304

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Agricultural Supply Functions

Estimating Techniques and Interpretation

- A compilation of essays on the problems, research techniques, and results of investigations dealing with agricultural supply functions in agricultural production.

- Dedicated to the proposition that progress is made by discussing hypotheses, theories, techniques, and illustrations of their use even though some are still in their formative stages.

- Published for use by students, researchers, teachers, and administrators who are daily confronted by problems in this important area.

Agricultural Supply Functions

—ESTIMATING TECHNIQUES and INTERPRETATION

Assembled and published
under the sponsorship of the

**NORTH CENTRAL FARM MANAGEMENT
RESEARCH COMMITTEE**

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Preface

THE MAJOR PROBLEMS of agriculture revolve around supply functions and relationships. This is especially true in highly developed economies, where commodity supply tends to grow faster than consumer demand. Evidently, too, elasticity of the short-run supply function is very low. Under these circumstances, and even though the structure of agriculture constantly changes in response to this supply-demand environment, commodity prices remain low, with low rewards going to resources used in farming. In less developed nations, the problem of agriculture is still supply or producer response, but mostly in terms of the slow rate of shift in supply and its low elasticity in relation to the real price of foods.

In the current decade of U.S. farming, basic questions of policy revolve around the supply function and its elasticity. The rate at which agriculture can adjust to the current complex of economic growth, under programs ranging from market freedom to public management, depends on supply elasticity and change. But more than the supply function for agricultural commodities is involved. The supply function for commodities is inseparably woven with the demand function for resources and the functional relationships of resource returns.

Improved knowledge of agricultural supply is necessary for effective policy formulation. Public policy of the past operated under the implicit assumption of certain supply relationships and magnitudes. Unfortunately, time has not always proved these assumptions to be correct, and empirical and factual knowledge is urgently needed on supply relationships for both agricultural commodities and resources.

Greater knowledge of supply is needed not only for improved policy formulation but also for better guidance and decision making of individual farmers. Knowledge is needed of the potential future supply structure under rapidly changing technology and factor prices. With this knowledge, communicated to farmers by the extension service, farm families can use their individual resources more wisely. Improved supply information can also aid greatly in annual outlook work. Finally, this information is needed for appraisal of problems and potentialities in interregional competition and area development.

Farm management and production economics specialists have long directed attention to analysis of the decision-making processes of farmers, the nature of production functions, and the structure of resource returns and prices. These are the very basis of commodity supply functions. We need to build upon this basic and well-developed foundation by relating commodity supply functions to production functions, decision processes, factor supply relationships, and technological change. Current interest for doing so is great, and prospects are for an expanded output in this area of research. Such research has not been lacking in the past, but it has not been of sufficient scale and coverage. With the prospects that this void will be filled shortly, the next problem is appropriate techniques for supply prediction.

Empirical prediction for supply is a difficult task. The supply structure changes continually, and techniques readily applicable to demand are complex and not suitable for application to supply. However, numerous empirical techniques and theories now need to be examined in terms of their promise in estimation. The conference represented by the chapters in this volume was organized accordingly. The Conference Committee hopes that the publication of the conference proceedings will both encourage greater research in supply analysis and lead to the improvement of theory and techniques.

This conference was organized by a subcommittee of the North Central Farm Management Research Committee in cooperation with the Farm Foundation. The Committee wishes to express its appreciation to the Farm Foundation for its financial support for the conference and for publication of the proceedings. It also expresses appreciation to the Agricultural Experiment Station Directors and the U.S. Department of Agriculture for the participation of their staff members.

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Howard G. Diesslin
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Contents

PART I

Introduction

Chapter 1

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Uses and Concepts in Supply Analysis*

PROBLEMS of supply have long been dominant in agriculture. Less developed countries have needed to understand supply phenomena in order to coax output to levels accommodating adequate human nutrition and larger populations and also to promote general economic development. In more developed economies, and particularly that of the United States, the major recent need has been greater understanding of supply phenomena in order to control surpluses and to raise farm prices and resource incomes. Fundamentally, the need even here is more basic knowledge which relates product output to factor inputs and provides a framework for adjusting production and resource employment to economic growth prospects or trends.

Aside from major developmental concerns with product supply and factor demand in agriculture, improved knowledge would also be useful in other directions. It would make possible more precise forecasts and predictions to aid farmers in making better short-run and long-run decisions on investments and planning. It would be useful in formulating policies directed toward greater stability in farm prices and incomes, and in developing the storage, price, and auxiliary mechanisms which contribute to this end. It would be useful for investment planning by firms producing inputs used by agriculture. On a less aggregative basis and in an interregional competition framework, improved knowledge of product supply and factor demand phenomena would provide a better basis for program projection by extension services and for planning by regional or community bodies. Finally, greater empirical output in respect to supply structure and response would help to satisfy the academic appetites of agricultural and other economists.[1] The major societal concern will, however, remain outstandingly that of gearing food output and resource employment of agriculture to economic growth goals.

In this conference, the questions posed to participants (and, presumably, the papers which they will present) deal quite largely with

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[1]Added information is forthcoming on supply. For example, see Nerlove (15), Heady and Dean (9), and Halvorson (7).

estimating supply parameters and structure. As an end *per se*, quantitative analysis of supply serves mainly to satisfy the academic appetite. Society, in the investment it makes in agricultural economics research for these purposes, would more nearly view quantitative analysis of supply as a means. Supply analysis should eventually serve as a means or foundation for adjusting or changing the structure of the agricultural industry in line with the more ultimate ends of concern to the relevant publics or sectors of society. Useful in this context, supply analysis would specify the relevant variables, along with magnitude of their associated coefficients, to be manipulated in bringing outputs, inputs, and prices of products and resources into line with goals held relative to farm family incomes, food targets, economic growth, or related ends. Certainly these concerns, including the surplus and income problems of the agricultural industry, provide the important reasons for intensification of research in agricultural supply and related functions.

More than ever before, answers to the major problems of American agriculture rest on supply response or output quantities and their inseparable relationships with resource inputs and prices. I emphasize this point because it illustrates certain of the most important needs in empirical work. Useful quantitative analysis of supply will need to extend beyond mere estimates of product supply or factor demand elasticities and coefficients. These are descriptive largely of quantities in past decades and are useful only for projections or forecasts over the short run where the major structure of agriculture might deviate insignificantly from the recent past. Most nations faced with problems of agricultural output, whether from the standpoint of surpluses or food deficits, wish to alter the supply structure, rather than to extend past structure into the future. Productive effort in the area of supply analysis would tell them how to do this and would need to extend far beyond the capacities of most empirical supply studies made to date.

Even in respect to guidance for individual farmers and communities, the major questions of importance are those of changes in supply structure. Largely, they rest on considerations of interregional competition and the relative changes in the structure of supply among regions. They are brought about by technical change, institutional innovations such as vertical integration, and changes in managerial skills of farm operators, as well as the more conventional observations on prices and other variables which economists conventionally have been able to measure and incorporate into their models. Forecasts relating to short-run outlook and year-to-year decisions on hog farrowing, potato acreage, etc., need depend much less, and perhaps scarcely at all, on models which recognize continuous change in basic supply structure. A model or equation based on time-series data allowing forecasts for a single period ahead may be relatively efficient for these purposes. But the set of ultra-short-run choices and decisions which it aids are of much less social import than those which can be aided only by estimates which consider changes in basic structure of supply.

SUPPLY, ADJUSTMENT, AND FACTOR DEMAND

Directly, the major problems of American agriculture are those of supply. They are manifested in commodity cycles with extreme price and income fluctuations for enterprises such as hogs and cattle, in growing surpluses and increasing public costs of storage programs, and in depression of farm income and low relative returns to resources in agriculture. But indirectly and more fundamentally, the basic problems of American agriculture have their roots in the various facets of demand for (use) and supply of resources by the industry. Conceptually, it is impossible to analyze product supply apart from factor demand and prices. Practically, in solving the major adjustment problems of American agriculture, product supply research has little meaning unless it relates closely to explanations of the short-run and long-run supply of and demand for factors in agriculture. In this same vein, it needs to relate to resource productivity, as inputs are used at different levels and in different mixes and as output varies accordingly. Finally, quantitative analyses of supply aimed at providing the solutions to major adjustment problems of agriculture must be made in conjunction with demand analyses, if we are to obtain a more general equilibrium view of the outputs and inputs which might be expected to prevail in a free market, under various alternatives in production and price policies or with alternative programs for altering the mobility of resources attached to farming.

Extremely refined estimates of supply coefficients and elasticities may be unnecessary as a basis for public decision. Most policies representing manipulation of variables to alter product supply and resource employment may well be put into effect before any major flow of new research results, useful for the purpose, is available. Certainly this will be true unless additional resources and vigor are injected into supply analysis. Even then, existing quantitative and qualitative knowledge may already be sufficient for development of improved or necessary policies in respect to product supply and resource employment of agriculture. Should this prove true, agricultural supply research will be restricted largely to the role of guidance for individual farmers and as an exercise in convenience. The competitive structure of agriculture lends itself to measurement and application of conventional empirical models better than other industries. But this writer has greater hopes. Supply research, although complex for the purpose, could provide the basis not only for guidance in general policy formulation, but more specifically for indicating the magnitude by which relevant variables need to be manipulated.

Ideally, models which generate estimates of these types would necessarily be complex and of general equilibrium types. They would entail estimation of the product supply functions, consumer demand functions, and factor demand functions for each major agricultural product and resource. Carried to logical extreme, a model of this sort would include relevant equations for each distinct product in agriculture and the rest of the economy. Such a massive general equilibrium model exceeds the resources of research economists and probably the time span for which the estimates would be most useful for agriculture. To be useful for important policy formulation and guidance of individuals over the next dozen years, a model of this general nature would need to include only agriculture and separate equations for the important product aggregates (those interrelated in surplus accumulation, low prices, and resource adjustment requirements) within the industry.

Even then, as those who have tried can testify, a regression system such as that implied and reliable for predictions is not generated with ease, nor always with signs and error terms for coefficients which cause the investigator great pleasure. Beyond these difficulties are those of formulating general equilibrium models which have utility beyond short-term forecasts and which incorporate the appropriate "shifters or adjusters" as reflected in degrees of fixed resources, in technological change, in uncertainty and knowledge conditions, or other variables which cause response elasticities to change over time but are not observed in the neat units of isolated economic theory. To be most useful in prescribing policies, educational and guidance programs relevant to specific geographic sectors of agriculture, product supply and factor demand equations would need to be estimated by major geographical regions. But again this degree of detail, incorporated into a model which surrounds only the products of the feed grain-livestock economy, becomes computationally complex.

Much can be contributed to knowledge on extent of adjustments needed in agriculture and action and educational programs to facilitate them with much less complicated and sophisticated models. Additional supply or response functions estimated separately by least squares methods for several major agricultural products, or groups of them, could serve for general guidance of policy and educational programs. Issues and policy directions would be clarified considerably if we had greater knowledge of supply elasticities for these products. Not only have agricultural economists expressed conflicting hypotheses about magnitudes of supply elasticities, but conflicting policies have been and are being proposed because different assumptions are implicitly made about these elasticities. For example, proposals for a free market at one extreme and rigid output quotas at the other extreme imply different elasticity magnitudes. The free market policy would suppose elasticities of product supply and factor demand to be large in the short run. Supposition evidently is that a new structure of prices would lead to rapid adaptation of agricultural output and resource employment and, therefore, that the burden of adjustment would be relatively short in duration and light in impact. In contrast, policies of restricting output or of providing price supports with no restraints on output evidently assume these elasticities to be extremely low. Educational programs which emphasize increased farming knowledge for a greater number of rural youth, without parallel opportunities in counseling or vocational education for other occupations, would assume high demand elasticities or low supply elasticities, or both, for labor in agriculture. Quite obviously, the outcome of a particular action or educational program depends on the supply functions for factors, as well as on the supply functions for those commodities which move directly into the consumer market.

What is needed or wanted is change *per se* in the structure of agriculture and supply. Accordingly, research into supply-related quantities must be much broader than the restraints imposed by past regression models. Research directly aimed at changing the parameters of supply is probably more important than that aimed at estimating the parameters of the past or present. For example, analyses which might estimate the rate or quantity by which labor now on farms would be furnished to agriculture or to nonfarm industries under various price relationships would provide useful information for some purposes. But if problems in particular areas of agriculture are to be solved more rapidly or permanently (or, even if our criterion is simply one of giving low-income persons in these areas greater opportunity for higher paying opportunities in rapidly growing sectors of the economy), we need research and services changing the relative supply of labor to agriculture under a given set of price ratios, other things remaining equal.

Conceptually, of course, all variables relevant to such change could be incorporated into a regression model. Practically, however, we must confess that observations and measurements are not available for quantitative operation of such models. To change certain of these parameters, supply-oriented research will include estimating the effects of investment in alternative forms of education and vocational guidance to increase the occupational and geographic mobility of farm people. Prediction of the effects of the variables involved is hardly possible from regression models based on time series data and will need to rest on other approaches. These will often involve greater judgment and less sophistication, but will still represent estimates in supply.

Similarly, other realms of needed information are not likely to be filled by conventional time series approaches. One example is in supply problems relating to interregional competition. If we knew more about the supply functions which will prevail over the future, or the relative changes which will take place among regions, we could do much better in counseling farmers on investment and occupational choices. We could provide improved credit, educational, and compensation programs in areas where production will either grow or decline. But insights into these relevant supply quantities again are not possible from conventional regression models. Broilers provide an example. Regression analysis of time series data prior to 1941 would not have provided much insight into the extent of postwar change in supply functions among areas. As a basis for occupational and investment decisions, many farm people ask whether the regional supply functions will change similarly among regions for hogs, beef, and cotton. Answers, refined or highly approximate, may need to be given by programming or budgetary approaches. Nevertheless, the empirical efforts which provide insights and answers to these changes and questions will still be those of supply analysis.

ALTERNATIVES AND REFINEMENT IN EMPIRICAL TECHNIQUES

We have tried to emphasize that an extremely wide range of information relating to supply functions is needed, both for policy formulation and educational guidance. A wide range of empirical techniques will have to be applied in providing these predictions. They will include regression analysis of time series and cross-sectional samples, mathematical programming, budgeting, less formal models for forecasting output, and others. They also will need to include analysis of national and regional aggregates of outputs and inputs, individual firm functions, and even relationships inherent in such "fixed plants" as an animal or acre. But these numerous alternatives in mathematical technique and degree of aggregation serve as supplements and complements rather than as rivals. The question is not a discrete one of which technique will prevail. Instead, it is a question of which technique is most appropriate for a particular purpose or set of estimates.

All techniques available to us at the present time have extreme limitations for particular purposes. Methodological implications and improvements relative to the data available have been fully explored for none. While the economic, statistical, and mathematical theory underlying the major empirical techniques is relatively satisfactory, the situation in respect to data availability and measurement is indeed different. If our only purpose were to find isolated bits of data to illustrate that certain facets of econometric technique can be applied to actual observations, we would be in fair position. But if we direct our research at the major relationships and problems of agriculture, the data and empirical problems take on quite a different dimension. The empirical approach will have to be decided as much or more on the data which are available or can be synthesized than on theoretical appeal.

Modifications and improvement of quantitative models will need to come especially from those who operate on data, because they are faced directly with the unique problems of aggregation, multicollinearity among variables, and others of available agricultural data which prevent coefficients and quantities from rolling out in neat form. Because many of these problems are inherent in the data rather than in the theory, we can find out what can and will work only by operating on the data available and shifting to another set of variables, degree of aggregation, or source of information if the first is not successful. Certainly, an important amount of effort must go into this trial and error phase of supply research methodology and a major portion must be done by those concerned with quantitative analysis and applied predictions.

MAGNITUDE OF RESEARCH AND DISCIPLINES INVOLVED

Research relating to product supply and its complement of factor demand has not been entirely lacking for agriculture. Many types of information are necessary for a moderately complete knowledge of supply. Much of this knowledge will come from research conducted several steps away from estimation *per se* of supply functions. Perhaps the greatest void over the past two decades has been that the several types of research which contribute to knowledge of supply have not been organized more systematically in this direction, or have not had the supply estimation or contribution as a more precise objective.

The economists who have worked most with the variables or phenomena underlying product supply and factor demand are those in farm management and production economics. Their central task has been analysis of the quantity and mix of the commodities farmers do produce and those which they should produce. Similarly, they have analyzed how these firms do use resources and how they should use them. They have estimated firm and technological production functions. They have analyzed farmers' expectations and planning procedures and the technologies used by farmers. At times they have aggregated food production possibilities and resource requirements for agriculture of the states and of the nation. They have studied decision-making processes of farmers and the effects of uncertainty and alternative economic goals on choices. These are basic data in knowledge of supply and factor demand. Basically, supply rests on these very micro relationships within agriculture. The foundation of product supply and factor demand is the firm's production function. But given knowledge or expectations of it, the commodities the farmer will produce and the resources he will use in any specified period are tempered by the nature of the farm family's economic goals, capital position, investment in fixed factors, expectations of prices, risk aversion, and related quantities or conditions.

The crucial supply information leading to improvement in agricultural policies and educational guidance must come in macro form, by important regional and national aggregates. We need to study the relationships and decision processes underlying individual output choices if we are to understand fully supply phenomena. Eventually, the quantities so derived must be aggregated or lead to improved procedures for estimating supply quantities from aggregate data. Farm management and production economists, the specialists most centrally concerned with the variables and phenomena leading to manifestation of agricultural supply, also have worked on aggregate supply response as much as three decades back. Such contrasting techniques as budgeting and regression analysis have been used, but micro and macro analyses must be related and integrated to improve knowledge of supply structure and improve forecasts of output and resource use.

In a sense, the entire social investment in biological and physical research for agriculture leads directly to the complex of commodity supply and factor demand. The quantities generated here stand at a level with, or beyond if degree of knowledge and uncertainty is considered, market forces in specifying the variables and quantifying parameters which are important in output and input responses. This statement is made not to suggest that economists need simply to count and measure physical scientists and incorporate them as a variable in their models to predict supply functions and resource demands, but to indicate that the group contains persons contributing information basic to supply knowledge. Should we be able to increase substantially our knowledge of conditions surrounding decisions on various types of innovations (e.g., choice of a new production function and expression of new factor demand functions for the farm firm), we could usefully and more readily project from discovery of a new innovation to future output quantities. In any case, the technologist provides necessary information in respect to supply and factor demand quantities for models which provide estimates of normative character. His efforts and cooperation will need to be enlisted for studies which probe supply relationships and prospective outputs beyond the extreme short run or those which deal with major change and interregional competition.

Sociologists, more than any other group of social scientists, have attempted to describe the processes involved in farmer adoption of innovations. Sociological and psychological sciences are importantly involved in providing information on how choices actually are made. Personnel from these fields should be encouraged to intensify their efforts, but they also should be encouraged to cooperate further with economists and vice versa. The sociologist perhaps needs to consider an innovation less as a discrete production function characterized by pure technical complementarity and unrelated to input and output and prices, uncertainty, and the investment period. Under this implicit assumption, adoption of innovations (e.g., selection of a new production function and choice of a different set of inputs and outputs) comes to rest too much on status, leadership, and similar roles and too little on prices and other important quantities.

Historically, other research and operations contributing to knowledge of supply, factor demand, and output have existed and contributed importantly to knowledge. The series of indices dealing with output, resources used, and employment initiated in the Division of Farm Management and Costs and continued under the Farm Economics Research Division of the USDA has provided certain aggregate measurements for these purposes, as well as projections or expectations of these quantities in the future. Estimates of ultra-short-run output responses have long been provided by the Crop and Livestock Reporting Division of A.M.S. These estimates and forecasts are only one step away from formal supply analysis. Dealing some with intentions to plant, farrow, etc., certain of these estimates are macro quantities paralleling the micro quantities provided in analyses of individual farm decision making and plans. Largely, the forecasts during the growing season are *ex poste* to the time of actual decision making and reflect the effect only of weather variables. Forecasting for these particular purposes would provide better farmer guidance if it could be incorporated into an extended time period and decision framework. At levels of national aggregation, demand studies of the past decade also have included supply equations in simultaneous models. Supply relationships have been included mainly for purposes of identification and specification in estimating demand relationships and structure. They have, however, had but minor emphasis and apply more nearly to an extremely short period wherein price and quantity variables are jointly determined. Finally, numerous interpretative analyses of American agriculture have included less technical approaches to agricultural supply, output, and factor demand. Generally, they have been quite usefully related to agricultural policy voids and needs.

This inadequate historic summary of research relating to supply has been included to indicate that our knowledge underlying agricultural supply is not totally lacking, although it is far from complete. Some major efforts, in terms of stage in development of estimating procedures and data, were invested in supply analysis more than three decades ago. Perhaps the lack has been more nearly one of systematic orientation of these various phases of research toward the supply pole. Needed in the future is more comprehensive and systematic research on the facets of production and choice which have supply relationships as their foci. Some of this research is not simple or it would already have been accomplished. Complex problems exist in (1) using aggregate time series data to provide more than a description of past relationships and a basis for short-run predictions (even with so-called long-run models) under technological and other revolutions in structure, (2) meaningful aggregation of estimates from firms and samples, and (3) establishing correspondence between normative and positive estimating procedures. The "shifters" in supply differ greatly from those in demand. Even population growth, per capita income, and income elasticities of demand do not have parallels in supply which can be measured and quantified in a simple, useful manner. It is unlikely that anyone will ever estimate an average price elasticity in supply which will have the same utility and degree of permanence as those which have or can be estimated for demand. To suppose that this can be done is either wishful thinking, hoping for the impossible, or assuming unlimited research resources. Yet it is extremely likely that much more useful quantitative knowledge can be derived with available time series data and empirical tools than has been accumulated to date.

THE PRODUCTION FOUNDATION

As mentioned previously, the production function is the foundation of supply. Under conditions of perfect knowledge in respect to all variables, a firm's static supply function could be derived directly from the production function, given a goal of profit maximization for competitive firms. Using the algebraic form in equation 1 for simplicity, we can illustrate the relationship of the production function to both the static short-run and long-run supply functions and the factor demand function. For the short run, we have the production function in equation 2 where $c = aX_2^{b_2}$ and X_2 is fixed at some specified level. The long-run total cost function is equation 3 when both factors are variable and p_1 and p_2 are prices of the respective factors. Substituting $k = p_2X_2$, the value of the fixed quantity of X_2, into equation 3, we obtain the short-run total cost function (equation 4).

(1) $Y = aX_1^{b_1}X_2^{b_2}$ (2) $Y = cX_1^{b_1}$

(3) $C = p_1X_1 + p_2X_2$ (4) $\bar{C} = k + p_1X_1$

Returning to the short-run production function (equation 2) and expressing input as a function of output, we obtain equation 5 where $n = b_1^{-1}$. Substituting the value of X_1 from equation 5 into equation 4, we obtain the short-run total cost function in equation 6. The marginal cost equation, the derivative of $\bar{C}$ with respect to Y from equation 6, thus becomes equation 7 and is a function of output. Equating equation 7 to p_y, the price of the product, and solving for Y, we obtain the form of the short-run supply curve in equation 8 when X_2 is fixed in magnitude.

(5) $X_1 = c^{-n}Y^n$ (6) $\bar{C} = k + p_1c^{-n}Y^n$

(7) $\frac{d\bar{C}}{dY} = np_1c^{-n}Y^{n-1}$ (8) $Y = \left(b_1c^n\frac{p_y}{p_1}\right)^{\frac{1}{n-1}}$

The magnitude of output is a function of the production coefficients b_1 and b_2, given the magnitudes of X_2, and the commodity prices.

Deriving a short-run factor demand equation, we can multiply equation 2 by p_y, the price of the product, to obtain a total value function (equation 9). Taking the derivative of V with respect to X_1, we obtain the equation of marginal value productivity in equation 10. Setting equation 10 to equal the price of the variable factor, p_1, and solving for X_1, we obtain equation 11, the static demand function for the factor.

(9) $V = p_ycX_1^{b_1}$ (10) $\frac{dV}{dX_1} = b_1p_ycX_1^{b_1-1}$

(11) $X_1 = \left(b_1^{-1}c^{-1}\frac{p_1}{p_y}\right)^{\frac{1}{b_1-1}}$

Returning to the long-run production function (equation 1) and cost function (equation 3), we can derive the long-run static supply function. First, we obtain the marginal rate of substitution of X_2 for X_1 in equation 12. Equating this to the ratio of factor prices, $p_1^{-1}p_2$, and solving for the expansion line for the given price ratio, we obtain X_1 as a function of X_2 in equation 13. Substituting this value of X_1 into equation 3, we obtain the long-run total cost equation in equation 14. Substituting the value of X_1 from equation 13 into equation 1, we obtain the long-run production function in equation 15 which supposes X_1 and X_2 always in

$$(12)\quad \frac{dX_1}{dX_2} = \frac{b_2 X_1}{b_1 X_2} \qquad (13)\quad X_1 = \frac{b_1 p_2}{b_2 p_1} X_2$$

$$(14)\quad C = \left(\frac{b_1 p_2}{b_2} + p_2\right) X_2 \qquad (15)\quad Y = a\left(\frac{b_1 p_2}{b_2 p_1}\right)^{b_1} X_2^{\,b_1+b_2}$$

proportions which minimize costs. From equation 15 we express X_2 as a function of output in equation 16 and substitute the latter value into equation 14 to obtain the long-run total cost equation in 17 where cost is expressed as a function of output. Now, taking the derivative of equation 17, the long-run marginal cost equation is 18. Setting equation 18 equal to p_y, the price per unit of product, and solving for Y, we obtain the form of long-run supply equation in 19.

$$(16)\quad X_2 = \left[a^{-1}\left(\frac{b_2 p_1}{b_1 p_2}\right)^{b_1} Y\right]^{\frac{1}{b_1+b_2}}$$

$$(17)\quad C = \left(\frac{b_1 p_2}{b_2} + p_2\right)\left[a^{-1}\left(\frac{b_2 p_1}{b_1 p_2}\right)^{b_1} Y\right]^{\frac{1}{b_1+b_2}}$$

$$(18)\quad \frac{dC}{dY} = \frac{1}{b_1+b_2}\left(\frac{b_2 p_2}{b_2} + p_2\right)\left[a^{-1}\left(\frac{b_2 p_1}{b_1 p_2}\right)^{b_1}\right]^{\frac{1}{b_1+b_2}} Y^{\frac{1-b_1-b_2}{b_1+b_2}}$$

$$(19)\quad Y = a^{\frac{1}{1-b_1-b_2}}\left(\frac{b_1}{p_1}\right)^{\frac{b_1}{1-b_1-b_2}}\left(\frac{b_2}{p_2}\right)^{\frac{b_2}{1-b_1-b_2}} p_y^{\frac{b_1+b_2}{1-b_1-b_2}}$$

The optimum long-run output, Y, supposing that price and the production function are correctly anticipated and the farmer maximizes profits, then is determined by the price of the two factors, p_1 and p_2, and the price of the product, p_y.[2]

[2]Given interest in elasticities of product supply and factor demand in relation to product price and factor price, respectively, we could compute the short-run elasticity of supply and demand, respectively, from equations 8 and 11. The long-run supply elasticity could be derived from equation 19. Similarly, we could derive a long-run resource demand function paralleling equation 19 and compute elasticities accordingly. However, we do not do so in order to conserve space and because their derivation is obvious.

Point of Departure

The static supply function above, derived from the relevant production function and set of commodity prices, thus provides a conceptual starting point in analysis of farmer output responses. By incorporating variables to represent new innovations, the knowledge of productivity coefficients for very particular resources previously thought to be zero, we could account for technological change. Of course, the assumptions implied in deriving supply and demand functions such as equations 8, 11, and 19 from the production function (equation 1) hardly square with decision-making conditions of the real world. If they did, we would only need derive production functions for farms of a sample, aggregate them by appropriate weights, and produce the regional or industry supply function. Or, under certain conditions, unlikely ever to be completely fulfilled, we could estimate the production function from an interfarm sample and derive a single supply function directly from it. Yet even though empirical operations of the latter type are not directly possible, the equations 1 through 19 generally provide the inventory of types of variables and parameters we try to use and estimate in deriving actual output response functions by means of regression procedures or in projecting possible responses by programming, budgeting, or related techniques. In fact, the normative supply functions derived by budgeting and linear programming generally employ the same assumptions as implied in going from equation 1 to the supply and demand equations in 8 and 11, respectively. However, they also include, as well as a moderate dose of subjective judgment, certain other assumptions about the nature of fixed resources and form of the production function. Use of normative procedures such as programming or budgeting does not obviate need for knowledge of the production function.

Complexities Relating to the Production Function

As stated above, supply functions could be derived directly from production functions if uncertainty, capital rationing, lack of knowledge, nonmonetary goals, and lumpiness of fixed factors did not exist. Absence of these and related conditions would allow us to estimate production functions first, then derive the product supply and factor demand functions. Even in the absence of these conditions, we would still be faced with empirical difficulties in estimating the underlying production functions from which the supply and demand equations must be derived.

One difficulty is that relatively few firms in agriculture produce single products. This fact would not bother us if (1) all products were competitive technically, produced together only because of the relationships between prices and substitution rates, and (2) the inputs used for each could be measured accurately and independently. But in most farming regions, commodities are produced in combinations because they are complementary and supplementary over some range. Too, services of many resources cannot be allocated very precisely among the several products into which they are transformed. This is true for durable types of assets which give rise to flows of input services regardless of the quantity of use for a particular product. Then, too, the degree of multicollinearity, difficulties of measurement, and inability to incorporate a large number of unique input categories into a satisfactory set of regression estimates necessitate aggregation of resources into a few gross categories for farm production function studies. The fact that some important resource categories are neither pure complements nor substitutes but serve as both, within the input magnitudes usually encountered, also complicates problems of estimation. Similarly, except in a few highly specialized and peculiar climatic areas, outputs must be aggregated by value transformations. These aggregation requirements themselves prevent derivation of clear cut commodity supply functions from production functions.

Many other measurement difficulties also prevent us from deriving production functions which can be used for computing clear cut normative supply functions. For this reason, economists have turned to budgeting and programming to estimate what farmers might produce under pure goals of profit maximization and perfect knowledge of production and price parameters. (These alternatives do not, as mentioned previously, eliminate need for knowledge of production functions.) Particularly bothersome are errors stemming from specification biases and inability to measure inputs such as management, information, and related items (cf. 6, 10). For resources clearly used up in a single production period, as seed for annual crops, measurement is simple. Slightly more difficult is measurement of inputs of fertilizer where some residual remains. At a higher level of difficulty are semidurable capital items such as machines and buildings which may provide service in proportion to some uses but which also depreciate even under nonuse. In the case of seed and, even though imperfectly, fertilizer, we measure capital input by value or input of the resource itself. We cannot measure input for machines and buildings similarly. We can attempt to measure input by services or depreciation during a particular production period. Yet given the mixture of stocks and flow services representing these assets, these efforts will usually lack complete accuracy. If interfarm differences in technology could be adequately identified and measured by input categories, farm production functions could be estimated, and product supply and factor demand equations derived from them much more readily and meaningfully.

Obviously, additional investigations are needed to establish a closer and more useful empirical linkage between production functions and supply. The major portion of public investment in physical and biological research relates to fully discovering or changing the production function. One major attempt to link knowledge of the agricultural production function and supply or output was the agricultural production capacity studies conducted in 1951 by agricultural economists and technologists (1). Black and Bonnen used essentially these data in projecting output to 1965, without measurement of possible effects of prices and other relevant variables in altering the mix over the next two decades, to point up the likelihood of a continuing surplus problem (3). Certainly, we could use a much more formal and systematic linkage between these major research efforts in the general realm of the production function and supply.

POSITIVE AND NORMATIVE APPROACHES

The slight excursion into the realm of output and supply has already brought us into contact with concepts of what farmers do and what they can, might, or should do. These are the two poles from which agricultural supply has been attacked in the past. They will continue to provide the two major directions from which empirical estimates are approached. Whether the one or the other approach is used will and should depend on the nature and purpose of the estimates. Each has its limitations, as well as advantages, for particular purposes and in respect to particular estimational objectives.

Terms which have come to broadly categorized the two separate approaches are positive and normative. This distinction stems partly, but not entirely, from J. M. Keynes' early discussion of methodology in political economy (13). Positive analysis has come, especially in considerations of supply by the North Central Farm Management Research Committee, to mean prediction of quantitative relationships among variables as they actually do exist at a point in time, or have existed over a period of time. Other terms sometimes used to describe this same type of empirical effort are descriptive and predictive. Within the limitations of technique, the analysis describes structure as it actually exists, and, hence, can be used to predict the magnitude of one variable from the magnitudes of others. In contrast, normative analysis refers to what ought to exist, under certain assumptions. The term normative departs from the Keynesian concept in the sense that it is not an ethical or value consideration, but simply an indication of what might be expected to happen if decision makers possess certain goals and knowledge and are free from certain resource and institutional restraints. Both the positive and normative approaches entail formulation of empirical models for use in predicting or estimating real world quantities. The efficiency of either thus depends on whether the relevant variables are included and accurately measured in the empirical model and how well they correspond with the real world conditions as they will exist during the period for which predictions are to be made.

The major tools for positive analysis are regression procedures, less refined methods of projection or others which attempt predictions from observations drawn out of the "actual operating world." The major tools for normative analysis include budgeting, programming, judgment, and related techniques. Here, certain assumptions are normally made about goals and actions of decision makers. Quantities consistent with these are derived. A somewhat pure example of this approach is illustrated in certain linear programming analyses of supply where the resource restraints are defined to represent different degrees of fixities and lengths of run, with programming used to specify optimum or profit-maximizing outputs at different levels of factor or product prices. Budgeting procedures such as those used by Mighell and others (14) are similarly normative, except that the estimates arising were more tempered, as one subjective linkage with positive aspects, with judgment of what farmers would do. How closely programming results parallel actual outcomes will depend, just as is true for budgetary analysis, on the manner that restraints are built into the model to correspond to the real world inflexibilities. Normative product supply and factor demand functions also can be derived from statistical production functions. The steps outlined in equations 1 through 17 illustrate the method.

Both positive and normative approaches have been and are being used because of the limitations of the estimates derived by each. Our conscience could rest if positive approaches existed enabling us to use coefficients generated in the actual process of farmer choice and decision in more accurately predicting production response at relevant periods in the future. But here is the major limitation of regression studies. Regression models based on time series observations cannot predict in light of new variables and structures, previously unencountered but known to exist for the future. They are necessarily tied to the past and are reflections of historic relationships. No satisfactory method is in sight for incorporating major changes in technology, institutions, and government policy into regression approaches. In supply, it is the quantity of the future, rather than the record of the past, that is important. The linkage is much weaker and less important in producer response than it is in consumer demand. True, most regression models of supply functions, of either the so-called short-run or long-run types, are useful and quite accurate for short-run predictions of aggregate outputs. This is particularly true for models where output in period t is regressed on output in t-1. Because of statistical necessity, regression models are highly aggregate in respect to inputs and cannot reflect quantitative effects of many specific variables of interest.

These limitations of regression models have caused research workers to turn to budgeting and related techniques. Models of the latter type allow analysis of the possible effect of new variables on the horizon and more detailed examination of specific variables. Estimates of product outputs and factor demands for more individual commodities can be analyzed. They also provide a method for estimating supply for firms where time series observations are not recorded or available and samples for cross-sectional analysis can provide only a set of mongrel relationships among short-run and long-run functions over an extremely small range of prices and similar parameters. Normative programming models also have an advantage over descriptive regression models in dealing with length of run as it relates to supply. Magnitude of output can be related quite precisely to extent and kind of fixed assets with programming, but not with regression models.

However, normative procedures, in turn, have had limitations not associated with the major positive procedures. One of these limitations concerns spatial aggregation. While national or regional aggregates can be handled quite readily by regression models, the same is not true with a programming model, for example. A programming model, using a single region or the nation as the aggregate producing unit, could be easily devised to meet all mathematical requirements of the technique; but the results might have little meaning. If restraints of sufficient quantity and variety are included, it might generate quantities paralleling those realized in the past. Yet these same restraints, devised to tell the historic story, would have the same limitations as a regression model in predicting a future subject to important technological or institutional changes. A regional or national model, formulated to represent a single producing unit and to allow a new environment of technology, institutions, and response, would be unlikely to provide a supply function approaching one representing the aggregate for individual firms producing under a variety of conditions in respect to soils, capital, tenure, fixed resources, and other variables which modify farmers' response to product and factor prices.

✓ In contrast to a programming model for a region as the producing unit, one can be derived for individual farms of a regional sample. A normative supply curve then can be computed for each farm, either separately or as part of a single computational model. If a representative sample is used and programming functions are computed for each farm, these can be aggregated directly, either after programming computations or in the computational process, to give a normative supply function for the region. (Use of "typical" farms gives rise to aggregation problems of greater complexity.) However, even though approaches such as these can be used in estimating an aggregate supply relationship of normative nature, the computational and financial burden would be great for aggregates at the national level.

While all normative and positive approaches have limitations unique to their type, each can add something to knowledge about product supply ✓ and factor demand in agriculture. Our current knowledge in respect to the effect of numerous variables on product supply and factor demand and use is relatively small. Even though they are tied closely to history, regression and other positive approaches are useful in giving some indication of the quantitative relationship between price and related changes and supply as they exist under actual decisions of ✓ farmers. Predicted for relatively small homogeneous regions, problems of the product and factor aggregation can be partly overcome. Similarly, a material increase in the magnitude of normative analyses may well provide means for overcoming difficulties inherent thus far in the procedure and for relating predictions from this method with those of regression estimates.

Additional Approaches and Comparisons Needed

Studies at various levels of geographic, product, and factor aggregation are needed, regardless of whether positive, normative, or related methods are used. Firm studies which can better quantify the decision-making process of individual farmers in respect to uncertainty, fixed assets or investment policy, technical innovations, and nonprofit goals also can lead eventually to greater knowledge of aggregate product supply and factor demand. Generally, these will need to be made over a considerable number of years with farm samples constructed to account for firms which both enter and leave the supplying scene. Such samples have been used for periods of 2 or 3 years (11, 17), but they will need to be extended over much longer periods if they are to provide detailed and dependable findings relating to the dynamics of supply. The decision-making processes of farmers and their plans in view of price and other expectations need to be linked more closely with their actual plans and outputs. On an aggregate and short-run basis, a partial linkage has been made in planned and actual inputs (and, hence, indirectly in outputs) through the crop and livestock estimates of the Agricultural Marketing Service. Data for intended and actual farrowing and planting are available as time series observations. However, these have not been sufficiently analyzed to indicate the quantitative effect of prices and other variables in causing deviation between plans and commitments or actual inputs and outputs.[3]

Linkage between normative firm supply functions derived by programming and statistical (sample) studies of farmers' intentions (decisions on inputs and outputs in respect to price and other expectations) and actual investment and outputs also is needed. Given this connection, we would have knowledge of the extent to which normative quantities must be discounted or otherwise modified to conform with (1) farmers' planned inputs and outputs and (2) actual investments and production response. This knowledge would provide an improved basis for projecting from major structural changes on the horizon to investments and supply or output over the longer run.[4] Models based on inventory (2) and decision theories (5, 12, 16, 18) may have some utility in making this linkage. Finally, as mentioned earlier, the normative supply functions which can be derived from statistical production functions and the supply functions derived from programming models need to be linked with actual decision-making processes of farmers.

