3.

Some Basic Theory of Resource Structure

WE HAVE examined changes taking place in the organization of resources and output in U.S. agriculture. Quantities and prices of factors and products are determined by parameters such as supply, demand and production elasticities in the resource structure. If we are to understand commodity supply and resource returns, we must know the conditions of resource demand and supply for the industry. The organization of the industry, in sizes and numbers of farms and in the amounts and proportions of resources used, rests heavily on resource demand and supply functions. This chapter will present some elementary but important theory of resource structure.

THE THEORY OF RESOURCE DEMAND

The static theory of the competitive firm is a useful starting point for construction of a structural model since: (a) in some respects agriculture is best represented by the purely competitive market structure, and (b) the firm is a logical beginning point for analysis of more general, dynamic market phenomena. We begin with the assumptions that the decision maker maximizes profits in an environment of known input/output and price ratios, instantaneous adjustments, divisibility of commodities (inputs or outputs) and unlimited capital. Furthermore, prices are given; individual decisions are assumed to have no influence on price under these competitive conditions.

For purposes of brevity and simplicity in presentation, we suppose that the factor demand and commodity supply functions for the industry are simply the summation of those for m firms. Hence, with X_{ik} being use of the i-th resource by the k-th firm and Y_k being the output of the k-th firm, we have the total employment of the i-th resource in (3.1) and the total output in (3.2).

(3.1) $$X_i = \sum_{k=1}^{m} X_{ik} \quad (i = 1,2, \ldots n)$$

(3.2) $$Y = \sum_{k=1}^{m} Y_k$$

We illustrate our discussion with factor demand conditions for the firm, but will not carry the subscript k. The static framework is an oversimplification of resource demand relationships for agriculture or any other industry. Space, however, restrains presentation of institutional and economic details relating to intricacies of supply and demand. Later in this chapter we summarize some elementary dynamic models, and in later empirical chapters we employ dynamic models which are major deviations from the simple ones presented in this chapter. These deviations are employed in an attempt at practical and realistic quantitative estimation of resource demand and supply relationships, in conformity with the time series observations available and limitations of regression models applied to these data. One of our ultimate interests is to measure the quantities defining the elasticity of demand of the i-th resource with respect to its own price and the cross elasticities of demand of this resource with respect to prices of the j-th factor and the product. Since these elasticities vary with time and the decision environment, they must be related eventually to dynamic models.

Profit Maximization and Resource Demand

The production function is (3.3) where $X_1, X_2, \ldots X_n$ are resources used in the production of output Y. From the production function, profit π can be defined in (3.4) as gross revenue, the magnitude of output Y multiplied by product price P_y, less the sum of costs. Costs are defined as the sum of resource prices P_i multiplied by resource quantities X_i.

$$(3.3) \qquad Y = f(X_1, X_2, \ldots X_n)$$

$$(3.4) \qquad \pi = f(X_1, X_2, \ldots X_n)P_y - \sum_{i=1}^{n} P_i X_i$$

Profit is maximized when all resources are used at levels such that their net marginal return is zero: use of more of the i-th resource would increase costs by a greater absolute amount than gross revenue. Hence, the conditions of profit maximization are defined in (3.5) by setting the partial derivatives of profit with respect to each resource equal to zero.[1] Alternative specifications of profit maximization, derived from equations (3.5), are presented in equations (3.6) to (3.9). Equation (3.6), found by shifting factor price P_i to the right side of equations (3.5), is the value of marginal product equated to factor price. If (3.6) is divided by product price P_y, the profit maximizing condition is defined as the marginal product equated to the inverse

[1] Cf. Heady, Earl O. Economics of Agricultural Production and Resource Use. Prentice-Hall, Inc. New York. 1952. pp. 1-200.

price ratio (3.7). The ratio of (3.7) for two resources, X_i and X_j, gives (3.8), which is the marginal rate of substitution of resource X_j for X_i equated to the inverse price ratio of the two resources. Equation (3.9) is a generalization of equations (3.5) when n resources are used to produce product Z in addition to Y. In static equilibrium, the value of marginal product of each resource must equal its price. Furthermore, (3.9) indicates that the marginal value product of a given resource must be equal for any product Z as well as for Y. A departure from these conditions must necessarily reduce profits.

$$\text{(3.5a)} \qquad \frac{\partial \pi}{\partial X_1} = \frac{\partial Y}{\partial X_1} P_y - P_1 = 0$$

$$\text{(3.5b)} \qquad \frac{\partial \pi}{\partial X_2} = \frac{\partial Y}{\partial X_2} P_y - P_2 = 0$$

$$\vdots$$

$$\text{(3.5c)} \qquad \frac{\partial \pi}{\partial X_n} = \frac{\partial Y}{\partial X_n} P_y - P_n = 0$$

$$\text{(3.6)} \qquad \frac{\partial Y}{\partial X_i} P_y = P_i$$

$$\text{(3.7)} \qquad \frac{\partial Y}{\partial X_i} = \frac{P_i}{P_y}$$

$$\text{(3.8)} \qquad -\frac{dX_i}{dX_j} = \frac{P_j}{P_i}$$

$$\text{(3.9)} \qquad \left(\frac{\partial Y}{\partial X_1} P_y\right)/P_1 = \left(\frac{\partial Y}{\partial X_2} P_y\right)/P_2 \ldots = \left(\frac{\partial Z}{\partial X_1} P_z\right)/P_1$$

$$= \left(\frac{\partial Z}{\partial X_2} P_z\right)/P_2 \ldots = \left(\frac{\partial Y}{\partial X_n} P_y\right)/P_n = 1$$

We have outlined the quantities which define the demand for resources where capital is unlimited. The magnitude of input of each factor depends on the technical coefficients in the production function (3.3) and the magnitude of prices for resources and products. Any change which increases the marginal rate of transformation of the i-th resource relative to the j-th resource, or to the product, will increase the demand for the first resource. A decrease in the price of the factor or an increase in the price of the product will increase the demand quantity of the factor, while an increase in resource price or decrease in product price will reduce the quantity.

In a static framework of perfect knowledge, capital would not be limited and the resource magnitudes, found by solving for X_i in (3.5), would specify the firm's demand for and use of resources. In agriculture the size of the farm in acres, as measured by a particular X_i, would be so specified. The amount and relative proportion of labor and capital, or particular capital items, would be similarly specified. However, without transition to dynamic and uncertainty models, we can specify the level of factor demand where capital is limited. (Presumably, capital is limited only under uncertainty.) Suppose that the firm has a given amount of funds, K, to spend on or invest in resources. Profit can be maximized and factors can be purchased or hired only under the restraint that total outlay does not exceed K. The profit equation then is redefined in (3.10) where λ is a Lagrange multiplier and the condition $\lambda(K - \sum_{i=1}^{n} P_i X_i)$ is used to restrain resource use so that expenditure does not exceed K. The magnitude of $\sum_{i=1}^{n} P_i X_i$ cannot exceed K, or the difference is set to equal zero, in the steps which follow. The partial derivatives of π with respect to X_i and λ for (3.10) are then set to equal zero as in (3.11).

$$\pi = f(X_1, X_2, \ldots X_n)\, P_y - \sum_{i=1}^{n} P_i X_i + \lambda\left(K - \sum_{i=1}^{n} P_i X_i\right) \tag{3.10}$$

The equations in (3.11) are solved for the value of the X_i and λ. The magnitude of any resource quantity then depends on the technical relationships in production (3.3), the prices of factors P_i and P_j, the price of the product P_y, and on the amount of funds K, available for investment. Dividing (3.11a) by P_1 and transposing $1 + \lambda$ to the right side of the equation, (3.12a) is formed. Equations (3.12) indicate that maximum profit is obtained when the ratio of the value of marginal product to the resource price is equal to $1 + \lambda$ for all resources. The condition, summarized in equation (3.13), indicates that λ is the rate of return on resource expenditures. Comparing equations (3.9) and (3.13) it is apparent that when capital is unlimited the rate of return λ is zero. As K becomes smaller the value of λ rises. A decline in the price ratio $P_i P_y^{-1}$, which increases income and equity of the firm and allows a larger K either from owned assets or from a larger borrowing base, will affect the quantities and combination of resources used.

$$\frac{\partial \pi}{\partial X_1} = \frac{\partial Y}{\partial X_1} P_y - P_1 - \lambda P_1 = 0 \tag{3.11a}$$

$$\frac{\partial \pi}{\partial X_2} = \frac{\partial Y}{\partial X_2} P_y - P_2 - \lambda P_2 = 0 \tag{3.11b}$$

$$\vdots$$

$$\frac{\partial \pi}{\partial X_n} = \frac{\partial Y}{\partial X_n} P_y - P_n - \lambda P_n = 0 \tag{3.11c}$$

$$\left(\frac{\partial Y}{\partial X_1} P_y\right)/P_1 = 1 + \lambda \tag{3.12a}$$

$$\left(\frac{\partial Y}{\partial X_2} P_y\right)/P_2 = 1 + \lambda \tag{3.12b}$$

$$\vdots$$

$$\left(\frac{\partial Y}{\partial X_n} P_y\right)/P_n = 1 + \lambda \tag{3.12c}$$

$$\left(\frac{\partial Y}{\partial X_1} P_y\right)/P_1 = \left(\frac{\partial Y}{\partial X_2} P_y\right)/P_2 = \ldots \left(\frac{\partial Y}{\partial X_n} P_y\right)/P_n = 1 + \lambda \tag{3.13}$$

The above relations specify the first-order conditions required for profit-maximizing use of resources. In specifying the quantities of resources, output also is specified through the production function in (3.3). These conditions and relationships are directly important and relevant for the firm and the agricultural industry in respect to capital items of biological nature, such as fertilizer, seed and insecticides. The conditions also suggest the size and number of farms in the sense that X_i represents the land resource. For the individual firm, resources and organization are subject to considerable changes in the long run and it is relevant to specify second-order conditions for profit maximization. The second-order conditions are especially important for specifying the size of the firm and, hence, the number of firms in the industry, even though the industry is based on relatively fixed input of a resource such as land. Setting n = 2, to simplify the presentation, the second-order conditions require that the second partial derivative of profit with respect to inputs for (3.5) is negative and in general that the principal minors of the corresponding Hessian determinant alternate in sign:[2]

$$\frac{\partial^2 \pi}{\partial X_1^2} < 0, \quad \text{and} \quad \begin{vmatrix} \dfrac{\partial^2 \pi}{\partial X_1^2} & \dfrac{\partial^2 \pi}{\partial X_1\, \partial X_2} \\[2ex] \dfrac{\partial^2 \pi}{\partial X_1\, \partial X_2} & \dfrac{\partial^2 \pi}{\partial X_2^2} \end{vmatrix} > 0 \tag{3.14}$$

Expanding the second determinant of (3.14), we have:

$$\frac{\partial^2 \pi}{\partial X_1^2}\frac{\partial^2 \pi}{\partial X_2^2} - \left(\frac{\partial^2 \pi}{\partial X_1\, \partial X_2}\right)^2 > 0 \tag{3.15}$$

to guarantee that π, profit, is decreasing with use of more of any single factor.

[2]For complete second-order conditions for profit maximization see Hicks, J. R. Value and Capital. Oxford University Press. London. 1946. p. 320.

