y_2 surface:

```
MTB > let c34 = 125.3 - .1076*c31 + .015*c32 + .00003270*c31**2 - &
CONT> .00375*c32**2 + .0004068*c31*c32
MTB > name c34 'yhat2grd'
MTB > contour c34 c32 c31

A = 42.0, 45.0      B = 48.0, 51.0      C = 54.0, 57.0
D = 60.0, 63.0      E = 66.0, 69.0      F = 72.0, 75.0
          - B,,,,,,,,,,,,,,,,,,AAAA,,,,,,,,,,,,,,,,,,BBBBBBBB......
x2grid    - BBBBBBBBB,,,,,,,,,,,,,,,,,,,,,,,,,BBBBBBBBBB.......CC
          - .....BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB........CCCC
          - CC..........BBBBBBBBBBBBBBBBBBBBBBBB...........CCCCCC
   112.0+ CCCCCCC....................................CCCCCCCC,
          - ,,,CCCCCCCCCC...........................CCCCCCCCCC,,,
          - ,,,,,,,,CCCCCCCCCCCCCC...........CCCCCCCCCCCCCC,,,,,
          - DDDD,,,,,,,,,,CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC,,,,,,,,
          - .DDDDDDD,,,,,,,,,,CCCCCCCCCCCCCCCCCCCCCCCC,,,,,,,,,,,
    96.0+ ....DDDDDDDD,,,,,,,,,,,,,,CCCCCCCCCCCC,,,,,,,,,,,,,,D
          - E......DDDDDDDDD,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,DDD
          - EEEE......DDDDDDDDD,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,DDDD
          - EEEEEE.......DDDDDDDDDD,,,,,,,,,,,,,,,,,,,,,,,,,DDDDD
          - ,,EEEEEE.......DDDDDDDDDDDD,,,,,,,,,,,,,,,,,,,DDDDDDD
    80.0+ ,,,,EEEEEE........DDDDDDDDDDDDD,,,,,,,,,,,,,,DDDDDDDD
          - F,,,,,EEEEEE........DDDDDDDDDDDDDDD,,,,,,,,,DDDDDDDDDD
            -----+---------+---------+---------+---------+-------x1grid
               640       800       960      1120      1280
```

Here's another plot using the S-Plus function "contour":



According to these surfaces, the x's that maximize y_1 subject to $y_2 \leq 55$ are approximately $x_1 = 800$ and $x_2 = 104$.

(e) The Minitab output needed for this problem is given below.

```
MTB > regress c3 5 c1 c2 c5 c6 c7;
SUBC> predict 800 104.

The regression equation is
y1 = - 17.9 + 0.0195 x1 + 2.24 x2 -0.000017 x1sq - 0.0121 x2sq
           - 0.000010 x1*x2

Predictor        Coef        Stdev    t-ratio        p
Constant       -17.92        63.19      -0.28    0.795
x1            0.01953      0.03979       0.49    0.657
x2              2.235        1.195       1.87    0.158
x1sq      -0.00001746   0.00001679      -1.04    0.375
x2sq        -0.012083     0.005871      -2.06    0.132
x1*x2      -0.0000104    0.0002212      -0.05    0.966

s = 3.321       R-sq = 91.5%     R-sq(adj) = 77.3%

                                  .
                                  .
                                  .

     Fit  Stdev.Fit         95% C.I.            95% P.I.
   87.41       2.22   (   80.35,   94.48)  (  74.70,  100.13)

MTB > regress c4 5 c1 c2 c5 c6 c7;
SUBC> predict 800 104 640000 10816 83200.

The regression equation is
y2 = 125 - 0.108 x1 + 0.02 x2 +0.000033 x1sq - 0.0037 x2sq +0.000407 x1*x2

Predictor        Coef        Stdev    t-ratio        p
Constant        125.3        161.2       0.78    0.494
x1            -0.1076       0.1015      -1.06    0.367
x2              0.015        3.050       0.00    0.996
x1sq       0.00003270   0.00004283       0.76    0.501
x2sq         -0.00375      0.01498      -0.25    0.818
```

```
x1*x2        0.0004068   0.0005645        0.72     0.523

s = 8.473        R-sq = 63.2%     R-sq(adj) = 2.0%
                                   .
                                   .
                                   .

      Fit   Stdev.Fit           95% C.I.             95% P.I.
    55.08        5.67   (   37.05,    73.11)  (   22.65,    87.52)
```

Use equation (9-56). The appropriate t is $t = Q_3(.95) = 2.353$ from Table B-4. The prediction interval for y_1 at $x_1 = 800$ and $x_2 = 104$ is

$$\begin{aligned} & 87.41 \pm 2.353\sqrt{(3.321)^2 + (2.22)^2} \\ = & \ 87.41 \pm 9.399474 \\ = & \ [78.0, 96.8]\ \%. \end{aligned}$$

The prediction interval for y_2 at $x_1 = 800$ and $x_2 = 104$ is

$$\begin{aligned} & 55.08 \pm 2.353\sqrt{(8.473)^2 + (5.67)^2} \\ = & \ 55.08 \pm 23.98914 \\ = & \ [31.1, 79.1]\ \%. \end{aligned}$$

Use equations (9-58) and (9-59) for the tolerance bounds. For the tolerance bound for y_1 at $x_1 = 800$ and $x_2 = 104$, $A = \frac{2.22}{3.321} = .6684734$.

Finally, using
equation (9-57)

$$\tau = \frac{1.28 + (.6684734)(1.645)\sqrt{1 + \frac{1}{2(3)}\left(\frac{(1.28)^2}{(.6684734)^2} - (1.645)^2\right)}}{1 - \frac{(1.645)^2}{2(3)}} = 4.4888$$

The resulting bound for y_1 at $x_1 = 800$ and $x_2 = 104$ is

$$87.41 - 4.4888\ (3.321) = 72.5\ \%.$$

For the tolerance bound for y_2 at $x_1 = 800$ and $x_2 = 104$, $A = \frac{5.67}{8.473} = .6691845$.

Finally, using equation (9-57)

$$\tau = \frac{1.28 + (.6691845)(1.645)\sqrt{1 + \frac{1}{2(3)}\left(\frac{(1.28)^2}{(.6691845)^2} - (1.645)^2\right)}}{1 - \frac{(1.645)^2}{2(3)}} = 4.489986$$

The resulting bound for y_2 at $x_1 = 800$ and $x_2 = 104$ is

$$55.08 + 4.489986\,(8.473) = 93.12\%.$$

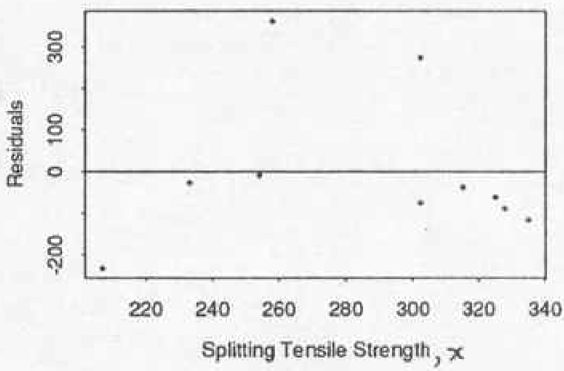
6. (a) From Ex. 16, Chap. 4 $b_1 = 12.45769$ and $b_0 = -927.6531$. The necessary computations for s_{LF} (the residuals) are also given there. Using equation (9-10),

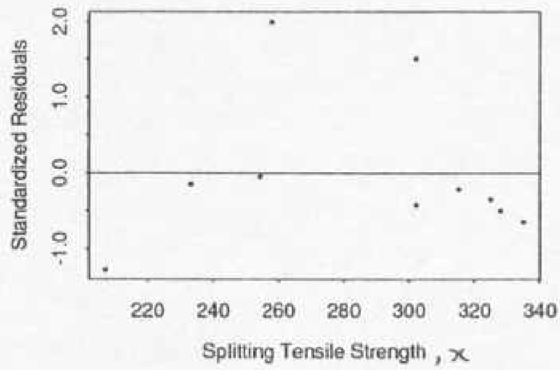
$$s^2_{\mathrm{LF}} = \frac{1}{10-2}(293948.6) = 36743.57,$$

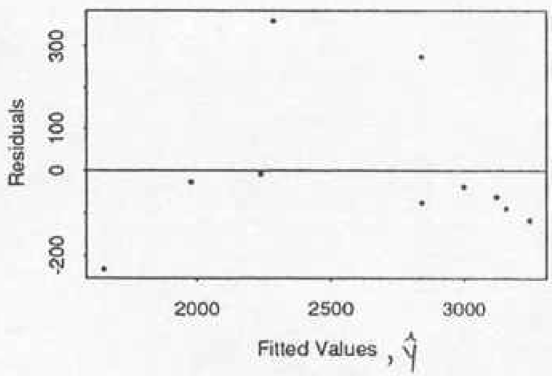
so $s_{\mathrm{LF}} = \sqrt{36743.57} = 191.69$ psi, with 8 degrees of freedom associated with it.

