Methodological Procedures in the Economic Analysis of Fertilizer Use Data

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Economic Analysis of Fertilizer Use Data

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Foreword

Economic considerations in fertilizer use are only now beginning to get the attention of research workers. Problems of fertilizer production and use have been studied for many years, but for the most part attention has been directed primarily at the technical aspects of the problems. As the fertilizer industry continues to adopt new processes to produce improved products, there will be a continued need for more technical studies. In addition, however, there is a tremendous need for research on the economic aspects of fertilizer use.

Fertilizer is a major item of expense on many farms and an increasingly important factor of production on many other farms in the United States. Farmers are interested in knowing how much and what kinds of fertilizer to use to maximize their profits. They also seek information on how to buy their fertilizer needs at the least cost. Agricultural extension workers and others, who provide production planning advice to farmers operating under widely varying conditions, recognize the need for more and better information on the economic aspects of fertilizer use. Research workers thus are being called on to conduct the research necessary to answer agronomic-economic questions basic to the development of practical fertilizer recommendations.

As an aid to these research workers, TVA sponsored a symposium in June 1955, bringing together a group of economists, agronomists, and statisticians. The papers which they presented have served as the basis for this book. The objective of this book is the same as that of the symposium — to present the most recent information and techniques bearing upon some of the important questions involved in studies of the economics of fertilizer use, thus facilitating the development of needed research.

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September, 1955

Preface

A technological revolution has occurred in American agriculture over the past quarter of a century. However, the role of chemical fertilizers in this change has largely escaped public notice. The public is generally aware that fewer Americans now are engaged in farming, but are producing more agricultural products; it knows that since 1930, American farm income has risen from depression depths to levels unattained in any other period in our history, or by farmers in any other part of the world. Common is the knowledge that mechanization and improvements in crops and livestock have contributed to these changes. The impacts of the tractor and of hybrid corn are well known. However, relatively few persons are aware of the phenomenal increases in fertilizer use associated with the increases in production and income.

Since the mid-1930's, fertilizer consumption in the United States has risen from 6 million to 23 million tons. In the Midwest, the increase in fertilizer use has been particularly striking, being 78-fold in Illinois and 130-fold in Iowa. In Tennessee, where fertilizers have been in common use for many years, the increase has been 6-fold during the period 1934-1954.

Another striking change in fertilizer use has been the rapid rise in average analysis. In the 35-year period 1900-1934, the plant food content of American fertilizer increased by only four units. However, in the 19-year period 1934-1952, the increase amounted to nearly nine units. This improvement in average analysis is a reflection of changes in fertilizer production technology. First the United States Department of Agriculture and then the Tennessee Valley Authority began programs of research on fertilizer production problems. United States Department of Agriculture laboratories at Beltsville, Maryland, and the Tennessee Valley Authority laboratories and pilot plants at Muscle Shoals, Alabama, made important contributions to fertilizer production technology. In addition, fertilizer firms expanded their investments in research. The result of all these research efforts has been an expanding fertilizer industry in which obsolete plants and processes are being replaced by facilities to produce better fertilizers at lower costs to farmers. Demand and use of fertilizer has grown similarly, and the trend will continue upward — given economic stability and further research and education in the production and use of chemical fertilizers.

The need for research on fertilizers and fertilization at a time when the Nation's warehouses are filled with stored food items and when production controls are in use may be questioned. However, the ultimate economic goals of a society are never reached by placing restraints on imagination and ingenuity in research. Moreover, the research with which this book deals is directed to problems for which solutions are to be sought five years or more in the future. With a rapidly growing population, and one which is increasing in urbanization, farm technologies need to be improved still further in order to increase farm output in the decades ahead. Then, too, a more efficient farm industry with more production from fewer resources is in line with national economic growth. Further strides in efficiency allow food production with a minimum of resources, so that the Nation can produce more of those semi-luxury and other goods which characterize a wealthy society.

Research on fertilizers and crop response to fertilizer use is greatly needed in other parts of the world, even more so than in the United States. Fortunately, the United Nations and the United States, with their technical assistance programs, have assisted in maintaining the increase of the world's food supply at a level equal to the need. This is not to say the problem of hunger has been completely solved. Without being famine-stricken, there still are millions of people who are chronically hungry — people who are alive but who are so poorly nourished that they lack the energy to work efficiently. More important, their physiological status prevents full development of the capacities of the human resource and the personal satisfactions which accompany such developments.

Hungry people fall easy prey to diseases, not only diseases of the body but also diseases of the mind, robbing man's faith in his ability to govern himself. If all mankind is to be provided with enough to eat, there is need for further increases in crop output. In many parts of the world, chemical fertilizers are a primary need to this end.

In recent years there have been rapid advances in the chemical and engineering aspects of fertilizer production. Agronomic research has been greatly improved; data from many experiments carried on more than five years previously are now obsolete. Still, as leading agronomists recognize, there is need for further improvement and expansion in agronomic research on fertilizer use.

Until very recently, economists have given little attention to fertilizer use as an area for empirical research. Still, nearly every elementary economics text uses fertilizer examples to illustrate the principle of diminishing returns. Because of lack of data, however, these examples have been based on hypothetical cases. While students may assume that research workers have thoroughly explored the production relationships between fertilizer use and crop use, such is not the case.

The early work of Mitscherlich and Spillman serves as a landmark on fertilizer response curves. While it appears strange that Spillman's work was not extended by any significant research on the fertilizer response economics until recently, there are sound reasons for this phenomenon. First, until quite recently, relatively few agricultural economists had enough training in mathematics and statistical techniques to use this type of analysis. Emphasis on econometrics in the graduate training has provided a larger number of economists with the requisite training. Second, there has been an overspecialization in agricultural research. Specialization can be an aid to efficiency, but in many state agricultural experiment stations, specialists have isolated themselves from each other by walls of administration and communication. In many instances, the barriers between agronomists and farm economists, for example, have grown too great for productive research in the physical response and economic use of fertilizer. Recent developments in interdepartmental cooperation promise to remove this barrier.

However, even though agronomists and economists realize a need to work together, there often is a formidable barrier in their conceptualization of the problem. Economists are inclined to look at a problem of fertilizer use as a production problem in which there is a functional relationship between input and output, with the relationship generally nonlinear in nature. On the other hand, some agronomists in designing their research have conceptualized their problem as one of comparison of discrete phenomena. Although these concepts are not necessarily inconsistent, they increase the difficulty of interdisciplinary cooperation. Economists find it difficult to understand why agronomists have not pushed their rate trials higher; why they are so insistent on numerous replications; and why they have avoided multi-variable experiments. Agronomists, on the other hand, have been dismayed by the complex terminology and models employed by economists to describe ideas which otherwise seem essentially simple.

The framework for carrying on fertilizer research, particularly that to be used in farm decision-making, appears to be on the verge of rapid change. Acceptance of the concept of the farm production unit in terms of the economist's model of a "firm" gives the new perspective to the role of physical research. Recommendations on all production practices and enterprises must fit together in an economic sense if the farm is to maximize returns. The farm operator, not the production specialist or economist, should make the choice of the types and combinations of production factors or practices to be employed, as well as the types and amounts of products to be produced. He must make these selections in terms of his capital, ability to stand risks, and family's goal as a consuming unit. The production specialist, or the economist, cannot supply a single "best answer" to a production problem if the farm is considered a firm-household combination. The farmer must be given data from which to fashion a plan to fit his own particular circumstances. Accordingly, the data from research may need to take special forms, such as that explained in the chapters which follow.

Farm production economists, because of their concern with the farm as a whole and the economic aspects of planning, are proving to be useful collaborators in many kinds of production experiments. Then, too, farm production research workers are gradually abandoning the practice of giving the results of completed research to economists with the request that they "analyze" the economic results of data which already take on a "predetermined" form. Production scientists are increasingly seeking the assistance of economists along with statisticians, in initiation of design of experiments and in statistical analyses which conform to the economic models used in decision-making.

This changing trend towards greater interdepartmental cooperation and increased use of economic models in fertilizer research was the basis for planning the Tennessee Valley Authority sponsored symposium on methodological procedures in the economic analysis of fertilizer use data. If economists are to contribute effectively to such research, they should have a broad understanding of agronomic research and of statistical methodologies, as well as the economic principles and methods of economic analysis. The symposium and this book were planned with these needs in mind.

Part I of this book includes statements on the over-all methodological problems involved in estimating and using fertilizer response functions. It indicates the practical uses which can be made of improved input-output data, the fundamental economic relationships in fertilizer responses, how these can be applied in considering prices and the capital situations of farmers, and how research workers from different disciplines can work together on designing and initiating fertilizer research. Part II deals with fundamental statistical problems involved in designing experiments and estimating functions of fertilizer response. It considers designs in relation to analytical models and statistical efficiency, alternative algebraic forms of functions as these relate to alternative designs and predictions, and discrete and continuous models in relation to both experimental design and farmer recommendations.

Part III relates to the agronomic problems of conducting experiments from which production functions can be designed. It considers the size and type of the experiment in relation to the resources and personnel available. It also includes detailed discussions of soil, moisture, cultural practices, and other variables as they relate to conducting experiments. The feasibility of using soil test data is discussed and examples of fitting standard curves to existing agronomic data are included. Part IV deals with the application of improved response data. It indicates the type of data needed in farm and home planning programs. Examples are included, showing how budgeting and linear programming can be used to relate fertilizer to the whole farm business. Finally, simple monographs are used to illustrate how complex estimates can be transformed to provide simple calculations for the extension worker or farmer.

Part V presents important trends in fertilizer use and costs. It indicates developments which have taken place in the relative price, production, and use of particular plant nutrients. It traces developments in the source and processes for nutrients. Finally, it outlines some of the prospective trends and problems in fertilizer use.

A debt of gratitude is owed particularly to those in the Tennessee Valley Authority's management who made possible this symposium and its reporting in this book, and to the Iowa State College Press, through which publication was effected. Appreciation is extended to Lois R. Carr, Helen P. Long, and Mary L. Robinette, Division of Agricultural Relations, Tennessee Valley Authority, for their fine cooperation in preparing the manuscript for publication.

The editors believe that the information presented in this book will contribute materially to the improvement and expansion of research in the economics of fertilizer use.

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Chapter 1

Methodological Problems in Fertilizer Use

THE central methodological problem in fertilizer use on a single crop is prediction of the mathematical form and the probability distribution of the response function. This is a task, of course, for various soils, crops, and climatic situations. However, there are other methodological problems which are auxiliary to this central problem. They include: (a) the design of experiments to allow efficient prediction of the response function, and (b) the estimating procedure for predicting the surface and optimum use of nutrients. Since the last two problems are being given detailed treatment in other chapters, this chapter will focus on the fundamental and basic problems which relate to estimating the response functions.

Practical Importance of Knowledge in Response Functions

Although this chapter has the main objective of treating methodological problems in fertilizer economics, some of the practical or applied aspects of these fundamental considerations need to be pointed out. First, greater knowledge of simple, single-variable response functions can encourage greater use of fertilizer. The slope of the response function represents the incremental or marginal yield due to small increases in fertilizer use. The farmer with limited capital needs this information in determining how much fertilizer to apply. Knowledge represented by a response function is more useful than knowledge represented by the mean yield increase of one or two fertilizer (level) treatments.

Suppose a farmer with limited capital can earn \$2.50 return on funds spent for other lines of his business (such as tractor fuel, mule feed, crop seed, or hog supplement). He is given information showing that one discrete level of fertilization, 30 pounds of nitrogen, will increase oat yield by 17 bushels. With oats at 70 cents per bushel and nitrogen application costing 18 cents per pound, the total return is \$11.90 and the total cost is \$5.40, a net of \$6.50. However, the return per dollar spent on fertilizer (\$11.90 ÷ \$5.40) is only \$2.20, and the farmer will allocate his scarce funds where he can get \$2.50.

Suppose, however, that the farmer is given even three points from a response function showing: the first 10 pounds of N has a marginal yield of 10 bushels; the second 10 pounds has a marginal yield of 5 bushels; and the third 10 pounds has 2 bushels marginal yield. With a unit costing \$1.80, the first 10 pounds returns \$3.89 per dollar invested in fertilizer, and the second returns \$1.95. Hence, since the farmer can realize only \$2.50 elsewhere in his business, he now is encouraged to invest in at least 10 pounds of N. With more detailed knowledge of the response function, he may even invest in 15 pounds. Obviously then, knowledge of the response function, coupled with information on the economics of fertilizer use, can encourage a greater investment in this resource on that great majority of farms with limited capital. (See Chapter 11 for indications of use of these notions in farm planning.)

Knowledge of the response function is equally important for the farmer who considers his crop in the environment of unlimited capital. This is the case of tobacco producers; it is becoming the case of many other farmers. It is known that the optimum or most profitable level of fertilization for these farmers is defined by equation 1 where the term to the left of the equality

$$\frac{dY}{dF} = \frac{P_f}{P_y} \tag{1}$$

is the marginal yield or response and the term to the right is the price ratio (price per unit of fertilizer divided by the price per unit of yield). The marginal yield is the derivative of yield in respect to nutrient; it is the slope of the response function for any particular input level. This is the type of information basic for making recommendations to farmers who seek to maximize profits in a decision-making environment of unlimited capital.

It is obvious that the most profitable level of fertilization changes as the term to the right of the equality changes. (Likewise the optimum level of fertilization will change for the limited-capital farmer previously cited, as the price of crop yield, fertilizer, or any other product or resource for his farm changes.) How much change needs to be made in fertilizer use, as prices change, again depends on the slope of the response function. If the slope changes only slightly over a wide range of fertilizer inputs, the loss (profit depression) from not shifting rates can be great;[1] if the slope changes greatly over a small input range, the farmer may lose but little in not adjusting his rates to price change.

Finally, greater knowledge of the response curve is needed as an aid in farm planning and linear programming, to allow improved predictions of how and where fertilizer fits into the program of the farm as a whole. If numerous points are known for the response curve, each suggested level of fertilization can be treated as an activity or investment opportunity. The optimum level of fertilization relative to (a) other investment alternatives (activities), and (b) complete farm organization can then be predicted. Data in a form for this purpose will generally encourage use of more fertilizer. The reason has been suggested

[1]This statement applies particularly where the previous price ratio was equal to a derivative of the function high (low) on the response function and the new price ratio is equal to a derivative low (high) on the curve.

already: Knowledge of high marginal returns for small fertilizer inputs can specify use of this resource, even by the farmer with very limited funds. This knowledge also will indicate how far in the use of fertilizer the farmer with more funds can profitably go.

The farmer is the only one who can make the decision as to the most profitable quantity of fertilizer to use. Optimum quantity is determined partly by the response function for his particular soil, tempered as it is by previous soil management, weather, insects and pests, and other variables which are both endogenous and exogenous to his decision-making environment. But aside from the purely physical and biological variables of the fertilizer production function, the optimum quantity is as much a function of present nutrient and future (crop) price ratios as it is of the response ratios. Since prices, and even yields, are held with uncertainty, the fertilizer recommendation must conform to the farmer's uncertainty or risk-bearing ability which includes (a) his equity position; (b) his psychological makeup; and (c) other phenomena which cause him to temper the quantity and kinds of the resources which he employs. Refined estimates of the fertilizer response function can help provide the basic data needed to guide these decisions which are unique to each farm.

Knowledge of multi-variable response functions also has great practical implications. Anyone knowing the basic principles of production recognizes immediately that the production coefficient for, and the return from, any one input category is a function of the amount and kind of other input categories with which it is combined. The economic potential in, and limits of, any one resource can be determined only by studies which consider numerous input categories as variables. These variables may include different fertilizer nutrients, seeding rates, seed varieties, irrigation, and various other technologies. A fertilizer rate study may show a much lower response curve for one nutrient, if it is varied alone, than if it is varied along with another nutrient. Similarly, a multi-variable response study may be applied productively when a new crop variety, which has a great yield-boosting effect, is discovered. In much of the Midwest higher-yielding varieties have little effect unless used with sufficient fertilizer nutrients. A simple single-variable response study may fail to "lift the lid on yield potential," under new varieties or other developments in technology. Finally, knowledge of isoclines from multi-variable studies provides a practical guide in fertilizer manufacture.

Methodological Problems in Single-Variable Functions

A few practical applications of fundamental fertilizer research have been presented above because (a) the practical problems and their solutions are the main goals of fundamental research and methodological considerations, and (b) fundamental research can result in a greater and more efficient use of fertilizer if it provides refinements for obtaining more practical recommendations for the individual farmers. (Practicality is characterized by recognition of the variables peculiar to each farm, including capital, equity position, risk considerations, and other economic variables, as well as physical and biological variables such as the crop and variety, alternative nutrients, soil conditions, etc.)

In discussing practical applications first, the cart has been put before the horse. The remainder of this chapter will deal with the fundamental science or methodological considerations — in this instance, the "horse."

Form of Single-Variable Function

For research on simple response functions with a single-variable nutrient, for a particular soil and management system, there are two basic methodological problems, viz., (a) the appropriate algebraic form of the response function, and (b) the between-year variability in the production function.

As far as this writer knows and as pointed out by Mason in Chapter 5, there is no biological proof that the fertilizer response function conforms universally to a particular algebraic form of equation. It is likely that the best-fitting form of the fertilizer production function varies by crop, year, soil, or other variables. One algebraic form which has been popular over time with research workers has been the Mitscherlich-Spillman type of function. One form of this function is equation 2,

$$Y = m - ar^F \;. \tag{2}$$

(Another form is shown in equation 2 of Chapter 5.) This function employs specific assumptions about the nature of the response curve: (a) It assumes that the elasticity of response is less than 1.0 over all ranges of fertilizer applications, a condition likely to be encountered in most situations but one which need not hold true universally (some experiments at particular locations show a short range of increasing returns). (b) It assumes that fertilization rates never become so great as to cause negative marginal products (i.e., declining total yields), since yield becomes asymptotic to the limit m. (c) It assumes the condition of equation 3,

$$\frac{\Delta_2 Y}{\Delta_1 Y} = \frac{\Delta_3 Y}{\Delta_2 Y} \cdots \frac{\Delta_n Y}{\Delta_{n-1} Y} \;, \tag{3}$$

namely, that the ratios of successive increments to total yield over all fertilizer inputs are equal. Lastly, (d) the function assumes that where two nutrients are involved, the maximum yield per acre can be attained with a large number of nutrient combinations (i.e., it does not allow the isoclines to converge at the point of maximum yield).

A function which also forces particular assumptions into the predictions, although these are considerably different from the Mitscherlich equation, is the Cobb-Douglas function, listed as equation 4. It does not

(4) $$Y = aF^b$$

assume that the ratios of marginal yields are equal. However, it does assume that the percentage increase in yield is constant and equal to b for all increments of fertilizer. This assumption, illustrated in equation 5 below, may be as realistic as the parallel assumption of the Mitscherlich equation.

(5) $$\frac{\frac{\Delta_1 Y}{Y}}{\frac{\Delta_1 F}{F}} = \frac{\frac{\Delta_2 Y}{Y}}{\frac{\Delta_2 F}{F}} = \cdots = \frac{\frac{\Delta_n Y}{Y}}{\frac{\Delta_n F}{F}} .$$

The Cobb-Douglas equation allows the yield to increase at either a diminishing, constant, or increasing rate, although the response curve can be represented by only one of these and never by a combination. If total yield increases at a diminishing rate, the function assumes negative marginal products and, therefore, that total yield becomes asymptotic to some limit.

Somewhat more flexible functions are the simple quadratic and square root forms indicated respectively as equations 6 and 7 below:

(6) $$Y = a + bF - cF^2$$

(7) $$Y = a + b\sqrt{F} - cF .$$

These equations do not force certain of the elasticity and marginal ratio restraints of the previous equations. Also, they allow the total yield to reach a maximum, followed by negative marginal yields. Equation 6 may apply particularly where a maximum is reached with relatively low fertilization level; equation 7 may apply where marginal yields change rapidly over low fertilization levels but "straighten out" for higher levels, if no other practices or inputs are limitational. But again these functions may have no unique biological base. Is there a unique biological base for response functions?

The research worker makes a biological (and at this stage of knowledge, a subjective) assumption when selecting a particular function. Methodological effort should be devoted to proving either that (a) biological responses do follow particular mathematical forms, or that (b) there is no unique algebraic response function for all situations. The hypothesis followed is that the latter will most likely prove correct. While fundamental greenhouse research may prove the first to have some validity, objective statistical tests may be used to specify which function is most appropriate under field conditions. This methodological problem merits further attention, since every fertilizer recommendation to farmers implies knowledge of the mathematical nature of the response function. Greater knowledge of the response form is needed for most efficient designs. If the mathematical form is known to be a quadratic equation, a Box design may be most efficient (see page 48, Chapter 3). However, another design may be more efficient if the mathematical form proves to be logarithmic or exponential.

Distribution of Response Functions

Conventionally, fertilizer recommendations are made as if the response or regression coefficients were single-valued. It would be convenient if farmers' decisions could be made in this framework of certainty in respect to both prices and yield increments. Unfortunately this is not true. A methodological problem arises in providing response information which recognizes that risk and/or uncertainty must be incorporated into farmers' decisions: The farmer is not faced with a single response function but with a distribution of response functions. He recognizes this situation and makes his decisions accordingly. Incorporation of risk-uncertainty and probability concepts into fertilizer research and recommendations would aid him in these decisions.

The problem can be brought into focus by viewing fertilizer response in the manner of the generalized production function represented by equation 8. Yield response (Y) is represented as a function of

$$(8) \qquad Y = f\,(F_1 \mid F_2 \; . \; . \; . \; F_n, X_1, X_2 \; . \; . \; . \; X_n \parallel Z_1, Z_2 \; . \; . \; . \; Z_n)$$

fertilizer nutrients F_1 through F_n and other types of inputs (practices represented by X_1 through X_n and Z_1 through Z_n). The last two categories of inputs (X_i and Z_i) are denoted by soil type, nutrients already in the soil, seed variety, cultural practices, number of cultivations, seeding rate, moisture of particular weeks, temperature at critical times, and other variables (resource inputs) which affect yield. In this case a single bar follows F_1, denoting that nutrient F_1 alone is the input in the production function which is variable or which can be controlled. All variables between the single and double bars, F_2 through X_n, are endogenous to the decision-making environment, (can be controlled by the farmer or decision-maker) but are held fixed for the particular production period (i.e., crop year). These represent seeding rates, number of cultivations, application of particular nutrients in fixed levels, etc. To the right of the double bar are variables, such as weather, which are exogenous to the decision-making framework and cannot be controlled by the farmer. These exogenous variables vary within and between seasons. Hence, the response curve for the single variable F_1 will take on a different height and slope with each change in the exogenous variables. The result is a distribution of response functions such as shown in figure 1.1. The most likely hypothesis is that the response functions are normally distributed. There have been suggestions, however, that this is not the case, at least over a period of a few years (the span usually relevant in a farmer's decisions). In case the response curves are not normally distributed, the mean may be represented by the dotted line in figure 1.1 and is above the mode (the "most probable"

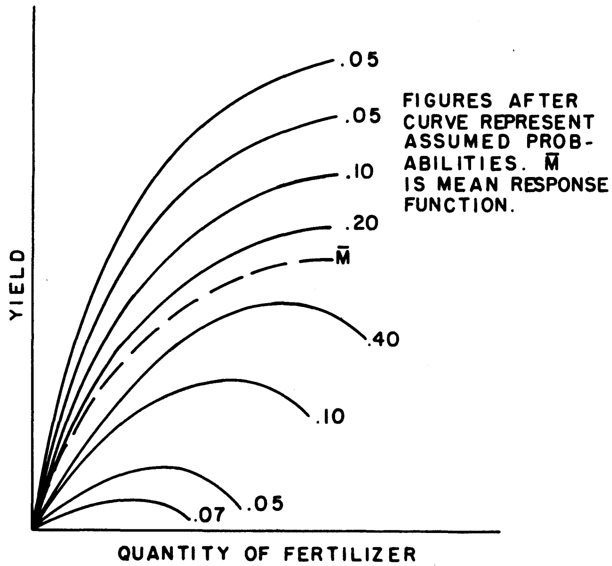


Fig. 1.1 — Possible effect of weather variations on the distribution of fertilizer production functions.

curve of any one year). (The mean might also fall below the mode, depending on the skewness of the distribution.)

How should the farmer make decisions when the response curve varies between years? Even though the distribution of functions might be established (and hence conform with Knight's (2)[2] risk concept), the curve of any particular year represents uncertainty. The answer depends on the individual farmer and his ability to bear risk as characterized by his capital, his equity position, and his aversion for risk. If he is a conservative individual with little capital and a low equity, he may wish to take few or no chances. In this case he may, in effect, count on the lowest possible response function and apply fertilizer accordingly. Using this type of "uncertainty precaution" (discount system), he feels assured that the probability is in favor of outcomes better than expected, and that there is slight chance of outcomes worse than predicted.[3] Undoubtedly, this type of uncertainty precaution causes farmers to use fertilizer in quantities smaller than conventionally recommended.

The farmer in a better capital position and with less risk aversion may make decisions on the basis of model response expectations. He

[2]Numbers in parentheses which appear in sentences refer to reference citations listed at the end of each chapter.

[3]Regardless of the decision and the outcome, the farmer is always faced with the possibility of two kinds of errors. First, he may assume "the best" and act accordingly. If he is wrong, he may be penalized by a depression of profits greater than if he had anticipated "the worst." Secondly, he can assume "the worst" and act accordingly. If he is wrong, his profits will be less than if he had used an alternative expectation and planned for "the best."

wishes the greatest probability of success in expectations and plans. He will, of course, never be 100 per cent correct. He will apply too little fertilizer for maximum profits in good years and too much in poor years.

Data on the distribution of the production function are lacking in most locations. To fill this gap in the farmer's decision-making environment, time sequences of fertilizer experiments are needed, with all endogenous variables (soil, seeding rate, previous management, etc.) held constant over a period of years. The exogenous variables then would be reflected in the distribution of functions, which would be useful in recommendations to, and decisions by, farmers. There is some preliminary indication that farmers believe the fertilizer response to "reflect the best yield to be expected" and, therefore, that deviations from this quantity are likely in the direction of lower yields.[4]

Information is needed to show whether the fertilizer functions are normally distributed and to indicate to farmers that "better incomes" are just as probable as "lower outcomes." But most important, this type of information would provide the decision-making basis for farmers who must use different plans because of variations in their ability to assume uncertainty. Table 11.3 (page 169) provides some insight into the need for variability data for farm planning.

Carry-over and Alternative Rates in Succeeding Years

Under the research needs outlined above, level of fertilization would be a variable handled similarly in a series of years. The focus here is on the distribution of functions, due to weather and other variations, without regard to: (a) carry-over effects or (b) the results of alternative fertilization rates in succeeding years. However, both of the latter are needed if fertilizer is to become a resource used to its full economic potential.

Leaching is great in parts of the Southeast and carry-over response is unimportant in economic decisions. In some localities, however, carry-over responses are important. Information on these residuals can increase the quantity of fertilizer used. With carry-over effects in years following the one of application, the optimum level of fertilization can be determined by equating the discounted value of marginal responses with the discounted value of marginal costs of each fertilizer increment. The value of the marginal response for any fertilizer input (i.e., the j-th input) then becomes, as shown in equation 9, the sum of the marginal response values

$$V_j = \sum_{i=n}^{i-1} \frac{R_i}{(1+r)^i} \tag{9}$$

[4]This statement is based on a survey of farmers' expectations being conducted by the writer.

divided by the discount coefficient. For example, suppose the third increment of fertilizer gives a response of 8 bushels in the first year, 4 bushels in the second year, and 2 bushels in the third year. The price of the crop is $1 and the farmer's discount rate is 10 per cent. With discounting for yearly periods, the present value of the sequences of yield response is:

$$\frac{\$8}{(1+.10)} + \frac{\$4}{(1+.10)^2} + \frac{\$2}{(1+.10)^3} = \$12.08 \ .$$

Without knowledge of residual responses, the first-year discounted marginal value of the third input is only $7.27. Obviously, then, more fertilizer will be used where residual effects exist and are made known to farmers. Knowledge of residual effects can reduce uncertainty considerations if the farmer knows that even though weather of the first year is bad, probabilities are high for getting a large residual effect in following years. He then will not be so timid about using fertilizer.

Finally, residual response functions allow farmers to discount fertilizer returns to fit their own particular capital and uncertainty situations. The magnitude of the discount rate should differ with each farmer. On the one hand, it will be a function of the alternative returns on capital in other parts of the farm business; the beginning farmer may discount at 40 per cent while the wealthy, established farmer may discount at 4 per cent. On the other hand, the magnitude of the discount rate will be a function of the subjective price and yield uncertainty in the farmer's mind. By supplying information on time sequences of yield responses, the research worker aids the farmer in using the fertilizer to fit his own unique circumstances.

A final phase of time should be mentioned. It is the effect of rate of fertilizer application in previous years on the response function in subsequent years. How much difference is there in the response function for corn this year on fields which received respectively 20, 40, 60, and 80 pounds of nitrogen last year?

Nature of the Production Surface

In order to be systematic, we have discussed single-variable functions or curves first. In following this procedure, the cart is placed before the horse. The reason is that one cannot know which single-variable curve is the appropriate one to predict unless he knows or assumes something about the response surface itself. Hence, he turns to the concepts and methodological problems involved in production functions involving two or more variables. Of course, what has been said about appropriate biological or algebraic forms of functions, about the distribution of the fertilizer response function, and other time considerations also applies to functions involving two or more variables.

When more than two nutrients can be variable for a single crop, two economic problems are involved: (a) the least-cost combination of nutrients for any given yield level, and (b) the most profitable level of fertilization, considering the nutrient combinations, which yields the lowest cost for each yield level. These decisions must be made by both the farmer with limited capital and the farmer with unlimited capital. If he has unlimited capital, then the optimum level of fertilization and the optimum combination of nutrients are simultaneously attained when the partial derivatives for both nutrients are equated with the crop/nutrient price ratio for each.

Using data for an Iowa corn experiment (1), for example, we have the two-variable response functions in equation 10. Using prices of \$1.40 per bushel for corn, 18 cents per pound for nitrogen, and 12 cents per pound for phosphorus,

$$Y = -5.68 - .316N - .417P + 6.35\sqrt{N} + 8.52\sqrt{P} + .341\sqrt{NP}, \tag{10}$$

the partial derivatives to equal the price ratios in equations 11 and 12 are set. From these, one solves for the quantities of the two nutrients in equations 13 and 14. Given this particular function, the optimum level of fertilization and combination of nutrients include 142.5 pounds of N and 156.5 pounds of P_2O_5.

$$\frac{\partial C}{\partial N} = -.316 + \frac{3.1756}{\sqrt{N}} + \frac{.1705\sqrt{P}}{\sqrt{N}} = \frac{.18}{1.40} \tag{11}$$

$$\frac{\partial C}{\partial P} = -.417 + \frac{4.2578}{\sqrt{P}} + \frac{.1705\sqrt{N}}{\sqrt{P}} = \frac{.12}{1.40} \tag{12}$$

$$N = 142.48 \text{ lbs.} \tag{13}$$

$$P = 156.45 \text{ lbs.} \tag{14}$$

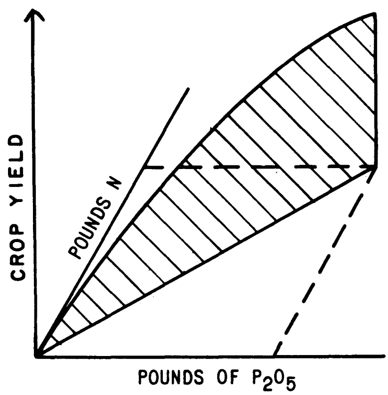
Even if the farmer has limited capital and cannot push fertilization to the point that the value of the last increment of yield is just equal to the cost of the last increment of fertilizer, he still needs to know the least-cost combination of nutrients for the particular yield to be attained. The least-cost combination is determined by equating the marginal rate of substitution of the two nutrients (the derivative of one nutrient in respect to the other with yield considered constant at a specific level) with the nutrient price ratio. Using the response function of equation 10, equation 15 is obtained, which defines the marginal rate of substitution between N and P_2O_5. Setting this equation of substitution rates to equal

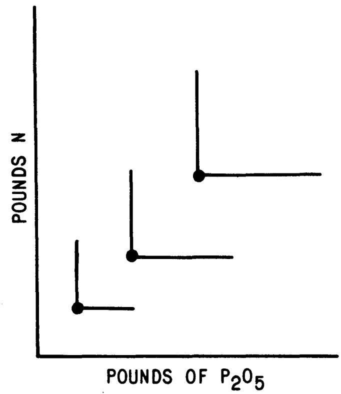
$$\frac{dN}{dP} = \frac{-.8348\sqrt{PN} + 8.5155\sqrt{N} + .3410N}{-.6323\sqrt{PN} + 6.3512\sqrt{P} + .3410P} = \frac{.12}{.18} \tag{15}$$

the $\frac{P}{N}$ price ratio of $\frac{.12}{.18}$, it is determined that for a 50-bushel yield, the least-cost nutrient combination includes 11.8 pounds of N and 24.3 pounds of P_2O_5; for a 100-bushel yield, the least-cost fertilizer ratio includes 79.3 pounds of N and 101.6 pounds of P_2O_5.[5]

The Nature of Yield Isoquants and Fertilizer Isoclines

The question of nutrient substitutability is now raised and, hence, the nature of the fertilizer production surface. Some concepts assume that nutrients are not substitutes in attaining a given crop yield. Liebig's classical *Law of the Minimum* assumed, for example, that the fertilizer yield surface reduces to a "knife's edge" as shown in figure 1.2. Higher yields can be attained only if higher rates of fertilization follow some limitational nutrient ratio. This also is the assumption employed in the so-called practical information which pictures crop production in the vein of a barrel, wherein yield cannot be raised above the shortest stave, namely, a particular fertilizer nutrient.





Figs. 1.2 and 1.3 — Production surface and yield isoquants for nutrients which are technical complements.

Now, for every yield surface, there is a corresponding map of yield isoquants or contours.[6] For the Liebig response surface, the yield isoquants take the form suggested in figure 1.3. Both nutrients are limitational in the sense that increasing one alone (a) neither reduces the

[5]In addition to knowing the least-cost nutrient ratio for a specified yield, the farmer with limited capital needs to use this information to determine the return per dollar invested in fertilizer as compared to other alternatives. This information will aid him in determining how much to invest in fertilizer.

[6]If the yield response for two nutrients is pictured as a surface or "hill" on a 3-dimensional diagram, it can be reproduced in 2-dimensional form just as a hill is reproduced by the soils expert on a topographical map, as a set or family of contours. Each contour represents a given yield level and the points on it represent the various nutrient combinations which allow attainment of this specified yield level. The yield contour, showing all possible combinations of nutrients allowing its attainment, is termed a yield isoquant (equal quantity).

amount of the other required to produce the given yield, or (b) increases the level of yield. This is denoted by the fact that the isoquant forms a 180-degree angle. However, if it is assumed that addition of one nutrient, without change in the other, causes toxic or other effects reducing total yield, the isoquants reduce to a single point consistent with the corner of the angles in figure 1.3.

However, a strict Liebig type of production surface is the exception rather than the rule. Otherwise agronomists would not have (or have been able to have) successfully conducted a relatively large number of single-nutrient experiments. Perhaps it is true that such distinct nutrients as nitrogen, P_2O_5, or K_2O do not substitute in the chemical processes of the plant (although close substitution may hold true for elements such as Na and K). However, availability of one nutrient may affect the ability of the plant to utilize other nutrients. Hence, in any case where variation of one nutrient, with another fixed at specific levels as in figure 1.4, results in different response curves, substitution does take place in the sense that different nutrient combinations can be used to attain a given yield. For example, if a 10-bushel response is attained with 20 pounds of N and 120 pounds of P_2O_5, with 60 pounds of N and 90 pounds of P_2O_5, or with 120 pounds of N and 40 pounds of P_2O_5, the given response can be attained with various nutrient combinations. It may be stated that nutrients are substitutes, at least at the level of farm decision-making.[7]

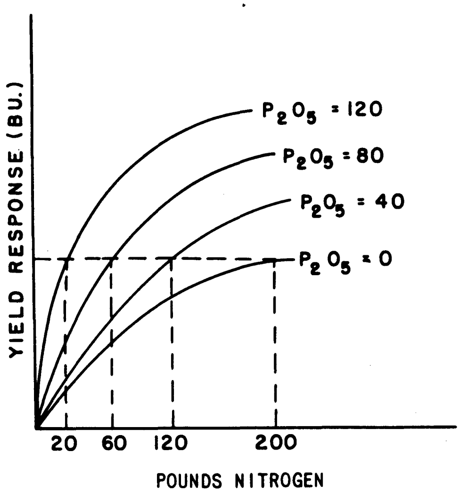


Fig. 1.4 — Yield resource curves for nitrogen with P_2O_5 fixed at different levels.

[7]These statements need, of course, to be conditioned in terms of plant composition and quality.

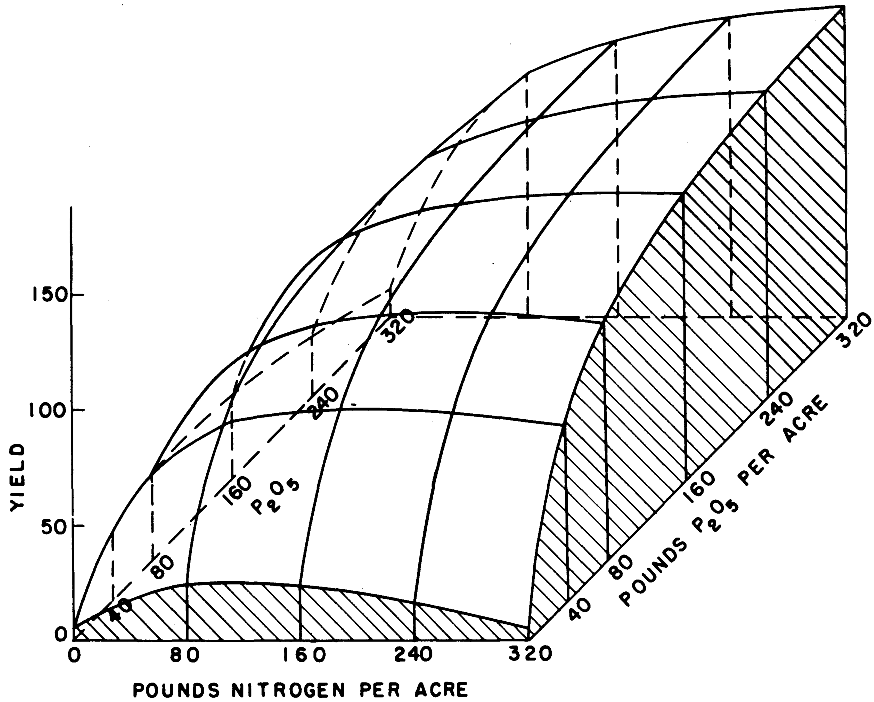


Fig. 1.5 — Predicted yield surface for corn. Source: Pesek, Heady, and Brown, Iowa Agr. Exp. Sta. Bul. 424.

The response surface for many crops and soils is more likely to parallel that shown in figure 1.5 for N and P_2O_5 on corn in western Iowa, or some modification of it (1). The corresponding family of yield isoquants is shown in figure 1.6. At high levels, the isoquants bend sharply to a purely vertical position at the "upper" end and to a purely horizontal position at the "lower" end. At these points of infinite and zero slope, respectively, the nutrients actually do become limitational or technical complements in the sense of Liebig; increase of one nutrient alone, at the vertical and horizontal points of the curves, will not result in reduction of the amount of the other nutrient, with yield remaining at the specified level or addition to the total yield. (Yield may actually be reduced if one nutrient is increased while the other is held constant at the level indicated at the points of infinite or zero slope.) However, between the two points of complementarity, the curves have a negative slope, denoting that they are substitutes in the sense that addition of one nutrient reduces the quantity of the other nutrient required to attain

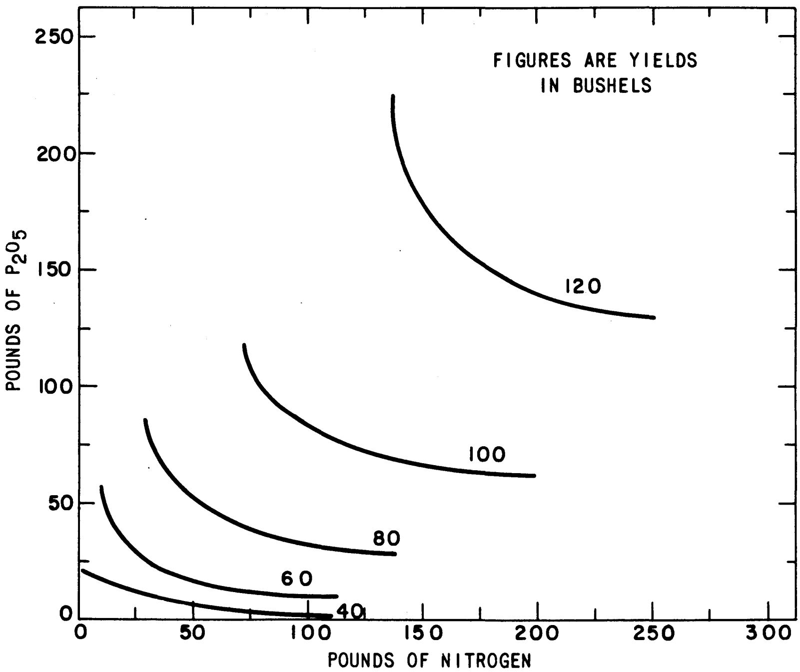


Fig. 1.6 — Predicted yield isoquants for corn (from Fig. 1.5).

(maintain) the given yield.[8] Furthermore, the curvature or slope of the isoquant changes, denoting that increasing quantities of the nutrient being added are necessary to offset constant decrements of the nutrient being replaced.

An important methodological problem in fertilizer research is that of obtaining more information on the slope and degree of curvature of the yield isoquants. If the slope changes only slightly and its length between the points of complementarity (i.e., the vertical point on the "upper" end and the horizontal point on the "lower" end) is great, the nutrients can be classed as "good" substitutes (i.e., "poor" complements). If the curvature is sharp (i.e., the slope changes rapidly) and the range between complementary points is narrow, the nutrients are poor substitutes (i.e., "good" complements). Now it is just as important to know that nutrients are "good" substitutes as it is to know that they are "good" complements. Perhaps too much research and too many recommendations have supposed that nutrients are only good complements. Given the meager knowledge which exists, the specialist making recommendations can suggest specific nutrient ratios with less burden on his

[8]For other alternatives in fertilizer production surfaces and isoquant maps, see (1).

conscience (and less profit depression to the farmer if the expert is not entirely correct) if he knows that substitution is "good" over a wide range.

If the slope of the isoquant is relatively constant over most of its range, and if this slope does not deviate greatly from the magnitude of the price ratio, a large number of nutrient combinations give costs and profits of fertilization which are quite similar. Here, again, the expert making fertilizer recommendations need not let his conscience be bothered greatly if he recommends a particular ratio such as 20-20-0 rather than a 10-20-0. However, if the curvature changes greatly between the complementary points and if the slope at either one or both ends deviates considerably from the magnitude of the price ratio, the expert needs to give particular heed to his recommendations on nutrient ratios. He will want to consider price ratios; he will want to consider the effects of nutrient prices on the optimum nutrient combination and the optimum fertilization level. The optimum nutrient combination will change with yield level, if the slopes of the yield isoquants differ greatly as successively higher yields are attained. Under these conditions, the recommendation on nutrient ratios should differ between farmers (a) who have funds for only low fertilization levels, and farmers (b) who have unlimited capital and can use higher fertilization ratios. Similarly, if slopes between isoquants change greatly with higher yields, the nutrient ratio will need to be changed as the price of the product changes (and higher or lower yield levels are profitable), even if the nutrient price ratio remains unchanged. The extent to which these facets of economics need to be incorporated into fertilizer recommendations depends on the nature of the production surfaces and isoquant maps. While they are fundamental science aspects of agronomic phenomena, knowledge is still too meager to determine where, and the extent to which, these considerations become important.

Fertilizer Isoclines

The slopes of isoquants change (i.e., the marginal rate of substitution between nutrients) as higher yields are attained. However, slope or substitution rate changes must be defined in a particular manner. They must be in reference to a fixed ratio of nutrients such as that illustrated in figure 1.7. The straight lines, A and B, passing through the origin, denote that nutrients are held in fixed ratios at higher fertilization levels. Changes in slopes or substitution rates on successive isoquants, in relation to needs for different nutrient ratios at varying yield levels, are measured at the point of intersection of the fixed ratio lines and the yield isoquants. If the slope of the isoquants were identical at all points where they are intersected by a fixed ratio line, the same fertilizer mix would be optimum for all yield levels. If the slope changes along a fixed ratio line, the nutrient ratio which is optimum for one yield level is not also optimum for another yield level.

A concept with perhaps greater application and more fundamental

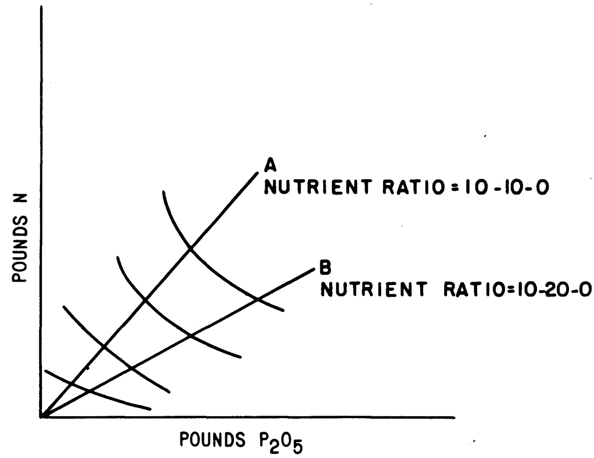
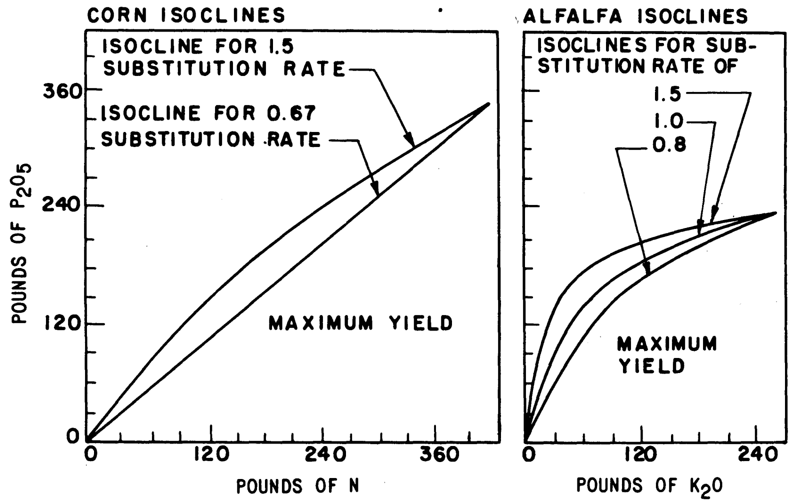


Fig. 1.7 — Isoclines showing equal nutrient ratios in relation to yield isoquants.

importance than the fixed ratio line is the fertilizer yield isocline. An isocline map exists for every fertilizer production surface. An isocline is a line connecting all points of equal slopes or substitution rates on a family of isoquants. In other words, it connects all nutrient combinations which have the same substitution rates for the various yield levels. There is a different isocline for each possible nutrient substitution rate. Of course, if the fertilizer production surface is of the Liebig knife-edge type illustrated in figure 1.2, the map reduces to a single isocline, denoting a zero substitution rate.

The isocline is also an expansion path, showing the least-cost and highest-profit combination of nutrients to use as higher yield levels are attained under a given price ratio for nutrients. In other words, it indicates whether the same nutrient ratio should be recommended and used regardless of the yield to be attained. Chapter 10 illustrates practical uses of this concept. Isoclines can be straight lines, such as A and B in figure 1.7. In this case they become identical with a fixed ratio line and the least-cost nutrient ratio will be the same for all yield levels. The expert need not inquire about the yield level to be attained when he makes his recommendation. However, an isocline map composed entirely of straight lines (fixed ratios) is very unlikely and perhaps impossible. Under maps of this nature, the isoclines would never converge but, instead, would spread farther apart at higher yield levels. Therefore, straight-line isoclines would indicate no limit to total yield level. Limits in total production exist only if the isoclines converge to the point of maximum yield and, therefore, are curved rather than straight (see Chapter 6 for other details on this point).

Isocline maps may take on many different forms. Little is known about them, and their nature can be established only by basic research. All isoclines for a given production surface may be bent in the same



Figs. 1.8 and 1.9 — Isoclines for corn and alfalfa showing convergence to maximum yield.

direction and none may be linear. Alternatively, one may be nearly straight while those above and below it bend in opposite directions. Different isocline maps, based on research in Iowa (1), are shown in figures 1.8 and 1.9. The two for corn, covering likely limits in price ratios for nutrients, are quite straight, with a slope relatively close to 1:1, denoting that recommendations of a constant nutrient combination may not deviate far from least-cost ratios for all yield levels. (Cognizance of the slight curvature in recommendations might cause more bother than savings in cost would merit.) In the case of the alfalfa data, however, the relevant isoclines bend rather sharply, suggesting that the least-cost nutrient ratio for one yield level may differ considerably from that for another yield level.

Two isoclines can be called "ridgelines" (see figure 10.1, page 153). They correspond to all points in figure 1.5, where the slope of the surface changes from positive to negative (i.e., the tops of the ridges denoting zero marginal responses). The ridgelines denote the points on successive yield isoquants where the nutrient substitution rate becomes zero. Since they denote technical complementarity of nutrients, they might appropriately be given the term "Liebig lines" because these are the limitational conditions which Liebig had in mind in his law of the minimum. The ridgelines (Liebig lines) converge, along with the other isoclines, at the point of maximum yield where nutrient substitution also is impossible.[9]

If (a) the ridgelines are not far apart, (b) the isoclines within their boundary are fairly straight, and (c) the yield isoquants for a particular

[9]The isoquant at the point of maximum yield reduces to a single point.

yield have only a slight curvature, with slopes not too different from the nutrient price ratio; several nutrient ratios, within the boundaries of the ridgelines, will give costs which are only slightly different (although only one will denote the least-cost ratio). If (a) the ridgelines are "sprung far apart," (b) the isoclines "bend sharply," and (c) the isoquants "curve greatly" away from the price ratios, the saving from changing nutrient ratios along an isocline can be quite considerable. Only basic research can indicate the frequency and extent of different isocline maps. The situation likely varies with soil, crop, year, and other variables.

Information of this nature not only has methodological importance but also practical significance. Therefore, the full economic potential of fertilizer use will be uncovered only by multi-variable response research. This is true since, as production economics logic has long suggested, the productivity of any one resource always depends on the level of input for other resources. While much of the logic is illustrated with two variables, analysis should be extended to variables which include other nutrients, seeding rates, moisture, quantities of nutrients already in the soil, soil type, and others. In other words, one should view the production function in the generalized form of equation 8. It is not inconceivable that soil typing and classification might be relative to the fertilizer production function. For example, with other inputs specified, economic distinction need not be made between soils where marginal response for parallel fertilizer inputs are the same. While they may be complex, steps to incorporate this concept into fertilizer research might obviate the need for considering experiments at isolated locations and in particular years as unrelated facts.

At the outset it was stated that the paramount methodological problem was that of the mathematical form of the fertilizer production function. Experimental designs and estimating procedures are auxiliary problems to it but at the same time are the foundation tools for establishing the mathematical characteristics of the function, at a given point in time and over time. To what extent is replication necessary when interest is in prediction of the response curve or function and the standard error which attaches to it, rather than the mean differences between treatment? Supposing that yield distributions are heteroscedastic in respect to variance; under what conditions would recommendations differ among regression lines predicted with nonreplicated treatments and means of treatments based on replications? What experimental designs allow both statistical and economic efficiency in estimate of complete surfaces, including isoquants, isoclines, and ridgelines? Is it unlikely that responses for different fertilizer inputs follow in the manner of a continuous function, and that other estimating procedures are necessary? There are hypotheses in respect to the answers of some of these questions; however, lack of time and space prevents the unraveling of their logic.

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Chapter 2

Interdisciplinary Considerations in Designing Experiments To Study the Profitability of Fertilizer Use

MORE interdisciplinary cooperation among agronomists, statisticians, and economists is an important need in agricultural research. Fertilization research should be looked at from an agriculturist's viewpoint rather than from the confined viewpoints of the farm management specialist, the soils specialist, the marketing specialist, the mathematical statistician, or the specialist in leguminous nitrogen fixation.

The Economics of Designing Experiments

Economics is concerned with the use of scarce resources in attaining multiple objectives. Experimental designs involving interdisciplinary research involve economic considerations. In designing interdepartmental experiments, some of the objectives pursued are in conflict; other objectives are complementary, i.e., attainment of one objective may make it easier to attain another. Such conflicts and complementarities occur both within and between the sets of objectives commonly of interest to agronomists, economists, and statisticians.

The job of agriculturists in designing an experiment is to approach the "best combination" of objectives in designing a particular fertilization experiment. The best combination of objectives should recognize any existing complementarity. Of course, the best combination of objectives depends on the relative costs of attaining the objectives. Mention of the cost of attaining objectives calls attention to the relationships among research resources and attainment of research objectives.

Pairs of Resources May Be Substitutes or Complements

If substitution is "near perfect," the designer should use the cheaper of the two resources in designing his study; i.e., if two identical fields are available, one for $400 an acre and the other for $350 an acre, he should use the latter. At the other extreme, pairs of resources may complement or contribute to the productivity of each other. For instance, an agronomist and a statistician working together may design an experiment which is superior to the product of either working alone. Their effort is then complementary. If two resources are perfect complements in the sense that they are unproductive used alone, or in only one proportion, the designers should take full advantage of this complementarity. The two research resources should be used in the one proportion.