DYNAMICS OR CHANGE IN SUPPLY

The major challenge in empirical supply analysis is to identify, measure, and express the quantitative effect of variables which cause

[3]These data, although extremely useful for the short-run projections intended, extend over a period which is too short for determining the quantitative importance of variables relating to longer-run investment decisions and the dynamics of supply.

agricultural supply to change with time. Some agricultural economists regard this, aside from short-run outlook projections which need be little concerned with the dynamics of supply and which can be based more on historic estimates of structure, as the only justification for large outlays for supply analysis. They would classify regression analysis of aggregate time series data largely as empirical doodling to illustrate certain logical arguments in mathematics and economics. Certain micro programming analyses would be similarly classified. The situation of agriculture and the pressing problems of the industry, they contend, call for "forward analysis," since past elasticities or the difference between short-run and long-run elasticities over past decades have little import for the future. They would emphasize that solutions of agriculture's problems depend on the changing structure of markets and supply, and on control in these structures by agriculture.

This writer would agree generally with this concept of the agricultural supply problem, particularly as one of projecting into the future and having weak links with data and coefficients of the past. Technological change, developments in market institutions and structure, government programs, increased educational and informational services leading to greater on-farm and off-farm mobility of resources, and related phenomena limit the usefulness of coefficients based on time series data. Yet it is largely by analysis of data available in this form that we can more fully understand the dynamics of supply — the change in supply over time or the relation of output in one period to the magnitude of variables (which can be measured) in earlier periods.

Fixed Resources

The existence of fixed resources, as simple as the concept might seem, poses important estimational problems in supply analysis. We are acquainted with the orthodox concepts of short-run and long-run supply, and the fan family of supply functions over different time periods as the restraints of fixed resources are lifted. Yet, to date, we have been unable to incorporate these types of relationships into regression analyses at either the macro or micro level. We can handle these relationships better with programming models, but we are still confronted with difficulties in deriving aggregate output responses for different periods corresponding to levels of fixed factors. The latter models are no better in supply prediction than the assumptions made in respect to fixed resources and technical coefficients. The usefulness of programming models in supply projections approaching reality will depend not only on the extent to which (1) appropriate statistical distribution of resource fixities has been used over time and among firms and (2) inputs of one period can be related to outputs in later periods, but on the extent to which (3) the effects of other considerations that place differential restraints on production over different time periods can be incorporated.

Fixity is a problem, especially because the period over which services are provided differs greatly among resources. Even fertilizer, a resource which would appear to have little fixity, applied at one time has residual response effects for different months within the season and between production years. Some resources consist mostly of flow services provided at particular rates in given time periods regardless of whether products are produced. Within the period, the prices of the resources or their services have little relationship to production response. Outputs of one period are supplementary to outputs of another period, and product prices between periods may have little relationship to the distribution of outputs over time. Services of buildings, machines, and labor with low mobility fall in this category, as do other resources in the extreme short run.

Other fixed resources represent stock services, with the amount available in one period depending on the amounts of services used and products produced in another period. Harvested feed for cash sales or livestock production is an extreme example, but certain of the services of machinery and land also fall in this category. The outputs of different periods, then, are competitive and can be related to prices of the same product in different time periods. The space services of land are so represented, and soil may be fallowed or cropped depending on the price in one year as compared with two years ahead. In contrast, other products may be complementary in respect to use of a resource or its services. Corn output, summed over a period of years, can be greater if the land is used for legumes this year because nitrogen and soil structure produced by hay become inputs for grain. Or, complementarity may surround the moisture services of a fixed farm acreage, with wheat output greater in t because land was fallowed in $t-1$ (8).

Even aside from other complexities surrounding changes in output response with time, we have few empirical measurements relating supply functions of different periods and their change with fixed resources. Of course, we have knowledge of the contrasting response of output to price within breeding and planting periods, when brood animals and planted acreage are fixed, as compared with interyear differences for individual commodities. But we have not yet been able to use regression analysis to penetrate much further into this general problem of time and fixed resources in relation to supply elasticity, especially in respect to agricultural output in aggregate. Some major conflicts in policy elements to remove surpluses and low incomes rest on suppositions in respect to the degree of fixity of resources, the nature of price alternatives for their services, and the corresponding output response in agriculture. The hypotheses which might be generated from Cochrane's work (4) as compared with current proposals of free market prices is an example.

Expectations and Uncertainty

Little connection also has been made to date between studies of farmers' expectations and uncertainty and the dynamics of supply. Starting at the other end, in his pioneering empirical work Nerlove has interestingly introduced concepts of distributed lags into aggregate regression analysis of supply to indicate how change in price in one period might be reflected in lagged producer behavior in later periods. Here the realistic assumption is used that farmers do not make full adjustments within a discrete period, but instead distribute their adjustments among future periods until they finally approach some optimum or maximum position. The supply elasticities are based on a model assuming certain characteristics of price expectations for farmers. Uncertainty surrounding price expectations provides one reason or basis for using a model supposing distributed lags in response.

Expectations in one period relative to prices in the following period might be held with great uncertainty and discounted accordingly. Hence, adjustment of production to this "expected" level would not be as complete as in the next period for which the same "most probable level of expectation" might be held but with less uncertainty because of knowledge gained over time. Hence, adjustment of production toward a given "most probable" or "normal" expected level of price should continue with time as knowledge is gained and uncertainty declines.

This approach appears especially appropriate for changes in plans prior to a response period. Assume for example, that a hog producer begins formulating his expectations for hog prices in May of year t in July of t-1. He is preparing plans for breeding in November of t-1, with farrowing and sale in March and September, respectively, of t. If the expectation of "normal" or "most probable" price formulated in July of t-1 is surrounded with great uncertainty, his adjustment in planned breedings and farrowings may be small. If he holds the same normal or most probable expectations of price in August, his planned breedings and farrowings may be adjusted nearer to a possible optimum or maximum. September, October, and November may lead to further adjustments toward this optimum if his knowledge increases and uncertainty declines regarding the same expectation of normal or most probable price. Similar adjustments may be highly realistic between years in building up dairy or beef herds where more time is required; knowledge may increase and uncertainty may decrease with time and the normal or most probable price expected remains similar between years.

But where prices fluctuate considerably and an entirely new normal or most probable magnitude of expected price arises frequently or before each period in which resources are recommitted, as continued or lagged adjustment toward a possible optimum or maximum probably does not occur. Hence, a similar degree of uncertainty may arise each year, rather than decrease over several years with further adjustment to the optimum ordered accordingly. Finally, adjustments which do take place for many products are not made against a price expectation in a single period but against those of several periods over which new investments must be made.

Some important new ideas relating to expectations and dynamic supply adjustments have been injected into the empirical streams by studies such as Nerlove's. However, much or most of the warp and woof of expectations and producer response is left to be unraveled. The task is difficult and may well be accomplished first at the micro and less aggregate level of analysis.

Technological Change

The truly important economic and adjustment problems of commercial agriculture revolve around the national aggregate of output. The important dynamic foundation of changes in aggregate output is the numerous variables encompassed by the phenomenon termed "technological change." These variables are difficult to measure and express in direct quantitative and logical relation to supply. New resources arise as specific capital items or innovations, and they do not have price observations tying them with time series observations of other variables. Even if they did, they are numerous and cannot be introduced separately in a model of modest aggregation. The production processes (research in private and public institutions) which give rise to them logically fit into the framework of supply and factor demand, but true quantitative relationships are thus far lacking. We have employed models with catch all variables such as time and lagged output (largely a substitute for time in input, output, and consumption series of the types typically analyzed by economists), but we have accomplished little in relating inputs and outputs of this general process and category to agricultural outputs in later periods.

SUPPLY OF FACTORS

The three considerations mentioned above (fixed costs, expectations and uncertainty, and technological change) provide the most important areas for research relating to producer behavior in different periods and change in supply over time. Perhaps equally important in explaining other unique characteristics of agricultural supply is study of the supply of factors to agriculture. When we can better explain the supply functions and reservation prices for such factors as labor, land, and capital improvements in farming, we will have gone most of the way in getting at some of the elasticity quantities which give rise to surplus and income problems within agriculture (and to debates among agricultural economists). We know so little about supply relationships for farm labor that we cannot predict the timing and income levels under which different price schemes would eliminate the surplus problem.. We do not fully understand why such large quantities of labor can be withdrawn from agriculture, as during the last decade, without decreasing total output, or why the process of migration and farm consolidation does not occur more rapidly. Similarly, we know little about price levels which would cause land and auxiliary resources to be shifted from crops in surplus to grass, forestry, and recreation areas. Neither do we know anything of importance about the dynamics of this supply situation and the lag with which shifts would take place, or the lag in labor migration and the persistence of income depression. Even though we are investing large quantities and directly paying prices to build up a supply of land in nonagricultural uses (land withdrawal and soil banks), we know practically nothing about the supply function of land in particular regions or the relationship of labor supply to land supply within this complex. Knowledge in these areas of factor supply can serve as the basis for guiding individual farm adjustments. Given more information on labor migration and land availability, we would know more about opportunities and costs and timing for farm consolidations and capital acquisition.

From the standpoint of major national farm problems, supply knowledge at the level of aggregate output for all commodities is more important than detailed knowledge of elasticities and coefficients for a large number of individual commodities. This aspect should not be forgotten, as it might, as momentum in supply analysis and producer behavior increases. Certainly, refined statistics for individual commodities and farming areas will increase our knowledge in the general area of supply. We need them for both individual guidance and policy. But unless an elaborate model of computational feasibility containing the appropriate numbers and forms of equations can be formulated, we will still know little about the forces molding the aggregate agricultural output. Our start here is probably in factor supply and its dynamics.

THE NEED

We have pointed out only a few of the major concepts, problems, and social implications relating to increased knowledge of agricultural supply and producer behavior. While the area of research is receiving increased attention, much is left to be done. This conference represents an attempt to focus emphasis on this need. It should serve as an aid in exchange of knowledge and hypotheses, as well as an interpersonal stimulation of imagination. This is the purpose for which it was designed, and its product should certainly flow forth in future years, even if only in distributed lag fashion.

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Discussion

HEADY'S PAPER provides an excellent springboard for this workshop on "Estimating and Interpreting Farm Supply Functions." It provides a comprehensive framework for an understanding of both the nature and the urgency of some of the improvements needed in analyses of supply response; the critical relationships between supply response and resource inputs, prices, and related factors; both the history and the current state of the arts as well as some of the obstacles which confront us in our efforts to unravel these relationships; and the advantages, shortcomings, and needed adaptations of the analytical tools generally in use in this broad area of research. That I have no major quarrel with the paper will be readily apparent.

Heady seems to agree with one of the conclusions of the conference on "Adjusting Commercial Agriculture to Economic Growth" called in Chicago in 1957, by the North Central Farm Management Research Committee namely, that instead of attempting to manipulate food demand, it is "the mechanism associated with achieving adjustment of farm production which should be analyzed in our efforts to solve the farm problem and achieve future economic adjustments." He does stress, however, the importance of factor demand and prices, and suggests that the basic problems of American agriculture have their roots in the demand and supply of resources used by the industry.

Our fundamental need is represented as the need for basic knowledge which will relate product output to factor inputs and provide a framework for altering the supply structure by adjusting production and resource employment with economic growth. Societal concern, rather than the individual firm, and forward-looking appraisals, rather than those of historic relationships, are stressed. A wide range of information relating to supply functions is said to be needed, as is a wide range of empirical techniques, with data characteristics and limitations frequently the major determinants of the most appropriate techniques to be used. The latter point, I think, is especially important today with all of the gaps and inadequacies in our data; but I hope the day is not too far away when we can have access to the data required to utilize fully our most appropriate techniques.

The production function is deemed to be the foundation of supply, but product supply and factor demand functions cannot be derived directly from production functions because of uncertainty, capital rationing, imperfect knowledge, lumpiness of fixed factors, nonmonetary goals, and various measurement difficulties. Normative analyses, that is, analyses of what producers would do if they had certain goals and knowledge and were free from certain resource and institutional restraints, have been developed in an effort to circumvent some of these difficulties. Inherent in such analyses, however, are both an aggregation problem and, even more important, a problem of relating normative supply functions to actual response. In contrast, positive or predictive analyses using regression procedures to quantify relationships among variables as they have existed or do exist at a point in time have their major limitation in the fact that they cannot predict effects of new variables and structures not encountered in the basic time series observations. Such models also are highly aggregative in respect to inputs and, as such, have limited ability to reflect quantitative effects of very many specific variables.

Heady concludes that despite their limitations, both normative and positive approaches can add something worthwhile to our knowledge about product supply and factor demand in agriculture. He suggests, and certainly I concur, that we need to increase our analyses of producer panels in which we attempt to link normative firm supply analyses with studies of farmers' actual response and thereby develop a basis for discounting normative quantities to conform with production decisions. He suggests the need for increased effort to relate normative predictions with those of regression estimates, and for increased attention particularly to the nature and effects on supply functions of fixed costs, uncertainty, and technological change. He especially emphasizes the need for analyses of factor supply. I have no quarrel with these suggestions.

Throughout his paper, Heady's emphasis is almost wholly on commercial agriculture and societal interest. However, we must continue to consider the interests of the individual producer. I question, for example, whether time-series analyses which allow forecasts for a single production period may be adequate as a guide for the year-to-year production decisions at the firm level. Despite the overriding importance of policy decisions requiring positive analyses, I suggest that we should not abdicate our responsibility to the individual firm, and that the individual producer has a real need for normative analyses indicating the economic consequences of alternative production decisions. I also believe that analyses of normative supply functions for representative firms and of means of quantifying the effects of various causal factors on variations between normative and actual response by such firms are among our most promising avenues to a better understanding of supply response.

The implied emphasis on commercial agriculture in Heady's paper also seems warranted. In this day of surpluses, we should probably concentrate on learning what makes the commercial farmer tick, or cease to tick. But does not the increasing importance to commercial farmers of income from employment at nonfarm jobs suggest that we need to make specific provision in our models for the modified and additional motivations and restraints inherent in this trend?

Similarly, do we not need to give a great deal more attention to economic conditions and to changes in the nonfarm sectors of the economy in our efforts to analyze the supply of factors to agriculture? For example, what effect does the business cycle have on labor transfers out of agriculture? More people have left farms since 1930 than now remain on farms, but how many would have left if business activity had been relatively limited throughout this period? What about the effect of nonfarm capital, our tax structure, and the business cycle on factor values? I am sure Heady would include such inquiries in his analyses of factor supply, but it seems to me that they need greater emphasis if we are to extricate ourselves from some of the traditional ruts inherent in overemphasis on intrafirm analyses.

Among other points that appear to need additional emphasis are those relative to the importance of cooperation with physical scientists in probing supply relationships and prospective outputs, and with sociologists and social psychologists in probing decisions. There also is need for emphasis on analyses of changes in the supply structure, by regions, and on the limitations imposed on our programming models in dealing with regional and national aggregates by a variety of conditions with respect to soils, capital, tenure, fixed resources, and other variables.

Finally, one might read into Heady's paper an implied need for a "bank" of production functions representing significantly different production situations, assembled and kept up to date over time as pertinent analyses are made, and available to all whose analyses strive to facilitate the adjustment of production and resource employment with economic growth. Certainly, others have expressed such a need as they have discovered the dearth of usable data, resulting in part from the fact that data simply were not assembled and retained in a form usable by others. Perhaps a part of our discussion should focus on the feasibility and the means of maximizing the product of the resources required in the tremendous job of assembling basic data.

PART II

Regression Analysis of Aggregative, Time-Series Data

Chapter 2

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Time-Series Analysis of the Supply of Agricultural Products

IN A RECENT SURVEY of the problems involved in the analysis of agricultural supply, the author and Bachman have classified these into five major categories: problems connected with (1) the complex structure of production, (2) technological change, (3) aggregation, (4) investment in fixed or quasi-fixed factors, and (5) uncertainty and expectations (1, pp. 9-23). While it is not proposed to repeat here what was said in this survey, all the problems mentioned are encountered in time-series supply analysis. It therefore seems appropriate to delineate them briefly.

The Complex Structure of Production

As is well known, agricultural production in the United States is composed of hundreds of different products and requires scores of different inputs. To a lesser degree the same is true at the level of the individual firm. It is rare to find a farm producing a single homogeneous product and using but a few well-defined factors of production. Furthermore, production of the same commodity may be carried out in a very different manner in one part of the country than in another. In the case of many industrial commodities, the relations among products and factors are relatively simple or can be well approximated by models which neglect many of the interconnections in the structure of production. However, in the agricultural sector, relations among products and factors are both strong and numerous at whatever level from firm to industry we choose to consider (19, especially chaps. 5 and 7). The complex structure of agricultural production leads to serious problems in time-series supply analysis for two reasons. First, time-series are generally short relative to the number of variables which it would be desirable to include in statistical analyses in the light of the complexity of agricultural production; hence, only relatively few may be taken into account. Second, because many time-series, particularly prices, tend to move together over time, the separate effects of even those few variables included frequently cannot be discerned.

Thus, we have of late begun to turn to the study of the production functions of individual firms or to time-series analyses related to geographical regions. While the value of this sort of investigation should not be denied, it should be pointed out that (1) most of the policy uses to which knowledge of agricultural supply is germane require knowledge of the behavior of aggregates over time, and (2) the link between results on a disaggregative level and the knowledge needed for policy purposes is by no means obvious and indeed beset with many problems. In the appendix, one of the ways in which aggregative time-series and other types of analyses can supplement one another and lead to greater knowledge of agricultural supply is pointed out.

Technological Change

Changes in technology are to supply analysis what changes in tastes are to demand analysis. The former are likely to be much more important, both in terms of their frequency and their impact on supply conditions. At a point in time or over a relatively short period, the assumption of unchanging technology may be a good approximation to reality. But over long periods of time, to which time-series supply analyses refer, it is clear that this assumption is a poor one. In time-series supply analyses for individual agricultural commodities, a simple trend has generally been used to take account of the effects of changing technology. In the analysis of the agricultural sector as a whole, and to some extent in the analysis of individual products, Cochrane and others (10, 11, 12, 39) have attempted to analyze the effects which prices and other factors have on the adoption of new technology. There is no doubt that this is an important first step. Further progress may lie in the recognition that the adoption of new technology and its effects on the productive process are closely related to the problems of uncertainty and investment. Because of the complex effects of technological changes on the ways in which individual commodities are produced, there is special need for studies on a disaggregative level.

Aggregation

Since time-series supply analysis generally deals with national aggregates which are of direct interest for policy purposes, it might be thought that aggregation problems play no role. However, this is incorrect for several reasons. As already remarked, the complexity of the structure of production in agriculture frequently leads us to study subsectors, which may be geographic, product, or both. Aggregation problems arise when an attempt is made to use such results for purposes of national agricultural policy. Furthermore, the necessity of confining attention to but a few relevant variables in time-series analysis is itself a form of the aggregation problem. Finally, because much of our knowledge of supply, both theoretical and empirical, is disaggregative, and because of our desire to supplement time-series analysis with such knowledge, the question of the connection between the two arises. The aggregation problems thus encountered are not inherent in time-series analysis, but are clearly related. A few tentative remarks on the question are made in the appendix, where a model which reveals the influence of an unequal distribution of fixed factors and technical knowledge on the elasticity of industry supply is discussed.

Investment in Fixed or Quasi-Fixed Factors

Fixed factors of production form the basis for the traditional distinction between short- and long-run supply response to price, as well as a similar distinction in the theory of costs at the firm level. Of course, fixed factors are not really fixed for all time but can and will be varied in response to economic forces. The study of such variation is the subject of the analysis of investment decisions. Glenn Johnson (24) has emphasized the importance of investment decisions to supply behavior and offers a number of constructive hypotheses about the relation among acquisition costs, salvage values, and expected marginal productivities which would appear to promise increased understanding of the effects of changes in price on resource use and output.

The principal criticism of this author of much of Johnson's recent work on this subject is that he does not pay sufficient attention to the smoothing effects of aggregation and overemphasizes the discontinuities found at the microlevel. However, this is a sin of omission rather than of commission.

The use of distributed lag models in aggregate time-series supply analysis is basically a very simple way of taking account of the effects of fixed factors on supply response (17, 31, 33, 35). It should be noted, however, that the model used is not well-founded on an explicit microtheory, nor, for this reason, are the results of analyses based on it capable of easy comparison with information about behavior on the level of the individual firm. The use of such models are subject to a number of difficulties in practice: (1) the empirical distributions of lag have been found to be unstable over time (17); (2) these distributions have also been found to imply unreasonably long periods of adjustment; and (3) there are severe problems in separating lagged adjustments of this sort from those resulting from the process of expectation formation (31, pp. 63-65, 236-54).

The simple sort of distributed lag model is particularly ill-adapted to the study of commodities in the production of which so-called quasi-fixed factors are involved. The line between final products and capital goods is not a sharp one. Agricultural outputs and inputs run the whole gamut from farm buildings and machinery, clearly durable capital goods, to fresh vegetables, which are clearly perishable final products. Although inventories of one kind or another play a role in the production of most agricultural commodities, their influence on supply is nowhere so evident as in the production of livestock and livestock products. In addition to feed inventories, livestock may be thought of as intermediate both to strictly capital goods and to final outputs. The multiple roles of livestock in the productive process are well described by Hildreth and Jarrett (22, p. 21).

> A given animal at a given time may be viewed as (a) a finished good, (b) a good in process or (c) a piece of fixed capital. This is perhaps most dramatically apparent for a young heifer, say 16 to 20 months old, of a beef or dual-purpose breed. If the animal has been well fed, she may be immediately marketable as medium or possibly good beef. Alternatively she might profitably be fed intensively for a short period with a consequent increase in weight and possibly in grade. A third alternative would be to retain her in the breeding herd to produce calves (or calves and milk if she were a dual-purpose heifer). Though a narrower range of alternatives exists for most other animals, it is typically true that selling livestock for slaughter reduces the productive capacity of the farm herd. Thus an individual producer or all producers as a group can increase current marketings either by increased feeding and production or by decreasing the productive potential of their herds.

The marked cycles observed in the numbers and prices of beef cattle, hogs, and sheep may be due in part to their special nature (3, 4, 13, 28, pp. 41-82).

D. Gale Johnson (23) has pointed out and analyzed in some detail the close connection between the problem of investment decisions in agriculture and the problem of uncertainty. His discussion is oriented primarily toward policy, but the implications for supply over time in the face of changing uncertainties is clear.

Uncertainty and Expectations

One of the chief problems in the empirical application of economic theory is the problem of specifying the correct, or at least a useful, relation between the constructs of the theory and the variables which can actually be observed. Economic theorists have been aware of this problem for some time, but much of the discussion has resulted in relatively sterile criticism of econometric work. Recently, however, there has been an increased recognition of the problem among econometricians who have consciously made an effort to state the relation between observable variables and theoretical constructs as an explicit part of the underlying theory. In time-series supply analysis one of the forms which this general problem takes is that of specifying the relation between observable events and the prices which farmers expect to receive for their outputs and expect to pay for their inputs. In the production of almost all farm commodities, inputs must be committed to a greater or lesser degree some time before output is realized. A farmer must therefore base his plans not on what he is currently receiving or has received in the past, but on what he thinks he may receive in the future. What a particular farmer thinks, or better what farmers as a group think on the average, is what is relevant to farmers' supply decisions, and is therefore the relevant theoretical construct for supply analysis. But what farmers think is a subjective matter and not directly observable over long periods of time; hence, we are faced in time-series analysis with an extremely difficult problem of the general type discussed here.

Uncertainty in agriculture has primarily been discussed in connection with farm management problems (25, 36). It is clear that if uncertainty affects how farmers ought to behave with respect to investment, farm organization, and production plans, it must also affect how they do behave, although perhaps in not quite the same way. This is just the other side of the economic coin. In this light, uncertainty raises issues for time-series supply analysis which go beyond the problem of relating theoretical constructs to observable variables. Technological developments, changes in market organization and structure, and government policies have altered the impact of uncertainty upon supply decisions. However, the problem of most immediate concern remains the one first mentioned; without some sort of workable solution to this problem, the other elements cannot be brought into the picture in a truly meaningful way.

The relation between the problems of uncertainty and investment decisions has already been indicated. What is less fully realized is how this relation can play a fundamental role in time-series supply analysis. One can appreciate the relation best in the context of the supply of livestock and livestock products because of their intermediate nature between true capital goods and final outputs and because of their greater degree of specificity. Ladd (27) and Breimyer (3) have pointed out in this connection, that the problem of specifying relevant expectational variables is greatly complicated in the case of livestock products because supply decisions can be made at many points of time, and decisions at many previous points in time affect current alternatives.[1]

A long paper could be devoted to each of the five problem areas discussed above. To attempt to delve further into all these areas would obviously be impossible within the scope of this paper and would furthermore infringe upon areas discussed elsewhere in this book. The rest of this discussion will be restricted to topics (4), investment in fixed and quasi-fixed factors, and (5), uncertainty and expectations. A discussion of specific techniques or achieved results will be minimized. Emphasis will be on needed areas of future research. Examples will be given of what the author believes to be fruitful ways of looking at the problems involved.

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[1]This point is nowhere better illustrated than in the discussion of the British pig cycle by Coase and Fowler (8, 9).

INVESTMENT IN FIXED AND QUASI-FIXED FACTORS

In this section, the question of how the presence of fixed factors of production produces lagged adjustment in supply and the difference between the short- and long-run will be examined. As indicated elsewhere (33), recognition of this difference is crucial to successful time-series supply analysis. Bachman and the author have discussed the relation between the investment problem and the problem of technological change elsewhere (1). A simple model is presented in the appendix which relates the fixed factor problem to the aggregation problem in connection with the question of how to use cross section information to check a time-series study. Perhaps the most serious omission in the present discussion is the lack of any consideration of the relation between the problems of uncertainty and investment which D. Gale Johnson (23) has cogently argued.

As indicated previously, many forms of fixed and semifixed inputs are used in the production of agricultural commodities. These range from things such as barns or tractors, which may clearly be treated as capital goods fixed in the short run, to an 18-month-old sow or a stock of seed. The sow or the seed may be considered as a final output or as capital goods to be used in the production of more of the same. Because capital goods consist of a tremendous variety of different things with greatly different durabilities, and enter the productive process in many different ways, simplification is both necessary and conceptually difficult.[2] For this reason, subsequent remarks are confined to a model in which the question of simplification does not arise by assumption.

The closest connection between supply analysis and investment in fixed factors appears in conjunction with the distinction between the long and short runs. To see how this occurs, it is useful to examine a simple example recently presented by Smith (41). Let us consider a firm which produces a single homogeneous output with two homogeneous factors of production, one a current input and the other a capital input. Suppose the capital input is like a "one-hoss shay" — it requires no maintenance and disintegrates at the end of a fixed period, L. We suppose that the current input may be measured continuously by some real number x_1 and that the stock of the capital input may also be so measured by X_2. For the moment we beg the question of whether the capital input is divisible or, if so, to what degree. Following Smith (41, p. 66), we write the production function, assumed to be continuous and differentiable, as

$$y = f(x_1, X_2) \tag{1}$$

where y is the continuous output of the productive process, x_1 is the continuous current input, and X_2 is the physical stock of the

[2] For some of the theoretical issues involved see Robinson (38). Griliches (16) gives an excellent account of the issues which must be faced on a practical level.

replaceable capital good.[3] We assume $f(x_1, X_2) = 0$ if either x_1 or $X_2 = 0$.

Suppose that the firm attempts to minimize the "current" cost of producing a given output (how output is determined is discussed later) given its production function and the prices it must pay per unit of the current inputs, w_1, and the fixed factor, W_2.[4] Under these circumstances and when discounting occurs continuously, Smith shows that current costs are

$$C = w_1 x_1 + \frac{r W_2 X_2}{1 - e^{-rL}}, \tag{2}$$

where r is the rate at which the future is discounted and L is the length of life of the fixed equipment. Minimization of equation 2 subject to equation 1 yields the conditions

$$\left\{ \begin{aligned} w_1 - \lambda \frac{\delta f}{\delta x_1} &= 0 \\ \frac{r W_2}{1 - e^{-rL}} - \lambda \frac{\delta f}{\delta X_2} &= 0 \end{aligned} \right. \tag{3}$$

in addition to equation 1, where λ may be interpreted as marginal cost, i.e., $\lambda = \frac{\delta C}{\delta y}$.

If equations 1 and 3 are satisfied for a given output, the firm is in long-run equilibrium. The value λ^0, which may be obtained by solving equations 1 and 3 for λ in terms of $w_1, \frac{r W_2}{1 - e^{-rL}}$, and the given output y is the long-run marginal cost for the output y. If the firm sells its output in a competitive market at a price p, it will be in long-run equilibrium only if

$$p = \lambda^0 . \tag{4}$$

[3]Actually there is some question about whether the stock of fixed productive factors should be included explicitly in the production function. Smith takes the position that both stocks of capital goods and flows of current inputs should be included. He says, "The direct objects of adjustment or action parameters of the firm are (1) the current inputs to current production, and (2) the physical stocks of the various kinds of capital goods employed. ...The distinguishing character of capital goods is simply that their presence, in the form of physical stocks, is required in order for production to take place. ...the inclusion of all current inputs in the production function permits one to account for the economizing consequences of variations in equipment utilization through the latter's impact upon the consumption of current inputs." (41, pp. 65-66.) The classical position, best expressed in Carlson (6), is that only flows of services should be included. In the appendix stocks and flows of the services of fixed factors are treated as parameters in the production function. I do not believe that these various positions are contradictory, but rather that one may be appropriate for one problem and another for a different problem.

[4]Current cost is taken to be the constant outlay stream per period which "...has a present value equal to that of all future cost outlays over the firm's planning horizon." (41, p. 67.) In order to compute this value a rate of discount is necessary. Smith does not, nor shall this paper, discuss what rate of discount should be used nor how it is determined.

Since λ^0 is a function of w_1, ${}^{r}W_2/(1 - e^{-rL})$ and y, equation 4 may be rewritten as

$$(5) \qquad y = S^0 \left(p, w_1, \frac{r W_2}{1 - e^{-rL}}\right) .$$

S^0 is the long-run supply function for this particular firm. It shows output supplied at a price p, given the prices of the current input and the capital input, the discount rate, and the length of life of the capital equipment. The important thing to note is that the long-run quantity supplied depends on the discount rate and the length of life of capital equipment, neither of which have generally played a role in time-series analysis.

The above formulation, however, cannot be applied directly to time-series supply analysis for the following reasons: (1) We do not generally observe firms on their long-run supply functions but only on their short-run functions. These differ according to the length of run considered and, as Smith shows, according to the nature of the physical capital involved, whether it is divisible, if so to what extent, and whether it has a resale value. (2) The formulation applies to the individual firm, and time-series analysis generally deals with industry aggregates. Firms in the industry will generally have equipment of various ages even if they all face the same prices and use the same rate of discount.

Let us suppose that (1) the capital good is perfectly indivisible, (2) it must be replaced in toto; and (3) current prices are expected to continue indefinitely into the future by all firms, each of which applies the same rate of discount, r, to the future. We also suppose that the production function f is the same for all firms. The relation between the short- and long-run supply functions for the individual firm may now be analyzed as follows: Let the firm's capital stock be of age A. For $A \neq L$ capital stock is fixed and unalterable at the level X_2. For this stock, it will choose x_1 to satisfy the equation

$$(6) \qquad w_1 = p \frac{\delta f}{\delta x_1} .$$

For very low prices, p, this may imply $x_1 = 0$, which means the firm is out of business. Given X_2, equations 1 and 6 may be used to eliminate x_1. Replacing X_2 by its numerical value and rewriting the result, we could obtain output as a function of p and w_1

$$(7) \qquad y = S' (p, w_1) .$$

S′ is the short-run supply function of the individual firm. Its relation to the long-run supply function is as follows: Suppose that prices are the same at the end of A years (when the capital stock suddenly disintegrates) as they were at the time initially considered. Then the firm would have the option of repurchasing an amount X_2 of the capital input and continuing in business, or of not repurchasing and going out of business. The matter would be decided on the basis of whether equation 1 and the following equations could be satisfied on the basis of an amount X_2 of the capital good:[5]

$$(8)\quad \begin{cases} w_1 = p \dfrac{\delta f}{\delta x_1} \\ \dfrac{r W_2}{1 - e^{-rL}} \leqslant p \dfrac{\delta f}{\delta X_2} . \end{cases}$$

In general there will be some price p_m for which the equality in the second of equations 8 just holds. This is the lowest product price consistent with repurchase and is a decreasing function of W_2, an increasing function of r, and a decreasing function of L and $\frac{\delta f}{\delta X_2}$.[6]

Let p'_m be the price at which the firm would discontinue business in the short-run. This price must, in general, be lower than p_m, the price at which the firm would discontinue business in the long-run and will be an increasing function of w_1. The short-run supply curve, given w_1 is perfectly elastic at p'_m; above p'_m, it is less than perfectly elastic but not perfectly inelastic. The long-run supply curve, given W_2, r, and L, is perfectly elastic at $p_m \geqslant p'_m$ and identical with the short-run curve thereafter.

What happens when we aggregate firms with equipment of different ages? Suppose that all firms have the same production functions and face the same prices, and let $\pi(A)$ be the proportion of firms with equipment age $A = 0, 1, \cdots L$. If the product price is less than p'_m, no firm will produce; hence, the industry supply curve is perfectly elastic at p'_m. If the product price is between p'_m and p_m, the short-run industry supply function is a weighted average of the short-run individual firm supply functions; the weights are determined by the proportions $\pi(A)$. However, at a price less than p_m no firm will repurchase equipment when that which it has wears out; hence, the long-run industry supply function must be perfectly elastic at p_m. Given a price less than p_m, the quantity supply will gradually drop off by a proportion depending on the distribution of firms with equipment of different ages. This can only stop when either industry output is zero or price has risen to p_m along the demand curve. This process is illustrated in Fig. 2.1. $D_1 D_1'$ is the demand curve after a shift has occurred.

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[5]Note that for the purpose of this example we are continuing to regard X_2 as continuously variable in the definition of f, even though we regard it as discretely variable in the problem at hand. The inequality reflects the latter; if the cost of X_2 is greater than the value of its marginal product at a price p, it will not pay to repurchase. If it is less it will. The possibility of setting up another plant, i.e., essentially another firm with X_2 of the fixed factor is disregarded for the moment.

[6]The proof of these propositions is elementary.

(Originally the demand curve passed through the point S′.) $p'_m SS'$ is the short-run industry supply function. The decline in output the first period, $y_1 - y_2$, is such that as a proportion of industry output, $\frac{y_1 - y_2}{y_1}$, it equals the proportion of firms with equipment age L - 1, and so on for subsequent periods. The gradual decline in output only ceases when industry output is y_m, at which output the price as determined by the demand curve is p_m. At this price producers are just indifferent as to whether they replace the capital equipment or not. By allowing different shifts in the demand curve, e.g., D_2D_2', D_3D_3' or D_4D_4', we can trace out the various short-run supply schedules, p_2p_2' p_2'' p_2''', p_3p_3' p_3'' p_3''', etc., appropriate to different periods of adjustment.

When the product price is above p_m, under the assumptions existing firms have the incentive to open new plants, i.e., purchase another unit of the capital good, and new firms will enter the industry. Output must increase until price falls to p_m along the demand curve. It follows that the short-run industry supply curve is perfectly elastic at the price p_m past the point S′. Thus the over-all one-period short-run industry supply function is given by the curve $p'_m SS' S''$.

In the long run, the industry supply function is perfectly elastic at a price p_m for the same reason it is perfectly elastic in the short run for upward shifts in demand. The long-run industry supply function is the curve $p_m S' S''$. As can be seen from the figure, for downward shifts in demand we have the usual fan of short-run industry supply functions emanating from S′ (except in this case they are bent lines with corners on the long-run supply function because of the discontinuities in age of equipment). These short-run curves approach the long-run curve from below.

Had we relaxed the assumption that the production function was the same for each firm, we would have found that the prices p_m and p'_m were different for different firms. In this case the adjustment pattern would be more complex, depending on the joint distribution of age and prices p_m and p'_m. In general, the long-run industry supply function would no longer be perfectly elastic. It would still be true, however, that upward adjustments would be instantaneous and downward would not be. Still further relaxation of the assumptions to restrict the purchases of new equipment and freedom of entry in the short run would produce less than instantaneous upward adjustments. A secondhand market for the capital good would induce faster downward adjustments. It is conceivable that a simple asymmetric distributed lag model could represent the phenomenon, at least approximately.

The main conclusion of this analysis is one which should be obvious from economic intuition: namely, that upward and downward adjustments are likely to be asymmetric in the short run. Black (2) and Cassels (7) suggested long ago that this might be the case.[7] However,

[7]It should be noted, however, that Halvorson (17) did not find a statistically significant difference in the case of milk.

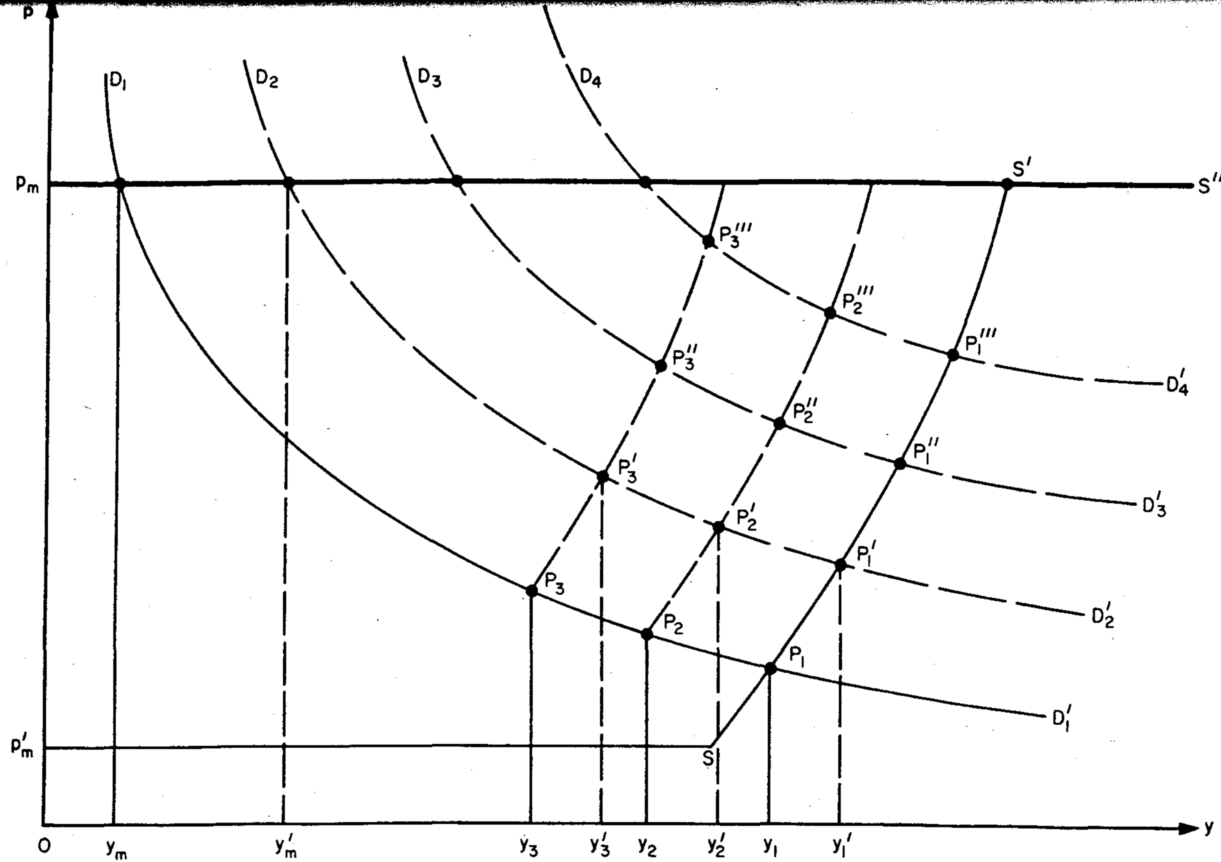

Figure 2.1. Illustrative derivation of short-run supply schedules.

it is hoped that the simple model discussed here will be a first step towards an engine of analysis which will permit the development of more realistic dynamic models of supply behavior which can be used in time-series supply analysis. Even the model discussed, as simple as it is, suggests several reasons why the distribution of lag in adjustment might be unstable over time. As noted above, the relation between the short- and long-run curves depends on (1) the durability of the capital good, and (2) the age distribution among firms. Over time the first is likely to change; but the second is almost certain to, because as the industry expands the average age will decline, and as it contracts, it will increase.