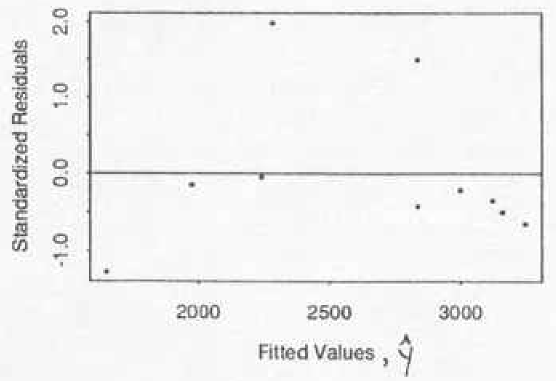
(b) The residuals were computed in Ex. 16, Chap. 4 . Use equation (9-12) to compute the standardized residuals. $\bar{x} = 285.9$, and $\sum(x-\bar{x})^2 = 17896.9$. The rest of the calculations are summarized below.

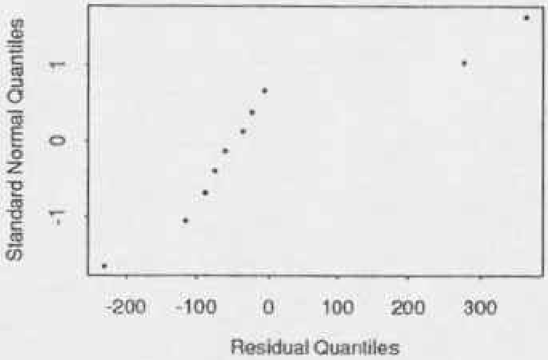
x	$\sqrt{1 - \frac{1}{10} - \frac{(x-285.9)^2}{17896.9}}$	e	e^*
207	.9510	-231.0884	-1.2677
233	.9502	-24.9883	-.1372
254	.9496	-6.5997	-.0363
328	.9474	-88.4687	-.4871
325	.9475	-61.0956	-.3364
302	.9482	275.4312	1.5154
258	.9495	363.5695	1.9976
335	.9472	-115.6725	-.6371
315	.9478	-36.5187	-.2010
302	.9482	-74.5688	-.4103

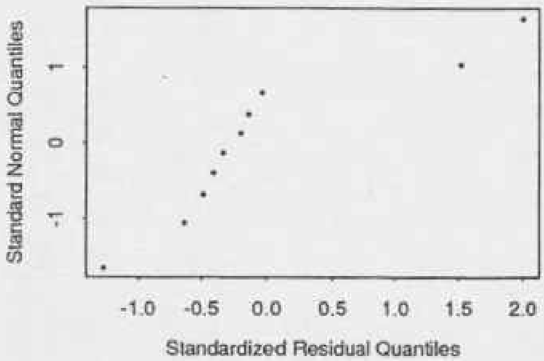












For each of the three types of plots, the residuals and standardized residuals look almost exactly the same.

(c) First make a confidence interval for β_1, and then multiply the endpoints by 5. Use equation (9-17). For 95% confidence, the appropriate t is $t = Q_8(.975) = 2.306$ from Table B-4. The resulting interval for β_1 is

$$\begin{aligned} & 12.45769 \pm 2.306 \frac{191.69}{\sqrt{17896.9}} \\ = \; & 12.45769 \pm 3.30416 \\ = \; & [9.153528, 15.76185] \text{ psi.} \end{aligned}$$

Multiplying each endpoint by 5, the resulting interval for $5\beta_1$ is $[45.77, 78.81]$ psi.

(d) Use equation (9-24). For 90% confidence, the appropriate t is $t = Q_8(.95) = 1.860$ from Table B-4. The resulting interval for the mean at $x = 300$ is

$$\begin{aligned} & 2809.653 \pm 1.860(191.69)\sqrt{\frac{1}{10} + \frac{198.81}{17896.9}} \\ = \; & 2809.653 \pm 118.8441 \\ = \; & [2690.8, 2928.5] \text{ psi.} \end{aligned}$$

(e) Use equation (9-26). The appropriate t is the same one used in part (d). The resulting prediction interval at $x = 300$ is

$$\begin{aligned} & 2809.653 \pm 1.860(191.69)\sqrt{1 + \frac{1}{10} + \frac{198.81}{17896.9}} \\ = \; & 2809.653 \pm 375.8217 \\ = \; & [2433.8, 3185.5] \text{ psi.} \end{aligned}$$

(f) Use equation (9-27). For $x = 300$, first using equation (9-29),

$$A = \sqrt{\frac{1}{10} + \frac{198.81}{17896.9}} = .3333296.$$

Finally, using equation (9-30),

$$\tau = \frac{1.28 + (.3333296)(1.645)\sqrt{1 + \frac{1}{2(8)}\left(\frac{(1.28)^2}{(.3333296)^2} - (1.645)^2\right)}}{1 - \frac{(1.645)^2}{2(8)}} = 2.41418$$

The resulting bound for $x = 300$ is

$$2809.653 - 2.41418\ (191.69) = 2346.877 \text{ psi.}$$

7. (a) The following output is from Minitab Version 9.1.

```
MTB > brief = 3
MTB > regress c1 2 c2 c3

The regression equation is
weight = 10.5 + 6.12 spacing - 3.57 xsq

Predictor       Coef       Stdev    t-ratio       p
Constant     10.5473      0.4143      25.46   0.000
spacing        6.118       1.020       6.00   0.000
xsq          -3.5707      0.5854      -6.10   0.000

s = 0.3471        R-sq = 32.6%      R-sq(adj) = 30.9%
     S_SF
Analysis of Variance

SOURCE        DF          SS          MS         F        p
Regression     2      4.4924      2.2462     18.64    0.000
Error         77      9.2771      0.1205
Total         79     13.7695

SOURCE        DF      SEQ SS
spacing        1      0.0101
xsq            1      4.4823

Obs.  spacing     weight        Fit  Stdev.Fit   Residual    St.Resid
                     .
                     .
                     .
 41      1.00    12.0000    13.0944     0.0526    -1.0944      -3.19R
                     .
                     .
                     .
```

$s_{SF} = .3471$ g. Assuming that the model is appropriate, this measures the variation in Weights for a fixed Spacing. Using equation (7-7), $s_p = .3448$ g. These two estimates are very close, giving no indication that the model is inappropriate.

(b) 1. H_0: $\beta_1 = \beta_2 = 0$.
2. H_a: not H_0.
3. The test statistic is given by equation (9-62). The reference distribution is the $F_{2,77}$ distribution. Large observed values of F will be considered as evidence against H_0.

4. The samples give (using Minitab output)

$$f = 18.64.$$

5. The observed level of significance is

$$P(\text{an } F_{2,77} \text{ random variable} > 18.64)$$

which is 0 to 3 decimal places, using the Minitab output. The null hypothesis means that the Weight does not depend on the Spacing at all. There is overwhelming evidence against this hypothesis; the model used is an improvement over a model in which Weight does not depend on Spacing ($y = \beta_0 + \epsilon$).

(c) This can be done using equation (9-47) or equation (9-66). It is easier to use the T test.

1. $\mathrm{H_0}$: $\beta_2 = 0$.
2. $\mathrm{H_a}$: $\beta_2 \neq 0$.
3. The test statistic is given by equation (9-46), with $\# = 0$. The reference distribution is the t_{77} distribution. Observed values of t far above or below zero will be considered as evidence against $\mathrm{H_0}$.
4. The samples give (using the Minitab printout)

$$t = -6.10.$$

5. The observed level of significance is

$$2P(\text{a } t_{77} \text{ random variable} < -6.10)$$

which is 0 to 3 decimal places, according to the Minitab printout. The meaning of this hypothesis is that Weight depends only linearly on Spacing (no curvature). There is overwhelming evidence against this hypothesis; The quadratic model is an improvement over the straight-line model $y = \beta_0 + \beta_1 x + \epsilon$.

