Chapter 7 Formula Sheet

$$y_{ij} = \mu_i + \epsilon_{ij}$$

$$\bar{y}_i = \frac{1}{n_i}\sum_{j=1}^{n_i} y_{ij}$$

$$e_{ij} = y_{ij} - \hat{y}_{ij} = y_{ij} - \bar{y}_i$$

$$s_{\mathrm{P}}^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2 + \cdots + (n_r-1)s_r^2}{(n_1-1)+(n_2-1)+\cdots+(n_r-1)}$$

$n = \sum_{i=1}^{r} n_i =$the total number of observations in all r samples

$$\frac{(n-r)s_{\mathrm{P}}^2}{U} \qquad \text{and} \qquad \frac{(n-r)s_{\mathrm{P}}^2}{L}$$

$$e^* = \frac{e}{s\sqrt{a}} \quad \text{in general, for unstructured studies } e_{ij}^* = \frac{e_{ij}}{s_{\mathrm{P}}\sqrt{\dfrac{n_i-1}{n_i}}}$$

$$\bar{y}_i \pm t\frac{s_{\mathrm{P}}}{\sqrt{n_i}}$$

$$\bar{y}_i - \bar{y}_{i'} \pm t s_{\mathrm{P}}\sqrt{\frac{1}{n_i} + \frac{1}{n_{i'}}}$$

$$L = c_1\mu_1 + c_2\mu_2 + \cdots + c_r\mu_r \quad \text{and} \quad \hat{L} = c_1\bar{y}_1 + c_2\bar{y}_2 + \cdots + c_r\bar{y}_r$$

$$\hat{L} \pm t s_{\mathrm{P}}\sqrt{\frac{c_1^2}{n_1} + \frac{c_2^2}{n_2} + \cdots + \frac{c_r^2}{n_r}}$$

$$\gamma \geq 1 - \big((1-\gamma_1) + (1-\gamma_2) + \cdots + (1-\gamma_k)\big)$$

$$\bar{y}_i \pm k_2^* \frac{s_{\mathrm{P}}}{\sqrt{n_i}}$$

$$\left(-\infty, \bar{y}_i + k_1^* \frac{s_{\mathrm{P}}}{\sqrt{n_i}}\right) \quad \text{or} \quad \left(\bar{y}_i - k_1^* \frac{s_{\mathrm{P}}}{\sqrt{n_i}}, \infty\right)$$

$$\bar{y}_i - \bar{y}_{i'} \pm \frac{q^*}{\sqrt{2}} s_{\mathrm{P}}\sqrt{\frac{1}{n_i} + \frac{1}{n_{i'}}}$$

$$F = \frac{\dfrac{1}{r-1}\displaystyle\sum_{i=1}^{r} n_i(\bar{y}_i - \bar{y})^2}{s_{\mathrm{P}}^2}$$

$$(n-1)s^2 = \sum_{i=1}^{r} n_i(\bar{y}_i - \bar{y})^2 + (n-r)s_{\mathrm{P}}^2$$

$$SSTot = (n-1)s^2 = \textstyle\sum_{i,j}(y_{ij} - \bar{y})^2$$

$$SSE = (n-r)s_{\mathrm{P}}^2 = \sum_{i=1}^{r}\sum_{j=1}^{n_i}(y_{ij} - \bar{y}_i)^2$$

$$SSTr = \sum_{i=1}^{r} n_i(\bar{y}_i - \bar{y})^2 = SSTot - SSE$$

ANOVA TABLE (FOR TESTING $\mathrm{H}_0\colon \mu_1 = \mu_2 = \cdots = \mu_r$**)**

Source	SS	df	MS	F
Treatments	$SSTr$	$r-1$	$SSTr/(r-1)$	$MSTr/MSE$
Error	SSE	$n-r$	$SSE/(n-r)$	
Total	$SSTot$	$n-1$		

$$R^2 = \frac{SSTr}{SSTot}$$

$$\bar{y}_{\cdot} \pm t\sqrt{\frac{MSTr}{mr}}$$

$$\hat{\sigma}_\tau^2 = \max\left(0, \frac{1}{m}(MSTr - MSE)\right)$$

$$\frac{1}{m}\left(\frac{MSTr}{U \cdot MSE} - 1\right) \qquad \text{and} \qquad \frac{1}{m}\left(\frac{MSTr}{L \cdot MSE} - 1\right)$$

$$\bar{\bar{x}} = \frac{1}{r}\sum_{i=1}^{r}\bar{x}_i$$

$$LCL_{\bar{x}} = \mu - 3\frac{\sigma}{\sqrt{m}} \qquad \text{and} \qquad UCL_{\bar{x}} = \mu + 3\frac{\sigma}{\sqrt{m}}$$

$$\hat{\sigma} = \frac{\overline{R}}{d_2}$$

$$\hat{\sigma} = \frac{\bar{s}}{c_4}$$

$$LCL_{\bar{x}} = \bar{\bar{x}} - A_2\overline{R} \qquad \text{and} \qquad UCL_{\bar{x}} = \bar{\bar{x}} + A_2\overline{R}$$

$$LCL_{\bar{x}} = \bar{\bar{x}} - A_3\bar{s} \qquad \text{and} \qquad UCL_{\bar{x}} = \bar{\bar{x}} + A_3\bar{s}$$

$$LCL_R = D_1\sigma \qquad \text{and} \qquad UCL_R = D_2\sigma$$

$$LCL_R = D_3\overline{R} \quad \text{and} \qquad UCL_R = D_4\overline{R}$$

$$LCL_s = B_5\sigma \qquad \text{and} \qquad UCL_s = B_6\sigma$$

$$LCL_s = B_3\bar{s} \qquad \text{and} \qquad UCL_s = B_4\bar{s}$$

$$LCL_{\hat{p}_i} = p - 3\sqrt{\frac{p(1-p)}{n_i}} \qquad \text{and} \qquad UCL_{\hat{p}_i} = p + 3\sqrt{\frac{p(1-p)}{n_i}}$$

$$\hat{p} = \frac{n_1\hat{p}_1 + n_2\hat{p}_2 + \cdots + n_r\hat{p}_r}{n_1 + n_2 + \cdots + n_r}$$

$$LCL_{\hat{p}_i} = \hat{p} - 3\sqrt{\frac{\hat{p}(1-\hat{p})}{n_i}} \qquad \text{and} \qquad UCL_{\hat{p}_i} = \hat{p} + 3\sqrt{\frac{\hat{p}(1-\hat{p})}{n_i}}$$

$$LCL_{\hat{u}_i} = \lambda - 3\sqrt{\frac{\lambda}{k_i}} \qquad \text{and} \qquad UCL_{\hat{u}_i} = \lambda + 3\sqrt{\frac{\lambda}{k_i}}$$

$$\hat{\lambda} = \frac{k_1\hat{u}_1 + k_2\hat{u}_2 + \cdots + k_r\hat{u}_r}{k_1 + k_2 + \cdots + k_r}$$

$$LCL_{\hat{u}_i} = \hat{\lambda} - 3\sqrt{\frac{\hat{\lambda}}{k_i}} \qquad \text{and} \qquad UCL_{\hat{u}_i} = \hat{\lambda} + 3\sqrt{\frac{\hat{\lambda}}{k_i}}$$