Diffraction of block substitution tilings

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For point sets and tilings that can are based on the cut and project method, one has a good understanding of the correlations in the structure, and of the corresponding diffraction; see [1] and references therein for details. In particular, cut and project systems with sufficiently nice windows are pure point diffractive.

The situation is less favourable for systems that are defined by substitution or inflation rules, in particular if one leaves the realm of Pisot substitutions. Indeed, examples for pure point diffraction (such as the Fibonacci chain or other systems that also allow a cut and project interpretation), singular continuous diffraction (for which the paradigm is the Thue-Morse chain [2]) as well as absolutely continuous diffraction (for example for the Rudin-Shapiro chain and its generalisations [3]) exist, so all spectral types are possible – and indeed also any combinations of spectral components of these types [4].

Here, we consider block substitution systems and study their correlations in terms of exact renormalisation equations in the spirit of [5], which gives access to the diffraction via Fourier transform. In particular, we are aiming at sufficient criteria to rule out the existence of absolutely continuous components. The renormalisation approach paves the way for tackling some seemingly very complex systems, in particular this applies also to tilings which do not have finite local complexity. While we mainly concentrate on block substitutions, we also consider a familiar example which lacks finite local complexity, the Frank-Robinson tiling. Using our methods, it can be shown that this tiling has purely singular continuous diffraction [5]. This approach applies to large classes of substitution tilings, as shown in [6] for family of primitive binary substitution tilings.

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