The difficult problems in selecting research resources arise, however, when resources are neither perfect complements nor perfect substitutes but are, instead, complements over wide ranges and substitutes over narrower ranges. In this case, the designer has to match the added costs of and returns from using another unit of one resource against the added costs of and returns from using another unit of an alternative resource. If a unit of one resource is more productive relative to its costs than another, it is logical to expand its use relative to the other. When research funds for a given experiment are limited, the best experimental design is one which yields equal additional returns for equal additional expenditures on the resources subject to the designer's control. If, as is very unlikely, there are unlimited funds to support the experiment, the best experimental design is one which yields additional returns equal to additional costs for all resources subject to the designer's control.

The designer should also ask himself whether (a) any part, or all, of any of the fixed resources could be disposed of (by sale or transfer to another experiment) at a net return in excess of what it would produce in the experiment under consideration, and whether (b) more of any of the fixed resources can be acquired at a net cost below what it would produce in the experiment. If the answer to either of these two questions is "yes" for a particular resource, the designer should cause the resource to become variable and adjust its use according to the rules previously considered.

In experiments on the economics of fertilization, a high degree of complementarity exists among the services of agronomists, statisticians, and economists. In most fertilizer experiments, agronomic (both in soils and in crops) and statistical training are complementary. And, if the experimental results are to be interpreted economically, the services of an economist complement those of the agronomist and the statistician. Thus, with the exception of a highly technical fertilization experiment intended to yield technical information for noneconomic application, most fertilization experiments can advantageously employ the services of agronomists, statisticians, and economists.

Reconciliation of Objectives

Agronomists, statisticians, and economists, as a result of their different training, comprehend and prefer to pursue objectives which are sometimes conflicting. Also, because research workers are specialists in different organizations or different parts of a given organization, their preferences and objectives may differ still further. These different objectives and preferences have to be reconciled and aggregated into group choices in designing cooperative experiments.

Generally speaking, *the reconciliation and aggregation process is a bargaining one,* with weights assigned to individual and institutional preferences on various bases such as, (a) the amount of resources contributed by the different organizations, (b) the professional repute of the individuals, (c) the democratic procedure of one vote per participant, or (d) the principle of "greasing the wheel which squeaks the loudest." If it were possible to price the objectives separately and produce research on some sort of a free enterprise basis, a free price system might be used instead of a bargaining process in making these decisions. Similarly, consensus or deference to recognized authority would make it unnecessary to use bargaining processes in arriving at these design decisions. But administrative authority is not well enough informed to make these decisions; uniformly recognized professional authorities do not exist and differences, not consensus, as to preferences are the rule, not the exception. Thus, the bargaining process seems inevitable in the committee meetings, Kaffeeklatsches, seminars, and informal cooperative arrangements in which experiments are designed.

The problem is not one of eliminating bargaining decisions in designing experiments. Instead, it is one of improved bargaining leading to design decisions. Such decisions can be improved *first* by appealing for agricultural statesmanship, rather than by encouraging competition among departments of institutions or among institutions. Agricultural research statesmanship, rather than destructive competition for personal position and prestige among individuals or ill-advised loyalty to one discipline among those serving agriculture, will lead to cooperative research which solves the problems of agriculture. A *second* important way of improving decisions on experimental design is to increase the knowledge of the designer (whether an individual or a committee) about (a) the nature and importance of objectives held by different research organizations, different disciplines, and different individuals, (b) the nature of different research resources and their usefulness in attaining the objectives listed in (a), and (c) research techniques or methods of value in using the resources considered in (b), to attain the objectives considered in (a).

In the remainder of this chapter, fertilization experiments in general will first be considered. Following this, special problems of making economic interpretation of data secured from fertilizer experiments will be considered along with the desirable characteristics of experimental data from the standpoint of economic analysis. Finally, a recently designed Michigan experiment will be reviewed. This outline will permit emphasis of two principle methods available for improving decisions on experimental design. They are (a) use of agricultural statesmanship, and (b) use of more knowledge about objectives, research resources, and research methods.

Specification of Function for Investigation

Most fertilization experiments involve investigation of a set of functional relationships such as that represented by equation 8 in Chapter 1.

This generalized function is, of course, too complex and extensive to be handled with the intellectual and physical resources of any research organization. Hence, the first step is to restrict the general area of investigation to a manageable size or number of input categories. This is commonly done in two ways. First, autonomous subfunctions within the function are isolated for study. The word autonomous here means that outcomes within the subfunction are not influenced by events in the remainder of the function. Choices among alternative subfunctions depend on the preferences of individuals and agencies and upon the comparative productivity of research resources in such alternatives. If, as is generally the case, such autonomous subfunctions are still too large to work with, controls have to be imposed on certain of the variables to limit further the realm of inquiry. Here the conflicting ends are "generality" and "accuracy." For given resources, the study can cover a larger subfunction with a low degree of accuracy or a smaller subfunction with greater accuracy. The designer must decide how much of one he is willing to sacrifice in order to get the other.

To illustrate the above two steps, consider the problem of setting up a fertilizer experiment within a generalized function, including all possible products, inputs, and associated technologies. This function could be cut down to, e.g., a corn, oats, and clover rotation which can be presumed to be independent of other rotations. This, however, would still require a very large experiment. There is almost an infinity of inputs to consider — land with all its variation, labor, nitrogen, different sources of phosphorus, potash, machinery, different technologies, varieties of oats, cultural practices, etc. If an attempt were made to study all of these factors at once, the resources required for the project would be spread very thinly, and only very inaccurate results (i.e., those with great variance) would be secured.

Restriction of scope can be attained by the imposition of controls, both selective and experimental. Here, many individual and organizational preferences must be considered. One agronomist may be particularly interested in corn over the cornbelt, while a cooperating colleague may be endeavoring to become a national authority on planting and fertilization practices for small grains. The experiment station director may know that agricultural leaders favor investigation of corn fertilization on a soil type within one state. Hence, all of these kinds of preferences and others, along with the conflict between generality and accuracy, enter into the series of negotiations leading to the final choice.

The final choice might involve, for example, (a) restricting the experiment to a rotation on the given soil type to include: (i) given varieties of corn, oats, and clover, (ii) given cultural practices, and (iii) given levels of available K_2O, and (b) restricting the experiment further to N and P_2O_5 as the primary variable inputs to be studied in application to corn only.

This last step would narrow the realm of inquiry to a consideration of only the following subfunction:

$$Y_c = f'(N, P_2O_5 \mid \text{oats, clover}, K_2O, X_f \ldots X_n) + u \quad . \tag{1}$$

This function reads as follows: the yield, Y_c, of a given variety of corn is a function of the amount of N and P_2O_5 applied to corn grown in a C-O-CL rotation with K_2O and other inputs $X_f \ldots X_n$ (such as, soil type, oats variety, clover varieties, cultural practices, etc.), fixed at specified conditions, or levels.

Unexplained Residuals

The "u" introduced in equation 1 stands for variations of actual yields from the functional relationship specified in (1) above. In practice, the u's are always, partially, functions of more or less uncontrolled and unstudied variables, such as lack of uniformity in soil types, variations in weather, and disease or insect infestation. So long as the u's behave substantially as though they are randomly and independently distributed with respect to the studied variables, they can be "averaged out" with statistical procedures. For instance, the method of least squares may be applied to secure estimates of equation 1 which minimize the sum of the squared deviations in the Y_c's. This procedure is appropriate so long as the u's can be interpreted as due to errors in measuring the Y_c's, or as random stochastic movements in the function, either with or without antecedent causes.

Another practical requirement is that the u's be small enough for the estimates of Y_c to be usable. At this point the objectives of the statistician and agronomist may come in conflict. Trained in estimating procedures, and perhaps charged by the experiment station director with responsibility for the statistical accuracy of estimates based on the data produced by the experiment, the statistician desires accuracy. Ordinarily, the agronomist does too, but not at the expense of what he may consider undue restriction of his work and expensive randomization and control procedures.

In investigating equation 1, the statistical conditions required with respect to the u's may be secured, in part at least, by (a) procedures which reduce errors in measuring X_j and Y_c, (b) controls on non-studied inputs and factors, and (c) procedures designed to randomize the incidence of unstudied and uncontrolled variables in the experiment and, hence, of the u's generated by those variables. Examples of the first set of procedures are doublechecking and the measurement of nutrients in the soil as well as those applied. The imposition of controls was illustrated above. Plot layouts to randomize the distribution of soil differences between plots are a common example of the third set of procedures. Decisions on such procedures must be made early in the experiment. As an earlier step, the total amount of resources to be devoted to the experiment has to be determined and allocated among such competing ends as: number of plots, measurement accuracy, search for uniform fields, etc. After the number of plots is determined, its use in producing accuracy versus generality must be determined.

Desirable Characteristics of Experiments for Economic Interpretation

To this point, the discussion has been general. It applies to purely agronomic experiments as well as to experiments to be interpreted economically. Experimental data to be used in agronomic analysis, however, may or may not possess certain desirable characteristics for economic interpretation. It is important that the nature of characteristics which are desirable for economic analysis be known *before* fertilization experiments expected to yield data of economic significance are designed. The nature of these desirable characteristics can be seen most clearly by examining the uses which an economist may wish to make of the data.

The first required modification of concepts used to this point, if economic analysis is to be carried out, is the introduction of input prices, P_{x_j}, and output prices, P_{y_i}, to produce a profit equation of the form:

$$(2) \qquad g(P_{y_1} Y_1 \ldots . P_{y_n} Y_n : X_1 P_{x_1} \ldots . X_m P_{x_m}) = \pi \,.$$

When narrowed down to manageable size, as previously done by isolation of an autonomous subfunction and imposition of selective and experimental controls, the following type of subfunction is secured:

$$(3) \quad g'(Y_c P_c,\ NP_n,\ P_2O_5\ P_{P_2O_5} \mid \text{oats, clover, } K_2O,\ X_f,\ \ldots,\ X_m) = \pi \,.$$

Application of maximization procedures (as taught in any elementary calculus course) to equation 3 or portions thereof, permits location of such economic optima as the quantity of Y to produce maximum profit and the least-cost combination of N and P_2O_5 to use in producing that amount of Y.

Corresponding applications also permit determination of how these optima shift with price changes. The laws of growth, of the minimum, or of diminishing returns (which are highly interrelated and are investigated by agronomists and economists alike) tend to assure the second order conditions necessary to locate these optima. The most important economic optima tend to occur on the function where the

$$\frac{\partial Y_c}{\partial X_i} > 0 \quad \text{are decreasing.}$$

As an example, when P_2O_5 is constant, $\frac{d\pi}{dN} = 0$ defines the most profitable amount of nitrogen to use with the constant amount of P_2O_5. Under ordinary competitive conditions $\frac{d\pi}{dN} = \frac{dY_c}{dN} P_{y_c} - P_n$. Thus, an estimate of $\frac{dN_c}{dN}$, which is the slope of equation 3 in the Y_cN dimension, is important to the economist attempting to ascertain the most profitable amount of input N to use.

Suppose, however, that the economist's interest is somewhat more complex. He may desire to find the best (most profitable) combination of N and P_2O_5 in producing a given amount of Y_c. The condition $\dfrac{\frac{\partial Y_c}{\partial N}}{\frac{\partial Y_c}{\partial P_2O_5}} = \dfrac{P_n}{P_{p_2o_5}}$ defines the least-cost combination of N and P_2O_5 to use in producing the amount of Y_c under consideration. As the

$$\frac{\partial \pi}{\partial N} = \frac{\partial Y_c}{\partial N} P_{y_c} - P_n \text{ and } \frac{\partial \pi}{\partial P_2O_5} = \frac{\partial Y_c}{\partial P_2O_5} P_{y_c} - P_{p_2o_5}$$

are the slopes of equation 3 in the Y_cN and $Y_cP_2O_5$ dimensions, respectively, also, slopes are crucial to the economist attempting to ascertain the most profitable (least-cost) combination of N and P_2O_5 to use in obtaining a given yield (Y_c = a constant) of corn. These steps parallel those of equations 11 through 14 in Chapter 1.

If the economist is considering the problem of a farmer with a given amount of money to spend on N and P_2O_5 then, instead of fixing Y_c, the relationship $\dfrac{\frac{\partial Y_c}{\partial N}}{\frac{\partial Y_c}{\partial P_2O_5}} = \dfrac{P_n}{P_{p_2o_5}}$ is solved simultaneously with

$P_nN + P_{p_2o_5} P_2O_5 = C$ (the amount of money which can be spent on N and P_2O_5), to determine N and P_2O_5. These values for N and P_2O_5 can, in turn, be substituted in equation 3 to determine Y_c.

In both this and the previous instance involving Y_c = a constant, the productivity of N may depend on the amount of P_2O_5 present (and vice versa) and the study should be designed so that the estimates of $\frac{\partial Y_c}{\partial N}$ and $\frac{\partial Y_c}{\partial P_2O_5}$ can reflect such relationships.

When the economist desires to determine the most profitable amounts of N and P_2O_5 to use and of Y_c to produce, he sets $\frac{\partial \pi}{\partial N}$ and $\frac{\partial \pi}{\partial P_2O_5}$ equal to zero, and solves simultaneously for N and P_2O_5. Having secured N and P_2O_5 in this manner, he then substitutes them in equation 3 and solves for Y_c. Alternatively, the optimum combination of N and P_2O_5 and the optimum level of Y_c can be solved in the manner of equations 11 through 14 in Chapter 1. As in the previous cases, $\frac{\partial \pi}{\partial N}$ and

$\frac{\partial \pi}{\partial P_2 O_5}$ involve estimates of $\frac{\partial Y_c}{\partial N}$ and $\frac{\partial Y_c}{\partial P_2 O_5}$, the slopes of equation 3 in the $Y_c N$ and $Y_c P_2 O_5$ dimensions, as the crucial values to be determined from the fertilization experiment.

Consideration of more complex subproduction functions involving more than two inputs reveals that, in each instance, $\frac{\partial Y_c}{\partial X_j}$ turns out to be crucial in estimating the most profitable quantities of Y_c to produce, and of the inputs X_j. The same is true if Y_c is fixed, or if the money which can be spent on the variable inputs is limited.

If the subfunction being investigated involves two products, Y_c and Y_L (corn and a legume) with the amount of Y_c produced affecting the productivity of resources used in producing Y_L (and vice versa), these influences should be measured and reflected in the estimates of the $\frac{\partial Y_i}{\partial X_j}$. In such subfunctions, an additional problem of determining the most profitable combination of Y_c and Y_L exists. Y_c and Y_L are in the most profitable combination and amounts when the following equations hold simultaneously:

$$\frac{\partial \pi}{\partial N(Y_c)} = O \tag{4a}$$

$$\frac{\partial \pi}{\partial P_2 O_5 (Y_c)} = O \tag{4b}$$

$$\frac{\partial \pi}{\partial N(Y_L)} = O \tag{4c}$$

$$\frac{\partial \pi}{\partial P_2 O_5 (Y_L)} = O \tag{4d}$$

where $\partial N(Y_c)$ stands for a change in the amount of N used in producing Y_c, as contrasted to a change in N used in producing Y_L, which is written $\partial N(Y_L)$, or a change in $P_2 O_5$ used in producing Y_c, which is written $\partial P_2 O_5 (Y_c)$. After solution of (4a), (4b), (4c), and (4d) for $N(Y_c)$, $N(Y_L)$, $P_2 O_5 (Y_c)$, and $P_2 O_5 (Y_L)$, these values can be substituted into equation 3 to determine the most profitable amounts of Y_c and Y_L to produce.

The above example involving two outputs Y_c and Y_L, and two inputs N and $P_2 O_5$, is easily generalized to "n" outputs and "m" inputs. In this generalized form, the same conclusion holds, i.e., *the crucial estimates required to determine high profit points, least cost combinations of inputs, and high profit combination of outputs are the estimates of* $\frac{\partial Y_i}{\partial X_j (Y_i)}$, such estimates to reflect interactions among the Y as well as among the X_j.

The economist's strong preference for accurate estimates of the

$\frac{\partial Y_i}{\partial X_j(Y_i)}$ where $\frac{\partial Y_i}{\partial X_j(Y_i)}$ are positive and decreasing may come into sharp conflict with the interests of agronomists in the early stages of interdepartmental negotiations on the design of fertilization experiments. The agronomist, after many earlier negotiations with statisticians, has a strong preference for accuracy in estimations yields for some combination of fertilizer nutrients; the economist has, for reasons expressed above, a strong preference for accuracy in estimates of $\frac{\partial Y_i}{\partial X_j(Y_i)}$. The agronomist is led to seek replications at points on the surface while the economist is led to seek less replication and more "spread" of the observations over the surface. These two objectives, while competitive over a narrow range, are also quite complementary over wider ranges since the standard error of estimate for yields is a component of the standard error for $\frac{\partial Y_i}{\partial X_j(Y_i)}$. In fact, an experimental design yielding low standard errors for $\frac{\partial Y_i}{\partial X_j(Y_i)}$, can be made to yield as low or even a lower standard error of estimate for Y_i than one in which the standard error of $\frac{\partial Y_i}{\partial X_j Y_i}$ is high. When the agronomist sees these complementarities and opportunities for cooperation, it is a relatively short step toward agreement and the presentation of a unified research proposal backed by personnel from both areas of work.

Alternative Agronomic Objectives and Linear Programming Determinations

Other objectives of agronomists, while not always complementary with those of economists, are seldom in sharp conflict. This is especially true if the need for full use of fixed research resources is considered, as well as the need for economy in the use of variable, or "out of pocket," research resources. For instance, fertilizer placement and tillage practices can be tested in subseries within a design with only a small increase in variable costs and probably no increase in fixed or overhead costs.

Another consideration involving slopes of function should be mentioned here. Some persons argue that economic interpretations of fertilization data can be made on a comparative budget and/or on a linear programming basis which does not require estimates of the $\frac{\partial Y_i}{\partial X_j(Y_i)}$ from continuous production functions. This is, of course, true. In such procedures profits are computed for each discretely estimated point on the relevant subproduction function for which an *estimate* of yield is available. Comparison of profits among such points permits the economist to determine the most profitable among them, as discrete opportunities. While these procedures do not make direct use of $\frac{\partial Y_i}{\partial X_j(Y_i)}$ estimates, they locate the "best" point by comparing finite difference between points. The smaller these differences, the more accurately the "best" point can be located. Thus, regardless of whether or not the economic analysis is to be based on point estimates or on estimates of derivatives from continuous functions, experimental observations should yield information on a multiplicity of points on that area of the surface where the derivatives are positive and decreasing.

Another point of similarity should be noted in the data requirements of economic analyses based on *point* versus *continuous function* estimates. In both instances, the "best" amounts of the different fertilizers to use vary with prices of the inputs and of the output. These variations occur in areas of the function where decreasing increments in yields result from equal successive increments in the variable inputs. This mutual characteristic of the different methods of economic interpretation further increases the desirability of having yield information over large areas of the surface, or on a multiplicity of points on the surface. Thus, we note again that the same complementarity which exists between the agronomist's desire for a low standard error of estimate for yields and the economist's desire for a low standard error of $\frac{\partial Y_i}{\partial X_j(Y_i)}$ also exists between the desires of (a) the budgeter or linear programmer on the one hand, and (b) the continuous function analyst on the other.

Economists carrying out continuous function analyses sometimes are devotees to certain functions. For instance, prior knowledge that one will predict a Cobb-Douglas, Spillman, or linear function creates the desire for special designs; i.e., a Cobb-Douglas analyst may want to avoid all zero rates of application since the $\log O = -\infty$. However, because of the current lack of knowledge of which function best fits the data, it appears desirable to avoid designs which confine the analysis to a particular function, unless resource limitations restrict the analyst to one of the simpler functions.

Methods of Attaining Desirable Characteristics for Economic Analysis

The objectives outlined above are attained in designing experiments by:

A. Ascertaining on the basis of existing information the range of combinations of X_j's for which the $\frac{\partial Y_j}{\partial X_i(Y_j)} > O$ and decreasing and concentrating experimental observations on these combinations.
B. Securing observations for a sufficient number of combinations in the area defined in (A) to give the economist flexibility in selecting

functional forms if he elects to use continuous functions or, if he elects not to use continuous functions, confidence that he has data on sufficient alternatives to make the relevant discrete comparisons. It should be recognized that while this may reduce the number of replications which can be made with given resources for any one combination there are complementarities between the desire of accuracy in Y_i estimates and accuracy in $\frac{\partial Y_i}{\partial X_j(Y_i)}$ estimates. This requirement insures that data on the interactions among the X_j's will be available.

C. Allocating experimental observations among the possible combinations of the X_j in such a way as to minimize the linear correlations among terms whose coefficients are likely to be estimated; i.e., if $Y_C = A + b_1X_1 + b_2X_1X_2 + b_3X_2 + b_4X_1^2 + b_5X_2^2$ is likely to be fitted, an experimental design which minimizes (with due consideration to the cost of minimization) the linear correlations among X_1 and X_1X_2 or betweem X_1 and X_2, or X_1 and X_1^2, etc., is desirable. Minimization of the intercorrelations among the variables whose coefficients are to be estimated reduces the standard errors of the estimated coefficients.

D. Allocating experimental observations among the possible combinations of the X_j's in such a way as to increase the standard deviation of the terms whose coefficients are likely to be estimated; i.e., if $Y = aX_1^bX_2^{b^2}$ is to be estimated linearly in the logarithms, then $\sigma_{\log Y}$, $\sigma_{\log X_1}$, and $\sigma_{\log X_2}$ should be kept large or, alternatively, if $Y = a + b_1X_1 + b_2X_2X_1 + b_3X_3^2$ is to be estimated, then σ_{X_1}, $\sigma_{X_1X_2}$, and σ_{X_3} should be kept large.

E. Controlling or measuring the influence of the Y_i's on each other's functional relationships with the X_i. This can be done if all but one of the Y_i is held constant or, if more than one of the Y_i is to be studied, by (a) measuring the by-products of each Y_i studied and the influence of these by-products on the production of the other Y_i, or (b) by simply letting the separate functions for the Y_i reflect the levels at which the other Y_i are produced. If by-products and/or "by-losses" involving humus, biologically fixed nitrogen, soil, nutrient removal, soil structure, erosion, etc., can be measured and incorporated into the functions, this is probably the preferable solution. Simply letting the separate functions for the Y_i reflect the levels at which the other Y_i are produced may cause estimates of the productivity of one or more of the applied nutrients to reflect either by-product losses or gains.

F. Maximizing, with available resources and in view of direct and opportunity costs, the number of observations made.

There are at least two important sets of interrelationships to be kept in mind in using the above methods. First, the objectives, both economic and noneconomic, being sought are in some instances competitive or conflicting while, in other instances, they are complementary with attainment of either or both of two objectives making it easier to attain the other. Second, the methods of agronomists, statisticians, and economists are in some instances competitive but are in many instances complementary.

Agronomic Methodologies

However, considering the interrelationships among objectives of economists and agronomists in some detail, certain agronomic methodological developments should be mentioned. The mechanization of plot work is extremely important in lessening some of the competitive aspects of objectives of experimental designs. In effect, agronomists are substituting especially adapted or constructed machines for much of the labor previously used in hand-weighing and measuring fertilizers and in hand-harvesting and measuring the crops produced. Use of such equipment calls for larger lanes and turning areas. Thus, these new technologies make it "profitable" to substitute both capital and land for labor in the research process.

This substitution tends to increase the overhead or fixed cost of an experiment but reduces the per-unit costs of adding plots to the design. It also makes possible an increase in the number of experimental observations. The increase in observations involves only a small increase in cost, with the advantage of spreading the fixed cost over more plots. Thus, designs are becoming increasingly possible whereby the agronomists can supply economists with the kinds of data needed for economic interpretations.

The work-simplification methods developed at Michigan State University can be mentioned as examples of techniques which make more elaborate experiments possible. One device is a fertilization attachment for corn planters, a mechanism both accurate enough for experimental work and for reducing the fertilizer cleaning work in moving from one plot to another. Another device is a one-row mounted corn picker which makes it possible to pick one row without knocking down adjacent rows. Accurate calibration of fertilizer drills also makes it possible to vary rates of application from plot to plot without hand measurement and weighing. Also, an accurately calibrated fertilizer drill on a garden tractor makes it possible to side-dress corn rapidly and efficiently. While machine work may be somewhat less accurate than hand work (though this is debatable if reliable labor is hard to get), reduced costs make it possible to offset these inaccuracies (if they exist) with more and larger plots. So promising are these developments that many experimental procedures need a thorough work-simplification study. The accuracy of machine work needs an equally thorough statistical evaluation.

An Example

The reconciliation process in designing an experiment for studying the economics of fertilization can be well illustrated with an example from Michigan. For some years there has been a rather close cooperation between members of the Agricultural Economics staff and the staff of the Soils Department. Also, there has been a fair interchange of graduate students, as well as a number of seminars and informal sessions. Thus, personnel involved have known and understood each other and, in general, there exists an environment favorable to agricultural statesmanship.

After some preliminary meetings, a decision was made to develop a joint project between the two departments to study the economics of fertilizing some of the major Michigan crops. Six people from the various departments actively designed the experiment. Statisticians, while not project members, were consulted and used, both directly and indirectly.

Decisions had to be made on: (a) crops to be fertilized, (b) range of fertilizer nutrients to be studied, and (c) soil types to be studied. The problem had to be confined to portions of an autonomous subfunction in order to make the problem manageable.

Preliminary discussions of objectives of the two departments and of the Michigan farmers tentatively indicated that three subprojects should be developed. The first of these was concerned with a corn, oats, wheat, and alfalfa-brome rotation on Miami silt loam, one of south central Michigan's upland soils. Another subproject dealt with corn under continuous cultivation on the Brookston series. The third dealt with the fertilization of pasturage on one of the pasture soils of north central Michigan.

Further consideration of the relative importance of these three studies and of the cost of doing experimental work at the different locations considerably modified the tentative conclusions. For instance, the pasture experiment was dropped because it was too far away from the campus to be conducted economically, and the pasture fertilization problem was less important to Michigan farmers than further strengthening of the continuous corn experiment. Also, it was decided to carry out the corn, oats, wheat, alfalfa-brome rotation on a soil in the Fox series because of the difficulty of getting a sufficiently homogeneous field of Miami soil. It was found that, after preliminary soil tests, the continuous corn experiments on Brookston would have to be moved to a more northern county from the county in which it was originally planned to locate them. The farmers in the original area had already fertilized the soil to such a high level that the response to fertilizer would be of little significance for economic analysis.

The continuous corn experiment on Brookston soil will be considered below. In this experiment, it was decided that each of the three nutrients would be applied at seven different levels including the zero rate of application. It was judged by the agronomists involved that these rates would fall mainly in the area where $\frac{\partial Y_c}{\partial X_i} > 0$, and decreasing.

The seven levels are presented in table 2.1.

TABLE 2.1. Rates of Fertilizer Application, Continuous Corn Experiment, Brookston Soil, Michigan, 1953

	Rate						
Nutrient	0	1	2	3	4	5	6
N	0	20	40	80	160	240	320
K_2O	0	20	40	80	160	240	320
P_2O_5	0	40	80	160	320	480	640

It was also possible to incorporate into the design, work of special interest to the agronomists (Fig. 2.1). Thus, the plots running up the main diagonal were replicated and split into two parts. On one-half of each plot in one replication, a different method of fertilizer placement was employed. This made it possible for one agronomist involved to gain certain information in which he was particularly interested. It should also be noted that all the plots were large enough to be split in subsequent years to absorb similar supplementary projects having to do with, e.g., type of fertilizer, variety of corn, planting rates, and various other cultural practices. Soil tests were made for each plot to enable both economists and agronomists to study the effects of difference in soil fertility on yields as well as accumulation of fertilizer residuals.

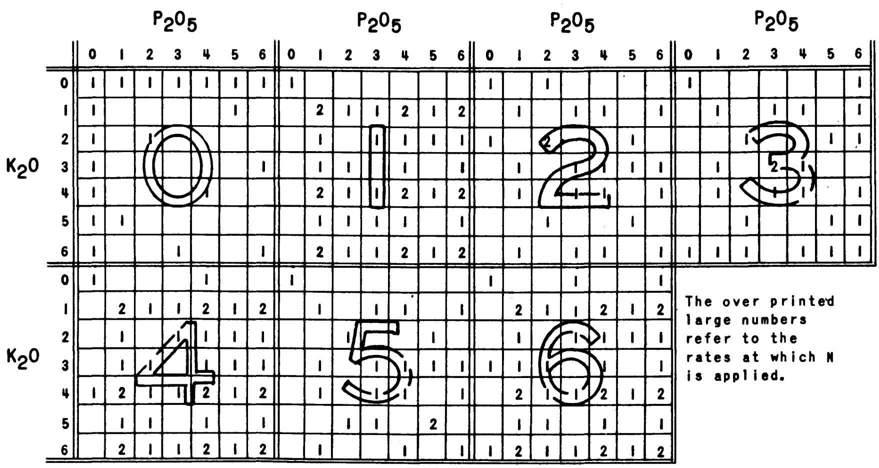

Fig. 2.1 — Schematic presentation of continuous corn experiment, Michigan State University, 1953.

A 7 x 7 x 7 experiment involves 343 different plots, if none of the plots were replicated. Several members of the committee had interests in replication of certain of the plots. For instance, the economists were interested in replication of the O, O, O plots for a number of reasons one encounters in fitting various alternative functions. The agronomists were also interested in having a replicated 3 x 3 x 3 factorial. After provision was made for 11 repetitions of the O, O, O plot and a replicated 3 x 3 x 3 factorial, it was obvious that project resources were inadequate (even after cancellation of the pasture experiment) to permit separate plots for each of the 343 cells in the design. It was decided, therefore, that the observations which could be afforded would be scattered throughout the sample space so as to keep the standard deviations for the three fertilizer nutrients large and to minimize the correlations coefficients among the three fertilizer nutrients applied.

Plans were made to control unstudied variables and to randomize the influences of those which could not be controlled. Controls were imposed in selection of the field and parts thereof as well as in selection of workers and equipment. Within the portion of the field selected by our soil classification expert as one being homogeneous, plot locations were randomized.

At this point a member of the Soils Department took active participation in the project and indicated to the economists that there were advantages of work simplification procedures in research. Hence, the number of plots were expanded somewhat. Some of the extra plots were scattered over the surface to be estimated. Others, however, were used to secure more information about the relationships between yields and each fertilizer nutrient considered separately with zero amounts of the other nutrients applied. The distribution of plots, while probably not ideal for fitting a given function, would give considerable flexibility in selecting functions for analytical purposes. The last requirement appears advantageous in view of certain modifications, which were developed at Michigan State University, in fitting modified Cobb-Douglas functions which are asymmetric and nonconstant, and have elasticities capable of reflecting more than one stage of a production function.

It is not claimed that the ultimate in experimental design has been secured. It is felt, however, that a moderately good job has been done in taking into account the various objectives of economists and agronomists. Experimental designs were used which reflect, rather satisfactorily, group choices (i.e., recognizing the wants, preferences, and objectives of the people and organizations concerned). The economists are pleased; the agronomists feel they will secure more than ample returns for their investment in the project. And both the experiment station administrators and the National Fertilizer Association administrators were favorable to financing the project. It has been shown that when representatives from various fields of work join forces and agree on a mutually advantageous research program to serve agriculture, such a program receives high priority in the minds of administrators charged with using research resources efficiently.

PART II

Fundamental Design and Prediction Problems

- Alternative Designs
- Appropriate Functions
- Continuous Functions and Discrete Models
- Estimational Procedures

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Chapter 3

A Comparison of Discrete and Continuous Models in Agricultural Production Analysis

Types of Experimental Procedures

Historical Review

IN a recent review article (1), this writer traced the development of multifactor experimental procedures. A brief resumé of this development seems desirable at this time: In the first multifactor experiments, a single factor was varied at a time. For example, with five factors, one might plan 5ℓ experiments, in which each of the factors in turn was used at ℓ levels while the other four factors were held at some starting level. Fisher (14) and Yates (25) encouraged the use of complete factorials and developed a large number of special designs involving them. In a complete factorial, all combinations of the factor levels are used, e.g., ℓ^5 for the above experiment. These designs were developed for experiments in which the experimental error could not be neglected. In order to estimate the magnitude of this error in each experiment, the experiment had to be repeated several times, e.g., r. These factorial designs were formed largely for useful field experiments in which sequential experimentation would be less than the laboratory experiments, and the factors were often of the discrete type, e.g., varieties or rations.

Because of the large number of factor combinations required in many field experiments, it was felt that some form of incomplete block design was needed to reduce the experimental error. This resulted in the so-called confounded designs, e.g., with 2^k, 3^k, 3×2^k, $3^k \times 2$, 4^k designs. These are described by Yates (26). More complicated factorial designs have been constructed by Nair (21, 22), Bose (4), Finney (13), and Li (20), among others.

When physical scientists and engineers became interested in multifactor experiments, they found that complete and confounded factorials required too many experimental units, especially since the experimental errors were often much lower than in field experiments. One method of reducing the number of experimental units was to use higher order interaction effects to estimate the error and hence avoid repetition of the design. Fisher (14) and Cornish (9) described the analysis of the singly replicated unconfounded factorial design and used the higher order interactions for this purpose. Jeffreys (17) and Kempthorne (18) have advance justifications for this approach. Then Finney (11, 12), Plackett and Burman (23), Kempthorne (18), Rao (24), and Davies and Hay (10) developed the fractional replication designs, based on using parts of the confounded designs. Yates (25) and Hotelling (16) had already mentioned the use of such designs.

Some General Considerations of Factorial Experiments

The results of multifactor experiments are usually summarized in various two- and more-way tables of means and an analysis of variance. For example, let us assume there are two factors (A and B), one with p and the other with q groups, each of the pq classes having r samples. Some characteristic, such as yield, is measured for each of the pqr samples. The results are summarized in a (p x q) table of class means ($\bar{Y}_{ij}$) with the corresponding (p + q) border means ($\bar{A}_i$ and $\bar{B}_j$).

		B				
		1	2	...	q	
	1	$\bar{Y}_{11}$	$\bar{Y}_{12}$	...	$\bar{Y}_{1q}$	$\bar{A}_1$
	2	$\bar{Y}_{21}$	$\bar{Y}_{22}$	...	$\bar{Y}_{2q}$	$\bar{A}_2$
A	.	.	.		.	.
	.	.	.		.	.
	.	.	.		.	.
	p	$\bar{Y}_{p1}$	$\bar{Y}_{p2}$	...	$\bar{Y}_{pq}$	$\bar{A}_p$
		$\bar{B}_1$	$\bar{B}_2$	...	$\bar{B}_q$	$\bar{Y}$

For example, the border means for A represent averages over all B-groups. There are two circumstances under one or both of which these A-means are of importance:

1. Differences between B-groups are the same for all A-groups, i.e., there is no AB interaction.
2. The experimenter desires to make inferences regarding A only when averaged over these particular B-groups.

If item 1 is true, one can set up the following model to represent the yield for a given sample:

(1) Y = (mean) + (A effect) + (B effect) + (error).

The A and B effects are estimated by computing the deviations of group means from the general mean, e.g.,

$$A_1 \text{ effect} = \bar{A}_1 - \bar{Y} .$$

The errors are assumed to be normally and independently distributed with zero means and same variances, σ^2. In this case, analysis of variance is:

Source of Variation	d. f.	S. S.	M. S.
A	p-1	SSA	MSA
B	q-1	SSB	MSB
Residual	(p-1)(q-1)	SSI	MSI
Error	(r-1) pq	SSW	s^2

In the above analysis of variance, SSA = $qr \ \Sigma \ \bar{A}_i^2 - pqr \ \bar{Y}^2$ and SSB = $pr \ \Sigma \ \bar{B}_j^2 - pqr \ \bar{Y}^2$. The residual sum of squares measures the failure of the A and B effects to be additive, i.e., presence of AB interaction. It is computed as:

$$r \ \Sigma \ \bar{Y}_{ij}^2 - pqr \ \bar{Y}^2 - SSA - SSB .$$

The error variance, σ^2, is estimated from the variability within classes. The mean squares are all computed by dividing the sums of squares by the corresponding degrees of freedom. One can test for the existence of interaction by use of F = MSI/s^2. Presumably, if this is significant, inferences about A effects must be confined to averages over these q B-groups. Otherwise one should consider the general model:

(2) $$Y = (\text{class mean}) + (\text{error}) .$$

Then each of the *pq* classes is considered separately and the simple analysis of variance is:

Source of Variation	d. f.	S. S.	M. S.
Treatments	pq-1	SST	MST
Error	(r-1) pq	SSW	s^2

$$SST = r \ \Sigma \ \bar{Y}^2 - pqr \ \bar{Y}^2$$

The same procedures can be followed for more than two classification variables. In this it is advisable to look at the individual contributions to the interaction: AB, AC, BC ... ABC .... In many cases it is even possible to subdivide SSA, for example, into pertinent single degree of freedom contracts; hence, SS(AB) can also be subdivided. This subdivision of SSI is useful in detecting particular aspects of nonadditivity which may be concealed in blanket tests of MSI/s^2. For more exact discussion of these problems, see Chapter 20 of Anderson and Bancroft (2).

Extension of Factorial Experimentation to Continuous Variables

In the past, even though the factors could be varied continuously, most analyses of experimental data have followed the same procedures as for discrete classifications. For example, if one had an experiment to study the effect of nitrogen (n) and potash (k) on the yield of corn, one might consider a simple 2 x 2 experiment with four treatment combinations: low *n* and *k* (00); low *n* and high *k* (02); high *n* and low *k* (20); and high *n* and high *k* (22).[1] Suppose each treatment were randomly assigned to *r* plots. The usual summary procedure would be to form the four-treatment totals and means in 2 x 2 tables. The totals are indicated as follows:

		Potash low	Potash high	
Nitrogen	low	(00)	(02)	N_0
	high	(20)	(22)	N_2
		K_0	K_2	G

The border totals are indicated by capital letters, with G for the grand total.

If one were unwilling to make any assumptions about the comparability of the four treatments, he would look only at the four-cell mean (cell totals divided by *r*) and use model 2 and the accompanying analysis.

If the experimenter feels that the effect of increased *n* or *k* is the same regardless of the level of the other element, he would use an adaptation of model 1 as follows:

(1′) Y = (mean) $\pm$ (*n* effect) $\pm$ (*k* effect) $\pm$ (error) ,

where the + sign refers to high level plots and the - to low level plots. For example, the average or expected yield for a plot receiving high *n* and low *k* is:

$$(\text{mean}) + (n \text{ effect}) - (k \text{ effect}) .$$

The *n* effect, for example, represents the expected increase in yield due to high *n* over the average of high and low *n*, and is estimated by

$$\frac{N_2 - N_0}{4r} = \frac{N_2}{2r} - Y .$$

The analysis of variance is the same as for model 1. The residual can be used to test the adequacy of the additive model 1′, i.e., test for the existence of an (NK) interaction. If this residual is significant, the

[1] 0 is used for the low level and 2 is used for the high level, so that 1 may be introduced as a middle level.

Source of Variation	Sum of Squares = Mean Square
N	$\frac{(N_2 - N_0)^2}{4r}$
K	$\frac{(K_2 - K_0)^2}{4r}$
Residual	$\frac{[(00)-(02)-(20)+(22)]^2}{4r}$

effect of increased n is not the same for low and for high k (and vice versa). Hence it is necessary to interpret each cell mean separately, i.e., use model 2.

Continuing this aping of the models for discrete factors, the following general model has been constructed for the 2 x 2 experiment:

(3) Y = (mean) $\pm$ (n effect) $\pm$ (k effect) $\pm$ (nk interaction effect) + (error), where the interaction effect receives a plus sign for the (0,0) and (2,2) plots and a minus sign for the (0,2) and (2,0) plots. For example, the expected yield for a plot receiving high n and low k is:

(mean) + (n effect) - (k effect) - (nk interaction effect) .

The interaction effect is estimated by:

$$\frac{(00) - (02) - (20) + (22)}{4r}.$$

If the response surface can be approximated by a simple mathematical function, it seems more logical to estimate the parameters of this function instead of main effects and interactions. In the present example, consider the following continuous model:

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_{12} X_1 X_2 + \text{(error)} . \quad (4)$$

X_1 and X_2 represent the respective levels of nitrogen and potash as deviations from the mean level in the experiment (X = -1 for low and X = +1 for high level); β_0 is the expected yield for n and k midway between the amounts applied in the experiment ($X_1 = X_2 = 0$); β_1 and β_2 are linear effects of added n and k; β_{12} is the interaction parameter. Using model 4, the cell totals (of r plots each) have these expectations:

	low k (X_2=-1)	high k (X_2=1)	Total
low n (X_1=-1)	$r(\beta_0-\beta_1-\beta_2+\beta_{12})$	$r(\beta_0-\beta_1+\beta_2-\beta_{12})$	$2r(\beta_0-\beta_1)$
high n (X_1=1)	$r(\beta_0+\beta_1-\beta_2-\beta_{12})$	$r(\beta_0+\beta_1+\beta_2+\beta_{12})$	$2r(\beta_0+\beta_1)$
Total	$2r(\beta_0-\beta_2)$	$2r(\beta_0+\beta_2)$	$4r\ \beta_0$

The estimators of the β's in equation 4 are:

Parameter	Estimator
β_1	$b_1 = \dfrac{N_2-N_0}{4r}$
β_2	$b_2 = \dfrac{K_2-K_0}{4r}$
β_{12}	$b_{12} = \dfrac{(00)-(02)-(20)+(22)}{4r}$
β_0	$b_0 = G/4r = \bar{Y}$.

The variance of each estimator is $\sigma^2/4r$. Note that these estimators are the same as for the effects of model 3. β_1, for example, measures the average difference in yield per unit change in n for these two k treatments, i.e., the change in Y for a unit change in X_1, neglecting interaction.[2] Also the analysis of variance produces the same three orthogonal sums of squares for treatments, using either models 3 or 4.

Hence it appears that models 3 and 4 are identical. However, there is a very important difference. Model 3 makes no assumption regarding the shape of the response surface, but model 4 implies a definite continuity of response; hence, one would feel free to use the results of model 4 to interpolate between the actual levels used in the experiment. If he did use model 3 for this purpose, he would actually be assuming the continuous model 4. One is often tempted to extrapolate the results beyond the levels used in the experiment; such extrapolation assumes the same response surface holds beyond the experimental levels. In other words, one uses model 2 or 1 if he does not wish to assume a quadratic response surface, but uses model 4 if experience or theory indicates such a surface would be satisfactory.

If the design is spread out so that the low and high levels differ by $2d$ units (instead of 2), b_i will have a denominator of $4rd$ and b_{12} a denominator of $4rd^2$. Hence the variance of b_i is reduced by a factor of d^2 and b_{12} by a factor of d^4. The only reason for not using extremely divergent levels is that the response surface may have a different shape at extremely large or small fertilizer applications.

If the continuous model 4 is used, it seems unreasonable to include a quadratic term involving X_1X_2 without also including terms involving X_1^2 and X_2^2. The shape of a response surface such as model 4 is rather grotesque. In other words one would be more likely to consider the following general quadratic model:

$$(5) \quad Y = \beta_0 + \beta_1X_1 + \beta_2X_2 + \beta_{12}X_1X_2 + \beta_{11}X_1^2 + \beta_{22}X_2^2 + (\text{error}) .$$

[2] It should be clear that the difference between low and high levels is a two-unit change, e.g., if low is 50 pounds per plot and high is 100 pounds per plot, a unit change is 25 pounds and β_1 and β_2 measure the linear effects of 25-pound increases.

If model 5 is the true continuous model, the expectations of the cell totals are:

	low k (X_2=-1)	high k (X_2=1)	Total
low n(X_1=-1)	$r(\beta_0-\beta_1-\beta_2+\beta_{11}+\beta_{22}+\beta_{12})$	$r(\beta_0-\beta_1+\beta_2+\beta_{11}+\beta_{22}-\beta_{12})$	$2r(\beta_0-\beta_1+\beta_{11}+\beta_{22})$
high n(X_1=1)	$r(\beta_0+\beta_1-\beta_2+\beta_{11}+\beta_{22}-\beta_{12})$	$r(\beta_0+\beta_1+\beta_2+\beta_{11}+\beta_{22}+\beta_{12})$	$2r(\beta_0+\beta_1+\beta_{11}+\beta_{22})$
Total	$2r(\beta_0-\beta_2+\beta_{11}+\beta_{22})$	$2r(\beta_0+\beta_2+\beta_{11}+\beta_{22})$	$4r(\beta_0+\beta_{11}+\beta_{22})$

It turns out that the estimates of β_1, β_2, and β_{12} are the same as for model 4 and are not mixed up with the quadratic terms (β_{11} and β_{22}), i.e., they are unbiased estimates. However, there is no method of estimating β_0, β_{11}, or β_{22}, since their sum is estimated by $\bar{Y}$. Hence this analysis indicates that it is safe if only statements are made regarding the treatments used in the experiment and no attempt is made to predict the results for other fertilizer levels.

Of course the solution to the above dilemma is to add other levels of n and k. The traditional design to estimate quadratic effects is the 3 x 3 complete factorial with the three levels of n and k equally spaced.[3] Assuming the middle values of n and k are the averages of the low and high levels used in the 2 x 2 experiment, i.e., if the low and high applications were 50 and 100 pounds per plot, the middle application would be 75 pounds per plot. In the factorial setup, the levels are designated as 0, 1, and 2 with X = -1, 0, 1, respectively. Henceforth, factor combination will be designated by the levels used, e.g., (-1, -1). Assuming r plots per cell and using model 5, the expectations for the (-1, -1), (-1, 1), (1, -1) and (1, 1) totals would be as before. The expectations for the other five class totals and the border totals would be:

(-1, 0)	$r(\beta_0 - \beta_1 + \beta_{11})$
(0, -1)	$r(\beta_0 - \beta_2 + \beta_{22})$
(0, 0)	$r\ \beta_0$
(0, 1)	$r(\beta_0 + \beta_2 + \beta_{22})$
(1, 0)	$r(\beta_0 + \beta_1 + \beta_{11})$
N_0	$3r(\beta_0 - \beta_1 + \beta_{11}) + 2r\ \beta_{22}$
N_1	$3r\ \beta_0 + 2r\ \beta_{22}$
N_2	$3r(\beta_0 + \beta_1 + \beta_{11}) + 2r\ \beta_{22}$
K_0	$3r(\beta_0 - \beta_2 + \beta_{22}) + 2r\ \beta_{11}$
K_1	$3r\ \beta_0 + 2r\ \beta_{11}$
K_2	$3r(\beta_0 + \beta_2 + \beta_{22}) + 2r\ \beta_{11}$
G	$9r\ \beta_0 + 6r\ (\beta_{11} + \beta_{22})$

[3] Equal spacing enables one to analyze linear and quadratic components in a simple manner, but it is not an essential, or even the most efficient, method of spacing.

The following estimators and variances are obtained:

Parameter	Estimator	Variance of Estimator
β_1	$b_1 = (N_2-N_0)/6r$	$\sigma^2/6r$
β_2	$b_2 = (K_2-K_0)/6r$	$\sigma^2/6r$
β_{12}	$b_{12} = [(-1,-1) - (-1,1) - (1,-1) + (1,1)]/4r$	$\sigma^2/4r$
β_{11}	$b_{11} = (N_2-2N_1+N_0)/6r$	$\sigma^2/2r$
β_{22}	$b_{22} = (K_2-2K_1+K_0)/6r$	$\sigma^2/2r$
β_0	$b_0 = [5(N_1+K_1)-(N_0+N_2+K_0+K_2)]/18r$	$5\ \sigma^2/9r$

Note that b_1, b_2, and b_{12} are the same as before; also, if the levels are (-d,0,d), the variances for the linear coefficients are again reduced by a factor of d_2 and for the quadratic and interaction coefficients by a factor of d^4.

The analysis of variance is as follows (ℓ stands for linear and q for quadratic component):

Effect	d.f.	M. S.
N_ℓ	1	$(N_2-N_0)^2/6r$
K_ℓ	1	$(K_2-K_0)^2/6r$
$N_\ell K_\ell$	1	$[(-1,-1)-(-1,1)-(1,-1)+(1,1)]^2/4r$
N_q	1	$(N_2-2N_1+N_0)^2/18r$
K_q	1	$(K_2-2K_1+K_0)^2/18r$
Residual	3	$[SST - SS(N_\ell + K_\ell + \ldots + K_q)]/3$
Error	9(r-1)	$s^2 = SSW/9(r-1)$

The residual mean square can be used to test for the adequacy of the model. If the 3 x 3 complete factorial is used, it turns out that these three degrees of freedom can be subdivided into three orthogonal components, which measure $N_\ell K_q$, $N_q K_\ell$, and $N_q\ K_q$ interaction effects $[\beta_{122}X_1X_2^2 + \beta_{112}X_1^2X_2 + \beta_{1122}X_1^2X_2^2$ is added to model 5].

Once again a factorial model similar to model 3 can be constructed with the same linear and quadratic effects as in model 5. However, there seems little reason for estimating such effects unless one is willing to assume a quadratic response surface. If he does not wish to assume a quadratic response surface, he has two possible factorial models:

1. Model 2 with nine treatments

2. Model 1′ with two effects for each factor: above and below the middle application or referred to either the high or low application.[4]

The analysis based on model 1′ would include a sum of squares attriutable to the interactions, giving a test of the adequacy of the model. These remarks hold for any number of factors and levels per factor.

If there is a mixture of classification variables (e.g., varieties) and continuous variables, a combined factorial and continuous model can be set up and analyzed in a manner analogous to covariance. This would assume that the parameters for the continuous variables were the same for each discrete classification; a test of this hypothesis can also be constructed.

The Use of Blocking Methods to Reduce Experimental Error

The use of blocking methods in the previous discussion has not been considered because they only complicate the presentation without altering any of the conclusions. However, one must consider the blocking procedure if there is confounding. Unfortunately, the procedures used in constructing such designs have been based on confounding certain parts of the higher order interactions which are not related to higher degree components. For example, the so-called I and J parts of the NK interaction in a 3 x 3 experiment do not pertain to any one of the four degree components, $N_\ell N_\ell$, $N_\ell K_q$, $N_q K_\ell$, or $N_q K_q$. One would prefer a design which minimized the confounding on $N_\ell K_\ell$.

A bulletin now in press by Binet, Leslie, Weiner, and Anderson (3) presents the confounding patterns in terms of degree components. This bulletin should be of use in three ways:

1. It presents short-cut methods of analyzing these confounded experiments when degree components are of interest.
2. Several new confounded designs are presented.
3. It presents the confounding patterns for various designs, so the reader can select the design which will be best for his problem.

To illustrate the procedures, suppose the nine treatments in the 3 x 3 experiment were put in 3 blocks of 3 plots each. One such arrangement would be (the treatments refer to levels, and B_i are block totals):

Block		
1	2	3
(1,-1)	(-1,-1)	(0,-1)
(-1,0)	(0,0)	(1,0)
(0,1)	(1,1)	(-1,1)
B_1	B_2	B_3

[4]Cf. Anderson and Bancroft (2), Section 20.5.

If b'_j represents the mean of the j-th block, then two block contrasts are formed:

$$2C_1 = b'_3 - b'_1 \text{ and } 6c_2 = b'_3 - 2b'_2 + b'_1 .$$

The least squares equations for the two block and four NK effects are:

C_1	$N_\ell K_q$	$N_q K_\ell$	C_2	$N_\ell K_\ell$	$N_q K_q$	Yield Sum
6	-6	6	0	0	0	$B_3 - B_1$
-6	12	0	0	0	0	$(N_\ell K_q)$
6	0	12	0	0	0	$(N_q K_\ell)$
0	0	0	18	-6	-18	$B_1 - 2B_2 + B_3$
0	0	0	-6	4	0	$(N_\ell K_\ell)$
0	0	0	-18	0	36	$(N_q K_q)$

The yield sum for $N_\ell K_q$, for example, is:

$$[(1,1) - 2(1,0) + (1,-1)] - [(-1,1)-2(-1,0) + (-1,-1)] .$$

The usual procedure in analyzing these results would be to assume the block contrasts and $N_\ell K_\ell$ were the only real effects. This leaves only one contrast for testing the model, since there are only four degrees of freedom in the above six equations. The method of analysis proposed in the bulletin is the abbreviated Doolittle method, which is also discussed in detail by Anderson and Bancroft (2). Obviously there is no estimate of error from this experiment. If such an estimate is needed, another replicate should be used, preferably one which has a different confounding pattern, as indicated in the bulletin.

For experiments with many factors, it is often possible to estimate the pertinent contrasts by use of fractional designs.

Special Designs To Estimate Parameters of Response Surfaces

The material by Binet et al. (3) furnishes a method of using existing confounded factorial designs to estimate the important degree components. However, for most experiments in which the experimenter has evidence that a smooth response surface is suitable, he should consider designs especially constructed to estimate the parameters of this surface and not to estimate class means for a classification model. Box (5) developed some general design principles for estimating the parameters of planar surfaces.

Box and Wilson (8) proposed a new design for estimating quadratic surfaces which gives more information on the quadratic effects and less on the high-degree effects. Their *composite design* would push the (0,1), (0,-1), (1,0), and (-1,0) points α units from the center of the design as indicated in figure 3.1.

