Physics of limit-periodic tilings

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The first known examples of aperiodic prototiles, due to Berger and Robinson, were sets that forced limit-periodic tilings. These are tilings that consist of a union of periodic sets of tiles with ever-increasing lattice constants, leading to a dense set of Bragg peaks in reciprocal space. I will present results on phase formation and diffraction properties of limit-periodic systems.

The Taylor-Socolar aperiodic prototile [1] forces a limit-periodic structure with hexagonal point symmetry. Lattice models for the formation of the limit-periodic pattern from a triangular lattice of randomly oriented tiles show that the limit-periodic structure can form in a robust way via a slow quench [2]. Even when some of the matching rules necessary for forcing nonperiodicity of the tiling are relaxed, the limit-periodic phase forms through an infinite sequence of phase transitions in which lattices with ever-increasing lattice constants crystallize one by one. These results suggest that real colloidal or atomic systems may form limit-periodic phases, though none has yet been found.

Limit-periodic tilings in 1D can be produced by substitution procedures analogous to the well-known methods for generating Fibonacci (quasiperiodic) tilings. As recent results in the study of hyperuniform point sets indicate, a feature of considerable import for the physical properties of noncrystalline systems is the scaling of amplitudes of long wavelength fluctuations, as reflected in the scaling of structure factor intensity as the wavenumber tends to zero, which requires careful analysis for systems with diffraction patterns consisting of dense sets of Bragg peaks [3]. Limit-periodic systems can exhibit the full range of scaling behaviors, from strongly suppressed to strongly enhanced long wavelength fluctuations as compared to completely random (Poisson) point sets.

1. J. E. S. Socolar & J. M. Taylor, *Journal of Combinatorial Theory, Series A* **118**, (2011) 2207.

2. C. Marcoux, T. W. Byington, Z. Qian, P. Charbonneau, J. E. S. Socolar, *Phys. Rev. E* **90**, (2014) 012136.

3. E. C. Oğuz, J. E. S. Socolar, P. J. Steinhardt, S. Torquato. *Phys. Rev. B*, **95**, (2017) 054119.

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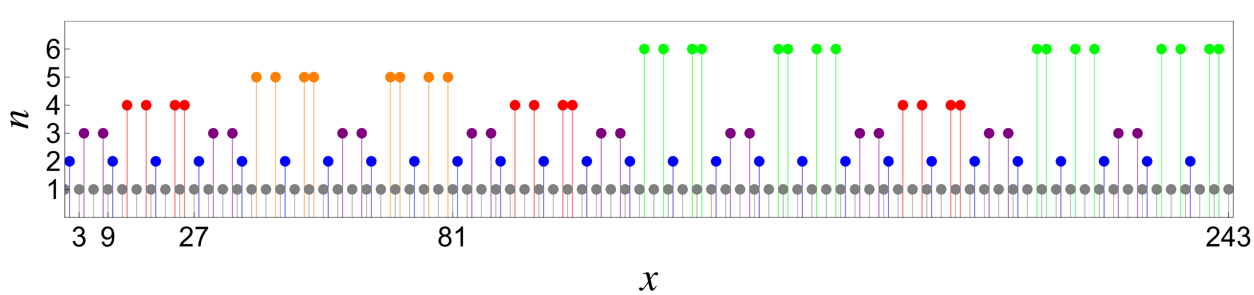

Figure 1. A portion of a limit-periodic 1D tiling with a novel type of scaling behavior.