(d) Use equation (9-53) and the Minitab printout. For a 90% one-sided interval, make an 80% two-sided interval and use the lower endpoint. The appropriate t for an 80% two-sided interval is $t = Q_{77}(.90) \approx 1.2926$ from Table B-4. For $x = 1.000$, the 90% lower confidence bound is then

$$\begin{aligned} & 13.0944 - 1.2926(.0526) \\ = & \ 13.0944 - .06799076 \\ = & \ 13.03 \text{ g.} \end{aligned}$$

(e) Use equation (9-56). The appropriate t is the same one used in part (d). The resulting lower prediction bound at $x = 1.000$ is

$$\begin{aligned} & 13.0944 - 1.2926\sqrt{(.3471)^2 + (.0526)^2} \\ = & \ 13.0944 - .4537839 \\ = & \ 12.64 \text{ g.} \end{aligned}$$

(f) Use equation (9-58). For $x = 1.000$, $A = \frac{.0526}{.3471} = .1515413$.
$p = .90$,

Finally, using equation (9-57)

$$\tau = \frac{1.28 + (.1515413)(1.645)\sqrt{1 + \frac{1}{2(77)}\left(\frac{(1.28)^2}{(.1515413)^2} - (1.645)^2\right)}}{1 - \frac{(1.645)^2}{2(77)}} = 1.608$$

The resulting bound for $x = 1.000$ is

$$13.0944 - 1.608\ (.3471) = 12.54 \text{ g.}$$

8. (a) The following printout was made using Minitab Version 9.1.

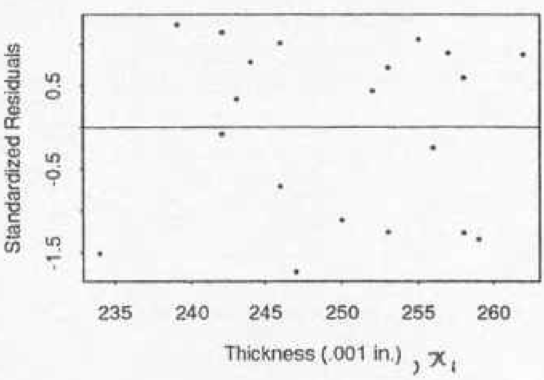
```
MTB > info c1-c3

Column    Name          Count
C1        y                20
C2        x1               20
C3        x2               20

MTB > brief = 3
MTB > regress c1 2 c2 c3;
SUBC> fits c4;
SUBC> resids c5;
SUBC> sresids c6;
SUBC> predict 260 380.

The regression equation is
y = - 1674 + 7.61 x1 + 2.59 x2

Predictor       Coef        Stdev     t-ratio        p
Constant     -1674.1        947.4       -1.77    0.095
x1             7.612        3.730        2.04    0.057
x2            2.5939       0.5698        4.55    0.000

s = 124.5         R-sq = 60.1%      R-sq(adj) = 55.4%
     SSF
Analysis of Variance

SOURCE        DF         SS          MS         F        p
Regression     2     397230      198615     12.82    0.000
Error         17     263358       15492
Total         19     660589

SOURCE        DF     SEQ SS
x1             1      76220
x2             1     321010

Obs.      x1         y        Fit  Stdev.Fit    Residual    St.Resid
                               .
                               .
                               .
 10      258    1374.0     1304.1       43.2        69.9        0.60
                               .
                               .
                               .
```

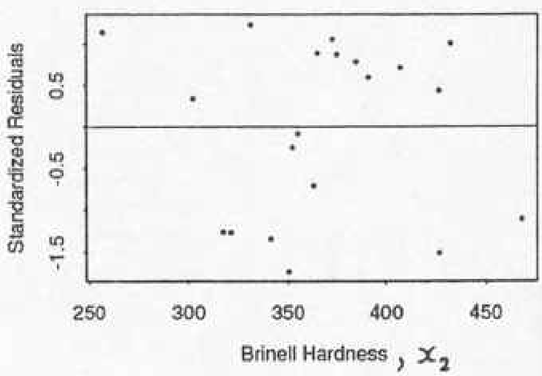
```
    Fit  Stdev.Fit         95% C.I.              95% P.I.
 1290.8       47.5    (  1190.5,  1391.0)  (  1009.6,  1571.9)

MTB > name c4 'fits' c5 'resids' c6 'stresids'
MTB > print c1-c6
```

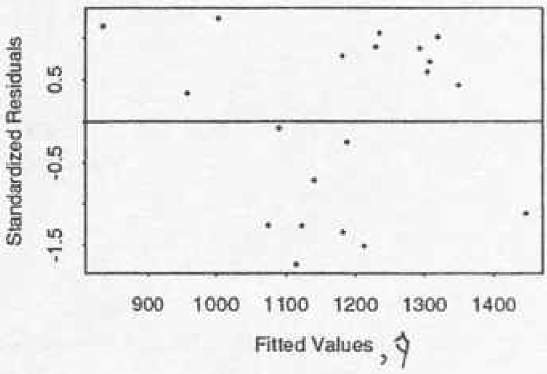
ROW	y	x1	x2	fits	resids	stresids
1	927	253	317	1074.07	-147.074	-1.25471
2	978	258	321	1122.51	-144.511	-1.26570
3	1028	259	341	1182.00	-154.001	-1.33623
4	906	247	350	1114.00	-207.999	-1.72597
5	1159	256	352	1187.70	-28.697	-0.24171
6	1055	246	363	1140.11	-85.107	-0.70644
7	1335	257	365	1229.03	105.970	0.89585
8	1392	262	375	1293.03	98.970	0.88026
9	1362	255	373	1234.56	127.444	1.06447
10	1374	258	391	1304.08	69.917	0.59897
11	1393	253	407	1307.52	85.476	0.72051
12	1401	252	426	1349.20	51.804	0.44525
13	1436	246	432	1319.09	116.914	1.02150
14	1327	250	469	1445.51	-118.509	-1.11311
15	950	242	257	834.71	115.294	1.14781
16	998	243	302	959.04	38.957	0.34462
17	1144	239	331	1003.82	140.183	1.24199
18	1080	242	355	1088.91	-8.907	-0.07572
19	1276	244	385	1181.95	94.052	0.79134
20	1062	234	426	1212.18	-150.176	-1.50501

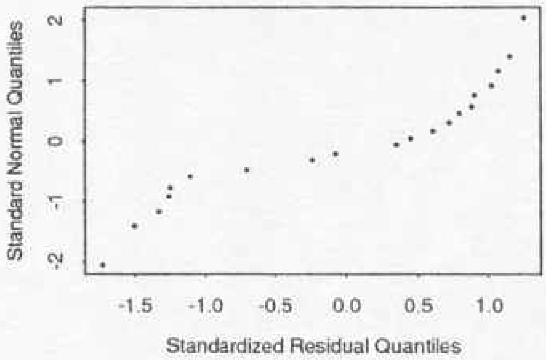
s_{SF} = 124.5 ft./sec.

(b)









The first 3 plots are patternless, giving no evidence that the model is inadequate. The normal plot is somewhat non-linear, giving some evidence that the normal part of the model assumption is incorrect.

(c) Use equation (9-47) and the Minitab printout. The appropriate t is $t = Q_{17}(.95) = 1.740$ from Table B-4. The interval for β_1 is

$$\begin{aligned} & 7.612 \pm 1.740(3.730) \\ = \; & 7.612 \pm 6.4902 \\ = \; & [1.1, 14.1] \text{ ft/sec.} \end{aligned}$$

The interval for β_2 is

$$\begin{aligned} & 2.5939 \pm 1.740(.5698) \\ = \; & 2.5939 \pm .991452 \\ = \; & [1.60, 3.59] \text{ ft/sec.} \end{aligned}$$

Now multiply the endpoints by 20. The increase in mean ballistic limit which accompanies a 20-unit increase in Brinell hardness number is [32.0, 71.7] ft/sec.