First-order conditions such as those in (3.5) and second-order conditions such as those in (3.14) are expected to have particular relevance in respect to the agricultural firm. In the first place, capital typically is limited. The individual farm accumulates capital during its life and thus can extend the expenditure and investment restraint. Enlargement of the restraint, K, is, for most production functions, likely to cause resource proportions to change along with output as the capital amount, K, in (3.10), grows, even where product and factor prices remain constant and the existing technology prevails. The proportional use of factors will change as K is increased, as long as the isocline $X_i = f_{ij}(P_i^{-1} P_j, X_j)$ is not linear through the origin of the input plane.[3] In a period such as 1940-55 when farm savings and assets grew rapidly and lifted the effective magnitude of K, we would expect the proportions as well as magnitudes of resources to change, even in the absence of new technical knowledge and change in price relatives. The isocline is not linear through the origin for farm resources, and a major change in the combination of resources did take place in the postwar period. A part of this change undoubtedly stemmed from a lifting of the capital restraint. At the same time, of course, prices of factors were changing relative to each other, changing the proportions of factors (3.8). Similarly, with commodity prices generally rising relative to factor prices in the war and immediate postwar years, increased input of resources are expected through (3.7). If the rise in productivity of a factor, X_i, is sufficiently large, inputs of X_i may increase even in the face of rising factor price P_i. An increase in P_y relative to the P_i is expected to change the proportions in which resources are used, as well as their amount, as the general price ratio $P_i P_y^{-1}$ declines, so long as the isoclines are not linear. Equally important, technology or the production function has changed over time to alter the dX_i/dX_j, causing factors to be substituted for each other in a manner suggested elsewhere in this study. Decrease in the $P_i P_j^{-1}$ price ratio, through a decline in P_i, is expected to have two effects: a substitution effect, X_i replacing some X_j as suggested in (3.8) and an expansion effect, with the magnitude of $\Sigma P_i X_i$ in (3.10) being lowered relative to K and the values of the X_i for (3.11) being larger.

When the productivity of a given resource is influenced strongly by the level of a second resource, the second-order condition (3.14) is particularly relevant. Although $d\pi/dX_i = 0$ for a given type and stock of machinery and cropland on a particular farm, it may be possible to increase profit by increasing farm size. Larger machines, with great labor replacement capacity, have given rise to increased productivity and profitability of machine investment. These conditions can prevail, of course, only if land input is extended to allow realization of the improved productivity of larger machines. Despite the increase in land price, it has been necessary for farms to extend land input if the joint

[3]Cf. Heady, Earl O., and Dillon, John L. Agricultural Production Functions. Iowa State University Press. Ames. 1961. Chaps. 1-4.

effect of greater use of machines and land is to be reflected and the conditions of (3.14) are to be approached or attained. Later empirical analysis shows that the demand for larger acreages to obtain cost economies is important in explaining the rise of land prices.

The Implicit Resource Demand Function

The demand functions for resources are found by solving the "equilibrium" equations (3.5) for X_i. The implicit demand function for the i-th resource (3.16) may be expressed as a function of the technical conditions of production, the factor/product price ratios and the level of fixed factors of production X_k.

$$X_i = f\left(\frac{P_i}{P_y}, \frac{P_j}{P_y}, X_k\right) \tag{3.16}$$

Prices, P_j, of <u>variable</u> resources are included in the demand function but quantities X_k of <u>fixed</u> inputs are included. Only prices or quantities of resources which interact with X_i are included in the demand function. The equations in (3.5) must be solved simultaneously for the X_i if interaction is present. If resources are independent in production, each equation (e.g., 3.6) can be solved individually for X_i and the resulting static demand function contains neither the price nor quantity of the unrelated j-th resource. With modifications for time lags and other real world conditions, (3.16) is the general basis for many of the empirical models of factor demand in this study.

It is generally agreed that farm commodity supply functions have low response to changing product prices in the short run, the response being lower in periods when commodity prices fall than when they rise.[4] Commodity supply response depends ultimately on factor quantities X_i; thus one reason for low commodity supply elasticity can be discussed in terms of equation (3.16). If X_i is supplied from nonfarm sources, factor price P_i is likely to be very stable and a rise in product price P_y would reduce the ratio P_i/P_y. Hence, X_i and output are reduced. But many factor prices are flexible in the short run and may have an imputed rather than a given, set price. The flexible price may be a function of product price, and consequently the two prices are highly correlated. The result tends to be a stable factor/product price ratio and input quantity despite changes in product price.

Small year-to-year variation in X_i or product could prevail where the prices of resources are flexible, with their movement being highly parallel or positively correlated with farm commodity prices.[5] Many factor prices are flexible in the short run. Examples are land and

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[4] Barker, R. L. The Response of Milk Production to Price: A Regional Analysis. Unpublished Ph.D. thesis. Library, Iowa State University. Ames. 1960.

[5] Heady, <u>op. cit.</u>, Chap. 23, and Johnson, D. Gale. The nature of the supply function for agricultural products. American Economic Review. 40:539-64. 1950.

buildings rented on a share basis and feed and livestock prices (with some lag related to the decision and production period). Resources or resource services with such flexible short-run prices are those which are produced, or have their origin, in the industry. Under these conditions, the commodity supply function can have high elasticity, but output will fluctuate little because of the conditions of factor pricing. But why are these factor prices so flexible? Generally because the supply functions, to be discussed later, of the resources themselves have low price elasticity.

Some controversy exists over the appropriateness of price ratios in empirical demand studies. Static theory (3.16) suggests the use of price ratios; dynamic economic theory raises doubts about the appropriateness of such forms. Farmers must make decisions of how much X_i to use on the basis of expected rather than actual product prices because of the length of the farm production period. The expected or normal price is a subjective estimate made by farmers on the basis of the permanent and transitory components of current and past prices. These components are of a different nature in output and input prices. It can be argued that the permanent component, the component upon which decisions tend to be based, is a much greater proportion of input price than of output price. When production plans are made, considerable uncertainty may exist about output price due to the time lag in production. Planning the level of use, purchasing and applying inputs are nearly concurrent acts, hence there need be little uncertainty about input prices. Also, the historic stability of input prices tends to create a large permanent component relative to the transitory component of input prices. The symmetric nature of price ratios implies that if output and input prices increase or decrease by the same proportion, the demand quantity remains unchanged. However, if farmers make decisions on the basis of the "permanent" component of price changes, a proportional increase in actual output and input prices could be expected to decrease the demand quantity since the permanent component of input prices is greater. For these reasons the use of price ratios in dynamic models does not appear justified in all cases.

Price ratios have certain advantages in statistical time series applications: (a) avoidance of errors from use of general price deflators (e.g., the wholesale price index), (b) reduction of multicollinearity and (c) increased degrees of freedom. Although use of price ratios is not strictly correct from a logical standpoint, the advantages may justify the use of ratios if the errors are not large. The results of empirical studies to date provide conflicting support for the hypothesis suggested by static theory that the price ratio is the decision variable used by farmers. The decision to use price ratios depends on the circumstances. If the sacrifice in higher intercorrelations, loss of degrees of freedom and errors from general deflators is considered less than forcing a symmetric response to input and output prices, the separate input and output price variables should be included in regression estimates.

Specific Forms of Resource Demand Functions

To provide a more specific model of resource demand variables and conditions, we now use a particular algebraic form. A Cobb-Douglas production function is selected for these illustrations, not since it typifies agriculture but because it minimizes space for presentation and algebraic manipulations.[6] Some conclusions drawn from it apply to other algebraic forms.

The production function of concern is (3.17) where the variables have the meaning specified earlier and n = 2. The corresponding marginal rate of substitution is (3.18) and the isocline equation derived from the latter is (3.19). (For the Cobb-Douglas function, the isocline is linear, the proportion of resources remaining fixed as more are used with a rise in P_y relative to the P_i and P_1 and P_2 remaining in fixed ratio. This condition does not necessarily prevail for other algebraic forms.)

$$Y = aX_1^{b_1}X_2^{b_2} \tag{3.17}$$

$$\frac{dX_1}{dX_2} = -\frac{b_2X_1}{b_1X_2} \tag{3.18}$$

$$X_1 = b_1b_2^{-1}P_1^{-1}P_2X_2 \tag{3.19}$$

With X_1 in (3.19) defined as a function of the technical coefficients, the prices of factors and X_2, the production function can be redefined as in (3.20). Since (3.19) defines the optimum or least-cost combination of the two resources, (3.20) defines output as a function of X_2 when resources are always so combined.[7]

$$Y = ab_1^{b_1}b_2^{-b_1}P_1^{-b_1}P_2^{b_1}X_2^{b_1+b_2} \tag{3.20}$$

Multiplying (3.20) by P_y, the price of product, to define the total value product, TVP, the marginal value product of the resource is defined as the derivative of TVP with respect to X_2 in (3.21).

$$\frac{d(TVP)}{dX_2} = (b_1 + b_2)\, ab_1^{b_1}b_2^{-b_1}P_1^{-b_1}P_2^{b_1}X_2^{b_1+b_2-1}P_y \tag{3.21}$$

Setting (3.21) to equal the factor price, P_2, to specify the profit maximizing use of the resource, and dividing both sides of the equation by P_2 and $X_2^{b_1+b_2-1}$, the static factor demand function is derived in (3.22). It specifies demand quantity for the resource as a function of the

[6] Heady and Dillon, op. cit., Chaps. 2-4.

[7] The steps employed here to derive factor demand are convenient for a two-variable production function. While they could be repeated for more variables, a more appropriate approach might be to solve the equations in (3.5) simultaneously for X_i. Insert these expressions $f(P_i/P_y)$ into the production function to form the product supply function.

technical coefficients of production, the prices of the resources and the price of the product. In general, an increase in the price of the particular resource will lower its use. Increase in the price of the product will increase demand quantity for the resource. The demand function (3.23) for X_2 variable, X_1 fixed, is derived by equating the marginal product dY/dX_2 from (3.17) to the price ratio $P_2 P_y^{-1}$ and solving for X_2. Note that when X_1 is variable (3.22), the price ratio $P_1 P_y^{-1}$ is included, but when X_1 is fixed (3.23), the quantity is included in the demand function.

$$(3.22) \qquad X_2 = \left[(b_1 + b_2)\, a b_1^{b_1}\, b_2^{-b_1}\, P_1^{-b_1}\, P_2^{b_1 - 1}\, P_y\right]^{\frac{1}{1 - b_1 - b_2}}$$

$$(3.23) \qquad X_2 = (a b_2 X_1^{b_1} P_2^{-1} P_y)^{\frac{1}{1 - b_2}}$$

From the static resource demand function in (3.22), the elasticities in respect to price may be derived. The price elasticities of resource demand indicate the percentage change in use of the factor associated with a 1 percent change in a particular price.

$$(3.24) \qquad e_{2,2} = \frac{dX_2}{dP_2}\frac{P_2}{X_2} = \frac{b_1 - 1}{1 - b_1 - b_2}$$

The elasticity of demand for X_2 with respect to its own price, $e_{2,2}$ (3.24), is the derivative of (3.22) with respect to P_2 multiplied by the ratio $P_2 X_2^{-1}$.[8] For the Cobb-Douglas production function in (3.17), the elasticity is a constant. The magnitude of the elasticity depends only on the coefficients of production, not on prices, the quantity of product produced and the amounts of factors used. For other algebraic forms, however, the elasticity is influenced by the magnitude of prices, output and other resources.[9] The elasticity of demand with respect to the resource's own price is negative where b_1 and b_2 individually, and in sum, are less than unity. If technology changes so that the productivity of the particular factor increases, the demand elasticity of the factor also increases for the logarithm type of demand function.

