Effect of Pendulation on an $SO(3)$-Based Attitude Estimator for Precision Pointing of an Atmospheric Balloon-Borne Platform

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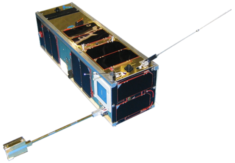
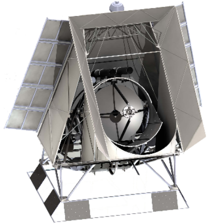
Background and Motivation

Attitude Estimation

- The process of estimating the orientation of a body from available measurements.
    - Rate gyros, accelerometers, magnetometers, sun sensors, etc.
- Required for autonomous maneuvers in many robotic vehicles.
    - Spacecraft, UAVs, and many balloon-borne platforms.
- Typically accomplished using an extended Kalman filter (EKF).



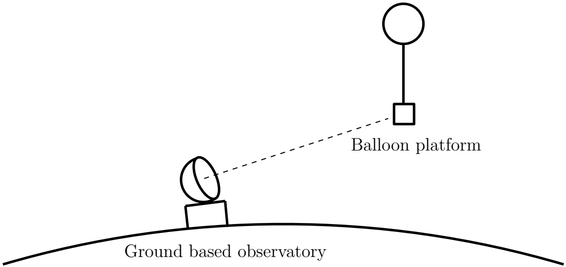


Credit: UTIAS and BLAST.

Background and Motivation

Motivating Example: McHAB

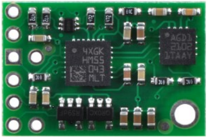
- Developing an atmospheric balloon platform to carry a calibrating microwave source.
- An adequate attitude control system that will enable precision pointing is needed.

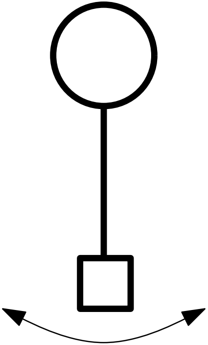


Background and Motivation

Motivating Example: McHAB

- Low cost IMU:
  - Accelerometer.
  - Magnetometer.
  - Rate gyro.
- Traditional EKF technique is difficult to apply robustly with poor quality sensors.
- Nonlinearities and non-Gaussian noise leads to poor performance.
- Motivated the use of a nonlinear estimator.





Accelerometer

- Low frequency disturbances (e.g. pendulation) can result in inaccurate measurements.

Background and Motivation

Background and Motivation

Attitude Parameterizations

- The rotation matrix uniquely and globally describes the attitude of a body.
  - Belong to $SO(3)$, $SO(3) = \{\mathbf{C} \in \mathbb{R}^{3\times3} | \mathbf{C}^\mathsf{T}\mathbf{C} = \mathbf{1},\ \det\mathbf{C} = +1\}$.
- Three-set parameterizations (Euler angles, Gibbs parameters, etc.).
  - Components are independent.
  - Presence of singularities.
- Constrained four-set parameterization (unit quaternion).
  - No singularities.
  - Non-unique.

Background and Motivation

This work is an investigation of the effects of pendulation on a nonlinear $SO(3)$-based attitude estimator.

- Model balloon-borne platform dynamics.
- Model IMU measurements.
- Review the nonlinear estimator.
- Test the robustness of the estimator in simulation.

Preliminaries

Rigid-Body Kinematics

- Poisson's equation,

$$\dot{\mathbf{C}}_{ba} + \boldsymbol{\omega}_b^{ba^\times} \mathbf{C}_{ba} = \mathbf{0}, \quad \mathbf{C}_{ba} \in SO(3).$$

- $\boldsymbol{\omega}_b^{ba} \in \mathbb{R}^3$ is the angular velocity.
- "Cross" operator: $(\cdot)^\times : \mathbb{R}^3 \rightarrow \mathfrak{so}(3),\ \mathfrak{so}(3) = \{\mathbf{S} \in \mathbb{R}^{3\times 3} | \mathbf{S}^\mathsf{T} = -\mathbf{S}\}$.

Dynamic Model

Model Kinematics and Dynamics

The platform is modelled as a rigid body constrained to a rigid pendulum.