It has been suggested that livestock are quasi-fixed factors of production. The analytical problems encountered are similar to those encountered in connection with the more usual types of durable goods with secondhand markets. The author hopes to present a theoretical framework for discussing this type of problem as well as a more complete over-all framework in the near future.

UNCERTAINTY AND EXPECTATIONS

The Notion of a Certainty Equivalent

In a recent survey of problems of uncertainty in relation to farm planning, Hildreth (21) distinguishes between probalistic and game theoretic approaches to the problem of decision making under uncertainty. The value of the latter appears at present to lie chiefly in normative applications at the level of the individual firm; but the former, particularly in connection with the recent reintroduction of the notion of certainty equivalence, may be quite useful in the analysis of observed behavior. Hicks (20) suggested that behavior under uncertainty could be studied by constructing, for each of various situations involving uncertain expectations, a related situation involving expectations held with certainty. Hart (18) rejects the certainty equivalent construct on the grounds that the most important behavioral manifestations of uncertainty, such as the maintenance of liquidity, postponement of decisions, and restriction of investment in specialized fixed capital, can be explained only by rather peculiar certainty equivalence models. It is clear, however, that in any application of the theory of behavior under uncertainty to time-series analysis of supply, it must be possible to summarize factors influencing behavior in a few variables. Thus, if several future prices, whose values are uncertain, are supposed to influence present supply behavior, it must be possible to take account of these effects by one or two variables for each price, e.g., the anticipated mean value and variance of the distribution of each.[8] Note that these need not be directly observable.

[8]It is interesting to note that even in normative applications of statistical decision theory, which is at present the most highly developed theory of optimal behavior under uncertainty, the need has been felt for such a reduction to more manageable proportions. See Reiter (37).

It is fortunate that Reiter (37), Simon (40), and Theil (43) have recently been able to demonstrate that a problem of decision making under uncertainty can, in some circumstances, be replaced by a much simpler problem involving many fewer parameters and which, in fact, can be thought of as a problem in decision making under certainty. A simple example will illustrate the principles involved. Suppose we are given a producer who produces a single homogeneous commodity. The producer must decide in advance how much to produce, q, at a time before that at which he can sell the commodity at a price, p_0, per unit. Furthermore, suppose that the total costs of producing q units depend upon the prices of factors, p_1 and p_2, which are also unknown at the time the decision is to be taken, i.e., total costs are

$$C(q, p_1, p_2) .$$

Let the prices, p_0, p_1, and p_2 have a joint (subjective) probability distribution $F(p_0, p_1, p_2)$. One possible formulation of the producer's decision-making problem under uncertainty is the problem of maximizing the mean value (or expected value in a statistical sense) of net revenue, i.e.,

$$\text{(9)} \quad E\,R(q, p_0, p_1, p_2) = \int \{p_0 q - C(q, p_1, p_2)\}\, dF(p_0, p_1, p_2) .$$

From the standpoint of statistical decision theory, the solution of this problem in its general form requires knowledge of the entire distribution function $F(p_0, p_1, p_2)$. Suppose, however, that the cost function is a function of the form

$$\text{(10)} \quad C(q, p_1, p_2) = a_0 + a_1 q + a_2 q^2 + b_1 q p_1 + b_2 q p_2 + b_{12} q p_1 p_2 .$$

In this case the problem of maximizing the mean value of net revenue reduces to a simpler problem which Reiter (37) calls the <u>surrogate</u> of form 9, i.e., maximize the expression

$$\text{(11)} \quad q \bar{p}_0 - \{a_0 + a_1 q + a_2 q^2 + b_1 q \bar{p}_1 + b_2 q \bar{p}_2 + b_{12} q \sigma_{12}\}$$

with respect to q, where $\bar{p}_0$, $\bar{p}_1$, and $\bar{p}_2$ are the means of the distribution of prices $F(p_0, p_1, p_2)$ and σ_{12} is the covariance of p_1 and p_2.[9]

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[9] This follows from the fact that if C is of the form 10,

$$ER(q, p_0, p_1, p_2) = q \int p_0 dF - \{a_0 \int dF + a_1 q \int dF + a_2 q z \int dF + b_1 q \int p_1 dF + b_2 q \int p_2 dF + b_{12} q \int p_1 p_2 dF .$$

Reiter (37) shows in general that, if x is a vector of decision variables, such as the quantity to be produced, and y is a vector of random elements, such as future prices, and if the function f(x,y), the expected value of which is to be maximized, is of the form

$$f(x,y) = \sum_{i=1}^{n} A_i(x)\, B_i(y) ,$$

The problem of maximizing form 11 with respect to q has the solution

$$q = \frac{\bar{p}_0 - a_1 - b_1 \bar{p}_1 - b_2 \bar{p}_2 - b_{12} \sigma_{12}}{2 a_2} . \tag{12}$$

and this is the solution of the original problem 9 when C is of the form 10. Thus, the certainty equivalent problem to 9 is 11, and the uncertain prices p_0, p_1 and p_2 are replaced by their certainty equivalents; namely the means of the distribution of prices and the covariance of p_1 and p_2, in this equivalent problem.

Theil (43) has shown that if the following conditions are satisfied: (1) The function to be maximized is a quadratic in the decision variables and uncertain variables, and (2) actions planned to be taken in the future do not affect the present values of the uncertain variables, then the certainty equivalents of the uncertain variables are simply the means of their distribution. In our simple example, Theil's assumption would be fulfilled if b_{12} were zero. While the assumption that the function to be maximized is a quadratic is certainly quite restrictive, it has merit as an approximation because it allows us to replace each uncertain variable by only one certainty equivalent variable.[10] In subsequent discussion, it will be assumed that Theil's conditions are fulfilled and therefore mean by certainty equivalent the mean value of the (subjective) probability distribution of the uncertain variable.

To this point the discussion has concerned a single decision maker, but in time-series supply analysis we are concerned with the collective results of decisions by a large number of individuals. Thus, in order to apply the theory of certainty equivalence to a group, we must adopt the Marshallian device of the representative firm, i.e., a possibly hypothetical firm whose certainty equivalent expectations, if held by all other firms, would result in the observed group behavior. The certainty equivalents of the representative firm may be thought of in some sense as group averages, but it is clear that in so doing a host of aggregation problems are raised. A discussion of these, however, would take us beyond the scope of this paper.

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then there is a simpler surrogate problem; namely to maximize

$$G(x_1 Z) = \sum_{i=1}^{n} A_i(x)\, Z_i$$

where $Z = (Z_1 \cdots Z_n)$ and $Z_i = \int B_i(y)\, dF(y)$.

Theil's results (43) for quadratic decision functions f(x,y) are a special case of Reiter's more general formulation in which it can be shown that the surrogate problem involves only the means of the distribution F (y).

[10]Reiter (37) points out that this approximation is better for normative applications of the theory than for positive, because, although the decision based on the surrogate problem may lead to a value of the function to be maximized not far from the true maximum, the values of the decision variables chosen may be very different. This is a basic difficulty inherent in the use of certainty equivalents in time-series supply analysis.

Summary. The device of introducing a certainty equivalent to an uncertain variable appears both useful and necessary in time-series supply analysis. To justify the use of a single certainty equivalent for each uncertain variable, it is necessary to make the following restrictive assumptions: (1) Group behavior can be adequately explained by treating it as the behavior of a single representative and hypothetical decision maker; and (2) the representative decision maker behaves as if he maximizes the expected value of a function which is quadratic in the decision variables and the uncertain variables. Theil's further assumption that future plans do not affect the present values of uncertain variables is hardly restrictive for the present purpose, although in the case of national planning it may be because of the effects of announced plans on the public.

Models of Expectation Formation

The certainty equivalent expectations held by the representative firm are both subjective and aggregative. They are not necessarily observable. The problem in time-series supply analysis is to construct an empirically useful hypothesis which relates these expectations to observable variables. In essence, this means constructing a model of expectation formation. This section will consider three types of models: (1) extrapolative, (2) adaptive, and (3) rational. The latter is actually a broad class of models of expectation formation stemming from the recently proposed "Rational Expectations Hypothesis" (29, 30). The rational expectations model is intimately related to the over-all model of behavior formulated. For that reason, the concluding section of this paper will be a discussion of rational expectations in a simple cobweb model.

Extrapolative

The classical approach in agricultural supply analysis is to suppose that expectational variables can be directly identified with some one past actual value of the variable to which the expectation refers. For example, the supply of an agricultural commodity at a future time depends on its price expected at that time. It might be assumed that this expectation is the current value of price, so that supply is simply related to lagged price. An extension of this approach, due to Goodwin (15), is to suppose that expected price in period t is actual price in $t-1$ plus (or minus) a fraction of the change in price from period $t-2$ to $t-1$:

$$p_t^* = p_{t-1} + \alpha (p_{t-1} - p_{t-2}) , \tag{13}$$

where p_t^* is the price expected in period t. Note that the term "expected price" is used for convenience in place of the term "certainty equivalent price for the representative firm." Elsewhere, the author has called the model generating expected prices of the form 13 the "intermediate model" and has discussed the estimation of supply functions when such a model is assumed (31, pp. 199-200). Muth (30) terms the expectation generated by form 13 "extrapolative." The classical model of expectation formation is a special case of form 13 when $\alpha = 1$.[11]

Adaptive

Expectations based on the extrapolative model do not forecast actual events very well. Furthermore, such expectations are theoretically unsatisfying in that they are determined by only two past items of experience and neglect information which we may feel is contained in other items of past experience. To develop a model of expectation formation useful in time-series analysis, the model must be kept relatively simple. A model, due originally to Cagan (5), which has greater intuitive appeal than the extrapolative model is the adaptive expectations model. According to it, expectations are revised periodically by some portion of the error between last period's expectation and what actually occurred. To use the previous example and notation,

$$p_t^* - p_{t-1}^* = \beta (p_{t-1} - p_{t-1}^*) , \tag{14}$$

where β is called the coefficient of expectations, it can be shown that this model leads to a representation of expected price as a geometrically weighted moving average of past prices[12]

$$p_t^* = \beta p_{t-i} + (1-\beta) \beta p_{t-2} + (1-\beta)^2 \beta p_{t-3} + \cdots . \tag{15}$$

Muth (29) has shown that adaptive expectations are optimal if the time series to be forecast is the result of two kinds of random components, one lasting a single time period and the other lasting through <u>all</u> subsequent time periods. Following Friedman (14), Muth calls these respectively the transitory and permanent components of the time series. The sense in which the forecasts are optimal is that they either give the means of the distribution of the actual values of the series or are a least-squares approximation to them. The former is the case when the permanent and transitory components are not statistically

[11] Goodwin (15) points out that the coefficient α can be considered as an average of the coefficients of individual subgroups of producers weighted by the elasticities of their respective supply functions.

[12] See Nerlove (32) where the problems of estimation are also discussed at some length.

independent;[13] and the latter is the case when the permanent and transitory components are independent. The second case can be illustrated by the following example: A price at time t is divided into two components $\bar{p}_t$ and η_t, respectively the permanent and transitory components,

$$p_t = \bar{p}_t + \eta_t . \tag{16}$$

If the transitory component is assumed to have mean zero and finite variance $\sigma_\eta{}^2$; and they are statistically independent of one another, i.e. $E\,\eta_t\,\eta_s = 0,\ t \neq s$; and if the changes in the permanent component are statistically independent of each other and of the transitory components, then form 15 is a least-squares estimate of the mean of the distribution of actual prices, provided β is a certain function of the relative variances of the transitory component and the changes in the permanent component (29, pp. 6-8). It is also true that the forecast for all future periods is the same, although the standard error of forecast becomes larger in the more distant future.

These results show that for certain kinds of time series, adaptive expectations are optimal in the sense of being good forecasts. Insofar as good forecasts are useful to the farmers, adaptive expectations are rational ones for such time series; but it is very doubtful that the time series of agricultural prices are of the random character necessary for this to be true. The reason is simply that agricultural prices are generated in large part by an economic mechanism; they are only in part the result of stochastic forces. It is highly doubtful that they can be well represented solely by the appropriate purely stochastic scheme.

Rational

Muth's discovery that adaptive expectations are optimal only under rather restrictive assumptions which can only doubtfully be applied to economic phenomena has led him to a formulation which he calls the "rational expectations hypothesis" (30). In some instances this does result in adaptive expectations, but in others it does not. From the standpoint of economic theory, the rational expectations hypothesis is the most attractive hypothesis concerning the formation of expectations which has been formulated to date and which is sufficiently simple to be

[13] When the actual value, for example, p_t, is a weighted sum of independent shocks, ϵ_t, $\epsilon_{t-1}, \cdots$, of the form

$$p_t = \epsilon_t + \beta \sum_{i=1}^{\infty} \epsilon_{t-i}$$

where $\qquad E\epsilon_t = 0,\ E\epsilon_t^2 = \sigma^2 < \infty$ and $E\epsilon_t\epsilon_s = 0,\ t \neq s$.

In this case, a weighted moving average of the form (7) gives the mean value of the distribution of p_t. Furthermore, this is true for all subsequent periods, $t + 1, t + 2, \cdots$. In short, p_t^* is an estimate of the permanent component and independent of the future date for which the forecast is made. See Muth (29), pp. 2-5.

used in connection with time-series analysis. It merits a detailed description.

Stated in concise form, the rational expectations hypothesis "... is that, expectations being informed predictions of future events, are essentially the same as the prediction of the relevant economic theory." Superficially the hypothesis thus stated might appear grossly unreasonable. After all, farmers and businessmen are not economists, let alone econometricians; how then could they formulate the appropriate economic model and use it in the best way possible to forecast future events? On closer examination, however, it appears to the author that this hypothesis is far more reasonable than it first sounds.

First, the rational expectations hypothesis does not require that every farmer or businessman formulate a correct and relevant economic model. Economists cannot even do that! What it does require is that the representative firm behave as if it had made predictions on the basis of the same economic model used by the economist to analyze industry behavior. It implies expectations which are constructs of the same nature as "certainty equivalents," "adaptive expectations," and "supply functions" — indeed almost any other economic concept. Furthermore, the expectations thus generated will be entirely consistent with the economic model used and will have the additional advantage of not assuming less rationality in the formation of expectations than in other forms of economic behavior. If one is prepared, for the purposes of a predictive model, to assume that on the average producers maximize profits, it does not make sense to assume that they err greatly in making forecasts on the average, or at least err more than the model used to predict their behavior.[14] The rational expectations hypothesis an attractive one from the aesthetic standpoint and because of its consistency both with general economic theory and the particular economic model underlying the statistical analysis undertaken.

Second, if expectations were not rational, at least on the average, then insofar as our economic model approximates reality we should tend to find a small group of individuals, whose expectations are better than those of the rest, gradually driving the others out of business. This is essentially the same argument used to support the hypothesis of profit maximization under conditions of competition: Those who do not maximize do not survive; therefore, those who survive must achieve maximum profits on the average.

Third, insofar as this argument is unconvincing, Muth shows that it is possible to introduce elements of irrationality into the picture. Such deviations from rationality are, of course, unimportant when they are unsystematic. This is what we mean when we speak of rational expectations "on the average." But if the deviations are systematic, biased expectations may result. Muth (30, pp. 17-19) gives an example of how

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[14] This, in fact, has been the chief criticism leveled at the cobweb theorem, namely that it rests on an extreme assumption of irrationality in expectation formation but assumes rationality in other aspects of behavior. A simple cobweb model will be discussed in the light of the rational expectations hypothesis.

such biases may be introduced. The approach of introducing errors in expectations explicitly as deviations from rationality has the advantage of making clear exactly how elements of irrationality enter the picture and precisely what effects they have on behavior, although introducing them in any way tends to destroy some of the simplicity inherent in the rational expectations hypothesis.

The acid test of any hypothesis is whether it proves useful in explaining actual behavior, not what it assumes and what it does not assume. The rational expectations hypothesis has only recently been proposed, and so faced the test of application to only a very limited extent. However, what limited evidence has been brought to bear tends to support the rational expectations hypothesis: Simple cobweb models which are based on extrapolative or adaptive expectations suggest that we should observe negative serial correlations in prices and cycles of relatively short duration. Both predictions are contradicted by experience. On the other hand, as Muth (30, pp. 32-39) shows, simple cobweb models based on rational expectations suggest that prices will exhibit positive serial correlation and cycles longer than three or four production periods (depending on which way cycles are measured).[15]

Application of the Rational Expectations Hypothesis to a Simple Cobweb Model

One of the attractive features of the rational expectations hypothesis is that the character of the implied expectations depends on the entire model. This feature makes the hypothesis difficult to explain out of a particular context. In this section, the results of applying the rational expectations hypothesis to a simple cobweb model are examined. These results and their derivation are reported by Muth (30, pp. 13-17) although the exposition here is thought to be somewhat clearer than Muth's, perhaps because it is less concise.

Consider a simple model of a market containing a supply equation,

$$q_t = a + b\, p_t^* + u_t \tag{17}$$

where q_t is the quantity supplied, p_t^* is the expected price and u_t is a random residual which reflects variations due to such factors as weather, technology, or other variables exogenous to the market in question. Note that the u_t need not be distributed independently of t,

[15] Breimyer (4, p. 765) suggests the hog cycle (trough to trough) may have a period between five and six years, and (3, p. 3) that the cattle cycle (trough to trough) may have a period between thirteen and seventeen years. In neither case do these periods correspond in any reasonable way with the "period of production" or "gestation period" which is much, much shorter than half the period of the cycle. Of course, as Goodwin (15) pointed out, coupling of two or more dynamic markets of the cobweb type may lead to irregular cycles and cycles longer than the individual markets would exhibit in isolation. However, it seems unlikely that such large discrepancies between theory and experience could be explained by coupling.

i.e., it need not be true that $E\, u_t u_s = 0$, $t \neq s$. Indeed, this assumption would generally be inappropriate. The model also contains a demand equation, which we will assume to be exact:

$$q_t = c + d\, p_t. \tag{18}$$

Under these circumstances the equilibrium price, $\bar{p}$, and quantity, $\bar{q}$, are obtained by setting $u_t = 0$, $p_t^* = p_t = \bar{p}_t$ and $q_t = \bar{q}_t$:

$$\bar{p} = \frac{c - a}{b - d}$$
$$\bar{q} = \frac{bc - ad}{b - d}. \tag{19}$$

Suppose the disturbance term u_t can be written as the weighted sum of independently and normally distributed random variables ϵ with zero means and common variance σ^2:

$$u_t = \sum_{i=0}^{\infty} w_t \epsilon_{t-i}. \tag{20}$$

If $w_0 = 1$ and $w_i = 0$ for $i = 1, 2, \cdots$, $u_t = \epsilon_t$ and the u_t are, therefore, independently distributed. On the other hand, more realistically, if the weights for previous periods are not all zero, u_t is serially correlated. By equating q_t in equations 17 and 18, replacing $(c - a)$ by $(b - d)\bar{p}$ from its value in equation 19 and rearranging terms, we find that the deviation of the actual price from the equilibrium price is the following function of the deviation of the expected price from the equilibrium and the independent disturbances ϵ_t:

$$p_t - \bar{p} = \frac{b}{d}(p_t^* - \bar{p}) + \frac{1}{d}\sum_{i=0}^{\infty} w_i\, \epsilon_{t-i}. \tag{21}$$

For simplicity, let p_t' and $p_t^{*\prime}$ represent the deviations $p_t - \bar{p}$ and $p_t^* - \bar{p}$ respectively. Then

$$p_t' = \frac{b}{d} p_t^{*\prime} + \frac{1}{d}\sum_{i=0}^{\infty} w_i\, \epsilon_{t-i}. \tag{21'}$$

According to the rational expectations hypothesis, stated in precise form, expectations are "...distributed...about the prediction of the theory," given the same information available to the decision makers (30, p. 3). Thus, if expectations are rational, their average value, $p_t^{*\prime}$, must equal the mean value of the distribution of the actual value of price, p_t', given past events. The past in this model is summarized in the values of $\epsilon_{t-1}, \epsilon_{t-2}, \cdots$, which are not directly observable. Thus, under the rational expectations hypothesis, we must have

$$
\begin{aligned}
p_t^{*\prime} &= E(p_t' \mid \epsilon_{t-1}, \epsilon_{t-2}, \cdots) \\
&= \frac{b}{d}\, p_t^{*\prime} + \frac{1}{d}\, E\Big(\sum_{i=0}^{\infty} w_i\, \epsilon_{t-i} \mid \epsilon_{t-1}, \epsilon_{t-2}, \cdots\Big) \qquad (22) \\
&= \frac{b}{d}\, p_t^{*\prime} + \frac{1}{d} \sum_{i=1}^{\infty} w_i\, \epsilon_{t-i}
\end{aligned}
$$

since $E(\epsilon_t \mid \epsilon_{t-1}, \cdots) = E(\epsilon_t) = 0$, by assumption. It follows that

$$
p_t^{*\prime} = \frac{1}{d-b} \sum_{i=1}^{\infty} w_i\, \epsilon_{t-1}, \qquad (23)
$$

i.e., the expected price for the t^{th} period is a weighted sum of past (unobservable) random shocks. The problem of relating these expectations to observable variables remains to be solved.

First, it is noted that if the u_t are statistically independent, $w_0 = 1$ and $w_i = 0$, for, $i = 1, 2, \cdots$. Therefore,

$$
p_t^{*\prime} = 0 \qquad (24)
$$

i.e., the expected price p_t^* is just the equilibrium price $\bar{p}$. In other words, under the rational expectations hypothesis with statistically independent shocks in the supply curve and a fixed demand curve, all variations in supply are caused by the random shocks. Variations in supply should exhibit no pattern and the observed elasticity of supply with respect to any observed price should be zero.

If the shocks u_t, however, are not statistically independent, more realistic results are obtained. To analyze this case, we express actual prices p_t' as a function of current and past values of ϵ_t by substituting equation 23 in equation 21′.

$$
p_t' = \frac{w_0}{d}\, \epsilon_t + \frac{1}{d-b} \sum_{i=1}^{\infty} w_i\, \epsilon_{t-i}. \qquad (25)
$$

Equation 25 is a linear difference equation in ϵ_t and can be solved for ϵ_t in terms of current and past values of p_t'. Thus, the general solution to the problem of expressing $p_t^{*\prime}$ in terms of observable variables, would be to replace $\epsilon_{t-1}, \epsilon_{t-2}, \cdots$ by their values in terms of $p_{t-}', p_{t-2}' \cdots$. However, instead of proceeding in this direction, which is not very enlightening, it is preferable to introduce more explicit assumptions about u_t at this point. Suppose, as is reasonable, the random shocks ϵ_t have a permanent and a transitory effect on u_t: The full value of the current shock ϵ_t is reflected in the current value of u_t. A positive fraction of this shock, for example β, permanently affects u_t. The remainder, $1-\beta$, has no effect in subsequent periods; it is transitory. Then u_t may be written

$$u_t = \epsilon_t + \beta\,\epsilon_{t-1} + \beta\,\epsilon_{t-} + \cdots \tag{26}$$
$$= \epsilon_t + \beta \sum_{i=1}^{\infty} \epsilon_{t-i}\,.$$

Comparison with equation 20 reveals that

$$\begin{aligned} w_0 &= 1 \\ w_i &= \beta,\ i = 1, 2, \cdots . \end{aligned} \tag{27}$$

Hence, from equation 25 we have

$$p'_t = \frac{1}{d}\epsilon_t + \frac{\beta}{d - b} \sum_{i=1}^{\infty} \epsilon_{t-i}\ ; \tag{28}$$

hence,

$$\epsilon_t = d\,p'_t - \frac{d\,\beta}{d - b} \sum_{i=1}^{\infty} \epsilon_{t-i}\,. \tag{29}$$

This is a difference equation in ϵ_t of a complicated sort, but instead of solving it directly we proceed as follows: Note that by equations 27 and 28 we may write expected price as

$$p^{*\prime}_t = \frac{\beta}{d - b} \sum_{i=1}^{\infty} \epsilon_{t-i}\,. \tag{30}$$

By equation 28, therefore,

$$p'_t = \frac{1}{d}\epsilon_t + p^{*\prime}_t\ . \tag{31}$$

It follows that

$$\epsilon_{t-i} = d\,(p'_{t-i} - p^{*\prime}_{t-i})\ . \tag{32}$$

Substituting equation 32 in equation 30 and lagging one period, we have

$$p^{*\prime}_{t-1} = \frac{d\,\beta}{d - b} \sum_{i=1}^{\infty} (p'_{t-1-i} - p^{*\prime}_{t-1-i})$$
$$= \frac{d\,\beta}{d - b} \sum_{i=1}^{\infty} (p'_{t-i} - p^{*}_{t-i}) - \frac{d\,\beta}{d - b}(p'_{t-1} - p^{*}_{t-1}) \tag{33}$$
$$= p^{*\prime}_t - \frac{d\,\beta}{d - b}(p'_{t-1} - p^{*\prime}_{t-1})\ .$$

Since the equilibrium price cancels out when a difference of primed variables is taken, equation 33 may be rewritten in the form

$$(34) \qquad p_t^* - p_{t-1}^* = m\,(p_{t-1} - p_{t-1}^*)$$

where $\qquad m = {}^{d\beta}/_{d-b}$.

Since $d < o$, $b > o$, and $o < \beta \leq 1$, $o < m \leq 1$. Equation 34 is just the equation which generates adaptive expectations. Thus adaptive expectations are also rational ones in this particular case. Note, however, because of the particular interpretation of the coefficient of expectations, m, the equilibrium must be stable.[16]

When other variables such as income are introduced in the demand equation or a random shock is added to it, the situation becomes more complicated. Muth indicates that rational expectations in such cases will depend on other observable variables in addition to past prices. Although the author has not yet worked out examples of this sort, he has no doubt that such examples would show rational expectations in more realistic cases to be quite different from the simple kind of adaptive expectations used in previous work on supply.

Also noteworthy at this point is that even in the simple case discussed here, the estimation techniques proposed for models based on adaptive expectations are inappropriate in the case of rational expectations, despite the fact that rational expectations turn out to be of the adaptive form. The reason for this is that the residuals u_t in equation 17 are serially correlated in a way different than that assumed in the development of the estimation procedure. If the recommended procedure (31) is used, the estimates of the elasticity of supply and of the coefficient of expectations will generally be biased. Suitable estimation procedures can be developed along lines suggested by Klein (26), but they will not be discussed here.

In conclusion, it may be said that rational expectations are difficult to find even for very simple economic models. This does not mean, however, that they are not worth finding. They have the property of being entirely consistent with the economic model into which they are introduced. The little qualitative evidence developed supports the rational expectations hypothesis. There is clearly a need for more evidence of a quantitative character.

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[16] See Nerlove (31), where it is shown that the condition

$$1 - b/d < 2/m$$

must be satisfied for stability. Since $2/\beta$ is obviously greater than 1, by our assumptions, the condition is always satisfied for $m = d\beta/d\text{-}b$. The fact that the equilibrium is always stable, however, does not preclude cycles because of the effect noted by Slutzky (42). These will generally be of an irregular character and substantially longer than the two period cycle predicted by the ordinary cobweb theorem.

CONCLUSIONS

The main conclusion of this paper is that there is the need for much more preoccupation with theory and less with estimation techniques in agricultural supply analysis. The foregoing discussion is, of course, greatly restricted in scope, not only to the areas of uncertainty and investment, but within those areas to a few simple models.

In connection with the problem of investment in fixed and quasi-fixed factors, a simple model showing how the relation between the short- and long-run industry response to price was determined by the presence of fixed factors of production was analyzed. Despite its simplicity the model suggested several ways in which time-series supply analyses should be modified. The need for further and more complete theoretical research is apparent.

In connection with the problem of uncertainty, it has been suggested that the theoretical notion of certainty equivalent is essential to time-series supply analysis. A more tentative suggestion has been that the rational expectations hypothesis, recently proposed by Muth, may be a very fruitful one for time-series analysis. This hypothesis implies that expectations depend on the theoretical model employed and are therefore as good as the model and no better. Thus economic theory plays a crucial role.

These remarks are not meant to imply that we should all concentrate on theory for the next few years and give up the task of measuring supply elasticities. Theoretical developments can only be fruitful in the context of real problems; empirical investigations suggest new theoretical approaches as well as vice versa. One may conclude that time-series supply analysis is greatly in need of more and better theory. Estimation techniques are only a means to an end. If there has been some tendency in the past to let techniques of estimation dictate the theory or the problems studied empirically, it should be corrected. Only in this way can the full interplay of theory and practice leading to a better understanding of supply behavior be realized.

APPENDIX: FIXED FACTORS AND AGGREGATION

The classical distinction between extensive and intensive margins of production is closely related to the differences among firms in their endowments of fixed factors and technical knowledge. Suppose we have knowledge about a number of "typical" farms hog-dairy and hog-cash grain, for example), and can deduce optimal production of hogs for these farms at various combinations of pork, dairy, and feed prices. We still know relatively little about the effects of changes in pork prices on aggregate hog production with unchanged dairy and feed prices or with specified changes in these prices. The reason is simply that we do not know what part of the aggregate supply response occurs on those farms which have that combination of fixed factors which makes them "typical" hog producing farms and what part occurs on farms which have combinations which make them "atypical."

In this appendix, a simple model suggesting how cross-section data can be used to estimate a short-run industry elasticity of supply is discussed. The purpose of this model is not to suggest a practical method of doing this, but to exhibit the connection between the fixed factor problem and the aggregation problem.

As noted in the text, the question of whether to include fixed factors in the production function is moot. For the present purpose, which concerns short-run response to price, it is convenient to include only variable inputs in the production function. Consider an industry, F, which is a population of firms, f. The question to be answered is: Can we, by taking a sample of firms in F at a point in time, determine the short-run industry supply function? The answer is yes in the simple case discussed below.

Suppose that each firm in the industry produces a single homogeneous output using two variable factors of production. Furthermore, suppose that each firm expects the prices it actually receives for its product and pays for its variable inputs and under these circumstances it acts in such a way as to maximize the return to its fixed factors of production. Let

p_0 = the price f receives for its product

p_i = the price f pays for variable input i, i = 1, 2

x_{of} = the quantity of output f produces

x_{if} = the quantity of input i f uses, i = 1, 2

and

$$y_{of} = p_0 x_{of}$$
$$y_{if} = p_i x_{if}$$

(Note that the prices paid and received are assumed to be the same for every firm.) We assume that each firm has a production function relating variable inputs to output of the same form, but that the parameters of this function differ from one firm to another reflecting the fact that different firms are possessed of differing amounts of technical knowledge and fixed factors:

$$x_{0f} = (a\, u_{0f})\, x_{if}^{\;a_1 u_{1f}}\, x_{2f}^{\;a_2 u_{2f}} \qquad (35)$$

where the terms u_{0f}, u_{1f} and u_{2f} reflect differences in the parameters of the production functions.[17] Under the assumptions each firm

[17] This type of production function has been used in connection with the problem of relative economic efficiency (34). The properties and meaning of the function are discussed more fully in this paper.

$$R = p_0 x_{of} - p_1 x_{1f} - p_2 x_{2f} \tag{36}$$

maximizes the return to its fixed factors subject to equation 35. If this is done perfectly, we must have

$$a_i u_{if} = y_{if}/y_{of}, \; i = 1, 2, \tag{37}$$

for all f.[18]

The supply function for the individual firm can be shown to be

$$x_{of} = K_f \left[p_0^{a_1 u_{1f} + a_2 u_{2f}} \; p_1^{-a_1 u_{1f}} \; p_2^{-a_2 u_{2f}} \right]^{\frac{1}{1 - a_1 u_{1f} - a_2 u_{2f}}} \tag{38}$$

where K_f is a function of $a_0 u_{of}$, $a_1 u_{1f}$, and $a_2 u_{2f}$.[19] Since the prices paid and received are assumed to be the same for every firm, the quantity supplied by a particular firm differs from that of any other firm because of differences among firms in their possession of fixed factors and technical knowledge, i.e., only because the u's differ among firms. Let x_0 be the quantity supplied by the industry and let $\varphi(u_0, u_1, u_2)$ be the joint density function of the distribution of the u's among firms, then the short-run industry supply function may be written

$$x_0 = \int_{u_0} \int_{u_1} \int_{u_2} x_{of} \, \varphi(u_0, u_1, u_2) \, du_0 \, du_1 \, du_2 . \tag{39}$$

Equation 39 is the short-run supply function because the fixed factors reflected in the u's are not allowed to vary when prices change. For simplicity, x_{of} has been written in equation 39 rather than the expression on the right-hand side of equation 38. Replacing x_{of} in equation 38 by this shows the dependence of industry supply on the prices and the distribution of the u's.

If we knew the production function for every firm in the industry, it would be a simple matter to determine the industry supply function. Knowledge of the production function for each firm could be obtained from a knowledge of the ratio of payments to variable factors to gross revenue for each firm by equation 37 and

$$a_0 u_{of} = \frac{x_{of}}{x_{1f}^{\,y_{1f}/y_{of}} \; x_{2f}^{\,y_{of}/y_{of}}} \tag{40}$$

[18] This is a well-known result for production functions of the general Cobb-Douglas form. The assumption of this form is what makes the subsequent discussion as easy as it is. Production functions of other forms could be used, but then the analysis would be much more complex. This is one reason for regarding the model discussed here as illustrative only.

[19] This result is derived in the appendix to Bachman and Nerlove (1).

But, of course, in any realistic situation it is unlikely that we will have this information for any but a small sample of firms. If this is the case, equation 39 suggests that we can still estimate the short-run industry supply function provided we can estimate the parameters of the density function $\varphi(u_0, u_1, u_2)$. This is true because the unknown u's are "integrated out" in equation 39.

Suppose that the u's have a joint lognormal distribution, i.e., if

$$w_{if} = \log u_{if} \text{ for } i = 0, 1, 2$$

then

$$(41)\quad \varphi(u_0, u_1, u_2) = \frac{\sqrt{1\Gamma^{-1}1}}{(2\pi)^{3/2}}\, e^{w_0 + w_1 + w_2}\, e^{-\frac{1}{2}(w_0\, w_1\, w_2)\,\Gamma^{-1}\begin{pmatrix} w_0 \\ w_1 \\ w_2 \end{pmatrix}}$$

where Γ is the variance-covariance matrix of the w's and the means of the w's are assumed to be zero.[20] It follows from equation 39 that the short-run industry supply function is

$$(42)\quad x_0 = \int_0^\infty \int_0^\infty \int_0^\infty \frac{x_0\, e^{w_0+w_1+w_2}\sqrt{1\Gamma^{-1}1}}{(2\pi)^{3/2}}\, e^{-\frac{1}{2}(w_0 w_1 w_2)\,\Gamma^{-1}\begin{pmatrix} w_0 \\ w_1 \\ w_2 \end{pmatrix}}\, dw_0 dw_1 dw_2$$

where x_0 is written as a function of the w's rather than the u's:

$$(43)\quad x_{0f} = K'\left[p_0^{a_1 e^{w_{1f}} + a_2 e^{w_{2f}}}\; p_1^{-a_1 e^{w_{1f}}}\; p_2^{-a_2 e^{w_{2f}}}\right]^{\frac{1}{1-a_1 e^{w_{1f}} - a_2 e^{w_{2f}}}}$$

Thus, leaving aside the difficulties of integrating a complex expression as appears in equation 42, an estimate of the short-run industry supply function can be obtained from a sample of firms if the observed variables in the sample can be used to estimate Γ, the variance-covariance matrix of the w's.

Suppose we have a random sample of N firms from F and know for each of them the values x_0, x_1, x_2, y_0, y_1, and y_{2f}. Let

[20] The assumption of zero means can always be made, since the a_i, $i = o, 1, 2$ can always be chosen so that

$$E \log a_i u_{if} = E \log a_i + E w_{if} = \log a_i + E w_{if} = \log a_i.$$

$$Z_{if} = \log ({}^{y_{if}}/y_{of}), \ i = 1, 2,$$

and

$$Z_{of} = \log\left(\frac{x_{of}}{x_{1f}^{\,y_{1f}/y_{of}} \; x_{2f}^{\,y_{2f}/y_{of}}} \right).$$

Now consider the sample variances and covariances of the Z's

$$s_{ij} = \frac{1}{N} \sum_{f=1}^{N} (Z_{if} - \bar{Z}_i)(Z_{jf} - \bar{Z}_j) \tag{44}$$

where

$$\bar{Z}_i = \frac{1}{N} \sum_{f=1}^{N} Z_{if}.$$

It can be shown that s_{ij} is the maximum-likelihood estimate of σ_{ij}, where σ_{ij} is the population variance or covariance between w_i and w_j,[21] i.e.,

$$\sigma_{ij} = E\, w_i\, w_j .$$

It follows that

$$G = \| s_{ij} \| \tag{45}$$

is the maximum-likelihood estimate of Γ, so that replacing Γ^{-1} by G^{-1} in equation 42 yields the maximum-likelihood estimate of the short-run industry supply function.

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[21] The proof is somewhat complicated and will not be given here.

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Discussion

MARC NERLOVE has provided us with a very useful survey of the major problems involved in the time-series analysis of supply functions for agricultural products. The quality of the essay is outstandingly high — it is rigorous and it presents important issues. In the past few years, Nerlove has made important contributions to the study of the supply of agricultural products; the present paper provides a strong basis for the expectation that such contributions will continue into the future.

Of the many important points discussed by Nerlove, I will comment on only two. One is the certainty equivalent and its rehabilitation, and the other is the rational expectations hypothesis.

The certainty equivalent has a very considerable appeal to the economist. This appeal rests in large part on the fact that if the certainty equivalent represents a useful simplification, the statistical analysis of important problems becomes possible. If we assume that the entrepreneur maximizes expected utility and does not place rather specific restrictions upon the forms of various functions, it is at once evident that if we are to explain that entrepreneur's behavior in an uncertain situation we must know a great deal. Even if we knew both the utility function and the cost function, and both were known with certainty by the observer, the required knowledge about the entrepreneur's expectations concerning prices of input and output in various time periods presents a staggeringly complex analytical and empirical problem. If one drops the apparently unreasonable assumption (at least in the case of agriculture) that the cost function is known with certainty, one greatly increases the complexity of the situation.