The demand response for a particular factor relative to the price of other factors also is important in determining the rate and magnitude

[8] The elasticity of X with respect to price P is defined as $\frac{dX}{dP} \cdot \frac{P}{X}$ or as $\frac{d(\log X)}{d(\log P)}$. The latter definition is useful for finding elasticities of Cobb-Douglas functions. For example, to compute $e_{2,2}$, simply take the log of (3.22), i.e. $\log X_2 = \log C + \frac{b_1 - 1}{1 - b_1 - b_2} \log P_2$. The elasticity is $\frac{d(\log X_2)}{d(\log P_2)} = \frac{b_1 - 1}{1 - b_1 - b_2}$.

[9] For example, see the elasticities derived in Chapter 6 for fertilizer production functions. For a comprehensive discussion of the influence of algebraic forms on demand and supply quantities and elasticities see Tweeten, Luther G., and Heady, Earl O. Short-run corn supply functions and fertilizer demand functions based on production functions derived from experimental data; a static analysis. Iowa Agr. Exp. Sta. Res. Bul. 507. 1962.

by which the structure of an industry changes. The cross elasticity of demand $e_{2,1}$, for X_2 in respect to price of competing resource P_1, and the cross elasticity $e_{2,y}$, with respect to product price P_y, are given respectively in (3.25) and (3.26), where the derivatives are from (3.22) and each is multiplied by the appropriate price/factor ratio.

(3.25) $$e_{2,1} = \frac{dX_2}{dP_1}\frac{P_1}{X_2} = \frac{-b_1}{1 - b_1 - b_2}$$

(3.26) $$e_{2,y} = \frac{dX_2}{dP_y}\frac{P_y}{X_2} = \frac{1}{1 - b_1 - b_2}$$

Relative Elasticities

As pointed out above, resource demand elasticities for all algebraic forms of production functions depend on the magnitudes of the technical coefficients. The resource demand elasticities computed from functions other than the Cobb-Douglas form also depend on the magnitude of factor and commodity prices and/or the amounts used of the particular resources. This point can be illustrated with the production function in (3.27).

(3.27) $$Y = aX_1 + bX_2 - cX_1^2 - dX_2^2$$

Following the steps in (3.17) through (3.22), we obtain the resource demand quantity for X_2 in (3.28).

(3.28) $$X_2 = .5d^{-1}(b - P_2 P_y^{-1})$$

Demand quantity is a function of technical coefficients of resources and of both factor and commodity prices. The corresponding elasticities of static demand for X_2 are given in (3.29) in respect to its own price, in (3.30) with respect to price of X_1 and in (3.31) with respect to commodity price.

(3.29) $$e_{2,2} = \frac{1}{1 - b P_y P_2^{-1}}$$

(3.30) $$e_{2,1} = 0$$

(3.31) $$e_{2,y} = -e_{2,2}$$

In general, the elasticity of demand for the resource declines as its own price P_2 decreases or as commodity price P_y increases. In comparing the two different functions in (3.17) and (3.27), the elasticity differs not only with magnitude of production coefficients for the latter but also with the form of production function characterizing each commodity.[10]

The elasticities for the Cobb-Douglas function in (3.17) and the quadratic form (3.27) show a uniformity. The cross elasticity of factor demand with respect to product price is equal numerically, with the sign changed, to the sum of elasticities of factor demand with respect to factor prices.[11] Thus, (3.31) is equal to the sum of (3.29) and (3.30) multiplied by -1. Also, for the Cobb-Douglas function, (3.26) is equal to the sum of (3.24) and (3.25) multiplied by -1.

This relationship stems from the fact that demand quantity for a resource is more exactly a function of the commodity/factor price ratio $P_y P_i^{-1}$. Regardless of the absolute magnitude of P_i or P_y, resource quantity will be identical for equal ratios. With the generalized demand function for resource X_1 in (3.32), the corresponding total derivative is (3.34), and the elasticity of demand with respect to commodity price is (3.35). The derivative and elasticity of demand for X_1 with respect to the price of the i-th variable factor are (3.36) and (3.37) respectively. The sum of the individual elasticities of X_1 with respect to all input prices P_i is equal to the elasticity of demand for X_1 with respect to the product price P_y (3.35), with sign reversed (equation 3.38).

$$X_1 = f\left(\frac{P_1}{P_y}, \frac{P_2}{P_y}, \ldots \frac{P_n}{P_y}\right) \tag{3.32}$$

$$f_i' = \frac{\partial X_1}{\partial(P_i / P_y)} \tag{3.33}$$

$$\frac{dX_1}{dP_y} = -f_1' \frac{P_1}{P_y^2} - f_2' \frac{P_2}{P_y^2} - \ldots - f_n' \frac{P_n}{P_y^2} = -\sum_{i=1}^{n} f_i' \frac{P_i}{P_y^2} \tag{3.34}$$

$$e_{1,y} = \frac{dX_1}{dP_y} \frac{P_y}{X_1} = -\sum_{i=1}^{n} \frac{f_i' P_i}{P_y X_1} \tag{3.35}$$

$$\frac{dX_1}{dP_i} = \frac{f_i'}{P_y} \tag{3.36}$$

$$e_{1,i} = \frac{dX_1}{dP_i} \frac{P_i}{X_i} = \frac{f_i'}{P_y} \frac{P_i}{X_1} \tag{3.37}$$

[10] If an interaction term, e $X_1 X_2$, is included in (3.27), the demand elasticity $e_{2,2}$ in (3.29) becomes a function of the magnitude of X_1 as well as of prices and coefficients. See Tweeten and Heady, ibid.

[11] These statements about cross elasticity of factor demand with respect to product price apply, of course, only to resource demand functions "built from the ground up" from underlying production functions. This exact connection does not exist among elasticities estimated statistically from time series where the observations do not directly express exact functional relationships among technical quantities and price.

$$\sum_{i=1}^{n} e_{1,i} = - e_{1,y} \tag{3.38}$$

$$e_{1,1} = - e_{1,y} \tag{3.39}$$

If factors other than X_1 are fixed, (3.39) indicates that the elasticity of demand with respect to product and factor price are equal but opposite in sign. Assuming as in (3.32) that resource X_1 is used only in production of Y, a given percentage increase in the prices P_i of all variable and related resources has the same influence on the quantity demanded X_1 as an equal percentage decrease in product price P_y. A proportional change in all prices leaves X_1 unchanged; the static demand function is homogeneous of degree zero. Stated alternatively, the sum of the demand elasticities with respect to own-price, the price of competing resources and commodity price is zero. It follows that a given percentage change in commodity price likely will cause a greater change in resource demand quantity than will an equal percentage change in the price of any single resource in the static demand function.

Elasticity of Substitution

The elasticity of substitution of resource i for resource j is defined as the percentage change in X_i associated with a 1 percent change in X_j, and mathematically is expressed as $e_{i,j} = \frac{dX_i}{dX_j}\frac{X_j}{X_i}$. Equation (3.8) indicates that in equilibrium $-\frac{dX_i}{dX_j} = \frac{P_j}{P_i}$. Multiplying this expression by X_j/X_i, it is apparent that the ratio of expenditures on X_i and X_j is equal to the elasticity of substitutions, i.e., $-e_{i,j} = -\frac{dX_i}{dX_j}\frac{X_j}{X_i} = \frac{P_jX_j}{P_iX_i}$. Since $\frac{dX_i}{dX_j} = -\frac{\partial Y}{\partial X_j} / \frac{\partial Y}{\partial X_i}$, and defining the elasticity of production e_i as $\frac{\partial Y}{\partial X_i}\frac{X_i}{Y}$, it follows that in equilibrium $-\frac{e_j}{e_i} = -e_{i,j} = \frac{P_jX_j}{P_iX_i}$. The ratio of production elasticities is equal to the elasticity of substitution and ratio of expenditures. The result indicates that introduction of a new input j with a high production elasticity and low supply price is likely to change appreciably the resource mix as equilibrium amounts are approached. If the ratio of production elasticities e_j/e_i is greater than one, in equilibrium more will be spent on the new input j than on input i. In agriculture, technologically improved purchased inputs have tended to have a large production elasticity relative to resources originating in agriculture such as labor. The consequence has been a sizeable substitution of capital for labor and consequent reduction in the factor share of labor. From the Cobb-Douglas production function (3.17), the elasticity of substitution of X_1 for X_2 is the respective ratio of production elasticities, or $-(b_1/b_2)$.

PRODUCT SUPPLY AND ITS RELATION TO FACTOR DEMAND

Resource demand functions indicate the quantities of resources that will be used by the firm at given factor/product price ratios. The production function dictates how much product will be forthcoming, given the above demand quantities of resources. If the demand equation for X_2 (3.22) and a similar function for X_1 are substituted into the production function (3.17), the Cobb-Douglas supply equation (3.40) is formed. If X_1 is considered fixed and the demand equation (3.23) for X_2 is substituted into the production function, (3.41) is formed.

$$(3.40) \qquad Y = \left[a b_1^{b_1} b_2^{b_2} (P_y P_1^{-1})^{b_1} (P_y P_2^{-1})^{b_2} \right]^{\frac{1}{1 - b_1 - b_2}}$$

$$(3.41) \qquad Y = a\left[(ab_2)^{b_2} X_1^{b_1} (P_y P_2^{-1})^{b_2} \right]^{\frac{1}{1 - b_2}}$$

Supply function (3.40), as the demand function discussed earlier, is homogeneous of degree zero in prices. The elasticities of supply computed from (3.40) with respect to P_1, $e_{Y,1}$; P_2, $e_{Y,2}$; and P_y, $e_{Y,y}$ are equations (3.42), (3.43) and (3.44), respectively.

$$(3.42) \qquad e_{Y,1} = -\frac{b_1}{1 - b_1 - b_2}$$

$$(3.43) \qquad e_{Y,2} = -\frac{b_2}{1 - b_1 - b_2}$$

$$(3.44) \qquad e_{Y,y} = \frac{b_1 + b_2}{1 - b_1 - b_2}$$

The elasticity of supply with respect to P_y is equal to the negative sum of the elasticities with respect to factor prices.[12] An equal proportional increase in product price and decrease in factor prices leaves the supply quantity Y unchanged.

Since the supply quantity is a function of the input magnitude and the technology of the production function, one might anticipate an exact theoretic relation between input demand, product supply and the production function. Tweeten and Heady show that the elasticity of supply $e_{Y,y}$ is equal to the sum of the cross elasticities $e_{i,y}$ of inputs X_i with respect to output price P_y times the elasticity of production $e_{Y,i}$ as in (3.45).[13]

$$(3.45) \qquad e_{Y,y} = \sum_{i=1}^{n} e_{i,y}\, e_{Y,i}$$

[12] For a general proof see Tweeten and Heady, ibid.

[13] Ibid.