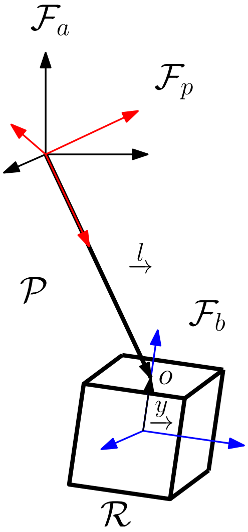
- Kinematics

$$
\begin{aligned}
\dot{\mathbf{C}}_{ba} + \boldsymbol{\omega}_b^{ba^\times}\mathbf{C}_{ba} &= \mathbf{0}, \\
\dot{\mathbf{C}}_{pa} + \boldsymbol{\omega}_p^{pa^\times}\mathbf{C}_{pa} &= \mathbf{0}.
\end{aligned}
$$

- Employing Lagrange's equation for constrained systems leads to the following dynamics:

$$
\mathbf{M}\dot{\boldsymbol{\nu}} + \boldsymbol{\tau}_{non}(\boldsymbol{\nu}) = \boldsymbol{\tau}^d + \boldsymbol{\tau}^c,
$$

where $\boldsymbol{\nu} = [\boldsymbol{\omega}_b^{ba^\mathsf{T}} \quad \boldsymbol{\omega}_p^{pa^\mathsf{T}}]^\mathsf{T}$.



Dynamic Model

Linear Acceleration

- Position of $\mathcal{F}_b$ relative to $\mathcal{F}_a$,

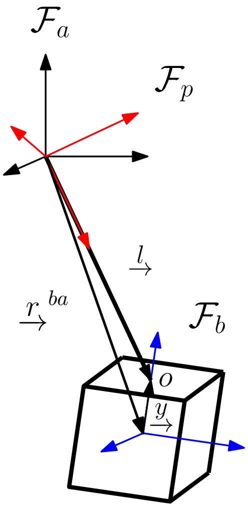
$$\underset{\rightarrow}{r}^{ba} = \underset{\rightarrow}{l} - \underset{\rightarrow}{y}.$$

- Acceleration,

$$\begin{aligned}\underset{\rightarrow}{a}^{ba} &= \overset{\circ}{\underset{\rightarrow}{\omega}^{pa}} \times \underset{\rightarrow}{l} + \underset{\rightarrow}{\omega}^{pa} \times \underset{\rightarrow}{\omega}^{pa} \times \underset{\rightarrow}{l} \\ &\quad - \underset{\rightarrow}{\omega}^{ba'} \times \underset{\rightarrow}{y} - \underset{\rightarrow}{\omega}^{ba} \times \underset{\rightarrow}{\omega}^{ba} \times \underset{\rightarrow}{y}.\end{aligned}$$

- Acceleration resolved in $\mathcal{F}_b$,

$$\begin{aligned}\mathbf{a}_b^{ba} &= \mathbf{C}_{bp}\left(\dot{\boldsymbol{\omega}}_p^{pa^\times}\mathbf{l}_p + \boldsymbol{\omega}_p^{pa^\times}\boldsymbol{\omega}_p^{pa^\times}\mathbf{l}_p\right) \\ &\quad - \dot{\boldsymbol{\omega}}_b^{ba^\times}\mathbf{y}_b - \boldsymbol{\omega}_b^{ba^\times}\boldsymbol{\omega}_b^{ba^\times}\mathbf{y}_b.\end{aligned}$$



Measurement Model

Measurement Model

- Rate gyro

$$\boldsymbol{\omega}^{y} = \boldsymbol{\omega}_{b}^{ba} + \mathbf{b} + \boldsymbol{\mu}.$$

- Magnetometer

$$\mathbf{m}_{b}^{y} = \mathbf{C}_{ba}\mathbf{m}_{a} + \boldsymbol{\mu}^{m}.$$

- Accelerometer nominally measures

$$\mathbf{g}_{b}^{y} = \mathbf{C}_{ba}\mathbf{g}_{a} + \boldsymbol{\mu}^{g},$$

  where $\mathbf{g}_{a} = \begin{bmatrix} 0 & 0 & -g \end{bmatrix}^{\mathsf{T}}$.

- We will consider

$$\mathbf{g}_{b}^{y} = \mathbf{C}_{ba}\mathbf{g}_{a} + \mathbf{a}_{b}^{ba} + \boldsymbol{\mu}^{g}.$$

$SO(3)$-Based Estimator

We will implement the estimator proposed by Mahony et al. (2005).