(d) Use equation (9-53) and the Minitab printout. The appropriate t is $t = Q_{17}(.975) = 2.110$ from Table B-4. The resulting interval is

$$\begin{aligned} & 1304.1 \pm 2.110(43.2) \\ = \; & 1304.1 \pm 91.152 \\ = \; & [1213, 1395] \text{ ft/sec.} \end{aligned}$$

(e) Use equation (9-56) and the Minitab printout. The t is the same as the one used in part (d). The resulting prediction interval at $x_1 = 258$ and $x_2 = 391$ is

$$\begin{aligned} & 1304.1 \pm 2.110\sqrt{(124.5)^2 + (43.2)^2} \\ = \; & 1304.1 \pm 278.06 \\ = \; & [1026, 1582] \text{ ft/sec.} \end{aligned}$$

(f) Use equation (9-58). For $x_1 = 258$ and $x_2 = 391$, $A = \frac{43.2}{124.5} = .346988$.

Finally,
using equation (9-57)

$$\tau = \frac{2.05 + (.346988)(\ 1.645 \quad)\sqrt{1 + \frac{1}{2(17)}\left(\frac{(2.05)^2}{(.346988)^2} - (\ 1.645 \quad)^2\right)}}{1 - \frac{(\ 1.645\ )^2}{2(17)}} = 3.09259$$

The resulting bound for $x_1 = 258$ and $x_2 = 391$ is

$$1304.1 - 3.09259\ (124.5) = 919.07 \text{ ft/sec.}$$

(g) This can be read directly from the printout as [1190.5, 1391.0] ft/sec. You can "check" this answer using equation (9-53). The appropriate t is the same as the one used in part (d).

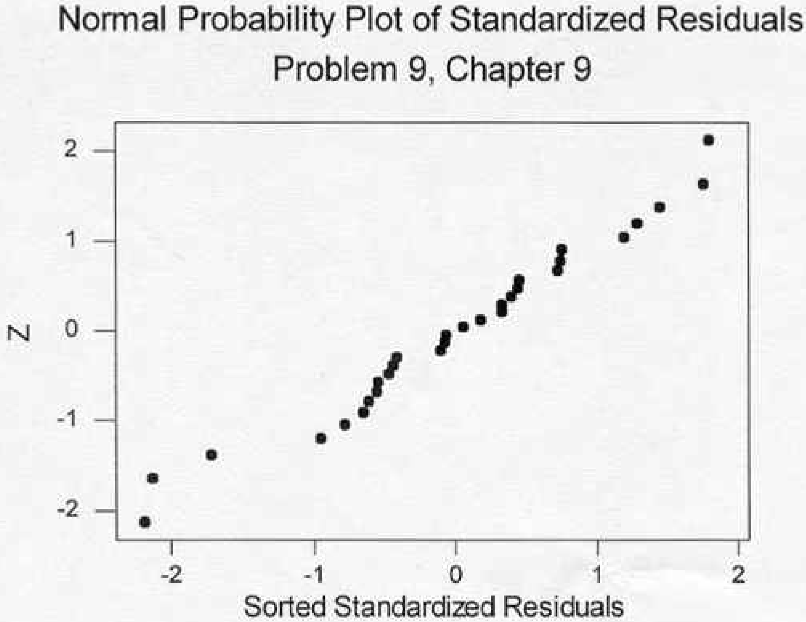
$$\begin{aligned} & 1290.8 \pm 2.110(47.5) \\ = \ & 1290.8 \pm 100.225 \\ = \ & [1190.6, 1391.0] \text{ ft/sec.} \end{aligned}$$

(h) This hypothesis means that Ballistic Limit is not related to Thickness or Hardness. The observed value of F and the p-value can be read from the printout: $f = 12.82$, and the p-value is zero to 3 decimal places. This is overwhelming evidence that Ballistic Limit is related to Thickness or Hardness.

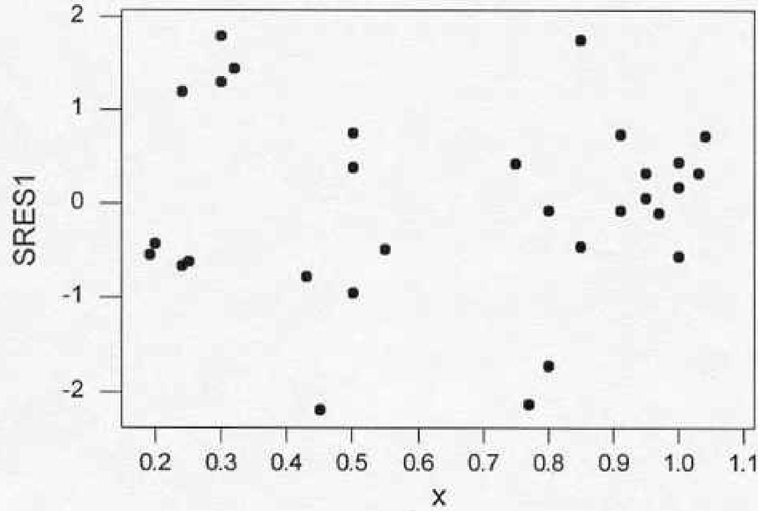
(i) This hypothesis means that a model with both Thickness and Hardness is no improvement over a model with only Hardness. The observed value of T and the p-value can be read from the printout: $t = 2.04$, and p-value $= .057$. There is moderate evidence that a model with both Thickness and Hardness is an improvement over a model with only Hardness.

(9)(a) $b_o = 2.144$, $b_1 = 3.276$, $\hat{\sigma} = .2284$, $s_p = .1997$. Since $\hat{\sigma} = .2284$ and $s_p = .1997$ are very similar, it seems the model $y = \beta_0 + \beta_1 x + \varepsilon$ is reasonable.

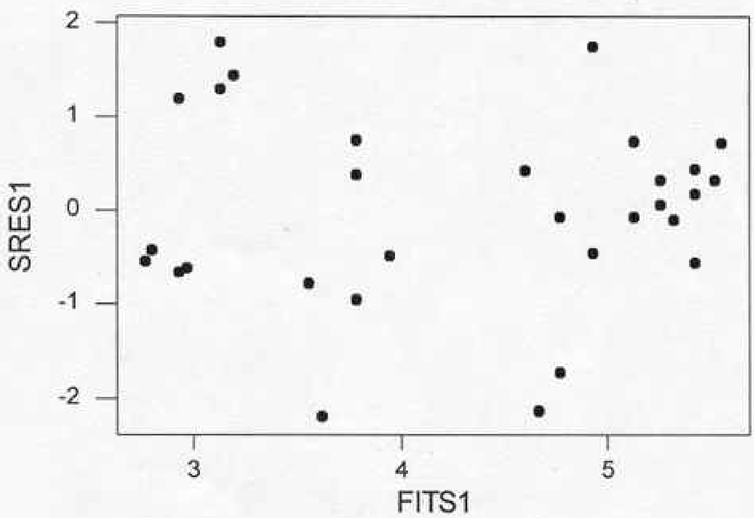
(b)



Residual Plot of Standardized Residuals vs. X
Problem 9, Chapter 9



Residual Plot of Standardized Residuals vs. Predicted
Problem 9, Chapter 9



Each of these plots suggest the standard assumptions with the straight-line model are reasonable.

(c) $b_1 \pm t_{28} s_{b1}$ becomes $3.276 \pm (1.701)(.14)$. Thus, the interval (3.0379, 3.514) is a 90% confidence interval for the slope.

(d) Using equation 9-24, at x = .65, the 90% confidence interval for the mean detonation velocity is $4.2745 \pm (1.701)(.0417)$ or (4.2036, 4.3454). Note: $\hat{y}(x = .65) = 4.2745$, $t_{28} = 1.701$ and the std deviation of fit is .0417.

(e) Using equation 9-26, x = .65, the 90% prediction interval for the next detonation velocity is (3.879, 4.669).

(f) Using equation 9-27, 9-29 and 9-30, $s_{LF} = .2284$, thus, from (d), A = (.0417)/.2284 = .18257. Since p = .95, $\gamma = .99$, n = 30, the numerator for τ is $1.645 + (.18257)(2.33)\sqrt{1+(1/2(28))[(1.645)^2/(.18257)^2-(2.33)^2]}$ = 2.2974. The denominator for τ is .903055. Thus, τ is 2.2974/.903055 = 2.544. The lower 99% tolerance limit that includes 95% of charges at x = .65 g/cc is 4.2745 - 2.544(.2284) = 3.693. The interval $(3.693, \infty)$ is the lower one-sided 99% tolerance limit for 95% of charges at x = .65 g/cc.