It is therefore possible to express output supply elasticity from knowledge of the production and factor demand functions. Equation (3.45) can be made dynamic and can be used to express elasticities over various periods of time by placing time subscripts on the supply elasticity $e_{Y,y}$ and on the input demand elasticity $e_{i,y}$. The relationship indicated by (3.45) is apparent in the simple case when only one factor, X_i, is variable (3.46).

$$(3.46) \qquad e_{Y,y} = \frac{dY}{dP_y} \cdot \frac{P_y}{Y} = \left(\frac{dX_i}{dP_y}\,\frac{P_y}{X_i}\right) \cdot \left(\frac{dY}{dX_i}\,\frac{X_i}{Y}\right)$$

$$(3.47) \qquad e_{i,y} = \frac{dX_i}{dP_y} \cdot \frac{P_y}{X_i} = -\frac{dX_i}{dP_i} \cdot \frac{P_i}{X_i} = -\,e_{i,i}$$

When only X_i is variable, the cross elasticity of demand $e_{i,y}$ for X_i with respect to product price is equal to the negative elasticity of demand $e_{i,i}$ for X_i with respect to input price (3.47). It follows that when one factor is variable, the static elasticity of supply is equal numerically to the elasticity of demand multiplied by the elasticity of production (3.48).

$$(3.48) \qquad e_{Y,y} = -\,e_{i,i}\,e_{Y,i}$$

If the firm is operating at the beginning of stage II (average product at a maximum), the elasticity of production is unitary ($e_{Y,i} = 1$) and the elasticity of product supply and factor demand numerically are equal but opposite in sign. As more X_i is used, the elasticity of production declines and the elasticity of supply is less than the elasticity of demand. As stage III (total product at a maximum) is approached, the elasticity of production approaches zero and the output supply elasticity $e_{Y,y}$ is very small relative to the factor demand elasticity $e_{i,i}$. A large percentage increase in factor or product price raises output very little when the increase occurs after input, output and relative prices $P_y P_i^{-1}$ already are very high. Since most production takes place in stage II, factor demand is expected to be more elastic than product supply.

The general relationship (3.45) may be verified in the specific example of the Cobb-Douglas production, cross-demand (3.26) and supply (3.44) elasticities. The production elasticities, b_1 and b_2, multiplied by the cross-input demand elasticities, $\frac{1}{1 - b_1 - b_2}$, do indeed equal the product supply elasticity in (3.49).

$$(3.49) \qquad \frac{b_1 + b_2}{1 - (b_1 + b_2)} = b_1\left(\frac{1}{1 - b_1 - b_2}\right) + b_2\left(\frac{1}{1 - b_1 - b_2}\right)$$

If the firm is in equilibrium and the value of marginal product of the i-th factor is equal to its price (3.50), then the factor share F_i (3.51) is equal to the elasticity of production $e_{Y,i}$ (3.52). The value of marginal product from (3.50) is substituted for P_i in (3.51) and the result (3.52) indicates that in equilibrium the factor share is equal to the elasticity of production.

$$\frac{\partial Y}{\partial X_i} P_y = P_i \tag{3.50}$$

$$F_i = \frac{X_i P_i}{Y P_y} \tag{3.51}$$

$$F_i = \frac{X_i}{Y P_y} \left(\frac{\partial Y P_y}{\partial X_i} \right) = \frac{\partial Y}{\partial X_i} \frac{X_i}{Y} = e_{Y,i} \tag{3.52}$$

$$e_{Y,y} = \sum_{i=1}^{n} E_{i,y} F_i \tag{3.53}$$

The equilibrium assumption permits substitution of F_i for $e_{Y,i}$ in (3.45) to form (3.53). The elasticity of product supply is equal to the sum of the cross elasticities of demand multiplied by the factor shares for each resource.

RESOURCE SUPPLY AND ELASTICITY

The resource structure of an industry depends not only on the nature of factor demand functions but also on the nature of the supply functions for resources. Commodity supply functions may have high or low elasticity depending on the supply elasticity of the factors which are used in agricultural production. With low supply elasticity of factors we expect high commodity prices and favorable resource returns when commodity demand increases relative to commodity supply, but the opposite when commodity supply increases more rapidly than commodity demand. Hence, the particular quantities and mix of resources used, with their effect on the commodity supply function, can be completely specified only if we know the supply functions of resources. The importance of factor supply functions to the mix and return of resources in agriculture can be illustrated with a few examples.

Consider the example of a supply equation (3.54) for a resource, X_1, used in the production of output Y where P_1 is the input price and b is the input supply elasticity.

$$X_1 = a\, P_1^b \tag{3.54}$$

Assume the production function is the Cobb-Douglas type (3.55) where output Y is produced by input X_1 and d is the elasticity of production.

$$Y = c\, X_1^d \tag{3.55}$$

Solving for P_1 in (3.54) and X_1 in (3.55) and substituting these into the total cost equation (3.56), the total cost becomes a function of variable cost $P_1 X_1 = f(Y)$ and fixed cost C. The derivative of TC with respect to Y is equated to product price from the assumption of profit maximization. Solving for Y in terms of product price P_y, the supply function (3.57) is formed. The elasticity of supply, $e_{Y,y}$, is specified in (3.58).

$$TC = P_1 X_1 + C \tag{3.56}$$

$$Y = \left[a^{\frac{1}{b}} c^{\frac{1+b}{bd}} \frac{bd}{1+b} P_y \right]^{\frac{bd}{1+b-bd}} \tag{3.57}$$

$$e_{Y,y} = \frac{bd}{1 + b - bd} \tag{3.58}$$

Several characteristics of the product supply elasticity are of interest. The two parameters which determine $e_{Y,y}$ are the input supply elasticity b and the production elasticity d. As the input supply elasticity b approaches zero, the product supply elasticity $e_{Y,y}$ also approaches zero. As the input supply elasticity becomes large and approaches infinity, the product supply elasticity becomes a function of the production elasticity d only and approaches d/1-d. A product supply equation (3.41), derived earlier without explicitly recognizing the input supply equation, provided the same estimate d/1-d of $e_{Y,y}$. The common practice of assuming input prices are given is comparable to assuming that the input supply elasticities are infinite. But from the example (3.58), it is apparent that for a given production elasticity d, the output supply elasticity is an increasing function of the input supply elasticity b. Ceteris paribus, the greater the value of b, the greater the value of $e_{Y,y}$. With constant returns to scale (d=1), then $e_{Y,y} = b$, and the input and output supply elasticities are equal.

To further illustrate the impact of factor supply elasticity upon employment and resource returns, we employ the following highly simplified empirical industry example where we do not detail production relationships relating factors and commodities, and our functions are linear. In (3.59) we suppose the consumer or commodity demand function, where demand quantity is a function of certain exogenous variables and magnitudes summarized in the constant and the commodity price.

$$Y_d = 1500 - 50P_y \tag{3.59}$$

$$Y_s = -240 + 150P_y - 50P_x - 40P_z \tag{3.60}$$

(For simplicity, cross-demand elasticities with respect to other commodities are not considered.) The commodity supply function is (3.60).

Short-run supply quantity is, given the production relationships, a function of product price and prices for two factors, X and Z. The conforming demand functions for the two resources are (3.61) and (3.62).

$$X_d = 2000 - 900P_x + 150P_z + 20P_y \tag{3.61}$$

$$Z_d = 2500 + 200P_x - 250P_z + 10P_y \tag{3.62}$$

(The factor demand functions are assumed to depend, given prices, on the production function in transforming factors into product.)

$$X_s = -200 + 600P_x \tag{3.63}$$

$$Z_s = 1800 + 50P_z \tag{3.64}$$

The supply functions for factors are (3.63) and (3.64) where we suppose quantity supplied to the industry to vary only with own-price of the factor. (In this oversimplification which does not allow simultaneity of factor supply quantities, or of factor supply price or income and commodity demand, we might suppose X to be fertilizer or machinery and Z to be labor.) The resource supply elasticities, in respect to their own prices, are respectively (3.65) and (3.66), the latter being smallest relative to equilibrium quantities determined later.

$$e_x = 600P_x X_s^{-1} \tag{3.65}$$

$$e_z = 50P_z Z_s^{-1} \tag{3.66}$$

Now, letting Y be the equilibrium demand and supply quantity of the commodity, X be the equilibrium demand and supply quantity of the first factor and Z be the same quantity for the second factor, equilibrium quantities of the market are specified by the matrix equality in (3.67). Designating the coefficient matrix as A, the vector of market prices and quantities as Q and the vector of constants as K, the equilibrium quantities are defined in (3.68) where A^{-1} is the inverse of the coefficient matrix.

$$\begin{bmatrix} 1 & 50 & 0 & 0 & 0 & 0 \\ 1 & -150 & 0 & 50 & 0 & 40 \\ 0 & -20 & 1 & 900 & 0 & -150 \\ 0 & -10 & 0 & -200 & 1 & 250 \\ 0 & 0 & 1 & -600 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -50 \end{bmatrix} \begin{bmatrix} Y \\ P_y \\ X \\ P_x \\ Z \\ P_z \end{bmatrix} = \begin{bmatrix} 1500 \\ -240 \\ 2000 \\ 2500 \\ -200 \\ 1800 \end{bmatrix} \tag{3.67}$$

$$Q = A^{-1} K \tag{3.68}$$

The equilibrium quantities so computed from (3.68) are included in column 1 of Table 3.1. At equilibrium, the supply elasticity of X with respect to its own price is $e_x = 1.2$ while that for Z is $e_z = .1$.

Table 3.1. Equilibrium Quantities and Prices for Example

Quantity	First Equilibrium (1)	Second Equilibrium (2)	Third Equilibrium (3)	Fourth Equilibrium (4)
Y	1000	1130	923	1073
P_y	\$10.00	\$9.45	\$9.48	\$10.49
X	1000	1359	994	1005
P_x	\$2.00	\$2.60	\$1.99	\$2.01
Z	2000	1980	1999	2001
P_z	\$4.00	\$3.60	\$3.98	\$4.02

Commodity demand now grows to (3.69) because of population increase, commodity supply grows to (3.70) because of change in technology, and the resource demand equations change to (3.71) and (3.72).

$$Y_d = 1650 - 55P_y \tag{3.69}$$

$$Y_s = -300 + 187.5P_y - 62.5P_x - 50P_z \tag{3.70}$$

$$X_d = 3000 - 1017P_x + 200P_z + 30P_y \tag{3.71}$$

$$Z_d = 2400 + 150P_x - 264.7P_z + 15P_y \tag{3.72}$$

The new equilibrium quantities are those of the second column of Table 3.1. Input of X has grown and its price has increased to \$2.60. Input of Z has declined and its price has fallen to \$3.60. While input of X has increased by 36 percent, input of Z has declined by only 1 percent because its supply elasticity is extremely low.

To further emphasize the effect of factor supply elasticity on input quantity and resource price or returns, suppose that commodity demand declines from (3.59) to (3.73) while all other supply and demand functions in (3.60) through (3.64) remain unchanged.

$$Y_d = 1350 - 45P_y \tag{3.73}$$

The equilibrium quantities then are those in the third column of Table 3.1. The equilibrium input and price of X drop .6 and .5 percent respectively from those in column 1. The equilibrium input for Z drops by .05 percent, as compared to the first equilibrium. Because the supply elasticity for Z is low, a relatively large quantity of Z continues to be employed even though the factor has a "large" decline in price or returns. Alternatively, suppose that all other demand and supply functions remain unchanged, but that commodity demand increases from (3.59) to (3.69). The equilibrium inputs and resource prices are those of the fourth column in Table 3.1. The quantity of Z increases but little while the price (return) increases because supply elasticity is low for this resource. Input of X increases by a larger percentage, but its price increase is expected to be relatively small because it has a higher supply elasticity.

Our example with a series of distinct short-run functions has been simple, setting forth certain outcomes for static-oriented market relationships for a competitive industry. Yet, it illustrates some of the basic structural problems relating to resource structure and factor income in an industry such as agriculture which has similar characteristics. Adding conditions to convert the model to a dynamic one would only accentuate the differences between resources X and Z where the former has high supply elasticity and increases in marginal productivity relative to the latter.