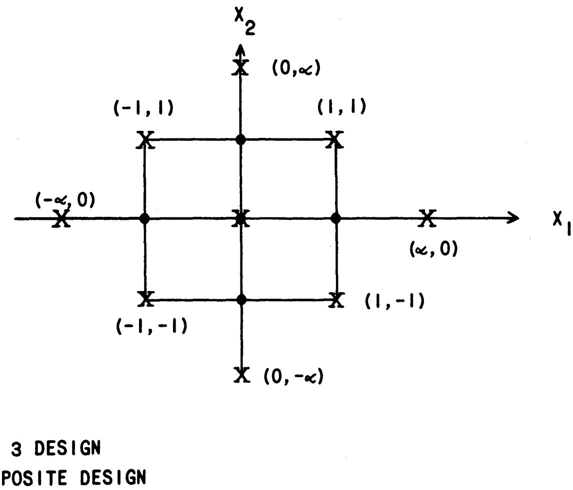

Fig. 3.1 — The Box and Wilson composite design for estimating quadratic surfaces.

If $\alpha = 2$, the expectations of the totals for the four altered cells are:

(-2,0)	$r\ (\beta_0 - 2\beta_1 + 4\beta_{11})$
(0,-2)	$r\ (\beta_0 - 2\beta_2 + 4\beta_{22})$
(0,2)	$r\ (\beta_0 + 2\beta_2 + 4\beta_{22})$
(2,0)	$r\ (\beta_0 + 2\beta_1 + 4\beta_{11})$

In this case one cannot analyze the results as for a 3 x 3 table, because it is an incomplete 5 x 5 factorial experiment. Here one must use the general least-squares approach. The matrix for the normal equations is:

	Coefficients of Estimators						Right hand side
Equation	b_0	b_1	b_2	b_{12}	b_{11}	b_{22}	
b_0	9r	0	0	0	12r	12r	G
b_1	0	12r	0	0	0	0	$g_1 = SX_1Y$
b_2	0	0	12r	0	0	0	$g_2 = SX_2Y$
b_{12}	0	0	0	4r	0	0	$g_{12} = SX_1X_2Y$
b_{11}	12r	0	0	0	36r	4r	$g_{11} = SX_1^2Y$
b_{22}	12r	0	0	0	4r	36r	$g_{22} = SX_2^2Y$

In the preceding, for example, $g_1 = (1,1) + 2(2,0) + (1,-1) - (-1,1) - 2(-2,0) - (-1,-1)$, where (1,1), etc., stand for class totals. The solutions and variances[5] of the estimators are:

Parameter	Estimator	Variance of Estimator ($X\sigma^2/r$) Composite	3 x 3	3 x 3 (Adjusted)
β_1	$g_1/12r$	1/12	1/6	1/12
β_2	$g_2/12r$	1/12	1/6	1/12
β_{12}	$g_{12}/4r$	1/4	1/4	1/16
β_{11}	$(30g_{11} + 18g_{22} - 64G)/384r$	5/64	1/2	1/8
β_{22}	$(30g_{22} + 18g_{11} - 64G)/384r$	5/64	1/2	1/8
β_0	$(10G - 3g_{11} - 3g_{22})/18r$	5/9	5/9	5/9

One gets the impression that there is a tremendous reduction in variances of estimators by use of the composite design instead of the 3 x 3 factorial. However, most of this gain is the natural result of using a wider range of X's; the incompleteness of the factorial in the composite design is not responsible for all the gain. This was indicated for the 2 x 2 experiment. One could adjust the coordinates of the 3 x 3 design so that the spread is the same as for the composite design. The variance of the coordinates for the latter (with $\alpha = 2$) is $[2(4) + 4(1) + 3(0)]/9 = 4/3$. Let the new coordinates for the 3 x 3 design be $(-d,0,d)$, so that the variance of these coordinates is $2d^2/3 = 4/3$; or $d = \sqrt{2}$. Hence, the variances of linear terms are reduced by 1/2 and of quadratic terms by 1/4. Therefore, the composite design has improved the quadratic estimators at the expense of the interaction one. Box and Wilson (8) show that this is desirable in estimating the optimal factor combination.

Another criterion of the relative efficiency of two different designs in estimating the parameters of a response surface would be the amount of information used to estimate the high degree coefficients, which are assumed to be unimportant.

Box and Hunter (7) have advanced another principle of a good surface-fitting design; it should be rotatable; i.e., the accuracy of the estimates of the parameters should not depend on the orientation of the design with respect to the true surface itself. They have constructed several incomplete factorial designs which meet this requirement.

Mason discusses in Chapter 5 some recent experiments in which the composite designs have been used.

[5]These are obtained by inverting the left-hand matrix. The abbreviated Doolittle or square-root method is usually used, although special pattern matrices can be used.

Sequential Experimentation

Much of the impetus for the Box-Wilson paper (8) came from a need to develop sequential procedures for determining optimal factor combinations. Various procedures have been summarized in Anderson's review article (1). Since then, Box (6) has published an extensive discussion of the entire problem. Although the use of these sequential methods may be somewhat limited in fertilizer experiments because of the length of time needed to obtain results, it probably would be desirable to develop a more systematic procedure of utilizing past experience in designing future experiments.

Better methods are needed to pool data from a series of experiments. Researchers should be encouraged to spend more time on these problems.

Some Special Comparisons of Discrete and Continuous Models

Comparison of Discrete Model 2 and Quadratic Model 5 Using 3 x 3 Design

1. *The quadratic model is correct.* In this case the estimated average yield for plots receiving X_1 units of N and X_2 units of K (measured from the mean level) is:

$$\hat{Y} = b_0 + b_1X_1 + b_2X_2 + b_{12}X_1X_2 + b_{11}X_1^2 = b_{22}X_2^2 .$$

In order to obtain the sampling variance of $\hat{Y}$, it requires the variances of the estimators given previously and the covariances. All of these could be obtained by inverting the matrix of sums of squares and products of the regression variables in the normal equations. This matrix is as follows:

	b_0	b_1	b_2	b_{12}	b_{11}	b_{22}
b_0	9r	0	0	0	6r	6r
b_1	0	6r	0	0	0	0
b_2	0	0	6r	0	0	0
b_{12}	0	0	0	4r	0	0
b_{11}	6r	0	0	0	6r	4r
b_{22}	6r	0	0	0	4r	6r

Since b_0, b_{11}, and b_{12} are the only correlated variables, consider them separately in a 3 x 3 matrix A, which when multiplied by its inverse C is the identity matrix.

$$A \begin{bmatrix} 9r & 6r & 6r \\ 6r & 6r & 4r \\ 6r & 4r & 6r \end{bmatrix} \cdot C \begin{bmatrix} C_1 & C_2 & C_2 \\ C_2 & C_3 & C_4 \\ C_2 & C_4 & C_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

There are only four different elements of C. These can be determined quite simply as follows:

$$\left.\begin{array}{l} 9r\,C_1 + 12r\,C_2 = 1 \\ 6r\,C_1 + 10r\,C_2 = 0 \end{array}\right\} \quad C_2 = -1/3r;\ C_1 = 5/9r$$

$$\left.\begin{array}{l} 6r\,C_2 + 6r\,C_3 + 4r\,C_4 = 1 \\ 9r\,C_2 + 6r\,C_3 + 6r\,C_4 = 0 \end{array}\right\} \quad \begin{array}{l} C_4 = -1/2r - \frac{3C_2}{2} = 0 \\ C_3 = 1/6r - C_2 = 1/2r \end{array}$$

Hence the matrix of variances and covariances of the b's is:

$$\frac{\sigma^2}{r}\begin{bmatrix} 5/9 & 0 & 0 & 0 & -1/3 & -1/3 \\ 0 & 1/6 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/4 & 0 & 0 \\ -1/3 & 0 & 0 & 0 & 1/2 & 0 \\ -1/3 & 0 & 0 & 0 & 0 & 1/2 \end{bmatrix}$$

The variance of $\hat{Y}$ is:

$$\sigma^2(\hat{Y}) = \frac{\sigma^2}{r}\,[5/9 + 1/6\,(X_1^2 + X_2^2) + 1/4\,(X_1^2X_2^2) + 1/2\,(X_1^4 + X_2^4) - 2/3\,(X_1^2 + X_2^2)]$$

$$= \frac{\sigma^2}{r}\,[5/9 + 1/2\,(X_1^4 + X_2^4 - X_1^2 - X_2^2) + 1/4\,X_1^2X_2^2]\ .$$

If the discrete model is used, every mean will have a sampling variance of σ^2/r. For even the most divergent points ($\pm$ 1, $\pm$ 1),

$$\sigma^2(\hat{Y}) = 29\,\sigma^2/36r$$

which is less than σ^2/r. Hence, if the quadratic model is correct, even the yields at the experimental points are estimated more accurately from the regression model instead of the simple average yield at that point. Of course $\hat{Y}$ is even more accurate for the other five points.

The same conclusions hold for comparing two mean yields. The largest variance using $\hat{Y}$ is the comparison of (1,1) and 1,-1), which is 5 $\sigma^2/3r$, as compared to 2 σ^2/r for model 2. Many of the comparisons using $\hat{Y}$ have much lower variances than this.

The results might be even more favorable if another design were used.

2. *The quadratic model is biased.* Suppose the true model is model 5 plus $\beta\,X_1^3$. In this case $\hat{Y}$ is too small by β when $X_1 = 1$ and too large by β when $X_1 = -1$. Some mean differences would be biased by 2β, others by β, and others not at all. However, the estimates using

model 2 would be unbiased. The problem of whether to use the biased estimates depends on a comparison of the suspected magnitude of the bias and the variances mentioned above. This problem may be even more serious if the form of the response equation is radically different from the quadratic, e.g., if it is exponential or logistic.

Returning to the bias of βX_1^3, it should be mentioned that at least one of the other β's will also be biased if this term is not considered in the estimation procedure (when $\beta \neq 0$); for example:

$$E(b_1) = \beta_1 + \beta \quad .$$

β is called an *alias* of β_1. Box and Wilson (8) consider possible aliases in evaluating various designs. It is possible to construct designs so that possible aliases will not have much effect on the estimates. This may be one of the chief reasons why agricultural experimenters have not considered continuous models. Hildreth (15) has considered an estimation procedure which is built on model 2, but uses certain inequality restrictions on the production function. The estimation procedure used by Hildreth is discussed in Chapter 4.

Pseudo-Interactions in Some Factorial Experiments

The tendency to follow the mechanical procedure of analyzing factorial experiments in terms of main effects and interactions can result in serious loss of information, often of a misleading nature. As an example, consider an experiment involving two levels of nitrogen (coded n = -1 and 1) and two different cover crops to be plowed under. Suppose C_1 supplies no nitrogen to the soil, whereas C_2 supplies 2 units of *n* (coded n = -1 and 1). In addition, the two crops supply other unspecified nutrients. Assume that the yield is a quadratic function of *n* plus some additive amount due to the unspecified nutrients in the soil and furnished by the two crops: $\beta_0 - \gamma$ for C_1 and $\beta_0 + \gamma$ for C_2 (γ may be positive or negative). Hence the model is:

$$Y = \beta_0 + \beta_1 n + \beta_{11} n^2 \pm \gamma + (\text{error}) \quad ,$$

where γ is added for C_2 plots and subtracted for C_1 plots.[6] The expected class and border total yields are:

	Crop 1	Crop 2	Total
n = -1	$r(\beta_0-2\beta_1+4\beta_{11}-\gamma)$	$r(\beta_0+\gamma)$	$2r(\beta_0-\beta_1+2\beta_{11})$
n = 1	$r(\ _0-\ )$	$r(\beta_0+2\beta_1+4\beta_{11}+\gamma)$	$2r(\beta_0+\beta_1+2\beta_{11})$
	$2r(\beta_0-\beta_1+2\beta_{11}-\gamma)$	$2r(\beta_0+\beta_1+2\beta_{11}+\gamma)$	$4r(\beta_0+2\beta_{11})$

[6]The center of the system is now one unit more than the average of the two nitrogen levels.

The estimators and their variances are:

Parameter	Estimator	Variance
β_1	$(N_1 - N_{-1})/4r$	$\sigma^2/4r$
β_{11}	$[(-1,1)-(-1,2)-(1,1)+(1,2)]\ /8r$	$\sigma^2/16r$
γ	$[(-1,2)-(1,1)]\ /2r$	$\sigma^2/2r$
β_0	$[(-1,2)+(1,1)]\ /2r$	$\sigma^2/2r$

Compare these results with those obtained by use of traditional factorial methods.

Effect	Yield	E(Yield)	E(MS)
Nitrogen	N_1-N_{-1}	$4r\beta_1$	$4r\beta_1^2+\sigma^2$
Crop	C_2-C_1	$4r(\gamma+\beta_1)$	$4r(\gamma+\beta_1)^2+\sigma^2$
N x C	$(-1,1)-(-1,2)-(1,1)+(1,2)$	$8r\beta_{11}$	$16r\beta_{11}^2+\sigma^2$

An N x C interaction is indicated if there is a quadratic effect of nitrogen; also the crop effect will be mixed up with the linear effect of nitrogen (this is satisfactory if one only wants to test for differences in yields and not to determine basic causes of such differences). But a major criticism is a failure to provide a method of estimating the quadratic effect of nitrogen. The N x C interaction effect is the least squares estimate of β_{11}, but this fact is concealed in a routine factorial analysis of variance.

This is a very simple illustration of the need for more basic models in discussing responses to treatments. Classification models may conceal basic response patterns. One might consider this problem when three instead of two levels of n were used. In this case the factorial estimate of β_{11} probably would be inefficient, because of neglect of the information from the N x C interaction.

Yates (26) presents a 2^3 experiment with 4 replications, the factors being N, K, and D (dung). Levels were none and some, the latter being 0.45 cwt. N per acre, 1.12 cwt. K_2O per acre, and 8 tons of D per acre: Assume that this amount of dung supplies the same as the "some" of n and k, plus "some" other nutrients (called d). Code these data with -1 for none and +1 for some. Hence, the values of the variables for the various plots are:[7]

[7]A unit of nitrogen is 0.225 cwt., of potash is 0.56 cwt., and of dung is 4 tons: the center is at 0.45 N, 1.12 K and 4 D.

N K D	n	k	d	Yield of 4 plots
0 0 0	-2	-2	-1	425
1 0 0	0	-2	-1	426
0 1 0	-2	0	-1	1118
1 1 0	0	0	-1	1203
0 0 1	0	0	1	1283
1 0 1	2	0	1	1396
0 1 1	0	2	1	1673
1 1 1	2	2	1	1807

Assume a quadratic equation in *n* and *k*, with *d* appearing linearly. Hence:

$$Y = \beta_0+\beta_1 n+\beta_2 k+\beta_{11}n^2+\beta_{22}k^2+\beta_{12}nk+\beta_3 d+ (\text{error}) \quad . \tag{6}$$

Because this experiment was not designed to estimate quadratic effects, it turns out that if a complete quadratic model was used with $\beta_{33}d^2$, $\beta_{13}nd$, and $\beta_{23}kd$ included, the following pairs of coefficients could not be separated: β_0 and β_{33}; β_{11} and β_{13}; and β_{22} and $\beta 23$. In other words the constant and d^2, n^2, and nd and k^2 and kd are aliases. It is assumed here that *d* is essentially a residual variable, which is unlikely to have any effect and especially not a quadratic one; however, one cannot be sure which of two aliases is responsible for an effect.

The matrix for the least squares equations for model 6 is:

b_0	b_1	b_2	b_{11}	b_{22}	b_{12}	b_3	Yield Sum
32	0	0	64	64	32	0	9,331
	64	32	0	0	0	32	3,320
		64	0	0	0	32	5,258
			256	128	128	0	18,984
				256	128	0	17,324
					128	0	8,928
						32	2,987

The forward solution of the abbreviated Doolittle method is as follows:

	b_0	b_1	b_2	b_{11}	b_{22}	b_{12}	b_3	Yield
	32	0	0	64	64	32	0	9331
b_0	1	0	0	2	2	1	1	9331/32
		64	32	0	0	0	32	3320
b_1		1	1/2	0	0	0	1/2	3320/64
			48	0	0	0	16	3598
b_2			1	0	0	0	1/3	3598/48
				128	0	64	0	322
b_{11}				1	0	1/2	0	322/128
					128	64	0	-1338
b_{22}					1	1/2	0	-1338/128
						32	0	105
b_{12}						1	0	105/32
							32/3	383/3
b_3							1	383/3

The variance-covariance matrix and the estimates are:

$$
\begin{array}{c}
\\
\frac{\sigma^2}{128}
\end{array}
\begin{array}{c}
\begin{array}{ccccccc} b_0 & b_1 & b_2 & b_{11} & b_{22} & b_{12} & b_3 \end{array} \\
\left[\begin{array}{ccccccc}
16 & 0 & 0 & -4 & -4 & 4 & 0 \\
 & 4 & 0 & 0 & 0 & 0 & -4 \\
 & & 4 & 0 & 0 & 0 & -4 \\
 & & & 2 & 1 & -2 & 0 \\
 & & & & 2 & -2 & 0 \\
 & & & & & 4 & 0 \\
 & & & & & & 12
\end{array}\right]
\end{array}
$$

	Estimates
b_0	310.75
b_1	10.41**
b_2	70.97**
b_{11}	.88
b_{22}	-12.09**
b_{12}	3.28
b_0	11.97*

Since the error variance in the experiment was 347.01 (with 21 degrees of freedom), $\sigma^2/128$ is estimated by 2.71. This is multiplied by the diagonal terms to obtain the estimated variances for the estimates. All linear terms and the quadratic term for k are significant (b_3 barely so at the 5% level) while b_{12} is about the same size as its standard error. The sum of squares can be compared with those of Yates as follows:

Effect	Yates	Here[8]
N_ℓ	3,465.3	172,225.0
K_ℓ	161,170.0	269,700.1
N_q and $N_\ell D_\ell$	810.0	810.0
K_q and $K_\ell D_\ell$	13,986.3	13,986.3
$N_\ell K_\ell$	344.5	344.5
D_ℓ	278,817.8	1,528.0
$N_\ell K_\ell D_\ell$	124.0	124.0

This is only an illustrative example, however, and should serve as an example of the procedure. There may be some questions concerning the use of the coded values. These are put in so that the estimators will be as nearly uncorrelated as possible; this enables one to better evaluate the usefulness of various predictors in the model. Box and Wilson (8) generally follow this procedure.

Problem of Adjustment for Available Nutrients With Continuous Models

One of the major needs in the determination of fertilizer response surfaces is a method of adjusting for nutrients available in the soil before the experiment is started. In a single experiment, it is usually assumed that the variation in basic levels is random, with the average level being taken account of by the constant term. If there are no essential differences between the basic levels in the plots for each of the treatments, the results of the experiment can be used to indicate treatment contrasts. However, if a continuous model such as the quadratic model 5 is used, the experimenter should be careful about extending the results to plots with different available nutrients.

If the effect of the available nutrients is to merely increase the actual levels of X, the results can be converted to a prediction equation in terms of the available plus added nutrients. In order to simplify the results, consider a quadratic prediction equation for an experiment involving only one nutrient,

$$E(X) = \beta_0 + \beta_1 X + \beta_{11} X^2 \,, \tag{7}$$

where X is the added amount of the nutrient. The actual amount (available plus added) of the nutrient in an experiment is designated as $N = X + d$ $(X = N - d)$. Then:

$$E(N) = (\beta_0 - \beta_1 d + \beta_{11} d^2) + (\beta_1 - 2\beta_{11} d)\, N + \beta_{11} N^2 \,. \tag{8}$$

Now try to apply the results of this experiment to a farm. The

[8]These are not adjusted sums of squares; i.e., N_ℓ is not adjusted for K_ℓ or D_ℓ; K_ℓ is not adjusted for D_ℓ; and N_q and K_q not for $N_\ell K_\ell$. Note the N_q = Yates' $N_\ell D_\ell$ and the K_q = Yates' $K_\ell D_\ell$, as indicated above.

predicted yield if X is applied is E(X). Suppose the value of d for this farm is d_0 ($N = X + d_0$); then the expected yield when X is added should be:

$$F(X) = (\beta_0 - \beta_1 d + \beta_{11} d^2) + (\beta_1 - 2\beta_{11} d)(X + d_0) + \beta_{11}(X + d_0)^2$$

$$= [(\beta_0 - \beta_1(d - d_0) + \beta_{11}(d - d_0)^2] + [\beta_1 - 2\beta_{11}(d - d_0)]X + \beta_{11}X^2 .$$

The bias in using E (X) instead of F (X) is:

$$E(X) - F(X) = (d - d_0)(\beta_1 + 2\beta_{11}X) - (d - d_0)^2 \beta_{11} . \tag{9}$$

One might suppose that even though the predicted yield is biased, at least the difference between the predicted yields for two different levels of farm application would be unbiased. Even this is not true. The bias in the predicted increase in yield for an application of X_2 instead of X_1 is $2\beta_{11}(X_2 - X_1)(d - d_0)$, which will be negative for $X_2 > X_1$ and $d > d_0$, since B_{11} is expected to be negative; hence, one would tend to underestimate the effect of added nutrients if the available nutrients at the farm are less than at the experimental plots.

These problems become further aggravated when one attempts to combine the results of experiments at two locations with different values of d. Suppose $d = d_1$ for one location and $d = d_2$ for a second location, but the same rates of application are used in each experiment, e.g., $X = -1, 0, 1$. If a quadratic model is used, the experimental model E(N) is:

$$E(N) = \beta_0^* + \beta_1^* N + \beta_{11}^* N^2 , \tag{10}$$

where β_0^*, β_1^* and β_{11}^* can be found from model 8 above. The values of the β^* are assumed to be the same for each experiment (neglecting other nutrients in this discussion); however, the values of β_0 and β_1 in model 7 are not the same. Let β_1' and β_1'' represent the values of β_1 in experiments 1 and 2, respectively.

Then solving for the β's in terms of the β^*'s, yields:

$$\beta_0' = \beta_0^* + d_1\beta_1^* + d_1^2\beta_{11}^* \text{ and } \beta_0'' = \beta_0^* + d_2\beta_1^* + d_2^2\beta_{11}^* ;$$

$$\beta_1' = \beta_1^* + 2d_1\beta_{11}^* \text{ and } \beta_1'' = \beta_1^* + 2d_2\beta_{11}^* .$$

On the basis of the above results, the experimenter would make one of two incorrect decisions if he did not take account of the inequality of the available nutrients for the two experiments:

1. He would conclude that the true response pattern was different at the two localities and publish two predition equations, each of which represents an inefficient use of the data in estimating the basic parameters. This may prevent the savings in extension work which overall recommendations entail. However, the biases mentioned above are less likely to be so important, because the experimenter realizes his prediction equation is different for different locations.

2. If the experimental error is large compared with $(d_2 - d_1)$, he might conclude that the differences in the estimates of the β's was a

chance difference, and use average β's for his prediction equation. This would produce the biased results mentioned above. However, the important point here is that the estimates of the parameters are quite inefficient because the large spread in N over both experiments is neglected. There is uncertainty as to which incorrect procedure is worse, since this is a matter of weighing the extra costs of a wide variety of recommendations against the inefficiencies and biases of over-all recommendations.

To illustrate the fact that one can obtain more information regarding the response surface by combining the two experiments, suppose only two levels of X (X = -1 and 1) are used for each experiment but the available coded levels are $d_1 = -1$ and $d_2 = 1$. If single estimates are made for each experiment, no estimator of β_{11} will be available; hence, if β_{11} is not zero, the separate estimators of the linear coefficient will be biased. However, in this case, the pooled estimator of β_1 will be unbiased because $d_1 + d_2 = 0$. Also, in this case, the objective is to compare the response surfaces in terms of the total nutrients (X + d). The number of plots for each level of N and the estimators of β_1^* and their expected values when $\beta_{11}^* \neq 0$:

	N				
Experiment	-2	0	2	b_1^*	$E(b_1^*)$
1	r	r		$(N_0 - N_{-2})/2r$	$\beta_1^* - 2\beta_{11}^*$
2		r	r	$(N_2 - N_0)/2r$	$\beta_1^* + 2\beta_{11}^*$

In both experiments, $\sigma^2(b_1^*) = \sigma^2/2r$. The pooled estimate of β_1^* is unbiased and has $\sigma^2(b_1^*) = \sigma^2/4r$.

If a combined analysis is made, $b_{11}^* = (N_2 - N_0 + N_{-2})/8r$, where N_0 is the sum of the yields of the $2r$ plots with N = 0; $\sigma^2(b_{11}^*) = \sigma^2/16r$. In this case $b_1^* = (N_2 - N_{-2})/4r$ with $\sigma^2(b_1^*) = \sigma^2/8r$; note that this variance is one-half the pooled variance. Even if the experimenter wants to assume different values of β_0^* in each experiment because of unequal amounts of other nutrients, he obtains the same estimate of β_{11}^* from the combined data, and the above pooled estimate of β_1^*.

If a more complicated model is considered, such as an exponential or logistic model, the experimenter will probably find that the inclusion of the available nutrients in the model is just as important. It may be that one of the reasons for obtaining such unrealistic production functions from combined data is the failure to adjust for the available nutrients. Also, this may account for the divergent shape of combined response surfaces when various mathematical forms are used. Someone might make studies similar to these for the more complicated production models.

If one can obtain more efficient and more nearly unbiased estimates by adjusting for available nutrients in several experiments in a combined analysis, why is this not done more often? In many cases, the answer may be lack of knowledge of how to make even the simple combined analyses. However, the real answer may be generally more complimentary to experimenters:

1. Statisticians have not developed easy and efficient estimation procedures for the more complicated models.
2. Procedures for determining available nutrients are not too well developed.
3. It is often difficult to calibrate available and applied nutrients.
4. Even though only a few nutrients are added in the experiment, admustments must be made for all available nutrients. This may result in a much more complicated analysis.
5. Research has not been well coordinated. As a result, computations may be complicated and total levels may not be spread out very much in the various experiments.
6. Adjustments for weather factors are also needed, especially when combining data from several years. Crop-weather and soil-weather relationships are even more poorly known than are crop-nutrient relationships.

Much of the computing difficulty will probably be relieved as more use of electronic computers is made. Hence, it should be recommended that coordination of efforts in the direction of setting up realistic models and measuring and calibrating available nutrients is needed.

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Chapter 4

Discrete Models With Qualitative Restrictions

IN statistical analyses, as in many other human endeavors, the product of a particular undertaking is closely related to the input. At the final stage of a statistical application, what one puts in are some observations and a specification; what one gets out are some statistical inferences, i.e., estimates, tests, and/or optimal decisions. Ways in which good observations contribute to useful inferences are generally well understood and are quite properly stressed in most applied statistics courses. The possible contributions, positive or negative, of alternative specifications are not as easily understood and, for many problems, have not been adequately explored by statistical theorists.

Specifications

Since the rationale for the procedure to be outlined and illustrated depends entirely on considerations of specification, a few general remarks on these matters may be helpful. First, a statistical specification is defined as the complete set of assumptions which are accepted as a basis for a particular statistical investigation. Another way of putting this is to say that a specification includes all statements about the underlying statistical population which the investigator accepts a priori.

Specification and model are nearly synonymous terms. According to a fairly well accepted usage (1, 6) observed here, the model is the class of all statistical populations which are consistent with the specification, i.e., which satisfy the a priori assumptions.[1] For most of the discussion the terms will be used interchangeably.

In general, an investigator's situation is such that if he adds assumptions to his specification (narrows his model), the prospective accuracy of his inferences is increased, provided the added assumptions are realistic. However, if the assumptions are unrealistic, biased inferences will generally result. Thus, a researcher should clearly use in his specification all of the relevant a priori information that he is sure is realistic.[2] In doubtful cases, the investigator may be helped by

[1]This statement would require some modification in contexts in which one needs to distinguish the statistical population from the theoretical structure which explains it. Such instances have arisen mainly in economics and psychology and need not be taken into account in the following discussion.

[2]Sometimes a researcher may ignore potentially useful a priori information to simplify computations. This possibility is left aside to keep from diverting the discussion.

theoretical research indicating the extent to which a particular added assumption may improve the inferences to be drawn and, on the other hand, the biases to which particular errors in the assumption will lead. If the possible biases are large relative to potential gains, a doubtful assumption should, of course, be rejected. If the prospects are reversed, a doubtful assumption might well be utilized. Considerable reliance upon the judgment of the investigator is unavoidable in all but the most routine applications, and good judgment combined with technical skill is what makes a good applied statistician.

That the contribution of a priori information differs from one problem to another may be observed by considering estimates obtained from a random sample from a normal population. If the investigator is primarily interested in a good estimate of the population variance, he may improve his estimate by specifying the population mean a priori, if it is known. This specification will substantially improve his estimate of the variance, if he has only a few observations, but will only be a slight improvement if he has many observations. Thus, if he (a) has a fair a priori notion of the mean but does not know it exactly, and (b) has a small number of observations, he might very well use his best a priori value for the mean; otherwise he may neglect his a priori notion.[3] On the other hand, if the investigator is primarily interested in estimating the mean, a priori knowledge of the variance is not of any help.

Clearly the difficult case is the one in which an uncertain assumption (a) may improve the analysis significantly if correct, and (b) damage it badly if incorrect.[4] A thorough knowledge of the field of application should help the research worker to judge the likelihood of bias. Sometimes a test of significance can be developed as an additional aid to judgment. However, this precaution has often been pointed out; i.e., to test an assumption and then use it (if not rejected) as part of the specification on which subsequent estimates and tests are based complicates the interpretation of the traditional probability statements that are later made about test statistics or confidence regions. While this statement is undeniable, it should not seriously inhibit use of preliminary tests. The basic difficulty is not that a preliminary test is performed but that the investigator is under pressure to utilize an uncertain assumption. Proceeding without attest does not remedy this basic difficulty.

The particular specification problem with which we shall be concerned is that of formulating appropriate assumptions about the form of a response surface. For convenience, a certain observable response, y, depends upon the magnitudes of certain observable, and sometimes controllable, variables, $z_1, z_2 \dots z_k$, and certain unobservable variables whose net effect may be approximately represented by a random variable, u. The unobserved variables may be partly controllable, especially in carefully conducted experiments. The assignment of the z's

[3]If his a priori information could be put in the form of a distribution function for the population mean, and the weight function for various possible errors in the estimate of the variance were taken into account, this could be handled as a statistical decision problem.

[4]A simple but suggestive example has been presented by Leonid Hurwicz (4).

may be randomized to assure that u is independent of the z's, and conditions can sometimes be held sufficiently stable from one observation to another that u will have a small variance.

Form of Equation

Familiar statistical procedures give the investigator two types of alternatives. He may assume a priori that an equation of a certain known form will represent the surface to a close approximation and use the observations to estimate several unknown parameters in the equation. Alternatively, he may forego the assumption as to form and regard each distinct combination of the z's as a different treatment, unrelated to the others in his statistical model. These alternatives correspond to the continuous and discrete models discussed by Anderson in Chapter 3. To use a discrete model it is necessary to make some specifications about the form of the function. Also, assumptions must be made about the way in which the random component, the u, enters. It is usually found desirable to make some assumptions about the interactions of the z's. There are, of course, an infinite number of models for each type of interaction from which an investigator might choose.

Advantages and Disadvantages of Continuous Models

Continuous models offer several potential advantages. There may be a substantial gain in efficiency in having a small number of parameters to estimate and in estimating response at a particular point (a particular combination of the z's) from all of the observations rather than just the observations at that point. The estimated equation provides a convenient means of interpolation and limited extrapolation. Furthermore, the form of the relation, once it is well established, may have interesting theoretical implications.

The principal disadvantage of continuous models lies in the biases which may accrue if an inappropriate form is used, and the difficulty of designing a satisfactory test of the appropriateness of a particular assumption regarding the form. It is particularly disconcerting that, in many instances in which several alternative assumptions have been investigated, alternative fitted equations have resulted which differ little in terms of conventional statistical criteria, such as multiple correlation coefficients or F tests of the deviation, but differ much in their economic implications (cf. 5, 9). It is also worth noting that bias due to inappropriate form does not decrease as sample size increases,[5] whereas inefficiencies in discrete or form-free methods become less important in large samples. In many contexts the convenience of interpolation offered by a continuous function may not be very important. Frequently the discrete alternatives analyzed will be sufficiently numerous to

[5]In general, bias will decrease if the range of the observations is increased along with sample size and, of course, can also be decreased by changing the assumed form as discrepancies become apparent.

determine an optimal decision to the degree of accuracy permitted by the data. In addition, when results of analyses are put to practice, there will always be relevant discrepancies between the conditions underlying the analysis and the conditions faced in commerical production on farms. Some judgment will of necessity be exercised at this stage; interpolation may be as effective as using a predetermined formula.

As noted earlier, there are many situations in which choosing a specification involves delicate judgment and a thorough knowledge of the particular field of application. Where judgment plays a large part, two different researchers may use somewhat different models and procedures without any existing way of labeling one, right, and the other, wrong. Instead of seeking "the" way to proceed in such instances, mathematical statisticians might better try to give the applied worker the means for employing any of a variety of models and procedures, thus enlarging the area over which judgment can be exercised.

Situations sometimes arise, for production economics analysis as elsewhere, in which the investigator does not find either the continuous or traditional discrete type of model to be ideal. He may feel that no particular form of function has been sufficiently well established in his area to give reasonable assurance against bias in a continuous model. He may have rather firm notions about some properties underlying the relation. These properties are ignored if he treats distinct input combinations as unrelated treatments. An economist might, for instance, strongly believe that a particular production function is characterized by diminishing returns; that a certain demand equation is homogeneous; that a certain supply curve slopes upward. To the extent that he knows these properties exist, it is wasteful to analyze statistical results that are inconsistent with them. For such situations it might be useful to have procedures enabling the researcher to include in his specification such qualitative properties as seem sufficiently well established, without forcing him to specify his relation as completely as a continuous model requires.

A Discrete Model

A possible approach is to formulate discrete models which include the appropriate qualitative restrictions and to work out appropriate statistical procedures for these models. Appropriate procedures can be found for a variety of such models. In an article by Hildreth (2), procedures were developed for obtaining estimates of points on a production surface under the assumption that inputs are subject to diminishing returns.[6] The work is now being extended and, while it is highly incomplete, a sketch of accomplishments may serve to suggest possibilities of the approach and the kinds of problems, mostly unsolved, which are encountered in using it.

[6]This exposition is marred by the inclusion of a hastily attempted generalization which can be shown to be false. A correction may be found in the December 1955 issue of the Jour. Amer. Stat. Assn. Fortunately, the false generalization does not affect the main result or the applications which have been developed.

The extensions have been worked out jointly by the author and A. P. Stemberger. They will be more fully reported in Stemberger's doctoral thesis. The data come from experiments on the response of corn yields to nitrogen, conducted by Krantz and Chandler (7).

The model initially employed was of the following form:

$$(1) \qquad y_{nt} = \rho\ (z_n) + u_{nt} \qquad\qquad \begin{array}{l} n = 1,2\ \dots\ N \\ t = 1,2\ \dots\ T_n \end{array}$$

where the N-observed levels of nitrogen have been arranged in ascending order and z_n is the pounds per acre in the n-th level ($z_{n+1} > z_n$). y_{nt} is the observed yield for the t-th trial (observation) with application z_n. T_n is the number of plots to which z_n pounds have been applied. u_{nt} is a random disturbance assumed to be independent of z_n.

The algebraic form of the production or response function, ρ (z), is regarded as unknown except that successive increments of z are assumed to increase y at a nonincreasing rate. In other words ρ (z) is concave, or $\frac{d^2y}{dz^2} \leq 0$ if the derivative exists. With only this assumption regarding form it is not generally possible to estimate the response to levels of nitrogen other than those (N in number) for which observations are available.[7]

Since there is no loss of generality in taking E $(u_{nt}) = 0$, the following may be written:

$$(2) \qquad \eta_n = E\ (y_{nt}) = \rho\ (z_n).$$

The assumption of diminishing returns then requires:

$$(3) \qquad \frac{\eta_{n+2} - \eta_{n+1}}{n+2\ -\ n+1} \leq \frac{\eta_{n+1} - \eta_n}{z_{n+1} - z_n} \qquad\qquad n = 1,2\ \dots\ N\text{-}2.$$

Regarding the η_n as the magnitudes to be estimated, the application of the method of maximum likelihood (if the u_{nt} are normally distributed) or the method of least squares leads to the problem of finding estimates, $\hat{\eta}_n$, which minimize the sum of squares:

$$(4) \qquad Q = \sum_{n=1}^{N} \sum_{t=1}^{T_n} (y_{nt} - \hat{\eta}_n)^2$$

[7] It is possible to estimate upper and lower bounds for all z such that $z_1 < z < z_N$; upper bounds can be estimated for $z > z_N$ or $z < z_1$.

when the restrictions, equation 3 above, believed to hold for the population parameters, are also required to hold for the estimates.

Thus the estimation problem is one of minimizing a positive definite quadratic form subject to constraints in the form of inequalities. Problems like this have been studied in activity analysis and in game theory. With the aid of a theorem by Kuhn and Tucker (8), it was possible to develop an iterative procedure for obtaining the required estimates.[8]

The use of this procedure to obtain yield estimates from the Krantz-Chandler data is described in the article mentioned previously. At the time of the estimates, only data pertaining to "good" weather and one type of soil were available. When access to the complete data was obtained, it was found that numerous other observations were available covering weather experience classified into three main categories: good, fair, and dry. Also, several soil types were available which could be placed in three fairly homogeneous classes: Piedmont, Coastal, and Drained Coastal.

The problem of using all of the data in a unified analysis was similar to problems sometimes encountered in combining data from different experiments. The model was modified to allow for soil and weather effects and could then be indicated:

$$y_{ijnt} = \alpha + \beta_i + \gamma_j + \eta_n + u_{ijnt} \tag{5}$$

$$\begin{aligned} l &= 1,2,3 \\ j &= 1,2,3 \\ n &= 1,2 \;\ldots\; 12 \\ t &= 1,2 \;\ldots\; T_{ijn} \end{aligned}$$

where:

y_{ijnt} = the t-th yield observed on soil i with weather j and nitrogen level n.

α = a general constant

β_i = the contribution to yield of soil i

γ_j = the contribution to yield of weather j

η_n = the contribution to yield of applying z pounds of nitrogen

u_{ijnt} = a random disturbance

T_{ijnt} = the number of observations with soil i, weather j, and level of nitrogen n.

The twelve levels of nitrogen were in 20-pound intervals from 0 to 220, inclusive.

[8]The computing procedure developed may also be used to solve a number of nonlinear programming problems, including some involving monopoly and risk elements.

Interaction Among Soil, Water, and Fertilizer

The model indicated by 5 assumes no interaction among soil, weather, and nitrogen effects. With observed yield as the dependent variable, this would mean, for instance, that dry weather should cut yield the same number of bushels on heavily fertilized plots as on lightly fertilized plots, and similarly for other effects. This assumption is not entirely plausible. A somewhat more promising possibility is the assumption that a change in weather has the same percentage effect on plots with various combinations of soil and fertilizer. To modify these assumptions regarding interaction, log y_{ijnt} is substituted for y_{ijnt} in equation 5.

For convenient future reference, write:

$$Y_{ijnt} = \alpha + \beta_i + \gamma_j + \eta_n + u_{ijnt} \tag{6}$$

where $Y_{ijnt} = \log\ y_{ijnt}$ and other symbols have meanings similar to their meanings in equation 5, except that the constants are now logs of factors in an expression for observed yield. Equation 6 is equivalent to

$$y_{ijnt} = A^{\alpha + \beta_i + \gamma_j + \eta_n + u_{ijnt}} \tag{7}$$

where A is the base of the system of logarithms used.

For several reasons it seemed desirable to initially analyze both equations 5 and 6 without imposing restrictions on the η_n. Before doing this it seemed reasonable to test the interaction assumption in equation 5. The restrictions on the η_n in equation 6 which would express diminishing marginal productivity are nonlinear; direct estimation of the coefficients of equation 6, subject to restrictions, would be even more difficult. While the interaction assumption implicit in equation 6 seems more plausible a priori than that in equation 5, it still seemed desirable to test this assumption before deciding what other analyses might be worthwhile. The data on which the analyses are based are given in table 4.1.

The tests for interaction confirmed the a priori belief that equal percentage effects were more plausible than equal absolute effects. The test showed significant interaction in equation 5 at the 0.01 level,[9] whereas the test applied to equation 6 shows no significant interaction, as can be seen in table 4.2. Accordingly, further analysis was confined to equation 6. The estimates of coefficients for equation 6 are given in table 4.3.

All of the indicated F ratios are significant at the 0.001 level, except for interaction which is not significant at the 0.05 level. In testing for

[9] For equation 5, the interaction mean square was 364.08, within cells mean square was 189.24, giving an F of 1.92. Degrees of freedom are 39 and 182 as in equation 6. The assistance of R. L. Anderson in performing these tests is gratefully acknowledged.

TABLE 4.1 Corn Fertilization Data

	Levels of Nitrogen in Pounds											
	0	20	40	60	80	100	120	140	160	180	200	220
A - Piedmont Soil												
	(Dry)	(Dry)	(Dry)	(Dry)	(Dry)	(Dry)	(Dry)	(Dry)				
	18.0	29.9	44.5	50.4	50.4	51.0	52.1	51.1	-	-	-	-
	33.3	41.4	51.1		62.3		60.2					
	35.6	48.5	37.0		37.7		39.2					
	31.7	42.0	54.9		54.5		57.8					
			63.8		67.1		74.5					
Cell means	29.6	40.4	50.3	50.4	54.4	51.0	56.8	51.1				
B - Drained Coastal Soil												
	-	(Dry)	-	-	(Dry)	-	-	(Dry)	-	-	-	-
	-	27.6			50.2			61.4				
	-	(Fair)	-	-	(Fair)	-	-	(Fair)	-	-	-	-
		66.0			86.5			88.9				
	(Good)	(Good)	(Good)		(Good)	-	(Good)	(Good)	(Good)	-	-	(Good)
	50.1	39.8	62.9		77.7		89.0	102.8	86.6			90.7
		80.8	63.5		78.1		114.3		90.0			
		22.2	57.0		111.5				123.2			
		84.4			110.2				123.9			
					70.3				86.7			
					90.6				99.4			
					102.4				114.5			
Cell means	50.1	56.8	61.1	-	91.5	-	101.6	102.8	103.5	-	-	90.7
C - Coastal Soil												
	(Dry)	(Dry)	(Dry)	(Dry)	(Dry)	(Dry)	(Dry)	(Dry)	(Dry)	(Dry)	(Dry)	(Dry)
	9.4	21.0	-	37.8	48.5	53.4	43.9	52.5	59.0	45.5	59.2	41.5
	21.3	44.6		52.6	60.8	60.5	61.3	40.0		68.6	63.0	81.2
	4.3	22.1		50.6	48.8	50.6	52.7	66.4		58.5	75.3	70.7
	14.4	29.6		42.0	57.5	68.2	80.9	62.6		50.7	61.8	70.7
	15.2	31.6		56.0		40.9	62.2			64.8		
	3.7			56.9								
	20.4											
	17.2											
	5.4											
	3.5											
	14.7											
	39.5											
Cell means	14.1	29.8	-	49.3	53.9	54.7	60.2	55.4	59.0	57.6	64.8	66.0
	(Fair)	(Fair)	(Fair)	(Fair)	(Fair)	(Fair)	(Fair)	(Fair)	(Fair)	-	(Fair)	-
	9.9	43.4	39.1	52.2	60.3	81.0	72.0	83.6	85.5		81.0	
	31.3		24.9	66.0	59.5		74.1	88.2			85.8	
	24.2			64.3	81.8		78.8					
	15.2			74.0	74.8							
	12.9											
Cell means	18.7	43.4	32.0	64.1	69.1	81.0	75.0	85.9	85.5		83.4	
	(Good)	(Good)	(Good)	(Good)	(Good)	(Good)	(Good)	(Good)	(Good)	(Good)	(Good)	(Good)
	19.8	35.7	40.2	59.0	59.8	77.2	80.8	86.9	81.5	115.7	72.9	107.8
	63.9	50.1	96.9	77.3	102.2	86.7	73.0		117.1	95.1	108.0	116.5
	22.8	56.0	52.1	58.5	81.7	83.6	117.0		102.3	110.3		74.7
	51.7	42.0	85.1	34.0	107.1	98.3	100.5		114.3	102.7		
	18.8	31.8	63.6	50.1	80.0	108.3	115.5		101.0	120.9		
	17.4	27.3	44.9	49.5	69.0		92.1		98.3	103.9		
	2.8	42.2	49.1	62.2	94.6		83.9		97.8	98.2		
	13.3	35.3			79.0		90.6		104.9	128.6		
	14.6				68.6		96.3		70.2			
	24.4				88.1		107.0					
	11.6				59.0		102.5					
	20.8				94.4		78.1					
	19.1				90.2		68.7					
	7.2				79.5		69.8					
	16.6						62.4					
	6.9											
	8.3											
	9.2											
	7.7											
	25.8											
Cell means	19.3	40.1	61.7	55.8	82.4	90.8	89.2	86.9	98.6	109.4	90.5	99.7
Treatment means	19.4	41.9	54.7	55.2	74.8	71.6	78.4	71.3	97.6	89.5	75.9	81.7

Soil means		Weather means		General means	
$\bar{y}_{1..}$	47.7	$\bar{y}_{\cdot 1.}$	44.5	$\bar{y}_{...}$	61.7
$\bar{y}_{2..}$	81.7	$\bar{y}_{\cdot 2.}$	55.0		
$\bar{y}_{3..}$	60.1	$\bar{y}_{\cdot 3.}$	72.3		

TABLE 4.2. Analysis of Variance for Equation 6

Source of Variation	d.f.	S.S.	M.S.	F. Ratio
Mean	1	691.358	-	-
Regression	15	17.882	1.192	47.11
Soils	(2)	(.364)	.182	7.28
Weather	(2)	(1.136)	.568	22.45
Nitrogen	(11)	(15.438)	1.403	55.45
Error	221	5.598	.0253	-
Interaction	(39)	(1.023)	.0261	1.04
Within cells	(182)	(4.576)	.0250	-
Total	237	714.839	-	-

TABLE 4.3. Estimates of Coefficients in Equation 6

Coefficient			Estimated Standard	
Symbol	Interpretation	Estimated Coefficient	Error of Coefficient	Antilog of Coefficient
α	General constant	1.2408	.0270	17.41
β_1	Piedmont soil	.12090	.0397	1.321
β_2	Drained Coastal soil	.08394	.0333	1.213
β_3	Coastal soil	0	0	1.000
γ_1	Dry W_X	-.17836	.0270	.663
γ_2	Fair W_X	-.02589	.0344	.942
γ_3	Good W_X	0	0	1.000
η_1	0# Nitrogen	0	0	1.000
η_2	20# Nitrogen	.38941	.0415	2.451
η_3	40# Nitrogen	.48193	.0470	3.034
η_4	60# Nitrogen	.56049	.0449	3.635
η_5	80# Nitrogen	.63435	.0370	4.309
η_6	100# Nitrogen	.67796	.0521	4.764
η_7	120# Nitrogen	.67381	.0382	4.719
η_8	140# Nitrogen	.66649	.0552	4.640
η_9	160# Nitrogen	.71981	.0467	5.246
η_{10}	180# Nitrogen	.75580	.0507	5.699
η_{11}	200# Nitrogen	.72706	.0618	5.334
η_{12}	220# Nitrogen	.73117	.0617	5.385

interaction, the within cells sum of squares was placed in the denominator. It has 182 degrees of freedom because only 55 of the 72 cells have any observations from which to estimate cell means. The error mean square has been used as the denominator for the other F ratios. This has been done so that both the estimates and the tests, other than the test for interaction itself, would be based on the same specifications. The adjusted R^2 is 0.72.

The estimates of coefficients are given in table 4.3, along with estimated standard errors and antilogs. The coefficients of equation 6 are not unique. The meaning of the equation would be unchanged if a constant were added to α, and the same constant were subtracted from all of the β_i, or all of the γ_j, or all of the η_n. This makes it possible to select arbitrary values for one coefficient for each type of effect. B_3, γ_3 and η_1 were set equal to zero.

The antilogs indicate how estimated yields change from one soil-weather combination to another. In going from Coastal soil to Piedmont, 32.1 percent was added to the estimated yield regardless of weather and nitrogen; in going from good weather and Coastal soil to fair weather and drained Coastal soil, 14.3 percent was added (1.213 x .942 - 1 = .143), etc.

It was desirable to obtain an estimate of the nitrogen effects subject to the diminishing returns restrictions. This estimate was complicated by the fact that cell frequencies were highly disproportionate and by the fact that the restrictions on the log of yield are nonlinear. The first difficulty is perhaps not too serious. Since the restrictions apply only to the nitrogen effects and since interaction is not significant, it seems a reasonable conjecture that imposing the restrictions would affect the soil and weather coefficients very little. The estimates of these coefficients are, in any case, unbiased but would be somewhat more efficient if estimated subject to the restrictions.

One might proceed by correcting the original observations on logs of yield by the estimated soils and weather effects and then re-estimate the nitrogen effects, treating these corrected values as observations. This procedure would go quite smoothly except for the second complication — the nonlinearity of the restrictions on log of yield. While the estimates subject to nonlinear inequalities can be developed, time has not been available, and therefore the author will not speculate as to how much the computations would be increased.[10] An approximation to the results

[10]It appears that quadratic restrictions would suffice for this problem.

Let $y = f(x)$, $\quad Y = \log y$

$$\frac{dY}{dx} = y^{-1}\frac{dy}{dx}$$

$$\frac{d^2Y}{dx^2} = y^{-1}\frac{d^2y}{dx^2} - y^{-2}\left(\frac{dy}{dx}\right)^2 = y^{-1}\frac{d^2y}{dx^2} - \left(\frac{dY}{dx}\right)^2.$$

Since y is positive

$$\frac{d^2y}{dx^2} < 0 \Longleftrightarrow \frac{d^2y}{dx^2} + \left(\frac{dY}{dx}\right)^2 < 0.$$

Thus, imposing the condition on the right is equivalent to imposing the condition on the left. While this relation only holds exactly at a point, its interval analogue will be sufficiently close for practical purposes and this will involve quadratic restrictions on the treatment effects in the log form.

that would be obtained under this procedure can be found by converting the corrected logs of yields back to yields and proceeding as in the original problem cited.

$$y^*_{nk} = 10\,(y_{ijnt} - \hat{\beta}_i - \hat{\gamma}_j) \tag{8}$$

where k runs from 1 to K_n, and K_n is the number of observations at the n-th level of nitrogen ($K_n = \sum_i \sum_j T_{ijn}$). Then, choose estimates $\breve{\eta}_n$ to minimize the sum of squares.

$$Q^* = \sum_n \sum_k (y^*_{nk} - \breve{\eta}_n)^2 \text{ subject to the restrictions} \tag{9}$$

$$\frac{\breve{\eta}_{n+2} - \breve{\eta}_{n+1}}{z_{n+2} - z_{n+1}} = \frac{\breve{\eta}_{n+1} \quad \breve{\eta}_n}{z_{n+1} - z_n} \quad n = 1,2 \; . \; . \; . \; N-2. \tag{10}$$

This procedure is not quite consistent with the assumptions implicit in equation 6 since the sums of squares of deviations are minimized in yields rather than in logs of yields. However, a comparison of restricted and unrestricted estimates in table 4.4 confirms that the error is not large. Estimates are presented for good and dry weather and Coastal soil. To obtain the estimate, either restricted or unrestricted, for any other soil-weather class and for any level of nitrogen, one could

TABLE 4.4. Estimates Responses to Nitrogen for Coastal Soil and Two Types of Weather

Nitrogen Level (inPounds)	Good Weather Estimates		Dry Weather Estimates	
	Unrestricted	Restricted	Unrestricted	Restricted
0	17.41	21.75	11.54	14.42
20	42.68	44.30	28.30	29.37
40	52.81	54.91	35.01	36.41
60	63.11	65.30	41.84	43.29
80	75.01	75.70	49.73	50.19
100	82.95	80.85	55.00	53.60
120	82.15	84.78	54.47	56.21
140	80.78	88.70	53.56	58.81
160	91.33	92.63	60.55	61.41
180	99.22	96.56	65.78	64.02
200	92.88	95.95	61.58	63.61
220	93.80	95.34	62.19	63.21

multiply the estimate from table 4.4 for good weather by the product of the antilogs, from table 4.3, of the coefficients for the desired soil-weather combination.

To become a generally useful tool, estimation subject to qualitative restrictions needs to be developed in several directions. Better procedures for handling transformation of variables are needed. It would be useful to have confidence regions and tests which take account of the restrictions.[11] As more variables are restricted, improved computational procedures will be needed.

Even when these developments take place, the procedures should be regarded as supplementing rather than supplanting existing techniques. There will still be the advantages of efficiency and convenience attached to continuous models when the appropriateness of a particular algebraic form can be rigorously established. However, criteria for goodness of fit are needed that take account of the implications to be drawn from fitted relations.

Certain other improvements in statistical capabilities are needed irrespective of the type of model chosen. In crop production studies, more effective procedures are needed for incorporating data on the initial condition of the soil into models and for relating response to specific observable weather variables.

There is one additional topic that should be mentioned, viz., the drawing of economic implications from our results. After estimating a continuous production surface for an economic unit, the natural procedure is to form a net revenue function with prices of inputs and outputs appearing as variables. This can be maximized with respect to inputs and outputs yielding the optimal quantity as a function of all of the prices. When the economic unit is a firm, these equations are the individual firm's supply and demand functions. More generally, these might be designated as the optimal decision relations.

When the analysis takes the form of estimation of response to a set of discrete alternatives, the natural analogue to the functions described above is a construction of a price map (3). If all possible prices of inputs and outputs are considered as points in a multidimensional Euclidean space, then the price map is a partitioning of this price space into regions which correspond to the production alternatives in such a way that a particular alternative (or combination of alternatives in extended analyses) is optimal whenever the actual price combination falls inside the corresponding region. A price map corresponding to the restricted estimates in table 4.4 is shown in figure 4.1. The procedure for determining regions is the same as that used for cotton fertilization data in the reference cited previously. Crosses show the price combinations which actually prevailed (on the average) in North Carolina in the indicated years.