Thus there is little wonder that economists, as rational beings, should search for simplifications that could reduce an insuperable task to one of manageable proportions. Some may argue that the use of the certainty equivalent in time-series analysis represents a high degree of simplification and may not present as accurate a representation of behavior as we would like. However, the use of certainty equivalents makes the analysis manageable and we can only find out its usefulness through actual empirical work.

In adopting the certainty equivalent in time-series analysis of agricultural supply functions, it must be recognized that there is always a possibility that farmers are confronted with a change in circumstances such that their behavior cannot be adequately predicted by the restrictive assumptions utilized. I am convinced, and I see nothing in Nerlove's presentation inconsistent with the position I take, that many of the investment decisions made by farmers cannot be adequately analyzed within the framework of the certain equivalent. If this is a valid position, what are its implications to the use of certainty equivalents in time-series analysis of supply responses? If the expected probability distributions of prices do not change appreciably over the relevant time period, I doubt if significant difficulties would arise. However, if there is a change to a price support system that really worked in the sense that price variability was reduced significantly, investment decisions would be modified even if the expected mean price remained unchanged. The change in investment would affect the cost functions and other variables entering decision functions. Thus it is likely that in the case of a period involving a significant change in the factors affecting price formation, the simplifying assumption of certainty equivalents may be an unsatisfactory one. It must be noted that such changes, at least in some cases, are quite obvious to the researcher, and gross mistakes should be avoidable.

A time-series analysis of supply will usually (hopefully) cover a period of two or three decades. During a period of that length, it is not at all unlikely that the income and wealth positions of farmers will increase by as much as 50 to 100 percent. As a result, it is possible that many farmers' attitudes about risk or uncertainty bearing may change during the period. The decisions made in response to given probability distributions of prices may not be the same at the end of the period as at the beginning. I suspect, though I doubt if I could present the evidence to support it, that many farmers in 1960 were much more willing to undertake ventures with a wide range of possible outcomes than farmers with the same general characteristics falling in the same relative wealth position in 1935 or 1940. However, it should be noted that I know of no other model that adequately takes account of the relationships between income or wealth and the various decision variables.

The rational expectations hypothesis is an inherently appealing one, since it rests on the presumption that economic theory is meaningful and important. The idea is new and it appears that it is worthy of serious attention, consideration, and application. Since I must play the role of a critic, I feel I must introduce at least several possible difficulties.

The first is that competition in agriculture need not result in profit maximization by most farmers. This is true because a large fraction of the resources used do not have a contractual price and "losses" can be absorbed for a considerable period of time as reduced returns to owned resources and perhaps even by resources rented on a share basis. As more and more of the resources used in agricultural production are purchased, the forces of competition are more likely to result in profit maximization as a condition for survival.

The second comment is not in any real sense a criticism of the rational expectations hypothesis, but it is an indication of the difficulties that may be involved. The example may be taken from corn and hog price relationships. Let us assume that price expectations for corn are uniform among all producers and have a small variance due to the price support and storage programs. The application of the rational expectations hypothesis is confronted with the rather serious difficulty that the supply function for corn is very elastic over a rather wide range. Thus the supply function for hogs is much more elastic than it would be in the absence of a price support and storage program for corn. The more elastic the supply function for a product, the greater will be the effect on planned output of a given change in expected price. If these simple statements are approximately correct, it means that the expected prices derived on the basis of the rational expectations hypothesis are likely to be subject to large errors due to the combination of very elastic supply and relatively low price elasticity of demand.

The above comments about the possible difficulties of utilizing the rational expectations hypothesis apply, with equal or greater force, to any other price expectations model. Perhaps the greater relevance of the comments is to problems of achieving price stability.

Chapter 3

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Regression Analysis of Supply Functions Undergoing Structural Change

BEFORE BEGINNING the discussion of alternatives in incorporating supply shifters and interpretation and evaluation of regression analyses under structural change, it is necessary to reach a common understanding concerning the distinction between a shift in supply and structural changes affecting supply. In spite of the widespread use in the literature on supply of the terms "shifts" and "structural change," there appears to be considerable confusion concerning their exact meaning. Later in this paper it will be argued that much of the difficulty surrounding supply estimation stems from confusion relating to these concepts.

DEFINITIONS AND CONCEPTS

As a means of setting the scene for the more rigorous concepts intended for later development, let us begin with an over-simplified illustration. We will define a supply function as a linear relationship expressing quantity as a function of price and, for the moment, one other variable (e.g., the price of inputs). That is, $Y = a + b_1X_1 + b_2X_2$, where Y is the quantity offered, X_1 is the price of the product, and X_2 is the price of inputs.

The simple graphic representation in Figure 3.1 illustrates this relationship, with the alternative curves S_1S_1 and S_2S_2 representing the price-quantity relationship for alternative values of X_2. The concept of a "supply-shifter" grows out of this elementary exposition of supply. The movement of the supply curve from S_1S_1 to S_2S_2 resulting from a change in the value of the "other" independent variable is conceived to be a shift in supply.

The concept of structural change is not so common at the elementary level, at least not in those terms. The most easily understood result of a structural change is a change in the slope of the supply function, i.e., a change in the slope of S_1S_1 and S_2S_2 in Figure 3.1. However, the concept of structural change is more inclusive. A change in b_2, the shifting effect of a change in the value of X_2, is also a structural change. And, if the value of "a" should change or the nature of the function should be modified (e.g., from a linear to a nonlinear form) this, too, is classified as a structural change.

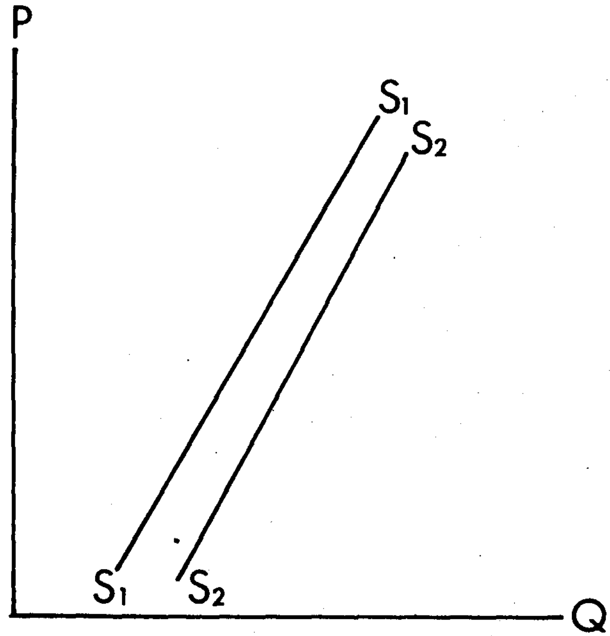

Figure 3.1. Illustration of linear function.

The concepts of shifts in supply and structural change perhaps can be more easily visualized at the micro level. We begin by looking at the concept of shifts in supply and structural change from the viewpoint of the individual firm.

A shift in the supply function for an individual firm is characterized by a change in the planned level of output at a given price without any change in the decision-making environment faced by the firm. As in the aggregate case, the supply shift is a parallel movement in the supply curve.

The requirement of an unchanged decision-making environment used here is analogous to the ceteris paribus condition in the aggregate case. Included in environment are factors such as the production functions faced by the firm, managerial abilities, and the institutional setting in which the firm operates. A change in any of these factors is likely to give rise to something more than a parallel movement of the firm's supply curve. Such a change may and probably does result in a change in the slope of the curve or in a change in the values of other parameters that define the firm's supply function. A new production function, for example, would most likely not only change the position of the supply curve but probably would also change the slope of the curve. ✓ In the same way, a new production function for a competing commodity is likely to alter the effect of a price change in the competing commodity on the firm's willingness to supply quantities of the commodity in question at various prices.

The above concepts are readily extended to an aggregation of all firms within the industry. However, in addition to technological relationships and prices, the number and distribution of firms also must be taken into consideration. Movement of firms in and out of production of a given commodity may be classified as shifting supply, a result of structural change in supply or one of the features which gives the static supply curve its characteristic slope. If, for example, a firm finds it profitable to produce at prices P_1 and higher, but unprofitable at prices below P_1, the entry and exit of this firm from production at P_1 is merely a component of the static supply curve; it is one of the phenomena which give rise to the supply curve's slope. The price (P_1) at which this firm enters production may change, however. If this results from a change in the value of a variable within a fixed enivronment, e.g., from changes in the prices of inputs or of prices of competing products in production, then the modification in the firm's behavior is properly classified as shifting the supply. But, if this change in the critical price (i.e., the price affecting entry or exit) results from a change in environment, e.g., a change in the production function for the commodity in question or of the production functions for competing commodities, or a change in the certainty with which prices may be expected, then the entry or exit is a manifestation of a structural change in supply.

In simple terms, this describes our conception of the distinction between supply shifts and structural change. Let us now consider this distinction under a more complex formulation of supply.

Let supply be a function involving n variables and described by m parameters.

$$\text{Supply} = F\ (x_1 x_2 \ldots\ldots\ldots x_n; \quad \alpha_1 \alpha_2 \ldots\ldots\ldots, \quad \alpha_m).$$

If this formulation is to be meaningful and useful in the usual static sense, we must also attach certain ceteris paribus conditions. These conditions relate to the complex set of factors referred to earlier as the decision-making environment. The most important of the ceteris paribus conditions commonly specified in supply analysis is, of course, a constant state of the arts.

Shifts in supply and structural change in the context of the above formulation are defined as follows: Shifts in supply result from changes in the values of any of the variables other than price and quantity. Structural change, on the other hand, results from some force which brings about a change in one or more of the parameters or a change in "F" (the form of the relationship). Structural changes therefore grow out of changes in one or more of the factors included in the ceteris paribus conditions.

PROCEDURAL PROBLEMS

It is from the background of these definitions that we want to discuss the problems of alternatives for incorporating supply shifters and interpretation of regression analysis under structural change.

A few ground rules should be made before proceeding further. It is not intended to deal specifically with the problem of long-run vs. short-run supply. To the extent that a distinction is applicable within our discussion we will be speaking in terms of the short run. Furthermore, it is not intended to discuss the problem of what specific price is to be used, i.e., the problem of expected price, responsible price, etc., will not be considered. This is not to deny the importance of such problems; they simply fall outside the scope of this paper.

SUPPLY SHIFTERS

Variables uniquely classified as supply shifters are relatively few. In fact, of the variables commonly considered in supply analysis, there are only two categories of variables which would qualify for such a classification within our definitional framework. These are (1) prices of inputs or factors, and (2) prices of commodities competing in production. Other variables often included as independent variables in supply analyses are either structural or quasi-structural in nature. In other words, we are arguing that strict interpretation of the static supply model presented earlier would restrict the variables to include only price and quantity of the commodity in question and prices of inputs and factors.

The problem of how to include prices of inputs and competing products in regression analysis of supply forces one to consider many interesting alternatives. In general, the alternatives include such things as whether to include individual prices or indexes involving several commodities and whether to include prices in a linear fashion or in some nonlinear manner, the most common of which is as a deflator.

The problem of multicollinearity among price series is such that one seldom attempts to include prices of more than one or two major competing products and more than one or two major inputs. It is assumed that the prices thus included adequately account for other less important commodities which might logically enter into a more complete model. In a hog supply analysis, for example, one might include the price of corn and the price of beef cattle, ignoring all other commodity prices. As an alternative, one could use an index of feed grain prices, or an index of prices paid by farmers for production items and an index of livestock prices other than hogs. A useful guideline to follow might be to use individual commodity price series where there is a single outstanding input or substitute product and to use an index where such a distinction is not well defined.

Whether to include the price series in a linear fashion or some other manner is also a problem for which no clear cut solution can be given. We are speaking here not so much of whether a linear in actual values or linear in logarithms formulation should be used but whether or not the price of the commodity should be included as a relative to some other price. A relative price is often used in studies of hog supply functions, for example. The hog-corn ratio is used as a single independent variable rather than the price of hogs and price of corn as separate variables. In defense of the use of the ratio, one could argue that this practice implicitly removes the influence of variation in the general price level and at the same time conserves one degree of freedom. If another undeflated price is used in a linear fashion, however, it raises additional questions concerning the influence of general price level.[1]

There are other significant questions which may be raised with respect to the use of the ratio. The interpretation to be placed on the regression coefficient for the price ratio forces one to accept that the price of corn has an almost equal effect on hog supply as the price of hogs. Further, it assumes the absolute level of hog and corn prices to be unimportant. This may be true within some range of prices, but it probably is not true over the observable range of prices for a given period of time. In any case, it is probably worth the price of one degree of freedom to let the data determine the separate effects of the individual prices rather than force conclusions such as those indicated above.

Weather is an especially difficult variable to handle in the framework employed here. Although it acts in many respects as a shifter, it also brings about changes in the production function albeit changes that can seldom be known at the time of decision making. Furthermore, weather does not affect supply over time in a perfectly random fashion, so cannot be relegated to the error term. However, it is an important variable in explaining variation in quantities supplied. Alternatives for its inclusion in regression analyses will be discussed now, with an understanding that it is not to be considered in the class of unique shift variables.

Weather is not a phenomenon which lends itself readily to quantitative measurement. This is particularly so in the context that it is used in supply analysis. Until recently, most attempts to include weather in supply studies have involved inclusion of some variable in the equation presumably highly correlated with weather effects, such as crop yields or moisture conditions. Another common and probably more satisfactory method has been to exclude from consideration variability in supplies due to weather. This may be done, for example, by using acreage rather than production as a quantity variable.

Stallings at Michigan State has developed a weather index that shows considerable promise for improving the ability to take account of weather in regression analysis of supply.[2] It is hoped that the USDA

[1] The problem of accounting for changes in general price level are not considered explicitly here. It is essentially a measurement problem as opposed to a conceptual problem. It is important, however, and should not be neglected in any meaningful supply analysis.

[2] J. L. Stallings. Indexes of the Influences of Weather on Agricultural Output, unpublished Ph.D. thesis, Michigan State University, 1958.

will follow up this pioneering effort with further refinements and make the development and publication of comprehensive weather indexes an integral part of their statistical program.

STRUCTURAL CHANGE

A supply equation containing only price, quantity, prices of inputs and substitutes, and weather is not a very meaningful estimating model for any extended period in American agriculture. The ceteris paribus conditions implied in such a model simply are not consistent with the facts of life in modern agriculture. In other words, structural changes are all-important forces in supply considerations for agricultural products.

The standard regression model does not and cannot fully allow for structural change. A basic assumption of the regression model is that the parameters of the system remain constant over the period of analysis. As shown in Figure 3.2, each price-quantity observation is considered a point on a supply curve with a slope equal to that of the "true" supply curve but shifted away from the "true" curve by some other variable. In addition, the very nature of regression analysis forces the assumption that the form of the supply relation does not

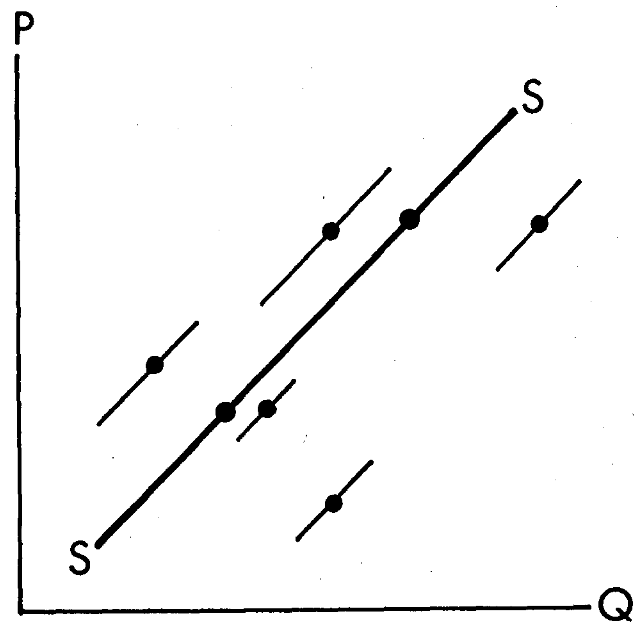

Figure 3.2. Hypothetical scatter diagram illustrating the basic shifting idea in least-square regression.

change over the period of analysis. This problem — the inflexibility of the regression model with regard to structure — is the principal one with which analysts must cope if they are to obtain meaningful supply estimates from time-series data. It is possible to properly account for the effects of changes in the values of variables which are strictly shift variables, but the effects of changes in structural variables can never be completely taken into account within the regression framework.

There are other difficulties. Confusion arises out of the fact that most variables which impinge upon the question of supply cannot be uniquely classified as giving rise to shifts, or to structural changes. Changes in some of the variables which we intend to discuss under the heading of structural change may and often do give rise to supply shifts of far greater significance than whatever structural impacts they might have. (This may be thought of in terms of changing the value of the parameter commonly referred to as the "constant term.") At the same time, variables may be employed in a regression analysis which are not strictly structural in nature but do affect structural variables or reflect structural changes in one way or another. These variables often substitute for nonmeasurable structural variables. For example, yield is often included as an independent variable in supply analysis for crops. This single variable, yield, may be expected to account for a whole complex of structural forces associated with technology and managerial skills.

The structure of an industry with respect to supply may be characterized as consisting of (1) the skills existing within the industry, both managerial and labor, (2) the technology or state of the arts, (3) the number and distribution of firms, and (4) the institutional framework surrounding the industry. We shall refer to these as the highest order structural variables.

Because the highest order variables are broad in scope and are not easily quantified, we typically deal with lower order structural variables which are associated with or influence one or more of the highest order variables. For example, these variables would include educational level of farm operators (associated with managerial skill) and asset position or current income position (associated with and influencing the rate of adoption of new technologies and the distribution of firms).

The most demanding of the structural variables is, of course, technology. Technological change is characteristic of American agriculture. Closely related and almost equally important over time has been the constantly improved level of managerial skills of farm operators. Other factors which have changed over time and have undoubtedly resulted in structural change are the size and degree of specialization among firms, government programs, and varying degrees of market integration. Uncertainty in decision making has been reduced through increased emphasis on outlook information and ability for more timely accomplishment of production tasks (e.g., spring plowing and seeding).

All these changes have had structural implications with regard to the supply of agricultural commodities.

How does one resolve the dilemma posed here? Essentially we have said that structural changes in agricultural production have been ubiquitous and overriding in recent decades. We also have said that structural change as we have defined it cannot be incorporated into a standard regression model.

Several partial solutions to the dilemma are available. The simplest is to ignore the problem, i.e., to assume that structural changes have not taken place or that those which have are relatively unimportant and are properly included in the error term of the regression model. Few would be willing to accept this solution except for very short time periods.

Even within the standard regression model, however, one can take some account of the effects of changes in structural variables. Procedures which make this possible do require, however, some very restrictive assumptions regarding the nature of the structural change.

Structural variables of any order may be incorporated in a supply analysis in the same manner as shift variables. For example, various indicators of the level of technology are often used in this way. This procedure may account for whatever shifting effect changes in the structure may have had on the supply relation. It will not, however, pick up the effect that such changes might have on the values of the parameters. To the extent that the values of the parameters might have changed during the time period studied, one probably obtains some sort of average of the true parameter values as a regression coefficient. If, for example, the true elasticity of supply varied from .5 to .2 during the period of analysis, one would probably estimate an elasticity somewhere between these values.

In addition to including various structural and quasi-structural variables in the analysis, a time variable is often used. This variable presumably accounts for shifting effects not adequately accounted for by other variables in the equation that have shown a relatively constant pattern of change over time. Indiscriminate use of time in regression analysis where the assumption of a linear pattern of change over time for some unmeasurable factor does not have *a priori* support is not good practice. Such a procedure can result in biased coefficients for other independent variables which exhibit a secular trend. In any case, simply including time as another independent variable cannot account for structural changes other than those that shift the supply curve.

It is possible to incorporate in the analysis an assumption that one or more of the coefficients is linearly associated with time. If, for example, one assumes that the price elasticity of supply has been changing at a reasonably constant rate during the period of analysis, an addition variable TP (time x price) could be included to account for such a change.[3]

[3] In the simple regression $Q = a + bP$, for example, we assume $b = c + RT$ (i.e., b is a linear function of time). The estimating equation including this assumption becomes $Q = a + (c + dT) P$ or $Q = a + cP + d (TP)$.

Existence of a "once and for all" structural change, e.g., institution of a government program during the analysis period, can often be accounted for by including a "dummy" variable with value zero during the free market period and value one when programs were operating. Again, such a variable will account only for shifting effects and not for any other structural changes.

Another procedure which may be used is to attempt to restrict the period of analysis to years in which it is believed that the structure is reasonably constant. This may involve the division of an original period of study into several subperiods with estimation of a separate equation for each subperiod. This procedure assumes that the pattern of structural change is not smooth over time, but tends to occur in "spurts", and that we can recognize a priori when the periods of relatively limited structural change have existed. It is somewhat more flexible than the earlier alternatives, however, since it permits one to deal with situations where any or all of the α's may have changed and, in fact, to measure such changes.[4]

SUMMARY AND CONCLUSIONS

This discussion suggests that regression analysis of time-series data is an imperfect tool for supply analysis where structural changes have occurred during the time period analyzed. Because structural change is a predominant characteristic of American agriculture, one might be tempted to discard regression from the tool kit of researchers concentrating on supply of agricultural commodities. Such action is not justified.

Regression analysis has rarely provided satisfactory supply estimates in the past, but this is also the case with other procedures. What has been lacking in regression, as in other techniques, has been sufficient understanding of structural changes and their impact upon supply. As this understanding is acquired, continued attempts to incorporate it in regression analyses of time-series data may prove fruitful in the quest for more accurate estimates of fundamental supply parameters.

This raises another question. Assume we are successful in properly accounting for past changes in structure on supply parameters and are able to accurately estimate the necessary parameters. Are we any further ahead as far as predictive ability is concerned? Must not we also be able to adequately predict future changes in structure? In other words, in addition to being able to detect and measure past changes in structure, we must understand the conditions which have given rise to those changes. In the terminology used earlier, we must be able to understand the relationship between the higher and lower order structural variables.

[4] This is, in fact, the type of analysis employed by Cochrane in his aggregate supply analysis. See Farm Prices: Myth and Reality, University of Minnesota Press, pp. 46-50.

This kind of knowledge is not likely to be gained from time-series analysis of macro-type data. It may be learned from micro data gathered over time. This may be exemplified by information of the sort collected in the producer panel research under the Lake States Dairy Adjustment Study. Many conceptual and procedural problems are inherent in such studies. It is hoped, however, that the information gained over time will be usable in future regression analyses. This, in turn, may lead to improved ability to interpret and evaluate regression analysis under conditions of structural change.

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Discussion

LEARN AND COCHRANE do a good job in pointing up the problems of the difficult task of incorporating supply shifters and structural change into regression analysis. Under their definitional framework, prices of inputs or factors, and prices of commodities competing in production are the only two categories of variables which uniquely qualify as supply shifters. Structural change, on the other hand, involves change in the ceteris paribus conditions implied in a simple supply equation containing only price, quantity, prices of inputs and factors, and weather. The authors emphasize the importance of structural changes, especially those due to advances in technology, in supply analysis. These definitions of supply shifters and structural change are simple and clear cut, and the emphasis on the importance of structural change is well taken.

The complexity of the problems under consideration was reflected in the authors' difficulty in making up their minds on one important point. At one place in their paper, they state categorically that the all-important structural changes, as they defined them, cannot be incorporated into a standard regression model. They then offer partial solutions for doing what they said could not be done.

None of the suggested alternative partial solutions seems satisfactory to the authors or to me. I am especially dubious of the alternative that involves selection of subperiods of years in which the structure of agriculture is judged to be reasonably constant. To me, it seems reasonably clear from the record that we have had marked changes in the rate of technological advance and in the rate at which other structural changes have occurred. I am reluctant to view these as "spurts" or jumps from the end of one subperiod to the beginning of the next.

The authors recognize the difficulty of handling the important weather variable in supply analysis. As might reasonably be expected, they do not come up with a satisfactory solution. In view of the dominant role of higher yields in our upsurge of farm production in the last two decades, I question particularly their suggestion of using acreage rather than production as a quantity variable.

In developing their paper, Learn and Cochrane were careful to point out the problems encountered in regression analysis and also its limitations as a specific tool in supply analysis. I wish they had chosen to give us a broader perspective that could provide a basis for better research programming in this important area. In the rest of this discussion, I shall give some of my general reactions which may contribute to such a perspective.

I am most concerned with the need for supply analyses which can provide a better basis for policy decisions and program formulation designed to alleviate agricultural adjustment problems. Needed are analyses of output response — how output varies with price under changing structure and other conditions over a given period of time. Analyses of this kind have many facets.

Obviously, the analyses should emphasize positive as opposed to normative aspects. Although of necessity they will need to be based on historical relationships, the analyses should be designed for maximum usefulness for predictive purposes. The latter objective can be served by formulations of models which permit the isolation of important variables and parameters whose future course may be independently projected rather than determined solely by historical relationships. The nonreversibility of the supply relation should be recognized and provided for in the formulation of models.

These are noble objectives for analyses of output response. Nevertheless, I believe that our research should be pointed more in this direction. How may regression analyses which incorporate supply shifters and structural change contribute to the attainment of such goals? Learn and Cochrane point up many of the limitations of regression analysis in meeting these objectives. I believe that more research emphasis should be given to approaches, such as the one outlined by R. H. Day, which synthesize time-series and linear programming models. Such techniques may prove more flexible and fruitful than traditional regression analysis in meeting our objectives.

I do not regret our inability, through regression analysis or other methods, to come up with supply analyses of the classical, *ceteris paribus* variety. Such supply analyses imply a reversibility which does not exist. Moreover, because of the rapid structural change of agriculture, the usefulness of such supply curves would be of short duration and could easily be subject to misinterpretation in policy decisions.

Chapter 4

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Evaluation of Time Series as Data for Estimating Supply Parameters

THE SUBJECT of this paper calls for appraisal of past regression work in light of: (1) the production structure for agriculture, including not only the interrelations within agriculture but also the relations between agriculture and other sectors of the economy; (2) the institutional, technological, and behavioral complex in which prices are determined and decisions are made; and (3) the effectiveness of the data selected and the methodological formulations employed in reflecting the operation of the above forces and thus, furthering our understanding of the system.

A demand function exists for the aggregate output of the agricultural industry. Some like to study the effects of the level or changes in the level of at least three types of demand. One type results from variations associated with the general level of employment and industrial activity (5). Others are short-run and long-run variations not associated with these measures. Long-run demand variability is related to consumer income, population changes, and consumer tastes. Variations associated with the general level of employment can be explained with National Income type of analysis.

An aggregate supply function for agricultural products also exists. If the function were not shifting, three supply schedules could be traced by plotting prices and quantities which would reflect responses of agriculture to the respective changes in demand described above. We know the shifts have taken place, and this type of analysis would be subject to the identification problem as discussed early by Working (51) and more recently by such writers as Koopmans (36). Causes of supply shifts are highly complex. Alternative hypotheses to explain these changes have been advanced. Various explanations include technology, investment in the human agent, more efficient combination of inputs, increased specialization, reduction of risk and uncertainty, "better" distribution of assets, and assets changing from fixed to variable. Such changes have implications upon the "length of run" as used in the traditional sense.

*Acknowledgment is made of Associate Professor Paul L. Kelley's suggestions for this paper, and of his help in advancing the author's thinking on supply analysis as a contemporary on a study employing another methodology. The assistance of Professor Walter D. Fisher is also appreciated.

Price is determined by the simultaneous interaction of supply and demand forces. In determining agricultural product prices, representations for these forces may not give a too erroneous solution, especially if corrections are made for institutional forces as government demands, controls, etc. As important, if not more so, the "length of run" or effects for elements that are fixed and that are varying must be included in the analysis.

Broad interrelationships can be stated, but the structure cannot be described so simply. Within and as a part of the interrelationship, agriculture is producing not one product but many and using not one resource but many. The structure can be disaggregated down to any level — such as from the industry to regions or to national type-of-farming regions or to states or to state economic areas or to counties on down to firms or even lower. Principles have been outlined which explain how resources are allocated to products in this situation of demands for each product, resource supplies, and prices, and possible substitution among products and resources in production. These apply to any level of aggregation. A full explanation can be made only after the effects of numerous "non-price" factors are included in the study. The family farm is one of the major institutional factors crucial in supply analysis.

Resources are held, changed both for combinations and total quantity, and allocated toward agricultural production by operators of a family farm. The unit consists of a firm and household, maximizing what we have called utility, and which has some, but not fully known, relation to and differences from income maximization (28). This decision-making unit allocates resources to the firm and household. While classical economic principles are guides, accompanying difficulties include imperfect information on prices, technical conditions, institutions and their effects, etc. Change per se involves cost, and these extend not only to decisions on input and output combinations but to levels of both, and to changes in technology. Predictions of supply involve predictions of responses to prices determined at the aggregative level (and possibly other specified changes) in the light of objectives, informational, institutional, and technical problems, costs of making changes, and quantity, quality, and combinations of resources held on these family farms.

With time-series data typically consisting of a number of very limited observations, the problem is to choose a set of representations large enough to realistically describe the real world but small enough to enable estimation of coefficients. With a limited number of observations and often nearly as many variables, the number of degrees of freedom left for error terms often is very limited.

Data available for aggregative time-series analysis are extensive. However, they are subject to inadequacies of a size that requires researchers to restrict their analysis in some cases and probably always to qualify their final results. With most of the data collected for administrative and further reasons other than for research, its inadequacies are not surprising. Excellent discussions of problems encountered in using crop acreage and production data and livestock and feed production data are given by Nerlove (41) and Hildreth and Jarrett (32), respectively.

Although such questions as the accuracy and length of particular series are often troublesome, the main difficulty is that the series are not relevant and logical representations for the effects of the "true" variables. Improperly constructed series may lead to errors or biases. Ladd (38) gives a theoretical discussion of problems of constructing production indexes, while Griliches (19) discusses problems associated with excluding variables and ignoring quality differences in inputs. Schultz (44) discusses the problems of measuring labor, including adjustments for quality, for leisure, for investments in the human agent, changing management, as well as problems of measuring land and capital. Heady (27) suggests the production function as a basis for classifying and aggregating land (and other resources as well). Bradford and Johnson (5) suggest the aggregation of inputs that are perfect substitutes or perfect complements, and that inputs substituting, but not at constant marginal rates, not be aggregated. Nerlove's work in relating "unobserved variables of a theory to variables which can actually be observed" is leading to a methodology which shows promise of providing more realistic representations for farmers' price anticipations (41).

Rules have been prescribed for the statistical analysis of data, e.g., the identification problem and counting excluded variables for a particular equation for simultaneous equations. The compliance with these rules, important as it is, is no guarantee of a meaningful analysis if *a priori* knowledge and experience are not properly built into the design.

THE INTERPRETATION OF INDIVIDUAL STUDIES

Aggregate General Supply Relations in Agriculture

Girshick and Haavelmo, and Tintner, mainly to introduce the method of simultaneous equations, illustrated their use in deriving simple linear aggregate agricultural demand and supply functions (18, 47). As part of their respective systems, such variables as farm prices and consumer income were interrelated.

Tintner related supply to current farm prices, national income, and a cost factor. His coefficients, derived by the variate difference method, were not statistically significant. He concluded that an index of lagged farm prices might have served better. Girshick and Haavelmo related aggregate farm production to farm prices (endogenous), to farm prices lagged one year (all on an index number basis), and to time. For data for 1922-41, they derived a positive coefficient for current farm prices and negative coefficients for lagged prices and time.

In spite of this beginning and also in spite of hypotheses in the literature, the number of more complex and realistic models constructed and tested have been very limited. Some are reviewed below:

Cromarty's Econometric Model for United States Agriculture

The hypothesized model for which empirical estimates were derived by Cromarty is the most comprehensive model attempted for the agricultural sector (12). While it is tied to the Klein-Goldberger model for the U.S. economy, it interrelates the supply, demand, and prices for 12 farm commodities. The short-run model largely abstracts from resource shifts between the agricultural and nonagricultural sectors, but permits the tracing of short-run changes in either sector on variables related to commodities produced in agriculture. The model included 35 structural equations plus a number of identities (the sample period covered was 1929-53), and with very few exceptions, equations were linear in original values of the variables. Cromarty illustrated the usefulness of the model by tracing effects of a change in the price of feed grains on feed grain production, hog production and price, beef cattle production and price, and the demand for feed grains of commercial, inventory and government types (2).

Without pursuing the power of the interrelated structure further, we may examine Cromarty's representations for variables measuring supplies, demands, and prices, and for variables causally related to them.

The argument for representations of various types was made above. These can be classified into groups: (1) economic, including price expectations, (2) institutional, (3) technological, (4) weather, and (5) resource levels. It appears that the representations for "economic" variables were more reasonable substitutes for the true variables than for most of the other categories. A number of the empirical relations were what would be expected, e.g., the influence of lagged absolute wheat prices on wheat production, lagged wheat-feed grain price ratios on feed grain production, milk and feed grain prices on milk production, hog prices on hog production, lagged egg prices on egg production, lagged broiler prices on the production of poultry meat, lagged soybean-corn price ratios on soybean production, tobacco prices on production, and similarly for vegetables.

The hypotheses on the demand side were not too different from those stated by others, but the reasonableness of the coefficients (size, sign, and significance) was generally disappointing compared to those for the supply relations. Demand was expressed (where appropriate) by dividing total demand into commercial, inventory, and government types. The incorporation of variables such as marketing charges also makes the demand equations more complete.

Results from using such variables as acreage allotments, and government demand for wheat, cotton, feed grain, and tobacco (all representing institutional influences) were good. Variables such as the number of Dairy Herd Improvement Associations and the number of birds in the National Poultry Improvement Plan, included as exogenous variables in the milk production and egg production equations respectively, are presumably representations for the state of technology. The coefficients attached to these variables suggest that the use of representations of this type strengthened the structure. The fertilizer variables used in wheat and feed grain equations may have been intended to be this type of representation. Some would reason, however, that these variables more nearly represent the intensity of use of variable inputs rather than technology.

Nearly all the crop production equations were corrected for weather. The measure was constructed in an attempt to reflect the influence of weather on the production of the specific commodity. A measure of the level of resource use was used as a predetermined variable in many equations, especially those used to estimate the production of livestock products. Examples include the inventory numbers of steers and calves, dairy cows and heifers, sows and gilts, and hens and pullets. For crops, the acreage for the preceding year could be interpreted similarly, as could such measures as numbers of combines.

The inclusion of measures as these add to the meaningfulness of supply functions. With the short-run analyses, these measures allow corrections for fixed resource levels. Then schedules of supply against the set of commodity and variable input prices can be derived, and the structure will hold for various resource levels. Some may object to the linear algebraic form for this context. They might argue that output changes non-linearly relative to changes in the level of these resources. Others might feel that the short-run analysis is inadequate and that the level of these resources is really the crucial variable. They would like to see them as endogenous variables with their level explained, not as a given.

Fox, in estimating the coefficients of what was for this study the parent Klein-Goldberger model, obtained reasonably similar results by the use of least squares regression as were originally obtained by limited information (16). It would be instructive here to gather empirical evidence on the necessity of using simultaneous equations by a further comparison of results from the two methods.

Other Studies of the Feed-Livestock Economy

Hildreth and Jarrett studied the feed-livestock economy where prices, feeds, production, and other data were considered for all classes of livestock as an aggregate (32). Five equations were fitted for data from 1920-49. The length of run again was essentially short, with roughage supplies and beginning inventory of livestock predetermined, but prices and quantities fed of feed grain and protein endogenous. However, the expression of the demand for livestock products equation on a per capita basis made some correction for long-run changes in demand. Other equations fitted were a production relation and three farm decision relations (farmer demand for feed grain and protein feed, and supply of livestock products), all linear in logarithms, in contrast to linear in values of variables as by Cromarty. The separation of the technological and behavioristic equations also distinguished the two analyses.

Their quest for a representation for anticipated prices did not uncover a variable in which the authors could place much confidence. In explaining livestock sales by current livestock prices, they derived a negative elasticity. They also found that livestock sales were positively related to the amount of livestock produced, the price of feed grain, farm wages, and beginning inventory numbers, and negatively related to the price of protein as well as the current price of livestock.

They were unable to find a variable to use to reflect the effects of changing technology in the industry although they made an empirical trial subsidiary to the main analysis. Less attention was shown for fixed assets such as equipment inventory. These differences, with the difficulties with price anticipations, describe the principal dissimilarities in handling difficulties with price anticipations and in handling different representations in the two studies. Feed grain production was determined in the Cromarty model, while in the Hildreth-Jarrett study, the equation was not complete because some variables could not be specified.

Their formulation does not give easily interpretable supply relations. The discussion of steps in building models, of problems inherent in the data, in the statistical procedures used, and the consequent interpretation given the final results, are all outstanding.

Foote's study is further analysis of the feed-livestock economy, where feed and livestock were likewise aggregated (15). Variables used, with data for 1922-42, were:

C = Price received by farmers for corn, cents per bushel.

S = Supply of all feed concentrates, in million tons.

A = Number of grain consuming animal units fed annually, in millions.

L = Price received by farmers for livestock and livestock products, index.

Q = Production of livestock and livestock products for sale and home consumption, index.

I = Personal disposable income, in billion dollars.

Using first differences of logarithms, coefficients were derived by least squares as follows:[1]

[1] Numbers in parentheses below coefficients on this and succeeding pages are standard errors for the regression coefficients.

$$\Delta C = .00373 - \underset{(.24)}{2.36\Delta S} + \underset{(.57)}{1.94\Delta A} + \underset{(.18)}{1.13\Delta L} \tag{1}$$

$$\Delta A = -.092 + \underset{(.040)}{.214\Delta S} - \underset{(.032)}{.185\Delta C} + \underset{(.036)}{.207\Delta L} \tag{2}$$

$$\Delta Q = .00369 + \underset{(.090)}{.562\Delta A} \tag{3}$$

$$\Delta L = .00578 - \underset{(.25)}{2.08\Delta Q} + \underset{(.08)}{1.45\Delta I} \tag{4}$$

In equation 2, the interpretation of the coefficient of ΔL is that for a 1 per cent change in L, A changes in the same direction by .207 per cent. In equation 3, a 1 per cent change in A results in a change of .562 per cent in Q. The effect of a 1 per cent change in price on sale of livestock equals (.207) x (.562 or .116 per cent), the supply price elasticity.

Although the author recognized the influence of technology on Q (and illustrates the influence on a flow diagram), the influence of this variable was not imposed on equation 3.

Hogs

Dean and Heady estimated a set of elasticities for the United States and north central region for 1924-37 and 1938-56 (13).

The authors concluded that most of the elasticity in hog production came from changes in the number of litters. They described changes in the industry that led them to hypothesize that the supply-price elasticity increased between time periods, especially for the north-central region. The hypothesis was not rejected for both the study of number of farrowings and weight of animals marketed.