Problems of overcapacity and low resource returns have roots in the nature of input supply functions and elasticities in agriculture. The process leading to overproduction and low returns on conventional farm resources can be described as follows: New inputs and improved conventional inputs representing advanced technology have a high marginal product (high marginal rate of substitution) relative to other conventional inputs. The new inputs often are supplied by nonfarm industries and the supply is highly elastic. Because the value of marginal product is high relative to input price and because input supply elasticity is large, the new inputs are introduced into agriculture at a rapid rate. Furthermore these technological inputs such as fertilizer and weed and insect sprays are easily introduced because they are divisible, do not require extremely large capital outlays and their adoption does not conflict with the value or institutional structure of farming. The rapid adoption results in increased farm output and depressed farm product prices and incomes. If the agricultural economy functioned perfectly, the depressed product prices would lower resource returns and cause conventional inputs to move into other industries until returns are equalized. But conventional farm inputs such as labor have a low supply elasticity because of values, institutions and training, and because of external factors such as national unemployment. Opportunities for supplies of farm real estate to move into nonfarm uses are extremely limited in the short run. The price may fall very far before large quantities of the resources leave agriculture, i.e. the supply elasticity is low.

For another major conventional farm resource, machinery, the supply is discrete or discontinuous and irreversible. When machinery quantities are moving into agriculture the supply elasticity is large, but when farm prices fall the machinery supply elasticity is low and essentially is governed by the rate of depreciation. The above conventional farm resources therefore tend to remain in agriculture during depressed periods, and accept low returns. The resulting cost-price squeeze may in some ways only enhance the difficulties. The late adopters of technologically improved inputs might be content to continue with old methods. But for the firm to survive in the face of falling incomes may require greater economies. Because the productivity of technologically improved inputs is great, the ratio of value of marginal product to input price may remain high despite a large drop in product price. The result is that perhaps the only way late adopters can raise income is to use more of the new inputs and consequently to increase output despite falling product prices. Those who have adopted new and improved inputs and techniques move only gradually to the profit maximizing level of use. The result is increased use of new inputs and rising output although prices received by farmers are falling. Because the supply of new inputs tends to be more price elastic than the supply of conventional inputs, the conventional inputs are unable to adjust to the influx of new inputs. Problems of low relative returns and overcapacity in agriculture result.

Because the farm labor supply elasticity is low relative to the rate at which commodity supply increases, labor has a lower imputed price than resources such as fertilizer, machinery and other items whose (a) supply elasticity is greater, (b) reservation price is high because of alternative uses in nonfarm sectors and (c) demand quantity increases even in a depressed industry. (Our simple example did not detail these interrelationships between economic sectors. Our quantitative estimates of later chapters attempt to do so, however.) In any case, our relatively simple example indicates the impact of factor supply elasticities on the quantities of resources used and their pricing or return. These parameters are equally important with those of resource demand functions in determining the resource structure of an industry such as agriculture. [14]

Resources Supplied by Nonfarm Industries

Because of the increasing importance in agriculture of inputs produced in other industries, and because of certain implications for empirical economic models of factor demand, it is desirable to discuss some characteristics of the supply function for nonfarm inputs. [15] The supply of nonfarm, nonhuman resources has been described as highly elastic in this chapter. Considerations which support this hypothesis might be summarized into the categories: (a) the historic input price-quantity relationships, (b) empirical studies of the cost structure of nonagricultural industries, (c) the type of competition among input-supplying firms, (d) the goals of these industries and (e) the relative importance of agricultural purchases in the sales of nonfarm firms.

The historic short-run stability of input prices gives some evidence that input supply is highly elastic. The fact that shifts in input demand due to weather and product price changes have not resulted in

[14] For other relationships of supply and demand elasticities for factors relative to change in production technology and consumer demand, as these relate to factor inputs and returns, see Heady, Earl O., Agricultural Policy Under Economic Development. Iowa State University Press. Ames. 1962. Chaps. 5 and 11.

[15] The elasticity of input supply may dictate whether a single or simultaneous model of factor markets in agriculture is necessary.

appreciable input price changes implies a high input supply elasticity, at least in the short run.

Empirical studies of major nonfarm firms reveal nearly constant or slowly rising average and marginal cost curves. Because the short-run industry supply curve is the horizontal summation of firm marginal cost curves, industry supply is likely to be highly elastic. Further, competition among nonfarm suppliers of agricultural inputs tends to be less than perfect. The actions of suppliers are interdependent, and in such instances of oligopoly, emphasis is placed on nonprice competition. The result tends to be a stickiness of prices at various quantity levels due to fear of recrimination by other suppliers.

Some economists indicate that goals other than maximum total profit are important in business decisions.[16] These goals include securing public good will, earning a stable return on investment, a fixed margin on costs of production and other goals. Despite an increase in marginal cost at higher output, a firm may not increase price for fear of losing public good will. When agricultural demand for an input increases, a supplier concerned with earning a stable return on investment may find it possible to maintain this return by maintaining or possibly by decreasing price. The latter case could give rise to a negative (but high in absolute terms) supply elasticity. If the manufacturer desires a cost-plus markup, the tendency could be to increase the supply elasticity. For example, a fixed margin above the marginal cost results in a "supply curve" more elastic than the marginal cost curve.

Finally, the importance of agricultural purchases in the total sales of the input supplier may influence the magnitude of supply elasticity. If a manufacturer sells only a small portion of his output to agriculture, an increase in agricultural demand may allow him to supply the increased quantity with little impact on the firm's cost structure. The change in input demand may be almost unnoticed, and the result is likely to be a highly elastic input supply. Since many firms supplying inputs to agriculture also supply inputs to other economic sectors, the declining nature of agriculture relative to other industries tends to increase supply elasticity. On the other hand, nonfarm inputs are substituting for farm produced inputs. Use of nonfarm inputs is increasing relative to farm output, and is rising in absolute amounts. This tendency, along with increased specialization of manufacturers in producing farm inputs, tends to reduce supply elasticity.

It seems reasonable to conclude that the supply of nonfarm inputs is highly elastic. A distinction might be made between supply at the industry and farm levels. Assuming a constant or decreasing margin at high prices, the industry supply is less elastic than supply at the farm level.

[16] Cf. Baumol, William. Business Behavior, Value and Growth. Macmillan and Company. New York. 1959.

Resources Supplied within Agriculture

The supply function for many farm resources is best described as (3.74) where the total supply is an aggregate of that from two sectors: from outside the industry, $f_N(P_i)$, and from inside the industry, $f_F(P_i)$.

$$X_i = f(P_i) = f_F(P_i) + f_N(P_i) \tag{3.74}$$

The total supply is a function of supplies from the two sectors because: (a) nonfarm supplies such as motor fuel, fertilizers, etc., are used to produce feed and livestock inventories — a complementary relationship, and (b) nonfarm supplies substitute for farm inputs, e.g., commercial fertilizer and farm manures or crop residues, tractor and horse power; commercial seeds and farm seeds. Furthermore machinery supply potential in a given period is composed of farm machinery inventories plus possible nonfarm purchases.

Resources supplied from outside agriculture have a higher supply elasticity than those furnished from within the industry. Despite the high supply elasticity of fertilizer, motor fuels and other inputs used to produce farm feed and livestock inventories, the input supply elasticity of feed and livestock resources is low in the short run. A long production period is required to increase inventories of breeding stock, and it is physically impossible to increase stocks of these resources rapidly in response to large price increases. Also the supply elasticity of intermediate farm resources such as livestock and feed is low because they are produced by farm resources such as real estate services with a very low supply elasticity.

Within restricted limits machinery and real estate services can be adjusted to price changes by inter-period shifts. In part, more services can be used next year and less this year. But an important part of machinery services, and almost entirely those of labor and buildings, are forthcoming at a constant rate in various years and little can be done, once they are fully employed, to squeeze more service out of them in a particular year. If these resources are highly specialized to agriculture, as labor skilled to farm production but little else, or steel forged into cultivators, their reservation price for use in agriculture is low because they have few alternative employment opportunities — or alternative opportunities provide low prices to the resources. A small amount of land can be furnished from the outside, but the major portion is furnished from within agriculture with low price elasticity and reservation price.

The implications of these different resource supply functions and their shifters can be illustrated as follows for a given period. Two sector supply functions exist for the resource measured by X_i. The supply function from the nonfarm sector is (3.75), the function for the farm sector is (3.76) and the total supply function of the resource to agriculture is (3.77) where P_x is price of resource (or service) for the period.

(3.75) $$X_n = bP_x - a$$

(3.76) $$X_f = 8bP_x - 4a$$

(3.77) $$X_t = 9bP_x - 5a$$

Corresponding elasticities of resource quantity in respect to own-price are given in (3.78) to (3.80).

(3.78) $$e_n = \frac{bP_x}{bP_x - a}$$

(3.79) $$e_f = \frac{bP_x}{bP_x - .5a}$$

(3.80) $$e_t = \frac{bP_x}{bP_x - (5/9)a}$$

While the supply elasticity is high for "outside" resources, it is low for "inside" resources and for resources in total.

An alternative view of the same phenomenon is the "pure" example of short-run resource supply functions in Figure 3.1. Disregarding the initial stock or supply of resources from inside agriculture, suppose that rs_1 is the supply function of resources from outside agriculture for a particular period. The resources may be machinery, buildings, breeding stock or similar durable items. The quantity purchased for the period is that indicated at q_1. These resources then, because they

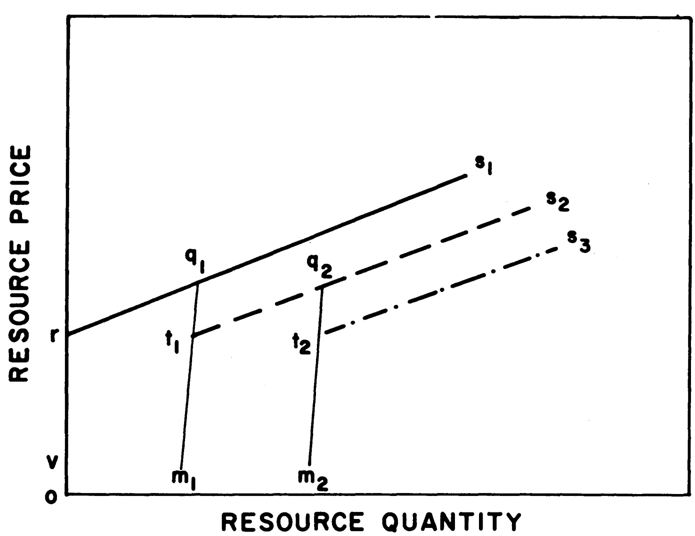

Figure 3.1. Resource supply functions from "outside" and "inside" sectors.

are specialized to farming, provide a "stock" of services within agriculture and their supply function becomes $m_1 q_1$ for the next period.[17] Hence, even if "outside" resources were banned from agriculture in the next period, a supply of $m_1 q_1$ would still exist. Consequently, the supply function of agricultural commodities will be the sum of these two factor supply functions. In expansion during the first period, the commodity supply function will have high elasticity, as does rs_1 for resources. But the commodity supply function in the second period is not also reversible from q_1 to r. Instead, it follows $m_1 q_2$. In a second period where economic conditions encourage further expansion in resources used, the resource supply function becomes $m_1 t_1 s_2$. If the resource is used at the level q_2 in the second period, the "inside" resources provide the supply function $m_2 q_2$ in the second period. The third period supply function is the sum of $m_2 q_2$ and $t_1 s_2$ and is $m_2 t_2 s_3$. But $m_2 q_2$ supply will exist even if no resources are purchased from "outside" agriculture. Because the short-run resource supply functions are not reversible after particular resources are added to agriculture, the commodity supply functions similarly are not reversible. Consequently important differences will prevail between short-run and long-run commodity supply elasticities as well as factor supply elasticities.