[11]It should be recognized that conventional tests which ignore the restrictions are unbiased even when the restrictions are known to hold. Utilizing the restrictions would generally increase the power of our tests.

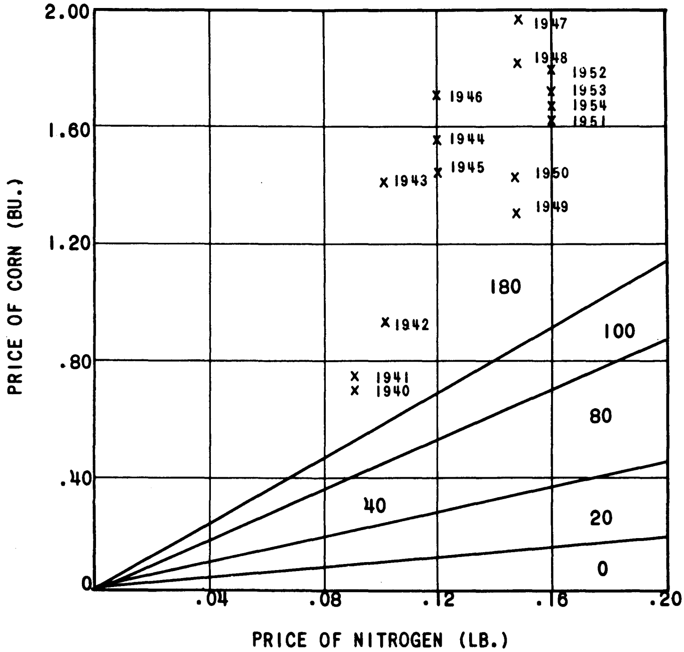

Fig. 4.1 — Corn-nitrogen price map.

Experimenters have increasingly accepted the desirability of taking statistical considerations into account, in planning their investigations, and of examining the statistical implications of their results. It now appears that a good start is being made toward assigning economic considerations and implications of their proper role. Actually, a set of optimal decision relations or its discrete counterpart, a price map, might well be regarded as just as necessary to a complete report of an investigation as the analysis of variance table.

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Chapter 5

Functional Models and Experimental Designs for Characterizing Response Curves and Surfaces

THE yield of a particular crop is a function of many possible factors, as has been pointed out in Chapters 1 and 2. The climate, variety, management practices, and soil factors are, in fact, broad categories which in themselves contain a number of subfactors, each of which may be modifying or limiting. This chapter is concerned primarily with the functional relationship between yield and a portion of the soil factor, that relating to the nutrient status of the soil.

Background

Even a superficial examination of the numbers and types of factors affecting crop yield will reveal that any function completely describing the relationship would be extremely complex. It is small wonder that widely different hypotheses have been developed and supported, since one may find almost any pattern of response, varying from strong positive linear relations to strong negative linear relations. From a statistical standpoint, the failure of hypotheses, purporting to have general application, to agree arises from failure of the experimenters to adequately sample the population to which inferences are made.

Russell (25) gives an excellent review of the historical development of the concepts of plant nutrition, and of the attempts to obtain rational explanations of various phenomena. Liebig, with his first publication in 1840 and subsequent papers and books on the subject, together with his heavy ridicule of the efforts of his predecessors and contemporaries, contributed much, particularly in the way of stimulating controversy and subsequent research. His law of the minimum, which he stated as "by the deficiency or absence of *one* necessary constituent, all the others being present, the soil is rendered barren for all those crops to the life of which *that one* constituent is indispensable," is perhaps his best remembered contribution.

The field experiment approach to the problems of plant nutrition and response initiated by Boussingault (about 1834) and Lawes and Gilbert in 1843 furnished positive evidence of the response of crops to natural and artificial manures. However, Russell reports that the controversy regarding the use of "chemical manures" went on for many years before their general acceptance was indicated. Even today a remnant of this controversy is evidenced by the "organic gardening" school of thought.

Mitscherlich's contributions, beginning in the first decade of this century, marked the first major attempt to formulate a general functional model. His experiments were made with plants grown in sand cultures supplied with "excess" of all nutrients excepting the one under investigation. His expression is commonly known by the descriptive term, "law of diminishing returns," and has the mathematical properties outlined in Chapter 1. Mitscherlich's work, like Liebig's, produced controversy and has both ardent supporters and critics. His function, together with modifications and contributions by other workers, will be given more quantitative expression in the following section. Spillman (26) later, but independently, developed the same function (in the algebraic form of equation 2 in Chapter 1) and extended the methodology to computation of economically optimum rates of fertilization. Spillman, as did Mitscherlich, suggested optimum experimental designs for obtaining data necessary for the estimation of the parameters of the model.

Anderson has adequately outlined, in Chapter 3, the procedural developments from the standpoint of the statistical approach of developing empirical polynomial functions to characterize the response. The development of the factorial experiment and appropriate methods of statistical analysis led to the definition and characterization of interaction between factors (also called complementarity, or joint effects). This, in turn, has led to the geometrical concept of a response surface as the realistic expression of the contribution of two or more nutrients to yield.

With the increased interest of production economists in the application of quantitative methods in the past several years, several papers have been concerned with the choice of a proper functional model for the characterization of input-output relationship in plant growth. Johnson (17), Heady (11), McPherson (18), Ibach and Mendum (16), Paschal (22), Hutton (14), Hutton and Elderkin (15), and Heady, Pesek, and Brown (12) have set forth, in varying degrees, bases of comparison and procedures for evaluation.

Functional Models for Single-Variable Response Curves

Two general approaches have been used in developing mathematical expressions for the relationship between the amounts of the various factors present, and the amounts of plant growth. They are:

1. Attempts to define a model which expresses basic laws of plant behaviour, and fitting the experimental data to this more or less rigid model.
2. The experimental data are studied by statistical methods and an empirical polynomial equation of "best fit" is developed, with no assumption or hypothesis as to the underlying causes.

The first approach is logically and intuitively more appealing. It has its counterpart in the simple physical and chemical systems where deterministic models are common and useful. However, even the simplest of biological systems is relatively complex, and together with errors of technique and measurement, exact relationships are to be viewed askance. Some of the more common functional models for which some biological justification has been claimed are first considered in the following paragraph.

The Mitscherlich Function

Expressing quantitatively the statement that the increase of crop produced by unit increment of the lacking factors is proportional to the decrement from the maximum, one has:

$$\frac{dy}{dx} = (A - y)c \tag{1}$$

where y is the yield obtained when x = the amount of the factor present, A is the maximum yield obtainable if the factor were present in excess, this being computed from the equation, and c is a constant. Upon integration, and assuming that y = O when x = O,

$$y = A(1 - e^{-cx}). \tag{2}$$

Mitscherlich maintained that the "c" values in his expression were constant for a given nutrient over different crops and growing conditions. Most of the early controversy about his work centered around his hypothesis concerning the "c" values. The workers subsequently mentioned as using the Mitscherlich-type equation have assumed that "c" is a parameter to be estimated from the data. This function is expressed in other algebraic forms by Spillman (26) and Stevens (27), and has been widely used by many workers. Ibach and Mendum (16) have detailed instructions for computations, together with examples, using the Spillman form. Monroe (19), Pimentel-Gomez (24), and Stevens (27) give simplified least squares procedures for estimation of parameters for solution, when the X levels are equally spaced. Also, standard errors may be computed for the estimated parameters.

Prior to the comparatively recent publication of the three references mentioned above, and a paper by Hartley (10), least squares estimates involved such heavy labor that they were seldom made. An interesting example of the application of the Mitscherlich model is given by Crowther and Yates (6)., in summarizing all published results of one-year fertilizer experiments conducted in Great Britain and the northern European countries since 1900, in order to formulate a wartime fertilizer distribution policy. Economic analyses, in terms of setting out optimum rates for maximum profit, were made of the data.

One of the other early criticisms of Mitscherlich's equation was that no allowance was made for possible yield depression by harmful excesses of the factor. Mitscherlich, after extensive study of his experimental data, introduced a modification of the following form to allow for such depressions:

(3) $$y = A(1 - 10^{-cx})10^{-kx^2},$$

with the constant "k" being called the "factor of injury." He felt that this would apply mainly to the response of grain crops to nitrogen. He provided estimates of "k" for several crops.

The Logistic Function

The logistic is the commonly used function for fitting growth curves in biological populations, and may be expressed in the form:

(4) $$y_t = \frac{k}{1 + b\,e^{-at}},$$

where a, b, and k are parameters to be estimated from the observed data, and y_t is the value of the growth character studied at point of time, t. For yield response models, x, for increment of fertilizer, would be substituted for t.

This curve has a lower asymptote of O, and an upper asymptote at k, and the point of inflection is at $y = \frac{k}{2}$, a point midway between the two asymptotes. Thus, we have the familiar S-shaped or sigmoid curve. Such a model would be useful to characterize the initial "lag" that may occur when the amount of the factor in the soil is very low, and small increments are applied in the low range. In the usual situation this initial lag is not observable. Nair (21) gives an extensive discussion of the logistic function together with methods and illustrations of fitting.

The Power Function (Cobb-Douglas)

The power function,

(5) $$Y = a\,X^b,$$

has been employed as the model in various economic investigations. In this equation, Y is the yield, a and b are constants, with X as the level of the factor. The equation may be written in the linear form as

(6) $$\text{Log } Y = \text{Log } A + b \,.\, \log X \,.$$

Hutton et al. (15) discuss the general characteristics of the Cobb-Douglas function, and suggest methods of analysis. Heady (11) and McPherson (18) also describe the various characteristics of this function and modified forms of the power function. If $b > O$, as would be the case in the yield response curve, y continues to increase as X increases.

The Polynomial

The terms in a polynomial equation may vary from one to n-1 where n is the number of levels of the factor X. In the single variable case, the number of terms and the degree of the equation are normally (but not necessarily) parallel. The first degree (or linear) equation describes a straight line, while the second degree (or quadratic) describes a monotonic curve. The degree less one indicates the number of times the curve may change direction. The usual forms are:

Linear : $Y = \beta_0 + \beta_1 X$

Quadratic: $Y = \beta_0 + \beta_1 X + \beta_{11} X^2$

Cubic : $Y = \beta_0 + \beta_1 X + \beta_{11} X^2 + \beta_{111} X^3$

General : $Y = \beta_0 + \beta_1 X + \ldots\ldots \beta_{\cdot\cdot(n-1)} X^{\cdot\cdot(n-1)}$.

The X may be transformed to the square root, logarithm, reciprocal, or other form, with the same general process of fitting applied. Methods of fitting such curves are straightforward. Discussion of fitting procedures, with examples, is given by Anderson and Bancroft (2) and other texts.

Discussion of Application of Exponential, Power, and Polynomial Models

The functions mentioned above are only a few of the better known of a large number of possible functions. Within the polynomial class alone an almost infinite number of possibilities exist. The problem, therefore, of choosing the "best" function is not soluble from a simple set of rules. By the use of least squares procedures the value of the constants for the equations may be computed. These procedures give the "best" fit for the particular form of functional model, in the sense of describing a curve from which the mean of the squares of the deviations of the individual points from that curve are a minimum.

It cannot be claimed that any of the functions represent fundamental biological laws of growth, although one may rationalize the form of a particular function in a particular situation. One procedure of choosing the "best" function, mentioned by Heady (11) and by Hutton et al. (15), is to examine possible applicable functions, and select the one that best fits the data. A useful procedure, where data are being examined from a replicated experiment (more than one observation at each increment), is to examine the size of the "lack of fit" term, as given in the analysis of variance. The following data, from Veits, Nelson, and Crawford (28), serves to illustrate the procedure.

If the lack of fit term is of the same order of magnitude as the experimental error, then the function is characterizing the data adequately. A significant lack of fit mean square indicates that the model is inadequate to describe the functional relationship.

TABLE 5.1. Observed and Predicted Yields by Three Functions For Corn Yields, 1952

Nitrogen Level (Lbs/A)	Observed Yields (Bu/A)	Estimated Yields Polynomial Equation	Estimated Yields Mitscherlich Equation	Estimated Yields Cobb-Douglas Equation
0	125.8	124.2	126.5	124.2
40	140.2	145.6	145.7	152.3
80	166.8	160.11	156.9	158.1
120	164.3	167.6	163.3	161.6
160	168.5	167.8	167.1	164.1
200	161.8	161.6	169.2	166.2

Source: Veits, et al. (28)

Analysis of Variance:

Source	d.f.	Polynomial M. S.	Mitscherlich M. S.	Cobb-Douglas M. S.
Treatments	5	1219.82	1219.82	1219.82
Due to regression	2*	2880.82	2673.74	2510.28
Lack of fit	3	112.48	250.53	269.63
Experimental error	21	152.6	152.6	152.6

*1 d.f. for regression for Cobb-Douglas, and 4 d.f. for deviation from regression.

Johnson (17) and Heady et al. (12) have examined the three functional models considered in table 5.1, for fitting response curves, and conclude that the quadratic polynomial model generally gives the better fit. Heady, Pesek, and Brown found that fit was improved by using a square root transformation of the X variate, in the quadratic model.

The Mitscherlich and Cobb-Douglas functions obviously give a poor fit when yield is depressed by the higher rates of application. This depression appears to be fairly common, particularly with higher rates of nitrogen. For example: a recent report by Hunter and Youngen (13), on a series of experiments on corn, shows that in six of seven experiments, where N rates were carried to 200 lbs. per acre, a yield depression resulted. This type of response is compatible with biological theory, although depression is more marked in cases of excesses of some of the minor elements.

An alternative that might be followed would be to discard those observations which fall beyond the maximum yield, and fit the exponential or power function, using the rationale that one obviously is not interested in that area of the curve. This would appear to be a poor practice statistically, since one is discarding information and introducing a degree of subjectiveness into the analysis.

One particular advantage claimed for the Mitscherlich function is that it gives plausible results when extrapolated for high values of X. Stevens (27) sharply warns against such extrapolations, and points out that the standard error of the predicted value becomes large as the asymptote is approached. It is necessary to have established the absolute generality of the formula, either by sound theoretical justification, extensive observation, or both, in order to extrapolate with confidence. Stevens points out that this is feasible in some physics models (e.g., Newton's law of cooling), but difficult in biology. Also, in viewing the general trend of economic-agronomic cooperation in experiments, it appears that the need for extrapolation will lessen.

Two more or less ulterior advantages may be claimed for the polynomial. First, it is easier to fit by least squares procedures and easier to provide estimates of standard errors of the parameters. Second, it is the most flexible of the three functions. This carries the added advantage of therefore being more generally applicable to a series of individual experiments conducted at a number of locations and years.

Functional Models for Characterizing Response Surfaces

The three functions considered in table 5.1 may be generalized to give a mathematical expression of the geometrical configuration of a response surface when two or more factors are considered. The principal points of contention regarding the relative suitability of the functions again centers on the restrictions placed on the form of the surface. The Cobb-Douglas and the Mitscherlich functions, as previously mentioned, do not have a declining phase and do not permit the reflection of changing ratios of nutrients for the optimum treatment when the level of yield is changed. Heady (11) illustrates this relationship diagrammatically by showing that the isoclines (a line connecting all points on the same slope of successive isoquants) are required to be linear.

Sufficient data have been accumulated from factorial experiments with fertilizers to give some indication of the nature of the interaction (complementarity) between nutrients. For example, Dumenil and Nelson (7) report on the results of 164 factorial experiments carried out in Iowa on corn, oats, and hay crops with N, P, and K, or two of the three nutrients in combination. Out of these, 62 showed some type of interaction significant. (It is likely that a greater number would have been found significant had individual degrees of freedom associated with particular coefficients been examined). Commonly the interaction between N and P was positive, while negative interactions were found between N and K. The authors conclude: "In view of the number and size of interactions encountered, the use of the factorial design, wherein the different fertilizer elements and rates are used in all possible combinations, appears highly desirable. The value of certain nonfactorial designs now in common use may lead to erroneous conclusions whenever interaction between the fertilizer elements occurs."

The Mitscherlich-Baule Function for Response Surfaces

Baule, according to Russell (25), generalized Mitscherlich's function while retaining the fundamental assumptions. He supposes that each of the factors influencing plant growth is in accordance with Mitscherlich's assumption and that the final yield is the product of the separate expressions. The equation then becomes:

$$y = A(1-e^{-c_1x_1})(1-e^{-c_2x_2})(1-e^{-c_3x_3}). \tag{7}$$

The equation requires, for example, that if two factors, L and M, vary simultaneously, each should produce its own effect independent of the other. Russell (25) illustrates this with the following reasoning and with data adapted from Mitscherlich's publication. If y and y′ represent the yields when x, x′ are the quantities of factor L, the quantity M remaining constant, then

$$y = A(1-e^{-cx})$$
$$y' = A(1-e^{-cx'})$$

where A is the maximum yield obtainable with any quantity of factor L at the given value of M. Now

$$\frac{y}{y'} = \frac{(1-e^{-cx})}{(1-e^{-cx'})} .$$

This ratio is therefore independent of the value of A; that is, it is independent of the level at which M was taken.

TABLE 5.2. Yield of Oats in Pot Experiments with Varied Phosphate Dressings and Varied Water

Calcium Phosphate (x)	Water 1 dose (y)	Water 2 doses (y′)	Ratio $\frac{y'}{y}$
0	6.4	11.0	1.72
1	14.6	25.6	1.75
2	22.6	36.6	1.62
4	29.7	53.1	1.79
8	41.3	70.5	1.71
16	50.8	77.5	1.53
32	55.7	88.5	1.59

Adapted from Mitscherlich.

It seems clear from this illustration that the Baule function accounts for "interaction" in the sense that the interaction arises from the failure of the *differences* between y and y′ to remain constant over the different levels of x. Another criterion of the applicability of the Mitscherlich-Baule function is a test of the deviations from parallelism of the curves when y is plotted as the logarithm of the yield increase, due to the i-th increment, and x as the log of the dose.

The Maskell "Resistance" Formula

Balmukand (3), not satisfied with the Mitscherlich function when applied to field data, critically examined the general problem of relating nutrient level to crop yield. He applies Maskell's formula, which by electrical analogy has been called the Resistance Formula. It may be stated as: Each activity of the plant (yield, etc.) is determined by a potential and a set of resistances, each of which represents one of the external factors. Maskell expresses the effect of nutrient supplies on yield as

$$\frac{1}{y} = F(N) + F'(P), + F''(K) + \dots + C, \tag{8}$$

y being the yield, and F(N), F′(P) and F″(K) being functions of these nutrients supplied, and they have the form,

$$\frac{a_n}{n+N}, \quad \frac{a_p}{p+P}, \quad \frac{a_k}{k+K},$$

where N, P, and K represent the amounts of these nutrients added; n, p, and k represent the amounts of the nutrients available in the soil, and a_n, a_p, and a_k are constants expressing the importance of the nutrients to the crop.

This expression, like Mitscherlich's, assumes each factor acts independently of all the others but fixes the difference of $\frac{1}{y} - \frac{1}{y}$ as constant.

Balmukand (3) illustrates the application of the function to data from replicated factorial experiments and gives least squares procedures for estimating the constants, together with appropriate estimates of the standard errors of the estimated constants. He obtains satisfactory fits of the response surface, using the magnitude of the lack of fit mean square as the criterion. However, the computations involved are heavy, compared to other functions considered. This may be the primary reason why this function has not been used.

The Cobb-Douglas Function

The Cobb-Douglas function may be generalized to

$$Y = aX_1^b \; X_2^c \dots X_n^n \tag{9}$$

where n is the number of factors considered. As mentioned previously, it may be put into the form:

$$(10) \qquad \text{Log } Y = \log a + b \log X_1 + c \log X_2 + \ldots + n \log X_{n-1}$$

for solution by least squares.

The Polynomial Function

The polynomial may take a wide variety of forms for a given number of factors, depending on the degree (highest exponent or products of exponents in a given term), and the scale in which the X variates are expressed. Some experience has been accumulated in the past few years on this model, both in the biological field and in industrial and engineering applications. Box (4) and Anderson (1) have reviewed the general approach to defining response surfaces and defining optimum operating conditions, using the general polynomial equation. Hanson, Hutton, and Robertson (9) have examined data from a $5^3 \times 2$ factorial experiment, with N, P, K, and lime as factors, and indicate the second degree polynomial equation is generally satisfactory. Heady et al. (12), after a detailed examination of possible functions, concluded that the generalized second degree polynomial with the X's scaled by a square root transformation was most satisfactory.

The generalized polynomial equation for two variables is

$$(11) \qquad y = b_0 + (b_1x_1 + b_2x_2) + (b_{11}x_1^2 + b_{22}x_2^2 + b_{12}x_1x_2) + (b_{111}x_1^3 + b_{222}x_2^3 + b_{112}x_1^2x_2 + b_{122}x_1x_2^2) + \text{etc.}$$

This corresponds to representing the function by its Taylor series. The brackets enclose the terms containing respectively the first, second, and third order terms. Thus, an equation containing all first order terms only defines a plane; one containing both the first and second order terms is a second degree equation and defines a quadratic surface, and so on. The number of constants to be fitted for functions of various numbers of factors for varying degrees is given in table 5.3, taken from Box (4).

TABLE 5.3. Number of Constants To Be Fitted for Equations of Varying Degree

Number of	Degree of Fitted Equation			
Factors	1	2	3	4
2	3	6	10	15
3	4	10	20	35
4	5	15	35	70
5	6	21	56	126

The equation is then fitted by the method of least squares (multiple regression). The procedure is in essence the application of multiple regression methods to observations in which the values for the independent variates have been planned. By proper choice of these levels the calculations may be simplified, particularly when the x variates may be coded, with the origin (O) at the center of the design.

Example of a Fitted Second Degree Response Surface

An example was chosen from summary tables presented by Hunter and Youngen (13), from a 4 x 4 factorial experiment, with nitrogen level and spacing (number of plants per acre) as the two factors. The experiment was run as a randomized block, with three replications. The above-mentioned authors have kindly supplied the necessary additional information about the experiment, including treatment totals and the analysis of variance, in order to allow a complete analysis of the data. The treatment means are given in table 5.4 below.

TABLE 5.4. Yields, Bushels per Acre, As Influenced by Variation in Plant Stand and N Levels

Nitrogen Lbs./A	Plant Population 15,400	17,000	(No. Plants per Acre) 17,900	21,500	Mean
0	106.1	96.7	94.6	98.0	98.8
50	121.9	120.4	126.4	110.8	119.8
100	128.9	129.3	134.2	130.4	130.7
150	119.4	134.5	138.4	140.2	133.1
Mean	119.0	120.2	123.4	119.8	

Second degree (or quadric) equations of the following forms were fitted:

$$(12) \qquad y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_{11} X_1^2 + \beta_{22} X_2^2 + \beta_{12} X_1 X_2$$

$$(13) \qquad y' = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_{11} X_1 + \beta_{22} X_2 + \beta_{11} X_1 X_2 \ .$$

The following equations were obtained:

$$(14) \qquad y = 128.19 + 5.6825 X_1 + .272864 X_2 - 1.1624 X_1^2 - .08675 X_2^2 + .345165 X_1 X_2$$

(a unit of X_1 = 25 lbs. N; unit of X_2 = 500 plants/A)

$$y' = 123.34 + 19.4913\ X_1 + 1.3692\ X_2 - 2.68875\ X_1 - 2.30082\ X_2 + 5.25987\ X_1X_2 \tag{15}$$

(a unit of X_1 = 50 lbs. N; unit of X_2 = 100 plants/A).

These two equations have their origin at the mean levels of N and spacing. An analysis of variance of the data is given in table 5.5. The F test of the lack of fit term for both equations shows that neither are significant at the 5 percent significance level, although the function with the X's in linear form indicates slightly better fit. No generalization, however, should be made from such a single comparison, particularly in view of other workers, Heady et al. (12) having indications to the contrary.

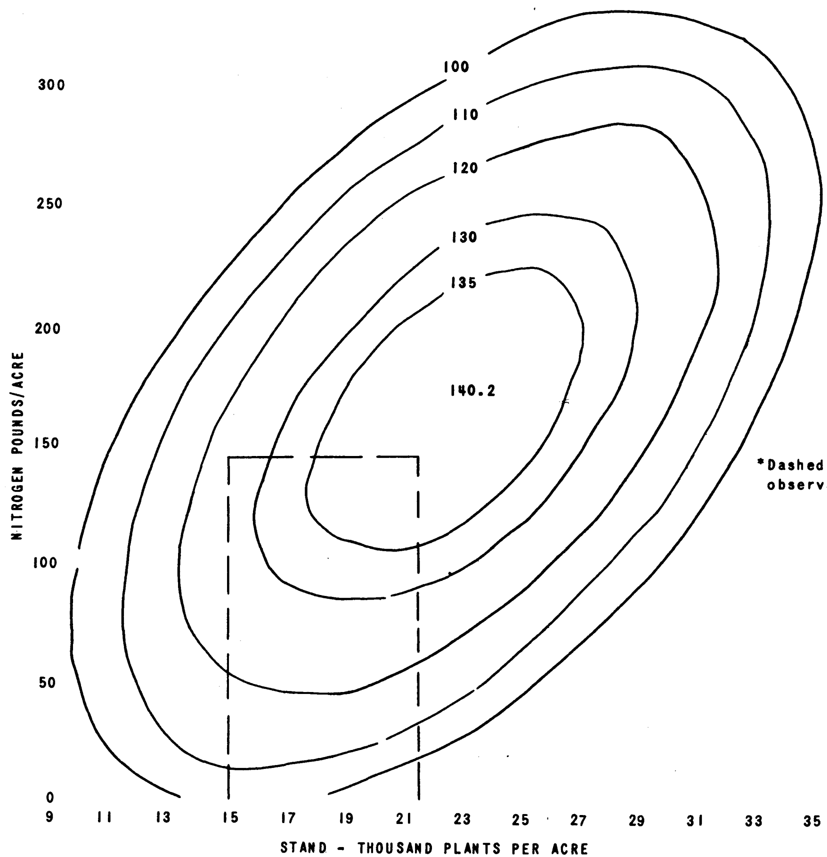
TABLE 5.5. Analysis of Variance of 4 x 4 Factorial Experiment (N Levels x Spacing)

Source of Variation	d.f.	M.S.	M.S. for $\sqrt{\ }$ Transform of X Variate
Replications	2	372.22	372.22
Treatments	15	683.16	683.16
Due to regression	5	1888.44	1823.97
Lack of fit	10	80.52	112.76
Experimental error	30	53.99	53.99
Total	47		

Figure 5.1 is given to illustrate the general picture of the joined yield contours as computed from equation 14. These joined contours show the symmetry required by the function used for fitting. However, the size of the standard errors of predicted points that are much removed from the area of the experimental observations clearly shows that such extrapolation is of little practical value. The importance of having the experimental points in the region of interest is indicated by considering the size of the seven standard errors of predicted yields (Y's) listed with the figure.

Figure 5.2 shows the portion of the yield contours (also computed from equation 14) within the area of experimental observations, together with the observed mean yields for the 16 treatment combinations. Figure 5.3 shows similar contours but computed from the "square-root" equation 15.

This example, incidentally, illustrates the importance of the consideration of subsidiary factors in fertilizer experiments. The data and figures 5.1 and 5.2 show that the response to nitrogen is appreciably modified by the choice of plant population.



STANDARD ERRORS OF SELECTED POINTS

	X_1	X_2	$\hat{Y}$	S.E.$\hat{Y}$	
1.	0	15.40	102.31	3.18	a
2.	75	17.95	128.19	2.15	b
3.	170	22.52	140.20	5.94	c
4.	250	25.40	131.78	19.91	d
5.	100	22.14	131.78	3.24	d
6.	100	17.32	131.78	1.89	d
7.	250	26.01	131.78	21.74	d

a. Lowest treatment combination in Expt.
b. Mean of treatment levels.
c. Maximum of surface.
d. Selected points on 131.78 yield contour.

*Dashed line encloses area of experimental observations. See Figure 5.2 for yields.

Fig. 5.1 — Yield contours (bushels per acre, corn) for 4 x 4 factorial experiment. Hunter and Youngen (13).

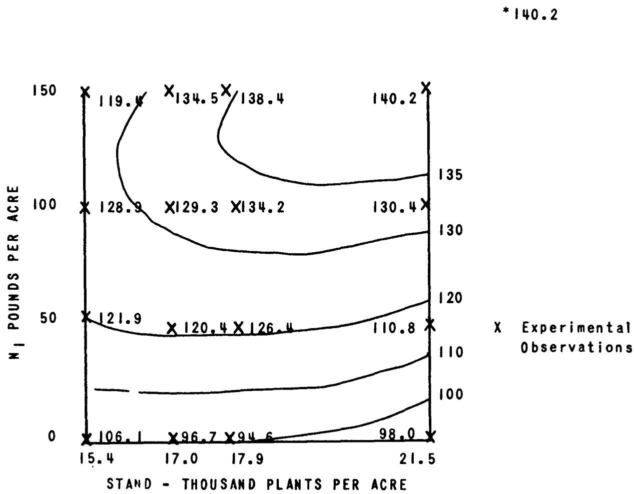


Fig. 5.2 —Yield contours, 4 x 4 factorial experiment with corn. (Data are in bushels per acre.) X's on linear scale.

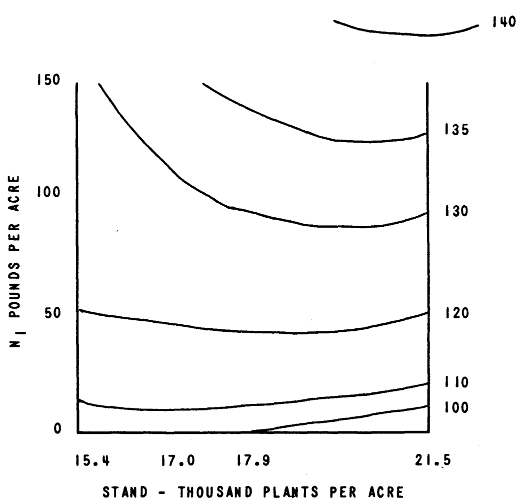


Fig. 5.3 — Yield contours, 4 x 4 factorial experiment, with corn, for square root transformation of X's.

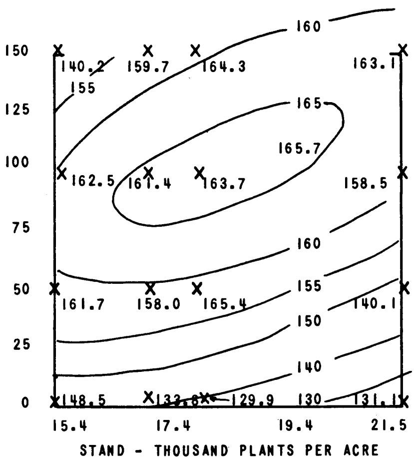


Fig. 5.4 — "Value" contours, after adjustment for cost of fertilizer and stand. Assumed constants: Corn = \$1.40 per bushel; nitrogen = 18 cents per pound; stand = \$1.00 per 1000 plants per acre.

Equation:

$$Y = 143.229 + 21.5042X_1 - 0.6609X_2 - 6.5014X_1^2 - 0.4834X_2^2 + 1.9245X_1X$$

where Y = predicted value (dollars per acre) above cost of fertilizer and stand.

$$X_1 = \frac{\text{lbs. N}}{50}$$

$$X_2 = \frac{\text{stand thousands per acre}}{1000}$$

β_0 in the equation is the expected yield at N = O, and stand = 15,400

The maximum point is:

X_1 = 110 pounds nitrogen
X_2 = 19,100 plants
$\hat{Y}$ = \$154.67, corresponding to a yield of 135.13 bushels per acre.

In figures 5.1 and 5.2 it may be seen that the predicted maximum yield was estimated to occur with a stand of 22,520 plants per acre and nitrogen application of 170 pounds N per acre. If no cost were attached to the N application and increased stand (an obviously rare situation), this point would be in the center of the region of maximum interest. However, consideration of the price of the N applied and the cost of increased stand would normally be expected to shift this region. Figure 5.4 shows the "value" contours, after accounting for the cost of N as 18 cents per pound and the cost of an additional 100 plants per acre as \$1.00. The value of the corn was computed as \$1.40 per bushel. (No overhead costs were considered; if constant overhead charges were assumed, the contour surface would not be changed, but the value attached to each contour would be decreased a constant amount). The "maximum return" of 165.67 is now estimated to occur at 19,100 plants per acre, with 110 pounds N. This corresponds to a yield of 135.1 bushels per acre. This point occurs well within the region of the experimental observations, a situation which is desirable.

Figure 5.4 and the accompanying computations are for illustrative purposes only. Heady et al. (12) and Chapters 1, 6, and 10 outline direct methods for estimating the economic optima, given a function, and input and output costs.

TABLE 5.6. Rates and Coded Values Used in Potato Fertility Experiment

Nutrient	Rates of Fertilizer Element for Coded Value				
	-2	-1	0	+1	+2
N	0	50	100	150	200
P_2O_5	0	60	120	180	240
K_2O	0	50	100	150	200

Table 5.7. Treatments, and Treatment Means for Three Replications, Potato Fertility

Treatment Number	X_1 N	X_2 P_2O_5	X_3 K_2O	Treatment Means (Lbs. U. S. No. 1's per 2-row, 25-ft. Plot)
1	-2	+1	+1	31.9
2	-1	+1	+1	41.5
3	0	+1	+1	54.2
4	+1	+1	+1	57.1
5	+2	+1	+1	51.9
6	+1	-2	+1	56.2
7	+1	-1	+1	56.0
8	+1	0	+1	49.8
9	+1	+2	+1	61.6
10	+1	+1	-2	45.6
11	+1	+1	-1	50.5
12	+1	+1	0	55.4
13	+1	+1	+2	53.2
14	-1	-1	-1	42.6
15	+1	-1	-1	52.3
16	-1	+1	-1	39.3
17	-1	-1	+1	31.4
				$\overline{X} = 48.6$

Example of a Response Surface for Three Factors Estimated from a Multifactor Design

The following data are from a multifactor NPK fertility experiment conducted in Watauga County, North Carolina, in 1954, by M. E. Harward. Table 5.6 gives the rates of N, P_2O_5, and K_2O used, together with their coded values given in the preceding table and following figure. The basic arrangement of the treatment combinations is that of having five levels of each of the nutrients, at a constant rate of the remaining two, with additional points added to form a 2^3 factorial design. Had the constant rate been in the center of the design (e.g., $P_2O_5 = o$, $K_2O = O$, for the rates of N) then it would come in the category of the central composite designs described by Box (4). With this point on one of the corners of the cube, Box describes this as a "noncentral" composite. Table 5.7 lists the treatment means (in terms of pounds of U. S. No. 1's per 2-row, 25-foot plot), together with the coded treatment combinations given in table 5.6. Table 5.8 gives the prediction equation together with the standard errors of the coefficients and the analysis of variance.

TABLE 5.8. Regression Coefficents and Their Standard Errors, and the Analysis of Variance for Second Degree Surface for Data in Table 5.7

Designation	Regression Coefficient ± Standard Error		
b_0	48.59	±	2.103
b_1	6.690	±	.943
b_2	1.254	±	.943
b_3	-.270	±	.943
b_{11}	-2.628	±	.832
b_{22}	1.832	±	.832
b_{33}	-1.231	±	.832
b_{12}	-1.640	±	.858
b_{13}	.786	±	.858
b_{23}	1.850	±	.858

Analysis of Variance:

Source of Variation	d.f.	M.S.
Replications	2	8.14
Treatments	16	232.88
Linear	3	882.08
Quadratic	6	145.52
Lack of fit	7	29.53
Experimental error	32	32.86

Figure 5.5 shows the geometrical configuration of the design. Figure 5.6 illustrates the yield contours for $X_1(N)$ and $X_2(P_2O_5)$, with $X_3(K_2O)$ fixed at the +1 (150 lb.) level. Thus, these contours apply to the front of the cube, and the extended points of N and P_2O_5, and the yields plotted on figure 5.6 may be located with respect to their position in figure 5.5.

Figure 5.6 illustrates the contours of a surface which is obtained when b_{11} and b_{22} are of opposite signs, and is termed a "saddlepoint"

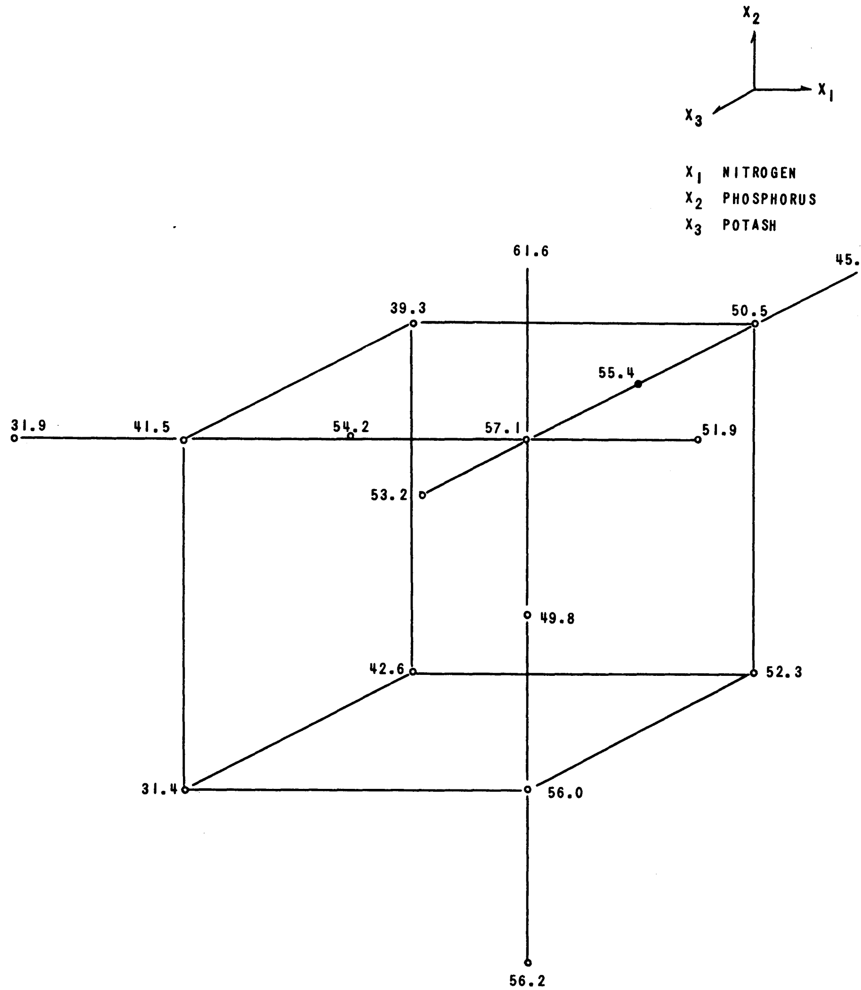


Fig. 5.5 — Design configuration and treatment means for multifactor potato yield experiment (yields are pounds, U.S. No. 1's, per 2 row, 25-foot plot).

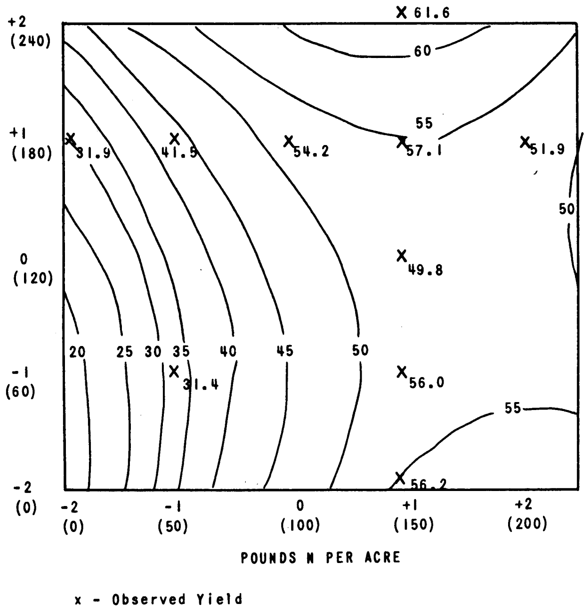


Fig. 5.6 — Yield contours for U. S. No. 1 potatoes, pounds per plot, for variations in X_1 (N) and X_2 (P_2O_5), with X_3 (K_2O) held at +1 (50 pounds per acre). X_1 (nitrogen)

(sometimes referred to as a "col" or "minimax"). Such a surface would appear to be difficult to interpret agronomically. One would certainly like some substantiation of this type of pattern before extending its application too far. A more complete sampling by observation points in the critical region is perhaps in order.

An illustration of the use of the logarithmic scale for the X variates is given in figure 5.7. These data are from one of a series of experiments to be reported on by Moore et al. (20). This example has little

TABLE 5.9. Analysis of Variance of Yield (Gms. Dry Wgt. of Lettuce Tops per 3 Plants)

Source of Variation	d.f.	M.S.
Linear terms	3	197.2
Quadratic	6	168.6
Lack of fit	5	6.2
Error	3	7.0

Source: Moore, et al. (20)

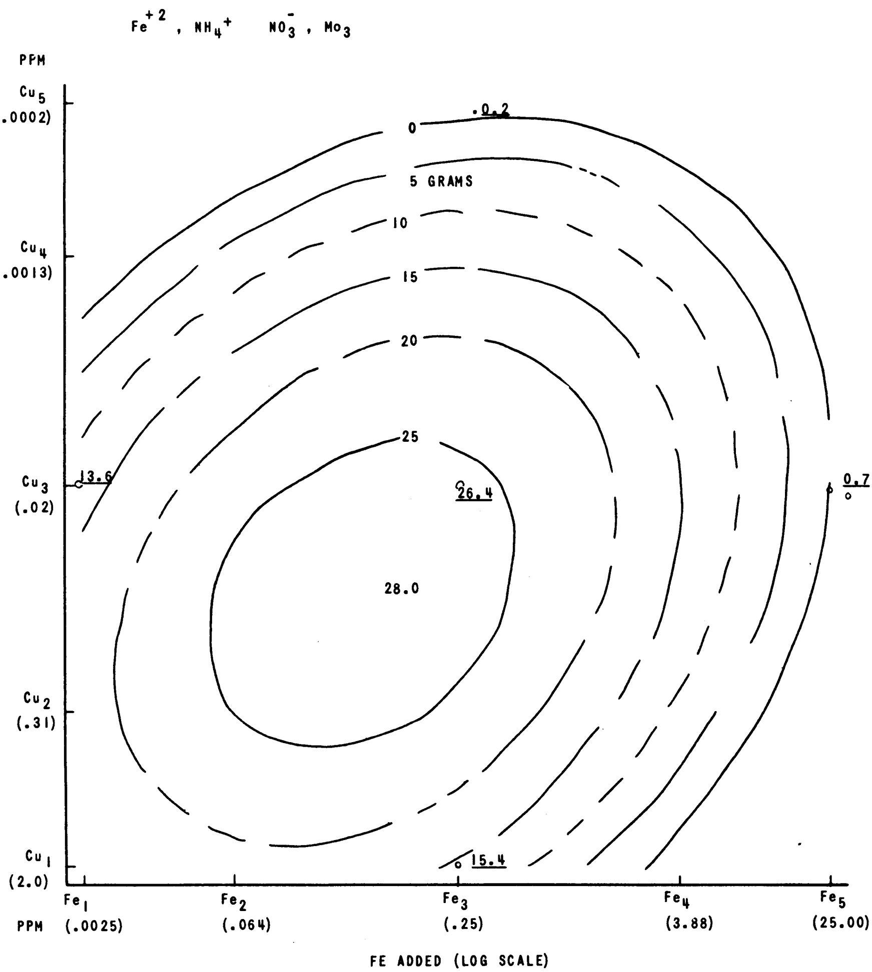


Fig. 5.7 — Yield contours of lettuce tops (gms. dry wt.) as affected by additions of Cu and Fe to nutrient solutions containing Fe^{+2}, $NH_4^+ + NO_3^-$ and the middle level of Mo. Observational points and yields are underlined. The point at the center of the contours is the predicted maximum yield.

direct application regarding economics of fertilizer use. It is part of a greenhouse solution culture study set up with the objective of obtaining a general perspective of the relationship of certain minor elements to the yield response of lettuce, and to obtain information upon which to base more detailed studies. The experimental design used here was one of the "rotatable" designs developed and reported by Box and Hunter (5). The actual levels and the coded values for Cu and Fe are given in figure 5.6. The third factor, molybdenum, had the same levels as copper, ranging from .0002 to 2.0 ppm.

The summary analysis of variance in table 5.9 indicates a reasonably good fit of the second degree surface to the actual data. However, it is realized that the experimental error mean square is poorly estimated with only 3 d.f.

Discussion of Functional Model and Design of Multifactor Experiments

Anderson, in Chapter 3, has stated that if the response surface can be approximated by a simple mathematical function then it seems logical to estimate the parameters of this function instead of main effects and interactions. Spillman (26) recognized this in his suggested treatment arrangements necessary for estimating the constants in the Mitscherlich-Baule function. In the approach used by Box (4) and associates, that of using the general polynomial function of the degree necessary to adequately fit the surface, designs of treatment combinations may be developed that have desirable properties compared to the complete factorial arrangement. They point out two disadvantages of the complete factorial arrangement: (a) estimation of the pure quadratic (β_{11}, β_{22}, etc.), with less precision than the mixed quadratic terms (β_{12}, β_{13}, etc.),; (b) complete factorial arrangements for estimation of many higher order mixed terms which ordinarily are of little interest, and which do not have the corresponding "pure" effects to go along with them. Box states, "To attempt to interpret two factor interactions without the corresponding quadratic effects is precisely analogous to considering covariances without the corresponding variances."

The composite designs also have been discussed by Anderson in Chapter 3. These designs appear to have good possibilities. Additional field experiments are now under way in North Carolina using the second order, composite designs, and it is expected that evaluation of these experiments will provide some measure of their utility. Composite designs have been used in industrial research for estimating cubic or 3rd degree surfaces, as indicated by Pike (23).

In fertilizer response studies it seems desirable to have replication of the treatments, both for providing the necessary precision for the individual points in the design and for checking on the adequacy of the model in characterizing the surface. Although the work by Heady et al. (12), and by Hanson et al. (9), and the illustrations already given in this chapter indicate that a second degree function seems to give an adequate fit of the surface, this needs to be further studied under a wider range of conditions.

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PART III

Agronomic and Related Considerations in Experiments and Fitting Functions to Existing Data

- **Size and Type of Experiment**
- **Soil and Moisture Conditions**
- **Soil Test Data**
- **Standard Curves**

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Chapter 6

Agronomic Problems in Securing Fertilizer Response Data Desirable for Economic Analysis

SINCE economic considerations so often determine the adoption of fertilizer use and other production practices, the individuals involved with the making of recommendations for fertilizer use need a workable economic framework within which to make their estimates. In research designed to determine alone the deficiency of a particular crop, the economics are simple, viz., either the quantity of fertilizer used returns a profit or it does not, and therefore, it is or is not recommended under similar conditions. The most serious limitation of this approach is that there is no way to know whether more or less fertilizer would have returned a higher net profit. In more complex experiments involving several rates of a fertilizer element, or several rates of two or more elements in various combinations, the simple economic procedure outlined above is not adequate. Although the simple procedure can be used in successive trials for the several rates and combinations individually, the flexibility and versability of a continuous production function are highly desirable.

Variations by Soils

Fertilizer needs of a given crop vary among the different soils, among fields on the same soil types, and under different levels of management. Because of this, some method of determining the fertilizer requirement of a particular crop on a given field is used. The chemical soil test is commonly used to accomplish this, although techniques employing the composition of plants or plant parts show promise of making better estimates of fertilizer needs possible.

Economic functions for decisions must take into account (a) the soil deficiencies in one or more elements, with the related crop responses to the combinations of these elements, and (b) the changing price ratios among input factors and between these and output. One object of this chapter is to review the types of experiments which are needed in estimating economic levels of fertilizer use. Another objective is to indicate some of the problems involved in conducting these experiments. The manner in which the experimental data might be used will be considered in view of attaining the best possible fertilizer recommendation for the farmer. Financing of research programs is usually a problem.

However, it will be assumed that required financial support exists, and a study of nonfiscal problems will be made.

Previous Work

Mitscherlich (9) and Spillman (14) proposed use of the logarithmic and exponential forms of an equation outlined in Chapters 1 and 5. Others (1, 7, 8, 11) have proposed a number of equations to express crop responses as a function of a single fertilizer nutrient variable. These several equations can be used to determine the optimum rate of fertilizer needed by the usual procedure of equating the first derivative of the particular equation to the price ratio of the input to the output, as indicated by equations 11 and 12 in Chapter 1 (page 12).

Ibach (5, 6) employed the Spillman function to determine the optimum quantity of nitrogen, phosphorus, and potassium by means of successive approximations. This procedure has the advantage of being mathematically simple. The most serious objections to the procedure are: (a) Only part of the data is utilized. (b) Three separate functions need to be written for the responses to the three elements, instead of a single equation with three variables. (c) No provision is made to allow the three elements to interact simultaneously. General limitations to the logarithmic and exponential equations are outlined in Chapters 1 and 5.

Both Mitscherlich (10) and Spillman (14) showed how to extend their single-variable equations to include more variables. More recently Heady et al. (3, 4), and Pesek and Heady (12) have proposed equations in two variables, to express the yield response of crops, which overcome the objections listed above. These investigators have also shown (a) how to employ these equations by simultaneous solution, in arriving at the optimum fertilizer rates and ratios for the experimental conditions under different assumed prices of product and fertilizers, and (b) how fertilizer ratios may change as production is expanded. They have also shown that, under given conditions, maximum yields can be achieved only by a particular combination of the two fertilizer elements. Similar equations in three or more variables can be written to express yield responses when the proper data are available.

Hanway and Dumenil (2) have related the response of corn to nitrogen in Iowa and the test for nitrifiable nitrogen in the soil. The Mitscherlich equation was used to express the response curve; a logarithmic equation was employed to express the response of corn to a given quantity of nitrogen as a function of the nitrogen soil test. These equations were combined to permit evaluation of optimum nitrogen needs for individual fields, and are currently used as the basis for nitrogen recommendations. Of particular interest was the fact that the data on hand for correlation purposes indicated that the experimental results did not deviate significantly from a general nitrogen response curve, and that as the nitrifiable nitrogen in the soil increased, the point of origin needed only to be translated along this response curve. The correlation is based on about 85 nitrogen experiments conducted over a period of 10 years, using those experiments in which there was evidence that the levels of other nutrient elements were adequate, and stand levels averaged 12,000 stalks per acre.

The use of the experiments above is pointed out not only to show how the soil test can and should be integrated with response data, but also to indicate the value of using data already collected (see Chapter 8 on estimation of functions from soil tests). A unique advantage of utilizing data collected over a past period of years is the fact that they include a sample of climatic variations not possible to achieve in any number of experiments in any two-year, three-year, or other short period. Since it is still necessary to predict fertilizer needs for average weather conditions, fertilizer response data over a period of years are invaluable.

The Multi-variable Response Function

Numerous experiments with two or more variables have been conducted. However, only recently has any successful attempt been made in expressing two variables in a general response equation and in applying principles of production economics to them. Yet there is a strong need to know the general nature of the multi-variable fertilizer response equations for various crops because they are the basis for fuller utilization of past and future fertility experiments within an economic framework. The number of combinations of soils, soil fertility levels, fertilizer grades, prices, and responses is so great that it seems improbable that rapid progress can be made in the absence of such empirical equations.

Nutrient Interaction

The approach by successive approximation by single-variable equations is possible under some particular conditions, but is likely to be inadequate under others because of the interaction of nutrients in producing yield increases. Interactions of nitrogen and phosphorus, phosphorus and potassium, lime and phosphorus, and population level and nitrogen are commonly observed. Interactions of three factors of production even appear frequently enough to merit attention in developing equations. It is necessary to utilize these interactions to take full advantage of the soil fertility, the applicable alternative fertilizer combinations, and varying price ratios of fertilizer elements and products in making the best fertilizer recommendation possible.

Although interactions are most often recognized when two or more of the nutrient elements are in low supply in the soil, it is quite probable that they also exist at higher fertility levels. They are less frequently identified at these higher levels because they are of smaller magnitude and the experiments usually are not precise enough to detect them, and because other independent factors limit the potential yield increases.

Requirements in Experiments

Keeping in mind the curvilinear nature of the normal response curve, and the remarks made above, it is possible to write the minimum terms required in a production function. For two variables it would be:

$$Y_2 = a + b_1 F_1 + b_2 F_1^n + b_3 F_2 + b_4 F_2^n + b_5 F_1 F_2, \tag{1}$$

where Y_2 is the yield, F_1 and F_2 are the two fertilizer elements, n is an exponent other than one, and b_1 through b_5 are constants. If only the response to fertilizer is considered, a becomes zero, and Y_2 becomes Y_2'. The equation may also require an additional term such as $b_6 F_1^n F_2^n$.

For a three-variable production function the equation would be:

$$Y_3 = a + b_1 F_1 + b_2 F_1^n + b_3 F_2 + b_4 F_2^n + b_5 F_1 F_2 + b_7 F_3 + b_8 F_3^n + b_9 F_1 F_3 + b_{10} F_2 F_3 + b_{11} F_1 F_2 F_3, \tag{2}$$

where F_3 is the third fertilizer variable and b_1 through b_{11} are constants. It may also be necessary to include such terms as $b_{12} F_1^n F_3^n + b_{13} F_2^n F_3^n + b_{14} F_1^n F_2^n F_3^n$.