Cromarty derived a short-run elasticity estimate of .130 (12). For the length of run implied by his equation, his estimate and those of Dean and Heady are not inconsistent. Where liveweight slaughter was related to hog prices and production of feed grains (year $t + 1$) (both endogenous variables in his model) and to the available supply of feed grains and January 1 inventory of sows and gilts (both predetermined), his model is more short-run than Dean and Heady's model explaining the number of spring farrowings but is less short-run than their within marketing period analysis. It is not clear how fall-farrowed pigs sold within the year will enter the Cromarty model. They too could be in a January 1 inventory measure. Likewise, another group will be farrowed within year t but will not be sold until year $t + 1$. The treatment of feed grain production, year $t + 1$ as endogenous, seems to be a realistic representation for an industry in which the production of hogs and corn is simultaneously determined. That is, the production of hogs is a function of, among other factors, corn supplies and prices.

	United States		North Central Region	
	1924-37	1938-56[2]	1924-37	1938-56[2]
Spring farrowings				
Expected price model	.46	.65	.53	.73
Equations 1 to 4	.50	.60	.58	.69
Fall farrowings[3]				
Equations 5 to 8	.28	.30	.41	.35
Within marketing period				
Aug. 1 - Feb. 1	.04	.08	---	---
Feb. 1 - Aug. 1	.05	.07	---	---

For equations 1 to 4, first differences in number of spring farrowings (December, year t-1 — May, year t) were explained by hog-corn price ratio, October-December, year t-1, by first differences of oats, barley, and grain sorghum production as a percentage of corn production, ratio of stocker and feeder cattle to hog price, October to December, year t-1 (for early period) and deflated margin between price of feeder cattle and slaughter cattle, October to December, year t-1 (for later period).[4]

For equations 5 to 8, number of fall farrowings were explained by the number of spring farrowings, hog-corn ratio, October to December, year t-1, ratio of price of slaughter steers to price of corn, March to June, year t, production of oats, barley, and grain sorghum (for later period) and by change in corn production from year t-1 to year t (for the earlier period). For the expected price model, the number of spring litters was explained by lagged hog-corn and beef-hog price ratios and lagged number of spring farrowings.

For the within marketing period equations, total liveweight of hogs slaughtered was determined from a supply and demand set of simultaneous equations. The supply equation, Q, (slaughter) was related to the price of hogs (August 1 to February 1 for spring farrowed pigs and February 1 to August 1 for fall farrowed pigs) and a set of predetermined variables affecting hog supply. For the demand equations, prices were related to slaughter and per capita, deflated, disposable personal income.

Similarly, corn production is a function, among other factors, of prospective profits from the hog enterprise.

In explaining total hog production, with a logarithmic function for data for 1924-51, Williams and Sherman achieved somewhat different results (50). For independent variables, all predetermined, they used:

X_1 = spring pig crop previous year.

X_2 = fall pig crop previous year.

X_3 = breeders intention regarding current spring pig crop.

[2]Omitted war years.
[3]Periods 1924-36 and 1937-56.
[4]The theoretical model conformed to the cobweb theorem.

X_4 = corn supply previous year.

X_5 = hog-corn price ratio previous September-December.

They derived estimates of coefficients as follows:

$$Y = 9.331X_1^{.1741} X_2^{.3886} X_3^{.2334} X_4^{.0592} X_5^{.0162},$$

where the standard errors for regression coefficients were .1230, .0674, .0921, .0729, and .0586, respectively.

For their set of independent variables they found total production explained by the fall crop of the previous year and breeder's intention regarding the current spring pig crop, while such variables as corn supply and the hog-corn price ratio did not influence production significantly. The influence of corn supply and hog-price ratios on the spring pig crop, as determined by Dean and Heady, necessarily implies an intercorrelation between X_4 and X_5 with X_3 and a consequent difficulty in detecting separate effects for the variables. It is likely that the influence of X_4 and X_5 was carried by X_3.

Cochrane's informal estimate of the short-run supply elasticity was .8 to 1.0 (9), which is fairly close to the measures derived by Dean and Heady (.46 to .73) for the number of spring farrowings. Cochrane (10) also derived an elasticity of .309 for farmers' intentions to produce spring pigs (data for 1921-56) where the number of sows farrowing, December, year t-1 to June, year t, was explained by hog prices, July through November, year t-1 and by corn prices, September through November, year t-1. Respective coefficients were positive and negative in sign, as would be hypothesized.

Beef

Research workers have probably had more difficulty deriving meaningful and realistic supply-price elasticities for beef than for any of the other commodities. The main difficulty, discussed by Hildreth and Jarrett (32, p. 104) and by Ladd (37) for livestock as an aggregate and by Wallace and Judge (48) for beef, seems to be that as prices increase (decrease), farmers hold back (dispose of more rapidly than normal) animals for breeding purposes, which causes further price movement in the same direction, and in which considerable time elapses before the real supply response, to the original price, reaches the market.

Wallace and Judge derive formulations which they call "supply of beef at the farm" and "supply of beef at retail." The authors explain the amount of farm production of beef as a function of time, number of cattle on feed, number of beef cattle and calves not on feed plus the number of dairy cows, and the production of corn for livestock. For a function linear in logarithms (data for 1925-55), coefficients derived by least squares were all positive.

For a function involving number of cattle slaughtered as dependent, a positive coefficient was derived for lagged price of corn and a negative coefficient for the lagged price of beef. This, the authors agreed, seemed opposed to theory. More meaningful results with supply-price elasticity implications were derived by explaining the weight of cattle slaughtered by a lagged price of beef, the production of corn for livestock, and the lagged weight of cattle marketed. All coefficients were positive, and here the supply price elasticity was .043. Cromarty derived an elasticity of .037 (12). These estimates appear unreasonably low; in fact, Cochrane believed the true elasticity might be .6 to .8 (9).

Milk

For his milk production equation, Cromarty (12) derived coefficients as follows:

$$Y_{41} = 198.188 + \underset{(.212)}{.869Y_{42}} - \underset{(.028)}{.116Y_{22}} + \underset{(.285)}{.598Z_{43}} + \underset{(.708)}{1.969Z_{44}} + \underset{(.510)}{2.425Z_{45}}$$

where

Y_{41} = production of milk in million pounds.

Y_{42} = price of milk in cents per hundredweight.

Y_{22} = price of feed grains (1910-14 = 100).

Z_{43} = number of Dairy Herd Improvement Associations operating on January 1, x 10.

Z_{44} = pasture condition as per cent of normal.

Z_{45} = January 1 inventory of cows and heifers, two years old or over, in hundred thousand head.

The coefficients were derived by the limited information method, with the coefficients for the Y's mutually determined by the system, while the Z's were regarded as predetermined variables. All the coefficients are acceptable. An increase in milk production can be predicted from an increase in milk prices and/or a decrease in the price of feed grain, a relation expected on a priori grounds. These are short-run responses, with technology, roughage supplies, and dairy cow numbers represented by Z_{43}, Z_{44}, and Z_{45}, respectively. These would become variable in a longer run, but here they are regarded as fixed.

The assumption that the Z's are not affected by the other variables might be difficult to argue. For example, changes in the number of cows milked might be explained to some extent by the milking (or allowing to nurse calves) of dual purpose cows on the margin. A large number of cows may be milked when price relationships are favorable, but not under other circumstances. Those who feel that the number of cows is the crucial variable explaining milk production will be disappointed with the short-run model and will be more interested in the results from use of a longer-run model in which this variable is endogenous.

Halvorson (using data for 1931-54) in a short-run analysis with a correction for cow numbers, pasture condition, and hay production, found that a 1 per cent change in the milk-price feed ration was associated with changes in milk production per cow per day (United States data) of .029 per cent during summer months and .135 per cent during winter months (24). He also found some evidence that farmers adjust grain feeding more in response to price increases than decreases.

A later study, encouraged by Nerlove's developments (with data for 1927-57 and for 1941-57) related milk production to a deflated lagged milk price and time, and successively added the variables milk production the previous year, total hay supply, supply of total concentrates, beef price, and hog price (25). As the number of independent variables was increased, he studied not only the coefficients and standard errors, but R^2, the coefficient of adjustment, and the short- and long-run elasticity. For the 1927-57 data, the supply elasticities for the alternative regressions were roughly .16 and .40 (but with data for 1941-57) the short- and long-run elasticities were both considerably lower for formulations including beef and hog prices. For formulations excluding these prices, he believed he detected evidence that farmers were more price responsive in the late period (elasticities of .286 and .526), compared to .157 and .398, short- and long-run elasticities respectively for 1927-57, which compares with the .212 estimate of Cromarty (12). The role of the price of beef toward explaining milk production was considerably greater during 1941-57. To the extent that the findings are valid, this represents a considerable change in structure. In an analysis, by years of rising and declining milk prices, he obtained the surprising results that the supply elasticity was greater (although not significantly) for years of declining prices.

Although the analyses of both led to results from which elasticities could be calculated, there were important methodological differences between the two studies. In the Cromarty model, milk and feed prices were endogenous variables, while for Halvorson's (25) single equation model, the price of milk and supply of concentrates were independent. Cromarty used variables for cow numbers and technology (to the extent that Z_{43} represented it). Halvorson used lagged milk production and prices for competing products in farm production.

Cochrane believes that the short-run elasticity for whole milk lies between .3 and .4 (9), but with quarterly data (1947-56), he derived a price elasticity of .030 where milk production by quarters was dependent, while milk prices and dairy ration prices (average for current and preceding quarter), annual production, and quarterly cow numbers were independent (10).

It is believed that the variables representing the effects of federal order markets are important omissions in these formulations. Also, the industry has made substantial changes since much of the data used in the analysis above were obtained. Farmers are investing in bulk tanks, pipeline milkers, and walkthrough milking parlors. It is anticipated that these developments, along with increased specialization and larger herds, will lead to marked structural changes for this industry.

Eggs

Where Y_6 is an estimate of the log of the index of per capita supply of eggs (for data for 1921-50), Judge (35) derived coefficients for an egg supply equation where other variables endogenous (Y's) and exogenous (Z's) were:

Y_5 = log of the index of prices paid to farmers for eggs, deflated by the cost of living index.

Z_4 = same as Y_5, lagged one year.

Z_3 = time.

Y_7 = log of the index of prices paid to farmers for meat, deflated by cost of living index.

Z_5 = log of index of cost of the poultry ration.

Z_6 = same as Z_5, lagged one year.

The estimated equation (in logarithms) was

$$Y_6 = 1.6727 + 1.1659Y_5 + .2298Z_4 + .0018Z_3 + .5438Y_7 - .9748Z_5 - .7769Z_6,$$

where, of course, the price elasticity of supply was 1.1659 for current prices and .2298 for lagged prices.[5]

Cromarty, relating egg production to the price of feed grain (endogenous) and to the number of birds in the National Poultry Improvement Plan (NPIP), January 1 inventory of hens and pullets, and the price of eggs December 15 of the previous year, all predetermined, derived an elasticity of .298 (12). These equations had elasticities about equal to and less than, respectively, the range of 1.0 to 1.2 presented by Cochrane. Both were determined by simultaneous equation methods, are an

[5] Estimates of coefficients, where a reduced form model was used, were:

$$\hat{Y}_6 = .3608\,Y_5 + .3033\,Z_4 - .0095\,Z_3 + .5375\,Y_7 - .4401\,Z_5 - .5397\,Z_6 + 1.6158$$

and by least squares,

$$\underset{}{\hat{Y}_6} = \underset{(.1674)}{.1924\,Y_5} + \underset{(.1441)}{.5295\,Z_4} + \underset{(.0287)}{.0743\,Z_3} + \underset{(.1280)}{.0149\,Y_7} + \underset{(.1523)}{.0067\,Z_5} - \underset{(.1538)}{.3956\,Z_6} + 1.3006.$$

These alternative derivations are examples of instability of estimates relative to estimating procedures.

interesting contrast. Judge made a correction for long-run changes in demand by stating supply in per capita terms. He allowed for the influence of competing commodities in production with variable Y_7, although the sign of the coefficient is unexpected. He also allowed for the influence of the current price of eggs.

Cromarty, on the other hand, made some correction for changing technology, to the extent that the number of birds in the NPIP represents the change. He also corrected for the number of layers with his January 1 inventory of hens and pullets. This omission in the Judge model makes the interpretation of the length of run difficult.

An additional study of the demand, supply, and price structure for eggs was published by Gerra (17). On the supply side, annual egg production was established as the product of size of laying flock and output per layer. The size of laying flock was determined by the rate of mortality and January 1 inventory of hens and pullets on farms (predetermined), by the number of layers sold (determined by egg and feed prices), and by the number of pullets started from January to June (determined by egg and feed prices for the same period). Output per layer was also determined directly by the same variables, but rate of a lay during the January to June period was assumed independent of egg and feed prices during that period. The influence of these prices was hypothesized to influence only the number of replacement pullets raised.

Estimates of supply — egg price elasticity — were not possible from this study. Other estimates presented were supply elasticity with respect to (a) the price of poultry ration, less than -.05 and -.3, (b) supply elasticity of pullets raised with respect to the egg-feed ratio, January to June, .40 to .44, and (c) the supply elasticity of layers sold with respect to the egg-feed ratio, annual average, -.40 to -.67.

It is believed that the method of handling feed and egg prices, inventory numbers, and changes in hen numbers in this study is realistic and leads to short-run relations more readily interpretable than some other treatments. The results must be interpreted as short run. Effects of more long-run factors as changes in population, tastes, etc. on the demand side and changes in technology (although drawn on the flow chart) on the supply side are omitted.

A further econometric study of the poultry industry was contributed by Fisher (14). Both deflated and nondeflated values were used for the price variables for a sample period 1915-40. Fisher estimated parameters for both a farm supply of eggs and farm supply of chickens against these variables. His short-run elasticity estimates for eggs ranged from -.11 to .217 and for chickens from -.18 to .31. Relating current quantities to lagged quantities and lagged prices, he derived long-run elasticities with respect to own price of 2.17 for eggs and .26 for chickens.

Cromarty's estimate of poultry meat production, explained by the price of feed grain (endogenous), the inventory of hens and pullets, index of poultry equipment, and lagged price of broilers (all predetermined) yielded an elasticity of .678 (12).

Price Expectation Models

An objective of Nerlove's work has been to provide more suitable representations for expected prices (41). By relating planned output (acres in most of his studies) not only to lagged price (deflated by prices received) but to lagged production (acres in his formulations), and in some cases to time, he obtained results he believed superior to earlier estimates based upon static analysis, where lagged production was omitted.

In a statistical sense, he found increased values for R^2, no positive serial correlation of residuals, and a reduction of importance of the trend variable for his dynamic model relative to the traditional method of acreage on lagged price regression. The short-run supply price elasticities were generally larger and more reasonable, he claims, although others might view them as being too high.

His early work concentrated on the expected price representation; and, by not allowing for the effects of other variables and possible simultaneity, his equations have been necessarily incomplete. A full appraisal of his advancements cannot be determined until more realistic models are studied. Some of the crop production equations of Cromarty (lagged price and lagged acreage variables), models of Halvorson explaining milk output, models of Dean and Heady for hog supply, and of Griliches on fertilizer demand were other examples of studies using essentially this procedure.

The representation would also be strengthened by empirical evidence of use of models of that type by farmers. The work of the Interstate Managerial Study indicated that farmers use price expectation models of a broad economic nature more and statistical models less than many had believed. The method would be strengthened by further economic interpretation. Ladd (37) did not believe the model would be appropriate as a representation for livestock prices, Brandow (6) expressed concern over the omission of variables, and Halvorson's experience was cited (25). Knowledge of a system's structure would be increased if effects carried by the adjustment coefficient associated with lagged output could be tied with changes in variables with more structural meaning.

Corn and Feed Grains

Nerlove's elasticity estimates were .09 and .18 for corn (40), and Cochrane's informal estimate was between .2 and .3 (9). Cromarty's estimate for feed grains was .430 (12). It is improbable that the effects of technology (hybrid seed, machinery, etc.) have been removed, and it is unlikely that the elasticities represent pure responses to price.

Wheat

For the Cromarty (12) model, the wheat production equation, derived by least squares, was

$$Y_{11} = -105.181 + \underset{(3.161)}{5.467Z_{13}} - \underset{(.518)}{.370Z_{12}} + \underset{(1.076)}{3.711Z_{14}} + \underset{(.909)}{2.632[Y_{12}]_{-1}}$$

where

Y_{11} = wheat production, in million bushels.

Z_{13} = seeded acreage of wheat for the previous year, or announced allotments when in effect, in million acres.

Z_{12} = fertilizer applied in North Dakota and Kansas, in thousand tons.

Z_{14} = index of weather influence in wheat areas (1943 = 100).

$[Y_{12}]_{-1}$ = higher, of price of wheat for the previous year or current year support price, in cents per bushel.

This equation represents both behavioristic characteristics of producers and technological characteristics of the wheat producing areas. For this reason we do not learn as much about the structure of the underlying relations as we would like.

Using New Zealand data for 1920 to 1953, Candler (8) explained wheat acreage (X_a) with the following variables:

X_b = wheat price, lagged or announced price.

X_c = fat lamb price.

X_i = red clover acreage.

X_j = number of rainy days at wheat planting time.

X_k = last year's wheat acreage.

The equation derived was

$$X_a = 155.0 + \underset{(0.165)}{0.269X_b} - \underset{(0.032)}{0.108X_c} - \underset{(0.079)}{0.145X_i} - \underset{(2.334)}{3.246X_j} + \underset{(0.167)}{0.507X_k}$$

Competitive crops were believed to be lambs and red clover, and a large value of X_j was hypothesized to make seeding difficult and to result in a reduced wheat acreage. Candler encountered a high degree of intercorrelation among his independent variables and expressed difficulty in using the equation for prediction if their interrelationships change in the future. The coefficient for the wheat price variable was significant at only the 63 per cent probability level, and he indicated there was no relation between his wheat price variable and acreage for the period beyond 1924.

Estimates of short-run supply price elasticity for wheat vary from near zero to .93. Bowlen (3), with regressions of first differences of acreage on first differences of lagged wheat prices, found the elasticity near zero (not different from zero statistically) for nine western Kansas counties and .315 for 44 eastern Kansas counties.[6] Nerlove (40), using data for 1909-32, derived an elasticity of .47 for acres seeded against a lagged deflated price and .93 by adding lagged acres as a variable. Cromarty (12) derived .370, and Cochrane indicates that the figure might lie between .1 and .2 (9).

One could safely assume that the true elasticity lies between the extreme estimates. From past estimates, we probably cannot make very accurate statements about the wheat supply-price elasticity. It is not likely that effects of technology, weather, reduced risks, etc. have been separated from the price effects.

Important changes in technology have been adopted in the wheat producing areas. The adoption of summer fallowing, methods of lessening soil blowing, purchase of new machinery, etc., have been at a time when the price of wheat was higher. The measures are not only correlated, but there is some causation (adoption during and following high prices). It is reasoned that part of the production response is explained by price per se and partly by the attending related developments, and that the elasticity estimates are probably biased upward. A decline in wheat prices would, in all likelihood, not be accompanied by a reduced level of use for some of these inputs or a readoption of the old cultural practices; in that sense the equation would not be appropriate for the irreversible portion of the supply curve.

It is believed that the short-run models neglect some longer-run factors at work on both the supply and the demand side for such factors as population changes, although Cromarty used a time variable in some demand equations to allow for changes in tastes. It may be that no correction has been made for some of these influences and that the resulting structure is a hybrid, influenced both by variables that are part of the system and others that were not specified.

Tobacco

After analyses of both a statistical and graphical nature, Johnson generalized that United States underplantings of burley tobacco acreage tend to "decrease around one thousand acres for each one cent increase in the real price of burley" (lagged one year, deflated by an index of prices paid) (33). In his same work he analyzed the effects of penalties on overplanting and of change in allotments. This is not only an illustration of the inclusion of representations for the effects of institutional arrangements (here penalties and allotments) in the analysis, but is

[6] Data for 1926-52, omitting 1938-43 and 1950.

further an example in which attention is pointed to unexplained variations in the dependent variable and consequent implications toward interpreting the results.

In another formulation, he offered an explanation of yield changes for the years 1935-49, where Y_1, United States average yield, was related to exogenous variables.

X_2 = logarithm of the lagged season average price of burley.

X_3 = squared index of prices paid for production items.

X_4 = time.

X_5 = index of weather effect on yields, constructed from yields on experimental check plots where cultural practices were unchanged from year to year.

X_6 = current years United States burley acreage allotment less average acreage of burley harvested in preceding six years.

The attention given to the algebraic form of the equation is interesting, especially regarding X_2 and X_3. A farm operator was assumed operating in stage II (diminishing marginal returns) and the use of logarithmic and squared values of the variables built the diminishing marginal returns concept into the function.

He derived statistical estimates of the influence of each variable. The supply-price elasticity (from yield changes) was roughly .11. The statistical analysis was followed by an examination of residuals unexplained by the statistical analysis. The problem of ascertaining the separate effects of X_2, X_3, and X_4 (which all moved closely together during the period) was examined. It was concluded that a modified interpretation of the statistical estimates was necessary. Also, the influence of certain "non-price" factors, as biological improvements, fertilizing improvements, and of increased price stability were major factors explaining yield changes (although they could not be incorporated in the statistical analysis).

Cromarty (12) derived a short-run elasticity of .381 for burley tobacco production. Coefficients both positive and significant were derived for independent variables, which were the higher of announced support price or price the previous year, time, and the higher of acreage allotment or last year's harvested acreage of burley tobacco.

The importance of "time" as an explanatory variable, and the many and complex set of factors it carries, makes interpreting the results from this equation difficult. As argued by Johnson and as in the discussion of the wheat production equations, it seems unlikely that the period with announced support price can be described by the same structure as the earlier period.

Cotton

Estimates of the elasticity for cotton range from roughly .2 as derived by Walsh (49) (acreage related to deflated, lagged prices for cotton and/or cottonseed) for 1910-24 and 1925-33, to .20 (acres = f (lagged, deflated price, time) and .67 (acres = f (lagged deflated price, time, lagged acres) by Nerlove (40). Cromarty's estimate was .361 and Cochrane's judgment estimate was .2 to .3 (9). Where Cromarty used lagged price and lagged acres plus representations for other influences, as climatic factors and prices paid by cotton growers for inputs, and allowed for simultaneity of supply and demand influences, his estimates stand on a stronger methodological footing.

While his equations do not contain the representations of Cromarty's, Brennan (7) has shown that for crops in which the substitute crops in production vary from region to region, there may be some advantage to geographical disaggregation. Using data for 1905-32 for regions he calls the Southeast, Delta, and Southwest, with acreage as a function of the past two years' prices for cotton and other crops believed substitutes in production, he derived regional elasticities of .33, .31, and .37. He also showed (for the period 1942-48) cross elasticities of cotton acreage with respect to the prices of substitute crops by states, where important substitute crops were hay and peanuts. While the prices of substitute crops explained cotton acreage from 1942-48, he found little such relation during the preceding nine year period. To the extent that his findings are valid (and he had data for only a limited number of years), he has shown some indication of (a) a change in structure over time and (b) differing structure (unlike substitute crops, own price elasticities nearly equal) across regions.

Unharvested Crops

Analyses by Suits (45), and Suits and Koizumi (46) with supply implications were developed for watermelons and onions in two stages. In the first stage, a crop available for harvest estimate was derived, while in the second stage the portion of the crop harvested was explained. For the first stage, least squares estimates were obtained by relating the watermelon crop available for harvest to lagged prices of watermelons, cotton, and commercial truck and dummy variables representing government cotton policy and war, and the onion crop available for harvest to lagged onion prices, lagged costs, and time. Elasticity estimates against own price were .581 and .324 respectively.

For the second stage, the unharvested acreage was related to current own price, (endogenous-price simultaneously determined with demand equations), to the size of the available crop and to harvesting costs. For watermelons, the price elasticity for the harvested supply, given the available crop, was about .2. For onions, the corresponding elasticity was given as E (elasticity) = $1.71\frac{u}{x}$ ($\frac{u}{x}$ = portion of available crop unharvested to harvested), and it was concluded since $\frac{u}{x}$ is usually low, that the elasticity of market supply is low and usually considerably less than the elasticity of crop supply.

Nerlove's analysis of abandoned acreage of corn, oats, barley, and grain sorghums in Kansas where he correlated harvested yields and percent of the crop abandoned, was a similar type of analysis. His argument was that there is some yield, considering the price of wheat and harvesting costs, at which the crop will be just worth harvesting, and at lower yields the crop will be abandoned. The "critical" yield in some cases may be higher than this. If the crop in question has as an opportunity cost another crop that could be planted on the land, the old crop may be "torn up" to prepare the ground for the new crop even though it might have been worth harvesting.

For some crops, especially wheat, some knowledge of eventual yields is known at seeding time from soil moisture readings, previous rainfall, etc. A proper analysis of wheat supply starting with seeded acreage should include representations for variables of this type.

Potatoes

Working with the supply for late spring potatoes in Kern County, California, Mundlak and McCorkle (39) found no relation between lagged potato prices and yields, nor did they find a relation between change in price and change in yield. They found more response to price in their study of acres of potatoes for 1929-53. For a linear equation, for lagged prices for potatoes, cotton, and alfalfa, elasticities of .376, -.137, and .460 were derived for the three respective prices, while for an equation linear in logarithms, elasticities of .237, -.229, and .450 were derived. Other formulations, such as the use of lagged potato prices and a lagged gross return over a two year period gave elasticities of .101, .223, and .277 along with negative elasticities for lagged prices and gross returns for cotton and positive elasticities for alfalfa. For all formulations they found cotton a competing crop and alfalfa a complementary crop to potatoes as they had hypothesized. This is the only study reported where the geographical area for the statistical analysis is the county.

Using data for 1921-41 and 1950-56, Cochrane (10) derived an elasticity of .246, where

X_1 = acres planted in current year, as an indication of intentions to produce, 10,000 acres.

X_2 = potato prices deflated by index of prices received for all crops for years t-1 and t-2, weighted equally, cents per cwt.

X_3 = yield per acre in current year, cwt.

Coefficients derived were

$$X_1 = 378.791 + \underset{(.2402)}{.6308X_2} - \underset{(.1499)}{1.8351X_3}$$

The Dry Bean Industry in Michigan

Hatheway (26) analyzed planted acreage of beans against lagged price, abandonment the previous year, expected income from competing crops, and costs. He found the supply, calculated to be roughly .26, to be relatively inelastic. In his yield model, he related yields to acres in current year (endogenous) and lagged prices and a weather index for beans. He did not detect a relationship between price and yields (which checked with their experience of no response in yields to fertilizer application), but he found a clear tendency for yields to decline with increases in acreage.

Both equations here are further illustrations of care used in algebraic form. Values of specified independent variables were expressed as logarithms and squared values to force certain properties which enabled a more realistic representation for the effects of the variables.

The Supply and Demand for Technology — Hybrid Corn

The implications of technology upon supply analysis have been well-hypothesized, but effects due to changes in technology have at best been treated in supply studies as exogenous. Griliches (20) sought an explanation for the supply and demand for one form of technology, hybrid corn.

Although his analysis of supply was to some extent inconclusive, he was convinced that "market density" or "market potential," involving corn acreage and adaptability of hybrids, both on a land area basis, explained the variation in beginning dates for different areas. For this particular form of technology, once seed was developed for an area, the supply soon became very elastic.

On the demand side, he explained independently (a) the rate of acceptance, and (b) the equilibrium level of use, both essentially in terms of the change in profitability of hybrid corn, and both in a "long-run" framework. For data for states and crop reporting districts, he explained the rate of acceptance using as variables the average increase in yield over open pollinated varieties, the long-run average pre-hybrid yield of corn, and average acres of corn per farm. Corn prices were so similar among areas that no variable was added for price. For equilibrium level of use, (the long-run percentages of the corn acreage that will be planted to hybrid seed), the independent variables were average corn acres per farm, pre-hybrid yield (as profitability measures), and capital per farm. The latter explained variations in equilibrium levels between crop reporting districts but not between states.

Fertilizer Demand

Griliches (21, 22), and Heady and Yeh (29), employing somewhat unlike methodology, derived fertilizer demand functions. In Griliches' formulation, level of fertilizer use was assumed a function of the real price of fertilizer (fertilizer price deflated by index of prices received for crops) and changes in use proportional to the difference between the "desired" and actual level of use, all units in logarithms. Equivalently, fertilizer use was assumed a linear function (in logarithms) of the real price and fertilizer use (both lagged). In the Heady and Yeh model,

Y_1 = total tons of commercial fertilizer consumed was assumed a function of[7].

X_1 = fertilizer price index at planting time, deflated by wholesale price index.

X_2 = crop price index, lagged, deflated as X_1[8].

X_3 = cash receipts from farming (crops, livestock, and its products and government payments) lagged one year.

X_4 = cash receipts from crops and government payments lagged one year.

X_5 = total acreage of cropland.

X_6 = time.

Heady and Yeh derived coefficients for a number of algebraic forms, and results for the functions linear in logarithms were published (as well as for some other formulations, as first differences).

For both studies, coefficients were derived not only for demand at the national level but by regions. Although they did not divide the United States into the same regions, some comparisons are interesting. The coefficients for lagged prices in the two studies may be interpreted as measures of short-run elasticities. Elasticities were considerably greater in the Heady and Yeh study than in Griliches', but there was some consistency in the ranking of the different regions for the two works. The coefficient for X_6 (time) in the study by Heady and Yeh and the "b" value (adjustment coefficient) for the Griliches study likewise have more or less the same interpretation — an indication of a long-run adjustment. The values found by Griliches varied from .04 to .28, while Heady and Yeh derived estimates ranging from .002 to 1.074, and

[7] Functions were also derived for quantities of nutrients the dependent variable.

[8] For the different tests, X_2, X_3, and X_4 were not all employed in the same function.

they were generally more erratic than those of Griliches. Here the two studies indicate the fertilizer demand function shifting to the right, although the regional changes indicated by the two sets of results did not form a clear pattern.

For the formulation of Heady and Yeh, the coefficients for X_3 or X_4 were significant at the 1 per cent level for each region. These variables are presumably representations for farmers' capital position and ability to make purchases of this type. This study then, is an example of an analysis at the aggregative level with a representation for the firm-household complex.

Demand for Machinery

For a stock equation, where machinery stock was related to a "real" price of machinery, rate of interest and lagged stock, Griliches (23) derived short-run elasticities of -.25 and -1.2 with respect to real tractor prices and the rate of interest and long run estimates of -1.5 and -6.9. For an investment equation the elasticities were still more extreme.

Using a somewhat different approach, Cromarty (11) derived elasticities of -1.0 and -2.5 for demand at the wholesale and retail levels, respectively. He related the value of manufacturers' sales and the value of farm machinery shipments (deflated) to a number of variables, such as a real price for machinery, a price for a substitute item in production, a measure of assets held by farmers, a measure of farm income, and a quantitative measure of government programs. Estimates were also derived for tractors and trucks individually.

SUMMARY INTERPRETATION

Any analysis of aggregative relations must be interpreted as the result of a considerable amount of averaging. The analyst is forced to choose variables selectively, hoping he has chosen those which are crucial and those which the influence of the omitted variables will not be major, will not bias the results, and will not increase standard errors appreciably.

Of the many problems in using time-series data for supply analysis, it did not appear that the available statistical techniques (mainly multiple regression and simultaneous equations) were real limitations. The degree of simultaneity and hence the need for the latter method are not known exactly. Hildreth and Jarrett (32) obtained roughly similar results from the two methods, while Judge's (35) results by using the two methods for his egg supply equation were quite unlike. Although there are particular problems in which the degree of simultaneity can be deduced, one cannot generalize on the basis of either arguments or comparative empirical results on the necessity of using a method which handles simultaneous relations.

Those working with regression methods must experience uneasiness when there are more or less erratic changes in coefficients with changes in algebraic form and specification of variables, and where there are very few a priori grounds for one form and/or specification over another. This situation can be aside from problems of single equation versus simultaneous equations. If there is a problem as to the degree of simultaneity, the "uneasiness" is increased still further.

For studies where all (or a large proportion) or the results (and we don't know how many "poor" results have been withheld) have been published, it is possible to find different degrees of these "within study" fluctuations in values for coefficients. Examples of studies for which alternative results were published include Candler (8), Fisher (14), Gerra (17), Mundlak and McCorkle (39), and Wallace and Judge (48). One of the more interesting examples for this context is the model of Halvorson (25), where variables were successively added.

Intercorrelation, mentioned explicitly by some researchers (Johnson (33) and Candler (8) are examples), undoubtedly was a problem in many other studies. The correlation of independent variables makes difficult the detection and quantification of effects traceable to changes in values of specific variables. While predictions can still be made (and the predictions of the structural coefficients will be unbiased), the reliability of the coefficients will decrease and the real purpose of the analysis, deriving coefficients to enable an understanding of the system, may be defeated. It is doubtful if sufficient attention has been given either to the construction of variables or the application of such mathematical methods as the total derivative for this context.

The serial correlation of the residuals was tested for many of the equations, and was a problem in some but not in others. For an analysis of the residuals, see Johnson (33). Nerlove and Addison (43) indicate that serial correlation of the residuals is much less of a problem with price expectation models than with other models.

The choice of a particular algebraic form automatically builds certain properties into the structure. For studies with imaginative forms for equations and particular variables, see Johnson (33) and Hatheway (26). The Cobb-Douglas power function was used in more studies than any other form. The primary reason for its use is that it yields curves (in terms of original values for the variables) which approximate the "theoretical form." For supply analysis, for example, it permits diminishing returns for an input applied to a set of fixed resources, and non-constant (again original values of variables) marginal rates of substitution between inputs and between products. The linear form has also been used by many. Those using it believe linear relations approximate the "true" relations closely enough to justify their use. First differences were used by several, and more specialized forms as the logistic have been employed.

Representations for the effects of variables of various types are frequently necessary to determine a complete and realistic supply structure. Effects for "economic" variables, such as prices for products, variable inputs and substitutes in production and consumption have probably been best represented among the list of types of effects. Their use in these analyses is suggested by economic theory, and results reported by Boyne and Johnson (4) from the Interstate Managerial Study confirm that farmers operating farms respond to changes in these types of data. Many workers have been forced to use deflated prices, e.g., adjusting the price of a commodity whose supply was being studied by an index of prices received. For a commodity with substitutes differing by regions, this kind of construction of representations is probably about all that can be done at the broad aggregative level.

Trained as they are in explaining the determination of price and in determining the chain of effects from a change in price, economists have encountered difficulties in explanations through empirical analyses. These difficulties have come not so much from improper handling of "price" as from locating and incorporating in the analysis effects for changes in "non price" variables.

This generalization holds for those models in which price was endogenous, and where "non price" effects, particularly those changing over time, were not handled explicitly. Difficulties with "non price" variables have been no easier (relative to problems of using "price") in deriving coefficients for equations from which elasticities are derived. Even with the explanation of price determination and effects of price changes as an objective, researchers may not be able to achieve it unless proper "corrections" are made for "non-price" effects.[9]

The handling of fixed resources (also an "economic" variable, and its relevance was confirmed (4) in the IMS for both farmers organizing and operating farms who indicated that they responded to fixed resources) has been a major problem in constructing and interpreting supply functions. Johnson has argued that we do not know what resources are fixed and the conditions under which they become variable (34). Although a complete analysis would treat resources as endogenous, past supply studies haven't gone far in this direction. In the study of supply functions for particular crops, the acreage as in many studies cited above has been explained, while in the hog supply study by Dean and Heady, the number of sows farrowing were explained. The explanation of resource commitment as in these studies is about as far as we have gone in explaining use of resources in incorporating level of resource use within our supply analysis models. The more advanced work of this type has been in the area of farmer demand for fertilizer and machinery.

Not only has progress in explaining resource use been slow, but for many studies the level of resource use is not well-represented in the formulations. This makes difficult the interpretation of the "length of

[9] This is not to say that effects of other changes should not be studied. In fact, effects of some of these "non price" changes may be as deserving of study as are effects of changes for "price." For studies where effects of "non price" changes were explicit, see Cromarty's (12) handling of technology, and Johnson's (33) study of the effects of penalties and allotments on tobacco production.

run" as the term is traditionally used. Among studies specifying levels of resource use are Hildreth and Jarrett (32), and Cromarty (12).

There have been problems in constructing representations for farmers as profit maximizers, but those representing other considerations to which they respond have as a whole been less adequate in the formulations. The institutional, technological, and environmental (e.g. weather) situations to which farmers respond in production of commodities have not always been specified in the statistical designs. Recent farm income measures used in the farmer demand studies — by Heady and Yeh (29) for fertilizer, by Cromarty (11) for machinery and capital per farm, and by Griliches (20) for hybrid corn — are examples of representations pertaining to the institutional arrangement the family farm and the fact that farm production decisions are made at that level. Beyond these examples (where in reality farmers are maximizing utility, are acquiring assets, are reluctant more or less to make changes, etc.), we have been unable in our aggregative supply analyses to more than use representations which imply that the farmer is maximizing profits over some time period. In this context the past studies must be interpreted as "averaging out" considerations of this type. It is likely that we not only are unable at the present time to incorporate the effects of some of these factors in our aggregative analyses but do not know as much as we need even at the level of the firm.

Variables for other institutional effects have been less inadequately specified. There are a number of studies which have included specifications for effects of farm programs. Examples include penalties and allotments in Johnson's study of tabacco control programs (33), a dummy variable for government cotton policy in Suits' (45) watermelon supply study, a quantified measure of government price support programs in Cromarty's (11) study of farm machinery and in his econometric model (12) for United States agriculture, a number of measures for acreage allotments, announced support prices, and equations to ascertain government demands for certain commodities (feed grains, wheat, cotton, and tobacco products).

Realistic analyses of supply must be designed in the context of the technical conditions of production for the commodity. Most of the equations which have been fitted are behavioral or a combination of behavioral and production function. Equations more pure in the latter would enhance our knowledge of these technical conditions and basic structure of the system. Examples where the two types of formulations were more separately determined where Hildreth and Jarrett's (32) "farm decision relations" and "production relation" and Wallace and Judge's (48) "supply of beef at retail" and "supply of beef at the farm."

The relevance of technical conditions of production to decision making and to the design of the particular models was discussed as a part of many studies. In particular, the studies of Dean and Heady (13) on hogs, describing the length of gestation for sows and length of time necessary before fat hogs reach the market, and Candler (8) in his New Zealand wheat supply study, discussing substitute crops and difficulties in planting (caused by rains at seeding time), are examples of studies in which technical conditions influenced the statistical design.

The role of weather, neglected in most earlier studies, was explicit in studies by Candler (8), Johnson (33), Hatheway (26) (for dry beans), and in Cromarty's (12) crop supply equations. Its use, except by Candler and by Hatheway in his use of a variable for bean abandonment, was largely in a production function sense. It is believed that more use of it in behavioral equations, especially for resource commitment equations for crops grown in dry areas, where expected profitability of crops is a function of prior rainfall and of soil moisture at seeding time, would enable a more complete design for the decision-making situation.

Aside from the adjustment coefficients derived with the expected price models, and from some coefficients for "time" in some other formulations, the supply analyses were essentially derived within a short-run framework. The results primarily apply to a situation in which certain resources are fixed.

For crops, a typical set of formulations consisted of derivation of coefficients (a) for an equation explaining acreage seeded, and (b) for a second equation explaining yields, acreage given. The situation in (b) was necessarily a more short-run situation, and most of the results were as expected — greater supply elasticity for (a) than (b). Examples of studies obtaining results of this pattern were Mundlak and McCorkle (39) for potatoes, and Hatheway (26) for dry beans. Results of Suits (45) with equations for watermelon production and proportion of crop harvested, followed the same pattern. A number of studies stopped with an explanation of acreage, and Cromarty's (12) crop production equations must be interpreted as being a mixture of both (a) and (b).