(10) (a) $b_0 = 51.6245$, $b_1 = .891739$, $b_2 = -.0149974$. $\hat{\sigma} = 12.476$, $s_p = 13.62$. Since s_p and $\hat{\sigma}$ are very close, the quadratic model seems reasonable. The estimate of σ measures the variation amongst all y values (torque) at a fixed x (depth).

(b) H_o: $\beta_1 = \beta_2 = 0$ vs. H_a at least one of the βs are not 0. $F_{2,16}$ calc = 2604.79/155.65 = 16.73, p-value = P(F > 16.73) ≈ 0. Conclude Ha. The question being asked is: when relating y (torque) to x (depth), does x play an important role (either in a linear or parabolic sense) in predicting y?

(c) Ho: $\beta_2 = 0$ vs. Ha: $\beta_2 \neq 0$, t_{16} = -.014997/.0051 = -2.94, p-value = 2 P(t_{16} > 2.94) $\approx .01$. Conclude Ha: $\beta_2 \neq 0$. This test answers the question: when relating y (torque) to x (depth), does a parabolic relationship exist?

(d) Using equation 9-53 and the quadratic model, the 95% two-sided confidence interval for the mean torque at x = 40 becomes (54.65, 71.95) where $\hat{y}(x = 40) = 63.298$ and s_{SF} A = 4.0812.

(e) Using equation 9-56 and the quadratic model, the 95% two-sided prediction interval for an additional torque at failure with x = 40 becomes (35.47, 91.13) where $\hat{y}(x = 40) = 63.298$ and s_{SF} A = 4.0812.

(f) An approximate $\gamma = 99\%$ lower tolerance bound for p = 95% of torques at x =40 is derived using equations 9-57 and 9-58. Since s_{SF} = 12.476, A = (4.0812)/(12.476) = .327. The numerator for τ becomes $1.645 + (.327)(2.33)\sqrt{1+(1/2(16))[(1.645)^2/(.327)^2-(2.33)^2]}$ = 2.615087. The denominator for τ becomes $1 - (2.33)^2/2(16)$ = .8303468. Thus τ becomes

2.615087/.8303468 = 3.1494. The 99% lower tolerance bound for p = 95% of torques at x = 40 is 63.298 - 3.14940(12.476) = 63.298 - 39.2920 = 24.006. The resulting interval is $(24.006, +\infty)$.

(11) (a) $b_1 \pm t_9 s_{b1}$ or $.99372 \pm t_9(.09458)$. Since t_9 is 1.833, the interval (.82035, 1.1671) is a 90% two-sided confidence interval for the β_1 coefficient.

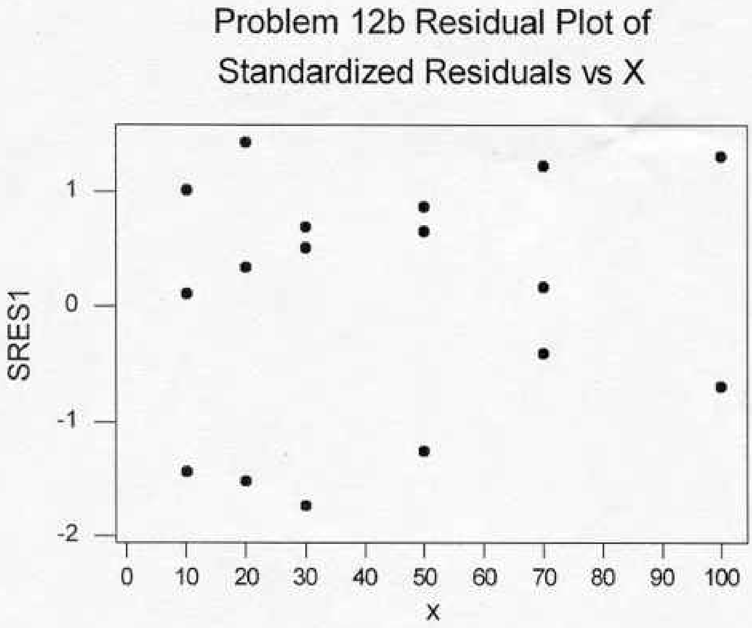
(b) Letting x_1 = .318, x_2 = .005, the 90% confidence interval for the mean log torque is (5.471, 5.6099).

(c) Letting x_1 = .318, x_2 = .005, the 95% prediction interval for a single log torque is (5.36986, 5.71104). Exponentiating these endpoints gives (214.832, 302.185) as a 95% prediction interval for torque at x_1 = .318, x_2 = .005.

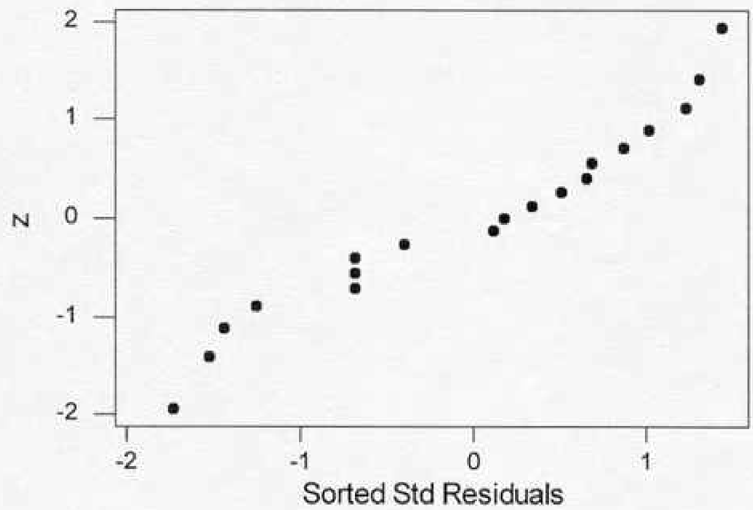
(d) The 95% confidence interval for mean log torque at x_1 = .3 in. and x_2 = .01 in./rev. is (5.9054, 5.9984).

(12) (a) b_o = 6.39925, b_1 = -.0102413, $\hat{\sigma} = .1234$.

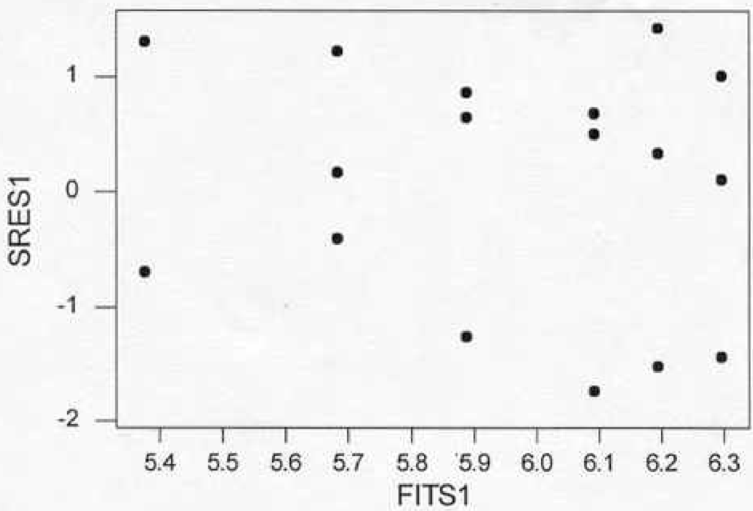
(b)



Problem 12b Normal Probability Plot of
Standardized Residuals



Problem 12b Residual Plot of
Standardized Residuals vs Fitted lnys



The new plots do not differ much compared to what we saw in part (h) of Exercise 25 of chapter 4. The normal probability plot in this example looks a little different.

(c) $10b_1 \pm 10\, t_{17}\, s_{b1}$ becomes $-.1 \pm (10)(1.74)(.0008749)$. The interval

(-.11522, - .0847767) is a 90% confidence interval for the change in mean log grip force that accompanies an increase in drag of 10%.

(d) The 95% two-sided confidence interval for the mean log grip force of a tire of this type under 30% drag (based on the simple linear regression model) is (6.0223, 6.1617).

(e) The 95% two-sided prediction interval for log grip force of a tire of this type under 30% drag is (5.8225, 6.3615). Exponentiating the endpoints gives a 95% two-sided prediction interval for grip force of a tire of this type under 30% drag (337.815, 579.114).

