Chapter 9 Formula Sheet

$$y_i = \beta_0 + \beta_1 x_i + \epsilon_i$$

$$s_{\text{LF}}^2 = \frac{1}{n-2}\sum(y-\hat{y})^2 = \frac{1}{n-2}\sum e^2$$

$$e_i^* = \frac{e_i}{s_{\text{LF}}\sqrt{1-\frac{1}{n}-\frac{(x_i-\bar{x})^2}{\sum(x-\bar{x})^2}}}$$

$$T = \frac{b_1 - \#}{\frac{s_{\text{LF}}}{\sqrt{\sum(x-\bar{x})^2}}}$$

$$b_1 \pm t\frac{s_{\text{LF}}}{\sqrt{\sum(x-\bar{x})^2}}$$

$$\hat{y} = b_0 + b_1 x$$

$$T = \frac{\hat{y} - \#}{s_{\text{LF}}\sqrt{\frac{1}{n}+\frac{(x-\bar{x})^2}{\sum(x-\bar{x})^2}}}$$

$$\hat{y} \pm t s_{\text{LF}}\sqrt{\frac{1}{n}+\frac{(x-\bar{x})^2}{\sum(x-\bar{x})^2}}$$

$$(b_0 + b_1 x) \pm \sqrt{2f}\, s_{\text{LF}}\sqrt{\frac{1}{n}+\frac{(x-\bar{x})^2}{\sum(x-\bar{x})^2}}$$

$$\hat{y} \pm t s_{\text{LF}}\sqrt{1+\frac{1}{n}+\frac{(x-\bar{x})^2}{\sum(x-\bar{x})^2}}$$

$(\hat{y} - \tau s_{\text{LF}}, \infty)$ or $(-\infty, \hat{y} + \tau s_{\text{LF}})$ for

$$\tau = \frac{Q_z(p) + AQ_z(\gamma)\sqrt{1+\frac{1}{2(n-2)}\left(\frac{Q_z^2(p)}{A^2} - Q_z^2(\gamma)\right)}}{1-\frac{Q_z^2(\gamma)}{2(n-2)}} \quad \text{where}$$

$$A = \sqrt{\frac{1}{n} + \frac{(x-\bar{x})^2}{\sum(x-\bar{x})^2}}$$

$SSR = SSTot - SSE = \sum_{i=1}^{n}(\hat{y}_i - \bar{y})^2$

$$F = \frac{SSR/1}{s_{\mathrm{LF}}^2} = \frac{SSR/1}{SSE/(n-2)}$$

ANOVA TABLE (FOR TESTING H_0: $\beta_1 = 0$)

Source	SS	df	MS	F
Regression	SSR	1	$SSR/1$	MSR/MSE
Error	SSE	$n-2$	$SSE/(n-2)$	
Total	$SSTot$	$n-1$		

$y_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \cdots + \beta_k x_{ki} + \epsilon_i$

$\hat{y}_i = b_0 + b_1 x_{1i} + b_2 x_{2i} + \cdots + b_k x_{ki}$

$e_i = y_i - \hat{y}_i$

$$s_{\mathrm{SF}}^2 = \frac{1}{n-k-1}\sum(y-\hat{y})^2 = \frac{1}{n-k-1}\sum e^2$$

$$e_i^* = \frac{e_i}{s_{\mathrm{SF}}\sqrt{a_i}}$$

$$T = \frac{b_l - \#}{s_{\mathrm{SF}}\sqrt{d_l}}$$

$b_l \pm t s_{\mathrm{SF}}\sqrt{d_l}$

$T = \dfrac{\hat{y} - \#}{s_{\mathrm{SF}} \cdot A}$ where $\hat{\sigma}_{\hat{y}} = s_{\mathrm{SF}} \cdot A$ comes from a printout

$\hat{y} \pm t s_{\mathrm{SF}} \cdot A$

$\hat{y} \pm \sqrt{(k+1)\,f}\, s_{\mathrm{SF}} \cdot A$

$\hat{y} \pm t\sqrt{s_{\mathrm{SF}}^2 + (s_{\mathrm{SF}} \cdot A)^2}$

$(\hat{y} - \tau s_{\mathrm{SF}}, \infty)$ or $(-\infty, \hat{y} + \tau s_{\mathrm{SF}})$ for

$$\tau = \frac{Q_z(p) + AQ_z(\gamma)\sqrt{1 + \frac{1}{2(n-k-1)}\left(\frac{Q_z^2(p)}{A^2} - Q_z^2(\gamma)\right)}}{1 - \frac{Q_z^2(\gamma)}{2(n-k-1)}}$$

$$F = \frac{SSR/k}{SSE/(n-k-1)} = \frac{R^2/k}{(1-R^2)/(n-k-1)}$$

ANOVA TABLE (FOR TESTING H_0: $\beta_1 = \beta_2 = \cdots = \beta_k = 0$)

Source	SS	df	MS	F
Regression	SSR	k	SSR/k	MSR/MSE
Error	SSE	$n-k-1$	$SSE/(n-k-1)$	
Total	$SSTot$	$n-1$		

$$F = \frac{(SSR_{\mathrm{f}} - SSR_{\mathrm{r}})/p}{SSE_{\mathrm{f}}/(n-k-1)} = \frac{(R^2_{\mathrm{f}} - R^2_{\mathrm{r}})/p}{(1-R^2_{\mathrm{f}})/(n-k-1)}$$

with $\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_k \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} b_{11} & \frac{1}{2}b_{12} & \cdots & \frac{1}{2}b_{1k} \\ \frac{1}{2}b_{12} & b_{22} & \cdots & \frac{1}{2}b_{2k} \\ \vdots & \vdots & & \vdots \\ \frac{1}{2}b_{1k} & \frac{1}{2}b_{2k} & \cdots & b_{kk} \end{bmatrix}$ $\mathbf{x} = -\frac{1}{2}\mathbf{B}^{-1}\mathbf{b}$

$\det(\mathbf{B} - \lambda\mathbf{I}) = 0$

$$x_i^{\mathrm{A}} = \begin{cases} 1 & \text{if the response } y \text{ is from level } i \text{ of A} \\ -1 & \text{if the response } y \text{ is from level } I \text{ of A} \\ 0 & \text{otherwise} \end{cases} \quad \text{and}$$

$$x_j^{\mathrm{B}} = \begin{cases} 1 & \text{if the response } y \text{ is from level } j \text{ of B} \\ -1 & \text{if the response } y \text{ is from level } J \text{ of B} \\ 0 & \text{otherwise} \end{cases}$$

$\vdots$

For 2^p studies and the "all high" effects use:

$$x_2^{\mathrm{A}} = -x_1^{\mathrm{A}} = \begin{cases} 1 & \text{if the response } y \text{ is from the high level of A} \\ -1 & \text{if the response } y \text{ is from the low level of A} \end{cases}$$

$$x_2^{\mathrm{B}} = -x_1^{\mathrm{B}} = \begin{cases} 1 & \text{if the response } y \text{ is from the high level of B} \\ -1 & \text{if the response } y \text{ is from the low level of B} \end{cases}$$

$\vdots$