Estimator Dynamics

$$
\begin{aligned}
\dot{\mathbf{C}}_{ea} &= -(\boldsymbol{\omega}^y - \hat{\mathbf{b}} + \boldsymbol{\sigma})^\times \mathbf{C}_{ea}, \\
\dot{\hat{\mathbf{b}}} &= -\frac{k_i}{k}\boldsymbol{\sigma},
\end{aligned}
$$

where $\mathbf{C}_{ea}$ is the estimate of $\mathbf{C}_{ba}$, $\hat{\mathbf{b}}$ is the estimate of $\mathbf{b}$, and $\boldsymbol{\sigma}$ is the innovation. The goal is to drive $\mathbf{C}_{ea}$ to $\mathbf{C}_{ba}$.

- Innovation

$$\boldsymbol{\sigma} = -k\left(k_g \mathbf{g}_e^\times \mathbf{g}_b^y + k_m \mathbf{m}_e^\times \mathbf{m}_b^y\right),$$

  where

$$\mathbf{g}_e = \mathbf{C}_{ea}\mathbf{g}_a \quad \text{and} \quad \mathbf{m}_e = \mathbf{C}_{ea}\mathbf{m}_a.$$

- Gains k_g and k_m are chosen based on the relative confidence of the measurements.

Control

Proportional-Derivative Control Law

$$\tau_c = -k_p \hat{\theta}_3 - k_d(\omega_3^y - \hat{b}_3)$$

- Yaw, $\hat{\theta}_3$, is extracted from $\mathbf{C}_{ea}$.
- ω_3^y is the third component of the measured angular velocity.
- $\hat{b}_3$ is the third component of the estimated bias.

Control

Simulation

Simulation Parameters

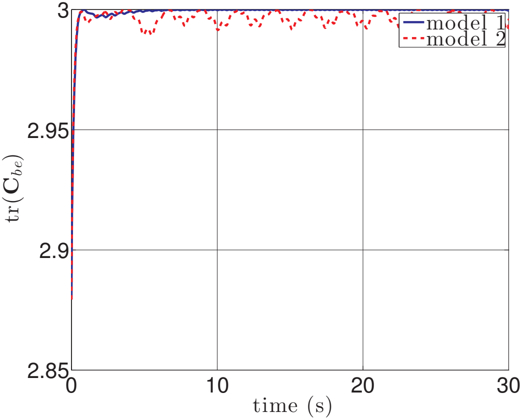
- $\boldsymbol{\omega}_b^{ba}(0) = [0 \;\; 0 \;\; 0.1]^{\mathsf{T}} \; (rad/s)$.
- $\boldsymbol{\omega}_p^{pa}(0) = [0 \;\; 0 \;\; 0]^{\mathsf{T}} \; (rad/s)$.
- $\mathbf{C}_{ba}(0) = \mathbf{C}_1(0^\circ)\mathbf{C}_2(0^\circ)\mathbf{C}_3(20^\circ)$.
- $\mathbf{C}_{pa}(0) = \mathbf{C}_1(-85^\circ)\mathbf{C}_2(0^\circ)\mathbf{C}_3(0^\circ)$.
- Disturbances are populated by flight data.

Estimator Parameters

- $k = 5$.
- First simulation: $k_g = k_m = 1$.
- Second simulation: $k_g = 0.1$ and $k_m = 1$.