(f) An approximate γ = 95% lower tolerance bound for the grip forces of p = 90% of tires of this design under 30% drag (based on the linear regression model for lny) can be found using equations 9-27, 9-29 and 9-30. Using equation 9-29, A = .0330/.1234 = .26742. Note .0330 is the std. dev. of fit at x = 30 and s_{LF} = .1234. Thus, looking at equation 9-24, A can be found as stated. The numerator for τ becomes

$1.285 + (.26742)(1.645)\ \sqrt{1+(1/2(17))[(1.285)^2/(.26742)^2-(1.645)^2]}$ = 1.84135. The denominator for τ becomes 1 - $(1.645)^2/2(17)$ = .9204. Thus τ becomes 1.84135/.9204 = 2. The 95% lower tolerance bound for p = 90% of lnys at x = 30 is 6.092 - 2(.1234) = 5.8452. Exponentiating (5.8452) gives 345.57 as the 95% lower tolerance bound for the grip forces (y) of 90% of tires of this design with 30% drag, i.e., (345.57, ∞).

(13)(a) 143.591= s_{SF} is the estimate of σ. s_p is 136.8. A quadratic model seems reasonable since s_p is "close" to s_{SF} .

(b) H_o: $\beta_1 = \beta_2 = 0$ vs. H_a at least one of the βs are not 0. $F_{2,21}$ calc = 1,105,273/20,618= 53.61, p-value = P($F_{2,21}$ > 53.61) ≈ 0. Conclude Ha. The question being asked is: when relating y (permeability) to x (asphalt content), does x play an important role (either in a linear or parabolic sense) in predicting y?

(c) Ho: $\beta_2 = 0$ vs. Ha: $\beta_2 \neq 0$, t_{21} = -64.53/11.75 = -5.49, p-value = 2 P(t_{21} > 5.49) ≈ 0. Conclude Ha: $\beta_2 \neq 0$. This test answers the question: when relating y (permeability) to x (asphalt content), does a parabolic relationship exist?

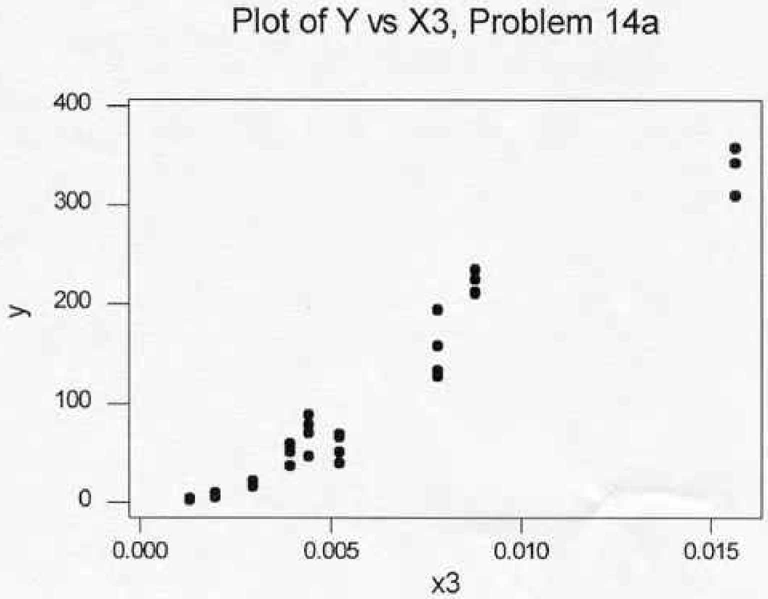
(d) A 90% two sided confidence interval for the mean permeability of specimens with 6.5% asphalt content is (793.2, 933.5).

(e) A 90% two-sided prediction interval for the next permeability measured on a specimen of this type having a 6.5% asphalt content is (606.5, 1120.2).

(f) 143.591= s_{SF} . At x = 6.5, the standard deviation of fit is 40.8. Thus, A = 40.8/143.6 = .2841 (see equation 9-53). The γ = 95% tolerance bound for p = 90% of specimens of this type having a 6.5% asphalt content can be found. Using equation 9-57, the numerator of τ becomes

$1.285 + (.2841)(1.645)\ \sqrt{1+(1/2(21))[(1.285)^2/(.2841)^2-(1.645)^2]} = 1.8424$.
The denominator for τ becomes $1 - (1.645)^2/2(21) = .93557$.
Thus τ becomes 1.8424/.93557 = 1.96928. The lower one-sided $\gamma = 95\%$ lower tolerance bound for p = 90% of specimens of this type having a 6.5% asphalt content is 863.3 - 1.96928(143.6) = 580.51, i.e., (580.51, ∞).

(14)(a)



(b) $x_3 = x_1^2/x_2$, y = axial breaking strength. $x_3 > .004$, $b_0 = -46.79$, $b_1 = 25{,}767$ $s_{LF} = 31.31$ and $s_p = 20.32$. Since there is a rather large difference between s_{LF} and s_p, perhaps the model is not appropriate.

(c) 98% two-sided confidence interval for the mean axial breaking strength when the diameter ($x_1 = .25$) and length ($x_2 = 8$) is (136.47, 172.55). Using equation (20), i.e., $\bar{y} \pm ts/\sqrt{4}$, becomes 153.02 $\pm$ t_3 (30.42)/(2) or 153.02 $\pm$ (4.541)(15.21), i.e., (83.95, 222.09). This interval is much wider than the interval derived using regression, both having the same confidence level of 98%.

(d) 98% two-sided prediction interval for y (axial breaking strength) of a single dowel ($x_1 = .25$ and $x_2 = 8$) is (72.58, 236.44). 98% two-sided prediction interval for y (axial breaking strength) of a single dowel ($x_1 = .25$ and $x_2 = 6$) is (139.21, 304.02).

(e) γ = 95% lower tolerance bound for breaking strengths of p = 98% of .25 inch diameter dowels 8 inches in length is found in the following fashion.
$\hat{y}(x_1 = .25, x_2 = 8, x_3 = .0078125) = 154.511$, s_{LF} = 31.31 and the standard deviation of fit = 7.07. Hence, A = 7.07/31.31 = .22581.

Using equation 9-30, the numerator of τ becomes

2.05 + (.22581)(1.645) $\sqrt{1+(1/2(18))[(2.05)^2/(.22581)^2-(1.645)^2]}$ = 2.715944.

The denominator for τ becomes 1 - $(1.645)^2/2(18)$ = .9248.

Thus τ becomes 2.715944/.9248 = 2.93679. The lower one-sided γ = 95% lower tolerance bound for p = 98% of dowels .25 inches in diameter and 8 inches in length 154.51 - 2.93679(31.31) = 62.559, i.e., (62.559, ∞).

Appendix A: More on Probability and Model Fitting

Section 1

1. (a) The process is stopped if the diameter is in the Red Zone, or if the diameter is in the Yellow Zone and a second diameter is in the Yellow or Red Zone.

$$P(\text{process stopped}) = .0730 + (.3023 + .0730).3023 = .1865.$$

The additions above are justified by axiom 3) of Definition A-6, because the events (diameter is in the Red Zone) and (diameter is in the Yellow Zone) are mutually exclusive for any particular diameter. The multiplication is an application of Proposition A-4. It also implicitly assumes that consecutive diameters are independent.

(b) Use Definition A-7.

$$P(A \text{ given } B) = \frac{(.3023 + .0730).3023}{.1865} = .6083.$$

2. There are 4 possible outcomes: (1/2 inch nut, 1/2 inch bolt), (1/2 inch nut, 9/16 inch bolt), (9/16 inch nut, 1/2 inch bolt), and (9/16 inch nut, 9/16 inch bolt).

(a)

$$\begin{aligned} P(\text{match}) &= P(\text{both } 1/2 \text{ inch } \textit{or} \text{ both } 9/16 \text{ inch}) \\ &= P(\text{both } 1/2 \text{ inch}) + P(\text{both } 9/16 \text{ inch}) \\ &= (.3)(.4) + (.7)(.6) = .54 \end{aligned}$$

The second line above is justified by axiom 3) of Definition A-6, because the two events are mutually exclusive. The third line above follows from Proposition A-4 and Definition A-8 because the nuts are selected independently.

(b) Use Definition A-7.

