Chapter 5 Formula Sheet

$f(x) = P[X = x]$

$F(x) = P[X \le x]$ (general) $F(x) = \sum_{z \le x} f(z)$ (discrete)

$EX = \sum_x x\, f(x)$ and $\operatorname{Var} X = \sum (x - EX)^2 f(x) \quad \left(= \sum x^2 f(x) - (EX)^2\right)$

Binomial $f(x) = \begin{cases} \dfrac{n!}{x!\,(n-x)!}\, p^x (1-p)^{n-x} & \text{for } x = 0, 1, \ldots, n \\ 0 & \text{otherwise} \end{cases}$

with $EX = np$ and $\operatorname{Var} X = np(1-p)$

Geometric $f(x) = \begin{cases} p(1-p)^{x-1} & \text{for } x = 1, 2, \ldots \\ 0 & \text{otherwise} \end{cases}$

with $1 - F(x) = (1-p)^x$, $EX = \dfrac{1}{p}$ and $\operatorname{Var} X = \dfrac{1-p}{p^2}$

Poisson $f(x) = \begin{cases} \dfrac{e^{-\lambda}\lambda^x}{x!} & \text{for } x = 0, 1, 2, \ldots \\ 0 & \text{otherwise} \end{cases}$ with $EX = \lambda$

and $\operatorname{Var} X = \lambda$

$$P[a \le X \le b] = \int_a^b f(x)\, dx$$

$$F(x) = P[X \le x] = \int_{-\infty}^x f(t)\, dt$$

$$\frac{d}{dx} F(x) = f(x)$$

$$EX = \int_{-\infty}^{\infty} x\, f(x)\, dx$$

$$\operatorname{Var} X = \int_{-\infty}^{\infty} (x - EX)^2 f(x)\, dx \qquad \left(= \int_{-\infty}^{\infty} x^2 f(x)\, dx - (EX)^2\right)$$

Normal $f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-\mu)^2/2\sigma^2}$ for all x

with $\Phi(z) = F(z) = \int_{-\infty}^z \frac{1}{\sqrt{2\pi}} e^{-t^2/2}\, dt$ and $z = \dfrac{x - \mu}{\sigma}$

Exponential $f(x) = \begin{cases} \frac{1}{\alpha} e^{-x/\alpha} & \text{for } x > 0 \\ 0 & \text{otherwise} \end{cases}$ with $EX = \alpha$, $\operatorname{Var} X = \alpha^2$,

and $F(x) = \begin{cases} 0 & \text{if } x \le 0 \\ 1 - e^{-x/\alpha} & \text{if } x > 0 \end{cases}$

Weibull $F(x) = \begin{cases} 0 & \text{if } x < 0 \\ 1 - e^{-(x/\alpha)^\beta} & \text{if } x \ge 0 \end{cases}$

with $f(x) = \begin{cases} 0 & \text{if } x < 0 \\ \frac{\beta}{\alpha^\beta} x^{\beta-1} e^{-(x/\alpha)^\beta} & \text{if } x > 0 \end{cases}$, $EX = \alpha\Gamma\left(1 + \frac{1}{\beta}\right)$,

$\operatorname{Var} X = \alpha^2 \left[\Gamma\left(1 + \frac{2}{\beta}\right) - \left(\Gamma\left(1 + \frac{1}{\beta}\right)\right)^2\right]$, and $Q(.5) = \alpha e^{-(.3665/\beta)}$

Probability plot $\left(x_i, Q\left(\frac{i-.5}{n}\right)\right)$ for an ordered data set

Normal plot $\left(x_i, Q_z\left(\frac{i-.5}{n}\right)\right)$

Exponential plot $\left(x_i, -\ln\left(1 - \frac{i-.5}{n}\right)\right)$

Weibull plot $\left(\ln x_i, \ln\left(-\ln\left(1 - \frac{i-.5}{n}\right)\right)\right)$

$f(x, y) = P[X = x \text{ and } Y = y]$

$f_X(x) = \sum_y f(x, y)$ and $f_Y(y) = \sum_x f(x, y)$

$f_{X|Y}(x \mid y) = \frac{f(x, y)}{f_Y(y)}$ and $f_{Y|X}(y \mid x) = \frac{f(x, y)}{f_X(x)}$

Independence $f(x, y) = f_X(x)\, f_Y(y)$ for all x, y

$P[(X, Y) \in \mathcal{R}] = \int\int_{\mathcal{R}} f(x, y)\, dx\, dy$

$f_X(x) = \int_{-\infty}^{\infty} f(x, y)\, dy$ and $f_Y(y) = \int_{-\infty}^{\infty} f(x, y)\, dx$

$f_{X|Y}(x \mid y) = \frac{f(x, y)}{f_Y(y)}$ and $f_{Y|X}(y \mid x) = \frac{f(x, y)}{f_X(x)}$

Independence $f(x, y) = f_X(x)\, f_Y(y)$ for all x, y

For $U = a_0 + a_1 X + a_2 Y + \cdots + a_n Z$, $EU = a_0 + a_1 EX + a_2 EY + \cdots + a_n EZ$

and $\operatorname{Var} U = a_1^2 \operatorname{Var} X + a_2^2 \operatorname{Var} Y + \cdots + a_n^2 \operatorname{Var} Z$

$E\overline{X} = \mu$ and $\operatorname{Var} \overline{X} = \dfrac{\sigma^2}{n}$

For $U = g(X, Y, \ldots, Z)$, $EU \approx g(EX, EY, \ldots, EZ)$

and $\operatorname{Var} U \approx \left(\dfrac{\partial g}{\partial x}\right)^2 \operatorname{Var} X + \left(\dfrac{\partial g}{\partial y}\right)^2 \operatorname{Var} Y + \cdots + \left(\dfrac{\partial g}{\partial z}\right)^2 \operatorname{Var} Z$

$$z = \frac{\bar{x} - E\overline{X}}{\sqrt{\operatorname{Var} \overline{X}}} = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$