The next problem is one of determining the combinations of treatments which give data satisfactory for equations, and of allowing ample measure of "goodness of fit." Remembering that data containing X points can be fitted with an equation with X - 1 constants, and the multiple correlation coefficient, R, will be equal to one, it is apparent that a two-variable experiment must have at least 7 treatments, and a three-variable experiment must have a minimum of 12. These experiments might be 4 x 2 and 3 x 2 x 2 factorials, respectively. While the former would provide one degree of freedom for deviation from regression, the latter would provide no measure of deviation from regression since the equation would pass through all points and there would be a "perfect" fit. Further objections to such small experiments arise from the fact that one variable in the former, and two variables in the latter could be fitted with linear terms. It is generally accepted that responses are curvilinear and provisions for this should be made in the design of the experiment. Therefore, the minimum types of experiments for two and three variables are 3^2 and 3^3 factorials, respectively, provided the general nature of the production function is known.

But one of the most important problems at present is to determine the production function. Two measures of the "goodness of fit" are the multiple correlation coefficient, R, and the deviation from regression. There seems to be no good figure for the number of degrees of freedom needed to give a reliable estimate of the mean square for deviation from regression. However, a number five or six times as great as the number of constants in the function would appear to be reasonable. Under these conditions a 6^2 or 7^2 factorial would be needed for an experiment in two variables, and a 4^3 or 4 x 4 x 5 factorial for three variables. These are about the minimum to satisfy the requirements of the statistical manipulations. When replicated twice, these experiments would contain 72 to 160 plots or require an area of about 0.9 to 2.0 acres in a corn experiment and would require up to one ton of high analysis fertilizer material. As hay or small grain plots, the area would need to be about one-fifth as large and would require proportionately less fertilizer.

In addition to the above, there are other characteristics which are desirable in experiments. The range of treatments should be wide enough to allow a diminishing total product at the higher treatments. It is not likely that the complete range of treatments on a soil low in fertility can be made with fewer than seven or more levels of the fertilizer elements without making the increments too great, especially at the lower levels. It is quite apparent that an experiment of 7^3 factorial would be about four acres in size and contain each treatment only once. Where the complete range cannot be included the lower range should be studied.

Some provision should be made to evaluate the residual effect of the fertilizer in terms of currently applied fertilizer. High rates of fertilizer are not usually expended in a single year, especially on heavy textured soils, and the residual value of even moderate applications is high. The value of the residual fertilizer should be measured in fertilizer equivalent during the second season as well as in terms of product yield. This gives a better evaluation of the residual effect because it cancels much of the seasonal influence. To include this phase in the study, the experiment will need to be made even larger.

Physical Problems

The agronomist conducting these experiments is faced with the need of experiments of ever-increasing size. The primary problem then is the size of the experiment and location of the experiment on a relatively uniform site. A uniform soil area is required in order that the results can be properly interpreted. Any soil survey map shows that soil types usually do not occur in large uniform areas. If the limits on soil survey maps were homogeneous, representing a particular soil type, the situation would not be so difficult. However, even on either side of the soil boundary line, there is a transition strip which is not representative of either soil type and cannot be used. With a heterogeneous system the variance will increase and interpretation and extension to other soils will be more complicated.

In any large area of a given soil type, other natural variations can cause nonuniform conditions. The depth of topsoil and the degree of erosion vary at different positions in the field. The slope can change by several percent and, furthermore, the aspect may change (i.e., in case of many soil types the north, south, east, and west slopes may be present and bring about variations in crop yields). When topography changes, internal drainage conditions, fertility, rainfall retention, evaporation, insolation, and soil temperature may also change. All of these variations contribute to errors of measurement which may become prohibitively large. In some cases, with small replicated experiments, as much as one-half of the total sum of squares has been due to replications. Consider what would happen to the precision in an experiment in which one replication was as large or larger than a small experiment such as the one above. It makes little difference whether the variance is measured by analysis of variance or deviation from regression, it remains if soils are not homogeneous.

Large areas of a given soil type or management system do not necessarily remain intact but are often broken up by man's activities. Parts of the area may be on different farms, in different fields, or divided by present or past crop boundaries. Differential treatments with respect to liming, fertilizing, manuring, crop removal, drainage, and tillage further subdivide the natural soil areas. Terracing, stripcropping, and contouring not only reduce the area into small strips, often too small for large experiments, but also present difficulties in laying out an experiment, with the consequence that such fields are often omitted from consideration as experimental sites.

In order that the experiment may be useful, it must be conducted in a field which will respond to fertilizer. If it responds to only one element, or only slightly to all elements, it is essentially worthless in describing the production function. Fortunately, enough work has been done with the soil test to make possible the selection of a field which will respond with a rather high degree of certainty. The lower the fertility and the greater the potential response, the more valuable is the experiment in determining with confidence the important characteristics of the production equation. Hence, fertility level is a primary consideration.

It appears from physical considerations that an experimenter should not expect to find suitable areas which exceed two acres in size, for use in a single experiment. Even so, uniform areas of some soil types as large as one-half acre are relatively few.

Miscellaneous Considerations

The geographical distribution of soil areas within a political or experimental unit presents problems in communications and control. Even in a small state such as Iowa, where the experiment station is centrally located, it is necessary to travel 200 miles to reach certain soil types. Experiments this distance from headquarters may not receive sufficient attention.

In addition to the above considerations, the field selected must be accessible both from the standpoint of location on a good public road and a good farm road where required. Means for controlling insects, diseases, and weeds must be present on the farm or within easy reach, since experiments may fail as a result of any one of these factors. Varieties capable of responding fully to the fertilizers with optimum plant populations have to be used. Poor varieties might lead to faulty conclusions about potential responses.

Plant population is one variable which may be studied; it is a factor which can influence the results markedly. Some crops with low stands respond less to fertilizers than crops with high stands. High stands on low levels of fertilizer treatments may affect the production adversely. There is always a question as to whether the stand on individual plots should be adjusted so as to fit expected performance (i.e., to adjust yields for stand). If these adjustments are made, the amount of labor needed will probably be doubled. For some crops, stands are less important. In other cases such as hay and some grains, stands already established are used for the experiments. Selection then is possible to meet the requirements of the trial.

Problems in Public Relations

Aside from the physical problems related to soils and their distribution, there are also problems in public relations. The research agronomist must depend upon some local person to help locate the possible sites for his experiments. Usually the local representatives of the college such as the extension directors or county agricultural agents are asked to help. For best results, it is important that the particular individual be highly interested in the type of work to be done, and that he understand the magnitude of the resources which will be expended in performing the work. Since he has to know what characteristics are wanted in the site, he should have an understanding of soil types and soil fertility in general. Unfortunately, a minority of county extension directors in Iowa, and possibly in other states, are graduates in agronomy, and real interest and good training in agronomy are needed. Finally, the local man must be willing to keep close contact with the experiment and the cooperator and to help answer the questions which always arise in the farmer's mind when the research agronomist is not available.

The cooperating farmer should have an interest in and an understanding of experimentation as a means of learning more about his own and his neighbors' problems, in addition to operating a farm with an acceptable area for the experiment which will be in the desired crop in a given year. He should have some willingness to accept a little inconvenience. Even though the investigator makes an effort to avoid interference with the normal operations on the farm, he is not always successful, and the farmer's understanding in the matter is very welcome. Usually the inconvenience is no more than having to plow, plant, or harvest in another field before the one in which the experiment is located, but even such small items could cause difficulties with some farmers. Inconvenience should be avoided where possible because a reflection on the experiment station would be inevitable. The man best suited to "size up" the cooperator is probably the county extension director.

In some cases, the experiment may cause a real or apparent loss in total product to the farmer. Usually this presents no special problems. Experience has been that the farmer was willing to accept a small loss on the check plots, low treatments for the increase in yield on the other plots, and the residual value of the fertilizer. Where this has not been acceptable, a reimbursement in cash or in kind has been satisfactory. Any additional operations requested in the execution of the experiment should be contracted with the farmer, and the farmer should be remunerated for any large removal of product which will not be returned. Oral agreements have been completely satisfactory in Iowa.

The seasonal nature of agronomic field experiments often presents problems in initiating and completing experiments because of weather. The experimenter must draw on experience to plan his program. He must be able to complete it in normal seasons.

The final problems in conducting these experiments relate to the personnel available at the experiment station to distribute the fertilizer, make notes, sample, harvest, and record the information. Well-trained technicians usually are not available to perform the operations. Much of the work has to be done by the researcher and the remainder by untrained persons. It is extremely difficult to instill into a day laborer the necessity for the careful and precise performance of all operations.

Some Solutions to Problems

The primary problem, which sets apart experiments designed to study the form of the production equation, is that of probable size and subsequent selection of sites. All the other problems are shared in common by smaller experiments which, of course, also have an important place in eventually providing sound bases for making the best possible fertilizer recommendations to farmers. The problem is to keep as many treatments as possible, provide for residual comparisons, and still keep the experiment within reasonable limits of size.

Since the study is concerned with determining a regression equation, the stress might be placed upon getting the largest possible number of points in the equation, rather than upon extreme precision in determining any one of fewer points. Hence, a large randomized factorial experiment without replication may be employed. Table 6.1 presents some results from three different experiments involving nitrogen and phosphorus fertilizers for corn, and phosphorus and potassium fertilizers for alfalfa and red clover. There were 57 treatments replicated twice. The data presented indicate that the variation as measured by deviations from regression agreed well with the results obtained among plots treated alike. As long as the number of degrees of freedom for deviations from regression is large compared to the number of constants in the equations fitted, it is not likely that the researcher will be misled with respect to the apparent precision of his experiment. For this reason it is felt that a nonreplicated factorial experiment is a definite possibility for purposes of studying production functions.

To supply a comparison for the residual study in the following year, a number of plots without treatment may be randomized among the factorial treatments. Since the equivalent residual effect does not usually exceed half of the initially applied quantity, the number of treatments can be fewer than in the original experiment. For example, in a 10^2

TABLE 6.1. Comparison of the Variation in Three Experiments as Determined by Deviations from Regression[a], and Among Plots Treated Alike

Crop	Source of Variation	d.f.	M.S.	F. Ratio[b]
Corn 1st year	Deviations from regression	51	215	1.38
	Among plots treated alike	57	156	
Corn 2nd year	Deviations from regression	51	123	1.28
	Among plots treated alike	57	96	
Corn Total	Deviations from regression	51	404	1.50
	Among plots treated alike	57	270	
Red Clover	Deviations from regression	51	215,406	.86
	Among plots treated alike	57	250,856	
Alfalfa	Deviations from regression	51	298,782	1.44
	Among plots treated alike	57	207,302	

[a] $Y = a + bF_1 + cF_1^{\frac{1}{2}} + dF_2 + eF_2^{\frac{1}{2}} + fF_1^{\frac{1}{2}} F_2^{\frac{1}{2}}$.

[b] "F" values for 50 and 55 degrees of freedom are 1.58 and 1.90 for 5 percent and 1 percent points respectively. cf. Snedecor (13).

Source: Heady, et al. (3)

factorial the previous year, perhaps a 4^2 or 5^2 factorial will be sufficient to cover the range of residual effects. It may be desirable to replicate these in the second year so it will be necessary to include 32 or 50 blank plots respectively for the above example. This means that in the first year there will be that many check plots. These extra plots have an advantage in the first year because they provide a good estimate of the check yield, and form an independent estimate of a in equations 1 and 2. Even when residual studies are not to be made it might be advisable to carry several check plots.

Some may not be willing to discard all replication but may want a wide range of coverage with their treatments. In this case it may be enough to replicate 5^2 factorial treatments of a 7^2 for larger factorial experiment and 3^3 factorial treatments in a three-element factorial experiment. The treatments would occur in a completely randomized experimental design; provision for the residual study above could still be included. The size of the experiment would be increased, but there would be an independent estimate of variance for checking the deviations from regression, and the precision of the replicated observations would be improved.

Another alternative in keeping the experimental area within limits and still retaining other desirable characteristics, such as including replication, is to discard treatments which are very likely to be outside the range of economic substitution (3, 4, 12). In a 10^2 nitrogen x phosphorus factorial, for example, it may be possible to discard some treatments in which the nitrogen rate is high and the phosphorus is low and also some in which the opposite is true. Possibly 30 of the 100 treatments may be omitted without serious consequences, and the same applies to experiments in three or four variables. Before proceeding in this manner it is important to have some knowledge of the probable outcome.

A final alternative is the one reported by Heady et al. (4), in which certain treatment combinations were left out in a systematic manner throughout all the factorial combinations, but all treatments retained were replicated. Where a fairly good prediction of the results is not possible this procedure has some advantage over the one above, as was indicated in the red clover and alfalfa experiments reported by these investigators.

Applications

Once the general form of the production equation is known and the important terms are determined on different soil types and under a range of weather conditions, the results may be employed to utilize other data. Because small experiments will always be less costly and easier to conduct under all conditions, they will be more numerous. As pointed out by Mason in Chapter 5, particular designs of smaller experiments can be formulated which will give the highest efficiency for any particular regression equation which applies. Actually, data for a large number of such experiments are already available. There may be several approaches to the problem after the production functions are determined.

To become useful, crop response data will have to be correlated with a soil test and plant tissue test (or a combination of these or other tests) to evaluate the soil fertility or the resources already on hand. It is as unreasonable to recommend fertilizers for a farmer, without some estimate of his soils' initial fertility, as it is to recommend ten extra dairy cows without knowing how many he already has, whether he can house them, or whether he is even interested in dairying. Too much fertilizer is recommended on this basis already. It it were not for the fact that fertilizers return 100 percent or more on the investment as compared to the 20 percent or less returned by some other enterprises, much more care would be exercised in making the recommendations.

For example, suppose that interest lies in soils which are deficient only in nitrogen and phosphorus, and also that there is interest in making recommendations for average operators. Suppose, too, that the smaller experiments have been on fields with average operators, and that the general fertilizer response function for this area is known. Let it be further assumed that four fertility levels of nitrogen and phosphorus are recognized by soil test: very low, low, medium, and high. There are, therefore, 16 possible nitrogen-phosphorus fertility situations which might occur in the area. One procedure would be to pool the results from all experiments conducted on soils in each of the 16 categories, and fit the production function to each group of pooled data. Since there is no longer interest in studying the production function, but rather in applying it, the experiments need not be large factorials, but they should be factorials. With the equations fitted, the optimum levels of fertilizer to use in individual cases could be determined by the procedures outlined by Heady et al. (4). The second possible procedure would be to relate the response to phosphorus and nitrogen fertilizers and the soil test levels for these two elements, using the pooled data from all the experiments. This pooled data would also be used to express the response part of the production function which might take the form of

$$Y' = c_1N + c_2N^n + c_3P + c_3P^n + c_4NP, \tag{3}$$

where N and P represent the pounds of N and P_2O_5 applied and c_1 to c_5 are constants.

This is the response surface starting with a soil that has no nitrogen or phosphorus available. To express the response on soils with nitrogen and phosphorus already present to a given degree, the response predicted by the equation needs to be decreased by the yield proportional to the fertility already in the soil. Under these restrictions both N and P in the above equations have two components: (a) the soil component, and (b) the fertilizer component designated by subscript s and f. Since the plant cannot distinguish between them, $(N_s + N_f)$ and $(P_s + P_f)$ can be substituted for N and P in the terms. But N_s and P_f are functions of the soil tests for these two nutrients which are designated as α and γ, and will give responses proportional to their magnitude with proportionality constants k and m. By making these substitutions, setting n equal to 2, and expanding, the equation becomes:

$$y = c_1k\alpha + c_1N_f + c_2(k\alpha)^2 + 2c_2k\alpha N_f + c_2N_f^2 + c_3m\gamma + c_3P_f + c_4(m\gamma)^2 + 2c_4m\gamma P_f + c_4P_f^2 + c_5k\alpha m\gamma + c_5N_fP_f, \tag{4}$$

where y is the yield increase on a particular soil.

Taking partial derivatives of y with respect to N and P and equating each to the price ratio of N to y and P to y respectively, the following is derived:

$$dy/dN = c_1 + 2c_2(k\alpha + N) + c_5P = \frac{X_N}{X_y}, \text{ and} \tag{5}$$

$$dy/dP = c_3 + 2c_4(m\gamma + P) + c_5N = \frac{X_P}{X_y}, \tag{6}$$

where X is the price of factor indicated by the subscript.

In this case k and m are evaluated experimentally along with the other constants. A simultaneous solution of both equations will yield the optimum rates and ratio of N and P under specified prices and given soil test values.

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Chapter 7

Methodological Problems in Agronomic Research Involving Fertilizer and Moisture Variables

MANY fertility experiments have been discarded or were never completed because soil moisture deficiency eliminated any chance for the different fertilizer treatments to exert their full potential. In fact, many fertilizer experiments conducted today tend to add more information about yearly variation to crop response due to season than they do to defining the actual limit of the fertilizer treatments. During years of adequate available moisture, the yield differences are often quite marked and a high production level is attained. However, during years of extreme drought little or no increase may be obtained, and the response to a given increment of fertilizer is much less than under conditions of better available moisture conditions. Plant nutrition is related in many ways to soil moisture, both directly and indirectly. This chapter presents some of the factors involved in the mineral nutrition of plants under variable moisture conditions.

The Soil System

A representative silt loam surface soil has been described by Lyon, Buckman, and Brady (30) as having 45 percent mineral matter, 5 percent organic matter, 25 percent water, and 25 percent air by volume. This means that the soil would have 50 percent of its space occupied by solids, the remainder being pore space that could be occupied by air or water or both. The volume composition values of four Tennessee Valley soils are given in table 7.1. These values do not deviate far from the average values presented by Lyon, Buckman, and Brady.

Soil particles possess the capacity to take up and retain moisture (8) which is distributed through the pore space of the soil and is held in the soil system by a combination of forces. It is possible under certain conditions for all of the soil pore space (except blocked pores) to be filled with water. Much of this water is unavailable for plant growth, however, as it is either lost through percolation or is held by the soil so tightly that the plants are unable to absorb it.

Figure 7.1 shows the volume composition of a Maury silt loam surface soil, and it may serve to illustrate the volume of the soil that is important in soil moisture-fertility relationships. The area A X B represents the larger pores of the soil. These pores are filled with

TABLE 7.1. Volume Percent Composition and Bulk Density of Four Tennessee Valley Soils

Soil Series	Sample Depth	Texture	Percent Pore Space	Percent Solids Mineral + Organic	Bulk Density
Maury	0-6"	Silt loam	50.9	49.1	1.38
	6-12"	Silty clay loam	49.8	50.2	1.47
Lindside	0-6"	Silt loam	47.7	52.3	1.43
	6-12"	Silt loam	49.8	50.2	1.38
Ennis	0-6"	Silt loam	56.7	43.3	1.25
	6-12"	Silt loam	54.4	45.6	1.32
Hartsells	0-6"	Fine sandy loam	54.0	46.0	1.31
	6-12"	Fine sandy loam	51.7	48.3	1.41

water during periods of heavy rainfall or heavy irrigation. However, this water is quickly lost through gravitational movement and is consequently of little use to plants. The area C X D represents that volume of water held by the soil so tightly that the plants are unable to extract

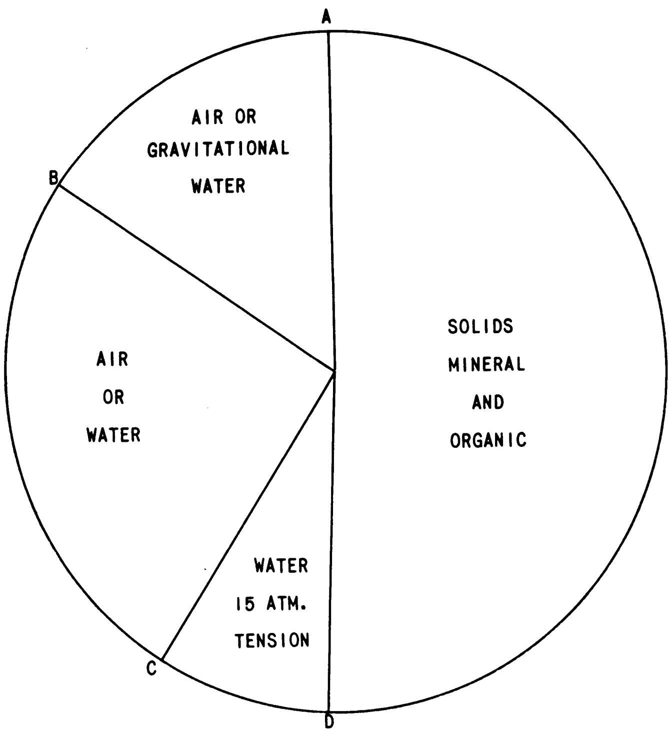

Fig. 7.1 — Volume composition of a Maury silt loam soil.

it from the soil. This is the moisture held under more than 15 atmospheres tension. Perhaps it is incorrect to say that plants are unable to extract this water from the soil because they can to a certain extent; however, they are unable to extract moisture from the soil in large enough quantities for normal physiological functions when the moisture is under a tension of 15 atmospheres or more. The area B X C represents that volume of a soil which may be filled by water or air. It is the moisture in this portion of the soil volume that is important in the growth and physiological behavior of crop plants. Point B could represent the field capacity and point C could represent the wilting point for the soil. The point for optimum growth of farm crops lies somewhere between B and C but is probably not very close to either B or C. The point for optimum growth may be different for different crops. The area A X D represents the mineral and organic portion of the soil. The size distribution and arrangement of particles, although they may vary quite widely for different soils, determine to a great extent the physical potentiality of the soil for crop production.

Moisture-Holding Capacity

There are several factors that determine the moisture-holding capacity and available moisture of a soil. Some are:

1. Texture and clay type
2. Organic matter
3. Osmotic effects
4. Total pore space and pore size distribution
5. Depth of soil profile

The effect of texture upon the moisture-holding capacity of a soil has been discussed by Richards and Wadleigh (40, 41). This effect is well illustrated by figure 7.2.

Note that for a given tension the heavier textured soils contain more water than the lighter textured soils. The bulk of available soil moisture in the loams and lighter textured soils has been depleted at tensions far less than 15 atmospheres, which is generally accepted tension corresponding to the wilting of plants in a soil system. This relationship may be of great economic importance in plant growth where moisture fertility relationships are involved.

The role which organic matter plays in the moisture-holding capacity of a soil is perhaps more indirect than direct. Jamison (23) concluded that organic matter did not increase the capacity of a soil to store available water except in sandy soils; however, the effects of organic matter on the structural development in a soil, its infiltration capacity, permeability, and other factors greatly aid its moisture-holding characteristics when considered in terms of crop production.

Osmotic Effects

The osmotic effects are those caused by soluble salts, exchangeable

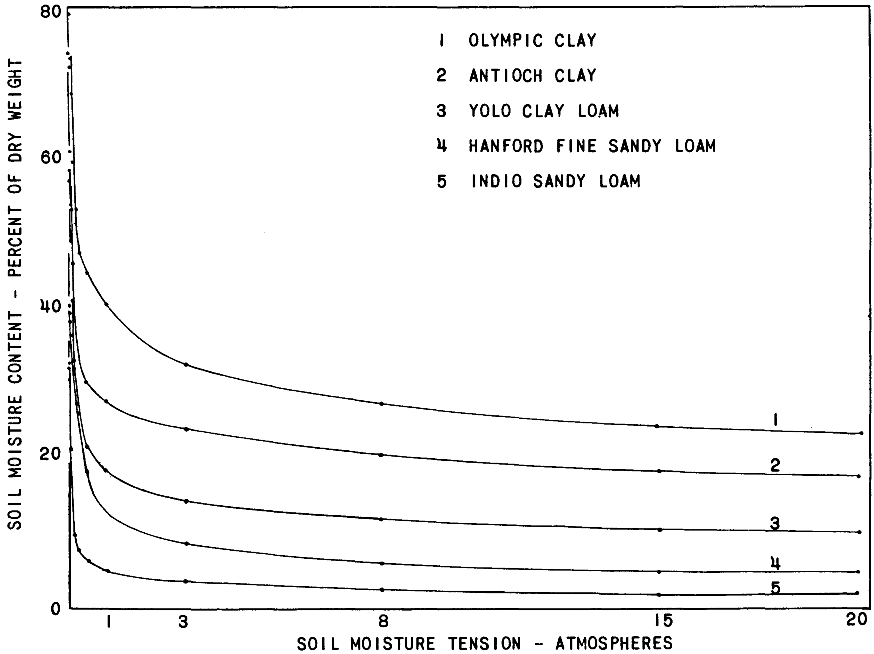

Fig. 7.2 — Curves showing the relation between the soil moisture tension and the moisture content of the soil. Richards and Wadleigh (40).

cations, and/or colloidal material suspended in the soil solution. These effects on the available moisture supply in a soil have been well illustrated (2, 29, 57). Increases in the salt and/or colloidal content in the soil solution will increase the permanent wilting percentage and thereby decrease the amount of water that the soil may retain which would be available for plant growth. White and Ross (60) have shown that an application of 1300 pounds of 3-9-3 fertilizer per acre to a Norfolk sandy loam increased the osmotic pressure of the soil solution to about 14 atmospheres. A similar application to a Cecil clay loam produced an osmotic pressure of only 3 atmospheres.

The total pore space is the factor that really determines the amount of water which may be found in a given volume of soil. It is this space which the water actually occupies. It is the pore size distribution that determines the relative amounts of the water which the soil will contain at different tensions. This property is determined by the texture, organic matter, and structural arrangement of these different components. The amount of water held by a soil, that is available to plants, depends upon the amount held per unit volume of soil. Thus, the depth of the soil explored by roots plays an important role in the total amount of water that may be available for plant growth within a given soil.

One very important factor in the moisture status of soils is the entry and movement of moisture into a soil (23, 45). The benefits of tillage, mulching, and crop cover upon water entry and movement in a soil are generally recognized. The slope of the soil is also important in determining the amount of water entry into a soil. This is especially true during periods of intensive rainfall. In cases where soil slope is an important factor in moisture entry into a soil, the infiltration capacity of the soil becomes increasingly important. A good example of this is the Dellrose soil on the slopes between the Highland Rim and the Central Basin in Tennessee and Kentucky.

One of the difficulties in evapo-transpiration studies is determining effective rainfall. A rainfall of two inches in an area may result in an effective rain of only one inch in a soil with some degree of slope. The colluvial soil adjacent to this area may receive over two inches, as it could receive runoff from adjacent areas.

Other factors play important roles in the moisture-holding capacity of a soil. However, many of these could be classified under one of the above factors.

Ionic Relations in a Soil System

The component in a soil that determines its ionic exchange capacity is the colloidal fraction. Although the silt fraction possesses some exchange properties, the magnitude of their effect on the total exchange capacity of a soil is relatively small. Organic matter accounts for considerable ionic exchange in a soil, even when present in small quantities.

Colloidal material is less than .002 mm in diameter. One of the properties of most soil colloidal particles is that they possess negatively charged surfaces and, when in a suspension, they are surrounded by a swarm of cations and anions that are in equilibrium within the system. Also surrounding these clay particles is a layer of water. The free energy of the vapor or liquid at the curved vapor liquid interface may be expressed as:

$$\triangle F = \frac{2\,\sigma V}{R}\ .$$

When σ is the surface tension, V is the specific volume, and R is the radius of curvature (8). Essentially this means that water is held more tightly as the radius of curvature decreases. Thus, the work that must be done to remove the water is inversely proportional to the radius of curvature of the liquid-vapor interface. If a large amount of water surrounds the clay particle, as would be the case when the soil is near saturation, little work will be required to remove a small increment of this water, but the amount of work required to remove each successive increment of water gradually increases.

It must not be concluded that the resistance a soil offers to water extraction by plants is entirely related to the radius of curvature of the liquid vapor interface; many other factors are involved. Among these are the osmotic forces, the gravitational forces, and the electrical forces of the charged clay particles. Many terms have been used to express the total effect of these forces on the soil moisture, the most recent one being total soil moisture stress.

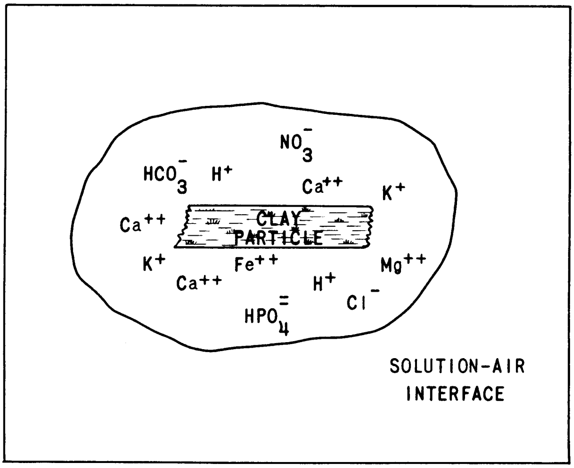

Fig. 7.3 — Clay particle surrounded by ion swarm.

Almost any surface possesses certain electro-chemical properties (1), and the surface of clay particles due to their structure is negatively charged. The magnitude of this charge and the water retention properties of clays are determined by their structure (3, 30, 31, 40, 41). Surrounding this charged surface, and in equilibrium with the soil solution, is an ion swarm. These ions largely constitute the nutrients that are absorbed and utilized by plants. Both cations and anions are contained in the ion swarm, and some of these may be "fixed" or absorbed on the surface of the clay particles (6, 54, 59). The extent of this fixation is determined by a number of factors. These will be discussed later.

Nutrient Absorption Under Variable Moisture Conditions

The soil system is made up of a number of particles, of various sizes, randomly distributed. Between the larger particles are the smaller particles. Surrounding all the particles is space that may be occupied by air or water. In order for plants to absorb nutrients they must extend roots through this air or water to contact the surface films of moisture surrounding the soil particles; it is in this film that the nutrients required for plant growth are found. Plants obtain certain of these nutrients through a process generally known as "contact exchange" (24), during which the plant root may absorb both cations and anions. Through normal respiration the plant root gives off CO_2, which unites with water to form H^+ and HCO_3^- ions. This maintains the electrostatic balance between cations in the soil solution and the negatively charged clay surface.

As the plant root absorbs nutrients from the surface of the soil particles, the equilibrium with the surrounding soil solution is disturbed. As a result, there is a net movement of other anions and cations toward the plant root and a net movement of H^+ and HCO_3^- ions away from the plant root. This exchange of ions results in a net increase in soil acidity.

Water is absorbed by the plant root, in addition to the mineral elements. As the moisture in the pores surrounding the clay particles near the root is utilized by the plant, there is a net movement of water toward this area from the adjacent pore spaces. This is because the water in a given unit of the soil system tends to be at equilibrium or reach the same energy state. As this progresses, the problems of keeping an adequate supply of nutrients and of water entering the plant become increasingly complicated.

It is evident from figure 7.4 that as moisture is withdrawn from the soil and the thickness of the surface films of water on the soil particles is reduced, the plant root will be in contact with fewer soil particles and thereby greatly decrease its supply of nutrients as well as moisture. Although the moisture in the soil as well as the ions in the soil solution tend to adjust themselves to a state of equilibrium, the distance over which this adjustment is made is not great (16, 32, 46). Also, as the

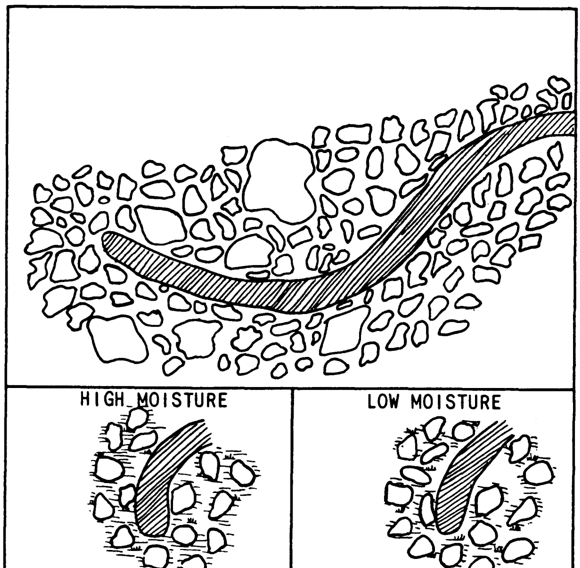

Fig. 7.4 — Plant root in contact with soil particles.

distance over which this adjustment is made is increased, the time for it to reach equilibrium is likewise increased. As the moisture decreases, the liquid surface contact between many soil particles is broken and the effective distance between them may be increased tenfold, even though they may be only a short distance apart in a linear sense.

It has been shown that when part of the roots of a plant are placed in dry soil, they will absorb nutrients (20, 21, 33, 55). However, it was also shown that the quantity of nutrients absorbed was small compared to the total supply needed for optimum plant growth. Plant roots, even in soils at 15 atmospheres tension, are in contact with many soil particles and may absorb nutrients, as well as small amounts of water, at this tension from a soil. Since the moisture film around the soil particles represents the zone of contact between plant roots and the soil particles, it is reasonable to assume that the number of soil particles that may be in contact with a plant root increases as the moisture tension decreases.

Perhaps one way to increase the probability of root contact with soil particles is to increase the bulk density of the soil. This is practiced to a certain extent. There are limits to which this may be done because too great an increase in bulk density results in a decrease in soil aeration. Unless the plant roots have an ample supply of oxygen, they will not absorb very much of anything. A good example of increasing soil density to benefit a crop is firm seedbeds. After the soil has been worked, it is usually loose and friable. After seeding, it is generally rolled or packed in some manner. This is to enable the roots of the young seedlings to come in contact with enough soil particles to survive during the period of germination when the soil moisture stress may be considerably below field capacity. Many a seeding has failed because of the lack of a firm seedbed.

When a soil is at field capacity and the ions in the soil solution are at equilibrium, there is a concentration, which may be designed as X, of these ions in the solution. As the soil becomes drier or as the moisture tension increases, there is an effective increase in the concentration of ions in the soil solution. If a soil contains 30 percent moisture at field capacity and 10 percent moisture at 15 atmospheres tension (wilting point), then the concentration of ions in the soil solution at 15 atmospheres tension will be 3X or 3 times as great as at field capacity (it being assumed that there was no net removal of ions from the soil solution).

As the concentration of ions in the soil solution increases, certain chemical reactions occur and some of the ions are precipitated, fixed, or absorbed on the surface of clay particles. Potassium may be fixed by 2-1 type clays (54, 59). Phosphorus may be precipitated (7, 49) or absorbed on the surface of clays (6). Boron may react with organic matter (36) or be precipitated as a borosilicate (18, 36). Calcium, magnesium, or iron may be precipitated if their concentrations are very high. Nitrates and chlorides are very soluble and generally remain in solution, often reaching very high concentrations.

Moisture Stress

Plants growing under conditions of moisture stress are generally high in nitrates, low in phosphate and potassium, and contain variable amounts of calcium, magnesium, and other elements (11, 12, 25, 28, 34, 37, 52, 53).

Why are plants grown under conditions of moisture stress high in nitrates? There are perhaps two basic reasons: (a) The concentration per unit of soil solution is much higher than when at field capacity. (b) The plant is unable to absorb enough phosphorus for rapid growth and, because of certain physiological processes, nitrogen accumulates in plants that are low in phosphate. It is, of course, recognized that the rate of plant growth is also a function of moisture stress.

Certain cases where phosphorus accumulates in plants under low moisture tension have been reported (25, 26, 33). In most cases, however, there is generally a lowering of the phosphorus content of plants growing under conditions of moisture stress (9, 11, 50, 51). It must be remembered in nitrogen and phosphorus nutrition of plants that phosphorus will accumulate in plants growing under nitrogen deficient conditions. Conversely, nitrogen will accumulate in plants growing under phosphorus deficient conditions. Therefore, it is entirely feasible for plants to accumulate phosphorus during periods of high moisture stress. Soils extremely high in phosphate, as the Maury soil, supply adequate phosphorus to plants during periods of moisture stress.

The low content of potassium in plants grown under conditions of moisture stress is best explained in terms of cation antagonism and plant root contact with fewer soil particles. The absorption of potassium by plants is inversely related to the calcium content of the soil. Increasing the soil moisture tension brings about an effective increase of both calcium and potassium in the soil solution. This increased concentration evidently depresses the absorption of potassium. The initial concentration of the two cations is important in determining this effect, and this varies quite widely in different soils. This perhaps accounts for the variation in results reported in the literature.

The concentrations of calcium, magnesium, and other elements vary quite widely in different soils. No general statements will be made at this time as to their behavior under periods of moisture stress. The initial concentration of these ions, as well as the concentrations of other ions, will determine their behavior under periods of moisture stress. The number of soil particles with which a root comes in contact will also affect the plant composition.

Root Activity and Penetration Under Variable Moisture Conditions

The moisture and fertility factors determine to a great extent the plant root ramification of a soil. Jordan, Laird, and Ferguson (27) found that corn plants fertilized with nitrogen during a dry year depleted the soil moisture to a depth of three feet. The unfertilized plots contained available moisture in the top three feet of soil. Painter and

Leamer (35) reported that the roots of grain sorghum removed moisture to a depth of at least 57 inches on plots where a moisture tension of 0.7 atmospheres or below was maintained at a nine-inch depth. On plots where the tensions were allowed to reach 12-15 atmospheres at nine inches before irrigation, moisture was removed to an approximate depth of 45 inches, with the greatest removal above 21 inches.

Hobbs (17) found that alfalfa utilized subsoil moisture reserves rather completely to a depth of eighteen feet in four years. Moisture reserves under fertilized stands of alfalfa were reduced to a lower level than those under unfertilized alfalfa stands. A shallow-rooted crop, bromegrass, did not seriously deplete the soil moisture below four feet. Soil moisture extraction data by Hagan and Peterson (13) indicate that, if the botanical composition remains nearly constant, there is no material effect on the distribution of absorbing roots in Ladino clover-grass and broadleaf trefoil-grass mixtures. Moisture extraction under clover-grass mixtures was largely confined to the top four feet of soil, while under the trefoil-grass appreciable extraction occurred throughout six feet of soil. Because of differences in effective depth of rooting, moisture extraction in the surface three feet of soil was most rapid under Ladino clover mixtures, less rapid under trefoil-grass, and slowest under alfalfa-grass. Land and Carreker (29a) measured the distribution of roots under corn and cotton and found that few roots of either crop penetrated below eighteen inches under either irrigated or unirrigated conditions. The proportion of feed roots of cotton in the top six inches was 55 percent with irrigation and 73 percent without irrigation. Approximately 85 percent of the corn roots were in the top six inches of soil.

The effective rooting depth may be defined as the depth to which a soil continues to lose moisture during a prolonged period of rainless weather (5, 39). Carey and Blake (5) reported that the effective rooting depth of sweet corn varied with soil type from 11 to 35 inches. When differences in rooting depth were combined with moisture per unit depth, tomatoes had about 4 times as much water at their disposal in a Sassafras loam as in a Nixon loam. Wheat had less than half as much as tomatoes when both were growing on a Sassafras loam.

Many factors may operate singly or in combination to determine the amount of water utilized by plants. The effects of these factors may not necessarily be constant from year to year as many of the factors are dependent upon the intensity or level of the other factors.

Fertilization of Farm Crops Under Variable Moisture Conditions

The effect of moisture tension on root ramification in a soil is quite evident. An intensive root system will enable the plant to exploit more extensively the native soil fertility and utilize more efficiently the nutrients added as fertilizers.

The stage of physiological development has a great deal to do with the results obtained from subjecting plants to periods of moisture stress

(15, 19, 22, 42, 48, 58). Generally the critical period is during the reproductive cycle of the plants involved. This is well illustrated by figure 7.5.

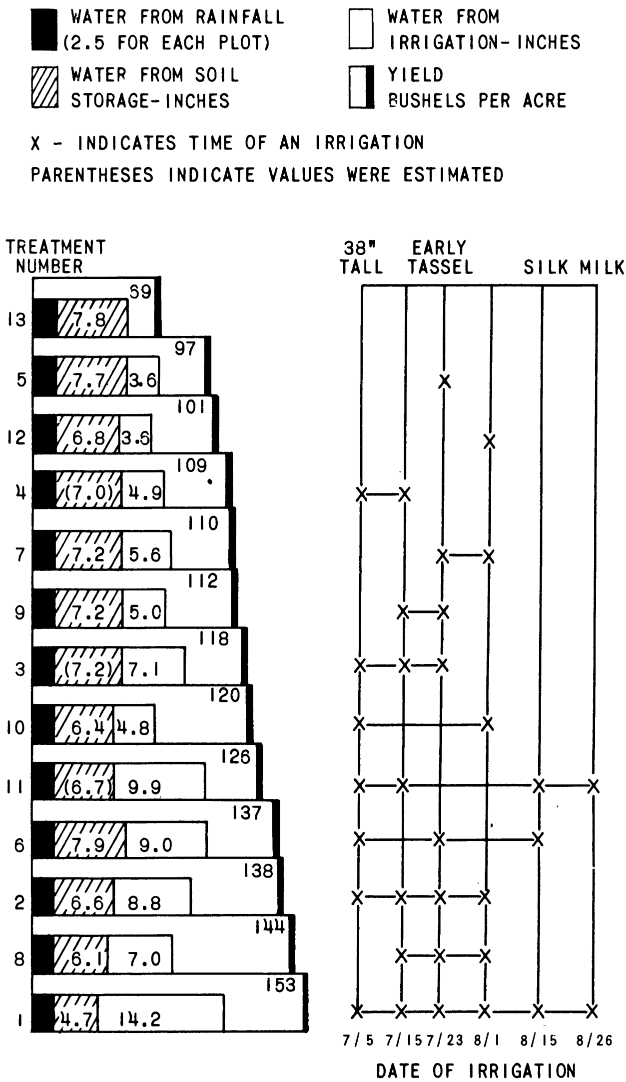

Fig. 7.5 — Yield of grain and water use by corn with different irrigation treatments. L. S. D. (5% level) for yield = 21 bu./acre. Howe and Rhoades (19).

Botanical Composition

As fertility and moisture treatments influence the physiological development of plants, it is reasonable to expect that they will affect the botanical composition of certain forage crop mixtures. Figure 7.6 shows the effect of different treatments on the percentage of clover and grasses in a meadow.

Both fertilizer and moisture treatments had an effect on reducing the amount of clover in the meadow. This change in vegetative

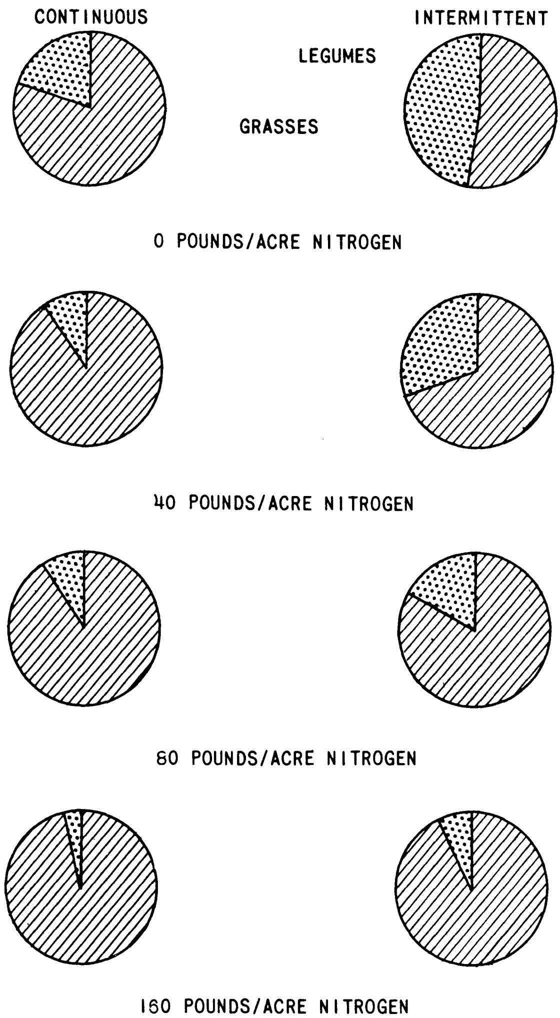

Fig. 7.6 — Effect of nitrogen and moisture variables on the composition of a meadow. Rouse (43).

composition of the meadow probably alters its nutrient value as a feed, as well as its nutrient and moisture requirement. As pointed out by Hagan and Peterson (13), the moisture requirement for an irrigation schedule must be based upon the moisture conditions within the root zone of the shallowest rooted component to be maintained in a forage mixture. With this difference evidenced in the rooting systems of forage mixtures, it is evident that the relative response of different species within a mixture to a given fertilizer treatment is quite different. In this particular case, a nitrogen-moisture variable was used, and this treatment combination favored the grasses in the forage mixture more than the legumes. Consequently, there was a net decrease in the percentage of legume composition. If the two extremes of the treatment combinations are taken as examples, the result is a comparison of a forage mixture of almost 50 percent clovers to one of less than 1 percent clovers. Since the primary objective of producing forages is to supply a feed for animals, the question immediately arises as to whether or not the two forages had equal feeding values and produced the same TDN per acre.

Chemical Behavior of Soil Elements

The chemical behavior of different fertilizer elements in a soil also enters into the magnitude of the response to a given level of that element under different degrees of moisture stress. This relationship, along with the different rooting characteristics and nutrient requirements of the various species of forage mixtures, enters into the results obtained from fertility-moisture variables.

Moisture Stress and Growth

The effect of moisture stress upon the growth of plants has been illustrated by the work of Wadleigh and Gauch (57).

They measured the length of cotton leaves daily and compared the rate of leaf growth to the intensity of the soil moisture stress. As is evidenced in figure 7.7, the rate of leaf elongation starts to decrease when the soil moisture tension reaches 9 atmospheres, and it is almost zero when the tension reaches 15 atmospheres. Under the conditions of this experiment, leaf elongation was expressed as a second degree function of the soil moisture stress for a given irrigation cycle.

The level of soil moisture which should be maintained for optimum growth of plants has been the objective of many irrigation experiments. It is recognized that it is physically impossible to maintain soil moisture in a soil of actively growing plants at a given tension. Therefore, the maintenance of a minimum soil moisture tension in a given zone of soil is the approach generally taken. In some cases the integrated soil moisture stress over the entire root zone is used (49a). Either approach provides a means for maintaining a minimum soil moisture level and thereby provides information on the moisture requirements for maximum growth of a crop under a given set of soil conditions. In some

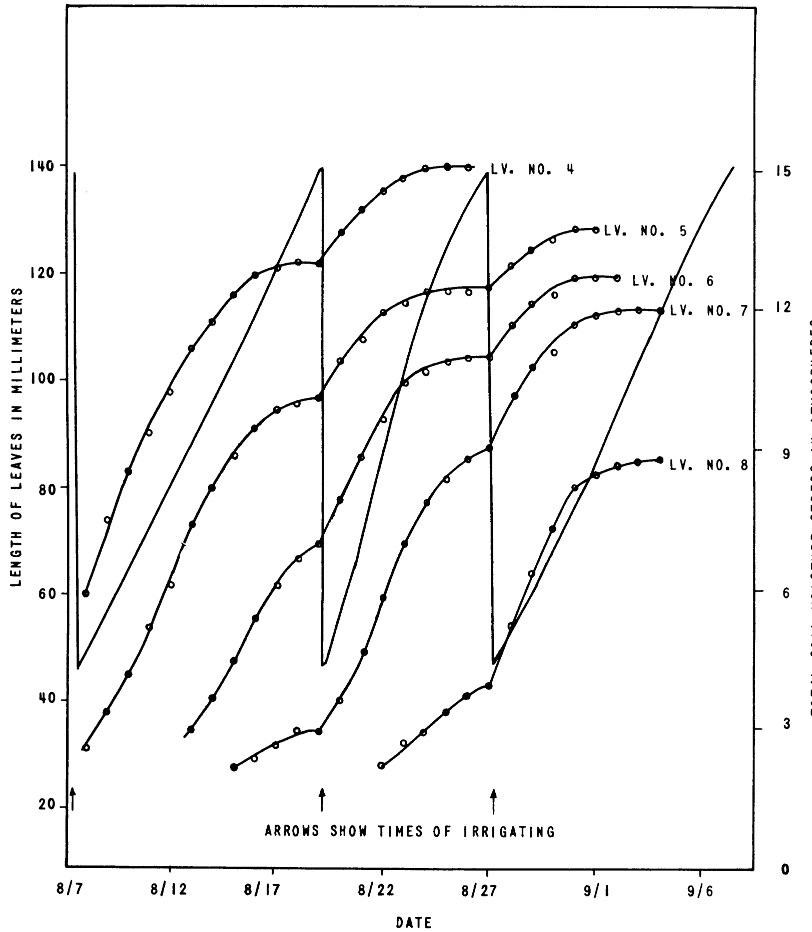

Fig. 7.7 — Effect of moisture tension on leaf elongation. Wadleigh and Gauch (56).

instances the "percent available moisture" level is used to determine moisture requirement and irrigation schedule. However, due to the different moisture-holding properties of soils, the relative energy by which moisture is held by soils is quite different. A sandy soil has lost over 90 percent of its available moisture at 1 atmosphere tension (see figure 7.2), while heavier textured soils do not lose this percent of available moisture until they reach a much higher tension.

Perhaps the fertilizer element that gives the greatest positive interaction when combined with moisture is nitrogen. This element is most often combined with moisture variables in the experimental results reported to date. Results shown in figure 7.8, by Painter and Leamer (35),

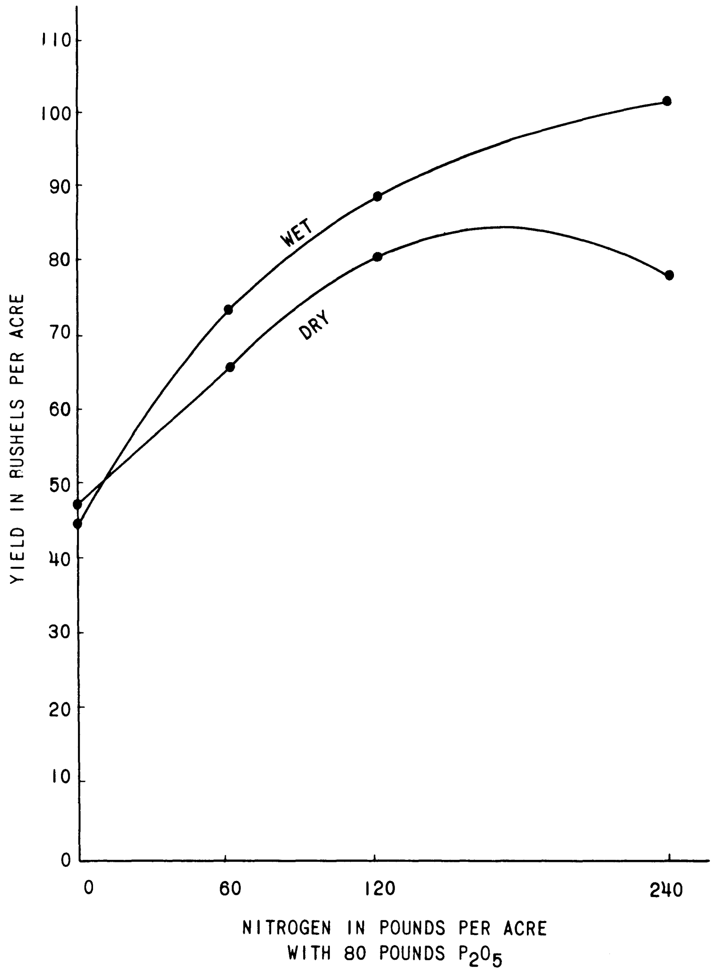

Fig. 7.8. — Effects of nitrogen and moisture on the yield of grain sorghum. Painter and Leamer (35).

represent the data obtained in corn experiments where nitrogen and moisture were varied. In the "wet" treatment, which was 0.7 atmospheres tension at 9 inches depth, the slope of the response curve obtained never reached zero. In the "dry" treatment, which was 12-15 atmospheres at 9 inches depth, the maximum yield was obtained at 120 lbs. N/A. A higher rate of N at this moisture level resulted in a slight decrease in yield.

Paschal and Evans (37) combined moisture, nitrogen, and spacing variables in an experiment with grain sorghums. They found that, as the moisture and nitrogen levels were increased, a greater yield was obtained by increasing the plant population. In this particular case the plant population was doubled. This brings out another problem in moisture fertility experiments, which is the desired plant population necessary to give an accurate measure of the treatment potential. In this particular case the yield obtained by M_1S_2 (moisture tension<0.7 atmospheres at 9 inches depth and 20,028 PPA) was quite different from that obtained by M_1S_1 (moisture tension<0.7 atmospheres at 9 inches depth and 43,560 PPA) at the higher nitrogen levels. The M_2 (<12-15 atmospheres at 9 inches depth until heading, then the same as M_1) moisture treatment indicates that grain sorghums are not as responsive as corn when moisture stress is reduced during the reproductive stages.

Once the response curve has been established, the point on the curve for the most economic yield may vary as prices fluctuate. However, the production surface should remain fairly constant for the conditions under which it was obtained.