$P(\text{nut is } 9/16 \text{ inch}|\text{nut and bolt match})$

$$\begin{aligned} &= \frac{P(\text{nut is } 9/16 \text{ inch } \textit{and} \text{ nut and bolt match})}{P(\text{nut and bolt match})} \\ &= \frac{P(\text{both nut and bolt are } 9/16 \text{ inch})}{.54} \\ &= \frac{(.7)(.6)}{.54} = .7778. \end{aligned}$$

3. (a) Define the events

A = specimen is of substance A
B = specimen is of substance B
C = one count is observed in one second.

Use Definition A-7.

$$
\begin{aligned}
P(A|C) &= \frac{P(A \text{ and } C)}{P(C)} \\
&= \frac{P(C|A)P(A)}{P(C|A)P(A) + P(C|B)P(B)} \\
&= \frac{e^{-3}(3)\frac{2}{6}}{e^{-3}(3)\frac{2}{6} + e^{-4}(4)\frac{4}{6}} \\
&= .5048.
\end{aligned}
$$

The numerator of the second line above uses Proposition B-4. The denominator of the second line uses the fact that the event C can be written as

$$C = (C \text{ and } A) \text{ or } (C \text{ and } B)$$

where (C *and* A) and (C *and* B) are mutually exclusive, so their probabilities can be added together (after again applying Proposition A-4 to each of them). The third line above uses the Poisson distribution, equation (5-10) (with $\lambda = 3$ and 4 respectively), and the fact that the specimen was chosen at random.

(b) Define the events

A = specimen is of substance A
B = specimen is of substance B
C = 10 counts are observed in 10 seconds.

Use Definition A-7.

$$
\begin{aligned}
P(A|C) &= \frac{P(A \text{ and } C)}{P(C)} \\
&= \frac{P(C|A)P(A)}{P(C|A)P(A) + P(C|B)P(B)} \\
&= \frac{\frac{e^{-30}(30)^{10}}{10!}\frac{2}{6}}{\frac{e^{-30}(30)^{10}}{10!}\frac{2}{6} + \frac{e^{-40}(40)^{10}}{10!}\frac{4}{6}} \\
&= .9984.
\end{aligned}
$$

The third line above uses the Poisson distribution, equation (5-10) (with $\lambda = 30$ and 40 respectively).

(c) No. Observing 10 counts in a 10 second check is much stronger evidence that the specimen is of substance A than observing 1 count in 1 second. Even though the average count of substance A is smaller than that for substance B, it is still not too unlikely that substance B could have produced 1 count in 1 second (relative to how likely A could have done this), especially since the original chance that a substance B specimen would be chosen was twice that of a substance A specimen. However, it is extremely unlikely that substance B could have produced 10 counts in 10 seconds (relative to how likely A could have done this), even though the original chance that a substance B specimen would be chosen was twice that of a substance A specimen. In both part (a) and part (b), the relative Poisson probabilities dominate over the original selection probabilities, but this is even more pronounced in part (b) because most of the evidence comes from the long observation.

4. Define the events

$A =$ chip is good,
$B =$ chip is bad, and
$C =$ chip tests good.

(a) Use Proposition A-4.

$$P(A \text{ and } C) = P(A)P(C|A) = (.8)(.95) = .76.$$

(b) Use axiom 3) of Definition A-6, and the fact that the event C can be written the union of two mutually exclusive events:

$$C = (A \text{ and } C) \text{ or } (B \text{ and } C).$$

$$\begin{aligned} P(C) &= P((A \text{ and } C) \text{ or } (B \text{ and } C)) \\ &= P(A \text{ and } C) + P(B \text{ and } C) \\ &= .76 + (.2)(.1) = .78. \end{aligned}$$

(c) Use Definition A-7.

$$\begin{aligned} P(A|C) &= \frac{P(A \text{ and } C)}{P(C)} \\ &= \frac{.76}{.78} = .9744. \end{aligned}$$

5. Define the events

$A =$ The ring meets specs on the first grind,
$B =$ The ring is above specs on the first grind,
$C =$ The ring is below specs on the first grind, and
$D =$ The ring meets specifications given B.

(a) Use axiom 3) of Definition A-6, and the fact that the events A and C are mutually exclusive.

$$P(A \text{ or } C) = P(A) + P(C) = .60 + .15 = .75.$$

(b) Use axiom 3) of Definition A-6, and the fact that the events A and $(B \text{ and } D)$ are mutually exclusive. Assuming that the first grind is independent of the second,

$$\begin{aligned} P(A \text{ or } (B \text{ and } D)) &= P(A) + P(B)P(D) \\ &= .60 + (.25)(.80) = .80. \end{aligned}$$

(c) Use Definition (A-7).

$$\begin{aligned} & P(\text{ring is ground only once}|\text{ring meets specs}) \\ = \;& \frac{P(\text{ring is ground only once } \textit{and} \text{ ring meets specs})}{P(\text{ring meets specs})} \\ = \;& \frac{P(A)}{.80} \\ = \;& \frac{.60}{.80} = .75. \end{aligned}$$

(d) Yes, since the answers to parts (a) and (c) are the same (see Definition A-8).

(e) Any two of the events A, B, or C as defined above are mutually exclusive. They cannot happen together.

6. Define the events

$A =$ Diameter is greater than 1.005 in. and
$B =$ Brinell hardness is greater than 210.

(a) Use Proposition (A-3). $P(A) = \frac{242}{1000} = .242.$

(b) Use Proposition (A-3) again. $P(A \text{ and } B) = \frac{95}{1000} = .095.$

(c) Use Proposition (A-3) again. $P(A \text{ or } B) = \frac{347}{1000} = .347.$

(d) Use Definition A-7 and Proposition (A-3).

$$P(A|B) = \frac{.095}{\frac{200}{1000}} = .475.$$

(e) No, because the answers to parts (a) and (d) are not the same (see Definition A-8).

(f) Define $C =$ Diameter is less than 1.000 in. Then the events A and C are mutually exclusive, because they cannot happen together.

7. Define the events

$A =$ First widget is good,
$B =$ Second widget is good,
$C =$ First widget is defective,
$D =$ Second widget is defective,
$E =$ an additional 3 widgets are sampled, and
$F =$ no adjustment is made

(a) Assuming consecutive widgets are independent,

$$P(A \text{ and } B) = P(A)P(B) = (.8)(.8) = .64.$$

(See Proposition A-4 and Definition A-8.)

(b) Use axiom 3) of Definition A-6, and the fact that the events (A *and* B) and (C *or* D) are mutually exclusive.

$$\begin{aligned} P((A \text{ and } B) \text{ or } (C \text{ or } D)) &= P(A \text{ and } B) + P(C \text{ or } D) \\ &= .64 + P(C) + P(D) - P(C \text{ and } D) \\ &= .64 + .05 + .05 - (.05)(.05) \\ &= .7375. \end{aligned}$$

The second line above uses Proposition A-2, and the third line uses Proposition A-4 and Definition A-8.

(c) The event F can be written as the union of 2 mutually exclusive events:

$$F = (F \text{ and } A \text{ and } B) \text{ or } (F \text{ and } E).$$

Using Proposition A-4 and axiom 3) of Definition A-6,

$$P(F) = P(F|A \text{ and } B)P(A \text{ and } B) + P(F|E)P(E).$$

$P(F|A \text{ and } B) = 1$, since there will certainly be no adjustment if both of the initial widgets are good. $P(E)$ can be evaluated as

$$\begin{aligned} P(E) &= 1 - P(\textit{ not } E) \\ &= 1 - P((A \textit{ and } B) \textit{ or } (C \textit{ or } D)) \\ &= 1 - (P(A \textit{ and } B) + P(C \textit{ or } D) \\ &= 1 - (.64 + .05 + .05 - (.05)(.05)) \\ &= .2625. \end{aligned}$$

The first line above is based on Proposition A-1. The third line is based on axiom 3) of Definition A-6, and the fact that the events (A *and* B) and (C *or* D) are mutually exclusive. The fourth line is based on Proposition A-2.

The only thing left that needs to be computed is $P(F|E)$. Given that 3 additional widgets are sampled, no adjustment is made if all 3 are good, or if 2 are good and 1 is marginal. These two outcomes are mutually excusive, so

$$P(F|E) = P(\text{all 3 are good}) + P(\text{2 are good and 1 is marginal})$$

by axiom 3) of Definition A-6. Using independence,

$$P(\text{all 3 are good}) = (.8)(.8)(.8) = .512.$$

Also,

$$P(\text{2 are good and 1 is marginal}) = 3(.8)(.8)(.15) = .288,$$

because there are 3 ways in which this can happen, all having probability (.8)(.8)(.15). Finally,

$$P(F) = (1)(.64) + (.512 + .288)(.2625) = .85.$$

(d) Use Definition A-7.

$$P(\text{no adjustment is made}|\text{only 2 widgets are sampled}) = \frac{.64}{.7375} = .8678.$$

(e) No, because the answers to parts (c) and (d) are not the same (see Definition A-8).