For livestock, production commonly was studied with livestock numbers given, and hence is a shorter length of run than where numbers are endogenous. Dean and Heady (13), with their equations for number of farrowings and weight of hogs marketed, considered two lengths of run and obtained a higher elasticity, as expected, for the former relation compared to the more short run context.

The omission of the technology variable undoubtedly makes necessary a modified interpretation of many past studies. The data used in deriving most of the coefficients were generated essentially in the 1930's and 1940's, years in which time, price, production, level of resource use, more stable farm prices for some products, and technology have been correlated. As some have already pointed out, the use of technology and/or employment of more resources, especially those whose initial cost is high and whose disposal value is considerably less, are not unrelated to periods of high prices and production and hence high farm income. Until recently, these interrelations had been argued. For empirical relations, see the results of Heady and Yeh (29), and of Cromarty (11), and the relation of previous income to purchases of fertilizer and farm machinery. For further empirical verification, there was evidence (4) in the IMS that farmers are more responsive to product price increases (than decreases) and to input prices decreases (than increases). For contrary evidence, Halvorson found slightly higher elasticities for years of falling milk prices than for years of rising prices (25).

Further arguments are that when product prices fall, the technology and/or the resources that were first employed under high price situations will remain in employment. It is not likely that many of the formulations have provided for this "irreversible" portion of supply curves. The period over which our data were generated, its intercorrelations, and probable imperfections in statistical design may make some of the estimates not too reliable, especially for period of product price decreases. In Cochrane's terms, many formulations represent response relations in contrast to supply relations (9).

It is not clear how much more knowledge of the basic structure a more disaggregated analysis would permit. The bulk of the analyses, conducted at the United States level of aggregation, necessarily yields results with a high degree of "averaging out." Brennan (7) found evidence that for cotton production, the substitute crops varied by region. Dean and Heady (13) determined a somewhat different structure for hog supply for the north central region than for the United States. Bowlen's (3) elasticity estimates for wheat acreage in eastern and western Kansas were different. Analyses of fertilizer demand indicated different functions by regions (22, 29).

At the aggregative level, regression analyses have been with time-series in contrast to cross-sectional data. The success of the many studies using inter-farm analysis on the many problems, including those with supply implications, to which they have been addressed suggests as potentially fruitful the extension of cross sectional analysis to aggregative data on the regional, state, or county, etc., level.

A further method of analysis, heretofore unused, and possibly a powerful method for future studies, is the method of analysis of covariance. This method enables the integrated analysis of time-series and cross-section data (for discussions of the method, see Hildreth [30, 31]). Although it builds degrees of freedom, it has even more important possibilities for examining effects due to years and to section as part of the over-all regression analysis.

While it is not clear how "far down" the analysis of disaggregative time-series should go, it likewise is not clear how "far up" farm-firm analyses such as budgeting and linear programming can be aggregated. The two methods are now being used for essentially the same purpose, e.g. deriving milk-supply functions for milksheds. The two methods can be used simultaneously for the same milkshed, for example, and yet be complementary (in contrast to competing) methods.

The understanding of the decision-making process and of factors to which farmers respond were necessary insights for designing realistic regression models. The studies at the level of the firm have been instrumental in the discovery of relevant variables and have been suggestive of appropriate forms for functions, and of directions in which static analysis needs modification to account for dynamic forces. It is unlikely that the processes of deduction and empirical testing of time-series data will alone ever lead to a sufficient amount of this knowledge, especially that associated with "non-price" elements.

The impact of farm-firm analyses, such as the IMS and the many linear programming studies, on analysis of aggregative time-series data has been to suggest needed representations for variables and the interrelations among them. The model of Cromarty (12), where the effects of changes in such variables as levels of resources, in technology, and acreage allotments, and government demands, was undoubtedly to some extent suggested by earlier studies at the farm-firm level.

The regression-linear programming work at the "regional" level contains potentialities for more integration than has been realized. The regression analysis could provide coefficients for spatial equilibrium studies; to date the regression work has not been on a disaggregated level. Unless the equilibrium model was for the United States level, this difficulty and the fact that coefficients for not many "clean" production function equations have been derived, have prevented the tieing of the methods thus far.

The applicability of the past estimates holds only until there has been a structural change in the system. The amount of such change for all the various product supplying segments is not known, but some changes have been argued and empirical evidence of changes in some sectors is available.

The role of high and stable prices and the adoption of technology in changing the structure in the wheat producing industry has been argued. Similarly, adoption of new technology in the dairy industry will lead, it is expected, to new relations. Johnson and Hatheway argued that, for tobacco and dry beans, announced support prices lead to a modified structure (26, 33). Brennan (7) found evidence of changes in the cotton industry for 1942-48 from 1933-41, as did Dean and Heady (13) for both sow farrowings and hog marketing weights between 1924-37 and 1938-56. Halvorson detected differences in the milk supply structure for 1941-57 over earlier years (25).

We have at hand estimates of parameters for supply equations and supply elasticities for most farm products, mostly at the United States aggregative level, subject to interpretations as set down. In the broadest sense, the product orientation of all but the most recent work on demand for factors has enlarged our knowledge of structural relations affecting that commodity and has enabled us to make better predictions. However, it has not yielded the more complete knowledge of the structure of the agricultural plant that would come by the use of models where resources were treated more explicitly and endogenously. The studies have been superimposed above a portion of the basic system — the resource base. Changes of a variety of types have been proceeding at that level — investment and disinvestment in land, reductions in labor, but increased investment in the human agent, increases in the quantity and quality of capital, etc. Here the institutions — the land, labor, and capital markets — probably serve most unsatisfactorily. And here, as was the experience of Heady and Yeh (29), and of Cromarty (11), representations for the effects of the family farm, as the locale of the decision-making process, can more easily be included in the formulations.

The work of Griliches (23) is illustrative of the derivation of product supply functions from derived demands for resources. Bachman and Nerlove have shown that cost, supply, and derived demand functions may be derived from production functions (1).

Hypotheses on the supply and demand for farm resources have not generally been tested, especially by regression analysis of time-series data. Approaching problems of supply through the product side has given some useful structural knowledge and more basis for making predictions of production in the immediate. It is believed it will be necessary to broaden the analysis to obtain knowledge of the ultimate structure.

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Discussion

I HAVE NO PARTICULAR QUARREL with anything that Knight has said, but instead I would like to extend his criticism of past work. He says that the objective in studying supply elasticities is to understand the structure of agriculture. However, in the context laid out by Heady in his paper and at the Adjustment Conference the objective is clearly more than this. The objective is to derive empirical estimates which can form a basis for policy decisions at a level other than at the firm or household level, i.e., it is to determine the effects of changes in various variables brought about by policy makers.

Taking this later context, there are three questions I would like to raise concerning the interpretation of supply estimates. First, what is required of a model to be used for the stated purpose? Second, how can this requirement be met? Third, what is the major obstacle to carrying out this procedure?

REQUIREMENTS OF A MODEL USED FOR POLICY PURPOSES

The requirement of a model where someone is to manipulate one or more variables in order to cause an effect in another variable is best characterized by a scientist's experiment. Here certain causal variables are under the control of the experimenter who varies these at will to determine the effects upon the variables in which he has particular interest. Through the design and carrying out of the experiment he determines causal dependence. These causal dependencies are what the policy maker wants to know. He, like the scientist, is interested in variables he can control. Hence, in this context one of the important requirements of models which provide supply estimates is that they show causal dependencies between variables. This means that in the construction of the model, in addition to the effect variables, at least those variables should appear which the policy maker can and will manipulate. Whether or not the models Knight has drawn upon meet this requirement can be determined through answering the question of how can this requirement be met.

PROCEDURE OF MEETING THE REQUIREMENT

Most persons in the social sciences claim that experiments are impossible, and hence the requirement is impossible to meet. Therefore, policy based on empirical results are as good as those based upon ethical considerations only. Laboratory experiments on social phenomena to a scale that are useful for broad policy purposes are not feasible today. We must content ourselves with the real world and the ways in which variables are changing either by chance or as can be varied in smaller experiments. By smaller experiments I mean such piecemeal social engineering as currently is taking place in one of our more prominent Midwestern agricultural states. Whichever way the causal dependencies get tested, the important thing is that they be tested empirically. By testing empirically I mean that a specific event can be predicted which can be compared to the actual situation in order to ascertain the accuracy of the prediction.

Studies of time-series data provide an excellent opportunity for such testing by the predicting of events through time. The prediction can then be compared to the actual data and causal dependencies as specified in the model tested. Since Knight mentioned that such predictions had not been made and tested, I cannot accept the models as being useful to policy makers.

EXAMPLES OF PREDICTION—TESTING METHODOLOGY

In some of the work Karl Fox has done on spatial equilibrium models, some testing of predictions has been attempted.[1] He estimated

[1]Fox, K. A., Econometric Analysis for Public Policy. Iowa State Univ. Press, Ames, 1959, pp. 170-91.

coefficients from 1921-1942 data and then predicted regional price differentials for corn after specifying the value of certain initial conditions for a number of years beyond 1942. He says the degree of conformity of actual prices with the model based on the correlation coefficient is encouraging, but then recommends extensions before the model is used for policy decisions.

Another example of the procedure I would like to see carried out in supply studies is that used by Carl Christ in his "Test of an Econometric Model for the United States, 1921-1947."

Christ, upon modifying the Klein model, made predictions of variables for 1948 and then compared them with the actual observed variables. The accuracy of the predictions from the Klein model was compared with the accuracy obtained by using a simple naive model, such as last year's values. The Chirst study demonstrates the methodology of testing predictions I believe must be applied to the models described by Knight before policy makers can have confidence in the causal dependencies displayed by the equations.

The fact that I selected two studies in which hypotheses (models) were not only stated (estimated) but also refuted was intentional because I want to point out another feature of the prediction-testing methods. The results of testing can be the selection of hypotheses which have stood up to the test or the elimination of those hypotheses which have not stood up to them. At this stage of our empirical work, I would say it is the elimination of the false hypotheses which is most crucial. No doubt we could find many systems of equations, as pointed out by Knight, which adequately describe the data from which the coefficients are derived.[2] For many of the models discussed by Knight, the high correlation coefficients and small standard errors of estimated parameters are an inconclusive test for the adequacy of the models for explanatory purposes. As Friedman has pointed out in his discussion of Christ's work, the fact that the equations fit the data from which they were derived is primarily a test of the skill and patience of the analyst. Instead of praising, we must try to find fault with the theory and models. We must try to falsify them. Only if we cannot falsify them, in spite of our best efforts, can we say that they have stood up to severe tests. This is the reason why the discovery of instances which fit a model means very little if we have not tried, and failed, to discover refutations. If we are uncritical, we shall always find what we want. We shall look for and find confirmations, and we shall look away from and not see whatever might be dangerous to our pet theories. This is the foundation for my belief that the method of selection by elimination is more crucial and insures that only the fittest of models will survive.

Knight mentioned the reluctance to publish "poor" results from studies using regression and simultaneous equation techniques.

[2]This was also pointed out by V. I. West in discussing some published results of George G. Judge.

However, in the prediction-testing context the results to be published are the painstaking efforts at falsification of hypotheses. Likewise, in setting up social experiments the most noble of design is one destined to be "unsuccessful."

THE MAJOR OBSTACLE TO PREDICTION AND TESTING

In answering the third question regarding the main obstacle to carrying out the prediction-testing methodology described above, I wish to state that no obstacles exist in the short run. However, in the long run, the situation is different. Here, as Knight discussed, we have technological progress. From his discussion and others that he noted, I conclude that the majority of researchers believe that a technological variable can be specified so that shifts in the supply curve can be predicted. I would argue that such is impossible, because what we are asking for in a technological variable is a variable describing the evolution of knowledge. It is the accumulation of knowledge and its subsequent application to everyday affairs which gives rise to the observed technological change. However, the process of accumulation of knowledge is an unique historical process; and hence any hypothesis formulated to describe it cannot be tested. If it cannot be tested, then it can only enter in the deduction of shorter-run predictions as a constant for a particular historical period. Since we cannot predict by rational or scientific methods the future growth of our knowledge, we cannot predict the future course of technological progress, i.e., we cannot anticipate today what we shall know only tomorrow. I do not doubt that we can find trends within any historical period of application and adoption of new techniques, but this confines our models to a given technological period.

In my discussion of Knight's paper, I have tried to show that the interpretation of supply models depends upon the predictive power of the model. I have said that this power can be ascertained through empirical testing of the model beyond the series of data from which the parameters were estimated. The main obstacle to making long-run predictions is the impossibility of finding a law of technological evolution.

Chapter 5

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Recursive Programming and Supply Prediction*

FOR ANALYSIS AND PREDICTION of aggregative production, it seems evident from past theoretical and empirical research that at least the following interrelated categories must be considered:

1. The interdependence of outputs using common inputs;
2. Technological change;
3. Planned or programmed policy actions;
4. Changes in both acreage and yield components in field crop production;
5. Uncertainty;
6. Demand, supply, and price interactions;
7. Adjustment over time;
8. The aggregate supply of production inputs;
9. Rates of investment in factors fixed in the short run; and
10. Regional specialization and competition.

Econometricians, in their use of multiple regression and simultaneous equation techniques have made considerable progress in accommodating variables and relations which reflect interdependencies among these phenomena. Yet there are certain fundamental difficulties in these techniques which send one in search of different, more suitable methods. This paper is an account of such a quest.

*The research on which this paper is based was begun while the author was research assistant at the Harvard Economic Research Project. It was continued while he was Teaching Fellow in the Economics Department at Harvard University and later while a member of the staff of the Farm Economics Research Division, ARS. At this writing, the author is on military leave from the latter organization. The specific contents of this paper have profited particularly from the comments of Professors James M. Henderson, Louis Lefeber, and Wassily W. Leontief, all of Harvard University; Hendrick S. Houthakker, Stanford University; and Dr. Glen T. Barton, Farm Economics Research Division, ARS.

SUPPLY EQUATIONS

Perhaps the simplest of all field-crop supply models is an equation which relates the acreage of a crop in a given year to its own price lagged one year. The simplest form in which this equation can be expressed is

$$X(t) = \alpha\ p(t\text{-}1)\ . \tag{1}$$

Suppose an acreage allotment is imposed on the crop. This specifies a constraint on the acreage of the crop which can be expressed

$$X(t) \leq a(t)\ , \tag{2}$$

in which a(t) is the acreage allotment in the year t. If $\alpha\ P(t\text{-}1) > a(t)$, i.e., if the acreage predicted by relation 1 is greater than the allotment, the two relations are inconsistent. This inconsistency can be removed if relation 1 is made into an inequality like that of relation 2. However, the system is now underdetermined.

The manipulation of instrumental or policy variables is not the only cause for the intrusion of inequalities in supply systems. A more fundamental cause consists of constraints on output arising from factors of production fixed in the short or long run. An example of the latter is an over-all land constraint in a developed region or country.

Suppose only two field crops are grown in a developed region whose acreages are $X_1(t)$ and $X_2(t)$, respectively. Assuming a lagged price supply relation as before, but including the competing crop's price as well as its own, a typical supply system might be written

$$\begin{aligned} X_1(t) &= \alpha_1\ p_1(t\text{-}1) + \alpha_2\ p_2(t\text{-}1) \\ X_2(t) &= \beta_1\ p_1(t\text{-}1) + \beta_2\ p_2(t\text{-}1)\ . \end{aligned} \tag{3}$$

On the basis of economic theory, it would be expected that $\alpha_1 > o$, $\alpha_2 < o$, $\beta_1 < o$, $\beta_2 > o$, i.e., that acreage increases with an increase in <u>own</u> price while it decreases with increases in a competitor's price.

The overall land constraint assumed for this example is

$$X_1(t) + X_2(t) \leq \bar{x}\ . \tag{4}$$

But now a situation analogous to the first example arises. Only if equation 4 holds as a strict inequality — i.e., only if part of the land is idle — can both equations of relation 3 hold. Again, the over-determinancy of the supply system could be avoided by making inequalities of the acreage lagged price relations.

If this is done, however, the model is underdetermined as before. Some kind of mechanism must be added if one is to decide in a meaningful way which of the two supply equations holds whenever the over-all land constraint holds. The mechanism which will resolve problems of this kind is the optimizing principle of economics. Rather than trying to force it on supply relations like those of the examples listed, it would seem to be more appropriate to follow the theory of production and to use it to derive supply relations from the underlying technical structure of production.

The suggestion that this principle be applied to predictive problems of supply is a little foreign to usual practice. Ordinarily, one attempts to estimate aggregative supply relations themselves without explicit reference to production structures and their choice mechanism. Even when this is done, the optimizing principle plays a role in the evaluation of the results. Thus, it is by means of this principle that one arrives at the conclusion that the response to "own price" will be positive while that to a competing commodity will be negative (7). Consequently, the explicit application of optimization is not as radical an innovation for supply response as it may at first appear.

The important problem is not whether it should be used but rather how it can be used without grossly misrepresenting the simple decision processes governing farm behavior. The attempt to solve this problem leads to a synthesis of time-series analysis and linear programming versions of production theory. It is to such a synthesis that the rest of this paper is devoted. We shall call it recursive programming.

A SIMPLE RECURSIVE PROGRAMMING MODEL: FLEXIBILITY IN CHANGING OUTPUT PATTERNS

An important application of linear programming to the problem of aggregative supply prediction is due to Professor James M. Henderson (3). The ingenuous innovation on which it rests is the specification of what we shall call flexibility constraints. These constraints specify that in any one year only a limited change from the preceding year's production can be expected. This hypothesis is based on the conglomerate of forces which lead to caution by farmers in altering established production patterns. Primary among them are uncertainty of price and yield expectations and restriction on the aggregative supply of production inputs. In short, they are the same factors which underpin Nerlove's adjustment equations (8). During this discussion, we shall split off the factors whose capacities are fixed in the short run for separate treatment. At this point, it will be supposed that the flexibility coefficients contain them as components.

The flexibility constraints can be expressed in dynamic notation as follows:

$$
(5)\qquad
\begin{array}{rrcl}
X_1(t) & & \leq & (1+\bar{\beta}_1)\, X_1(t-1) \\
 & X_2(t) & \leq & (1+\bar{\beta}_2)\, X_2(t-1) \\
-X_1(t) & & \leq & -(1-\beta_1)\, X_1(t-1) \\
 & -X_2(t) & \leq & -(1-\beta_2)\, X_2(t-1)
\end{array}
$$

in which $X_1(t)$ and $X_2(t)$ have the same meaning as before and in which we shall call the β's flexibility coefficients.

The first equation of relation 5 asserts that the acreage of the first crop will not exceed the previous year's acreage plus some proportion of its determined by the upper flexibility coefficient $\bar{\beta}_1$. Equation 3 of relation 5 asserts that the acreage of the first crop must not be less than an amount determined by the lower flexibility coefficient, β , and the preceding year's acreage. Equations 2 and 4 of relation 5 have the same meanings, respectively, for the second crop. (This example follows the preceding one, assuming that two crops only are grown in the region or country in question.)

The over-all land constraint (relation 4) should also apply here, further limiting the possibilities for change. Together with the inequations of relation 5 this gives a total of five constraints on change in output patterns. These five constraints form a system of linear nonhomogeneous difference inequations.

Now let $\pi_1'(t)$ and $\pi_2'(t)$ be the expected per acre net returns to the first and second crops, respectively. The system consisting of relations 4 and 5 can be resolved by an appropriate application of the optimizing principle, thus

$$\text{(6)} \qquad \text{maximize } \{\pi_1'(t)\ X_1(t) + \pi_2'(t)\ X_2(t)\}$$

subject to relations 4 and 5. That is, choose $X_1(t)$ and $X_2(t)$ so that total net returns are as great as caution and fixed factors will allow. The flexibility constraints are now seen to enclose the profit motive in a web of dynamic adjustment.

A fundamental theorem asserts that the solution to a linear programming problem is such that the number of constraints which hold as equalities is just equal to the number of nonzero variables. Translated into recursive programming language, this means that a supply system is governed by exactly as many dynamic equations as there are positive variables selected by the optimizing principle.

In our example, at least two variables must be positive because of the lower bounds, relations 2 and 4 of equation 5. As there are only two variables in the system for time t, we know that two equations will govern the behavior of the system over time. Which two there will be for any time period will depend upon which is greater, $\pi_1'(t)$ or $\pi_2'(t)$, and upon the relative magnitudes of the five constraints.

To actually obtain the solution, we must begin at a base period t=o. The initial conditions are then X(o). Then as the $\pi_1'(t)$ and $\pi_2'(t)$ are formed (exogeneously so far), a linear programming problem becomes available for each period that can be solved by the usual techniques.

A change in the equations which "govern" the system is called a phase change and the period of time during which the same equations hold a phase. The operation of this system over time will tend, in general, to exhibit multiple phases (6). During a given phase, simple first-order difference equations will determine the time paths of acreage.

The solution to such an equation is

$$y(t) = \lambda^{(t-t_j)}\, y^{(t_j)} \tag{7}$$

in which $y^{(t_j)}$ is the value of y(t) holding in the time period just prior to a phase change.

To visualize how time paths of acreage might appear, suppose that with each new year, the first crop is expected to be the more profitable ($\pi_1'(t) > \pi_2'(t)$,), and that net returns from both crops are positive. Suppose also that the acreage of the first crop is much smaller than that of the second, and that there is some idle land.

The following phases are a possible outcome.[1]

$$\begin{array}{ll} & \text{Phase I} \\ & \begin{aligned} X_1(t) &= (1+\bar{\beta}_1)^t\, X_1(o) \quad (t=1,\ldots,t_1) \\ X_2(t) &= (1+\bar{\beta}_2)^t\, X_2(o) \end{aligned} \\ (8) & \text{Phase II} \\ & \begin{aligned} X_1(t) &= (1+\bar{\beta}_1)^t\, X_1(t_1) \quad (t=t_1+1,\ldots,t_2) \\ X_2(t) &= \bar{X} - X_1(t) \end{aligned} \\ & \text{Phase III} \\ & \begin{aligned} X_1(t) &= \bar{X} - X_2(t_2) \qquad\quad (t=t_2+1,\ldots) \\ X_2(t) &= (1-\beta_2)^t\, X_2(t_2) \end{aligned} \end{array}$$

A graphic representation of these phases is shown in Figure 5.1.

In phase I, the acreage of idle land is sufficient to allow both crops to increase at the maximum rate allowed by caution and growth in the aggregate supply of factors. In phase II, the over-all land constraints render inconsistent the maximal growth of both crops, and the less profitable crop merely takes up the slack. Finally, in phase III, the maximal rate of growth demands that more land be released from crop 2 than farmers are willing to release, so that the maximal abandonment rate for the relatively unprofitable alternative dominates supply response.

A linear program has a <u>dual solution</u>, as well as the <u>primal solution</u> discussed above. The <u>dual variables</u> express (in this example) the marginal net revenue productivities of unit changes in the constraints. Call $\rho_i(t)$, $i=\bar{X}, \bar{\beta}_1, \bar{\beta}_2, \beta_1, \beta_2$, the dual variables for land, the upper flexibility and lower flexibility constraints, respectively. The dual results are

[1] In general, the phases will depend upon net returns, the initial conditions, and the flexibility coefficients.

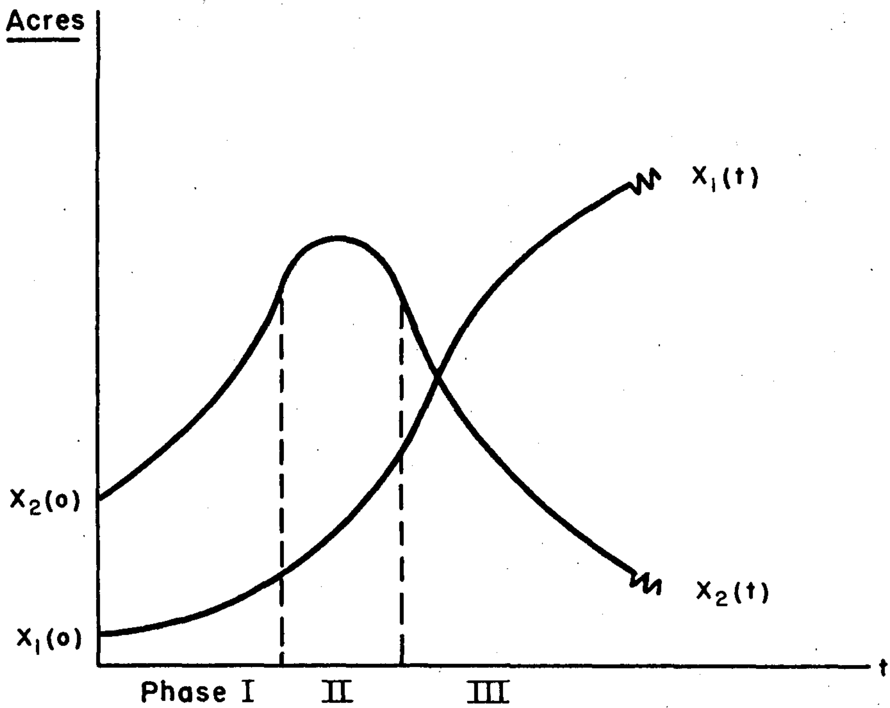

Figure 5.1. Acreages of both crops expand geometrically until land supplies are exhausted. After this, willingness of farmers to specialize is governed by the upper bound on the first crop (in Phase II) and the lower bound on the second crop (in Phase III).

$$
\begin{array}{llllll}
 & \text{Phase} & & \text{I} & \text{II} & \text{III} \\
 & \rho_{\bar{x}}(t) & = & 0 & \pi_2'(t) & \pi_1'(t) \\
 & \rho\bar{\beta}_1(t) & = & \pi_1'(t) & \pi_1'(t) - \pi_2'(t) & 0 \\
(9) & \rho\bar{\beta}_2(t) & = & \pi_2'(t) & 0 & 0 \\
 & \rho\beta_1(t) & = & 0 & 0 & 0 \\
 & \rho\beta_2(t) & = & 0 & 0 & \pi_1'(t) - \pi_2'(t) \,.
\end{array}
$$

Notice that in phase III the marginal return to the lower flexibility constraint for the second crop is positive. This illustrates how lower bounds can be made to reflect the unwillingness of farmers to abandon too rapidly relatively unprofitable alternatives in the face of uncertainty.

The optimizing principle in this application does not imply that long-run or even short-run optima are obtained. Rather, it expresses the empirical fact that when farmers change, they cautiously improve their economic positions according to their current uncertain expectations.

The example above may be said to be an "open" model — open with respect to output and input prices. No mechanism was allowed for determining net-return expectations. In aggregative models, this openness is a drastic limitation, for neither net returns, nor their expectations, can be assumed to be independent of past prices and, therefore, of past output. It is to this interdependence we shall now turn.

NET RETURN EXPECTATIONS AND INTERACTION WITH AGGREGATIVE DEMAND

Net returns are a function of prices and of outputs, inputs, and the technical structure of production. Thus, net returns can be expressed as

$$\pi_i(t) = p_i(t)\, y_i(t) - C_i(t) \tag{10}$$

in which $p_i(t)$, $y_i(t)$ and $C_i(t)$ are the actual price, the yield, and the cost (which is a function of input prices and technical coefficients) for the i^{th} crop in the year t. We could submit net returns to some kind of expectation model, for example, Nerlove's price-expectation model (7). As it seems unlikely that farmers have much notion of what their "long-run equilibrium price" is (even conceding that such a price exists), it may be advisable to use a simple function of past net returns. This can be done while still preserving the properties of the analysis presented so far. What is more important, output plans are independent of current demand, a result that would not be true if no lag were presumed. In the latter case, we would have a model which would represent a region as a monopolist who had complete knowledge of his demand curves and not an agglomerate of atomistic sellers.

Of course, the simplest expectation function is obtained when net-return expectations are equal to the preceding year's actual net returns $[\pi'(t) = \pi(t-1)]$. As this is sufficient to illustrate the generality of recursive programming, we shall hypothesize the validity of this model.

Suppose that the demand structure for our two commodity regions is

$$\begin{aligned} Y_1(t) &= a_1 p_1(t) + a_2 p_2(t); \\ Y_2(t) &= b_1 p_1(t) + b_2 p_2(t), \end{aligned} \tag{11}$$

in which $Y_1(t)$ and $Y_2(t)$ are the demands for production and in which, according to the theory of consumption, we would expect a_1 and b_2 to be negative and a_2 and b_1 to be positive. As with expectations, this model is chosen because it is just sufficient for our present purpose. Suppose further (for simplicity) that yield is constant for each crop: $y_1(t) = y_1$ and $y_2(t) = y_2$ all t. Then, if the market is free to clear itself,

$$\begin{aligned} Y_1(t) &= y_1 X_1(t); \\ Y_2(t) &= y_2 X_2(t). \end{aligned} \tag{12}$$

Substituting relation 12 into relation 11 and solving for $p_1(t)$ and $p_2(t)$, the following expressions could be obtained:

$$(13)\qquad \begin{aligned} p_1(t) &= a_1' X_1(t) + a_2' X_2(t)\,; \\ p_2(t) &= b_1' X_1(t) + b_2' X_2(t)\ , \end{aligned}$$

in which the coefficients of the X(t)'s are determined by the coefficients of relation 9 and the yields. This closes the model with respect to output price, though not with respect to costs. The latter could be treated similarly, but to avoid further complexities let it be supposed that $C_1(t)$ and $C_2(t)$ are constant over time. This gives the closure needed to develop explicit dynamic solutions for acreage, price, and marginal returns over time.

Returning to phase II, there is a corresponding phase for prices. The reader can verify that it is

Phase II

$$(14)\qquad \begin{aligned} p_1(t) &= (a_1' - a_2')\ (1 + \bar{\beta}_1)^{t-t_1} X_1(t_1) + a_2'\bar{x} \\ p_2(t) &= (b_1' - b_2')\ (1 + \bar{\beta}_1)^{t-t_1} X_1(t_1) + b_2'\bar{x} \end{aligned}$$

Similarly, price movements can be found for any phase.

By means of relation 10, these price movements can be converted to expected returns. Thus, though they have not exact knowledge of it, farmer expectations follow an inexorable law which is based on the aggregative demand functions for their products.[2] At some place in the course of phase III, for example, net returns will reverse their relation; crop 2 will become less desirable to produce, and farmers will begin a response to the changed price expectations by transferring land from crop 2 to crop 1, thus reversing the former trend. The effect of this process on prices is shown in Figure 5.2. The price lines cross before the end of a phase because of the lag in expectations and the role of costs and yields. The dual variables can also be expressed as functions of time. For example, in phase II equation 14 can be substituted into equation 10; for $i = 1$ and $i = 2$.

The following results seem most important. First, prices and acreages, ergo, net returns, marginal revenues, and outputs undergo multiple phases in which rates of change over time change in each phase. Second, the phases begin to repeat themselves. This is called phase periodicity[3] and the results tend to resemble dampened sine and cosine curves! Third, phases occur in which output of a commodity may increase while its price is falling!

---

[2] Needless to say, this law is inexorable in a statistical sense. In stochastic processes, dynamic laws determine not variable values but rather their probability distributions over time. The rather complicated stochastic processes underlying recursive programming have not been explored very fully as yet. The term "dynamic law" is still used in its stochastic sense.

[3] Again, these may be stochastic laws.

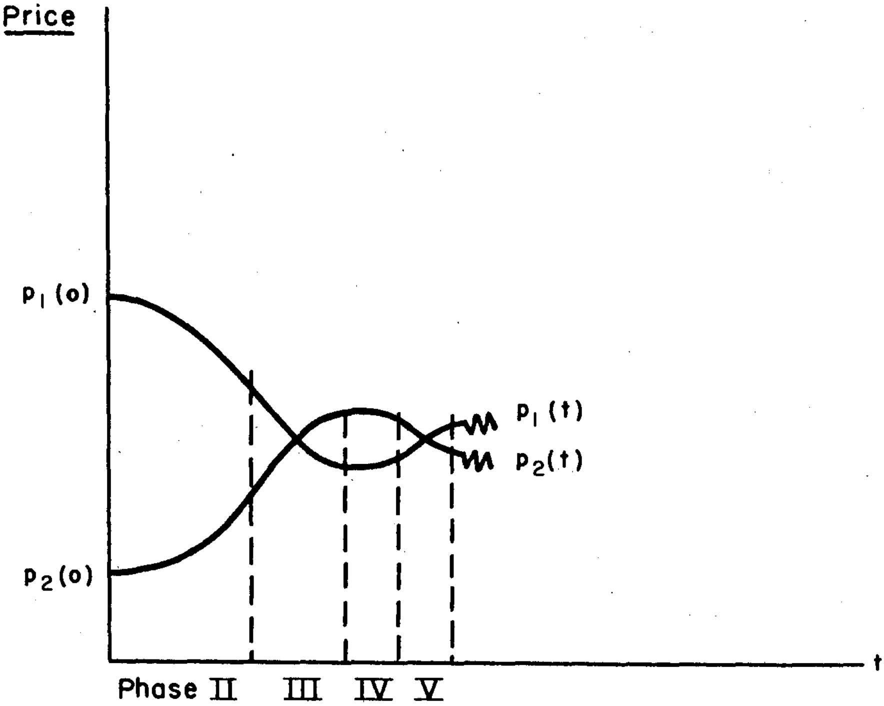

Figure 5.2. The time paths of output prices in this hypothetical example resemble dampened sine and cosine curves whose periods become shorter with the passage of time.

The implication of the first result is that the elasticity of supply is not a very stable parameter for prediċing response over time. The second result is the attainment of a multivariate cobweb cycle, which is likely to be highly stable (for crops) because of the quick change in phase when relative returns change. The third explains the enigma of the downward sloping supply curves sometimes obtained with time-series data! This result stems from the lag in expectations linked through the reaction of output on demand to a production structure with a finite number of alternatives.

This explanation of the inverse supply relation over time is consistent with the Marshallian, positively sloped, short-run supply curve. Such curves are obtained by holding constant everything except the price of a given commodity. The latter is varied continuously over a wide range to obtain the relation between output plans and price for a given time period. This same type of relation can be obtained with this model, which, for a given time period, is a straightforward linear programming problem. "Price mapping" or "parametric programming" is the technique which gives the desired supply functions. These functions will, of course, be step functions which increase discretely. However, the analysis (and synthesis) of this section reveal that the only conditions under which such curves have any real meaning is when the supply system is relieved of the influence of demand on price. Hence, Marshall's purely theoretical construct is useful operationally only for predicting the effect of artificial prices, such as those created by law. In the market, prices and production must be determined by dynamic laws derived from technical and demand structures (1).

INVESTMENT, CAPACITY, AND TECHNOLOGICAL CHANGE

To treat the relation of aggregative investment to output, the constraints on production expansion can be split into two components, one expressed by the flexibility constraint and the second by capacity constraints. The former expresses the reluctance of farmers to specialize too rapidly in a given product. The latter expresses farmers' unwillingness and inability to invest in any particular method of production at a rate greater than some maximum. This inability may come from limitations which are imposed by the rate of expansion of farm machinery and related industries or from external credit rationing. The former might be expressed as internal credit rationing.

Suppose, for example, that the first commodity can be produced by either of two methods. The first of these has been introduced in the recent past and as yet accounts for only a small portion of current practice. Let $X_1^I(t-1)$ and $X_1^2(t-1)$ be the actual capacities in number of acres utilized during the year (t-1). Let $I_1^*(t)$ and $I_2^*(t)$ be maximal investment patterns potentially observable during the year (t). Now let α_1 and α_2 be the investment coefficients in the two capacities, respectively. Now suppose that maximal potential investment can be related to the immediate past levels of capacity utilization by

$$(15)\qquad \begin{aligned} I_1^*(t) &= X_1^1(t) - X_1^1(t-1) \leq \alpha_1 X_1^1(t-1); \\ I_2^*(t) &= X_1^2(t) - X_1^2(t-1) \leq \alpha_2 X_1^2(t-1), \end{aligned}$$

Expressed as inequalities, these relations determine the maximal potential rate of investment. Predicted capacity in either process is thus constrained by the relation

$$(16)\qquad X_1^i(t) \leq (1 + \alpha_i)\, X_1^i(t-1), \quad (i=1,2).$$

The new dynamic production model including both kinds of constraints can be written as:

$$(17)\qquad \max \{\pi_1^1 (t-1)\, X_1^1(t) + \pi_1^2 (t-1)\, X_1^2(t) + \pi_2 (t-1)\, X_2(t)\}$$

subject to

$$
\begin{aligned}
X_1^1(t) + X_1^2(t) + X_2(t) &\leq \bar{X} \\
X_1^1(t) + X_1^2(t) &\leq (1+\bar{\beta}_1)\,[X_1^2(t-1) + X_1^2(t-1)] \\
X_1^1(t) &\leq (1+\alpha_1)\,X_1^1(t-1) \\
X_1^2(t) &\leq (1+\alpha_2)\,X_1^2(t-1) \\
-X_1^1(t) - X_1^2(t) &\leq -(1-\beta_1)\,[X_1^1(t-1) + X_1^2(t-1)] \\
X_2(t) &\leq (1+\bar{\beta}_2)\,X_2(t-1) \\
-X_2(t) &\leq -(1-\beta_2)\,X_2(t-1)
\end{aligned}
$$

Actual investment patterns are then predicted by the model. Omitting demand functions to simplify the argument and returning to phase III conditions, the change in capacity might follow the time paths shown in Figure 5.3. It is presumed that for the period considered, expectations are such that $\pi_1^2(t) > \pi_1^1(t) > \pi_2(t)$, that is, that the second (newest) way of producing crop 1 is most profitable, while the older method is more profitable than production of the second crop.

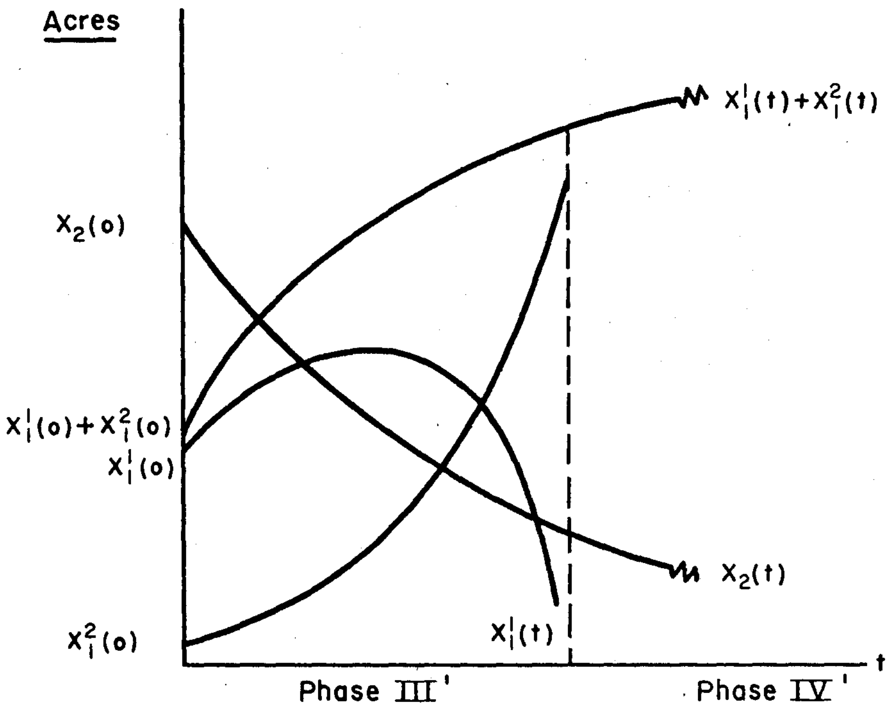

Figure 5.3. The acreage of crop 1 handled by the older method increases but eventually decreases until the method is entirely abandoned by Phase IV'. Acreage of crop 1 grown under the new method rapidly replaces acreage devoted to the old method until Phase IV' when uncertainty and other forces constrain aggregate production of the crop.

