17

Index Numbers

The next few chapters deal with "parity prices." These parity prices are index numbers, and an understanding of parity prices requires first some understanding of how index numbers are constructed. Accordingly, this chapter deals with the theory of index numbers, and applies the theory to computation of parity prices. The following chapters then deal with parity prices directly.

INDEX NUMBERS ARE WEIGHTED AVERAGES

The prices of any single commodity, such as hogs, may be recorded over periods of time and analyzed directly in their original form. But the prices of farm products, as a group, cannot be studied so simply. If we want to know how the prices of farm products, as a group, have changed over the years, we must first add up or average the prices of the different products into a single figure for each year. This single figure is called an index number.

The averaging process needs to take into account the fact that some products are more important than others. A simple averaging process would give equal weight to each item; but a change in the price of beef cattle is more important, and should carry greater weight in the index, than a change in the price of avocados.

Many different formulas and systems of weighting can be used in constructing index numbers, and most of them give different results. Good economic and statistical judgment is required, therefore, in selecting the formula and weights to be used.

DIFFERENT FORMULAS

The two chief kinds of index number formulas are (1) the ratio of aggregates and (2) the average of relatives.

In the formula for the ratio of aggregates, prices of the different products are added up (aggregated) each year. The series of sums then is the series of index numbers. The formula is simply:

$$P = \frac{\Sigma p_1}{\Sigma p_0}$$

P is the price index
p is the price of a product
$_1$ refers to the given year which is being compared with the base
$_0$ refers to the base period from which changes are measured.

But this simple formula would give all the products equal weighting. In order to give each product its proper weight, commensurate with its importance, its price should be weighted (multiplied) by the quantity sold.

The use of quantity weights neatly compensates for the erratic effect of the choice of the size of the unit used to measure the quantities. If the quantity weights were expressed in pounds, that would give the item 2,000 times as much weight as if it were expressed in tons. But this is exactly compensated for by the fact that the price for a pound would be only one 2,000th as high as the price for a ton.

A question arises whether the price should be weighted by the quantity sold in the base year, or the given year, or some intermediate period. A widely used formula, the Laspeyres formula, uses quantities during the base year. This formula is:

$$P = \frac{\Sigma p_1 q_0}{\Sigma p_0 q_0}.$$

Table 17.1 gives a simple illustration of this formula, showing how to compute a price index based on two products, *A* and *B*.

The central part of Table 17.1 shows that the sum (aggregate) in the base year was 14. In the given year, the sum was 18. It is easy to see how much prices have risen in the given year if the index in that year is expressed as a percentage of the index in the base year. When this is done here, it shows that prices have risen from an index of 100 in the base year to an index of $(18 \times 100)/14 = 125.7$ in the given year; that is, they have risen 25.7 per cent.

AVERAGE OF RELATIVES

The computation of the index numbers by the use of the other formula, the average of relatives, is shown in the right-hand part of Table 17.1.

The relative change in the price of commodity *A* is $3/4 = 0.75$. For commodity *B*, it is $4/2 = 2$.

TABLE 17.1
COMPUTING INDEX NUMBERS BY TWO FORMULAS

			Ratio of Aggregates			Average of Relatives		
Prices			Quantities	Weighted prices		Relatives	Value weights	Relatives weighted by values
	Base year p_0	Given year p_1	Base year q_0	Base year p_0q_0	Given year p_1q_0	$\frac{p_1}{p_0}$	p_0q_0	
A..........	4	3	2	8	6	0.75	8	6.0
B..........	2	4	3	6	12	2.0	6	12.0
Sum........	....	....	5	14	18	......	14	18
Index.......	....	....	....	100	125.7	......	100	125.7

When weighting these relatives, we need to compensate for the erratic effect of the choice of the size of the quantity units, as in the other formula. For this purpose, quantity will not do, for the number of the quantity units is not offset by the size of the price; the price relative is unaffected by the size of the quantity units. So we weight by base-year values (*prices* $\times$ *quantities*) instead of by quantities alone, as shown in the last column of Table 17.1.

The index of prices, the average of relatives, in the given year comes out to be $(18 \times 100)/14 = 125.7$, exactly the same as with the ratio of aggregates formula. The two formulas, in fact, are merely different forms of the same formula, for the average of relatives formula with base-year values is:

$$\frac{\Sigma \frac{p_1}{p_0} p_0 \, q_0}{\Sigma \, p_0 \, q_0}$$

which by cancellation of the p_0's in the numerator reduces to the Laspeyres ratio of aggregates formula:

$$\frac{\Sigma \, p_1 \, q_0}{\Sigma \, p_0 \, q_0}$$

The average of relatives weighted by the product of base-year prices and given-year quantities ($p_0 \, q_1$) reduces to the Paasche formula. (The Paasche formula is like the Laspeyres formula but uses given-year weights instead of base-year weights).

The choice between the two formulas depends chiefly upon the desire for relatives showing the movement of different prices or subgroups of prices separately. If there is no need for these relatives, the ratio of aggregates is the simplest and involves the least amount of computation. But in many cases the relatives of different prices or subgroups of prices are useful and worth the extra computation involved.