INDUSTRY SUPPLY, DEMAND, INTERDEPENDENCE AND CAUSALITY

Economic theory of the competitive industry introduces additional concepts which must be considered in any empirical estimation of the resource structure. For a small segment of agriculture, the price of nonfarm inputs may be assumed as given or exogenous in the input demand functions. That is, the actions of a small group of farmers have little influence on the prices of resources supplied by the nonfarm sector, and input supply is perfectly elastic. The action of one farmer or a small group of farmers also has little influence on the prices they receive for farm products. Thus, prices may be assumed exogenous, i.e., determined by forces outside the system being examined. Only farm output and resource inputs are endogenous (determined within the system), and the quantity of any input may be estimated as a monocausal function of prices and fixed factor levels as in demand equation (3.16). Also, the supply of farm products from a small group of farmers may be considered a simple function of prices and other exogenous variables.

[17] Figure 3.1 has meaning only for durable resources. The assumption is that depreciation is negligible. If depreciation is sizeable, a portion of $m_1 q_1$ would be to the left of that indicated in Figure 3.1. For resources such as fertilizer or seed which have a high "depreciation," $m_1 q_1$ would move to the vertical axis and rs_1 would again be the supply curve for the second period. The irreversibility of the supply curve depends on the extent of durability in resources.

The most general model of industry supply and demand is the Walrasian general equilibrium system. According to the Walrasian system, prices and quantities of commodities are determined interdependently by a system of demand and supply equations. The complete Walrasian system includes demand and supply functions in the entire economy. Even if the simultaneous system is considered pertinent, empirical models necessarily must abstract from the more remote markets in the entire economy and must emphasize the markets for agricultural inputs and outputs.

The type of economic (and statistical) model chosen to represent the market structure of agriculture depends strongly on the underlying causal framework. A direct relationship exists between the nature of causality specified in the economic model and the type of statistical model chosen to estimate the parameters. For present purposes we avoid an extended discussion of the ontological aspects of causality. Rather we consider only the immediate, pragmatic aspects of causality and emphasize those considerations necessary in constructing economic models.

The static equilibrium models of Walras, Marshall and others stress the interdependence of supply and demand in determining equilibrium price and quantity. The early econometric analysis of supply and demand from time series, however, assumed a monocausal relationship. That is, price (or quantity) was chosen as the dependent (effect) variable, and was considered a function of the quantity (or price) and other independent (causal) variables. Econometricians such as H. Schultz and Working were uncomfortable with this simple cause-effect relationship.[18] They realized that only under certain conditions could the structural demand or supply function be identified using the single equation, least-squares statistical model. This led to the development of statistical procedures which allowed for the simultaneous determination of price and quantity by supply and demand, and thus for the identification of structural economic relationships in an interdependent system.[19]

The new statistical techniques satisfied the basic premise of interdependence derived from static economic theory, and economists hailed the new methods as a greatly improved tool for analyzing supply and demand. Possibly due to the computational burden and other shortcomings of the newly developed statistical techniques, economists began to re-examine the adequacy of least-squares single equations.[20] The

[18]Schultz, Henry. The Theory and Measurement of Demand. The University of Chicago Press. Chicago. 1938. pp. 72-114; and Working, E. J. What do statistical "demand curves" show? Quarterly Journal of Economics. 41:212-35. 1927.

[19]Cf. Haavelmo, Trygve. The statistical implications of a system of simultaneous equations. Econometrica. 11:1-12. 1943.

[20]Bentzel, R., and Hansen, B. On recursiveness and interdependency in economic models. Review of Economic Studies. 22:153-68. 1954-55; Bentzel, R., and Wold, H. On statistical demand analysis from the viewpoint of simultaneous equations. Skandinavisk Aktuarietidskrift. 29:95-114. 1946; Fox, Karl A. Econometric Analysis for Public Policy. Iowa State University Press. Ames. 1958.

nature of the causal structure underlying economic variables in the real world was the fundamental point in the re-examination. In particular, the Stockholm school questioned the basic premise of simultaneity in dynamic economics. The fact that decisions take time led them to conclude that economic decisions are not made simultaneously. Instead, they conceive of the recursive model as the most fundamental at an abstract level of economic theory. The recursive model is composed of a sequence of causal relationships.[21] The values of economic variables during a given period are determined by equations in terms of values already calculated, including the initial values of the system.

Much intuitive appeal lies in the disequilibrium nature of the recursive system. For example, in agriculture it seems logical that the current supply quantity often is determined by past price, and the current year price is a function of the predetermined current quantity. Commodity cycles, conceptualized in this type of recursive system — the cobweb model — give strong support for the disequilibrium model in agriculture.[22] Simultaneous equations that include only current price and quantity are dynamic equilibrium models, and may not be appropriate where production is predetermined and cycles are apparent. The conclusion is that if the economic model is sufficiently detailed and adequately specified, and if the time period is sufficiently short, the recursive model may be appropriate.

Surprisingly, the real basis for interdependent models does not seem to arise from the static economic equilibrium models of Walras et al., but from the exigencies of empirical data. One example is aggregation of data over time. Suppose that A determines B, B determines C, and C determines D through time. If A is aggregated with C, and B with D, then a joint "causal" relationship exists between the aggregate A C and B D.

SIMPLE DYNAMIC MODELS

Resource employment in agriculture does not respond immediately to changes in factor prices and technical coefficients. Even where quantities do change, the extent of short-run response is seldom consistent with the magnitude of change in price and production coefficients. Several years pass before the industry adjusts fully to a new set of price relatives or marginal resource productivities. There are many reasons why this is true. Time itself and the durability of resources help to prevent it. Farmers do not discard a building, machines and power units as soon as more efficient ones are developed,

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[21]There may be more than one endogenous (jointly determined within the system of equations) variable in a recursive equation. The matrix of coefficients of endogenous variables must be triangular, however.

[22]For an example of an industry strongly characterized by cobweb-type cycles see Tweeten, Luther G. Variability in Broomcorn Prices and Land Use Adjustments in Southcentral Oklahoma. Unpublished M.S. thesis. Library, Oklahoma State University. Stillwater. 1958.

partly because those already employed have further use, and especially because the supply elasticity and price of those already in use merit their employment as substitutes for the new items. Capital limitations, as these revolve around time and uncertainty, also prevent immediate adoption of new input forms where large new investments are required. The existence of uncertainty also discourages "immediate adoption" where the return on a durable resource purchased in the current period depends on product prices and productivities (weather, technology) in future periods. To varying degrees, farmers wait for more information to better predict the outcome of a new technology and price trends. Many resources are fixed to the firm and complement another resource which emerges as a new technology. Consequently, use of the new capital form awaits sufficient depreciation of the "fixed" resources (actually resources with low reservation prices and low supply elasticity to agriculture). While new feed handling or livestock equipment may be productive, full investment in and use of it may await depreciation of an old barn and investment in a new one. The input of one resource will generally affect productivity of others. Hence, as the "fixity" of some durable resources is relaxed, demand will grow for other resources.

The process of acquiring knowledge gives rise to lagged response for agriculture in aggregate as it responds to changes in prices and production coefficients. On an aggregate basis, farmers undoubtedly acquire knowledge or form expectations in a manner described by a logistic curve: A few with proper knowledge and favorable expectations react immediately, but the process picks up speed with increasing and "chain reaction" contacts among farmers. Eventually, the rate of change slows as the majority of farmers have adopted the resource or practice and the remaining farmers adopt the resource slowly and reluctantly. Too, the uncertainty in expectations of an individual farmer causes him to use only a small amount of some new resources (or resources with lowered prices) in a first period. He may use slightly more in a second period, then move towards a profit maximizing quantity in a later period. The purely psychological resistance to change affects the time path of adjustment to new stimuli. Institutional arrangements in farm size, tenure and contract arrangements and other customs also alter the time path describing response in inputs and outputs to changes in technical and economic variables. Decisions in agriculture also are complicated by the fact that specific investment decisions are made at many points in time, and each investment affects the productivity of past, current and future investments.

These considerations and others cause a distributed lag in adjustment of resource purchases to changes in price, technical coefficients, knowledge and other variables in the economic environment. They cause the response elasticity to be greater in the long run than in the short run.

Adjustments in use or demand for particular resources may follow numerous adjustment paths. Some alternatives are illustrated in

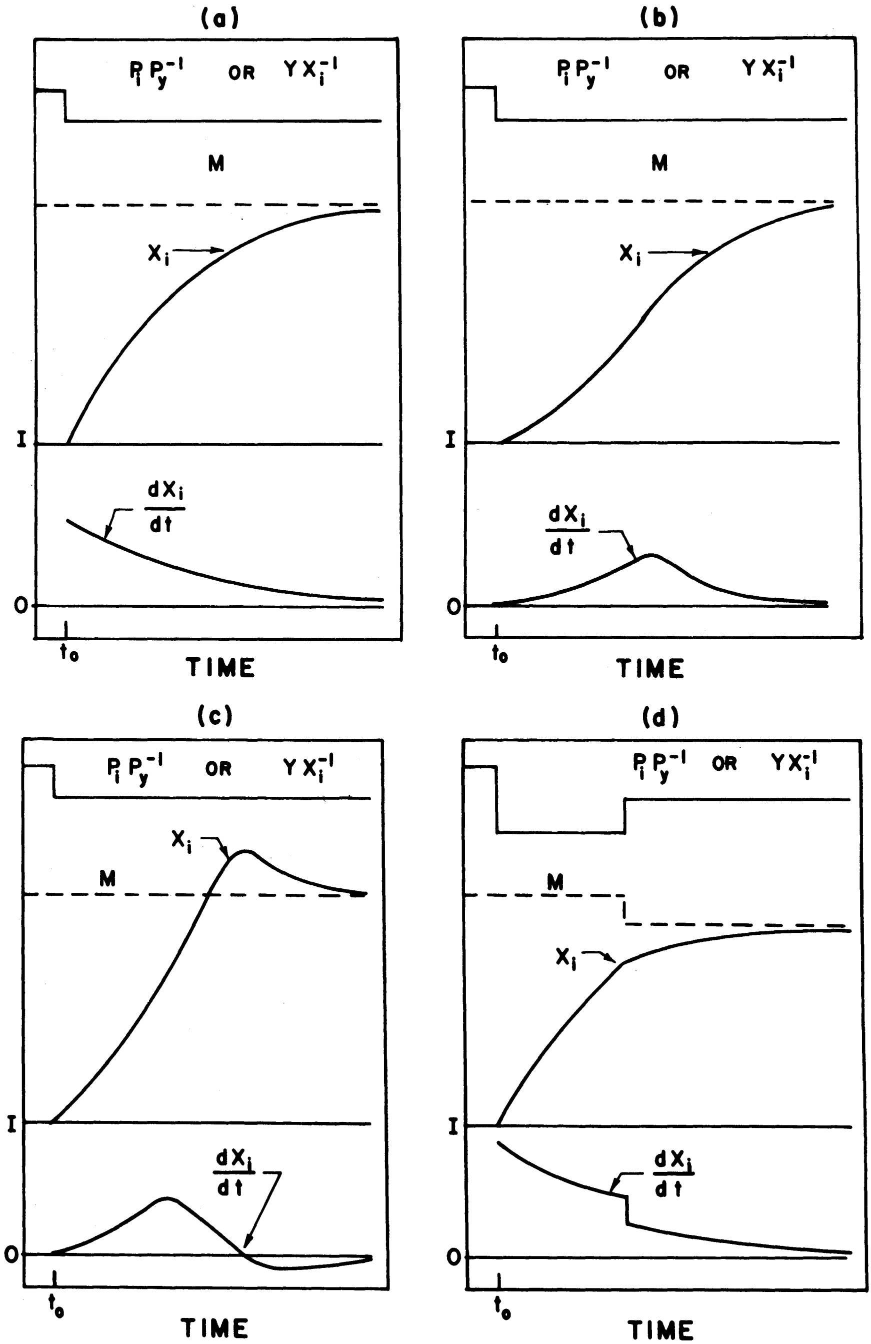

Figure 3.2. Some alternative adjustment rates and time paths in resource demand.