Here are the main results. First, actual capacity expansion (investment) and abandonment are predicted simultaneously with production patterns. Aggregate production is constrained by the forces acting on the rate of change in output patterns and by the forces which determine maximal potential growth. Second, it is likely that when the capacity of a superior production process is small, investment may occur in a relatively inferior process until sufficient growth has taken place in the former (phase III). Finally, after investment has proceeded long enough in the superior process, not only may capacity of the inferior process be abandoned at an increasing rate, but the unwillingness to alter output patterns beyond a certain rate will prevent investment in a superior process from achieving its maximal potential rate (phase IV). Thus variables may move on paths devious to their "long-run equilibrium" positions as calculated by relative profits alone.

Technological change can be split into three components — invention, innovation, and diffusion. While invention and innovation appear still to belong to historical analysis, technological change, insofar as it is a diffusion process, is solidly within the boundary of economics itself (1). Suppose that in the year t=o, the second process for producing crop 1 was innovated. The capacity $X_1^2(o)$ was an historical fact which could not have been predicted, but the diffusion process is expressed now by the same theory under which general investment patterns were predicted. <u>Diffusion is an investment process</u>. The growth of knowledge is simply an added component acting on internal and external credit rationing. Its effect may not differ vastly from "normal" investment processes, which probably always contain a knowledge component. If this is true (as we suspect), we need not search too far outside economics for exotic theories of technological change. Further, its effects on output response are traced by considering production decisions as determining investment and capacity abandonment simultaneously with changing output patterns.

PLANNING OVER TIME, REGIONAL COMPETITION, AND OTHER GENERALIZATIONS

While the optimizing principle is the criterion of micro-economic action, it is applied to the regional unit. This is done in a way which does not truly optimize economic action for a region but rather reflects the time distribution of aggregative response to current average expectations. The model has nothing to say about which production units will change in a given year, but only that specific proportions of the region's resources will be reallocated by a corresponding proportion of the region's producers with the passage of time. Such proportions could be interpreted as probabilities of change for the allocation of individual resource units. The peculiarities of individual decision criteria are subsumed in statistical averages.

The model presents a similar attitude toward planning over time.

If a given process continues to be relatively attractive as an investment opportunity for some extended period of time, the model predicts a growing rate of investment in it. Again, the individual peculiarities of planning are subsumed. Whatever the varied time horizons among producers may be, the model projects investment for the region, indicating that particular budget limitations and time horizons will lead over time to growing aggregative investment.

For certain applications, however, it is likely that planning over time should be accounted for more explicitly. In doing this, we should not like to sacrifice the rather realistic picture of sequential decision making developed so far. Dynamic programming, as it is currently applied, derives the time distribution of production and investment as the result of a single optimizing decision. Aggregative economic processes, however, do not terminate after some finite period of time in achieved terminal objectives. The ubiquitous presence of uncertainty, the accumulation of knowledge, and the play of more or less fortuitous events prevent such grandiose scheming.

The model can be generalized to include planning over time, but in a way that would preserve the yearly reevaluation of production and investment plans. For this purpose, consider a time horizon of two periods. Relations 4 and 5 are still adequate to express possibilities in the region for the first (imminent) time period. For the second (future) production period for which production and investment plans are projected, a second set of relations is required. It consists of the relations of 4 and 5 advanced one time period. The resulting 10 restrictions on production and capacity change form the following recursive programming system:

$$\text{(18)} \qquad \begin{aligned} \max \{ & \pi_1'(t+1)X_1(t+1) + \pi_2'(t+1)X_2(t+1) \\ & + \pi_1'(t)X_1(t) + \pi_2'(t)X_2(t) \} \end{aligned}$$

subject to

$$\begin{array}{ccccccl}
X_1(t) & + & X_2(t) & & & \leq & \bar{x} \\
X_1(t) & & & & & \leq & (1+\bar{\beta}_1)X_1(t-1) \\
 & & X_2(t) & & & \leq & (1+\bar{\beta}_2)X_2(t-1) \\
-X_1(t) & & & & & \leq & -(1-\beta_1)X_1(t-1) \\
 & & -X_2(t) & & & \leq & -(1-\beta_2)X_2(t-1) \\
 & & & X_1(t+1) & + \; X_2(t+1) & \leq & X \\
-(1+\bar{\beta}_1)X_1(t) & & & X_1(t+1) & & \leq & 0 \\
 & & -(1+\bar{\beta}_2)X_2(t) & & X_2(t+1) & \leq & 0 \\
(1-\underline{\beta}_1 X_1(t) & & & -X_1(t+1) & & \leq & 0 \\
 & & (1-\underline{\beta}_2)X_2(t) & & -X_2(t+1) & \leq & 0
\end{array}$$

in which $\pi_1'(t+1)$ and $\pi_2'(t+1)$ are expected net returns for the future period. The latter might be linked through an expectation model to a demand structure to obtain a closed system. While production and investment in time t is conducted with an eye for the future, the plans made for t+1 may be changed as a new plan is generated. For a given year t, the plan is a dynamic linear programming problem of the usual kind, but it is dynamic not only in the Hicks sense, but also in the Frisch-Samuelson sense (4). This methodology can be summarized in Leontief's words (5):

> " . . . [An] economic [process] is . . . a continuing, unending process the path of which is determined by a never-ending sequence of choices. Particularly important for this point of view is the fact that the explicit time-horizon of each one of these successive choices is much shorter, in principle infinitely shorter, than the span of time covered by the dynamic process as a whole. Thus while each step . . . satisfies certain maximizing conditions, the sequence as a whole does not.

A dynamic process of regional competition can be formulated too. For illustration, suppose there are two regions. Disregarding time-horizon and demand aspects of the model, two sets of relations, 4 and 5, one for each region, might be specified, with the variables labeled with superscripts I or II for the first or second region. Apart from demand, the regions might be interrelated through the growth in the regional capacities of short-run fixed factors and labor. Thus the farm labor force and investment in machines would flow in the direction of highest marginal returns as reflected in the dual variables. Augmented in this way, the model is

$$(19)\qquad \max\ \{\pi_1^{I}(t)X_1^{I}(t) + \pi_2^{I}(t)X_2^{I}(t) + \pi_1^{II}(t)X_1^{II}(t) + \pi_2^{II}(t)X_2^{II}(t)\}$$

subject to

$$\begin{array}{llllcl}
X_1^{I}(t) & + X_2^{I}(t) & & & \leq & \bar{X}^{I} \\
X_1^{I}(t) & & & & \leq & (1+\bar{\beta}_1^{I})X_1^{I}(t-1) \\
 & X_2^{I}(t) & & & \leq & (1+\bar{\beta}_2^{I})X_2^{I}(t-1) \\
-X_1^{I}(t) & & & & \leq & -(1-\beta_1^{I})X_1^{I}(t-1) \\
 & -X_2^{I}(t) & & & \leq & -(1-\beta_2^{I})X_2^{I}(t-1) \\
 & & X_1^{II}(t) & + X_2^{II}(t) & \leq & \bar{X}^{II} \\
 & & X_1^{II}(t) & & \leq & (1+\bar{\beta}_1^{II})X_1^{II}(t-1) \\
 & & & X_2^{II}(t) & \leq & (1+\bar{\beta}_2^{II})X_2^{II}(t-1) \\
 & & -X_1^{II}(t) & & \leq & -(1-\beta_1^{II})X_1^{II}(t-1) \\
 & & & -X_2^{II}(t) & \leq & -(1-\beta_2^{II})X_2^{II}(t-1) \\
X_1^{I}(t) & & + X_1^{II}(t) & & \leq & (1+\alpha_1)[X_1^{I}(t)+X_1^{II}(t-1)] \\
 & X_2^{II}(t) & & + X_2^{II}(t) & \leq & (1+\alpha_2)[X_2^{I}(t-1)+X_2^{II}(t-1)]\ .
\end{array}$$

This points a way to analysis of the well-known relation between regional competition and technological change.

The applied linear programmer is familiar with the rich variety of production relations which can be accommodated in the linear programming framework. An important generalization for this aggregative model of production response would be to include distinct processes representing several levels of fertilizer application for each basic technological process or "type." If a relation which would determine aggregative fertilizer stocks (purchases for a given year, for example) could be established, the yield component could be subjected to the same analysis as the acreage component of production.

Like other empirical techniques, the generality of recursive programming is determined in practice by a judicious compromise among logical structuring, data availability, and the research budget.

ESTIMATION PROCEDURES

Although simple examples have yielded interesting, theoretical results, it remains to be seen whether an operational tool exists. Given that they are both meaningful and relatively stable, can parameters of a recursive programming model be estimated? The dynamic nature of the model can be invoked to answer the question in the affirmative. The approach to be suggested is closely related to familiar time-series analysis, but it involves some unfamiliar techniques and problems.

Consider the simple model of section 3. During phase I the regional acreages of the two crops follow two simple equations. Therefore, time-series estimates of aggregate acreages can be used to estimate the coefficients $\bar{\beta}_1$ and $\bar{\beta}_2$. Notice, however, that the coefficients β_1 and β_2 cannot be estimated with data from this period. In phase II, time-series data for the first crop can be used to increase the efficiency of the $\bar{\beta}_1$ estimate, but no additional information can be added to the estimate of $\bar{\beta}_2$. In phase III, time-series data for the second crop can be used to estimate β_2.

In summary, the progress of regional production has revealed sufficient information to permit estimation of the upper flexibility coefficients of the two crops and of the lower flexibility coefficient of one of them. Remaining unidentified is the coefficient β_1 which determines the lower flexibility constraint for the first crop. Thus, the model introduces a new kind of identification problem.

Having information with which to estimate some of the coefficients is quite different from knowing how to use it. We cannot know exactly which phases actually hold over time. Consequently, two distinct sets of hypotheses are involved. Given the set of structural inequalities defining the dynamics of the model (which are, of course, hypotheses too) one must first guess which equations actually determined the system for particular periods of time. Second, using the usual time series techniques (least squares, perhaps) one must estimate the parameter (or parameters) of each equation so "identified." A "good" guess can be made by a study of relative net returns and of the data on acreage and production in the region, and with the help of an intimate knowledge of the region's economic conditions.

Having obtained estimates of some of the parameters in this way, one returns to some initial date and begins the model running as described above. If the optimizing principle selects the same phases as those guessed, and if the model estimates explain a fairly large percentage of the total variation in the several variables of interest, then the model's hypotheses appear to be useful approximations of reality. On the basis of this test, future projections could be made and revised with the passage of time to accommodate the latest information and newly revealed structural relations.

Recursive programming does not replace existing statistical methods but rather performs a synthesis between them and explicit choice criteria and modifies the sphere within which their application is valid.

CURRENT APPLICATIONS[4]

A recursive programming model has been developed by the author for the analysis and prediction of production from 1940 to 1959 in one of the major cotton production regions, the Delta area of Mississippi. The study includes eight commodities, four technological "stages," three soil classes, and four fertilizer levels. The production structure is represented by 103 distinct processes and 38 dynamic inequalities. Among these are investment constraints for each technological stage, capacity constraints for regional labor and fertilizer, over-all stocks of the three soil classes, and cropland acreage constraints reflecting acreage allotments.

With respect to prices, the model is open. Closure in this sense could not be achieved at this level of aggregation because of the national character of demand for Delta commodities. This shortcoming is not serious historically. In recent times, production has been independent of demand because of the incidence of high price supports. The model will generate estimates of production, acreage, and average yields for each commodity, investment patterns and rates of diffusion of existing and newly innovated technological stages, regional land, and other input utilizations. All parameters have been estimated and model predictions are currently being derived. As all time-series information used for

[4] The applications described in this section are being conducted in the Farm Economics Research Division, ARS. The structuring of the Delta model was conducted with the close cooperation of production experts both in Washington and at the Delta Branch Experiment Station at Stoneville, Mississippi. It was during the course of this empirical investigation that many of the important theoretical characteristics of recursive programming became evident. Among those to whom the author is particularly indebted are E. L. Langsford and Grady B. Crowe, both of the Farm Economics Research Division, ARS. The results of this investigation will be reported at a later date.

estimation came from 1954 or earlier, estimates from 1955-59 are true predictions. Consequently, some idea of the model's predictive power as distinct from its explanatory utility can be acquired.

A study of aggregate production response in the irrigated Far West has been initiated.[5] Plans call for tailoring the recursive programming method for this purpose. In addition, a projected study is the interrelation of several major cotton-producing regions in a dynamic interregional competition model.

The considerable flexibility of the approach lends itself to analysis and prediction at the regional level. Its broad structure, some essentials of which have been presented here, can accommodate all the inventive ingenuity brought to it for any particular application. Its union of statistical methodology and production theory seems to promise much for the analysis and prediction of aggregative supply of agricultural and other commodities.

PROOF OF THE PUDDING

It is too early to pass judgment on the empirical usefulness of recursive programming for the study of production response. At this stage only its promise can be described. It is an operational tool constructed to reflect production structures and to simulate explicitly the aggregative implications of decision processes at the firm level. While there is (as yet largely undeveloped) a theory of statistics by which estimates and hypotheses can be evaluated formally, the most attractive feature of recursive programming is its direct relation to the theory of production. Its foundation is not an esoteric theory of statistical decisions, but rather a highly plausible theory of economic action.

In addition to the lack of extensive empirical testing and a well-developed statistical theory, a thorough exploration of the bias of applying a micro-decision criterion at an aggregative level is lacking. Any study of aggregation must begin with a theory of the firm. Recursive programming seems to be well-suited to the job. The fact that certain other statistical methods are not derived from some explicit production structure does not exempt them from aggregation problems. Rather it implies that even the highest correlations do little to illuminate their essentially obscure micro-structural foundations.

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Discussion

DAY HAS PRESENTED an interesting application of linear programming to regional analysis. As he suggests, the empirical applications now being conducted will serve as a useful evaluation of the technique. Although I have no specific arguments to raise regarding his analysis, I do have one or two general remarks.

The aggregate supply of agricultural products can be traced to production on individual farms. Variations in the quantities of output produced on individual farms are often attributed to such factors as:

1. Technological developments causing changes in costs or output.
2. Variations in prices of agricultural commodities, including the expected duration of a price change.
3. Changes in institutional factors, including credit, tenure arrangements, and farm programs.

Moreover, in any given year, a farmer might base his decision to grow a crop on one or more of the following criteria:

1. The adaptability of his soil to a given crop. (Thus, farmers tend to think of "corn" or "bean" land.)
2. His ability to grow a certain type of crop. (A farmer tends to grow a crop he has had good luck with in the past. This is, in part, a quest for income security.)
3. The type of specialized equipment available to him.
4. The amount of operating capital available.
5. Expected occurrence of disease, weeds, and insects.

6. Expected prices after harvest time.
7. Type of storage available.
8. Necessity to meet fixed annual payments.
9. Planned livestock program.
10. Ability to withstand possible losses.

Other factors could be listed. There are a multiplicity of forces, all affecting supply partially and none affecting supply completely, which must be considered.

Reflection on the above criteria will illustrate the load which must be carried by the flexibility coefficients. A listing of factors determining farmers' decisions to invest or disinvest, not included here because of space limitations, would similarly illustrate the duties of the investment coefficients. Therefore, I feel the crux of Day's method lies in the estimation of the flexibility and investment coefficients. Because of their importance, I would have liked a more detailed discussion of their estimation.

As Day suggested, the meaning of the coefficients should be more fully explored. At first glance, one wonders if the coefficients can adequately estimate the effects of the many forces affecting supply. Upon reflection, however, a coefficient reflecting an aggregate rate of adjustment appears to have considerable utility. Obviously, we can never hope to quantify all the forces which influence production decisions on individual farms. It is not clear that such quantification, even if possible, would supply the answers sought. On the aggregate level, we do not need detailed knowledge of the supply response on individual farms, but rather we need a general knowledge of the supply response of all farmers in the region. Because of interactions within groups of farmers and the effects of aggregate supply and demand, the summation of individual farm responses may not be equal to the regional response. Thus, the flexibility coefficients, representing an over-all response for a region, could reflect effects of forces not apparent at the farm level.

It would also be useful to know the stability of the coefficients with respect to both time and technology. If the rate of adoption of a certain type of technological change, such as mechanization, is found to follow a characteristic trend through time in a given region, the effects of new machinery developments on the supply of the region could be predicted.

Interpretation of the flexibility and investment coefficients poses another, more general problem. Apparently our empirical techniques and available data often do not lend themselves to the estimation of our well-known economic parameters. Thus, we should study the available techniques and data with a view towards estimating parameters which are meaningful in a dynamic setting and useful for prediction and policy decisions. These parameters may not always be the familiar ones presented in classrooms and textbooks. The flexibility coefficients appear to be of this type. We must be careful, however, not to limit our thinking to problems or techniques which can be solved using known data sources and electronic computers.

The linear programming model used by Day is, in a predictive sense, fundamentally deterministic. This means that at any given point in time, it has a single solution. The question I would like to ask is: Should the predictions of analytical models aggregated on a regional level be regarded as completely determined or should they be presented in a probability framework? At what level of aggregation should the deterministic approach be used? Certainly, output prediction at the farm level can only be asserted with some probability. If supply predictions are eventually stated in terms of probabilities, continual effort would be needed to evaluate changes in the probabilities caused by changes in technology and other factors.

The present workshop was stimulated by problems in supply response. The immediate problem in agriculture is that of supply control. However, we should also be prepared to deal with other supply problems as they might arise. One very important problem both now and in the future is that of policy implementation. Thus, I would have liked to have added one or more additional papers to the program of this workshop. They would deal with problems of implementing policies which are founded in economic logic and validated by empirical analysis.

Chapter 6

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Supply Functions Estimated in Demand Studies

SINCE THE TITLE of this paper may appear puzzling, we shall try to justify it, at least in terms of definitions, before proceeding to the principal topic: the usefulness of such equations in studying supply.

DEMAND STUDIES

For a long time, the individual consumer has been postulated to regulate his consumption habits according to some system of ordered preferences so that his demand per unit of time for some particular one of the n commodities available could be expressed as a function of the prices of the n commodities and his "income."

$$D_i = D_i (p_1, p_2, \cdots, p_n, I) \tag{1}$$

Under more restrictive assumptions it has been usual to assume that the market demand resulting from the aggregation of such functions is a stable and meaningful relationship in the structure of the economy.

These functions are the principal object of estimation in demand studies.

$$Q_i = Q_i (p_1, p_2, \cdots, p_n, y) \tag{2}$$

At an earlier time when these functions were universally estimated from time-series or cross-sectional data, by minimizing the sum of squared deviations of computed quantities from observed quantities during some "sample" or observation period, classification of studies as to whether or not they were demand studies was relatively simple. Although some studies may have considered such questions as how much of the price change from one period to another was due to changes in demand and how much to changes in supply, the estimating phases of the work could be segregated into those devoted to the estimation of the demand functions, and those devoted to the estimation of the supply functions.

The advent (c. 1943) of "simultaneous equations methods" has made such classifications difficult. Titles such as "Econometric Analysis of Supply and Demand," "Statistical Analysis of the Feed-Grain Economy," or "Estimating Elasticities of Supply and Demand" are more common. These titles, although more or less hypothetical, emphasize the problem of trying to define demand studies in such a way that we may speak meaningfully of "supply functions estimated in demand studies." If the estimating techniques are such that a supply function is estimated, the study is likely to have a symmetry in the treatment of supply and demand so that the authors would not be willing to call it a demand study.

We should probably emphasize that the reference to the advent of simultaneous equations methods does not mean that students of demand were unaware of the supply function up to that time. Even the pioneers (H. L. Moore, E. J. Working, H. Schultz, et al.) were aware of the essential role of supply response in generating the observed data, and their treatment of demand may not have suffered so much from "blind analogy" to experimental situations as has been alleged.

THE ROLE OF "OTHER EQUATIONS" IN DEMAND STUDIES

Demand studies intended to estimate the parameters of equation 2 have been criticized because they do not always take explicit precautions to prevent the resulting estimates from reflecting the shape of the remainder of the system which generates the observations.

This argument may be illustrated by reference to a simplified model.

$$Q = a_1 + b_1 P + u \tag{3}$$

$$Q = a_2 + b_2 P + v \tag{4}$$

In estimating the least squares relation between quantity and price, it is not clear (unless additional information is available) whether the regression coefficient will approximate b_1 or b_2 or some combination of the two. This information may come in the form of knowledge or assumptions about the variables u and v which are assumed to be unobservable. The impact of assumptions in this area may be assessed from the relation which shows the expected value of this regression coefficient as a function of the parameters of the model and the distribution of u and v.

$$E\left[b_{qp}\right] = \frac{b_1\sigma_v^2 - (b_1 + b_2)\,\sigma_{uv} + b_2\sigma_u^2}{\sigma_v^2 - 2\sigma_{uv} + \sigma_u^2} \tag{5}$$

Thus, information indicating that $\frac{\sigma_u^2}{\sigma_v^2}$ is small will assure that the least-squares regression coefficient will approximate the slope of the demand curve. (If $\frac{\sigma_v^2}{\sigma_u^2}$ is small it will approximate the slope of the supply curve.) As long as σ_u^2 is not zero, the estimates will be subject to what has been called Haavelmo bias. It is usually called least-squares bias.

If, in this model, equation 3 is replaced by

$$Q = a_1 + b_1 P + C_1 Y + u' \tag{6}$$

it can be shown (assuming $\sigma_{yu'} = \sigma_{yv} = 0$) that

$$E\left[b_{qp\cdot y}\right] = \frac{b_1\sigma_v^2 - (b_1 + b_2)\,\sigma_{u\,v} + b^2_{2u'}}{\sigma_v^2 - 2\sigma_{u'v} + \sigma\sigma^2_{u'}}. \tag{7}$$

This is the same as equation 5 with u replaced throughout by u′.

Even though the ratio $\frac{\sigma_u^2}{\sigma_v^2}$ may not be small enough to cause b_{qp} to be a useful estimate of b_1, it may be that the ratio $\frac{\sigma^2_{u'}}{\sigma^2_v}$ will be small enough to make $b_{qp\cdot y}$ a useful estimate of b_1.

Thus, even in the absence of simultaneous equations methods, the assumed presence of a varying supply function plays a fundamental role in the model for estimating the parameters of the demand function. This must reflect upon the accuracy of the usual designation of demand models fitted by the method of least-squares as uni-equational complete models. Although the estimating procedure is dependent only upon assumptions and observations made upon variables of a single equation, the "other equations" of the model must be assumed to justify these assumptions.

A more usual exposition of the role of the other equations may be illustrated by the following statements which refer to the model consisting of equation 6 and equation 4.

If we are unwilling to make any assumptions about $\frac{\sigma^2_{u'}}{\sigma^2_v}$, but are willing to assume $\sigma_{yu'}$ and σ_{yv} are zero, we cannot fit equation 6 by least-squares because of equation 7. These assumptions will, however, justify fitting.

$$Q = a_{qy} + b_{qy}Y + W_{qy} \tag{8}$$

$$P = a_{py} + b_{py}Y + W_{py} \tag{9}$$

These two planes intersect in a line which also lies in the plane defined by equation 6 and in the plane defined by equation 4. If neither b_1 or b_2 is zero these planes are distinct from those defined by equations 8 and 9. There are, however, an infinite number of planes which contain this line, and we must ask whether there is a way to distinguish equation 4 and equation 6 among this family of planes. Equation 4 can be distinguished. It is the one plane which contains this line and is also parallel to the Y axis. No such restriction holds for equation 6 so that in this model it cannot be distinguished. Thus equation 6 cannot be identified in any formal sense.

These simple examples may serve to illuminate the role of "other equations" in demand studies. This role is usually expressed in the general case as:

> A parameter estimate cannot be identified unless the number of variables effectively present in the system, but absent from the equation in which the parameter is estimated, is at least equal to one less than the number of endogenous variables present in the system.
>
> A parameter estimate can be identified if the coefficients of these excluded variables in the other equations of the system form a matrix of that rank.

The role of the "other" equations is to identify the demand function, and to make possible the estimation of relations in which untenable or unsupportable assumptions seem required for estimates analogous to equation 7.

DEMAND STUDIES AND ECONOMETRIC STUDIES

Reference has already been made to the difficulties of classification. The finished product may be formally the same whether the principle objective of the research is to estimate the functions (2) or whether it is to estimate functions of some other sort. The estimates of any one relation depend upon the structure of the other relations.

The distinguishing characteristics may be the structures permitted in various equations, but I should not like to be adamant on this point. Some illustrations may serve to indicate the differences in structural precision attempted in the various equations.

In 1953 Fox (1) estimated the demand and supply for pork as

$$p = -1.14q + .90y$$

$$q = -0.07p + .77z$$

p = price of pork
q = consumption of pork
y = consumer income
z = the estimate of production that would be arrived at based on predetermined variables alone.

Since the coefficient of p is not significantly different from zero, Fox concludes that this demand function can be fitted by least-squares. It should be noted, however, that this short-run supply elasticity of zero would not be very helpful in appraising supply response and that the structure of the variable z, which is not specified in this exposition, contains all the information on factors which affect supply.

Ladd and Tedford (3) fitted the demand equation in the model:

$$q_t = a + a_0 P_t + a_1 P_{at} + b_0 y_t + b_1 y_{at} + u_1 t$$
$$q_t = c + c_0 P_t + c_1 X_t \qquad\qquad + u_2 t$$

q_t = log per capita meat consumption, time t
P_t = log average retail meat prices, deflated, time t
P_{at} = log average retail meat prices for n (=3, 5 or 9) time periods preceding t
Y_t = log per capita disposable income, time t
Y_{at} = log average per capital income for n (=3, 5 or 9) time periods preceding t
X_t = log meat marketing charges, deflated, time t.

Here we have a relatively elaborate formulation of the demand function to accommodate dynamic characteristics of demand with respect to both price and income. The supply equation was not fitted, but suppose it had been. It is not reasonable to assume that the quantity of meat supplied to stores depends upon retail prices and marketing margins except to the extent that this is a behavior equation for the marketing sector. If it is, then one must wonder why X_t is treated as exogenous. This is a demand study!

An older example, also concerned with the demand for meat, is reported in an appendix to Demand for Meat by E. J. Working (4).

$$\begin{array}{llll}\text{Demand} & X_1 = 157.462 - .9135X_2 + .5998X_3 - .5217Y_0 + .0706Y_7 + u_1 \\ \text{Supply} & X_1 = 28.675 + .0356X_2 + .7392Y_4 + .0914Y_6 + u_2 \\ \text{Income} & X_3 = 55.324 + .2049Y_5 + .3033Y_6 + .3158Y_7 + u_3\end{array}$$

X_1 = quantity of meat demanded per capita in a given year
X_2 = real retail price of meat in a given year
X_3 = real disposable personal income per capita in a given year
Y_4 = production of meat per capita
Y_5 = real investment expenditures
Y_6 = time
Y_7 = lagged real disposable personal income per capita.

This study used quantity produced as a predetermined variable in the supply at retail and hence also leaves the question of why these amounts were produced out of consideration.

All of these examples had both expository and empirical objectives so that they should not be regarded as examples of what can be done in demand analysis.

Judge (2) fitted the first and fourth equations in a 12-equation model of demand and supply relationships for eggs. The equations of the model are:

1. The demand for eggs at retail;
2. The supply function for eggs at retail;
3. Demand for eggs by the commercial sector;
4. Supply of eggs by farmers;
5. Demand for meat in the retail market;
6. Supply function for meat in the retail market;
7. Demand for meat by the commercial sector;
8. Supply of meat by producers;
9. Demand for food other than meat and eggs at retail;
10. Supply of other food at retail;
11. Demand for other food by the commercial sector;
12. Supply of other food by farmers.

Is this a demand study? It is certainly not subject to the adverse comments made about some of the other supply estimates. The supply is elaborated even more than the demand in discussing the model.

The supply of eggs by farmers is apparently the equation most closely related to supply response in the usual sense since this is where the "production," as contrasted to "marketing," activities take place.

Let us, therefore, look at the structure of this equation. The following supply equations were obtained by Judge. Since the data used were in logarithms, the coefficients are in terms of elasticities:

(a) Limited information:

$$y_6 = 1.17y_5 + .54y_7 + .002z_3 + .23z_4 - .97z_5 - .78z_6 + 1.67,$$

(b) Reduced form:

$$y_6 = .36y_5 + .54y_7 - .01z_3 + .30z_4 - .44z_5 - .54z_6 + 1.62,$$

(c) Single equation:

$$y_6 = .19y_5 + .01y_7 - .07z_3 + .53z_4 + .01z_5 - .40z_6 + 1.30,$$

Where:

y_6 = index of per capita supply of eggs by farmers;
y_5 = index of prices paid to farmers for eggs;
y_7 = index of prices paid to farmers for meat;
z_3 = time with the origin at 1920;
z_4 = index of prices paid to farmers for eggs lagged one year;
z_5 = index of the cost of the poultry ration;
z_6 = index of the cost of the poultry ration lagged one year.

The author concludes that the limited information estimates are "reasonable" and the signs are "consistent with the economic theory of the firm."

Table 6.1 presents equations (a) and (b) in a way intended to emphasize the effect of eliminating some variables from the other equations.

Table 6.1. Comparison of Limited Information and Reduced-form Estimates of Supply Elasticities

Elasticity of supply with respect to	Limited information (equation a)	Reduced-form (equation b)
Egg price	1.17	.36
Livestock price	.54	.54
Egg price lagged	.23	.30
Feed cost	-.97	-.44
Feed cost lagged	-.78	-.54
Time	-.002	-.01

These have been several examples of supply functions estimated in demand studies or studies in which the study of demand is at least as important as the study of supply. They are included, not as being typical, but only as illustrations.

Limitations or possible limitations include:

1. Inadequate specification of the supply function, resulting in estimates which may be useless for supply studies and the improper specification of the supply function may have serious implications for the demand functions to be estimated.
2. Sector aggregate or average orientation contrasted to firm orientation which is appropriate for some objectives of "supply research."
3. The use of "consistent" estimating procedures in lieu of approximate identification and least-squares may have led to an inappropriate distribution of effort between the formulation and computation phases of research.

In supply studies for policy purposes sector aggregate orientation may be appropriate. Even in these cases, the econometric approach (or demand study) has not been as helpful as one could wish because of the low state of information on the factors which determine supply response and their applicability. Most people who work with these functions will probably say that their supply functions turned out to be less "reliable" or less "reasonable" than the demand functions.

We continue to hope that synthetic supply estimates generated from engineering and accounting data may replace statistical supply functions in demand studies. This hope may be held more strongly because of ignorance of the pitfalls along that road.

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Discussion

WEST DIRECTED his paper toward an evaluation of the usefulness of systems of equations involving estimates of both supply and demand functions. To achieve this end, he found it necessary to differentiate between "demand" and "demand-supply" studies. Essentially, the criterion used to differentiate between such studies was the type of estimating technique employed. He writes: "If the estimating techniques are such that a supply function is estimated, the study is likely to have a symmetry in the treatment of supply and demand so that the authors would not be willing to call it a demand study." Presumably, if the estimating techniques do not result in the estimation of a supply function, the research would be considered a demand study. While this classification is useful for West's purpose of pointing out the interrelationships between demand and supply studies, the more general application of this classification is questionable. If there should be substantive reason for classifying supply and demand studies, it would appear more reasonable to do so on the basis of their primary orientation or objectives rather than on the basis of empirical technique. If the primary objective of a study was the estimation of a demand function, it would be found desirable (for identification or other purposes) to employ a model involving a supply function and it would seem logical to identify it as a "demand" study. Parallel but opposite cases would be identified as "supply" studies. Those with the dual objective of estimating both demand and supply functions would be identified as "demand - supply" studies. This would avoid the difficulty of classifying demand (supply) studies in which the supply (demand) function is not empirically estimated but rather assumed or taken as given.

West's paper contains one rather subtle point which appears to be of extreme importance to a group of researchers working in the supply response area. This point is the necessity of explicitly recognizing the demand factor when attempting to estimate supply functions. Apparently, some early attempts to estimate demand functions were not completely satisfactory because the "supply blade of the scissors" was not adequately incorporated into the models used. In attempts to come up with more satisfactory demand estimates, researchers have moved in the direction of more elaborate models which take this factor into account. The similarities between problems encountered in estimating supply functions and demand functions suggest that researchers on the supply side can profit greatly from the lessons learned by their predecessors working on demand studies. One of the most important of these lessons is that every scissors has two blades and that a description of one blade tends to be meaningless unless it involves the relationship of the pair. West has made a worthwhile contribution by calling this fairly obvious, yet easy to overlook, notion to the attention of this conference.

In his evaluation of supply functions estimated in demand studies, West indicates that such functions may be limited in usefulness because of inadequate or improper specification of the supply function, their aggregate nature and an inappropriate distribution of effort between formulation and computation. He does not elucidate on these possible limitations but leaves one with the impression that he is fairly pessimistic about the usefulness of supply functions estimated in demand studies. It would have been helpful if West would have given us his thoughts concerning (a) the kinds of problems upon which such supply function estimates might shed light, (b) more specific ways in which the usefulness of such estimates might be increased and (c) additional pitfalls which have been encountered in demand studies which might be wise to circumvent in supply studies.

PART III

Supply Estimates Derived from Individual Farm Data

Chapter 7

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Determination of Supply Functions from Cost and Production Functions

THIS PAPER will explore some of the possibilities for using firm level production and cost functions as a basis for estimating agricultural product supply functions. It is further confined to certain aspects of this problem when cross section farm records and surveys are used as basic data for estimation.

It is assumed that the supply response of an industry may be looked upon as the aggregation of individual firm experience and action. Further, cross section estimates of sector or industry relationships are based on an assumption that individual units of observation are relatively homogeneous in certain respects, e.g., with respect to production functions. Compared with time-series analysis, the problem of estimation is one of accounting for spatial heterogeneity rather than changes or shifts in relations over time.

Since the production function is basic to both estimation of cost relationships and of supply functions, some of the theoretical production relationships and their implications for estimation of supply response or functions will be discussed first. The theory in this area is well-developed for static situations, i.e., the analysis abstracts from time as a variable. No attempt will be made to exhaustively present this static theory, but a sketch of some pertinent relationships is made as a basis for further discussion. This theory is presented mostly in the context of certainty of knowledge with respect to prices, quantities of the productive services of factors of production and their relationships to the forthcoming products. There is no well-known and well-developed body of theory in this area beyond this static level. The lack of such theory is, of course, a bottle neck to empirical research. The gaps on the theory side as listed by Bachman and Nerlove (1, pp. 3-4) are:

1. An adequate theory of aggregation of firm supply functions.
2. An adequate theory of behavior under uncertainty.
3. An adequate operational theory of investment for the firm, that is, an empirically useful theory of how so-called fixed factors are varied over time in response to economic forces.
4. A theory of, or at least techniques of measuring, the diffusion of technological changes and their specific effects on the production possibilities open to the firm.

THEORY

Single Product Situation

We suppose that a single product forthcoming in a given period is some function of the input of variable factor services given certain amounts of fixed factor services which may be considered as a group or technical unit. Variable and fixed designate services the use of which respectively affect a change in costs or do not affect costs as output is changed given the period in question (4, p. 12). Relationships between the product and factor services indicate the maximum amount of product forthcoming from any combination of factors (technically efficient production). Accordingly, with appropriate secondary conditions we can assert the economic theorem (12, p. 60) that in order for total cost to be a minimum for any

$$\frac{\frac{\partial Y}{\partial X_1}}{P_1} = \frac{\frac{\partial Y}{\partial X_2}}{P_2} = \ldots = \frac{\frac{\partial Y}{\partial X_n}}{P_n} = \frac{1}{\lambda}$$

where $\frac{\partial Y}{\partial X_i}$ — are the partial derivatives of the above mentioned production function and represent the marginal productivities of the variable factor services, X_i. P_i represent the factor service prices.

Given output, marginal productivity of the last dollar input ($1/\lambda$) must be equal in every use. The combination of points of minimum cost for different levels of output is termed the expansion path. A firm that increases production in the least costly manner remains on its expansion path which, especially in the short run, may not be linear. Marginal cost is the addition to total cost brought about by increasing output by one unit while remaining on the expansion path. There is no reason why all firms should have the same marginal cost curves. In fact, it is expected that firms will generally have different short run cost curves. Lacking perfect knowledge in times past and present the firms have been and are being organized in various ways involving different combinations of fixed assets.

The marginal cost curve of the firm under conditions of perfect competition is looked upon as the supply curve of the firm. If marginal cost is above the average variable cost per unit of output the lowest cost at which the firm will offer a given quantity of product is the marginal cost of the corresponding output. Furthermore, if individual firms are ruled by the profit maximization motive the supply curve of the industry is the simple sum of the individual firm supply functions, other things being equal. The <u>ceteris paribus</u> conditions are (1) the firms do not affect their factor markets, i.e., the changes in quantities of factors demanded by the firms of the industry as a result of shifting levels of output do not affect the price of these factors or factor services to the firms and (2) the number of firms in the industry is given. If the conditions (1) and (2) are not in effect, the static marginal cost curves of the firms will not sum to the industry supply curve.

In the event that the total demand of the industry for factors of production affects the price of various factor services to the individual firm, the flexibility of factor prices must be incorporated into any method of aggregation of firm level production response to product price change. If farmers have imperfect knowledge of price and production relationships, or if farmers are under capital rationing pressures, simple summation of marginal cost curves need not lead to industry supply functions. Similarly, technological change may lead to unforeseen changes in firm behavior. If the profit maximization motives of the farmer are qualified by or in competition with household goals, elements other than the marginal cost functions must also be considered in aggregating the individual firm actions to obtain supply response of the industry.

Multiple Product Situation

The theory of production and costs has been extended to include firms producing more than one product (4). If the products are independent in production, i.e., if the production of one product does not affect the costs of producing any other and vice versa,[1] the supply curves of the products at the firm level may be considered as specified in the same manner as for the single product firm. The production response of each product may be considered separately in the case of independent production. However, there would probably be no incentive for production of multiple products in such situations.

Also, in the joint product situation in which the products are forthcoming in fixed proportion there is no difference between the single product and joint product situation at the production level. The products may be combined and considered as a single product for purposes of analyzing cost and production functions.

The multiple product situation differs from the single product production case if products are interdependent because of technical and/or service price conditions. Technical interdependence occurs if the marginal productivity of one product is a function of the level of output of another product and/or the levels of service inputs of other products. If service prices are not constant the marginal cost of one product may be affected by the level of service input of other products when cost is assumed a function of levels of output. It may be noted that these interdependent situations describe products which while independent at certain levels of production of the products may be interdependent at others.