WEIGHT-BASE PERIODS

Practical complications arise when the quantities used do not remain constant over a period of years. Tastes change, and new products partially replace the old. We now use more tractors and fewer horses and eat more fruits and vegetables and less bread.

The quantities used in the base year, then, gradually become inaccurate for the given (current) years. How can this be avoided?

One suggestion is to use given-year weights instead of base-year weights, using the Paasche formula:

$$\frac{\Sigma\, p_1\, q_1}{\Sigma\, p_0\, q_1}.$$

But this merely means that the quantities used in the current year are inaccurate for the base and other early years.

Furthermore, both formulas are likely to be not only inaccurate, but biased. If tastes remain constant, but the relative costs of producing different goods change, the relative prices of the goods will change too. Consumers then will buy less of the high priced items and more of the cheaper ones. The Laspeyres index with base-year weights would show no change, when actually consumers would be getting the same amount of satisfaction for less money than before; the Laspeyres formula then has an upward bias.

The Laspeyres formula has a downward bias if tastes change but relative costs remain constant. In both cases, the Paasche formula has the opposite bias. These biases may be combined to offset each other, at least in part. Marshall and Edgeworth suggested using crossed (that is, average) weights. Their formula is:

$$\frac{\Sigma p_1\ (q_0 + q_1)}{\Sigma p_0\ (q_0 + q_1)}.$$

Irving Fisher made a different suggestion. He recommended using the geometric mean of the Laspeyres and Paasche formulas

$$\left(\frac{\Sigma p_1\, q_0}{\Sigma p_0\, q_0} \cdot \frac{\Sigma p_1\, q_1}{\Sigma p_0\, q_1}\right)^{1/2}$$

This is sometimes referred to as Fisher's "Ideal" formula.

Among these formulas, the Fisher formula meets two tests — the time-reversal test and the factor-reversal test. That is, it gives consistent results forward and backward, and with the factors (prices and quantities) interchanged.

But the practical objections to these formulas are formidable. It is difficult to say just what an index number computed by either of these formulas does measure. The fact that the formulas average two opposite biases or inaccuracies does not guarantee that they provide accurate answers. Furthermore, the computations involved in the use of the Fisher formula are more than twice as laborious as those for the Laspeyres or Paasche formulas; and data showing the quantities of the different goods purchased, to be used as current weights, never are actually current but are gathered only at irregular intervals by special surveys, usually several years apart.

Accordingly, the USDA worked out what it considered to be the least unsatisfactory solution.[1] It has adopted a formula for the computation of its indexes of prices received and paid by farmers which uses neither base-year weights nor current-year weights. The USDA recognized that neither base-year weights nor given-year weights, nor any single combination of the two, could be appropriate over a long period; so it used two different sets of weights based on averages for two different periods. It used weights based on averages for the period 1924–29 for its indexes up through March, 1935, and weights based on averages for the period 1937–41 from March, 1935 thereafter, the indexes being linked at March, 1935. In January, 1959, the weight-base period for the index of prices *paid* was moved up to 1955. The weight-base period for the index of prices *received* was moved up to 1953–57.

BRITISH INDEXES OF AGRICULTURAL PRICES

The British indexes of agricultural prices originally were based on 1906–08, and then 1911–13, with 1908 gross value quantity weights. This base became increasingly inaccurate with the passage of time. Accordingly, after 30 years, the British Ministry of Agriculture revised its formula. In 1938, it shifted the price base to 1927–29, and shifted its weight base, not to a more recent *fixed* period, but to a moving average of the five years immediately preceding the current year, each year.

[1] B. Ralph Stauber, Nathan M. Koffsky, and C. Kyle Randall, "The Revised Price Indexes," *Agricultural Economics Research,* BAE, USDA, Vol. 2, No. 2, April, 1950.

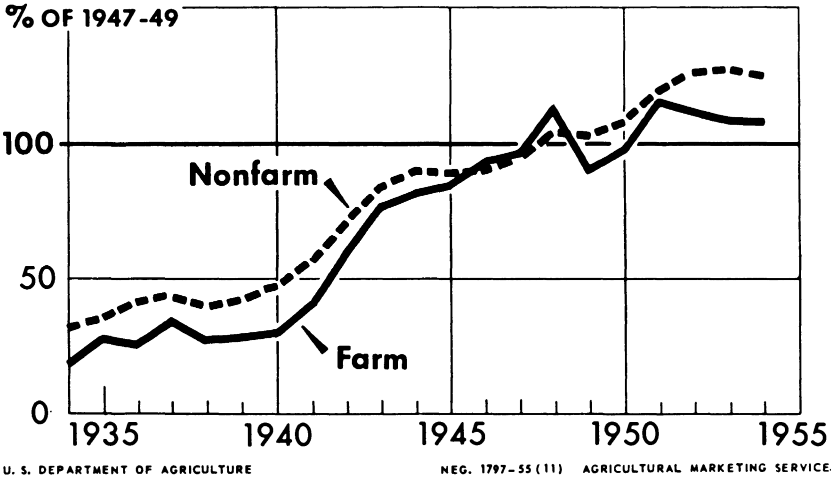

Fig. 17.1 — Indexes of per capita farm and nonfarm income, computed on a 1947–49 base.