Figure 3.2 which includes four quantities: (a) the magnitude of the factor/product price ratio, $P_i P_y^{-1}$, or the magnitude of the output/input coefficient, YX^{-1}, (b) the optimum level, M, of resource use under the new price or technology, (c) the quantity of the resource used, X_i, and the change in resource use relative to time, $\frac{dX_i}{dt}$. For purposes of generality, we suppose changes in X_i and $\frac{dX_i}{dt}$ to be continuous, although we lift this assumption in later discussion to emphasize realistic conditions for agriculture. We suppose that X_i is at an initial equilibrium level at time, t_o, but that the new equilibrium level, M, exists as price and technical coefficients change. Following Koyck, the X_i curve is the adjustment path and $\frac{dX_i}{dt}$ is the time shape of the reaction of X_i to $P_i P_y^{-1}$ or YX_i^{-1}.[23] Graph (a) illustrates the type of adjustment an individual firm might make, due to the numerous restraints mentioned above, to a reduction in factor price or the input/output ratio or an increase in product price. (The adjustment for the firm would be discrete movements for a resource represented by separate units such as a tractor or building and for fertilizer where the production period is discrete, but would represent a "smooth curve" for the industry.) With the price or technical change taking place at time t_o, a new optimum or profit maximizing quantity of the resource comes about and is represented by line M. The firm, however, does not adjust input immediately to this level, but gradually approaches it with time. As illustrated by $\frac{dX_i}{dt}$, the rate of change slows down with the passage of time. Alternatively, the firm may adjust as illustrated in graph (b). Here the rate of adjustment speeds up initially due to increased knowledge, lessening of "fixed factor" restraints and others. After reaching a peak, the rate of adjustment slackens and approaches zero as use of the resource approaches the optimum level. While graph (a) might represent the adjustment path for the firm, graph (b) may represent the corresponding path for the industry. This would be the case where a "chain reaction" exists in adoption of a new practice: the rate picks up as more "neighbors" are contacted, but declines as there are fewer remaining farmers who have not adopted the practice.

Graph (c) illustrates a possible outcome as farmers overestimate the productivity of a practice relative to prices, or the realized magnitude of $P_y P_i^{-1}$. Investment exceeds the optimum level in a short time period, then declines towards the profit maximizing level after improved knowledge is acquired. (Graph [c] also may depict the outcome for a resource with zealous salesmen.) While elasticity of expectations is not discussed, graph (c) might relate to particular elasticities of expectations attached to the initial change. Graph (d) suggests the break in the adjustment path as the price or technical effect is first extremely

[23]Cf. Koyck, L. M. Distributed Lags and Investment Analysis. North-Holland Publishing Co. Amsterdam. 1954. Chap. 2.

favorable; then becomes less favorable, but still remains at levels above that at t_0. The X_i curve in graph (c) also might describe the path of adjustment when the factor price or output/input coefficient first declines, then rises to a level more favorable than at the outset.

These few illustrations suggest the many different time patterns resource adjustment might take. It is fortunate for a geographically dispersed industry such as agriculture that the distributed lag pattern is followed. With an instantaneous change in resource demand as implied in equations (3.22) and (3.23), a tremendous social and economic shock and uprooting would take place. Labor and families would be displaced from agriculture more rapidly than could be absorbed by communities and employment opportunities. This statement means not at all that magnitudes of prices and technical coefficients are unimportant in resource demand, but only that some period of time, depending on the resource and its period of production, are required before adjustment to these various stimuli approach their limit in effect and change. Of course, the time paths in Figure 3.2 best explain the adjustments when the discrete change in coefficients is expected to endure. Where coefficients are subject to repeated change and great uncertainty is attached to their values, full adjustment is even less likely because of strategies adopted to meet risk. Too, precautions to meet uncertainty give rise to patterns and discounts in adjustment which depart from those illustrated in Figure 3.2.

Algebraic Examples

Lag in adjustment to price and technical coefficients, or even to institutional and other variables affecting resource demand, will be distributed in various algebraic forms. Suppose that the demand function of a resource is the general equation (3.81) where P can be taken as a resource price, although it also can refer to other variables of the demand function. The magnitude of resource use in the current time period is X_t and is a function of resource price in the current period, P_t; in the previous year, P_{t-1}; and in general the i-th previous period, P_{t-i}.

$$X_t = f(P_t, P_{t-1}, \ldots P_{t-i}, \ldots P_{t-n}) \tag{3.81}$$

Linear in original observations, or in logarithmic transformation, the distributed lag function can be written as

$$X_t = a_0P_t + a_1P_{t-1} + a_2P_{t-2} + \ldots + a_iP_{t-i} \ldots + a_nP_{t-n} = \sum_{i=0}^{n} a_iP_{t-i} \tag{3.82}$$

where the a_i (the $\frac{dX_i}{dt}$ values in Figure 3.2 if we consider continuous changes in X), are the extent of change in X_t associated with each P_{t-i}, and initial equilibrium is disturbed as P changes to a new but constant level after a disturbance. In other words, X_t is the sum of adjustments occurring in the current year, the previous year, through t-n year. The series Σa_i in (3.82) converges as X approaches equilibrium level, with a_n approaching the limit zero when n becomes large (or $a_n \to 0$ as $n \to \infty$ and the value of P remains constant after an initial change.

As pointed out previously, the adjustment to a rise in the factor/product price ratio may be quite different from a decline. This condition prevails particularly for multiperiod resources such as machines, buildings and breeding stock. Suppose that a_i is the reaction coefficient for a ratio decline in $P_i P_y^{-1}$, and b_i is the reaction coefficient for a rise in $P_i P_y^{-1}$ and that $a_i = b_i$. Then the adjustment path or curve, X_t, will be symmetric and reversible: a given decrease in $P_i P_y^{-1}$ will cause the same absolute change in X_t in a subsequent period as would the same rise in $P_i P_y^{-1}$. This condition is very unlikely for agricultural resources, even those such as fertilizer and new seeds. It is possible for the inequality $a_i \neq b_i$ to prevail but still for $\Sigma a_i = \Sigma b_i$. In this case, the adjustment path or curve of X is asymmetric but reversible. If, however, $a_i \neq b_i$ and $\Sigma a_i \neq \Sigma b_i$, the X_t curve or adjustment path is asymmetric and irreversible. The condition of asymmetry and irreversibility does not mean that a reversal of $P_i P_y^{-1}$ will not cause an opposite change in the value of X_t, but only that the declining phase of the adjustment path will not be a "mirror image" of the rising phase. Figure 3.3a provides an example. Starting from the initial level I, the resource quantity X_i increases over time with a lag

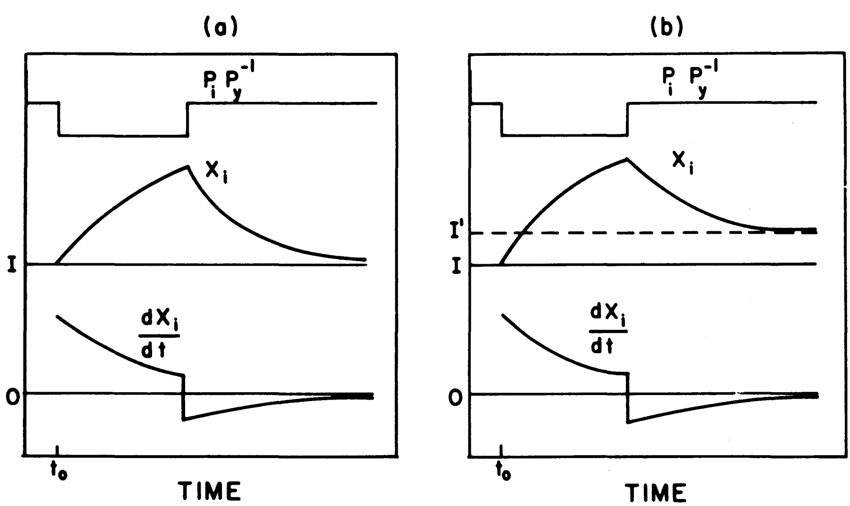

Figure 3.3. Asymmetric and irreversible adjustment paths in resource demand.

in response to an initial decline in $P_i P_y^{-1}$. However, an absolute increase in $P_i P_y^{-1}$ of the same magnitude gives a slower decline in X_i, because it is a multiperiod resource, must be used with other resources with longer lives and is restricted by custom or institutions. The case characterizing many resources of agriculture is that of Figure 3.3b. While the initial level of X_i is I, resource input does not converge towards this level, but is I′ after $P_i P_y^{-1}$ first falls then rises. Its failure to fall to I results from the reasons enumerated earlier, or because other variables such as knowledge, complementary resources or psychological restraints are changed. As outlined earlier, this type of irreversibility causes the commodity supply to have low elasticity and to remain greater during a period of rise in $P_i P_y^{-1}$ than during a period of decline in the ratio.

The adjustment paths and time shape for resource use in Figures 3.2 and 3.3 suggest that the elasticity of demand will change among production and investment periods. The elasticity in reaction or adjustment of X_i with respect to the price ratio $P_i P_y^{-1}$ will have the value over the first period in (3.83) and over the first and second periods in (3.84). These are short-run elasticities.

$$e_1 = a_0 P_i P_y^{-1} X_i^{-1} \tag{3.83}$$

$$e_2 = (a_0 + a_1) P_i P_y^{-1} X_i^{-1} \tag{3.84}$$

The long-run elasticity is (3.85). (Tinbergen restricted short run to refer to the elasticity in [3.83] and the long run to that of [3.85].)[24]

$$e_L = (a_0 + a_1 + \ldots + a_\infty) P_i P_y^{-1} X_i^{-1} \tag{3.85}$$

Obviously, then the relative elasticity or reaction in demand for a factor relative to prices and other variables can differ greatly between short-run and long-run periods.

The analysis above does not link the prices on which plans are made for one period with prices of other periods. Instead the time aspects are reflected in the physical, psychological and institutional factors which link outputs of different periods. In farming particularly, the prices among periods are themselves linked, not only in the structure of the economy, but also in the expectations of farmers.[25] Viewed in alternative fashion, we can compare short-run and long-run adjustment and elasticity coefficients for particular resource demand functions. We can attempt to link prices on which plans are based and prices of other periods. In the preceding figures and equations, changes in price were assumed to be known and permanent. (This assumption also was

[24] Tinbergen, Jan. Long-term foreign trade elasticities. Metroeconomica. Vol. 1. 1954. pp. 20-31.