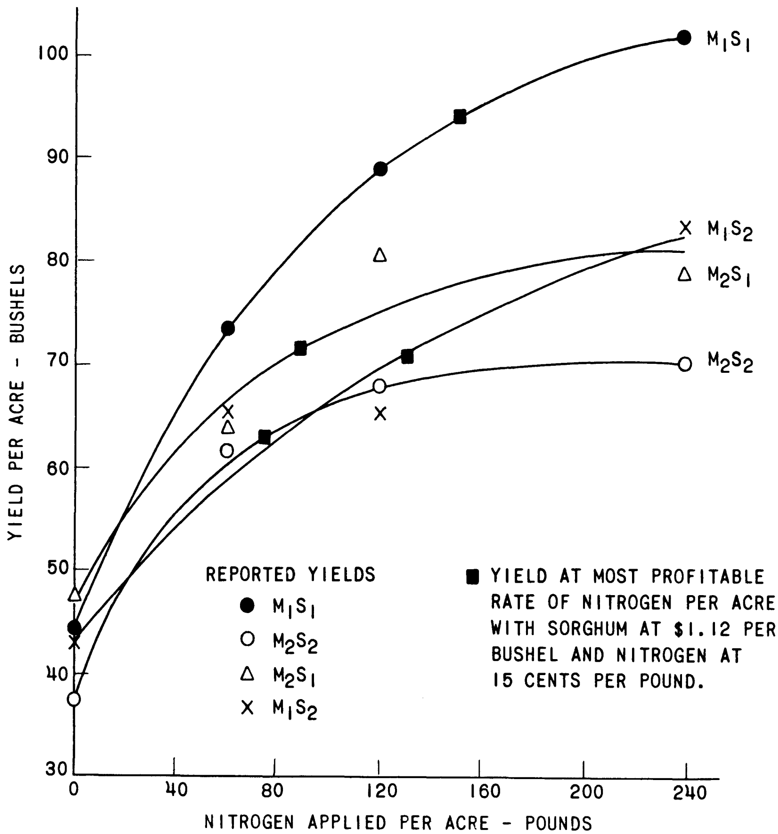

Fig. 7.9 — Effect of nitrogen, moisture, and spacing on the yield of grain sorghum. Paschal and Evans (37).

Fisher and Caldwell (10) of Texas have reported some interesting results from the application of nitrogen to Coastal Bermuda-grass under conditions of heavy irrigation. Increases in yield of dry forage were obtained from each increment of nitrogen up to 800 lbs. N/A (5 separate applications). Of particular importance is the rapid decrease in the amount of water needed to produce a ton of forage as the level of nitrogen is increased. This is of great importance in the efficient use of water for supplemental irrigation in the humid region.

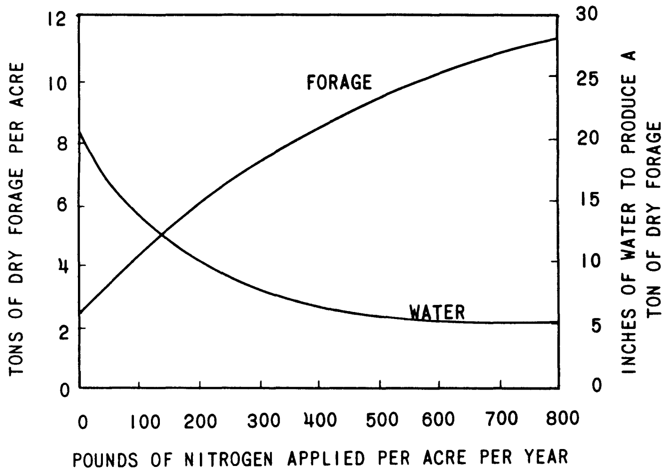

Fig. 7.10 — The effect of nitrogen on forage yield and water utilization of Coastal Bermuda grass. Texas A & M Progress Report 1731.

Experimental Designs for Combined Moisture and Fertility Experiments

In the study of variable moisture levels for crop production there are two primary objectives: (a) Determine the minimum moisture tension for maximum yield. (b) Determine the fertility requirement for this moisture level. In many instances, these two objectives are combined in the same experiment. Such a combination works very well in certain experimental designs, but the area involved often gets very large if many levels of each variable are studied.

Irrigated plots must be large because of moisture movement in a soil, problems of application, and lateral root extension by plants. It is possible to have fertility plots that are much smaller and still have a reliable measure of the treatments under study. A split-plot experimental design fits this combination of variables and has been found to work very well in experiments conducted to date.

There are several factors that may affect the fertility level desired for optimum yield when the moisture regime of the crop is controlled.

Some of these factors tend to counterbalance others, but considerably more information is needed to determine where the equilibrium point lies. One school of thought is that, with an ample supply of available moisture, larger and more rapid plant growth will result, bringing about a need for a greater supply of plant nutrients. On the other side, tending to counteract this increased nutrient requirement, there is a more extensive root system. This enables the plant to feed from a larger volume of soil and results in an increased efficiency in use of nutrients in the soil and of those added as fertilizers. There is a great need for information that will evaluate these counteracting factors.

In most experiments involving the production of farm crops, an effort is made to obtain the maximum yield from a specific element under a given set of conditions. It is recognized that this maximum yield may not be the most economical yield; but once the response curve has been established, the point for the most economical yield may very well move up or down the curve from year to year.

There are many variables which, when combined, give a significant interaction. Also, when a number of factors are involved, the same product may be obtained by altering the levels of the different factors. During periods of price change, it may be desirable to alter the level of the various factors to obtain the desired yield in the most economical way.

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Chapter 8

Some Problems Involved in Fitting Production Functions to Data Recorded by Soil-Testing Laboratories

IF all variables were known and measurable there would be one production function for each crop. All soil types, soil conditions, and fertilizer techniques could be thrown together. However, it is practical to estimate a different production function for each technique of fertilizer application and each separate soil type. For example, row application and broadcast methods may achieve different results from the same levels and combinations of fertilizer. Evidently some variable, such as the distance the plant travels to obtain the fertilizer, is involved. Since present knowledge does not furnish good scales for the effects of and the measurement of such factors, it is necessary to limit our estimates of production functions to homogeneous classes of application situations.

Data to derive production functions of the type hypothesized are limited. The Soil Testing Laboratory has considerable information available in its records regarding the fertility status and yields of individual fields. A complete cropping history for the past year or two and information about drainage, slopes, textures, and fertilizer application are on record.[1] Controlled experiments of a design and scope to secure the necessary data would be more desirable from the viewpoint of insuring the range in data needed to estimate the production surfaces (function). Furthermore, all measurements and sampling techniques could then be supervised by trained personnel. Noncontrolled variation could be reduced to a minimum through proper experimental design. However, the soil testing data have the advantage of availability in quantity and for a period of years. Many of our laboratories collect samples each year, running into the tens of thousands. The data, therefore, warrant examination as to the possibilities for production function derivation. First, however, some attention should be paid to the specific nature of the function to be fitted.

Crop Yield Functions

While there are few problems of mathematical function selection

[1]Reference throughout this paper to specific information available in Soil Testing Laboratories is based on conditions at Purdue University. The situation varies to some extent from laboratory to laboratory.

which are peculiar to estimating crop production functions from soil laboratory records, the selection of variables is limited to those for which information is recorded. Production function estimation from soil test records is limited to the crops for which yield estimates are recorded. Corn, soybeans, and wheat are among these. Hay yields are not often recorded. Fertilizer applications are sufficiently well recorded so that the nutrient elements N, K_2O, and P_2O_5 applied the year of the crop and the year before may be estimated. Soil test results are available for P_2O_5, K_2O, and pH but not nitrogen. Hence, a production function may be derived which states that corn yield depends upon soil nutrient levels of K_2O, P_2O_5, fertilizer elements N, P_2O_5, and K_2O applied in each of two years, and pH. Further information is available to sort the data on the basis of soil type, texture, drainage, past cropping history, and other such variables as will permit a fairly homogeneous grouping. Technique of fertilizer application or machine used for application is also recorded. Plant population, soil nitrogen, and moisture are among the more important variables for which information is lacking. At the same time, certain peculiarities of the data give rise to statistical problems of deriving any specific function chosen.

Peculiarities of Soil-Testing Laboratory Data and Their Implications in Fitting Production Functions

THE SAMPLE

Soil samples are sent to the soils laboratories on a volunteer basis. These samples may not be representative of the area and/or fields from which they are drawn. McCollum and Nelson (3) have examined the possibility of fields volunteered being higher in some fertility elements and lower in others than those of a systematically drawn sample. They indicate the differences are small although statistically significant.

The fact that the average phosphorus or other nutrient level for fields in the sample is lower or higher than for the area sampled is not of major concern, however. From the standpoint of deriving a production function, equally good estimates of all portions of the production surface are desirable. Some conditions which are scarce in the soil type area may be relatively heavily represented in the best sample for deriving a production function. The observations in the sample must be representative of these various portions of the population sampled, however. That is, if fields with relatively low K_2O content are scarce in a particular area, the sample observations from such fields should be typical of the low K_2O fields. Any predictions as to yields would otherwise be meaningless when made for other farmers with those field conditions.

In order to study the effects of fertility levels on yield, the soil samples from which fertility estimates are made also must be representative of the plots or fields for which yields are estimated. This problem is of importance in soil sampling and testing as well as in deriving production functions. Any sample which is considered sufficiently representative of a field to make a soil test representative of the field should be satisfactory for production function fitting. The systematic type of field sample designed for the farmer's use is reasonably efficient in this respect.

Since no random sampling procedure is used in selecting the farmers who have soil tested, there is always an unpredictable possibility that the farmers who do have their soils tested are not representative of the entire population of farmers in the area under consideration. This situation may or may not create a problem. If the fields from which samples are sent to the soils laboratory are representative of the general soil type for which economic recommendations are to be made, even though the farmers are not, nothing is necessarily lost. However, there should be some indication of the farm management practices of fertilizer application used by these farmers in order to be able to tell other farmers how to attain the same results. It is likely that the methods of applying fertilizer are as important in determining yield response as the quantity of fertilizer used. More work is needed on this problem.

Farmers' Soil Sampling Procedures

More important is the fact that farmers take the samples. While specific directions for taking soil samples are given to the farmers, one cannot be sure that they are followed exactly. With such a large number of untrained people drawing samples, there is always the possibility of inaccurate sampling. This problem is probably resolved to some extent by the errors averaging out over the large number of samples taken.

Yield Estimates from Years Previous to Soil Test

Another peculiarity of our information is that yield estimates are made on crops raised the year previous to the soil tests. Hence, the recorded soil tests may not be the correct ones to associate with the yield data available. Unless the soil tests are reasonably stable from one year to the next, it becomes hard to distinguish soil nutrient effects from fertilizer effects in this situation. Heavy fertilizer applications can affect soil tests taken later. In the case of corn, the fertilizer applications will probably not be high enough to cause any great difficulty. Total fertility changes, in this case, would be small from one year to the next. This problem may be more serious with other crops and other fertilization techniques. It amounts to the same thing as errors of measurement of the independent variables, as discussed below.

Farmers' Yield Estimates

To complicate matters further, the yield data for crops are estimates and not necessarily actual measurements made by the farmers. Most farmers do not weigh their corn or make accurate checks on the yields. Many of these yield estimates are rough guesses by the farmers to give the soils laboratory an approximation from which to start analysis. A great deal of variability may therefore be introduced into the dependent variable — yield — by this process. If the farmers are not biased up or down in their estimates of yield, the large numbers of available observations will tend to resolve this problem. Their errors of estimation will offset each other. The importance of any possible bias would also depend on its nature, such as whether it is a constant or a relative deviation. If it were a relative amount proportional to yield level, less accurate estimates of yields at high levels would occur then at low levels. A constant overestimation or underestimation would not be too serious from the standpoint of affecting the accuracy of estimates of the additional yields that would be produced by additional amounts of fertilizer. However, the total yield estimates would be in error.

Uncontrolled and Unmeasured Variables

Many variables are unmeasurable in a cardinal sense. Texture groups, drainage groups, slopes, color, etc., can be classified but good quantitative measurements cannot be made of them. As previously mentioned, production functions can be fitted for various homogeneous groups sorted from such data. On the other hand, there are some variables which are not measurable and some, though measurable, for which measurements have not been made. These variables include moisture measurements, plant population, nitrogen test levels, and management factors.

At present there are not nitrogen tests that are universally accepted. Many soil laboratories will not record soil nitrogen except as indicated by cropping history, soil color, texture, and drainage. In deriving a production function, the data can be sorted according to these factors to achieve relatively homogeneous situations with respect to soil nitrogen. A separate function would have to be fitted to each situation.

Plant population is also uncontrolled and unmeasured in soil testing laboratory data. The number of plants per acre can usually be profitably increased as the level of fertilizer application is raised. If farmers take advantage of this situation, a relatively high correlation may exist between fertilizer application and plant population. The soil laboratory data do not indicate the extent of this problem.

At least two possibilities regarding these unmeasured factors are: (a) All or some unmeasured variables are uncorrelated with the measured independent variables and their effects may be normally and independently distributed. (b) Some or all unmeasured variables may be correlated to some degree with certain measured variables. It is the latter situation that is of concern.

In this instance the plant population, for example, may be correlated with fertility level, fertilizer application level, or method of fertilization. If such is the case, any increase in yield may not be a result of increasing one or more of the available plant nutrient supplies, but a result of increasing both the amount of plant nutrients and the plant population. A recommendation based on such an estimate will fall short when presented to and used with a lower plant population than assumed with the recommended fertilization program. This situation limits the use of the production function. However, the limitation may not be of concern in the short run. If farmers actually increase plant population and fertilizer application together, the estimate of fertilizer effects would be fairly accurate for prediction of yields under farm conditions as long as the relationship between fertility level and plant population is maintained. Unfortunately, no information as to the extent of this situation is available in the records. Some indication of the true situation might be obtained by examining data from experiments in which plant population was controlled and by making comparisons with portions of the production function derived from soil-testing records.

Fertilizer Nutrient Pounds Not Equivalent to Soil Nutrient Test Pounds

Neither the pounds of nutrients in fertilizer nor those indicated by soil tests measures the pounds of nutrients which are available to plants; both are functions of available nutrients. These variables, fertilizer elements and soil nutrient levels, may have to be considered as separate variables because the relationship between them is not known with certainty. This situation is not a serious disadvantage since there is no interest in the results of adding fertilizer nutrients to different soil fertility levels. The function and its variables in terms of the units commonly employed do not require any conversions to a common unit. The chief disadvantage is that the production function will contain more terms than would be the case if available plant nutrients such as P_2O_5 and K_2O could each be looked upon as one variable rather than several. This condition adds to the computational costs.

Correlation of Independent Variables

Perhaps one of the most troublesome problems in fitting functions to data other than those from controlled experiments is that of correlation between independent variables. Even without error of measurement of any variable, this condition is a serious limitation. It may occur because there is a correlation of independent variables in the population from which the sample is drawn or because of chance situations in sampling; therefore, difficulty results when estimating any large portion of the production surface with respect to two variables. For example, if P_2O_5 and K_2O are highly correlated, only a band on the production surface can be derived (figure 8.1). The width of this band also has an effect on the accuracy with which estimates can be made of the effects of changing amounts of K_2O or P_2O_5 within this area. Although the regression coefficients derived in the absence of measurement error will be unbiased, their variances will rise as the degree of correlation increases. Measurement errors of the dependent variables will accentuate this condition but still permit unbiased estimates.

Correlated Fertility Levels

In the event that the fertility levels found in the soil are so highly

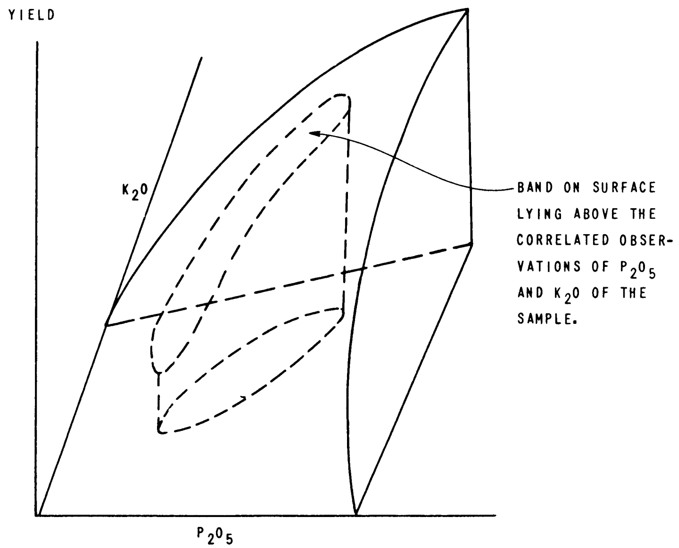

Fig. 8.1 — Correlation of independent variables.

correlated that it is not possible to separate the individual effects of nutrients, it is reasonable to use a function of the pair of nutrients as a single variable. Of course, the production function furnishes insufficient information to cover a situation where it is necessary to predict yield with these variables in some other relationship to each other than found in these data. Correlation of soil fertility levels would not be as serious as correlation of fertilizer applications in this respect. If the fertility levels of the soil are highly correlated in the population there could be a justification of a combined soil fertility level for a given soil type. On the other hand, failure to achieve a range in fertilizer application prevents analysis of shifts in the kind and amount of fertilizer nutrients that would be profitable under a variety of agronomic and economic circumstances. Hence, if possible, a wide range in the amount of any factor of production in the sample is desirable in order to examine its effects on yield, but for best results it has to be independent of other production factors in its variation over this range.

Correlated Fertilizer Application Levels

Many farmers may already be following fertilizer recommendations of the extension personnel and soils laboratory. Farmers with a given soil fertility situation may therefore be applying essentially the same fertilizer combinations and amounts; hence, the correlation between fertilizer elements applied may be high. The effects of adding a particular plant nutrient are then difficult to assess and the possibility of substituting one fertilizer element for another will be missed. There is no other solution to this situation unless it is possible to locate soil test records with relatively wide uncorrelated variation of the independent variables and to include them in the sample.

Errors in Measurement of Independent Variables

From a statistical viewpoint another possible problem is the inexact measurement of independent variables (1). The methods of function fitting usually used (least squares regression) depend upon an assumption that the independent variables are measured without error, if unbiased estimates of the regression coefficients are to be obtained (4). Such a condition rarely (if ever) is met in practice. If the errors are relatively small, the bias may be small. Insofar as the variability of laboratory tests on a particular soil sample is concerned, this condition is probably the case. However, the sample comes from an entire field from which a number of subsamples are systematically selected (5). These subsamples are mixed and the result is associated with the yield given for the field. Hence, sampling variability enters the estimate of the independent variables. Similar problems arise with pH tests. Fertilizer applications would not be a problem in this light if the fertilizer and soil nutrient levels were always evenly distributed. The rate per acre would then be the actual rate that went with the yield in question, starting from a particular level of soil fertility. If the distribution is uneven, however, a situation exists where some parts of the field may be receiving a much higher rate and some much lower. The combinations of these average to the rate used or recorded, but these do not necessarily give the same yield response as if the average rate were evenly applied. Similarly, if the fertilizer rates may be assumed equally, original fertility will vary throughout the field with similar implications.

There are methods of weighted regression which can be used to overcome the errors of measurement problem (6). Present methods do not account at the same time, however, for both errors of measurement and errors in the equation, i.e., omission of independent variables.

The measurement problem is most serious when the substitution rates between various nutrient elements are desired. In order to derive the substitution rates, unbiased estimates are needed for the function's parameters. These are very difficult to obtain from the situation involving serious measurement errors of the independent variables.

A further complication can arise if these measurement errors are combined with correlation between independent variables (2). Often, estimates result which may look reliable by use of the standard significance tests of the regression coefficients and examination of standard deviations. Unfortunately, the results can be unreliable.

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Chapter 9

Evaluating Response to Fertilizer Using Standard Yield Curves

THE term "standard" curve implies acceptance of some particular hypothesis or yield function for evaluating response to fertilizer. For purposes of this presentation, the standard curve technique based on the exponential function will be used. But his in no way implies satisfaction with the current state of knowledge as to appropriate yield functions for fertilizer (cf. Chapters 1, 5, and 6).

Standard yield curves based on the exponential function are prepared from a table of values of $1\text{-}R^x$, in which R, a fixed ratio of successive increments in yield, has been assigned a specific value, 0.8 in this instance. Each value of x (unit of fertilizer) is associated with a specified value of $1\text{-}R^x$. The higher the value of $1\text{-}R^x$ the nearer the curve approaches maximum. On the standard yield chart, M, maximum yield, is coincident with the top of the $1\text{-}R^x$ scale, that is, when $1\text{-}R^x = 1.0$. A large number of standard yield curves should be prepared by anyone who uses this form of the graphic method. The curves will have different shapes by varying the scale on the x axis, but for each tabulated value of x the value of $1\text{-}R^x$ is always the same. Instead of finding the value of R that represents best fit to the data, R is standardized and a fit is obtained by varying the size of a unit of application. Each standard curve is based on a different size of unit. The decimal fraction $1\text{-}R^x$, when multiplied by 100, represents the percentage of maximum yield.

Given an adequate set of standard yield curves, some part of one of them can be found to describe yield responses to fertilizer from good rate experiments. This may be considered to be a fair statement, but it might well be added that it is true provided the exponential function fits the data. Fitting a standard curve to the reported yield is done by plotting the latter and overlaying on a standard yield curve. Usually the obvious choice is between 2 or 3 portions on one or two curves. From that point further refinement can be attained by recording plus and minus deviations of reported yields from the curve. A little practice will enable one to locate a fit at which the sum of the plus and minus deviations approximates zero.[1] If a good "tool kit" of standard curves has been

[1] A forthcoming U. S. Department of Agriculture publication, "A Graphic Method of Interpreting Response to Fertilizer," includes a more complete description of the method.

prepared, results from this process can be made rather quickly to approach the accuracy found by more precise time-consuming methods.

The constant M is read on the overlay at the point at which the yield scale coincides with the value 1.0 on the $1\text{-}R^x$ scale of the standard yield curve. The $1\text{-}R^x$ value for any yield read from the curve is obtained as y/M. Two readings from the curve provide two $1\text{-}R^x$ values, and the x value of each of these is found in the table used in preparing the standard yield curves. The number of pounds of fertilizer that represents the range between the two yields read from the curve, divided by the difference between the two x values, results in the number of pounds per unit required to obtain the fit, when R = 0.8.

Example

M = 119 bushels, read from standard yield chart,

	y	$1\text{-}R^x = y/M$	Units of x (from table)
y_{120} lb. =	92 bu.	.77311	6.65 = n+a
y_0 lb. =	17 bu.	.14286	0.69 = n, "soil content"
		Difference	5.96 = a, applied portion

Size of unit, $u_a = 120/5.96 = 20.13$ lbs.

a, in units = lbs. applied/u_a

x value in table = n^{+a} in units. $y = M(1\text{-}R^x)$.

Graphic Versus Mathematically Fitted Curves

Some indication of the approach to accuracy that can be obtained by this method is indicated by results obtained when used in analyzing three 12-rate experiments involving nitrogen on irrigated corn. Results are compared with those obtained from use of the mathematical solution for least squares suggested by Stevens (2). These comparisons are presented in table 9.1. Results obtained by the two methods are equivalent for purposes of recommendations. This is true whether all 12 rates were used, or whether only 5 or 6 rates distributed over the range were used in fitting the curves. In the Oregon and Nebraska experiments, rates were carried to 320 pounds of N per acre; in the Washington experiment, to 520 pounds. In all instances the curves become decidedly flat at the higher rates.

In 3 of the 6 comparisons between the graphic and the Stevens methods, differences in yields at the most profitable rates were smaller than 1 standard error in the yield at MPR on the Stevens fitted curve. Sums of squared residuals are approximately twice as great for the graphic as for the Stevens fitted curve. Obviously, if the problem were one in which precision rather than a basis for field recommendations were needed, the mathematically fitted curve would be necessary. But if the exponential equation is suited to the data, the graphic method of fitting the curve is useful for those who need reliable answers quickly.

TABLE 9.1. Comparison of Results from Graphic and Mathematical Solutions for Least Squares Fit When Applied to Three 12-Rate Experiments Involving Nitrogen on Irrigated Corn (Based on Unpublished Data Supplied by Soil and Water Conservation Research Branch, ARS, in Cooperation with Indicated State Experiment Stations)

Experiment and Method of Analysis	Number of Rates[a]	Most profitable rate	Yield at MPR	$s_{\hat{Y}}$ at MPR	Σe^2	M	R	s_M	s_R	Return per Acre at MPR	
		(Pounds)	(Bushels)	(Bushels)	(Bushels)			(Bushels)		(Dollars)	
Ontario, Oreg., 1952											
Graphic	6	168	140.7	-	85.4	-	-	-	-	173	-
Stevens	6	151	139.0	5.0	75.0	-	-	-	-	173[b]	173
Graphic	12	168	142.0		270.3	150.1	0.75544	-	-	175	-
Stevens	12	164	141.7	1.7	254.8	150.2	.75000	6.7	0.0204	175[b]	175
Hardy, Nebr., 1952											
Graphic	6	136	119.1	-	76.4	-	-	-	-	147	-
Stevens	6	153	119.9	1.4	27.3					146[b]	145
Graphic	12	135	119.0	-	331.3	123.9	.63653	-	-	148	-
Stevens	12	164	121.6	1.4	161.7	127.8	.70652	5.0	.0520	146[b]	146
Prosser, Wash., 1953											
Graphic	5	233	137.9	-	121.3	-	-	-	-	159	-
Stevens	5	267	140.9	2.7	52.8	-	-	-	-	157[b]	156
Graphic	12	231	135.3	-	520.0	145.3	.89660	-	-	155	-
Stevens	12	269	140.2	2.0	307.2	152.7	.83504	6.5	.0435	156[b]	155

[a] When only 5 or 6 rates were used, they were the same for each method and were scattered over the range.

[b] Calculated from Stevens' fitted curve at MPR on graphic curve.

The last column of table 9.1 shows returns per acre above cost of fertilizer at the most profitable rate as determined from the graphic curve and from the Stevens fitted curve. Then, using the constants of the Stevens fitted curve, the return above cost of fertilizer was calculated for the rate indicated as most profitable on the graphically fitted curve. The differences are negligible.

Results for Two Nutrients in a Factorial Design

Methods have not been developed for simultaneous solution for values of the constants of the exponential equation when two or more independent variables are involved. For combinations of independent variables, results are obtained by using constants derived by fitting each regression curve at specified levels of each of the other variables. Thus, in calculating yields for a production surface, this equation is used under the assumption that the rate (R) of response to a nutrient is the same at different levels of the other nutrients. Results are shown for this equation applied in this way to three factorial experiments. Results obtained in these instances are compared with results from use of the quadratic square-root equation used by Heady and Pesek.[2]

Comparisons are shown in tables 9.2 and 9.3. Table 9.2 shows the sums of squared residuals explained by regression, and the coefficients of correlation resulting from use of the exponential and quadratic square-root equations as applied to three 9 x 9 partial factorial experiments.

TABLE 9.2. Sums of Squared Residuals Explained by Exponential and Quadratic Square-Root Equations as Applied to Three 9 x 9 Partial Factorial Experiments

Experiment	Sums of Squares				Coefficients of Correlation	
			Explained by Regression			
	Total	Treatments	Exponential[a]	Quadratic Square Root	Exponential	Quadratic Square Root
Corn	242,707	233,811	222,927	222,899	0.9764	0.9764
Alfalfa	29.80	26.75	20.78	22.98	.8694	.9229
Red Clover	17.85	13.66	9.69	11.52	.8425	.9184

[a]Based on constants derived from only 17 of the 57 treatment combinations, as no simultaneous solution is available for the equation. The SS are computed for all 57 treatment mean yields.

[2]See Heady and Pesek (1). When N and P are the independent variables, the quadratic square-root equation used by these authors is written as:

$$y = a + b_1 N + b_2 P + b_3 \sqrt{N} + b_4 \sqrt{P} + b_5 \sqrt{NP}.$$

TABLE 9.3. Sums of Squares Reported from Calculated Yields

	Entire Surface		Deducting "0" and "320" Layers	
Experiment	Exponential	Quadratic Square Root	Exponential	Quadratic Square Root
Corn	5,442	5,485	2,296	2,366
Alfalfa	3.27	1.77	.75	.81
Red clover	1.98	1.40	.64	.62

Only 17 of the 57 treatment combinations were used in finding constants of the exponential equation, because of lack of method for simultaneous solution.

In contrast with the exponential equation, which approaches though theoretically does not reach the calculated maximum, the quadratic square-root equation has merit in the ability to calculate reduced yields after the maximum has been reached.

Sums of squares of deviations reported from calculated yields are shown in table 9.3 for the entire production surface and for the more relevant portion — after deducting deviations at the extremes, 0- and 320-pound levels in these experiments. As indicators of reliability of results for use in recommending rates of application, deviations assume importance primarily around the section of the surface that includes combinations reasonably close to those found to be most profitable. A high percentage of the deviations occurred in the "fringe" area of the surface. If the deviations that occur there are deducted, there is no difference in the sums of squared residuals resulting from use of the two equations. Of course, as mentioned by Mason in Chapter 5, few persons would use this procedure because of the high degree of subjectivity involved. In this sense, the quadratic equation would appear to be the best fit.

Differences in yields calculated by the two equations at the most profitable combination were small in relation to the standard error of the yield at MPR as determined by the quadratic square-root equation. Differences in returns per acre above the cost of fertilizer were substantially less than the value of the units represented by one standard error of the yield.

The many facets of these comparisons are discussed more fully in the reference cited.[3] These few illustrations are presented merely to indicate comparisons based on the three experiments with no wish to imply definite conclusions. Certainly one job of methodological research that might well be undertaken is that of finding a simultaneous solution for values of the constants of the exponential equation when two or more independent variables are involved. Also, it would be well if someone

[3] A graphic method of interpretive response to fertilizer, op cit.

would determine the nature of the distribution of the constants of the exponential equation, so that standard errors computed from the one-variable form of this equation could be predicted with more certainty. Standard errors are now based on the assumption of normal distribution.

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PART IV

Application of Data

- Simple Nomographs
- Farm Planning
- Budgeting
- Linear Programing
- Price Considerations

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Chapter 10

Practical Applications of Fertilizer Production Functions

BEFORE practical use can be made of fertilizer production functions, the basic experiments must have first been designed to allow satisfactory estimates of the production surface. Complete factorial experiments, in which every level of one fertilizer is combined with every level of other elements, can be recommended for surface estimation if the required number of treatment combinations is not too large. Where complete factorial experiments require excessive treatment numbers, the composite design appears to offer a promising alternative.[1]

Interaction of Nutrients and Design

In Iowa experiments, fertilizer elements have often interacted to produce an added effect not due to either element alone (2). For positive interaction, as between nitrogen and phosphorus on corn yield, fertilizer rates should go high enough to cause a decrease (or at least a leveling out) of average yield for each nutrient. In table 10.1, average response to N and P_2O_5 has leveled out or started to decline at the heaviest rates. Unfortunately, the "incompleteness" of this experiment makes interaction effects harder to isolate. The average yields of 200, 240, and 280 pounds of P_2O_5 presented in table 10.1 are 97.66, 101.79, and 106.03 bushels, respectively. However, it is hard to tell whether the increase in yield is due to: (a) the heavier level of P_2O_5; (b) the higher levels of N at the 280-pound level of P_2O_5 as compared to lower levels of N at 240 and 200 pounds of P_2O_5; (c) the interaction of N and P_2O_5. The average responses in table 10.1 which are strictly comparable are only those which include *all* levels of the other nutrient, or the 0-, 160-, and 320-pound rates. Therefore, a "complete" factorial experiment would seem preferable to the incomplete type in table 10.1, even though it might be necessary to space some of the fertilizer rates farther apart to keep down the total number of treatments.

Highly significant N · P interaction in the data of table 10.1 was indicated by an F(40, 57 d. f.) of 4.25. The strong interaction is indicated in table 10.1. At low levels of N, yield of corn is restricted despite large P_2O_5 applications. With N at 160 and 320 pounds, response to P_2O_5

[1]See Chapters 3 and 5 for a detailed discussion of these points.

TABLE 10.1 Bushels of Corn per Acre for Varying Levels of Fertilizer on Calcareous Ida Silt Loam Soil in Western Iowa in 1952. (Each Cell Represents the Average of Two Observations) (2).

Pounds of P_2O_5	Pounds of Nitrogen								
	0	40	80	120	160	200	240	280	320
0	15.35	17.85	17.55	24.80	10.75	8.65	11.50	17.25	22.05
40	28.15	71.35			101.50	95.40			79.15
80	26.35		107.45		94.25		119.00		105.50
120	33.05			108.35	107.85			122.05	103.80
160	23.00	88.45	105.35	128.85	123.00	110.60	127.40	133.05	129.15
200	33.95	66.05			119.00	141.25			127.10
240	36.50		102.50		126.70		117.65		137.90
280	29.90			127.35	131.00			135.95	119.45
320	11.60	59.60	100.25	128.60	122.80	132.40	133.40	121.85	123.35

Average Response to Pounds of Nutrients

Pounds	0	40	80	120	160	200	240	280	320
P_2O_5	26.65	60.66	86.62	103.59	104.09	97.66	101.79	106.03	105.27
N	16.19	75.11	90.51	95.02	107.65	97.87	104.25	108.73	103.76

is comparatively great. Likewise, nitrogen response is definitely dependent upon the P_2O_5 application.

Regression estimates of the data in table 10.1 confirm the importance of interaction. A t value of 8.85 for the interaction term in regression equation 10 in Chapter 1 is significant at the .00001 level of probability.

Although many Iowa experiments have shown important interaction between nutrients, what if a factorial experiment is run and no significant interaction is found? The independent responses can then be computed with a considerable gain in efficiency; each level of one nutrient can be used as a replication in estimating the response from the other element (1). Thus, complete factorial experimentation is efficient if treatment effects are independent and experimental error does not increase markedly with large treatment numbers. If treatment effects are not independent, some type of factorial experiment is necessary to estimate treatment interaction.

Practical Application

Assuming that the basic experiment was properly designed and that the estimating function gives a good "fit" and is logically and statistically satisfactory, how can these fertilizer production functions be used?

Marginal rates of substitution and isoquants can be computed as illustrated in Chapter 1 and give relevant economic information. It is also possible to present in one chart the economic information needed for decision-making. In figure 10.1, added isoquants and isoclines have been derived for equation 10 in Chapter 1. Along the isocline labeled $P_n = 1.5\ P_p$, 1 pound of nitrogen produces the same yield as would 1.5 pounds of P_2O_5. Therefore, when the price of nitrogen is 1.5 times the price of P_2O_5, the N and P nutrient combination should fall on the isocline labeled $P_n = 1.5\ P_p$. (The isocline can be thought of as the optimum "fertilizer mix" curve).

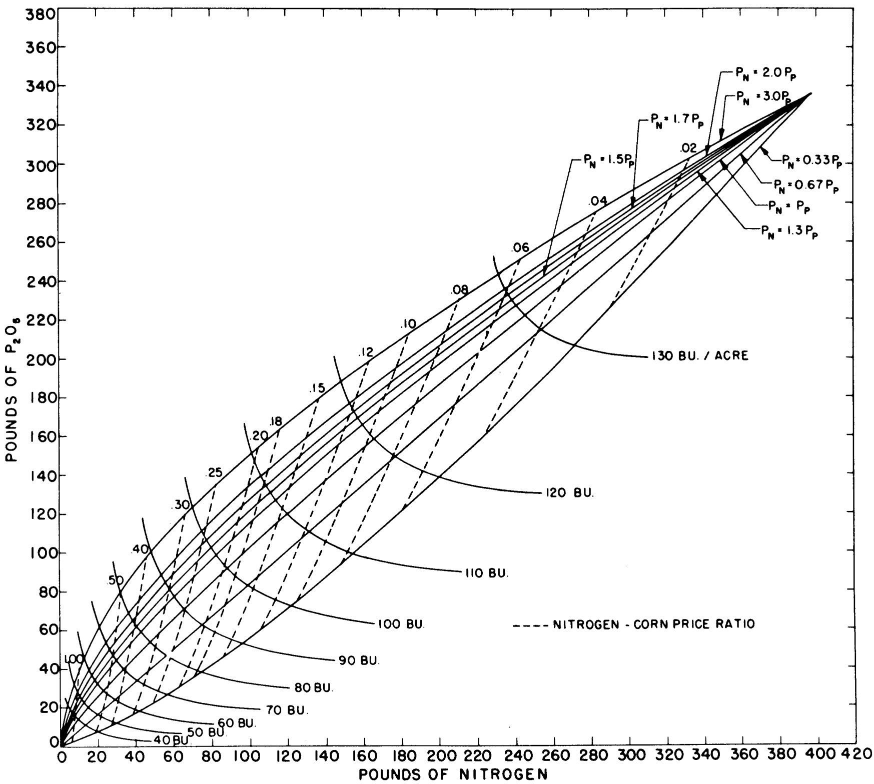

Fig. 10.1 — Yield isoclines and isoquants for corn on Ida-Monona soil, Iowa. Optimum rates are indicated by dashed lines representing the nitrogen-corn price ratio.

Figure 10.1 was designed to provide a simple method of determining the most profitable rates of nitrogen and phosphorus to apply to corn. (Construction of the chart required calculus, but its use by farmers or extension personnel requires only short division). The following price situation illustrates use of the chart:

Price of corn	\$1.00 per bu.
Price of element N	0.15 per lb.
Price of available P_2O_5	0.10 per lb.

The price relationship of nitrogen to phosphorus is $P_n = 1.5\ P_p$. Therefore, the line leading from the lower left corner to the upper right corner (labeled $P_n = 1.5\ P_p$) is chosen. This line gives the optimum nitrogen-phosphorus combinations for all levels of production when nitrogen is 1.5 times as expensive as P_2O_5. To find how far to go on this line, it is necessary to determine the nitrogen-corn price ratio. In this case, $P_n/P_c = 0.15$. Therefore, the line labeled $P_n = 1.5\ P_p$ is followed until the dashed line labeled 0.15 is reached. Then by dropping straight down from this point, a reading of about 125 pounds of nitrogen is obtained. Likewise, by reading straight to the left from the 0.15 point on the fertilizer mix line (isocline), about 142 pounds of P_2O_5 is indicated. (When the price ratios are not exactly those of the chart, it is possible to interpolate between the nearest values given).

Determination of corn yield for the indicated inputs of N and P_2O_5 is easily made from the yield lines or isoquants. For the preceding optimum inputs, a yield of about 113 bushels is predicted. At \$1 per bushel, a total value of about \$113 per acre is estimated. Gross return and fertilizer cost figures are given below:

Value of corn	\$113.00
Cost of nitrogen	18.75
Cost of P_2O_5	14.20
Margin over fertilizer	80.05

Although the dashed lines in figure 10.1 represent various nitrogen-corn price ratios, they also include the phosphorus-corn price ratio. For example, the intersection of the dashed line labeled 0.15 with the isocline labeled $P_n = 1.5\ P_p$, was found by the optimum solution from equations 11 and 12 in Chapter 1.

Solving equations 11 and 12 from Chapter 1 simultaneously, N = 124.7 and P = 141.8 are used as the point of intersect of the dashed line labeled 0.15 with the fertilizer mix line labeled $P_n = 1.5\ P_p$. The rest of the points on the chart are located in the same way.

Alternative Solutions

How should such a chart as figure 10.1 be used by agronomists in making recommendations to farmers? Figure 10.1 is based upon empirical results which would apply only to farmers who had calcareous Ida silt loam soil with the same general fertility level as the experimental field. If the farmer does have a similar soil, recommendations can be made directly from the chart as in the preceding sample. However, several alternatives to the "optimum" probably should be presented to the farmer. If the price of nitrogen is expected to remain 1.5 times the price of P_2O_5, all alternatives should be selected from the fertilizer mix line marked $P_n = 1.5\ P_p$. For example, the farmer in the preceding price situation might be so restricted on capital that he could apply only \$12 worth of fertilizer per acre. The "restricted" optimum can still be located for the farmer by laying a straight-edge from 120 pounds on the P_2O_5 axis to 80 pounds on the nitrogen axis. (Any point lying along the straight edge would represent \$12 worth of fertilizer for N at 15 cents per pound and P_2O_5 at 10 cents per pound). The intersection of the straight-edge with the isocline labeled $P_n = 1.5\ P_p$ at about 60 pounds of P_2O_5 and 40 pounds of nitrogen is the best that can be done with \$12 worth of fertilizer per acre. A yield of approximately 79 bushels is estimated from figure 10.1 for these inputs of N and P_2O_5.

Value of corn	\$79.00
Cost of N	6.00
Cost of P_2O_5	6.00
Margin over fertilizer	67.00

Although the farmer's margin over fertilizer cost in the above restricted case is reduced by about \$11 per acre as compared to the first "optimum" inputs, farmers may still be justified in "holding back" because of capital shortage or uncertainty. Farmers probably should be permitted to choose their own "optimum" from a range of alternatives presented to them by extension agronomists. Thus, if a farmer could realize \$1.50 on each dollar of his limited capital invested in some other enterprise, he could roughly equalize this return with his last dollar expended for fertilizer.

For a tenant farmer who pays one-half share rent and must bear all fertilizer cost, a different optimum solution is indicated for the preceding price situation. For the tenant, the relevant nitrogen-corn price ratio would be $\frac{0.15}{0.50} = 0.30$. At the most, it would pay him to apply only about 58 pounds of N and 81 pounds of P_2O_5, which would result in a predicted yield of about 90 bushels per acre.

If some new technique should alter the nitrogen-phosphorus price ratio, another isocline (fertilizer mix line) should be chosen. For example, if nitrogen and P_2O_5 were the same price per pound, production should be expanded along the isocline labeled $P_n = P_p$ in figure 10.1.

Application to Alfalfa

A similar production "map" in figure 10.2 could supply relevant economic information to a farmer growing alfalfa on Webster loam soil in north central Iowa. If alfalfa is assumed to be \$20 per ton, P_2O_5 is 10 cents per pound, and K_2O is 8 cents per pound, the fertilizer mix line labeled $P_k = 0.8\ P_p$ should be selected. Following this isocline until the dashed line labeled 0.005 (representing the phosphorus-alfalfa price ratio) is reached, an optimum input of about 21 pounds of K_2O and 69 pounds of P_2O_5 is indicated. A yield of about 3.15 tons is predicted by the isoquants for this input of K and P.

If the farmer is limited on capital or is a tenant, he may be rational in not applying as much fertilizer as indicated by the phosphorus-alfalfa

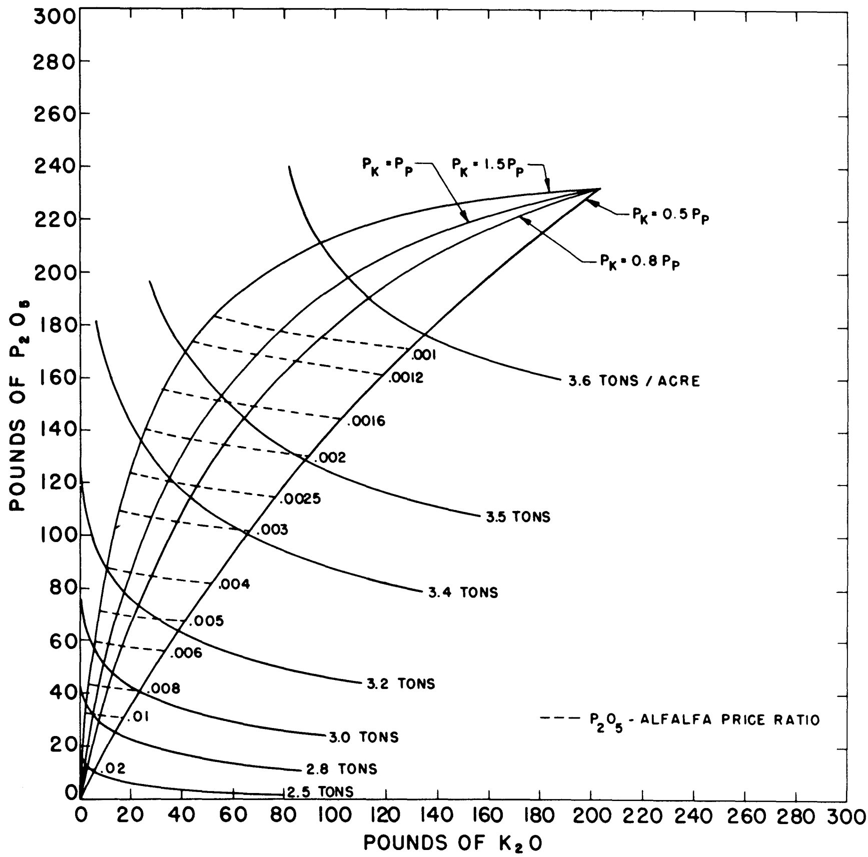

Fig. 10.2 — Yield isoclines and isoquants for alfalfa on Webster soil, Iowa. Optimum rates are indicated by dashed lines representing the phosphorous-alfalfa price ratio.

price ratio. Whatever the level of production, however, it should be obtained along the isocline representing the appropriate phosphorus-potash price ratio.

Limitations of Existing Data

One limitation of the procedure followed by the use of figures 10.1 and 10.2 is that it does not include residual or carry-over response in the second year for fertilizer applied in the first. The residual problem is analyzed for corn by Heady, Pesek, and Brown (2) and could be incorporated into figures 10.1 and 10.2. However, residual response in the second year may partly reduce the response to new fertilizer applications in the second year. Consequently, the first year's response alone may be a fair approximation of the response to be obtained year after year, including the next year's residual response. Of course, more empirical information regarding fertilizer response in succeeding years is needed.

Another limitation to recommendations made from production functions such as those represented by figures 10.1 and 10.2 is that the recommendations are based on a single year's result. Response on the same soil type could be much different in another year under different growing conditions. Confidence interval limits can be set up which may be "narrow" for last year's experiment, but these confidence limits do not really apply to next year's crop — which is what the farmer cares about. Also, even when the farmer has the same soil type, his fields will seldom be of exactly the same fertility level as that of the experimental plot.

A possible solution to these related problems is to apply the principle of continuity between experiments as well as within. Thus, a number of experiments run on the same or similar soil types but with varying fertility levels could be "pooled." A more general production function could be obtained in this manner, and it would include soil test measurements as variables as well as fertilizer applications. Then, results of the farmer's sample soil test could be "plugged into" the general production function to predict expected fertilizer response. Another advantage to such a procedure would be that if experimental results over a number of years were "pooled," an estimate of response variability could be obtained which would relate to *next* year's crop.

Before more general estimating functions can be tried, results of many factorial experiments, along with soil tests, must first be accumulated. Meanwhile, results of individual experiments could be utilized where applicable through use of charts such as figures 10.1 and 10.2.

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Chapter 11

Organizing Fertilizer Input-Output Data in Farm Planning

FERTILIZER is a major cost item on north Georgia farms. Using general fertilizer recommendations, this item amounts to 40 to 60 percent of the cash cost and 20 to 35 percent of the total cost of corn production. Correctly then, farmers are interested in using fertilizer to gain maximum profits. Most farmers in Georgia know that they must use fertilizer efficiently if they are to make a reasonable profit from farming. Many have compared the 10 to 20 bushels of corn per acre obtained from land not fertilized with the 50 to 100 bushels of corn obtained on land that is well fertilized. Many also know that yields from 100 to 150 bushels per acre are possible on some of the best land, if larger amounts of fertilizer are used and recommended management practices are followed.

Steps in Decisions

From these and other production possibilities, the farmer must somehow make decisions as to which practices and fertilizer quantities are best for his particular situation. There is considerable information concerning the logic involved in the process of making rational decisions but there is limited information on how different farmers actually do make decisions (3, 4, 5, 7, 8). The first step in actual decision-making is that of observation. Some knowledge of possible fertilizer responses is obtained from personal experience, observation, and discussion of the problem with other farmers. While these kinds of observations do not give complete information, evidence (1, 2, 9, 12) suggests that this is the extent of information obtained by many farmers. Aside from the farmer's own experience and his observation of other farmers in the area, information may be obtained from the various agricultural agencies, magazines, newspapers, radio, commercial organizations, and college bulletins.

How does a farmer analyze the information he obtains? Here, too, research information is limited. Certainly much of the information he obtains is, or seems to be, conflicting. One neighbor may be convinced that 250 pounds of fertilizer per acre is the right amount on corn at planting time, while another is equally convinced that 700 pounds is needed. One article in a farm magazine may infer increasing returns;

another, decreasing returns in using fertilizer. Yet farmers somehow resolve these differences and come to a decision.

Response and Economic Use by Soil Type

Most farmers know that the response to fertilizer varies with the type of soil. Crops on some soils are not able to use large amounts of fertilizer efficiently. Good bottom soils without serious erosion, poor structure, or drainage and drought hazards give better response to fertilizer than soils with limitations other than fertility. This relationship is illustrated in figure 11.1 for a Talladega soil area, a Hayesville soil, and a Transylvania soil. When corn is worth \$1.50 per bushel and nitrogen costs 15 cents per pound, only 40 or 50 pounds of nitrogen can be used economically on the Talladega soil by the farmer with ample funds.[1] On the Transylvania silt loam, however, an application of over 90 pounds is economical.

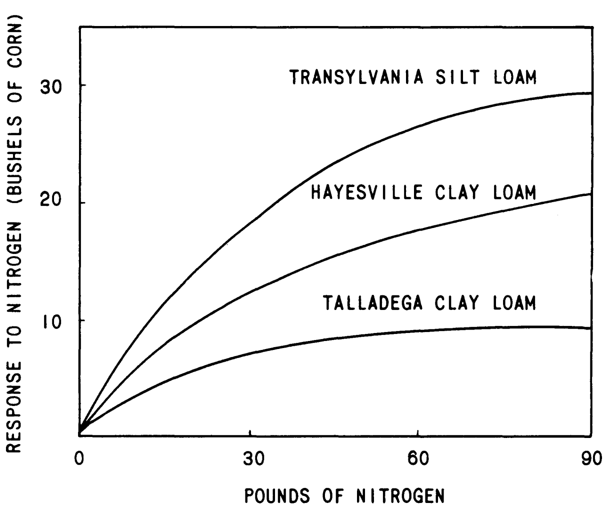

Fig. 11.1 — Corn yield response to nigrogen for three soil types. Source: Adapted from Ga. Exp. Sta. Bul. 264 (response estimated for Talladega clay loam).

Response to fertilizer also varies with the fertility level of a particular soil type. This situation is illustrated in figure 11.2 and table 11.1, for Hayesville clay loam of low, medium, and high fertility.[2] Using 60 pounds of P_2O_5 and 60 pounds K_2O and assumed prices of 15 cents per

[1] Expressions such as "most economical rate" or "most profitable rate" for farmers with unlimited capital denote that application which would result in highest net return per acre (where marginal cost equals marginal revenue).

[2] Quadratic square-root equations were computed for the yield data which included three consecutive crop seasons. Since the experiment did not include over 90 pounds of nitrogen, prediction beyond that level was impossible. Also there are some doubts about the reliability of the curve fitting for the medium fertility soil because of the limited range of the data.

TABLE 11.1. Corn Yield Response to Nitrogen on Hayesville Clay Loam When 60 Pounds P_2O_5 and 60 Pounds K_2O Are Applied per Acre

Nitrogen	Total Yield	Additional Corn from 10 Lbs. Nitrogen	Value Additional Corn $1.50 per Bu.	Cost Additional 10 Lbs. Nitrogen 15¢ Lb.
(Pounds)	(Bushels)		(Dollars)	
		Low Fertility		
0	12.4	-	-	-
10	26.5	14.1	21.15	1.50
20	31.0	4.5	6.78	1.50
30	34.0	3.0	4.42	1.50
40	36.1	2.1	3.20	1.50
50	37.7	1.6	2.42	1.50
60	39.0	1.2	1.86	1.50
70	40.0	1.0	1.43	1.50
80	40.7	.7	1.11	1.50
90	41.2	.6	.83	1.50
100	41.6	.4	.61	1.50
		Medium Fertility		
0	53.6	-	-	-
10	59.6	6.0	8.96	1.50
20	62.8	3.2	4.85	1.50
30	65.6	2.8	4.17	1.50
40	68.1	2.6	3.82	1.50
50	70.5	2.4	3.60	1.50
60	72.8	2.3	3.44	1.50
70	75.1	2.2	3.32	1.50
80	77.2	2.2	3.23	1.50
90	79.3	2.1	3.15	1.50
100	81.4	2.1	3.08	1.50
		High Fertility		
0	71.2	-	-	-
10	76.3	5.1	7.63	1.50
20	78.4	2.2	3.30	1.50
30	80.2	1.8	2.58	1.50
40	81.6	1.5	2.22	1.50
50	83.0	1.3	1.98	1.50
60	84.2	1.2	1.82	1.50
70	85.3	1.1	1.68	1.50
80	86.4	1.1	1.58	1.50
90	87.4	1.0	1.50	1.50
100	88.3	1.0	1.43	1.50

Source: Woodworth, R. C., and Brooks, O. L., Economics of fertilizer use on north Georgia farms, unpub. ms., Dept. Agr. Econ., University of Georgia, Athens, Ga.

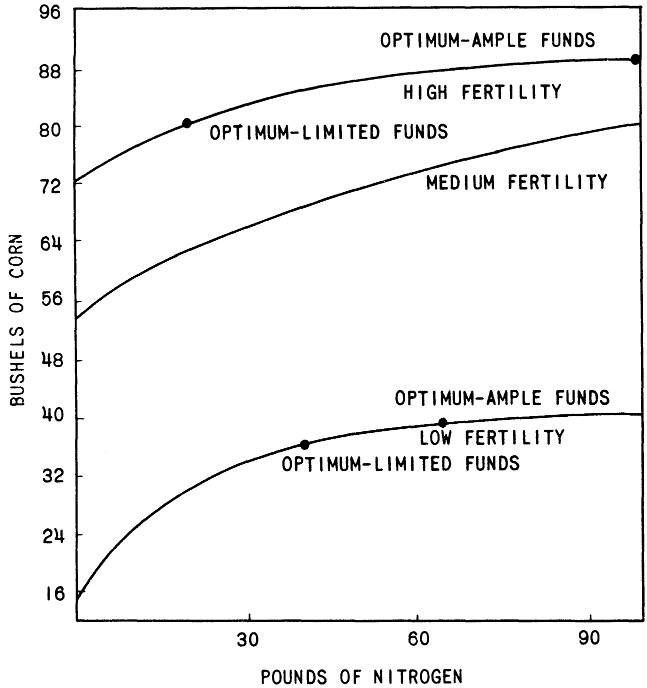

Fig. 11.2 — Corn yield response from nitrogen for three fertility situations on Hayesville clay loam. Source: Woodworth, R. C., and Brooks, O. L., Economics of fertilizer use on north Georgia farms. Unpubl. ms., Dept. Agr. Econ., University of Georgia, Athens.

pound of nitrogen and $1.50 per bushel of corn, 65 pounds of nitrogen is most profitable for the farmer with ample funds on low fertility land. For the medium fertility soil, the profitable level is apparently well over 90 pounds of nitrogen. For the high fertility soil, the profitable level is slightly over 90 pounds.