"For example, the index number for 1937 would be obtained by using weights derived from the average annual output of the five years ending 1935–36. Weighting would thus change each year, the latest year's output being added and the earliest year's output being dropped. For the purpose of calculating the index number for each year, the output chosen for that year would be valued at the prices of that year and of the base year; the index number would represent the ratio between the two values. Under such a system, however long the series were continued, it would be possible throughout the series to make accurate comparison between prices in years not very far part, and as satisfactory a comparison as possible, without making separate calculations, between two years separated by a long period, bearing in mind that in agriculture the changes in composition of the total output are gradual."[2]

This formula is a kind of Paasche formula, with current (five-year average) weights. A similar formula is used in New Zealand.

EFFECTS OF USING DIFFERENT BASE PERIODS

The selection of a base period only moves a curve up or down in relation to the base line or to another curve on a chart. But this is a very important movement. We already saw in the first chapter (Fig. 1.1) that using an earlier base would give agricultural prices the appearance of running higher than nonagricultural prices most of the time, instead of lower, as it does in Figure 1.1.

[2] C. T. Houghton, "A New Index Number of Agricultural Prices," *Journal of the Royal Statistical Society,* CI (Part II), pp. 294–95, 1938.

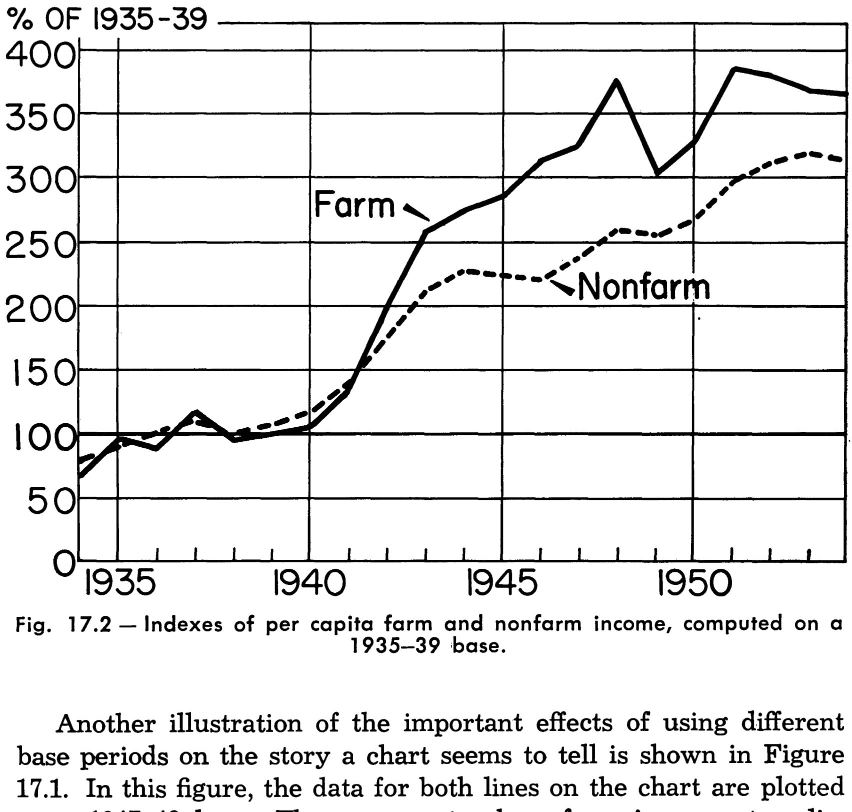

Fig. 17.2 — Indexes of per capita farm and nonfarm income, computed on a 1935–39 base.

Another illustration of the important effects of using different base periods on the story a chart seems to tell is shown in Figure 17.1. In this figure, the data for both lines on the chart are plotted on a 1947–49 base. They appear to show farm income at a disadvantage relative to nonfarm except for a brief period following World War II. The caption for the original USDA chart was "Income per person of farmers lagging behind that of nonfarm people."

The same data are converted to a 1935–39 base in Figure 17.2. This is done simply by dividing each item in the series by the average value for that series in 1935–39. Nothing is changed but the relation, the height of the two lines relative to one another. But the effect on the story the chart seems to tell is striking. It makes farm income look superior to nonfarm income most of the time, and so much superior that the decline, so prominent in Figure 17.1, is hardly discernible in Figure 17.2.

Both charts are equally illusory, though in opposite directions. Figure 17.1 is based on a period when farm incomes were unusually high relative to nonfarm incomes, so that the relation in other years looks unfavorable to agriculture. Conversely, Figure 17.2 is based on a period when farm incomes were unusually low. The reader of any charts of this nature needs to study them carefully before accepting the conclusions they imply.