(f) The events A and C as defined above are mutually exclusive, because they cannot happen together.

(8)(a) .675 = (2.1N + .6N)/4N, where N is the size of the smaller lot.
(b) .175 = (.6N + .1N)/4N, where N is the size of the smaller lot.
(c) .857 = Prob(Lot 1 given blemished(useable)) = [.6N/4N]/[.7N/4N] where N is the size of the smaller lot.

(9)(a) $(1-p)^2$
(b) $2(1-p)(p)$
(c) $(1-p)^2 / [(1-p)^2 + p^2] = P$
(d) Using "P" from (c), P^4 is the answer for part (d).

(10) (a) P[A and B and not C] = 0, P[A and not B and not C] = .1,
P[not (A or B or C)] = P[not A and not B and not C] = .1
(b) P[A and B] = .1
(c) P[B|C] = .4285
(d) No, P[B]P[C] = .28 does not equal P[B and C] = .3.

Section 2

(1) $k = 5$, $.999 = r^5$ or $r = .99979$.

(2) $.99 = 1 - (1-.9)^k$, solving for k gives $k = 2$.

(3)(a) A series of three sets of two A's in parallel. Cost = 6($8)=$48. Reliability is (.9996)(.9996)(.9996) = .998.
A series of three sets of three B's in parallel. Cost = 9($5) = $45. Reliability is (.999)(.999)(.999) = .997.

(b) A series of three sets of an A and B in parallel.
Cost = 3($8) + 3($5)=$39. The reliability is (.998)(.998)(.998)=.994.

Section 3

1. (a) Use equation (A-16) with $n = 100$ and $r = 10$.

$$\binom{100}{10} = 1.7310309 \times 10^{13}.$$

(b) Use equation (A-16) and Proposition A-5.

$$\binom{10}{1}\binom{20}{2}\binom{70}{7} = 2.2776719 \times 10^{12}.$$

(c) Use Proposition A-3.

$$\frac{\binom{10}{1}\binom{20}{2}\binom{70}{7}}{\binom{100}{10}} = .1316.$$

2. (a) Assuming that the bolts are independent, use Proposition (A-4) and Definition (A-8).

$$(.2)(.2)(.2)(.5)(.5)(.5)(.3)(.3)(.3)(.3) = .0000081.$$

(b) This is an application of axiom 3) of Definition A-6. Each of the mutually exclusive outcomes consisting of 3 short, 3 good, and 4 long have the same probability of occuring (the answer to part (a)). The total number of these outcomes be found using equation (A-16) and Proposition A-5:

$$\binom{10}{3}\binom{7}{3}\binom{4}{4} = 4200.$$

So .0000081 must be added 4200 times to get the desired probability

$$4200(.0000081) = .03402.$$

3. (a) Use Proposition (A-5).

$$(26)(26)(26)(10)(10) = 1,757,600.$$

(b) Use Proposition (A-5) and Proposition (A-3). The total number of ways of ordering the 3 letters is

$$P_{3,3} = 3! = 6.$$

The total number of ways of choosing the two numbers is $(10)(10) = 100$, so the total number of possible names consistent with Joe's memory is $(6)(100) = 600$. Since only one of these names is correct, the probability that he selects his own name is $\frac{1}{600} = .00167$.

(c) Now the total number of ways of choosing the numbers is 10, so the total number of possible names consistent with Joe's memory is $(6)(10) = 60$. Since only one of these names is correct, the probability that he selects his own name is $\frac{1}{60} = .0167$.

4. (a) Use equation (A-16), Proposition (A-5), and Proposition (A-3). The total number of ways of choosing 4 meters from the 10 is

$$\binom{10}{4} = 210$$

and the total number of ways of choosing one miscalibrated and 3 calibrated meters is

$$\binom{3}{1}\binom{7}{3} = 105$$

so the desired probability is $\frac{105}{210} = .5$.

(b) Use proposition A-4. Define the events

A = Exactly one miscalibrated meter is found in the first 4 checked and
B = The fifth meter checked is miscalibrated.

$$\begin{aligned} P(A \text{ and } B) &= P(B|A)P(A) \\ &= \frac{2}{6}(.5) = .1667. \end{aligned}$$

$P(B|A) = \frac{2}{6}$ because, given A, there are 6 meters left, 2 of which are miscalibrated, and one of these 6 will be chosen at random.

(5)(a) $.125 = (1/2)^3$
(b) $.72 = 1(9/10)(8/10)$
(c) $512/1000 = .512$

(6)(a) $72 = 4 \times 2 \times 3 \times 3$
(b) $8 = 2 \times 1 \times 2 \times 2$
(c) $36 = 8 + (4)(1)(1)(1) + (4)(1)(3)(2)$

Section 4

1. (a) See also equation (5-24) and just below it.

$$E(X+Y) = EX + EY = 15 + 5 = 20 \text{ hrs.},$$

and

$$\mathrm{Var}(X+Y) = \mathrm{Var}X + \mathrm{Var}Y = (15)^2 + (5)^2 = 250$$

so the standard deviation of $X+Y$ is $\sqrt{250} = 15.811$ hrs.

(b) This can be written as $P(Y \le t - X)$. The joint density is the product of the marginal densities because X and Y are independent (see equation (5-50)).

$$f(x,y) = \frac{1}{15}e^{-\frac{x}{15}}\frac{1}{5}e^{-\frac{y}{5}}$$

for x and y greater than zero. To find the desired probability, integrate $f(x,y)$ over the region where $y \le t - x$.

$$\begin{aligned} P(Y \le t - X) &= \int_0^t \int_0^{t-x} f(x,y)\,dy\,dx \\ &= 1 - \frac{3}{2}e^{-\frac{t}{15}} + \frac{1}{2}e^{-\frac{t}{5}} \end{aligned}$$

for $t > 0$.

(c) The answer to part (b) is the CDF of T. Using equation (5-17), the probability density of T is

$$f(t) = \frac{dF(t)}{dt} = \frac{1}{10}\left(e^{-\frac{t}{15}} - e^{-\frac{t}{5}}\right)$$

for $t > 0$.

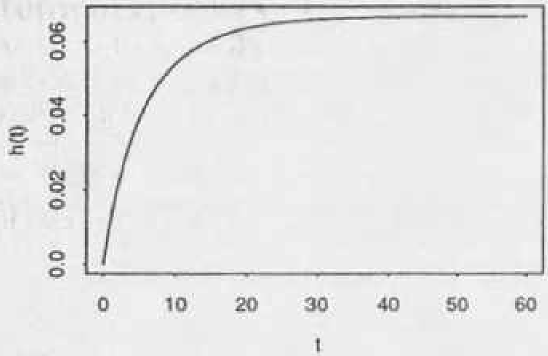
(d) From definition (A-13), the survivorship function is

$$S(t) = 1 - F(t) = \frac{3}{2}e^{-\frac{t}{15}} - \frac{1}{2}e^{-\frac{t}{5}}.$$

From definition (A-14), the force of mortality function is

$$\begin{aligned} h(t) &= \frac{f(t)}{S(t)} \\ &= \frac{\frac{1}{10}\left(e^{-\frac{t}{15}} - e^{-\frac{t}{5}}\right)}{\frac{1}{2}\left(3e^{-\frac{t}{15}} - e^{-\frac{t}{5}}\right)} \\ &= \frac{1}{5}\left(\frac{e^{-\frac{t}{15}} - e^{-\frac{t}{5}}}{3e^{-\frac{t}{15}} - e^{-\frac{t}{5}}}\right). \end{aligned}$$

The function is graphed below.



The graph shows that the force of mortality function is not constant. It starts at zero, and increases to an asymptote of $\frac{1}{15}$. For $t > 30$ hrs., the function is approximately constant.

(2)(e) h(t) is zero at t=0 and increases then eventually decreases. When the mean is 0 and sigma is 1, h(t) generally decreases (except for very small t). This is not good for in-service replacement policy.