Using Fertilizer Under Limited Capital

Economic logic suggests that a farmer attempting to make the best use of limited resources should spend funds as follows: Use fertilizer until a point is reached where a greater return can be obtained by investment elsewhere in the business. Hence, the "right" amount of fertilizer to apply is less for a farmer short of capital, with many alternative productive investment opportunities, than for a farmer with ample capital. The farmer short on capital has great investment and consumption uses relative to his funds. Unfortunately, very little information is available to enable the farm operator to make wise decisions on the most efficient use of his limited capital. Should he apply three-fourths

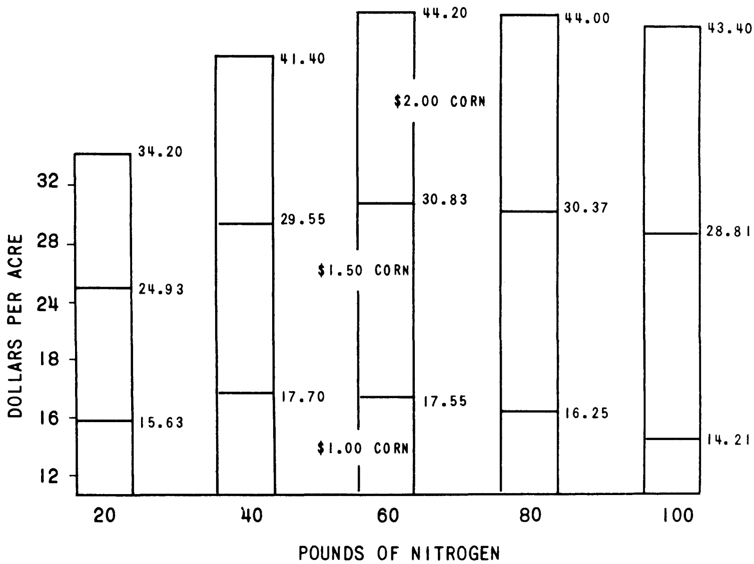

Fig. 11.3 — Returns above nitrogen cost from the use of nitrogen for corn on low fertility Hayesville clay loam. Source: Woodworth, R. C. and Brooks, O. L. Efficient use of fertilizer on north Georgia farms. Unpubl. ms., Dept. Agr. Econ., University of Georgia, Athens.

of the recommended fertilizer and purchase an extra brood sow or cow? or would some other division of investments increase his net farm income?

Using the illustration shown in figure 11.2 for a situation where a farmer has limited capital and needs a 2-dollar return for each dollar invested in nitrogen fertilizer, he should apply 40 pounds on the low fertility soil, or 25 pounds less than if he had ample funds; he should apply 20 pounds on the high fertility Hayesville clay loam, 80 pounds less than if he had ample funds.

Alternative price expectations cause the optimum nitrogen application to vary. On the low fertility land, corn priced at $2 would specify use of about 70 pounds of nitrogen; corn priced at $1 would specify 50 pounds. On the highly fertile land, over 100 pounds of nitrogen would have been economical with $2 corn; 36 pounds would be optimum for $1 corn. The graph in figure 11.3 shows returns above nitrogen costs on low fertility Hayesville clay loam when nitrogen is 15 cents and corn priced at $1.00, $1.50, and $2.00. In this case, 60 pounds of nitrogen would not miss maximum returns by enough to be termed important by many individuals, regardless of historical price relationships.

The situation is quite different for the farmer with limited capital, however. When the price of corn is low, he can apply 30 pounds of

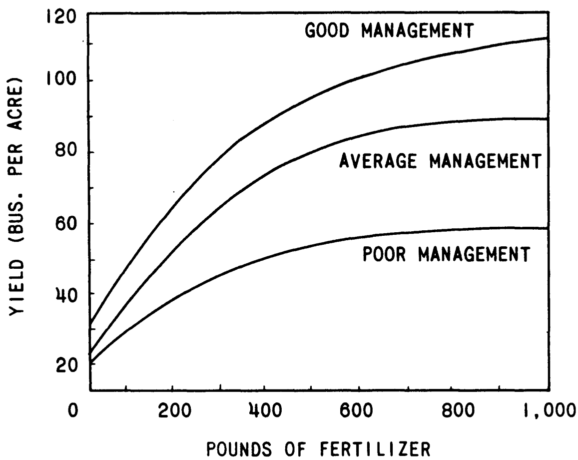

Fig. 11.4 — Effect of management on response to fertilizer. Source: Virginia Farm Economics, No. 141, Feb. 1955, p. 17.

nitrogen and sacrifice little income, compared with using 60 pounds. But when corn prices are high relative to nitrogen, he loses much more by not using 60 pounds. With $2.00 corn, he loses $4.20 per acre by not investing another $4.50; with $1.20 corn, he loses about $3.00; and with $1.00 corn he loses about 50 cents.

Management Levels and Risk

The extent to which general management levels are important in determining economic use of fertilizer has been explored by Plaxico and Loope (11) at Virginia. In their investigations (see figure 11.4), a superior manager with unlimited capital could use 800 pounds of fertilizer per acre under stated conditions; a poor manager could use only 500 pounds. Intangible as management measures are, fertilizer is apparently more productive under superior management which includes efficiency in timeliness of operations, choice of varieties, and other recommended cultural practices. Increased use of fertilizer is most effective on many farms only if improved cultural practices are used at the same time. Management considerations also involve adjustments to risk.

Risk Considerations

Very little information is available to show the farmer how much risk is involved in alternative fertilizer investments. Yet this, too, is part of the judgment a farmer undertakes when he decides how much fertilizer to use. The farmer short of capital is more likely to use less fertilizer because a one-in-ten chance of a $50 loss would be more serious than for a farmer with ample capital. Since many farmers borrow money for fertilizer, these risks must be considered in relation to "staying in the business."

Some indication of risk due to differences in seasons is shown in figure 11.5 for low fertility Hayesville clay loam. In two of the three years involved, 1947 and 1948, over 90 pounds of nitrogen would have been profitable for the farmer with ample funds. The other year, 1946, turned out to be a dry growing season. For this year, only about 35 pounds of nitrogen would have been economical. This situation is clearly recognized by many farmers. They would like to know the nature of risks involved in fertilizer use (see discussion in Chapter 1). In the absence of reliable information tailored to their needs, many farmers perhaps discount expected returns too severely.

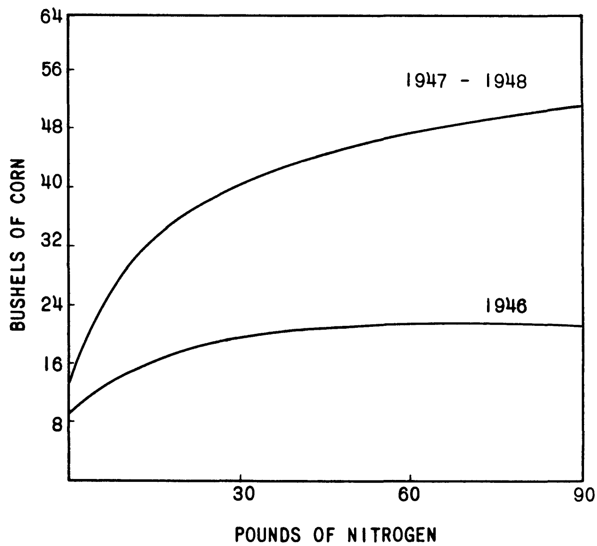

Fig. 11.5 — Corn yield response from nitrogen on low fertility Hayesville clay loam for different years. Source: Ga. Exp. Sta. Bul. 264.

Organizing Information for Planning

How can fertilizer input-output data be organized for greatest efficiency in farm planning? There is no one answer for all areas and all uses. Generally, fertilizer input-output data are needed for: (a) agricultural workers and farmers to illustrate economic principles in fertilizer use; (b) agricultural workers and farmers to assist in making specific decisions; (c) budgeting or linear programming in whole-farm planning; (d) other micro- or macro-economic analyses dealing with resource allocation in agriculture.

If greater progress is to be made with the "hard-to-reach" farmers, who do not come regularly to agricultural workers for help, more and better basic input-output estimates are needed. One should start with a particular farmer's situation in terms of resources, interests, and abilities. The farmer will have to be provided with the know-how to determine the consequences of various courses of action. He may need to solve his own problems so that he can make future decisions without assistance. This type of educational approach places heavy demands on research in various subject matter fields.

In teaching farmers some of the basic principles of fertilizer application, this approach is used. The farmer is informed that an investment in fertilizer is similar to any other investment. He is asked: "If you invest $2, how much can you expect to get back? How much risk is involved? If the farmer can invest $2 profitably after considering returns from other investments, why not invest a second $2, a third $2, and so forth?" The farmer is shown that the major difference between investing in fertilizer and in a savings account is that the rate of return for each additional $2 spent for fertilizer and the risk of loss will depend very definitely on how many 2-dollar units are invested.

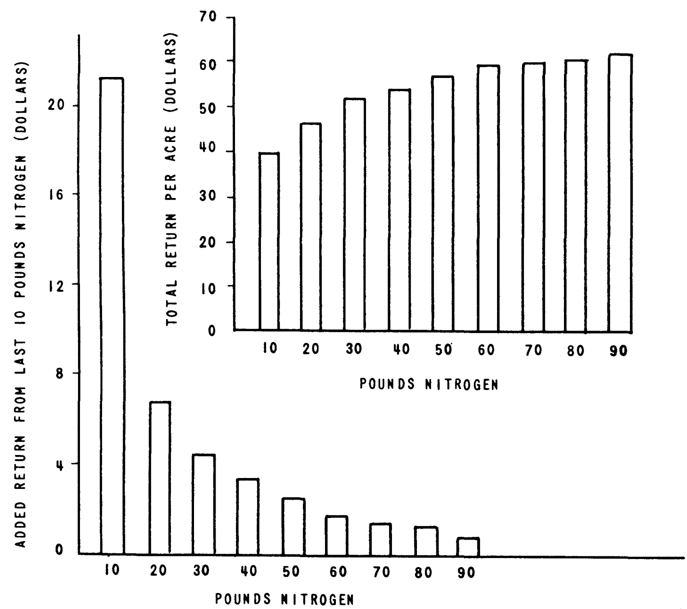

Fig. 11.6 — Returns from the use of nitrogen on corn for low fertility Hayesville clay loam. Source: Woodworth, R. C., and Brooks, O. L. Efficient use of fertilizer on north Georgia farms. Unpubl. ms., Dept. Agr. Econ., University of Georgia, Athens.

While research work along this line is conducted informally, it is felt that a large proportion of farmers can be effectively reached with this sort of logic. There is evidence of its effectiveness, because the typical response is, "But how can I tell when another $2 will not be profitable?" A chart such as that of figure 11.6 is highly effective in illustrating economic principles for farmers or agricultural workers.

Whole-Farm Business

Research and educational programs designed to promote agricultural development must deal with the whole-farm business as well as the parts that make up the whole. Acceptance of whole-farm planning techniques hinges on confidence in the integral parts. In linear programming, as in budgeting, the assumptions, the inputs and outputs, must be accepted as realistic if the conclusions are to be accepted and put into use. For use on specific farms, specific fertilizer input-output data are needed which will be realistic for the particular situation.

With the introduction of refined techniques of estimating input-output relationships under experimental conditions (6), and with an increasing need for these farm planning guides, greater attention should be given to problems of inference. A primary consideration is to determine the population of soil conditions for which particular estimates apply. How does one make the best use of limited research funds when attempting to provide information on different soils? Intensive and refined research conducted on a particular soil experiment provides maximum information for that particular field; but it provides only limited inference for other conditions. However, plot research funds usually are not sufficient to provide data from all soil-mapping unit conditions found in a particular area.

If a technique could be developed which would allow interpolation between soils and plot applications in predicting yield, experiments could have a greater range of applicability to different soils. From this standpoint, the logical starting point in assembling fertilizer input-output data would seem to be in the area of soil classification. Since traditional classification schemes were designed for other specific functions (such as erosion control or similarity in physical properties), additional considerations need to be given to schemes which are based directly on crop response. One such scheme was developed by Osgood (10) in Mississippi. Preliminary investigations in Georgia using a similar concept have indicated the possible usefulness and need for further research on ways and means of organizing soil mapping units for efficiency in farm planning.

Use of Plot Information for Farm Planning

The work in Mississippi indicated that, for a particular crop, soils could be arrayed according to ability to supply moisture (other variables may be associated with moisture), and this array could be used advantageously to specify soil differences in organizing and presenting input-output data for farm planning. The conceptual framework for this scheme is illustrated in figure 11.7 for corn and cotton. In this arrangement, the dependent variable is the array of soils from wet bottom and well-drained bottom to the good terrace and upland soils and to the poorer upland soils with erosion or drought hazards. "Benchmark" soil-mapping units can then be selected at strategic intervals across the range of soil conditions.

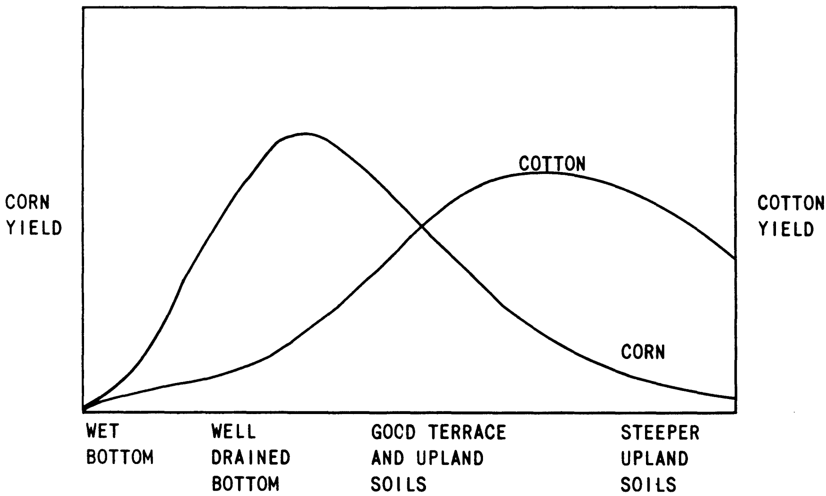

Fig. 11.7 — Relationship between different soils and yield of cotton and corn when fertilizer and other production factors are used in "optimum" amounts.

If this or a similar scheme would allow prediction of the effect of different soils on yield with a desired degree of reliability, research dollars would provide more information. Each plot experiment could be placed geographically to provide maximum efficiency in predicting results for a variety of soil conditions. One could interpolate between areas for soils where specific research has not been conducted.

Three major problems exist in making inferences to farms from experimental plots. First, present knowledge of soil differences is not sufficient to allow refined estimates of reliability for this scheme in predicting yield. Still, these differences may not be of sufficient magnitude to be important. For preliminary testing of this procedure for local soil conditions in Georgia, (a) soil mapping units were rated for suitability to produce specific crops, (b) mapping units were combined with similar ratings, and (c) these groups of soils were arrayed according to the "model" illustrated in figure 11.7. The results are presented in table 11.2 and are sufficiently encouraging to warrant further development.

TABLE 11.2. Soil Characteristics – Fred Nichols' Farm of 160.6 Acres in Catoosa County

Soil Series and Types	Lindside Silt Loam	Huntington	Pope	Dewey-Decatur	Clarksville	Clarksville	Decatur	Clarksville	Colbert
Topographic position	Wet bottom	Bottom	Draw	Upland	Upland	Upland	Upland	Upland	Upland
Slope	0-2	2-6	2-6	2-6	2-6	6-10	10-15	10-15	10-15
Acres	32.0	5.6	19.5	9.2	5.8	19.0	19.8	44.2	5.5
Drainage, surface	Poor	Good	Fair	Good	Good	Good	Good	Good	Poor
Entire profile	Poor	Good	Fair	Good	Good	Good	Good	Good	Poor
Erosion hazard	None	None	Slight	Slight	Slight	Moderate	Mod.-Severe	Mod.-Severe	Moderate
Drought hazard	Slight	Slight	Moderate	Moderate	Moderate	Moderate	Severe	Severe	Severe
Suitability for:[b]									
Corn	Fair[a]	Excellent	Very good	Very good	Good	Good	Fair	Fair	Very poor
Small grain	Poor	Excellent	Very good	Excellent	Very good	Good	Good	Fair	Very poor
Alfalfa	Not suited	Poor	Excellent	Very good	Very good	Very good	Good	Good	Very poor
Temporary pasture	Fair	Excellent	Excellent	Excellent	Very good	Good	Good	Good	Poor
Grass – Ladino	Excellent	Excellent	Excellent	Excellent	Excellent	Very good	Very good	Good	Fair
Cotton	Very poor	Poor	Fair	Very good	Good	Good	Fair	Fair	Poor

[a]Water hazard, assumes adequate control.
[b]Assumes adequate erosion control.

Source: Woodworth, R. C.; Perkins, H. F.; McIntyre, O. L., Dept. of Agr. Econ., Dept. Agron., University of Georgia, and Soil Scientist, SCS., respectively.

A second problem concerning inferences for farm planning purposes from experimental plot experiments is that of differences in management practices. If responses to fertilizer, when all factors but management are held constant, could be arrayed for a population of farm operators, presumably responses obtained on an experiment station would fall in the upper quartile of these; inferences drawn from the experiment would involve increasing error as applied to average and below-average managers (see fig. 11.4). Here survey data and soil-testing histories may be used to fill gaps in knowledge. Perhaps plot experiments and check-row data on case-study farms also can be an aid.

A third inference problem is associated with most current fertilizer input-output data. It stems from a lack of knowledge about the dynamic effects of fertilizer on soil fertility over a period of time. It also is mentioned in Chapter 1. Information of the effects of time on responses is needed if economic analyses are to be applied with a desired degree of confidence. Information is needed on economical rates of fertilizer application for a return in one crop year, two crop years, or three crop years to fit particular "time horizon" attitudes of farm operators in different capital positions.

TABLE 11.3. Response to Fertilizer Corn on Hayesville Clay Loam, Georgia, 6-10% Slope, Soil Test P_2O_5 Low, K_2O Medium

Fertilizer per Acre[a]			Range[b]	Average	Dry Year	Favorable Year	Years in 100 Yield Would be as Much or More Than (Bu. per acre)			
N	P	K	Bus./Acre	Bus./Acre	Bus./Acre	Bus./Acre	40	60	80	100
X	X	X	X-X	40	X	X	X	X	X	X
X	X	X	X-X	50	X	X	X	X	X	X
X	X	X	X-X	60	X	X	X	X	X	X
X	X	X	X-X	70	X	X	X	X	X	X
X	X	X	X-X	80	X	X	X	X	X	X
X	X	X	X-X	85	X	X	X	X	X	X

[a]Most efficient fertilizer combination to produce given yield for specific or average price ratios.
[b]Range in long-time average yield to reflect variations in management.

Table 11.3 indicates some of the basic types of information which are needed for farm planning. First, the data should refer to particular soil and fertility conditions. Second, alternative levels of fertilizer application should be indicated to assist in making decisions for particular capital situations and goals. Third, ranges of outcomes in expected long-time average yields (first and third quartile of a population of farm yields) are needed to assist in making judgments for management differences on particular farms. Fourth, expected yields for favorable and unfavorable seasons are needed. Fifth, a measure of relative frequency of expected given yields over time with average management should be included.

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Chapter 12

Selecting Fertilizer Programs by Activity Analysis

THE central notion of activity analysis or linear programming is that of an activity or a process. In the usual type of production planning problem employing activity analysis, an activity may be described in terms of its resource requirements and the product, or products, it generates. Thus in a farm planning problem, e.g., cattle feeding or haymaking are considered to be activities. The product of each activity is assumed to be a linear function of the resources used in its production. Although the assumption of linearity or fixed technical coefficients for a given process or activity may seem unduly restrictive, such is not the case. For example, activities may be added to correspond to different points on a production surface. Thus, feeding cattle to the same weight and quality by three different methods may represent three points on a conventional isoquant. These three methods may be represented by three separate activities. Likewise, several rates of fertilizer application may be treated as distinct corn-growing activities or processes. Lowering of per-unit labor requirements, as the size of an enterprise increases, can be treated in the same fashion.

The economic problem is, of course, one of selecting optimum activity levels. This is accomplished by maximizing or minimizing a linear criterion relation (4), usually some type of profits or costs, subject to such restraints as resource supplies, product requirements, and definitional restrictions that may be appropriate.

The particular application of activity analysis that is treated in this paper might best be termed farm planning or budgeting.[1] Activity analysis has an advantage over less formalized budgeting techniques in that it assures that there is no better organization possible within the set of restrictions considered. Activity analysis is more appropriate than conventional budgeting in relatively complex planning situations where it may be desirable to consider many alternative activities with a large number of restraints on profit maximization. Accordingly, activity analysis is more likely to be useful in selecting a fertilizer program when, for example, outlays for fertilizer must compete with many other

[1] For an application of activity analysis in problems of selecting a minimum-cost fertilizer to meet a given set of requirements at the farm level, cf., G. A. Peterson (6). Another application is that of mixing ingredients, cf., E. R. Swanson (9).

enterprises of the farm business for a limited amount of capital. On the other hand, farm problem situations with only a single crop and few alternative methods of production may best be handled by conventional budgeting.

The model that is to be presented in this chapter, along with the results, should be viewed only as an illustration of the technique. Thus, the model serves chiefly as an expositional device to suggest the usefulness and the limitations of activity analysis in selecting fertilizer programs. It will be noted that several of the properties of the model are not as realistic as desirable, or perhaps even presently possible. Some of these simplifying assumptions are due to the inadequacy of data while others are made for computational convenience. However, it is not believed that these characteristics of the model seriously interfere with our central purpose, i.e., to indicate the general methodology of selecting fertilizer programs by activity analysis.

Farm Situation

An owner-operator is located on 200 acres of Muscatine silt loam, a highly productive level soil found in Illinois and Iowa. He is interested in selecting a farm plan starting in 1955. He presently has no livestock. Soil tests indicate that potash is adequate, but that three tons of lime per acre is needed for satisfactory legume stands and that the available phosphate is low (5). Barn space is available for feeding cattle, but equipment will be needed. Hog houses and equipment will need to be purchased if hogs are to be included in the farm plan. Adequate power and machinery investment have already been made for this acreage. Three hundred hours of labor per month is assumed to be available during peak labor periods.

Activities Considered

Cropping Systems

Six cropping systems are considered as separate activities. Cropping systems are defined as a combination of a rotation (2, 7), or crop sequence, and a fertilizer plan. Thus we attempt to account for the interdependence of the rotation and the fertilizer treatments. Two alternative rotations are considered: (a) three-year rotation of corn-corn-oats (clover catch crop), and (b) four-year rotation of corn-corn-oats-clover. For each of these two rotations three different fertilizer plans are considered, thus giving the total of six cropping systems. The fertilizer plans presented in table 12.1 are based on the "build-up" and "maintenance" concepts of a soil fertility program (8).[2] Thus, it can be seen that the various plans for a given rotation constitute points on

[2]The yield estimates and fertilizer requirements were furnished to the author by E. H. Tyner, Department of Agronomy, University of Illinois.

iso-yield curves for each year. Ideally, other yield levels should be included as alternatives. Adequate experimental data are not presently available to estimate other yield levels as alternatives. Although the analysis was simplified by using one yield level, additional activities for other yield levels could have been employed had the data been available.

All three of the fertilizer plans are similar in their limestone and nitrogen applications for a given rotation during the "build-up" phase of the program. They differ in the type of phosphate materials and annual rates of application of the phosphate materials. Plan A of fertilizer treatment relies solely upon superphosphate as a source of phosphate; the "build-up" is accomplished very slowly; seven years is required for the three-year rotation, and nine years is required for the four-year rotation. However, initial outlays for capital are not as great as for the other two fertilizer plans.

Plan B uses rock phosphate as the sole source of phosphate. Large applications are made in the initial years. These supplies are depleted down to the level at which a "maintenance" program may be initiated after seven years in the case of both rotations. In order to make the plans comparable in terms of ending in the same year, a "maintenance" program of superphosphate was started in the eighth year of Plan B. This procedure avoided the problem of placing values on the unexpended rock phosphate that would normally have been applied that year. Hence, all plans have the same asset valuation at the end of the nine-year period.

Plan C, the third alternative fertilizer plan, consists of a combination of rock phosphate and superphosphate. Under this scheme the "build-up" program is completed in four years for the three-year rotation, and after six years for the four-year rotation. "Maintenance" requirements are the same as for the other plans for their respective rotations. Note in table 12.1 that in the initial years of each plan relatively heavy applications of nitrogen are made. This is done to bring the yields up to the specified level prior to procurement of full legume nitrogen effects.

Livestock Enterprises

Only two livestock enterprises are considered as alternatives: (a) a two-litter hog system and (b) a feeder-cattle enterprise. Thus, opportunity is provided for competition from livestock for limited resources to affect the fertilizer plan adopted. Further, the inclusion of livestock provides a means for implicity solving the question of whether legumes are a cheaper source of nitrogen than commercial fertilizers. Taking this problem out of the context of the farm business may prove to be misleading because of the joint-product characteristics of legume production. The appropriate value of legume roughage produced must include consideration of its marginal value productivity in livestock production. In activity analysis, the problem of placing explicit values on such intermediate inputs may be by-passed and the "built-in" pricing mechanism may be relied upon to yield the optimal total farm plan. Finally, another relation between livestock and fertilizer programs is that

TABLE 12.1. Lime and Fertilizer Requirements for "Build-Up" and "Maintenance" (Pounds per Rotation Acre) Muscatine Silt Loam, Starting with "Low" Phosphate Test, a 3-Ton Limestone Requirement, and Adequate Potassium (Treatments for Each Specific Crop Appear in the Appendix

Year	Materials	Rotation					
		Corn-Corn-Oats (Clover)			Corn-Corn-Oats-Clover		
		(Plan A)	(Plan B)	(Plan C)	(Plan A)	(Plan B)	(Plan C)
1955	Limestone	2,000	2,000	2,000	1,500	1,500	1,500
	Rock phosphate	-	1,300	333.3	-	975	250
	0-20-0	300	-	133.3	250	-	150
	N	80	80	80	50	50	50
1956	Limestone	2,000	2,000	2,000	1,500	1,500	1,500
	Rock phosphate	-	-	333.3	-	325	250
	0-20-0	300	-	116.7	250	-	125
	N	73.3	73.3	73.3	50	50	50
1957	Limestone	2,000	2,000	2,000	1,500	1,500	1,500
	Rock phosphate	-	-	333.3	-	-	250
	0-20-0	300	-	100	250	-	87.5
	N	70	70	70	45	45	45
1958	Limestone	-	-	-	1,500	1,500	1,500
	Rock phosphate	-	-	-	-	-	250
	0-20-0	300	-	100	-	-	87.5
	N	70	70	70	45	45	45
1959	Limestone	-	-	-[a]	-	-	-
	Rock phosphate	-	-	-	-	-	-
	0-20-0	300	-	130	250	-	150
	N	63.3	63.3	39	37.5	37.5	37.5
1960	Limestone	-	-	-	-	-	-
	Rock phosphate	-	-	-	-	-	-
	0-20-0	300	-	130	250	-	127.5
	N	56.7	56.7	39	37.5	37.5	20.5
1961	Limestone	-	-	-	-	-	-[a]
	Rock phosphate	-	-	-	-	-	-
	0-20-0	133.3	-	130	250	-	105
	N	56.7	56.7	39	37.5	37.5	14
1962	Limestone	-[a]	-[a]	-	-	-[a]	-
	Rock phosphate	-	-	-	-	-	-
	0-20-0	130	130	130	250	105	105
	N	39	39	39	37.5	14	14
1963	Limestone	-	-	-	-	-	-
	Rock phosphate	-	-	-	-	-	-
	0-20-0	130	130	130	125	105	105
	N	39	39	39	37.5	14	14

[a]Indicates first year that all fields under a given cropping system are on "maintenance" program.

of the effect of manure. In the illustration presented in this chapter, capital outlays are reduced in the succeeding year by the value of the manure produced in any given year.

Other Activities

Hay-making activities are employed to process pasture into hay. In addition, corn-buying and corn-selling activities are introduced to permit both purchase and sale of corn. Since the model embraces several time periods, it is necessary to make some provision for transfer of income from one period to a later one. This transfer is accomplished by what might be called "saving" activities. The levels of the saving activities represent the dollars of income in each year (1955 through 1963) above a given amount specified for fixed costs and household withdrawals. This particular model does not permit transfer of income to any period beyond the one immediately following its generation. More complex assumptions could be made at the expense of added computational burden.

Criterion for Selection of Farm Plan

The criterion for selecting a farm plan for a period of time requires a dating of each outlay and income. Accordingly, cash outlays are assumed to be made on January 1 of each year, and the income from production during that year is assumed to occur on December 31. After specifications of the dates of the outlays, incomes, and given price expectations[3] (including the interest rate, r), the present value of a stream of net income over the relevant horizon is constructed as the criterion equation. In this case the present value (P V) of the plan is maximized and is denoted for a nine-year horizon as follows:

$$\text{(1)} \qquad P\,V = \sum_t \sum_i P_{it}\, x_1 \frac{1}{(1 + r_t)t} \qquad \begin{array}{l} t = 0, 1, 2, 3 \ldots 9 \\ i = 1, 2, 3 \ldots 13 \end{array}$$

The 13 activities or processes entering into the criterion equation are as follows:

x_1 = acres of C-C-O (C1) with superphosphate build-up program (Plan A)

[3]Price expectations for all periods were assumed to be at the following levels: corn, \$1.48 per bushel; oats, \$0.74 per bushel; March hogs, \$19.25 per cwt.; September hogs, \$21.00 per cwt.; feeder calves, \$23.00 per cwt.; choice steers, \$26.10 per cwt.; rock phosphate, \$24.00 per ton (spread); limestone, \$4.50 per ton (spread); 33-0-0, \$104.00 per ton (spread); 0-20-0, \$46.00 per ton (spread); protein supplement, \$88.00 per ton; cash costs other than fertilizer (fuel, repairs, seed, etc.): for three-year rotation, \$8.60 per acre; for four-year rotation, \$7.85 per acre; cash costs for livestock (equipment, protein feed, veterinary and medicine, original outlay for gilts, annual purchase of feeder calves): hogs, \$13 for the first year and \$4.50 for each of the subsequent years; cattle \$105 for the first year and \$100 for each of the subsequent years; manure credits: hogs, \$3.80 per animal; cattle, \$16.70 per animal.

x_2 = acres of C-C-O (C1) with rock phosphate build-up program (Plan B)

x_3 = acres of C-C-O (C1) with combination superphosphate and rock phosphate program (Plan C)

x_4 = acres of C-C-O-C1 with superphosphate build-up program (Plan A)

x_5 = acres of C-C-O-C1 with rock phosphate build-up program (Plan B)

x_6 = acres of C-C-O-C1 with combination superphosphate and rock phosphate program (Plan C)

x_7 = number of hogs produced

x_8 = number of good-to-choice feeder calves fed

x_9 = tons of hay produced in period May 15-June 14

x_{10} = tons of hay produced in period July 15-August 14

x_{11} = tons of hay produced in period August 15-September 14

x_{12} = bushels of corn equivalent sold

x_{13} = bushels of corn equivalent purchased.

Restraints

Capital

The restraints on maximization of the present value of the total farm plan is considered. Initial capital available for cash outlays will constitute the first restraint. In the solutions presented, this quantity will vary to note the effect on optimum farm organization. Since the focus is on the time shape of capital outlays for various fertilizer programs, the annual cash outlay for lime and fertilizer for each of the six cropping systems is presented in table 12.2. Expenses for the livestock enterprises are presented in a previous footnote referring to prices. Letting α_{1i} be the 1955 capital requirements of the various activities, the first restraint is:

(2) $$\sum_i \alpha_{1i}\, x_i \leqslant \text{capital available January 1, 1955.}$$

In order to make income available for investments in subsequent periods, the savings activities are utilized as previously mentioned. The levels of these activities are denoted as x_{14} through x_{21}. Writing these capital requirements and supply relationships for the year 1956 through 1963, the second through the ninth restraints are:

(3) $$\sum_i \alpha_{2i} x_i - \beta_{2,14} x_{14} \leqslant 0$$

(4) $$\sum_i \alpha_{3i} x_i - \beta_{3,15} x_{15} \leqslant 0$$

. . . .

. . . .

. . . .

(10) $$\sum_i \alpha_{9i}x_i - \beta_{9,21}x_{21} \leqslant 0.$$

The α's refer to the capital requirements of each activity in each year. The fraction of income above a specified level available for spending by the firm in the following year is designated as β. An arbitrary value of 0.6 was selected for β.

TABLE 12.2 Total Capital Outlays for Lime and Fertilizer Required for Various Phosphate "Build-Up" Programs (per Rotation Acre)

	Rotation					
	Corn-Corn-Oats (Clover)			Corn-Corn-Oats-Clover		
Year	Plan A	Plan B	Plan C	Plan A	Plan B	Plan C
	(In Dollars)					
1955	23.10	32.60	24.00	16.90	22.90	17.60
1956	22.10	15.90	22.60	16.90	15.10	17.10
1957	21.60	15.40	21.70	16.15	7.00	15.40
1958	17.00	10.90	14.00	15.00	5.90	14.00
1959	16.00	9.90	8.90[a]	11.60	5.90	9.30
1960	15.00	8.90	8.90	11.60	5.90	5.90
1961	12.00	8.90	8.90	11.60	5.90	4.60[a]
1962	8.90[a]	8.90[a]	8.90	11.60	4.60[a]	4.60
1963	8.90	8.90	8.90	9.30	4.60	4.60

[a]Denotes first year all fields under a given cropping system are on a "maintenance" program.

Income

The minimum level of income required in any year is specified in the next set of restraints. These restrictions may be viewed in terms of a further description of the time shape preference of the income stream. A minimum requirement for fixed costs and household withdrawals of 4,000 dollars was specified for this problem. These restraints were handled as equalities:

(11) $$\sum_i \alpha_{10i}x_i - x_{14} = \$4{,}000 \text{ (1955)}$$

(12) $$\sum_i \alpha_{11i}x_i - x_{15} = \$4{,}000 \text{ (1956)}$$

. . . .

. . . .

$$(18) \qquad \sum_i \alpha_{17i} x_i - x_{21} = \$4{,}000 \text{ (1962)}$$

α's indicate dollars of income per unit of activity level, and activities 14 through 21 act to transfer income to their respective subsequent years.

Labor

In addition to the capital and income restrictions on the choice of a high-profit program, other resources were considered. Labor in three periods during the year was considered fixed at 300 hours. These were considered to be periods when the labor supply would be critical and also when the hay-making operation would be performed. The crop and livestock labor requirements were secured from *Illinois Farm and Home Development Reference Book* (3).

$$(19) \qquad \sum_i \alpha'_{18i} x_i \leqslant 300 \text{ May 15-June 14}$$

$$(20) \qquad \sum_i \alpha_{19i} x_i \leqslant 300 \text{ July 15-August 14}$$

$$(21) \qquad \sum_i \alpha_{20i} x_i \leqslant 300 \text{ August 15-September 15}$$

Land

The land available for cultivation also needs to be added as a restraint

$$(22) \qquad \sum_i \alpha_{21i} x_i \leqslant 200$$

where $\alpha = 1$ for each of the six cropping systems and $\alpha = 0$ for other activities.

Grain and Hay

A grain relation is also added to permit feeding or sale of grain produced as well as purchase of grain. No beginning inventories are assumed, hence:

$$(23) \qquad \sum_i \alpha_{22i} x_i = 0$$

where the α's indicate the production, consumption, purchase, and sale coefficients of their respective activities.

A set of three relations concerning pasture is specified which permits the direct consumption of pasture by animals or its transformation into the intermediate product of hay. Again, no beginning inventories are assumed; however, surplus pasture days are permitted, i.e., more pasture days produced than consumed.

(24) $$\sum_i \alpha_{23i} x_i \geqslant 0$$

(25) $$\sum_i \alpha_{24i} x_i \geqslant 0$$

(26) $$\sum_i \alpha_{25i} x_i \geqslant 0$$

Finally, a relation is specified which accounts for the hay production in each of the three hay-making periods and its consumption by the two classes of livestock. No hay is assumed to be available at the beginning of the period and none will be produced unless it is necessary for consumption by the livestock enterprises.

(27) $$\sum_i \alpha_{26i} x_i = 0$$

Results

Proceeding directly to the results (table 12.3) of maximizing equation 1 subject to the relations 2 through 27, it is noted that the fertilizer programs selected depend on the levels at which some of the assigned constants are arbitrarily set. The method of computation may be found in Charnes, et al. (1). Situations I through IV show the effect of various amounts of initial capital on the optimum farm plan. A nine-year horizon is considered in the discounting procedure in order to make the various fertilizer plans comparable without placing values on unexpended nutrients. This does not mean that the farmer is committed to a single plan once it is adopted for 1955. It merely means that the 1955 plans are dependent upon expectations of incomes and outlays in the eight succeeding years as well as the expectations for 1955.

A discount rate, r, of 5 percent is used in the first four situations. As the starting capital available for cash outlays decreases from $13,000 (Situation I), to $11,000 (Situation II), a shift is made from cropping systems employing rock phosphates as the sole source (Plan B) to a combination rock phosphate-superphosphate plan (Plan C). Also, hog numbers decrease and a shift is made toward more acres in the three-year rotation as beginning capital decreases. The effective restraints (i.e., limiting resources) level of $13,000 is land and labor. In the remaining situations, starting capital and land are the effective restrictions. Thus, surplus capital for cash outlays in 1955 exists only in Situation I.

This general pattern of increased acres in catch crops and decreased livestock numbers continues as beginning capital decreases (Situations III and IV). Only 34 hogs remain in the farm plan in the $7,000 beginning capital situations. Obviously, these plans would need a certain degree of adjustment in their adaptation to a specific farm situation. For example, small fields in a rotation may not be feasible. Additional restraints on profit maximization may be imposed to prevent more than one rotation from being chosen. Such restraints were not considered in this problem.

In Situation V the discount rate is changed from Situation IV. A

TABLE 12.3. Optimum Cropping and Livestock Systems for Various Situations on a 200-Acre Muscatine Silt Loam Farm

Situation	Available Capital Jan. 1, 1955	Relevant Horizon	Discount Rate	Cropping System		Hogs	Beef Cattle
	(Dollars)	(Years)	(Percent)	(Rotation and Fertilizer Plan)	(Acres)	(Head)	(Head)
I	13,000	9	5	C-C-O-(Plan B)	116	395	4
				C-C-O-Cl-(Plan B)	84		
II	11,000	9	5	C-C-O(Plan C)	137	376	-
				C-C-O-Cl-(Plan C)	63		
III	9,000	9	5	C-C-O-(Plan C)	174	173	-
				C-C-O-Cl-(Plan C)	26		
IV	7,000	9	5	C-C-O-(Plan C)	195	34	-
				C-C-O-Cl-(Plan C)	5		
V	7,000	9	20	C-C-O-(Plan C)	195	34	-
				C-C-O-Cl-(Plan C)	5		
VI	7,000	9	5	C-C-O-(Plan B)	50	(no livestock	
				C-C-O-(Plan C)	150	considered)	
VII	9,000	2	5	C-C-O-(Plan A)	168		
				C-C-O-Cl-(Plan A)	32	217	-

change from 5 percent to 20 percent does not alter the optimum organization. Outlays in the original year still appear to dominate the program selection at this level of beginning capital ($7,000).

It is also of interest to investigate the effect of considering no livestock alternatives. Accordingly, in Situation VI livestock are not considered. With beginning capital at $7,000, the optimum plan calls for the three-year rotation on the total acreage and a mixed fertilizer plan. Had sufficient capital been available in this cash-grain situation, the cropping system of the three-year rotation and fertilizer Plan B would have been selected. This would have required about $8,200. Since only the $7,000 was available, Plan C (combination rock phosphate-superphosphate) was selected for three-fourths of the farm. An alternative plan lying between B and C might have been chosen, had such a plan been considered.

Finally it is of interest to examine the effect of a shorter horizon. Accordingly, if the relevant period is reduced from nine years to two years, Plan A, or the straight superphosphate program, appears to be the optimum solution. It should be mentioned that no account was taken of the differences among plans in the valuation of soil nutrient assets in considering the two-year horizon. It will be recalled that in the nine-year horizon the levels of fertility for each cropping system were assumed to be identical at the end of the nine-year period.

12A.1. Field Treatments per Acre for Various Cropping Systems

Year	Materials	Corn-Corn-Oats (Clover) with Fertilizer Plan A		
		Field 1	Field 2	Field 3
		Oats	Corn	Corn
1955	Limestone	3 T	-	-
	Rock phosphate	-	-	-
	0-20-0	300 lbs.	300 lbs.	300 lbs.
	Elemental N	40 lbs.	100 lbs.	100 lbs.
		Corn	Oats	Corn
1956	Limestone	-	3 T	-
	Rock phosphate	-	-	-
	0-20-0	300 lbs.	300 lbs.	300 lbs.
	Elemental N	80 lbs.	40 lbs.	100 lbs.
		Corn	Corn	Oats
1957	Limestone	-	-	3 T
	Rock phosphate	-	-	-
	0-20-0	300 lbs.	300 lbs.	300 lbs.
	Elemental N	100 lbs.	80 lbs.	30 lbs.
		Oats	Corn	Corn
1958	Limestone	-	-	-
	Rock phosphate	-	-	-
	0-20-0	300 lbs.	300 lbs.	300 lbs.
	Elemental N	30 lbs.	100 lbs.	80 lbs.
		Corn	Oats	Corn
1959	Limestone	-	-	-
	Rock phosphate	-	-	-
	0-20-0	300 lbs.	300 lbs.	300 lbs.
	Elemental N	60 lbs.	30 lbs.	100 lbs.
		Corn	Corn	Oats
1960	Limestone	-	-	-
	Rock phosphate	-	-	-
	0-20-0	300 lbs.	300 lbs.	300 lbs.
	Elemental N	80 lbs.	60 lbs.	30 lbs.
		Oats	Corn	Corn
1961	Limestone	-	-	-
	Rock phosphate	-	-	-
	0-20-0	100 lbs.	150 lbs.	150 lbs.
	Elemental N	30 lbs.	80 lbs.	60 lbs.

1962 Maintenance program with following requirements per rotation acre: Elemental N 39 lbs. 0-20-0 130 lbs.

12A.2. Field Treatments per Acre for Various Cropping Systems

Year	Materials	Corn-Corn-Oats (Clover) with Fertilizer Plan B Field 1	Field 2	Field 3
		Oats	Corn	Corn
1955	Limestone	3 T	-	-
	Rock phosphate	1,300 lbs.	1,300 lbs.	1,300 lbs.
	0-20-0	-	-	-
	Elemental N	40 lbs.	100 lbs.	100 lbs.
		Corn	Oats	Corn
1956	Limestone	-	3 T	-
	Rock phosphate	-	-	-
	0-20-0	-	-	-
	Elemental N	80 lbs.	40 lbs.	100 lbs.
		Corn	Corn	Oats
1957	Limestone	-	-	3 T
	Rock phosphate	-	-	-
	0-20-0	-	-	-
	Elemental N	100 lbs.	80 lbs.	30 lbs.
		Oats	Corn	Corn
1958	Limestone	-	-	-
	Rock phosphate	-	-	-
	0-20-0	-	-	-
	Elemental N	30 lbs.	100 lbs.	80 lbs.
		Corn	Oats	Corn
1959	Limestone	-	-	-
	Rock phosphate	-	-	-
	0-20-0	-	-	-
	Elemental N	60 lbs.	30 lbs.	100 lbs.
		Corn	Corn	Oats
1960	Limestone	-	-	-
	Rock phosphate	-	-	-
	0-20-0	-	-	-
	Elemental N	80 lbs.	60 lbs.	30 lbs.
		Oats	Corn	Corn
1961	Limestone	-	-	-
	Rock phosphate	-	-	-
	0-20-0	-	-	-
	Elemental N	30 lbs.	80 lbs.	60 lbs.

1962 Maintenance program with following requirements per rotation acre:
Elemental N 39 lbs. 0-20-0 130 lbs.

12A.3. Field Treatments per Acre for Various Cropping Systems

Year	Materials	Corn-Corn-Oats (Clover) with Fertilizer Plan C Field 1	Field 2	Field 3
		Oats	Corn	Corn
1955	Limestone	3 T	-	-
	Rock phosphate	1,000 lbs.	-	-
	0-20-0	-	200 lbs.	200 lbs.
	Elemental N	40 lbs.	100 lbs.	100 lbs.
		Corn	Oats	Corn
1956	Limestone	-	3 T	-
	Rock phosphate	-	1,000 lbs.	-
	0-20-0	150 lbs.	-	200 lbs.
	Elemental N	80 lbs.	40 lbs.	100 lbs.
		Corn	Corn	Oats
1957	Limestone	-	-	3 T
	Rock phosphate	-	-	1,000 lbs.
	0-20-0	150 lbs.	150 lbs.	-
	Elemental N	100 lbs.	80 lbs.	30 lbs.
		Oats	Corn	Corn
1958	Limestone	-	-	-
	Rock phosphate	-	-	-
	0-20-0	100 lbs.	150 lbs.	150 lbs.
	Elemental N	30 lbs.	100 lbs.	80 lbs.

1959 Maintenance program with following requirements per rotation acre:
Elemental N - 39 lbs.; 0-20-0 - 130 lbs.

12A.4. Field Treatments per Acre for Various Cropping Systems

Year	Materials	Corn-Corn-Oats-Clover with Fertilizer Plan A			
		Field 1	Field 2	Field 3	Field 4
		Corn	Corn	Oats	Clover
1955	Limestone	-	-	3 T	-
	Rock phosphate	-	-	-	-
	0-20-0	300 lbs.	300 lbs.	400 lbs.	-
	Elemental N	60 lbs.	100 lbs.	40 lbs.	-
		Corn	Oats	Clover	Corn
1956	Limestone	-	3 T	-	-
	Rock phosphate	-	-	-	-
	0-20-0	300 lbs.	400 lbs.	-	300 lbs.
	Elemental N	100 lbs.	40 lbs.	-	60 lbs.
		Oats	Clover	Corn	Corn
1957	Limestone	3 T	-	-	-
	Rock phosphate	-	-	-	-
	0-20-0	400 lbs.	-	300 lbs.	300 lbs.
	Elemental N	40 lbs.	-	40 lbs.	100 lbs.
		Clover	Corn	Corn	Oats
1958	Limestone	-	-	-	3 T
	Rock phosphate	-	-	-	-
	0-20-0	-	300 lbs.	300 lbs.	400 lbs.
	Elemental N	-	40 lbs.	80 lbs.	30 lbs.
		Corn	Corn	Oats	Clover
1959	Limestone	-	-	-	-
	Rock phosphate	-	-	-	-
	0-20-0	300 lbs.	300 lbs.	400 lbs.	-
	Elemental N	40 lbs.	80 lbs.	30 lbs.	-
		Corn	Oats	Clover	Corn
1960	Limestone	-	-	-	-
	Rock phosphate	-	-	-	-
	0-20-0	300 lbs.	400 lbs.	-	300 lbs.
	Elemental N	80 lbs.	30 lbs.	-	40 lbs.
		Oats	Clover	Corn	Corn
1961	Limestone	-	-	-	-
	Rock phosphate	-	-	-	-
	0-20-0	400 lbs.	-	300 lbs.	300 lbs.
	Elemental N	30 lbs.	-	40 lbs.	80 lbs.
		Clover	Corn	Corn	Oats
1962	Limestone	-	-	-	-
	Rock phosphate	-	-	-	-
	0-20-0	-	300 lbs.	300 lbs.	400 lbs.
	Elemental N	-	40 lbs.	80 lbs.	30 lbs.
		Corn	Corn	Oats	Clover
1963	Limestone	-	-	-	-
	Rock phosphate	-	-	-	-
	0-20-0	150 lbs.	150 lbs.	300 lbs.	-
	Elemental N	40 lbs.	80 lbs.	30 lbs.	-

1964 Maintenance program with following requirements per rotation acre:
Elemental N - 14 lbs; 0-20-0 - 105 lbs.

12A.5. Field Treatments per Acre for Various Cropping Systems

Year	Materials	Corn-Corn-Oats-Clover with Fertilizer Plan B			
		Field 1	Field 2	Field 3	Field 4
		Corn	Corn	Oats	Clover
1955	Limestone	-	-	3 T	-
	Rock phosphate	1,300 lbs.	1,300 lbs.	1,300 lbs.	-
	0-20-0	-	-	-	-
	Elemental N	60 lbs.	100 lbs.	40 lbs.	-
		Corn	Oats	Clover	Corn
1956	Limestone	-	3 T	-	-
	Rock phosphate	-	-	-	1,300 lbs.
	0-20-0	-	-	-	-
	Elemental N	100 lbs.	40 lbs.	-	60 lbs.
		Oats	Clover	Corn	Corn
1957	Limestone	3 T	-	-	-
	Rock phosphate	-	-	-	-
	0-20-0	-	-	-	-
	Elemental N	40 lbs.	-	40 lbs.	100 lbs.
		Clover	Corn	Corn	Oats
1958	Limestone	-	-	-	3 T
	Rock phosphate	-	-	-	-
	0-20-0	-	-	-	-
	Elemental N	-	40 lbs.	80 lbs.	30 lbs.
		Corn	Corn	Oats	Clover
1959	Limestone	-	-	-	-
	Rock phosphate	-	-	-	-
	0-20-0	-	-	-	-
	Elemental N	40 lbs.	80 lbs.	30 lbs.	-
		Corn	Oats	Clover	Corn
1960	Limestone	-	-	-	-
	Rock phosphate	-	-	-	-
	0-20-0	-	-	-	-
	Elemental N	80 lbs.	30 lbs.	-	40 lbs.
		Oats	Clover	Corn	Corn
1961	Limestone	-	-	-	-
	Rock phosphate	-	-	-	-
	0-20-0	-	-	-	-
	Elemental N	30 lbs.	-	40 lbs.	80 lbs.

1962 Maintenance program with following requirements per rotation acre:
Elemental N - 14 lbs.; 0-20-0 - 105 lbs.

12A.6. Field Treatments per Acre for Various Cropping Systems

Year	Materials	Corn-Corn-Oats-Clover with Fertilizer Plan C			
		Field 1	Field 2	Field 3	Field 4
		Corn	Corn	Oats	Clover
1955	Limestone	-	-	3 T	-
	Rock phosphate	-	-	1,000 lbs.	-
	0-20-0	300 lbs.	300 lbs.	-	-
	Elemental N	60 lbs.	100 lbs.	40 lbs.	-
		Corn	Oats	Clover	Corn
1956	Limestone	-	3 T	-	-
	Rock phosphate	-	1,000 lbs.	-	-
	0-20-0	200 lbs.	-	-	300 lbs.
	Elemental N	100 lbs.	40 lbs.	-	60 lbs.
		Oats	Clover	Corn	Corn
1957	Limestone	3 T	-	-	-
	Rock phosphate	1,000 lbs.	-	-	-
	0-20-0	-	-	150 lbs.	200 lbs.
	Elemental N	40 lbs.	-	40 lbs.	100 lbs.
		Clover	Corn	Corn	Oats
1958	Limestone	-	-	-	3 T
	Rock phosphate	-	-	-	1,000 lbs.
	0-20-0	-	150 lbs.	150 lbs.	-
	Elemental N	-	40 lbs.	80 lbs.	30 lbs.
		Corn	Corn	Oats	Clover
1959	Limestone	-	-	-	-
	Rock phosphate	-	-	-	-
	0-20-0	150 lbs.	150 lbs.	300 lbs.	-
	Elemental N	40 lbs.	80 lbs.	30 lbs.	-

1960 Maintenance program on Fields 1, 2, and 3 with following requirements per rotation acres: Elemental N - 14 lbs.; 0-20-0 - 105 lbs.
Field 4: Elemental N - 40 lbs.; 0-20-0 - 150 lbs.
1961 Maintenance program as above on all fields

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Chapter 13

Fertilization in Relation to Conservation Farming and Allocation of Resources Within the Farm

FERTILIZER has long been a resource of particular interest to agronomists and economists. Some of the better known historical literature has revolved around this resource, as a single category of input. Examples of classical studies include agronomic phases such as Liebig's law of the soil or law of the minimum and work by other early soil chemists such as Woolny and Mayer. Also included are economic phases, such as Ricardo's evaluation of rent; von Thunen's discussion of intensity; and Spillman's further development of the principle of diminishing returns. Most classical studies have tended to treat fertilizer and fertilizer use as a resource and practice apart from other resources and practices. While less prevalent than 100 years ago, this tendency still exists.

There are unique reasons why fertilization can be, and tends to be, treated as an isolated practice. From the standpoint of classical economics, fertilization fits remarkably well into conceptual frameworks including variable proportions, marginal analysis, fixity of particular resources, and others. This particular resource and its use serve well for classroom examples of several basic theorems: It is realistic to consider this resource as one which is variable and can be allocated in varying patterns relative to fixed resources. From the agronomic standpoint, fertilization allows expression of systematic biological laws, perhaps better than any other agricultural practice. Then, too, this important economic resource can be used conveniently without entanglement in the total organization of the farm and without requiring a large investment in complementary resources.