Simulation

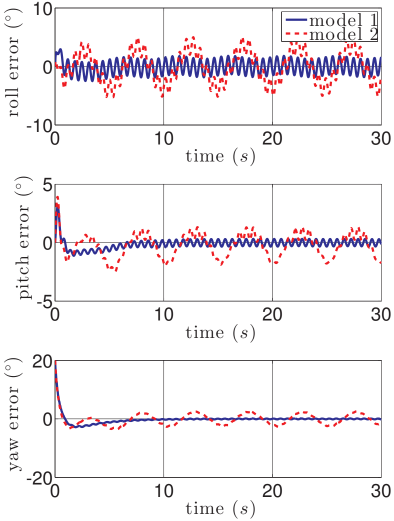
$k_g = k_m = 1$



$$\mathbf{C}_{be} = \mathbf{C}_{ba}\mathbf{C}_{ea}^{\mathsf{T}}.$$

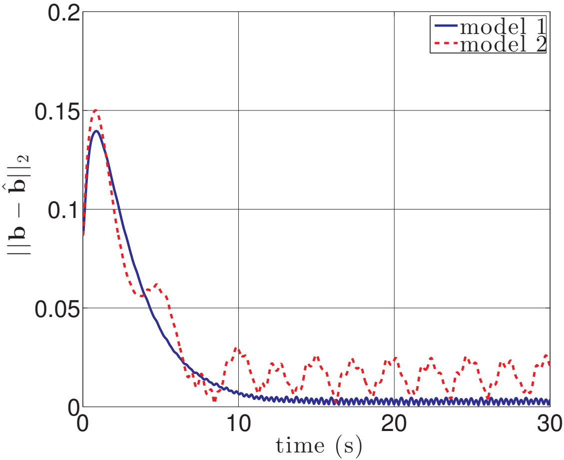
Simulation

$k_g = k_m = 1$



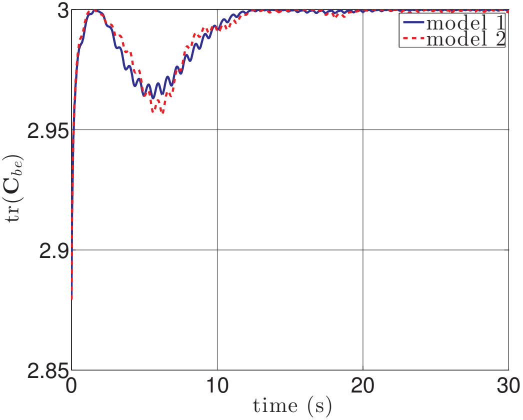
Simulation

$k_g = k_m = 1$



Simulation

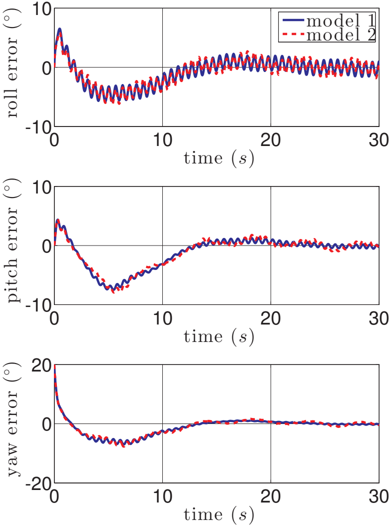
$k_g = 0.1$ and $k_m = 1$



$$\mathbf{C}_{be} = \mathbf{C}_{ba}\mathbf{C}_{ea}^{\mathsf{T}}.$$

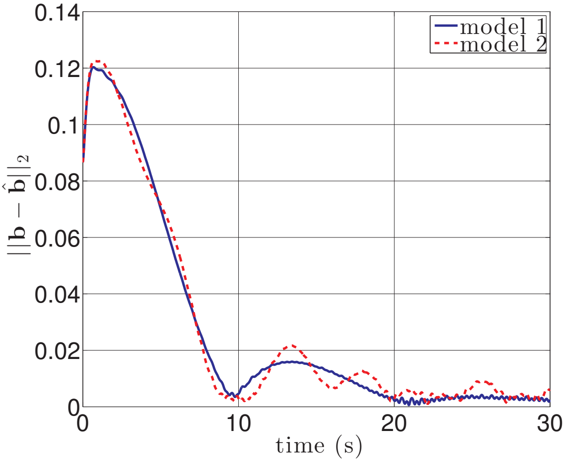
Simulation

$k_g = 0.1$ and $k_m = 1$



Simulation

$k_g = 0.1$ and $k_m = 1$



Closing Remarks

- The effects of pendulation on a nonlinear $SO(3)$-based estimator has been investigated.
- A dynamic model of the balloon-borne platform was derived.
- Acceleration of the platform was included in the accelerometer measurement.
- Effects of oscillation can be mitigated by reducing the estimator gain associated with the accelerometer.
  - Not ideal as magnetometer measurements can be unreliable.
- Future work includes the development of an estimator that directly takes acceleration due to oscillation into account.

Questions

Questions?

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Presentation created using LaTeX and Beamer.