[25] For example, see Heady, Earl O. Economics of Agricultural Production and Resource Use. Prentice-Hall. New York. 1952. pp. 475-95. Some of the simple models presented here are perhaps more widely used than those discussed later in the book.

implied in the classical static demand analysis presented earlier.) But in dynamic models it is necessary to search for the price expectations which are relevant to resource demand quantities. One concept in relevant price is that of expected normal price.[26] Here we will concern ourselves with factor/product price ratios and let $R = P_i P_y^{-1}$ be the actual price ratio where we designate it as R_t, R_{t-1}, ... for the current period, the preceding period, etc. The average or long-run expected normal price level is designated as $\overline{R}$. As one alternative, the expected normal price of the current period, $\overline{R}_t$, may then be related to the expected normal price, $\overline{R}_{t-1}$, and the actual price, R_{t-1}, of the previous period as in (3.86) and (3.87) where e is the elasticity of expectation and $0 < e \leq 1$.

$$\overline{R}_t - \overline{R}_{t-1} = e(R_{t-1} - \overline{R}_{t-1}) \tag{3.86}$$

$$\overline{R}_t = eR_{t-1} + (1 - e)\overline{R}_{t-1} \tag{3.87}$$

In terms of (3.86) the relationship between expected price for year t and t-1 is the difference between the actual price and expected price in period t-1 multiplied by e. If e is zero, the actual price of previous periods have no effect on expected price.

On the other hand, if e = 1, expected normal price would be equal to the t-1 actual price. In other words, the expectation model then is simply one which extends the value of the current year into the future. The error, E, of this expectation model can, in classical statistical terminology, be indicated as (3.88),

$$E = 2\sigma^2(1 - \rho) + b^2 \tag{3.88}$$

the mean square difference between realized price and expected price, and σ^2 is the equivalent of the usual variance computation. This outcome is specified between the extremes of no trend and a linear trend. If there is no trend, with ρ as the correlation coefficient for price observations between years, and b, the regression coefficient of price on time, are both zero, the expectational error is $2\sigma^2$. The farmer would be better off to use the mean price (perhaps of a previous period, if this population were to be repeated in the future) as his expectation of price since its error measured in the same manner would be only σ^2. If the farmer used a normal price, based on concept of normality in a particular period, and the normal price differed from the mean of the price universe by c, the expectational error, measured as the mean square difference between R_t and R_{t-1}, over time would then be (3.89).

$$E = \sigma^2 + c^2 \tag{3.89}$$

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[26]Cf. Nerlove, Marc. The Dynamics of Supply: Estimation of Farmers' Response to Price. Johns Hopkins Press. Baltimore. 1954. pp. 25-27; and Hicks, J. R. Value and Capital. Oxford University Press. Oxford. 1946. pp. 204-6.

In cases such as this, we would expect the magnitude of E (c and b) to affect the rate at which resource input is altered in response to price change because of the error and uncertainty involved. Hence, the e in (3.86) cannot completely explain the price upon which decisions are based, R_t likely being discounted in relation to input decisions depending on the value of E. While these considerations and the use of other expectation models are of obvious importance in linking the prices and resource investments of different periods, we continue the discussion in the somewhat less realistic framework which does not incorporate them.

Seldom, of course, are the prices of one period linked only to those of the previous period. Given the value of R_t in (3.87), we would expect the similar linkage for $\overline{R}_{t-1}$ in (3.90).

$$\overline{R}_{t-1} = eR_{t-2} + (1 - e)\overline{R}_{t-2} \tag{3.90}$$

Continuing the linkage and substituting (3.90) into (3.87), the value of $\overline{R}_t$ then is logically (3.91).

$$\overline{R}_t = eR_{t-1} + e(1 - e)R_{t-2} + (1 - e)^2\overline{R}_{t-2} + \dots \qquad 0 < e \leq 1 \tag{3.91}$$

We now define a resource demand function in period t as (3.92) where $\overline{X}_t$ is the desired or optimum level of input, given the expected factor-product price ratio $\overline{R}_t$.

$$\overline{X}_t = a + b\overline{R}_t \tag{3.92}$$

If input or resource demand in t is (3.92), (for the purpose of simplicity we do not include the random error term u in the demand equation of this chapter), the expression for $\overline{R}_t$ from (3.91) is substituted into (3.92) to form (3.93), where desired input level is linked to price ratios of the past.

$$\overline{X}_t = a + b[eR_{t-1} + e(1 - e)R_{t-2} + (1 - e)^2\overline{R}_{t-2} + \dots] \qquad 0 < e \leq 1 \tag{3.93}$$

Many other values might exist for $\overline{R}_t$, in its linkage to the past, as in (3.94) for example.[27]

$$\overline{R}_t = n + e(R_{t-1} - R_{t-2}) + e^2(R_{t-2} - R_{t-3}) + \dots \qquad 0 < e \leq 1 \tag{3.94}$$

We could substitute the equivalent expectation values of R_{t-1}, R_{t-2}, ... into equation (3.94). Eisner and others have applied such alternatives.[28]

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[27]The resource demand equation in period t then becomes:

$$X_t = a + bn + b[e(R_{t-1} - R_{t-2}) + e^2(R_{t-2} - R_{t-3}) + \dots]$$

[28]Eisner, R. Expectations, plans and capital expenditures. Conference on expectations, uncertainty and business behavior. (Edited by M. J. Bowman, Univ. of Chicago); and Yeh, M. H. Fertilizer Demand Functions. Unpublished Ph.D. thesis. Library, Iowa State University. Ames. 1958.

However, for purposes of brevity, we consider further only some of the more orthodox expectation and lag models below.

The fact is, even apart from the expectation of price in the decision period and its linkage to the past, inputs may be linked between production periods as illustrated in (3.82). A model paralleling the earlier price model also may be relevant and facilitates the explanation of differences between short-run and long-run elasticity coefficients. For any one price situation, a long-run normal or desired (some concept of optimum) resource input, $\overline{X}_t$, may exist. It is not, as pointed out above, attained in a single period. We also suppose that the actual input for the current or short-run period X_t, that being planned, will be related to both (a) this optimum or desired level, $\overline{X}_t$, in the long run and (b) the actual input, X_{t-1}, of the previous period.

$$(3.95) \qquad (X_t - X_{t-1}) = g(\overline{X}_t - X_{t-1}) \quad 0 < g \leq 1$$

In (3.95) the difference between actual input in t and actual input in t-1 is stated to be a g proportion of the difference between desired input in t and actual input in t-1. We will call g the adjustment coefficient. This formulation supposes a given price level, with a gradual adjustment of input X to the desired level of use. The adjustment is gradual because of physical, psychological or institutional restraints. As the difference between $\overline{X}_t$ and X_{t-1} becomes smaller with time, the ΔX_i or resource addition for a particular year also will decline. By defining X_{t-1}, X_{t-2}, ... in a similar manner to (3.96), X_t can be defined as a function of inputs in a sequence of other periods, although the particular algebraic form may have less logic for agriculture than many other models (see Chapter 10) which can be specified.

$$(3.96) \qquad X_t = X_{t-1} + g(\overline{X}_t - X_{t-1})$$

At the outset of some innovations, investment in successive years may be an increasing function of resource use in early years, with the increment of investment later declining. This might be the expected case as the farmer "makes some tries" and initially gains experience plus increased capital for further investment. It is possible to combine the adjustment and expectation models by substituting the value of X_t in the resource demand equation (3.93) into (3.96) to obtain the value of X_t taken with a distributed lag. Resource input in the current period is linked to those of previous periods and in relation to a rate of input adjustment indicated by g and an expected current price ratio linked to past price ratios by an expectation coefficient e.

$$(3.97) \qquad \overline{X}_t = a + b\overline{R}_t + c\overline{F}_t$$

Instead, we extend the demand equation to (3.97) where $\overline{R}_t$ is the expected ratio of price of the i-th factor to commodity price, $\overline{R} = P_i P_y^{-1}$, and $\overline{F}_t$ is the expected ratio of the i-th factor price to the j-th factor price, $F = P_i P_j^{-1}$, for the period t and $\overline{X}_t$ is the desired or optimum level of inputs. Substituting this resource demand function into the equation (3.96), we obtain:

$$X_t = ag + bg\overline{R}_t + cg\overline{F}_t + (1 - g)X_{t-1} \tag{3.98}$$

Demand or input in the current period, then, is a function of the expected factor/product and factor/factor price ratios of the same period and of the actual input of the previous period. Where the quantity and price ratios are measured in logarithms, bg is the short-run elasticity of resource demand with respect to the expected factor/product price ratio, and cg is the short-run elasticity with respect to the expected factor/factor price ratio. With knowledge that $1 - g = \lambda$ (λ estimated as a regression coefficient in quantitative analysis), we can compute the adjustment coefficient as $g = 1 - \lambda$. From (3.98) it is apparent that when g is zero, adjustments are never made and the demand quantity in the current period is equal to that of the previous period. If g equals 1, all adjustments are made in the current period and current resource demand is not directly linked to the value of X in the previous period. The long-run elasticities b and c in equation (3.97) can be found merely by dividing the least-square coefficients bg and cg in (3.98) by the adjustment coefficient g (the variables are assumed to be in logarithms). If g is small (λ is large), the long-run elasticity is much greater than the short-run elasticity of resource demand relative to factor/product or factor/factor price ratios. A large value of g means that most of the adjustment in resource input is made in the first period and the long-run demand elasticity is only slightly larger than the short-run elasticity.

We have outlined some simple models suggesting the linkage of resource demand in one period with inputs and prices of earlier periods. These simple dynamic models are perhaps elementary in respect to those most appropriate for real world situations. They are, however, realistic steps: (a) beyond the static models discussed earlier in explaining changing demand and use of resources over time, and (b) exposing some possible models for quantitative estimates. (Where the variables in (3.98) are not measured in logarithms, elasticities must be computed other than directly as the coefficients of the variables.) In later chapters empirical estimates of resource demand functions and other relationships relating to commodities and factors are made by numerous variations of both the static and the dynamic models outlined in this chapter.

Additional Conditions Suggesting the Need for Expectation and Adjustment Models

Aside from uncertainty, trends in economic growth and factor prices also change demand for specific factors through their effect on resource structure and scale economies. Physical and institutional restraints cause lagged adjustment in resource employment even where subjective certainty exists in the minds of decision makers. Under economic growth, prices of capital items fall relative to the price of labor. Because machinery and equipment come in large, discrete units, they have greater advantage than horse or manpower only when used with greater inputs of complementary resources such as land. The supply of land is fixed in farming communities, and firms can expand only as other farm businesses are liquidated and their land is relinquished. Individual farms can only add to land input in discrete and discontinuous fashion, and the aggregate of remaining farms can only distribute this adjustment over time as farm operators retire or themselves express distributed lag reaction in their eventual decision to sell at higher land prices.

Additions of complementary resources such as more land or livestock typically take place only as investment capital availability is increased. For both individual farms and the aggregate of remaining farms, the adjustment is distributed with a lag over time, thus causing a similar lagged pattern in increased demand for resources specialized to particular products, in the size and numbers of farm business units and in the size of the farm work force.