However, to view fertilizer as an isolated resource, with main relation only to soil management practices other than fertilization itself, causes the broader role and economic significance of fertilizer to be overlooked. Also, it causes some of the broader methodological problems to be overlooked. Over much of the nation, fertilizer (at least nitrogen) from chemical sources is a substitute for nutrients furnished by crops. Hence, the optimum combination of crops and the optimum fertilization plan must be determined jointly. The problem is hardly scratched in analyses such as have been made when designs, response surfaces, and optimum ratios and rates have been determined for a single crop. The big problem is still ahead, namely, the exploration of fertilizer use when crops and other enterprises are variable. A few examples can be used to illustrate some of the broader uses of fertilizer input-output data. Research needs will then be explained.

Fertilizer in the Total Farm Picture

Fertilizer is a resource which can give returns in a relatively short period of time. Because of this fact, it can be used advantageously as an *income complement* in those major farm adjustment ploblems, most of which involve a considerable time span for investment and before important increments are made to income. Most major farm adjustment problems involve several years of planning before they can be put fully into operation and the income flow can be increased. Some examples include:

1. Adjustment from cash crops to livestock farming in the Southeast. Not only is a large increase in funds needed for this type of change, but also, establishment of pastures, erection of buildings, and creation of livestock herds require from 5 to 10 years before operations can get into high gear.
2. Adjustment from exploitative to conservation farming systems in the Midwest.
   Plans commonly recommended by the Soil Conservation Service involve shifts from continuous row cropping, or rotations with only a small amount of meadow, to rotations including a considerable percentage of meadow. To get rotations into effect and obtain their yield benefits, cycles of rotations as long as 5 to 10 years are required. A similar time period is required in gearing livestock, buildings, and other investments to the new cropping system.
3. Adjustments from dry-land to irrigation farming.
   Leveling land, laying out ditches or sprinkler systems, putting new rotations into effect, and acquiring organizational and managerial skills ordinarily require 5 or more years before most dry-land farmers complete the shift to fairly efficient operations.

Other examples could be cited. However, most of these are the same, viz., several years is required before effective adjustment can be made in organization of the farm; capital investment must be built up over a considerable period and maintained at a higher level than previously; most major adjustments require some sacrifice in current income as the shift is made. Sacrifices in income are made as land is planted to forages or nurse crops, rather than to immediate cash return crops. Income is not replenished immediately by the livestock or soil development investments which require several years for their outturn of market product. In cases of some farm adjustment, income under the recommended system may never return as much as the current system. In other cases, however, income will be increased in the distant future, but with the sacrifice of income in the few years ahead.

Time Effects

Time requirements of farm adjustments cause shifts, such as those outlined above, to be discounted in the farmer's decision-making process because of (a) a shortage of capital, (b) the opportunity costs of using funds, and (c) the uncertainty relating to outcomes in the more distant future. A resource such as fertilizer, with a relatively short period for transforming investment into profit, can help overcome some of these effects and facilitate adjustments.

However, before empirical examples of this process are given, the effects of time on the "current outlook" for profits spread over time are examined. Suppose that a budget to determine the "average expected income" of the farmer's *present system* is constructed. Income per year is predicted to be \$3,000. Income under a *recommended farming system,* after it is put fully into effect, is predicted to be \$4,000, with an added investment of \$10,000. The *revised plan* appears most desirable, profitwise. However, no consideration has been given to the time required to get the plan into effect. Income may be lower than \$3,000 at the outset and income of the distant future comes at the expense of income in the near future. Consequently, the stream of income under the old plan may be preferable to the stream of income for the revised plan.

This point can be illustrated by principles explaining the present (discounted) values of alternative income streams. Suppose that the farmer has limited capital, but that he can obtain funds for the increased investment required by the new plan. His task is to determine whether the "old" or "new" plan gives the greatest present (discounted) value of future incomes. (Future incomes relate to the length of period which is relevant for the farmer's decisions, i.e., until income of a particular year is discounted to zero). The present value (PV) of income under each plan, supposing that only time with no uncertainty is involved, can be defined by equation 1 where R

$$PV = \sum_{i=1}^{i=n} \frac{R_i - C_i}{(1 + r)^i} \tag{1}$$

refers to the annual revenue of the i-th year, C refers to annual costs of the i-th year and r refers to the interest or discount rate. A dollar of income in the near future has much greater value than a dollar in the distant future, because the "discount coefficient" grows with time.

For example, a dollar forthcoming at the end of the next year has a present value of $\frac{\$1}{(1.10)^2} = \frac{1}{1.21}$, or 86 cents under a 10-percent discount rate. In other words, 86 cents invested at 10 percent will amount to \$1 in two years. The 10 percent is the return which can be realized (or is sacrificed) from investments other than the one under consideration. A \$2 amount forthcoming in 20 years, at the same discount rate, is worth only $\frac{\$1}{(1.10)^{20}}$ or 15 cents today. In other words, a dollar forthcoming in two years has a much greater present value than $2 forthcoming in 20 years.

Under the same discount levels, a farming system which returns $4,000 per year for the first 10 years and $1,000 per year for the next 10 years will average $2,500 over the 20 years. However, it has a higher present or discounted value than an alternative plan which returns $1,000 per year for the first 10 years and $7,000 per year for the second 10 years, an average of $4,000 over the 20 years. Thus, farming plans which include "quick turnover" investments, such as fertilizer and cash crops, have a strong economic advantage over long-time investments with delayed incomes. However, if capital is available, use of resources such as fertilizer can be added to "long-lived" plans to boost incomes in the near future, hence increasing the relative advantage of "long-time" plans which eventually increase returns.

All alternative resources and practices, even though some of these appear remote to the main adjustment in question, need to be considered in recommendations. Fertilizer can be important in this respect. Its increased use can serve as an "income catalyst" in adjusting to conservation farming in the Midwest, irrigation in arid regions, and other farming shifts. An empirical example in conservation farming is selected to illustrate this point.

Role of Fertilizer in a Conservation Plan Involving Time

An extreme problem in conservation is to be found on the Ida-Monona soils of western Iowa. The steep topography and the structure of these soils give rise to serious gully erosion. However, the shift toward soil-conserving farming systems has been small, even though education on needs has been fairly intensive and considerable public funds have been used on dams, conservation subsidies, and for Soil Conservation Service personnel in each county. As one travels through the area, he sometimes wonders if farmers have ever heard the word *conservation*. Of course, they all have, and they know the adjustments which are recommended to stop the extreme gullying found on most farms. But mainly they do not shift because, under their capital limitations and discounting, current farming systems have income advantages. Time-sequence budgets have been constructed on some of these farms to examine income prospects under current farming systems and those being recommended under educational and action programs. The purpose of these comparisons is to determine the length of time required for conservation systems to become profitable (and, as well, to determine if they are profitable), and the effect of different discount rates on the relative profitability of different plans.

There is, of course, no unique discounting rate for all farmers. While agricultural economists quite often use the market rate of interest for discounting calculations (i.e., land valuation, etc.), this magnitude applies only to a farmer with unlimited funds. It does not apply, as an opportunity cost, to the farmer who is limited on funds to invest in his own business because of either (a) capital rationing by lending firms, or (b) a risk aversion on the part of the farmer himself. The appropriate discount rate (i.e., opportunity cost rate) for this farmer is the return which can be earned within the year on some other enterprise.

An auxiliary objective of calculations was to determine planning procedures which would facilitate conservation farming systems by serving as "income complements" over time. Fertilizer was selected as one of the most promising opportunities in this respect.

Nondiscounted Incomes Under Two Alternatives

Budgeted incomes over a 15-year time period for one farm in the Ida-Monona soil association are presented in figure 13.1. The solid line shows predicted income, if the farmer continued to follow his current soil-exploitative farming system. Prices are assumed to be constant through the period and computations are based on average weather for each year. This farm is 160 acres and has soils typical of the area. It has several large gullies, also typical of the area. Budgets were made under assumptions of declining soil productivity, but these are not shown because of time and space limitations. General conclusions are not different, however, from those apparent in data of figure 13.1.

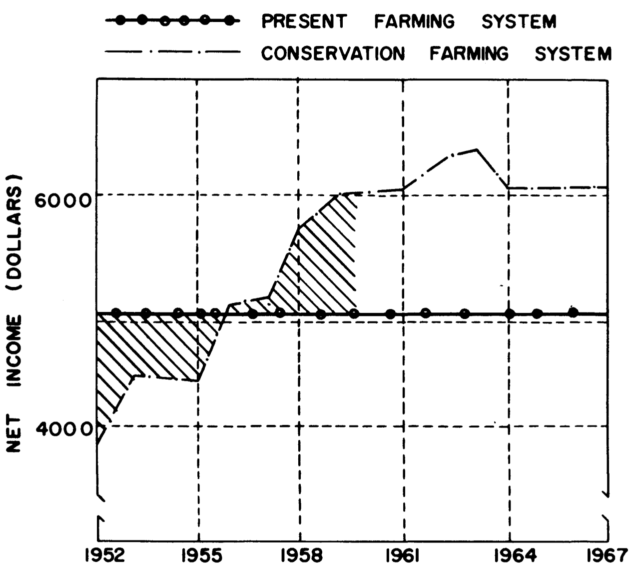

Fig. 13.1 — Net income predicted for typical 160-acre farm on Ida-Monona soils of western Iowa. (Actual income without discounting.)

The broken line shows the path of income predicted if the farmer were to follow the cropping plan suggested by the Soil Conservation Service, and if the farmer were to adapt livestock to it. The plan supposes that all farm-grown feed would be used under the revised plan. (Some was sold for cash under the existing plan). The revised plan averages about 40 acres of corn, 25 acres of oats, and 50 acres of hay. The existing plan included 66 acres of corn, 34 acres of oats, and 12 acres of hay. With terraces and contouring, per acre soil loss would be reduced to 6.63 tons; it was predicted to be 73.38 tons under the old system. The main livestock system for the new plan includes yearling steers wintered, grazed on pasture, and "fed out" in dry lot. Hogs are used as a supplementary enterprise to use the remaining corn. (Budgets were also made for nine other livestock systems, but are not presented because conclusions are similar).

As the two lines of figure 13.1 indicate, a shift to the conservation plan entails a decline in income for the four years: 1952, 1953, 1954, and 1955. Then, income of the revised plan moves above income of the existing plan in the fifth year. The "eventually greater income" under the conservation plan comes mainly from two sources: (a) gains in yield from a complete rotation cycle, (b) a larger livestock program with lesser amounts of the crops sold for cash. (The latter represents the largest portion of the income increase). Under the revised plan, fertilizer is not used in a "commercial" manner, but only in quantity and kind needed to get forages under way in the rotation.

While *income under the conservation plan* moves above *income under the existing farming system* in the fifth year (1956), it requires until the ninth year (1960) for the yearly sums of incomes under the former to exceed the yearly sums under the latter. That is to say, the surplus of four more years is required to make up the deficit of the first five years under the conservation plan. (In figure 13.1 the shaded area between the lines before they cross is greater than the shaded area after they cross, up to 1960). Hence, a total of nine years is required before the farmer can "break even" on his conservation plan. If the farmer is paying off a mortgage and has a low equity, is pinched for living funds, or has educational and other emergencies to meet, nine years is a long time.

If he discounts incomes, the picture is even less encouraging, as is illustrated in figure 13.2. The lines in figure 13.2 represent the same incomes as in figure 13.1, except that they have been discounted at the rates indicated. Different discount rates have been used to represent the outcome for farmers with different degrees of capital limitations. At a 5-percent discount rate, the sum of the surpluses of the revised plan, over the existing plan, is greater than the sum of the deficits for the 15 years represented. At 30 percent, however, the opposite is true, even up to 20 years. At discount rates of 20 and 30 percent, those common for many farmers, emphasis must be placed on resources which give a quick return. Fertilizer, hogs, and similar alternatives with production periods of a year are examples.

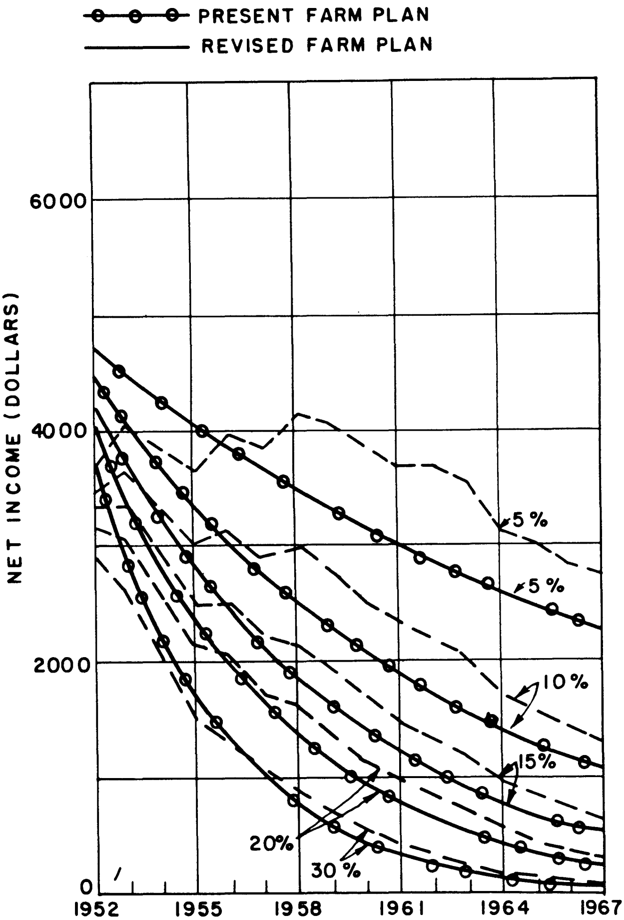

Fig. 13.2 — Net income on typical farm using variable discount rates.

Fertilizer to Close the Income Gap

The "income gap" in the first few years after adopting a conservation plan, prevents many farmers from shifting to an erosion control farming system. Hence, alternatives for removing this gap are examined. Two possibilities seemed important: (a) lengthening the expanse of time over which various practices are put into effect, and (b) using nitrogen fertilizer, or other farm practices, to give an immediate boost in production and income, where the practices themselves are profitable. Generally, these are practices which would be profitable even if the whole farm organization were not changed to a soil conservation system. The added income from them should not be viewed as resulting from the conservation plan. Along with conservation adjustments, these practices are simply part of the over-all farm management system. By offsetting income reductions due to shifts from grain to forage, these added practices may facilitate the adoption of conservation farming systems by a greater number of farmers.

Additional fertilizer was considered to be applied to corn acreage on the Ida and Monona soils of the farm so as to increase the annual yield to as much as 70 bushels per acre. No additional fertilizer was considered for Napier soil (although it could, perhaps, profitably use some). The yield increase in oats and hay (which would undoubtedly occur) was omitted in making calculations. Its value would more than counteract the cost of harvesting the additional corn yield. In spite of this conservative estimate of increases, net income can be increased considerably. Of equal or more importance to the farmer with a low income, is the fact that the increased income generally occurs in the year in which the fertilizer is applied.

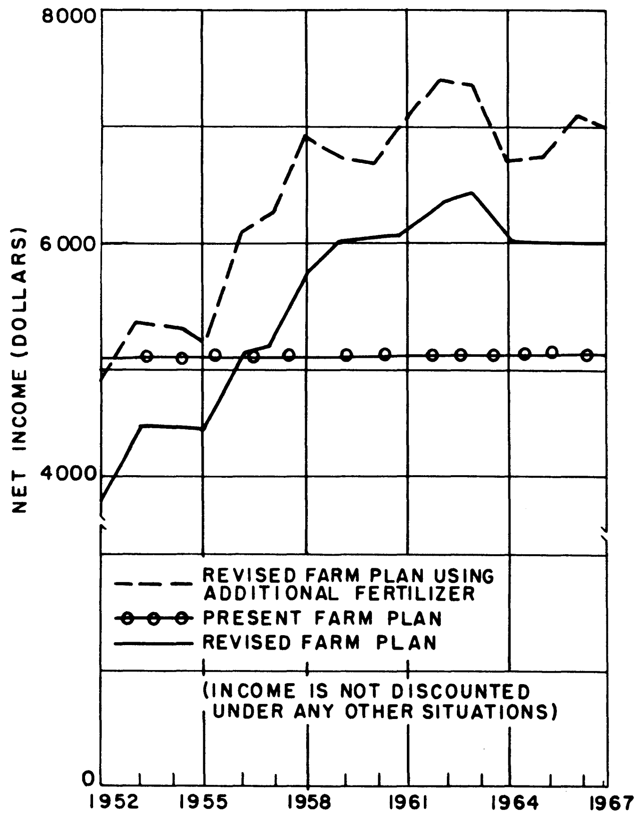

Fig. 13.3 — Use of additional fertilizer to reduce the income gap on farm shown in Figs. 13.1 and 13.2.

TABLE 13.1. Description of Processes or Activities

Activity or Process Number	Enterprise	Description	
		Types of Rotation Supplying Hay Requirement[a]	Types of Pasture Used
P1	Renting out unimproved pasture		
P2	Beef cow-calf	CCOM	Unimproved Kentucky bluegrass
P3	Beef cow-calf	CCOM	Improved Kentucky bluegrass
P4	Beef cow-calf	CCOM	Phosphate-lespedeza impr.
P5	Beef cow-calf	CCOM	Renovated pasture
P6	Beef cow-calf	CCOMM	Unimproved Kentucky bluegrass
P7	Beef cow-calf	CCOMM	Improved Kentucky bluegrass
P8	Beef cow-calf	CCOMM	Phosphate-lespedeza impr.
P9	Beef cow-calf	CCOMM	Renovated pasture
P10	Beef cow-calf	CCOM+F	Unimproved Kentucky bluegrass
P11	Beef cow-calf	CCOM+F	Improved Kentucky bluegrass
P12	Beef cow-calf	CCOM+F	Phosphate-lespedeza
P13	Beef cow-calf	CCOM+F	Renovated pasture
P14	Beef cow-calf	CCOMM+F	Unimproved Kentucky bluegrass
P15	Beef cow-calf	CCOMM+F	Improved Kentucky bluegrass
P16	Beef cow-calf	CCOMM+F	Phosphate-lespedeza
P17	Beef cow-calf	CCOMM+F	Renovated pasture
P18	Yearlings fed in drylot	CCOM	None
P19	Yearlings fed in drylot	CCOMM	None
P20	Yearlings fed in drylot	CCOM+F	None
P21	Yearlings fed in drylot	CCOMM+F	None
P22	Yearlings full fed on pasture	CCOM	Unimproved Kentucky bluegrass
P23	Yearlings full fed on pasture	CCOM	Improved Kentucky bluegrass
P24	Yearlings full fed on pasture	CCOM	Phosphate-lespedeza
P25	Yearlings full fed on pasture	CCOM	Renovated pasture
P26	Yearlings full fed on pasture	CCOMM	Unimproved Kentucky bluegrass
P27	Yearlings full fed on pasture	CCOMM	Improved Kentucky bluegrass
P28	Yearlings full fed on pasture	CCOMM	Phosphate-lespedeza
P29	Yearlings full fed on pasture	CCOMM	Renovated pasture
P30	Yearlings full fed on pasture	CCOM+F	Unimproved Kentucky bluegrass
P31	Yearlings full fed on pasture	CCOM+F	Improved Kentucky bluegrass
P32	Yearlings full fed on pasture	CCOM+F	Phosphate-lespedeza
P33	Yearlings full fed on pasture	CCOM	Renovated pasture
P34	Yearlings full fed on pasture	CCOMM+F	Unimproved Kentucky bluegrass
P35	Yearlings full fed on pasture	CCOMM+F	Improved Kentucky bluegrass
P36	Yearlings full fed on pasture	CCOMM+F	Phosphate-lespedeza
P37	Yearlings full fed on pasture	CCOMM+F	Renovated pasture
P38	Deferred feeding of yearlings	CCOM	Unimproved Kentucky bluegrass
P39	Deferred feeding of yearlings	CCOM	Improved Kentucky bluegrass
P40	Deferred feeding of yearlings	CCOM	Phosphate-lespedeza
P41	Deferred feeding of yearlings	CCOM	Renovated pasture
P42	Deferred feeding of yearlings	CCOMM	Unimproved Kentucky bluegrass
P43	Deferred feeding of yearlings	CCOMM	Improved Kentucky bluegrass
P44	Deferred feeding of yearlings	CCOMM	Phosphate-lespedeza
P45	Deferred feeding of yearlings	CCOMM	Renovated pasture
P46	Deferred feeding of yearlings	CCOM+F	Unimproved Kentucky bluegrass
P47	Deferred feeding of yearlings	CCOM+F	Improved Kentucky bluegrass
P48	Deferred feeding of yearlings	CCOM+F	Phosphate-lespedeza
P49	Deferred feeding of yearlings	CCOM+F	Renovated pasture
P50	Deferred feeding of yearlings	CCOMM+F	Unimproved Kentucky bluegrass
P51	Deferred feeding of yearlings	CCOMM+F	Improved Kentucky bluegrass
P52	Deferred feeding of yearlings	CCOMM+F	Phosphate-lespedeza
P53	Deferred feeding of yearlings	CCOMM+F	Renovated pasture
P54	Spring farrowed hogs	CCOM	None
P55	Spring farrowed hogs	CCOMM	None
P56	Spring farrowed hogs	CCOM+F	None

TABLE 13.1 (Continued)

Activity or Process Number	Enterprise	Description	
		Types of Rotation Supplying Hay Requirement[a]	Types of Pasture Used
P57	Spring farrowed hogs	CCOMM	None
P58	Fall farrowed hogs	None	None
P59	Laying flock	None	None
P60	CCOM, entire production sold on market	None	None
P61	CCOMM, entire production sold on market	None	None
P62	CCOM+F, entire production sold on market	None	None
P63	CCOMM+F, entire production sold on market	None	None

[a]A +F sign on the rotation indicates fertilization has been included as a practice. If this sign does not follow a rotation notation, fertilization is not included on the field crops.

As figure 13.3 illustrates, fertilizer used in conjunction with a conservation plan eliminates, from the very outset, the drop in income which would otherwise accompany the adjustment. The amount of fertilizer, now included as a resource in corn production rather than alone for establishing forages in the rotation, is not yet at the most profitable level (i.e., where MC=MR for fertilizer investment). However, enough is used to cause a plan which might not otherwise appear economically desirable to be adopted. Of course, other short-lived investments, such as more hogs, could serve similarly. However, the data suggest that "adjustment practices" of the farm business should not be treated in isolation, but should be treated in the fashion of over-all farm organization and resource allocation. Fertilizer has an important role in this over-all planning of farms in about all but the arid regions of farming. Even with discounting as high as 20 and 30 percent, our figures show that by using fertilizer as an income complement, conservation can be made *currently profitable* with the addition of fertilizer on corn.

Other Aspects of Over-All Farm Planning

Fertilizer also fits into the total farm program in other ways. It is one of many alternatives in which the farmer can invest. If profits are maximized, each dollar of capital and unit of other resource should be used where it gives the greatest marginal return. In other words, profitable fertilizer use cannot be established apart from the rest of the farm. In many cases, the return on fertilizer can outcompete many other investments in adding profit to the farm; at some level of fertilization, other investment opportunities may have profit priority over fertilizer. The farmer must decide whether scarce funds will add the most to profits if used for breeding stock, livestock feed, more buildings, new crop varieties, or fertilizer.

Linear programming provides a convenient method of testing the best investment plan for the farm, and in deciding what proportion of scarce funds should be invested in fertilizer. For example, a linear programming study for beginning farmers on Clarion-Webster soils in Iowa shows this: With very limited funds, a beginner would be better off to farm 80 acres and grow a corn-corn-soybean rotation fertilized at an intermediate level, rather than to farm 160 acres without fertilization. He would invest nothing in livestock if he maximized profits. With an intermediate amount of capital, he would farm 160 acres, grow a corn-soybean-corn-oats-meadow rotation fertilized to an intermediate level; he would raise 40 litters of pigs. With a larger amount of capital, he would use a corn-corn-oats-meadow rotation fertilized to a somewhat lower level per acre; he would raise 30 litters of pigs and feed out a carload of cattle.

Another study can be cited to illustrate a role of fertilizer in the program of the farm as a whole. This linear programming solution was worked out for a 160-acre farm in Clark County, Iowa, with associations of Grundy-Shelby-Haig and Seymour-Edena as the main soils. Limitational resources include labor of the operator and family, capital at various levels, building space for poultry and cattle, and cropland and pasture land. Considering two rotations of corn-corn-oats-meadow

TABLE 13.2. Optimum Plans Under the Various Capital Situations[a]

Investment Capital Situations	Optimum Program
$1,000	110 acres CCOM+F rotation; 38 acres unimproved pasture rented out; 148 hen laying flock; 10 litters of spring hogs. ($3286 net return)
$2,000	110 acres CCOM+F rotation; 38 acres unimproved pasture land rented out; 148 laying flock; 15 litters of spring hogs; 5 litters of fall hogs. ($4481 net return)
$4,000	110 acres CCOM+F rotation; 9 acres unimproved pasture rented out; 148 hen laying flock; 15 litters of spring hogs; 10 litters of fall hogs; 10 yearling steers on a deferred feeding program with unimproved pasture. ($5526 net return)
$8,000	110 acres CCOM+F rotation; 148 hen laying flock; 15 litters of spring hogs; 10 litters of fall hogs; 31 steers full fed on pasture; part of pasture improved, and part left unimproved; 6 steers on a deferred feeding program. ($6718 net return)
$10,843	110 acres CCOM+F rotation; 148 hen laying flock; 15 litters spring hogs; 10 litters fall hogs; 20 steers deferred feeding program;[b] 19 steers full fed on pasture.[b] ($4917 net return)

[a]All values have been rounded to nearest whole number.

[b]These enterprises utilized completely renovated pasture. Beef enterprise in preceding plans were on unimproved Kentucky bluegrass pasture.

and corn-corn-oats-meadow-meadow (both with and without a *discrete* amount of fertilizer), fall pigs, spring pigs, chickens, and four beef cattle systems, there are 63 possible activities when pasture can be (a) rented out as unimproved bluegrass, (b) used as unimproved bluegrass, (c) used as improved bluegrass, (d) improved with lespedeza and phosphate fertilizer, and (e) completely renovated with a pasture mixture and fertilization; and crops can be sold for cash. These 63 activities (processes or investment opportunities) are shown in table 13.1.

The linear programming solutions are shown in table 13.2 for different capital situations. These quantities refer to funds available beyond that required for land and building machinery, and regular cropping expenses for the rotation land. With funds very limited (i.e., $1,000 in capital beyond the amounts mentioned above), profits are greatest if the farmer uses a CCOM rotation with fertilization, and rents his pasture out. He would keep 148 hens, and raise 10 litters of pigs, but the greatest proportion of the farm's crop production would be sold for cash. Fertilization of rotation crops would, in fact, be more profitable than investment in any livestock practices. As capital availability increases, it becomes profitable to invest in more livestock and, finally, to use pasture improvement, or renovation. However, pasture renovation does not come in partially until capital is at $8,000. Pasture is completely renovated with $10,843 in capital.

Since pasture renovation also requires fertilization, an important point has been illustrated: Fertilizer use is a practice giving both a higher (i.e., on field crops) and a lower (i.e., on pasture) return than alternative investments at low capital levels. Hence, its optimum use cannot be determined without planning or programming of the farm as a whole. These programming analyses need to go even further than illustrated here, and allow consideration of different rates of fertilization on different crops.

Research Needs

The "farm solutions" mentioned above suggest some of the kinds of research information for farm-and home-planning programs, or other recommendations fitted into the farm as a whole. To be certain, they present difficult research problems, but they are much needed for the types of educational programs being intensified today.

One major problem is to determine the time effects or response for fertilizers, including:

1. The response sequences for fields or farms where a fertility build-up is taking place. If rates of 40, 80, and 120 pounds of nitrogen are applied in the first year, what will be the marginal products in the second year if these same three rates are superimposed on each of the same three rates of a previous year? What happens if this process is continued over several years? How much time is required and what are the effects of different rates in moving from a low fertility level to a level of economic maintenance?

2. The residual response functions for fertilizer applied at different rates. How much response can be expected in the second, third, and later years? What rates of discount should be used in figuring optimum fertilizer programs where incomes extend into the future?

3. What variations in response and incomes can be expected from weather variations as a farmer shifts between major organizations and uses fertilizer as an income complement?

To analyze the role of fertilization in the total farm program, research data are needed which predict the effects of different nutrient rates and ratios for different rotations. If the farmer can select among five different rotations such as CSC, CCOM, CSCOM, COM, or CCOMM, several rates and ratios of nutrients are needed on each crop in the rotational sequence (and on the first and second year of particular crops) before it can be determined which rotation and fertilization program is best. Of course, the optimum plan for the farm as a whole cannot be determined until the response functions for fertilizer are known, when this resource is varied in relation to other management practices or inputs such as livestock and, hence, manure, crop varieties, seeding rates, etc. The problem is only begun after having decided on the best empirical designs for determining the response function for a single crop, entirely apart from other inputs, other crops, and other management practices.

PART V

Trends in Use and Manufacture of Fertilizer

- **Quantity and Costs**
- **Sources of Nutrients**
- **Processes for Fertilizer**
- **Future Trends and Problems**

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Chapter 14

Our Changing Fertilizer Technology

ABOUT 100 years ago, nitrogen, phosphorus, and potassium were identified as essential plant nutrients. Previously, fertilization was practiced on an empirical basis. Substances such as bone, wood ashes, and organic materials were used because they were found to be beneficial to crop yields, but it was not known why they were beneficial.

Nitrogen, phosphorus, and potassium became known as the primary plant nutrients. Although many other elements have been found since to be essential to plant growth, the three primary nutrients continue to be the main concern of the fertilizer industry. Other elements usually are present in adequate quantities in the soil or may be supplied more economically by means other than inclusion in fertilizer mixtures. For instance, large quantities of limestone, dolomite, and gypsum are applied directly to the soil to supply calcium, magnesium, and sulfur. Elements needed in smaller quantities are sometimes included in mixed fertilizers. They are often applied separately, since soil deficiencies in these elements are not often widespread enough to warrant inclusion in fertilizer mixtures that are offered for general use.

Although the fertilizer industry has been classified for many years as a chemical industry, it was, until the early 1950's, primarily mechanical in nature. Phosphate rock was mixed with sulfuric acid to make superphosphate. Superphosphate was mixed with potash salts and organic materials to make mixed fertilizer. Chemists and chemical engineers were conspicuously absent from many fertilizer manufacturing establishments.

But the fertilizer industry is changing rapidly. It is fast becoming a full-fledged chemical industry. New processes are being studied and adopted. New forms of fertilizer and new chemical compounds are appearing. The chemical industry is noted for its progressiveness and the avidity with which it seeks and adopts new processes. Many more changes in the fertilizer industry in the near future may be expected.

Trends in Quantity and Cost of Fertilizers Used

Fertilizer production and consumption have increased very rapidly in the United States, particularly since 1945. Table 14.1 and figure 14.1

TABLE 14.1. U. S. Fertilizer Consumption and Composition

Year	Thousands of Tons of Primary Plant Nutrients				Percent of Total Primary Plant Nutrients Supplied as Mixed Fertilizers	Concentration of Mixed Fertilizer, % $N + P_2O_5 + K_2O$
	(N)	(P_2O_5)	(K_2O)	(Total)		
1910[a]	+46	499	211	856	59	14.80
1920	228	660	257	1145	49	13.90
1925	279	680	282	1241	62	16.00
1930	377	793	354	1524	66	17.90
1935	312	597	306	1215	68	18.32
1940	419	912	435	1766	62	19.90
1941	458	993	467	1918	62	20.22
1942	399	1131	546	2076	67	20.32
1943	508	1238	643	2389	72	20.68
1944	635	1405	649	2689	68	21.12
1945	641	1435	753	2829	71	21.74
1946	759	1671	854	3284	73	21.50
1947	835	1775	878	3488	74	21.58
1948	841	1842	956	3639	75	22.14
1949	911	1884	1065	3860	72	22.78
1950	1126	2073	1215	4414	70	23.58
1950-51[b]	1238	2107	1383	4728	72	24.19
1951-52[c]	1422	2199	1581	5203	72	24.86
1952-53[c]	1637	2271	1738	5646	72	25.85
1953-54[c]	1847	2242	1806	5896	71	26.87
1954-55[d]	2126	2286	1841	6253		

[a]From 1910 through 1950: USDA, Agricultural statistics, table 710, p. 705 (1952).
[b]W. Scholl and H. M. Wallace, 1955 (USDA). Commercial fertilizer consumption in U.S.
[c]———, ———, and E. I. Fox 1955 (USDA). Domestic fertilizer consumption, Commercial Fertilizer and Plant Food Industry 90:35.
[d]A. L. Mehring and C. A. Graham, 1955 (USDA). Fertilizer situation for 1954-55, Commercial Fertilizer and Plant Food Industry 90:44.

present statistics showing this rapid increase. The quantity of primary plant nutrients used in the form of commerical fertilizers more than doubled during the period 1945-1955; it quadrupled during the period 1930-1955.

As shown in table 14.1, the use of nitrogen and potash increased more rapidly than the use of phosphate fertilizers during the period 1945-1955. Nitrogen use tripled during this period; potash use more than doubled, and phosphate use increased by about 60 percent. The ratio of plant nutrients ($N:P_2O_5:K_2O$) was about 1:2:1 for many years prior to 1945; in 1955, this ratio approached 1:1:1. It appears likely that nitrogen use will exceed that of either of the other plant nutrients.

The price of fertilizers decreased considerably when compared on a basis that takes into account the decreased purchasing power of the dollar. The adjusted cost per unit of the primary plant nutrients in terms

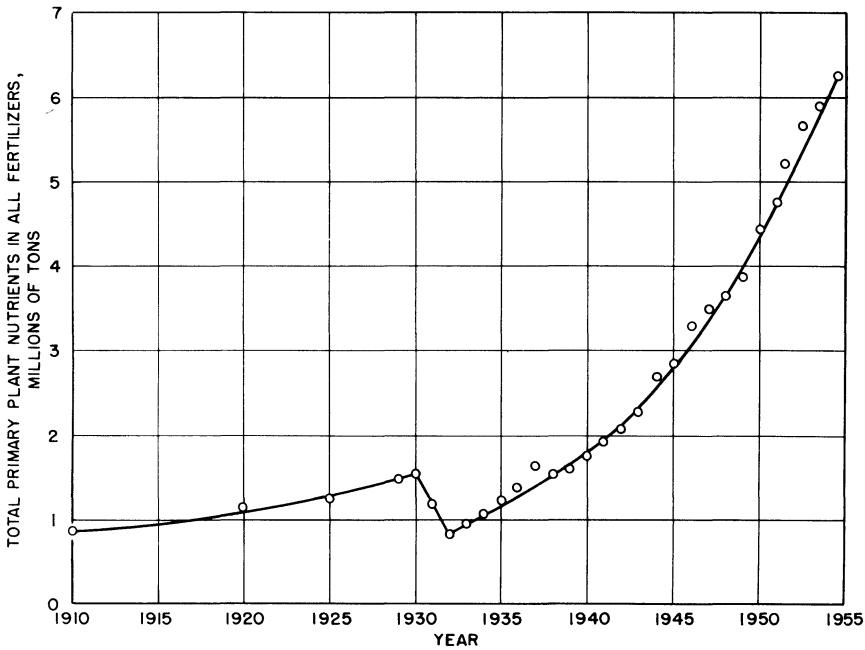

Fig. 14.1 — Annual U. S. consumption of primary plant nutrients in fertilizer.

of the 1955 dollar is presented in table 14.2. Past prices were adjusted by the Bureau of Labor Statistics wholesale price index. The basis for these data was wholesale market prices of the principal fertilizer materials, in bulk carload lots, f.o.b. point of production.[1] In calculating an average price of a unit of one of the nutrients (such as nitrogen), an average of the principal forms of that nutrient was taken and weighted in proportion to the quantities of these forms that were used.

The data in table 14.2 indicate that the 1954-1955 price of a unit of nitrogen in fertilizer materials was only about one-third of the adjusted 1920 price. A unit of potash cost only one-fifth of the adjusted 1920 price. The adjusted price of a unit of P_2O_5 decreased about 27 percent.

The decrease in the relative price of two of the three primary plant nutrients has probably been an important factor in the increased use of fertilizer and in the shift in proportions of plant nutrients. Changes in costs of plant nutrients also may result in changes in optimum farm practices. For instance, in 1920 when one unit of nitrogen cost about four times as much as one unit of P_2O_5, the use of phosphate fertilizer on nitrogen-fixing crops would yield a quite different return, in

[1]These price relationships may not accurately reflect the relative costs of the plant nutrients to farmers because of locational factors, especially differences in transportation costs.

TABLE 14.2 Adjusted[a] Wholesale Price, Bulk, F.O.B. Works or Ports, 1955, Dollars per Unit of N, P_2O_5, or K_2O

Year	N	P_2O_5	K_2O	Average
1920	4.63	1.23	2.18	2.68
1930	4.30	1.06	1.39	2.25
1940	3.42	1.26	1.17	1.95
1950	1.79	0.82	0.40	1.00
1955	1.56	0.90	0.40	0.95

[a]Wholesale price index of Bureau of Labor Statistics, U. S. Department of Labor: Monthly Labor Review, Feb. 1952 and May 1955, and Statistical Abstracts of the United States, 1937 and 1954.

comparison with the use of nitrogen fertilizer, than by the mid 1950's when chemical nitrogen was relatively cheaper.

Sources of Nitrogen

Typical wholesale prices for the various forms of nitrogen during a 35-year period (1920-1955) are shown in table 14.3. Prior to 1920, natural organics were the principal sources of nitrogen, and generally the cheapest. Ammonium sulfate, a by-product from the steel industry, was an important source of nitrogen in 1920. This material came into use with the adoption of by-product coke ovens around 1900. The quantity of available ammonium sulfate increased as the by-product ovens replaced the older beehive ovens and as the steel industry expanded. Nitrate of soda was a principal source of nitrogen from 1920 to 1950, but this source supplied a much smaller percentage of the total nitrogen used in 1955.

Since 1920 the use of organics has declined. In many instances, new uses have been found for these materials in which they are processed into products of higher value.

The first successful synthetic ammonia plant was started in 1921. Several other plants were built in the next few years. These plants supplied nitrogen materials to both chemical and fertilizer markets. About one-third of the fertilizer nitrogen consumption was supplied from synthetic ammonia in 1930, and about the same proportion in 1940. After 1945, greatly increased proportions of nitrogen fertilizer were derived from synthetic ammonia, and in 1955, it was estimated that about 88 percent of the nitrogen fertilizer used was derived from this source. Nitrogen fertilizer materials supplied from synthetic ammonia include anhydrous ammonia, ammonium nitrate, ammonium sulfate, urea, ammonium phosphates, sodium nitrate, and nitrogen solutions which are composed of ammonia, water, and either ammonium nitrate or urea.

Ammoniation of superphosphates was started in 1928 and increased slowly at first. In 1940, 60,000 tons of nitrogen was used for

TABLE 14.3. Estimated Consumption and Adjusted Wholesale Prices of Fertilizer Nitrogen, Phosphate, and Potash by Sources[a]

Fertilizer	Consumption in 1000 Tons of N, P_2O_5, or K_2O, Adjusted Wholesale Prices (in parentheses) in Dollar per Unit									
	1920		1930		1940		1949-50		1954-55	
Nitrogen										
Ammonium sulfate	78	(4.49)	128	(3.58)	137	(3.02)	179	(2.39)	382	(2.10)
Ammonia	-		(in "others")		60	(1.16)	107	(0.95)	502[b]	(1.04)
Nitrogen solutions	-		(in "others")				252	(1.20)	470[c]	(1.30)
Ammonium nitrate	-		-		-	-	154	(1.72)	484	(1.94)
Sodium nitrate	47	(4.88)	71	(4.98)	119	(3.63)	100	(3.17)	(in "others")	
Organics	77		60	(5.00)	48	(4.00)	42	-	33	-
Others	26		118		55		199	-	255	
Total nitrogen	228		377		419		1033		2126	
Phosphate										
Normal superphosphate	528	(1.23)	690	(1.06)	675	(1.21)	1601	(0.81)	1521	(0.88)
Concentrated superphosphate	13		40		155	(1.51)	272	(0.88)	471	(0.98)
Others	119		63		82		187		294	
Total phosphate	660		793		912		2060		2286	
Potash										
Muriate, 60%	-		28		291	(1.17)	866	(0.40)	1675	(0.40)
Muriate, 50%	69	(2.67)	149	(1.48)	69	(1.17)	96	(0.43)		
Manure salts	72	(1.71)	85	(1.24)	18	(1.27)	21	(0.21)	1	(0.19)
Others	116		92		57		86		165	
Total potash	257		354		435		1069		1841	

[a]Data for 1920, 1930, and 1940 from Fertilizer Technology and Resources in the United States, Academic Press, Inc., New York, 1953. Data for 1949-50 from "Commercial Fertilizer Yearbook," Walter W. Brown Publishing Co., Inc., Atlanta, Georgia, Sept. 1950. Data for 1954-55 from "fertilizer situation for 1954-55," Commercial Fertilizer and Plant Food Industry 90: No. 6, p. 44, June, 1955.

[b]390,000 tons were used for direct application.

[c]75,000 tons were used for direct application.

ammoniation as ammonia and as solutions. In the 1954-1955 season, about 507,000 tons of nitrogen was used as ammoniating materials. This amount is over eight times the 1940 usage and about 24 percent of the total nitrogen used in the 1954-1955 season.

The use of ammonia and nitrogen solutions for direct application and the use of solid ammonium nitrate have grown spectacularly since 1945, and by mid 1950's accounted for about 22 and 23 percent of the total nitrogen use, respectively. Large-scale production of ammonium sulfate from synthetic ammonia was started in 1945 and, 10 years later, ammonium sulfate production from this source was about equal to the production of the by-product material. Ammonium sulfate from both sources accounted for about 18 percent of the nitrogen fertilizer consumption.

The production of urea was increasing sharply by mid 1950's, and processes that produce compound fertilizers such as ammonium phosphates and nitric phosphates also were coming into increased use.

Phosphates

Normal superphosphate, made from phosphate rock and sulfuric acid, has been and continues to be the principal form of phosphate fertilizer. It contains about 20 percent available P_2O_5. However, the use of concentrated superphosphate has been increasing rapidly, as shown in table 14.3. This material is made from phosphate rock and phosphoric acid and contains 45 to 50 percent available P_2O_5. Either electric-furnace or wet-process phosphoric acid may be used. In the 1954-1955 season, about twelve times as much concentrated superphosphate was used as in 1930, with the use of this material increasing more as new plants come into production. In 1930, normal and concentrated superphosphate supplied 87 and 5 percent of the available P_2O_5 consumed; in the 1954-1955 season, the proportions were 66 and 21 percent, respectively. Although concentrated superphosphate is slightly more expensive than normal superphosphate at the point of production, the savings in transportation costs make the concentrated product less expensive in many consumer areas. TVA has contributed significantly to the farmer's acceptance of concentrated superphosphate and of high-analysis mixtures made from it. Test-demonstration and educational-sales programs have helped to create a market for these products.

The development of the electric-furnace process, in which TVA has been active, has permitted the use of low-grade phosphate rock that otherwise might not be economically usable. The quantity of low-grade rock in our reserves far exceeds the high-grade rock. Much of the low-grade rock cannot be beneficiated economically by presently known processes.

Except for TVA's operations, there has been little production of phosphate fertilizer by electric-furnace methods, although large quantities of phosphorus compounds for other uses are made by this process. However, by the mid 1950's there was an important quantity of commercial electric-furnace phosphoric acid used in the production of fertilizer.

Fertilizer processes using significant amounts of electric-furnace phosphoric acid are the production of nitric phosphates, liquid fertilizers, diammonium phosphate, and enriched superphosphate. Calcium metaphosphate is made from phosphorus produced in the electric furnace. Whether the electric-furnace route will become a much greater factor in the production of phosphate fertilizers depends on a large number of factors that cannot be predicted; however, conditions appear favorable for some expansion.

Companies that produce most of the wet-process phosphoric acid use it to make concentrated superphosphate and ammonium phosphates. There are indications that wet-process phosphoric acid may become more generally available to other fertilizer manufacturers. Reductions in freight rates on fertilizer-grade acid should encourage this trend. The increased availability of wet-process acid may encourage its use in mixed fertilizers in conjunction with ammoniation, in enriched superphosphate, and in liquid fertilizers.

Noteworthy advances have been made in the technology of mining and beneficiating phosphate rock. More efficient techniques have helped keep the price of phosphate rock at a reasonably low level and have greatly increased the recovery of usable phosphate from our reserves.

Potash

Prior to 1933, the United States was dependent principally on European sources for potash materials. The large New Mexico deposits were discovered in 1925, and production started in 1931. Domestic production has increased rapidly, until only about 2 percent of our potash is imported.

There have been excellent advances in methods of mining, beneficiation, and refining of potash salts, which have been reflected in increased use of high-grade salts and decreased cost. The data of table 14.3 show that, in 1930, only 9 percent of the fertilizer potash was in the form of the 60 percent K_2O muriate of potash; 41 percent was in the 50 percent K_2O grade; and 50 percent was in the form of lower-grade salts such as manure salts, which contain about 25 percent K_2O. In contrast, in 1953 about 90 percent of the fertilizer potash was supplied in the form of potassium chloride containing at least 60 percent K_2O (6).

Production of Mixed Fertilizer

Mixed fertilizer has been and continues to be the principal form of fertilizer used by the farmer. About 70 percent of the plant nutrients used are in the form of mixed fertilizer. As shown in table 14.1, there is no definite trend in the percentage of plant nutrients supplied in the form of mixed fertilizer, although the percentage was somewhat greater in the 1950's than prior to 1942.

At first, the production of mixed fertilizers was a mechanical operation consisting of dry mixing of various fertilizer materials and conditioners. Ammoniation of superphosphates as an integral part of mixed fertilizer production was started in 1928 and has been increasing greatly. There also has been an increase in the use of sulfuric or phosphoric acid during ammoniation. These practices place increased emphasis on the chemical aspects of mixed fertilizer production.

The principal advantages of ammoniation are: (a) ammonia and nitrogen solutions are the cheapest forms of nitrogen available to the fertilizer manufacturer; and (b) since these solutions are highly concentrated forms, they facilitate the preparation of high-analysis fertilizers. These advantages have proved to be sufficient to stimulate widespread efforts by fertilizer manufacturers to use as much ammonia or ammoniating solutions in their formulations as possible. TVA has conducted pilot-plant development (10) of methods and equipment for incorporating more than the usual proportion of nitrogen in superphosphate by ammoniation. Since the extent to which superphosphate can be ammoniated imposes a limitation on the amount of ammoniating solution that can be incorporated in mixtures, many manufacturers add sulfuric or phosphoric acid to the mixture to absorb more ammoniating solution.

There has been much interest by fertilizer producers in the TVA continuous ammoniator. Over 40 companies have been given royalty-free licenses to use the process or to manufacture the equipment.

Granulation of mixed fertilizers and production of high-analysis fertilizers have been increasing rapidly. The two trends are related, since most high-analysis fertilizers contain a large proportion of soluble salts which would cause caking unless the mixture is granulated. Granulation greatly decreases the caking tendency, but does not necessarily eliminate it. Drying, curing, conditioning, and packaging in "moistureproof" bags also help prevent caking. Granulation also is effective in preventing segregation of the fertilizer ingredients. It facilitates more uniform distribution in the field and decreases dust losses of fertilizers. About 100 fertilizer plants were producing granular fertilizers of some kind by the mid 1950's. Most of these plants were above average in production capacity. TVA actively studied granulation of high-analysis fertilizers, and the majority of plants producing granular materials made at least some use of information from TVA's research and development program.

There is little standardization in the particle size of granulated fertilizer; many plants produce a partially granulated product containing some fines. Experiments sponsored by TVA have been set up to determine the agronomic effect of particle size for materials of several degrees of water solubility.

The concentration of primary plant nutrients in mixed fertilizers has increased steadily, as shown in table 14.1, and in 1954 averaged about 27 percent. In some areas the trend toward high-analysis products has been much more pronounced. For instance, in 1954 the average concentration was about 35 percent in the west north-central states as compared with 22 percent in the south Atlantic states (7).

Granulation and ammoniation have resulted in a change in the chemical character of mixed fertilizers to the extent that the chemical compounds present in them may be far different from the compounds present in the ingredients. A few grades of typical mixed fertilizers were examined by TVA for identification of chemical compounds. The phosphate compounds identified, roughly in the order of decreasing abundance, were dicalcium phosphate, ammonium phosphate, apatite, and monocalcium phosphate. The kind of apatite was not identified, but it is probably that both fluorapatite and hydroxyapatite were present. The principal nitrogen compounds identified were ammonium chloride, potassium nitrate, ammonium phosphate, and ammonium sulfate. The principal forms of potash were potassium nitrate and potassium chloride.

Combination Fertilizer Processes

There has been a sharp increase in combination fertilizer plants in which fertilizers containing two or three of the primary plant nutrients are produced directly, without the usual steps of first producing separate fertilizer materials and then mixing them in another operation. Most of these plants are either of the nitric phosphate type or the ammonium phosphate type.

In 1943, TVA started experimental work on the treatment of phosphate rock with nitric acid for making phosphate and nitrogen fertilizers. The principal purpose of this work was to develop lower-cost processes for producing fertilizers. Four nitric phosphate processes were developed through the pilot-plant state (1, 2, 4, 5, 9). Stanfield (8) has discussed the economics of these processes in comparison with conventional processes.

Most of the TVA development work was done without knowledge of similar work in progress in Europe, since information on this work was not published until later. However, similar processes were developed in Europe and have come into extensive use, particularly in France.

In the early 1950's the sulfur shortage stimulated intense interest in the nitric phosphate processes in the United States. Sixteen companies were given certificates of necessity to build nitric phosphate plants. As the shortage of sulfur was alleviated, interest in the nitric phosphate processes waned and only two plants were built. By 1955, other companies were considering construction of nitric phosphate plants.

Combination fertilizers based on ammonium phosphate have been produced in several plants. Wet-process phosphoric acid is combined with ammonia to produce impure monoammonium or diammonium phosphate or mixtures of these salts. Sulfuric acid and additional ammonia are added to form ammonium sulfate when higher $N:P_2O_5$ ratios are desired. Potassium chloride is added if desired. The resulting products are homogeneous, granular materials of high water solubility.

Liquid Fertilizers

The direct application of anhydrous ammonia, aqua ammonia, and nitrogen solutions has proved very popular. According to estimates, about 22 percent of the total nitrogen used in the 1954-1955 season was applied in these forms (3). The use of liquid fertilizer mixtures containing two or more plant nutrients has shown rapid growth. About 150 companies produced liquid fertilizer mixtures in 1955 or had them registered for sale. Although the quantity constituted a very small proportion of total fertilizer consumption in most areas, the rapid growth of liquid mixed fertilizer production may indicate important future possibilities.

Some of the advantages claimed for liquid mixed fertilizer are: greater ease of application to the soil, lower cost of preparation and handling, elimination of bagging costs, and elimination of the faults of dry fertilizer such as caking and segregation. One of the disadvantages is that the raw materials for the liquid mixes often are more expensive. Storage of raw materials or products is more expensive. Differing opinions are held regarding agronomic effectiveness. Many producers claim that liquid fertilizers are more effective than solids, at least for some crops or areas. More rapid soil fixation of the phosphorus content of liquid materials is advanced as a possible reason for lowered effectiveness. More data are needed on this point.

Most liquid fertilizer manufacturers use electric-furnace phosphoric acid as the source of phosphate. The supply of this material is limited. If an important proportion of fertilizers is to be supplied in liquid form, either electric-furnace production of phosphoric acid must be greatly expanded or an alternative source developed. Wet-process phosphoric acid is not widely used in liquid mixes because its impurities precipitate on neutralization and might cause trouble by clogging handling and distribution equipment. Perhaps this difficulty can be overcome.

Most manufacturers of liquid mixtures operate on a highly seasonal basis. The amount of raw materials obtainable during a short peak season is limited. If a large proportion of fertilizer is to be supplied in liquid form, problems of storage of raw materials or products will have to be solved.

Some Present and Possible Future Trends and Problems

There is an evident trend toward large-scale production of fertilizers in modern chemical plants. This type of fertilizer production offers economic advantages of lower processing costs. Processes that are too complex for small manufacturers may be carried out economically by larger manufacturers. A disadvantage of the large-scale chemical plant is that its economics usually require more nearly continuous operation than is characteristic of the mixed fertilizer industry. It is not economical to allow the large investment required to remain unproductive during the greater part of the year. Skilled operators are required, and steady employment is necessary to keep them. The need for steady operation of the manufacturing facilities and the highly seasonal demand for the product are responsible for a serious storage problem.

Another characteristic of a large chemical plant for fertilizer production is that it usually is not feasible to make a large number of grades. As fertilizer plants grow in size and complexity, it may become desirable to decrease the number of grades.

Divergent trends are evident in the water solubility of the phosphorus content of fertilizers. Fertilizers of lower water solubility are produced by the nitric phosphate processes and by heavy ammoniation of superphosphates. The production of highly water-soluble materials such as ammonium phosphates and liquid fertilizers is also expanding.

There is a need for close coordination of the work of chemists, engineers, agronomists, economists, and others in evaluating new processes and products and determining optimum direction of change. In many cases, compromises may be necessary between manufacturing economics and the economics of fertilizer use and distribution. These points have been emphasized previously in Chapter 2.

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