[1] If service prices are assumed constant, technical independence is specified. It is possible for products to be interdependent in production because of (a) technical interdependence and/or (b) service prices varying with levels of product output.

Even with constant service prices this is true if certain factor services are fixed. Such a possibility may have implications of importance to short-run and relatively short-run analysis.

A more complete development of the foregoing points may be found in the references cited (7, 11).

ESTIMATION PROBLEM

The estimation of supply functions from cross section data consists generally of (1) estimation of individual firm supply relationships and (2) aggregation of individual firm relationships into a supply function for the commodity in the industry or sector of the economy. Attempts to estimate firm supply relationships from cross section data may take either of two directions. In the first, underlying production functions may be estimated and then the firm's cost and supply functions are derived from the estimated production functions. In the second, supply relationships from the firm are obtained from cost relationships estimated directly from financial data of the individual producing units. The first approach has the advantage of being more general. That is, production functions, if sufficiently detailed and with inputs and outputs in physical units, may have alternative prices attached in order to analyze various cost and price situations. Theoretically, this approach permits one to arrive at a set of relations that will remain valid or change in known ways under a wider variety of circumstances than would an approach involving estimation of supply relationships directly from recorded cost data. The direct cost analysis approach has the advantage of requiring a type of data which is usually more easily acquired. Financial records are generally used for income tax filing and other purposes so the data can usually be had by survey or by simple record keeping of a type understood by farmers. However, cost relations estimated directly from such records reflect specific cost and price situations and are not easily modified to account for changes. The direct cost analysis approach also leads to problems in the case of multiple product firms because of the difficulty of allocating certain fixed costs among the various enterprises.

Single Product Firm Production Function Approach

If the output of a single product is considered as a function of certain resource inputs, the conventional procedure of predicting the total output curve or surface as a regression equation may be followed. On the product side, the output may be measured in physical units or as a value product that is a constant multiple of the physical product. Except in controlled experimental situations, the number of possible variables on the input side is too large to permit working with all variables. Hence, researchers have aggregated the factor services (or investments in factors) into categories on the basis of their being technical complements and near perfect substitutes (9). This categorization has led to the measurement of input categories such as machinery service in value terms. The specification of the input categories and the measurement of the appropriate variables raises the same problems of estimation considered in some detail in the literature concerned with the estimation of production functions for farm management or intra-firm purposes.[2] As has been pointed out elsewhere (6), biases may result from failure to include important variables such as management service. As usual, the problem of multicollinearity will continue to plague researchers trying to obtain production function estimates from cross section data. These problems may be no greater in estimating supply relationships than in the usual production function analysis where they have caused considerable concern. However, few attempts have been made to carry production function estimation at the firm level to supply functions on an industry basis. The implications of these problems for aggregate level analysis are not yet spelled out.

As noted, discussion of production function estimation opens a Pandora's box. It is not the purpose of this paper to review these problems in detail. Most of them are well-known to researchers who have tried to estimate a production function. The articles cited offer possibilities for solutions to the problems at least in certain instances.

However, the use of cross section data gives rise to the suspicion that each observation may represent a point on a different production surface, especially since firms are found using different technologies. A possible solution under relatively short run conditions lies in the selection of the cross section samples. If strata are delineated in a manner to represent firms with common technologies and/or other factors likely to affect technology, separate production function estimates may be made for these situations. For example, firms may be classified into strata by size, age of operator, production practices such as use of milking parlors, etc. This stratification procedure, besides aiding in identification of the production functions, may be useful in reducing the biases caused by failure to include variables such as management. One intuitively feels that in such situations management and other nonincluded variables are more likely to cause randomly distributed disturbances. That is, the correlation between management factors and included variables is apt to be relatively less given the extent of the variation in these variables within the homogeneous strata. Among strata, management might be expected to vary in proportion to capital inputs, but when size of firm is specified, and hence the corresponding capital inputs are specified within limits, the correlation will ordinarily be lower.

If in the short run one can assume that the number of firms in each

[2] A number of production function estimation problems were reviewed in Heady, Earl O., and others, eds. (3, 9.) Reference cited by a number of authors in this book may be used to further follow up the problem.

strata will remain relatively constant, the firm supply relations may be estimated and subtotaled for the strata and then added to obtain the supply function of each other stratum. (It is assumed that appropriate sampling techniques are used within strata so that the resulting firm supply function for each stratum is representative for that stratum.) More specifically, this procedure involves first estimating the outputs for various factor combinations within each stratum by the use of the production function. One generally uses the combinations of factors corresponding to points on the expansion path defined by the production function and expected factor service prices (values). The levels of certain factor service inputs would be fixed according to length of run considerations. Since the relationship of various levels of output to the combined (minimum) value of the factor services required to produce these outputs is the variable cost curve of the firm, the strata marginal costs are obtained by estimating with this function the additional cost that accompanies each added unit of output. Multiplying the successive output levels by the number of firms in each stratum and relating to the corresponding marginal costs leads to a stratum supply curve.[3] To obtain the industry or sector supply curve, the strata output levels corresponding to the specified marginal cost situations are simply added together. An aggregation problem arises here.

It is, of course, likely that the firms as a group will influence factor prices by their combined action. Factor price flexibilities or the group output effect on prices may be taken into account in the single product model. As long as one assumes that no individual firm influences price perceptibly, it is only necessary that the price be specified for the industry or group of firms as a whole for each total (group) output level. Conceptually, one has only to set these prices at their expected level for the output in total and then maneuver the individual strata models to a point on the respective expansion paths that leads to the desired total output for all strata but keeps the marginal cost the same for all firms among strata.

As output levels for the group of firms is changed, marginal costs to the individual firm within strata shift, e.g., upward as suggested in Figure 7.1. OA represents the individual firm share of total output given the initial price and total output specification for the group of firms. OB specifies the individual firm share with a second specification and similarly for OC. It is the sum or aggregation of curves such as FG which becomes the supply curve of the industry or group of firms rather than the aggregation of marginal cost curves *per se*, as was the case when the factor service prices did not change.

It is possible that even when firms know their production cost relationships and are able to adjust outputs to maximize short run profits, they may choose not to do so. Various reasons are hypothesized for such action, e.g., influence of holding other goals than profit

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[3] For a mathematical derivation in the static case of the individual firm supply function from a Cobb-Douglas type production function, see Bachman and Nerlove (1), pp. 39-41.

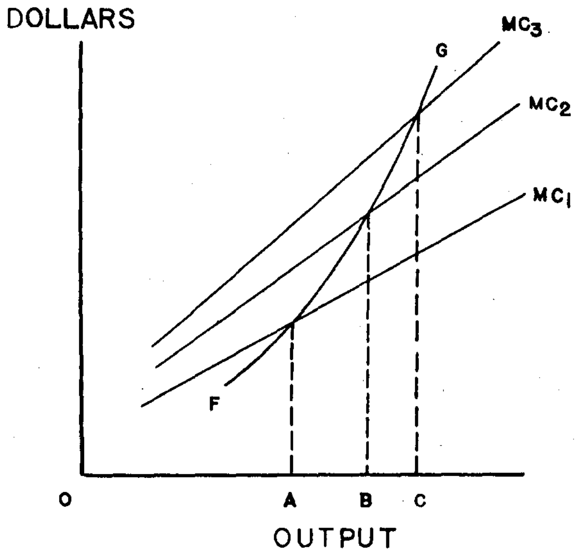

Figure 7.1. Cost and supply relations for hypothetical average firm in a strata. (Linear relationship used for convenience in presentation.)

maximization. If the suggestions for deriving a supply function are used, various adjustment factors might be devised. Similarly, the effects of risk and uncertainty modify any such analysis. Nevertheless, it is felt that until more general theories of firm behavior under risk and uncertainty and conflicting goals are devised, approaching supply analysis through static classic production functions may offer worthwhile insights to the researcher and policy maker. Differences between estimates made with the static model and the supply responses in reality may be analyzed or rationalized in the light of such factors as risk and uncertainty as a first step toward more adequate analysis of the effects of these factors.

In the long run, changes in the factors by which strata are specified may become of major interest. Technological change cannot be ignored as an important factor in supply analysis. Since there is no well-developed body of theory upon which to draw here the problems of estimation are going to be more difficult. One might suggest, as does Klein (10, pp. 236-40), that cross section data might be taken over time to form a time series of cross section data. Such a procedure might permit the introduction of such techniques as distributed lags (5) or possibly analysis of covariance (8) and other techniques. It must also be recognized that a related problem is the changing form of fixed assets within individual firm that takes place both with or without changing technology.

Single Product Firm Cost Analysis Approach

Elementary theory indicates that the supply curve for an industry is, under certain conditions, the sum of the individual firm marginal cost curves. Thus, it appears possible to estimate the cost curves directly from the cross section firm records rather than starting with the underlying production function. The differences between the cost-output positions of the cross section of firms as shown by appropriate financial records might be considered as data from which to estimate a total cost function. However, even when the firms operate with the same production functions, it is not necessary that they have the same marginal cost curves. Marginal costs depend upon the level and distribution of fixed costs, or rather upon the nature of the fixed factors and their levels within a firm, i.e., upon length of run under consideration. Some way of grouping the firms according to the nature of the marginal cost function is needed. In other words, homogeneous fixed plants must exist before a cross section would indicate points on the same variable cost structure.

If one aggregates all variable costs and estimates the function the costs are of output and fixed cost category levels, an assumption is made that farmers do combine resources to produce on the expansion path of the firm (at minimum cost per level of output). Otherwise, some hybrid cost curve is obtained which would be difficult to interpret meaningfully. In practice firms vary rather widely in their fixed asset structure. In production function estimation the differences in services flowing from such fixed factor situations were taken into account in the cross section estimate by considering such factors as variable from farm to farm and hence within the individual firm, perhaps on a long run basis. Since marginal cost functions will vary depending upon the fixed asset structure, one could either (1) stratify the data to obtain comparable fixed asset structures and estimate individual cost functions for each or (2) include more than one fixed cost category in the model, each with varying levels, just as was done with factor services in the production function model. In a competitive model the production function may theoretically have per unit values attached to the services and any one of the categories made a function of output and the other categories. Elementary theory indicates the total cost function as the inverse of the production function which has undergone a value transformation on the input side. This is the same as working with a production function in which value inputs are used. Hence, the detailed cost analysis approach comes back to the ordinary production function analysis approach.

Most agricultural products are produced on multiple product farms. Interest in the single product firm cases is somewhat limited. It has been presented in some detail to give an insight into the overall problem and because historically cross sectional estimation of production functions for multiple product agricultural firms has seldom proceeded beyond the use of single product variations.

Multiple Product Firm Production Function Approach

The multiple product case, while of most general interest, is unfortunately the most involved and difficult to handle. Some sort of interdependence of the products in production or marketing exists or there would be little incentive of firms to produce more than one product. The nature of this interdependence affects the possibilities and methods of estimation that must be used to discover the production functions and related supply response estimates. Although few attempts have been made in agriculture to connect production functions to supply analysis directly, most firm level production function estimates have been based on multiple product firm data. However, the interdependence problem has not been satisfactorily handled in general. Researchers have often tried to avoid the issue by either selecting firms as nearly homogeneous in their output complex as could be obtained and then aggregating the product outputs in terms of gross income, or they have divided the product outputs into several relatively homogeneous categories such as livestock and crops and then fitted separate production functions for these categories using various accounting procedures to allocate an appropriate share of the input services to each output category. Once output and input categories have been designated, estimation has proceeded as in the single product case, usually by least squares regression fits of an equation. The usual choice has been a variation of the Cobb-Douglas production function.

The use of independent estimates of the separate production function by enterprise is theoretically feasible, as pointed out by Beringer (3), if the production functions are technically independent. In practice, as noted before, separating the products in the multiproduct firm for the purpose of estimating individual production functions leads to accounting-allocation problems on the input side. It is difficult to determine how much of a feed floor service or building service to allocate to the production of each enterprise or product. On the other hand, treating the entire complex or portion of the products as a group leads to an index number problem.

An example of this problem can be illustrated. If two products, A and B, are aggregated into a single value product category, one can easily obtain biased estimates of the production coefficients, especially labor. For, if the cross section sample is one in which considerable substitution of the two products has occurred one (B) may be a larger user of labor than the other (A). The use of labor may be correlated with the substitution of this product (B) for the other (A). If prices of the products are such that product B leads to lower total value from the same resources, it is possible as B increases in the cross section data, other resources constant, that gross income decreases; i.e., those farms with other resources comparable but having more labor may tend to substitute B for A. A cross section estimate of the production function will often show negative labor coefficients in this case although the addition of a unit of labor in the production of either product may be positive and the marginal value product of a unit of labor used in B higher than the corresponding marginal value product in A.

Intra-firm analysis information of considerable interest can often be obtained when this problem is avoided by limiting the analysis to farms with relatively the same proportions of A and B in production. In supply analysis it is the substitution effects among the products that may be of major concern. The supply function of B cannot be considered separately from possible substitution possibilities between A and B.

A possible solution to such problems is suggested by Klein (10, pp. 226-36). He uses a production function model in which he includes the various output categories. Some are designated as independent variables for purposes of regression estimation. In the example he gives, a Cobb-Douglas variation is used. As he points out, a theoretically unacceptable relation between the products under pure competition, i.e., diminishing marginal rates of substitution, occurs with this model. He suggests the possibility of other equations, indicating that he has tried one which at least permits a constant marginal rate of substitution among products.

Multiple Product Firm Direct Cost Analysis Approach

Again, a direct cost analysis approach might be tried. Total cost functions may be derived in the multiple product case. Average cost functions are not meaningful, but marginal cost function derivation is possible. However, such costs are subject to the conditions implied by the output levels of the other products. One might, therefore, set out to estimate the total costs of a production process as a function of the output levels of the various products. Suppose a cross section of accounting records supplies the basic data. Presuming that an appropriate mathematical model has been chosen to represent the relationships involved and that it has been successfully fitted, marginal costs of any product given specific levels of others may be calculated as a first derivative of cost with respect to output.

Assuming that the firms have the same cost structure given the same factor prices, cross section estimates of the total cost function are based upon inter-firm differences in cost and output. If all firms were operating efficiently they should all be at the same point in their cost relationships. Only a point estimate could be achieved. To identify the cost structure, it must further happen that the firms differ with respect to output positions for some reason that does not prevent them from having the same cost structure. If firms had and have different expectations, such a situation might exist. Some firms might overproduce and others might underproduce relative to the "optimum level." Under these conditions inter-firm differences would reflect differences in the total cost structure of a nature found within individual firms. However, when estimating such cost functions from cross sectional data one may not even then be approximating the desired cost function because many firms may have had ex ante expectations which led to factor service combinations other than on the expansion path. It would be difficult to identify the real cost functions.

Beside difficulties in meeting all the foregoing assumptions, cost curves developed from cross section cost data reflect a given set of cost conditions (factor service prices and production function). At best, short-run firm supply curves may be obtained by examining marginal cost of a particular product. Difficulties arise when analysis is to be made for a length of run which permits the factor service prices to vary as a function of the output levels.

The marginal cost curves for individual products can be specified only if the interrelationships of costs among the products are specified. It is difficult to assess the impact of changes in service prices upon these curves, since one assumes that the firms tend to operate upon the expansion paths designating service combinations for each product and also upon the expansion path indicating an optimum combination of products. Any cost complex is a shifting function of the underlying physical production function relations of substitution and the changing factor service prices. Hence, this approach has very definite limitations, especially beyond the short run.

The foregoing notes are not intended as a complete coverage of the problem under discussion, but it is hoped that they will serve as basis for productive discussion in this workshop.

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Discussion

KEHRBERG has done a commendable job of bringing together in a brief, concise manner the theory and methods of determining supply functions by means of production functions and cost analysis approaches. He has outlined procedures involved and discussed objectively both the strong points and the limitations of these approaches. I found no particular area in which to disagree with his analysis, but as I studied the paper one thing kept bothering me—how can this theory be adapted to an actual farm supply response study?

Kehrberg's paper is based on static, profit maximizing, equilibrium theory in which the firms have virtually complete knowledge. This he acknowledges. It therefore provides a good starting point in conceptualizing the problem, but I was a little disappointed that he did not try to bridge the gap between profit maximization theory under complete knowledge and the more realistic decision making behavior under uncertainty conditions that must be resolved in empirical farm supply response studies. I realize that this is no slight task. Perhaps I should have been satisfied with the contribution Kehrberg made. (I feel it definitely was a contribution.) But any exploration into this area, no matter how fragmentary, would have been of considerable value as meat for discussion in this workshop. It must be remembered that we are dealing with farmers who do not have complete knowledge and are not necessarily motivated by the usual concept of the profit maximizing goal.

In the multiple product firm, Kehrberg indicates that unless there was product interdependency because of technical and/or service price conditions "there would probably be no incentive for production of multiple products." His assumption of complete knowledge rules out multiple product organization as a means of combating uncertainty (income security maximizing goal for example). However, he ignores the resource adaptability limitation (for example, land characteristics which prevent producing continuous corn in certain areas) that would necessitate a multiple product organization or idle resources, even with perfect knowledge.

I agree completely with Kehrberg that it is necessary to classify farms into strata based on size of business, "level" or combinations of technology, or other criteria to aggregate individual supply functions. This is true, I believe, no matter how the data for estimations are collected. I would like to suggest another set of criteria for stratification, or at least point out situations under which farmers would possibly react differently to price changes.

Kehrberg's paper, and most of the others I have examined, dwell primarily on the situation in which the farmer is now producing the designated commodity or commodities in approximately the amount and proportions he has in the past and will for a certain period in the future (if product and factor prices remain constant). But what about the farmers who are in the process of expanding the production of a certain commodity, or those in the process of cutting back? It is certainly conceivable that their supply functions would be considerably different from those of farmers in a more or less equilibrium position. We also need to determine how much production is called forth by designated price increases from producers not currently producing the commodity, as well as the converse. The input structure, particularly investments, affecting the ease of getting into and out of production of a certain commodity is a major consideration in this regard. There is, of course, wide variation among commodities. Technological changes have probably caused a considerable decrease in the number of "inners and outers," particularly for certain livestock enterprises, but it is still a factor. Probably more important than the "inners and outers" in this entire question of noncurrent producers are the interregional aspects—the possibility of entering the production of a given commodity.

Still another consideration in this stratification process concerns the level of production in relation to "possessed" capital resources. I believe different supply responses exist, even assuming that farmers are combining resources in the manner that will maximize profit if (1) expansion requires no additional investment, (2) additional investment can be accomplished using accumulated capital, and (3) additional investment requires expanded use of credit.

The discussion on the relative merits and weaknesses of using a production function versus a direct cost analysis approach is very well presented. The inadequacies of the records kept by many farmers for use in cost estimation and allocation cannot be overstressed. Differences in farmer evaluations of noncash costs is a major hurdle to be considered.

One other point that might be open for question is the effect of stratification on management inputs. This would depend, among other things, on the basis of stratification selected.

There is very little question but that Kehrberg fulfilled the objectives outlined in his opening paragraphs. My only regret is that he did not try to penetrate the area of developing production and supply functions under conditions of less than perfect knowledge.

Chapter 8

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*Programming Intra-Farm Normative Supply Functions**

THE PURPOSE of the normative supply function is to describe the optimum relation between the quantity of a product supplied and its price, relative to some given norm. The maximization of farm profits is one possible norm upon which estimates of normative supply functions may be based. It is the norm assumed in the following discussion. The normative supply function is thus an estimate of the optimum supply reaction to product price changes in terms of the stated norm, rather than an estimate of actual supply reactions made by producers to changes in product price (1).

Estimates of normative supply functions become of particular importance in the evaluation of major resource allocation alternatives on individual farms. Although the actions taken by an individual farm usually have little impact upon the industry, the collective actions of all competing farms will have a considerable impact. The optimum allocation of resources on an individual farm will be dependent in part upon (1) what constitutes the optimum allocation of resources on competing farms in the same area as well as in competing areas and (2) the effect this has upon the aggregate supply of the product and the product price. The aggregation of individual farm normative supply functions to form the aggregate supply function provides a means of taking into account the effect of changes in aggregate supply in evaluating the resource allocation alternatives of individual farms when related to the appropriate product demand function. The results obtained by this approach are optimum only in terms of the assumed norm, or set of norms, used in the analysis. However, such results provide a point of reference against which divergent uses of resources and divergent goals or norms can be compared.

The objective of this paper is to discuss the problems of using linear programming in the derivation of individual or intrafarm

*Journal Article 2543 from the Michigan Agricultural Experiment Station. Many of the ideas presented in this paper were evolved in the course of the development of the Lake States Dairy Adjustment Study involving the states of Michigan, Wisconsin, Minnesota, Illinois, and Iowa. The study consists of a group of individual state projects coordinated on an informal basis through the Farm Economics Research Division, ARS, USDA. The many discussions that have taken place among the persons associated with these projects have contributed greatly to the content of this paper. However, the responsibility for the validity and accuracy of the statements made rests entirely with the authors.

normative supply functions used to construct an estimate of the aggregate normative supply function. The aggregate supply function is here regarded as being obtained by the horizontal summation of the supply functions of the individual farms. Hence, the basic unit of inquiry is the individual farm and the main use to be made of linear programming is to derive the normative supply functions of the individual farms. The problems of deriving the normative supply function of an individual farm using linear programming will be considered first. Following this, the problems of deriving the individual farm supply functions as related to construction of the aggregate supply function will be discussed. The main problems connected with the use of linear programming for this purpose are more nearly ones of definition and theoretical development of the problem than they are of programming methodology per se.

THE LINEAR PROGRAMMING PROBLEM

Basically, linear programming is simply a method of solving a particular type of mathematical problem (2). Stated in general form, this problem is,

$$
\begin{array}{ll}
\text{maximize} & f(x) = c_1 x_1 + c_2 x_2 + \dots + c_j x_j + \dots + c_n x_n \\
\text{subject to} & a_{11} x_1 + a_{12} x_2 + \dots + a_{1j} x_j + \dots + a_{1n} x_n \leq b_1 \\
& a_{21} x_1 + a_{22} x_2 + \dots + a_{2j} x_j + \dots + a_{2n} x_n \leq b_2 \\
& \vdots \\
& a_{i1} x_1 + a_{i2} x_2 + \dots + a_{ij} x_j + \dots + a_{in} x_n \leq b_i \\
& \vdots \\
& a_{m1} x_1 + a_{m2} x_2 + \dots + a_{mj} x_j + \dots + a_{mn} x_n \leq b_m \\
\text{and} & x_j \geq 0
\end{array}
$$

Any empirical problem that can be represented in terms of this general mathematical statement is amenable to analysis by linear programming. Linear programming is thus a general analytical technique that may be usefully adapted to the analysis of a variety of empirical problems. The major problem of using linear programming concerns how its mathematical format is adapted to the conditions of a specific application. To determine this, it is necessary to look to the conditions of the specific problem as it is defined and the relevant body of theory that applies to it. Linear programming only specifies the general mathematical form in which these conditions and the pertinent theoretical concepts must be expressed.

In estimating an intrafarm normative supply function, the general approach would be to construct a mathematical model of the farm in terms of the above mathematical format. Since the criterion is the maximization of farm profits, the f(X) would represent the total profits of the farm. The x_j would represent the level of the various production alternatives or activities to which the farm's resources may be allocated. The c_j would represent the profit per unit of each activity. The a_{ij} would represent the resource requirements per unit of each of the activities. The b_i would represent the quantity of each of the various resources available to the farm or such other restrictions as limit the use of resources on the farm. As the mathematical model consists of a series of linear equations, nonlinear relationships such as diminishing marginal physical productivity or decreasing marginal rates of substitution must be handled by approximating the relationship by linear segments. The mechanics of dealing with these and similar problems in constructing a linear programming model representing the resource allocation possibilities of a farm have been treated in considerable detail in several recent references (4).

To derive an estimate of the normative supply function for one of the products that could be produced by the farm, the various solutions to this model would be obtained over a range of values of the appropriate c_j, using the procedure of variable price programming. The quantity of the product produced at each price level assumed can be readily obtained from these solutions of the model to form the supply function (9).

SCOPE OF THE INDIVIDUAL FARM MODEL

There are a number of major conceptual problems involved in the construction of this mathematical model of a farm. The first of these concerns the scope of production alternatives to be considered. This becomes primarily a matter of problem definition. In any analysis of overall farm organization, the range of production alternatives considered is likely to be rather large.

At a given point in time, a farm operator will have a certain stock of resources under his control making up the farm's current production organization. At the same time, he will have before him an array of possible alternative courses of action that he might take with respect to the organization of these resources to produce income in subsequent production periods. One possible course of action would be to leave the current organization of the farm unaltered. Other alternatives range from completely disbanding the farm and directing the use of the resources to nonfarm alternatives, to making any of a number of possible investments to expand or modify the farm's existing organization. Some of the possible investments may consist simply of purchases of single period inputs — inputs completely utilized within a single production period — which would involve only minor changes in the farm organization. Other investments might consist of major changes in multiple period inputs. These inputs provide services over a series of production periods, and could involve organizing the farm around entirely new lines of production or adoption of entirely different production technologies. Given that profit maximization is the principal criterion of choice from among these alternatives, there is no theoretical basis for restricting consideration of allocation alternatives to some subset of this full range of possibilities. That is, at any point in time, there is no reason why action cannot be taken to alter any aspect of the farm organization should it be profitable to do so and provided that means for accomplishing the change are available (6).

Although it is true that reorganizations requiring construction and equipping of a new set of buildings will take somewhat more time to accomplish than a reorganization such as changing the composition of a ration, this is not the type of time consideration that is of main concern in a normative analysis of optimum farm organization from the standpoint of profit maximization. The purpose of the analysis is simply to determine from the array of alternatives what changes, if any, should be made in the organization of the farm at a particular point in time to maximize the profits accruing to that farm's managerial unit over subsequent production periods. Whether or not a particular alternative would be included in the optimum organization of the farm would depend upon (1) the current asset structure and organization of the farm, (2) the current investment required to adopt the alternative, and (3) the expected future pattern of price relationships.

In constructing the programming model for this type of problem, the list of activities to be included would theoretically cover all alternatives germane to the agriculture of the particular farm's locale. The list would not only include the various products that might be produced in the area, but also the various technologies by which they might be produced.[1] Furthermore, the techniques of production considered would include those currently being used in the area as well as those known and commercially feasible but not as yet generally adopted. Obviously, such an all-inclusive analysis would be extremely difficult to carry out. Not only would the size of the programming model be near prohibitive but the available data are not likely to be sufficiently reliable to justify attempting to distinguish between alternatives in such extreme detail. Therefore, the exact list of activities making up the model will necessarily be arrived at by arbitrary decisions based on the analyst's judgment and knowledge with respect to the problem. However, the above reasoning with respect to the scope of the model is nonetheless valid and provides the conceptual basis for designating appropriate activities for the type of problem outlined.

The relevant restraints to be incorporated into the model as well as the activities will depend partly upon the amount of detail that can be "reasonably justified" in considering the organization of a farm.

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[1] When a set of alternatives or activities pertains to production of the same product, those having a greater requirement for all inputs per unit of output need not be included in the model. Such alternatives would be technically inferior and would never be included in an optimum solution.

Again, this is a matter of judgment. Conceptually, at least, one could go to the point of breaking each possible farm enterprise into tasks such as seedbed preparation, row-crop cultivation, grain harvesting, corn harvesting, etc., and designating every different way of performing a task as a separate activity. By the same token, the capacity of each distinct type of machine or facility could be designated as a separate restriction or equation in the model. The feasibility of this depends partly upon whether or not meaningful differences of this magnitude are reflected by the data and partly by the resources available to carry out the analysis.

The relevant restrictions for a programming model to be applied to the analysis of the overall organization of a farm might be classified into three broad groups: (1) The resource restrictions; (2) the institutional restrictions; and (3) the technical restrictions.

Resource Restrictions

The resource restrictions refer to the physical resources of the farm, such as land, labor, capital, equipment, and facilities. A separate equation would be included in the model for each distinct resource. The general rule of thumb applies here that resources which are perfect or near perfect substitutes in all uses can be treated as the same resource and be combined into a single equation in terms of their least common denominator. For example, all feed grains could be combined into corn equivalents on the basis of TDN. Resources that are imperfect substitutes or perfect compliments would have to be dealt with in separate equations. Resources such as labor provide a continuous flow of services over a production period and are used in varying amounts by a number of different activities at different times within the period. They are most appropriately handled by designating separate equations relating to the use of the resource in the different subparts of the production period. The initial level of each of the resource restrictions, the b_i, would be the current stock of each resource that is on the farm. Except for the capital equation, the construction and use of these equations in the model would follow the usual lines of the many applications made of linear programming to the analysis of the resource allocation problems of farms (4).

As the problem posed concerns not only how the present resources of the farm should be used but also whether or not the level and form of the assets should be altered, the level of the restraint for each equation would not be regarded as absolutely fixed at their initial level. The model would be constructed to allow for either the purchase or sale of any resource, depending upon whether or not it would be profitable to do so. This is accomplished by including in the model a purchase activity and a sale activity for each separate resource (8). Whether or not a resource such as a farm building has any sale value is a question of empirical fact. If its sale value is zero, that is, it has no value other than in direct use on the farm, its selling activity would become the usual slack or disposal activity with $C_j = 0$ that appears in the programming model when the initial restriction is stated as an inequality.

The capital equation would express the amount of capital required per unit of each of the activities, just as in the case of any of the other resource equations. However, as the analysis deals with new capital investments in multiple period resources as well as the possible disinvestment in existing assets, some unique problems arise. The initial level of the capital restriction, b_i, would represent the amount of liquid assets currently available to the farm exclusive of credit. The model would include a capital sale activity that would account for the opportunity cost of capital in nonfarm uses. Purchase of capital would involve the use of available sources of credit expressed in the model as capital borrowing activities. To the extent to which different sources of credit are available at different interest rates and/or involving different repayment schedules, separate activities can be defined to take this into account. The capital equation thus deals with the allocation of both the liquid assets of the farm and any capital that the farm may find profitable to acquire from credit sources. The coefficients of the capital equation would express the total capital requirement per unit of an activity for both single or multiple period inputs. The capital coefficients thus reflect the full cost at the current point of time of introducing an activity into the farm organization and operating it over one production period.

As all resources are regarded as potentially variable in this formulation of the problem, the availability of capital and credit constitutes the principal resource limitation to the organization of the farm. The availability of credit is based largely upon the entrepreneurs equity in his assets. The slope of all factor supply functions to the farm is generally regarded as being zero. In instances in which this assumption does not hold, an upward sloping factor supply function can be approximated by a step function incorporated into the model (5). A separate activity and a separate equation would be required for each step in the function. As in the case illustrated in Figure 8.1, there would be three activities, X_1, X_2, and X_3, for which the corresponding Factor prices would be C_1, C_2, and C_3. There also would be three equations stating the range in factor purchases over which each price applies and for which the corresponding restrictions would be b_1, b_2, and b_3. This construct probably would be most pertinent in the case of credit when there are a number of possible sources of credit which differ in terms of the interest charge, repayment rate, and quantity available.

Institutional Restrictions

The institutional restrictions would reflect those that bear upon the

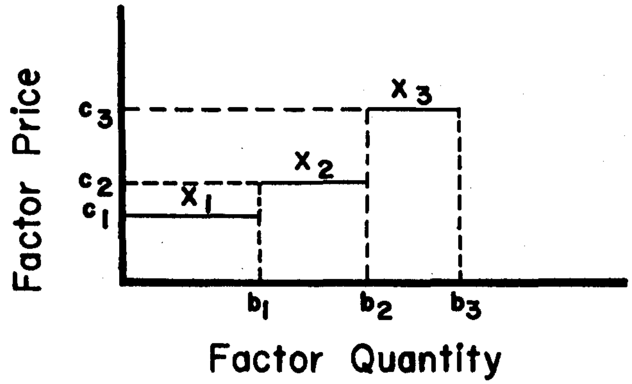

Figure 8.1. Step factor supply function.

organization of the farm and arise from the institutional setting within which the farm must operate. The most obvious of these would be crop acreage limitations or marketing quotas imposed by government programs. Depending upon how they are established, these limitations may or may not be altered by increasing or decreasing the land area in the farm. Contractual obligations would be another type of restraint in this group. Limits on the availability of credit from various sources and of various types might also be regarded as among the institutional restraints.

Technical Restrictions

The technical limits within which the farm must be organized are expressed here as the technical restrictions. For example, it is not advisable, in certain areas at least, to plant field beans on the same field two years in a row because of disease problems. Therefore, in areas in which field beans are a cropping alternative, one technical restraint would be that the acreage of field beans in any one year could not exceed 50 per cent of the total cropland acreage of the farm.[2] The basis for specifying such limits may be the soil characteristics, topography, or disease problems of the particular locale. The number and nature of this type of restraint required in any particular programming model will depend upon the technical conditions pertaining to the particular farm programmed.

Where activities may be combined in vertical sequence in the production process and more than one possible combination exists, a

[2]This type of limitation may also be taken into account by specifying crop activities in terms of rotations. Rotation activities involving field beans in two successive years would not be included or would reflect the low yields on the second year of field beans making it a relatively unprofitable alternative.

series of technical restraints are required to express the nature of the relationship between the activities. This is the case when the output of one activity may become an input for one or more other activities. A common example of this is when feed crops produced on the farm may be either sold or used as inputs for livestock activities. Here, a separate equation would be required for each distinct feed category. The output of the feed from the crop activities would be expressed as a negative input in the appropriate equation. The use of the feed by other activities would be handled in the same equations in the same manner as any other input requirement. The same kind of situation arises when enterprise alternatives are separated into tasks and activities are defined for each alternative way in which a particular task may be performed. In this case, the output of the activity is not a product in the usual sense but a condition that is a necessary prerequisite to the performance of other tasks occurring later in the production process. However, the way in which it is handled, in terms of specifying equations in the model, is exactly the same as in the case of intermediate products, where inventories may exist in which the initial level of the restraint would be the inventory of the product currently on hand.

CONSTRUCTION OF THE PROFIT EQUATION

The construction of the profit equation for this type of programming model presents some problems as the model takes into account possible changes in the quantity of both single- and multiple-period resources used on the farm. If all multiple-period resources were regarded as fixed at their current levels and only the purchase of single-period resources were considered in the model, construction of the profit equation would be relatively simple. The profit equation would include expenditures on all single-period resources used during the production period and all revenue from the sale of products during the same production period. The costs associated with the multiple-period resources that have been assumed to be fixed in quantity, for example taxes, repairs, etc., would not be included in the profit equation as they would not be affected by the pattern of organization that may be adopted.[3]

As the production processes involve some lapse of time between the initiation of production and the output of the product, the maximization of profits will be made with reference to some specific span of time. The choice of time span is an arbitrary choice, although the calendar year is customarily used as the accounting period. Any other span of time could just as well be used. However, the calendar year does conform reasonably well to the cycle at which operations are

[3] For convenience these fixed costs may be entered into the profit equation in a lump as a negative constant so that profits (gross returns less fixed and variable costs) of the optimum organization would be obtained directly in solving the model.

repeated in agricultural production processes. Using the calendar year as the accounting period, all expenditures, revenues, and resource requirements would be stated in terms of this 12-month interval and profits would be maximized in terms of that interval.

When the purchase or sale of resources having different lengths of useful life are considered in the analysis, construction of the profit equation becomes somewhat more complicated. When the useful life of a resource extends over several accounting or production periods, charging its full purchase price or crediting its full sale value to a single accounting period in most instances would mean that profits would be maximized by selling all assets of the farm. If the concern is only with maximizing the profits obtained in the current production period, the sale of all the farm assets probably would be the optimum course of action when there are assets that can be sold. This raises the question as to what constitutes the appropriate length of planning period. Is it a single production period or a series of production periods into the future? To some extent, this question goes beyond the bounds of a normative analysis of farm organization in terms of profit maximization and gets into consideration of individual values and preferences, as well as other characteristics associated with the individual such as age, family status, etc. As it is conceived here, the normative analysis of farm organization completely abstracts from these aspects of the problem related to the human factor. The analysis is concerned specifically with the choice of the most profitable set of alternatives as of the present point in time in relation to the technological and institutional circumstances surrounding the individual farm. Under these conditions, the appropriate approach is to seek to maximize profits of the immediate production period because the results that can be achieved in subsequent production periods will depend partly upon how effectively the farm is organized in the preceding production periods.[4] Therefore, the model still would be cast in terms of the single production period or calendar year, even when considering the purchase and/or sale of resources of widely different lengths of useful life. However, the price of the multiple-period resources must be pro rated over the series of production periods in which their services are available.

The cost of a multiple-period resource that is borne in a single production period would be related to the proportion of all the stock of services provided by the resource that are utilized in the single production period. An estimate of this cost may be obtained by deducting the estimated scrap value of the resource at the end of its productive life from the total initial investment and dividing the remaining value

[4]This presumes that exploitive practices which would impair the productivity of the resources in future production periods are not to be considered. Under some circumstances, such practices may be justified but this aspect of the problem has not been taken into account here. See Heady (3).

by the estimated number of years of useful life.[5] The sale of a multiple-period resource would be treated similarly. Using these "annualized" prices for multiple-period resources, the profit equation would be stated as before with one exception. Any additional costs incurred as a result of resource ownership (for example, property taxes) would also need to be included in the coefficient of the profit equation. These costs would be added to the purchase price of the multiple period resource. They would also be added to the sale price of a resource.

The basic assumption implicit in this statement of the profit equation is that the prices of all inputs and all product prices hold over all production periods extending into the future. Under this assumption, the optimum choice of the alternatives would apply not only to the current production period but also to each subsequent production period, provided the profits are not reinvested in the farm, growth resulting from capital accumulation. Derivation of a normative supply function from this model involves considering different assumptions as to the price of a particular product over this series of production periods relative to factor prices and the prices of other products. This supply function describes optimum adjustments in resource allocation and farm organization to price relationships at a particular point in time, the present, under the assumption that the farm's profits are to be maximized. Further, the nature of this supply function is related directly to the present asset structure and organization of the farm. If the purhcase and selling prices of all resources are equal, that is, there are no price discontinuities, this normative supply function would be perfectly reversible and would apply equally well to all production periods extending into the future. With an increase in product price, the farm would move up this normative supply function. With a decline in price, the reverse movement would occur along the same function.

In the presence of price discontinuities, the supply function no longer would be reversible and if there has been any change in the organization and asset structure of the farm in the meantime, there would be a unique normative supply function for each subsequent production period (6). Having made an adjustment in resource use to one set of price relationships, the conditions under which adjustments can be made to changes in that set of price relationships will have been altered. When the price obtained from the sale of an asset is less than its original cost by more than the value of the services used, a reverse movement in a price of the same magnitude that made one course of action profitable would not return the farm to the same position it held prior to any price change. Once having made an investment, it may be profitable to continue using it in the face of substantial price declines because of the loss that would be sustained by disinvesting or by failing

[5] This procedure is an approximation of the appropriate distribution of the costs of the multiple-period resources. The duration of the expected price, rate of obsolescence and rate of use, as well as other factors will affect the period of time over which this cost should be pro rated.

to use the asset.[6] It should be clearly understood that the normative supply function describes optimum adjustments to alternative product price levels as of a particular point in time and does not relate to adjustments made over time. Over time, the nature of the normative supply function of the farm changes because of changes made in the farm organization, as well as because of technological developments.

At any one point in time, there is a single normative supply function that describes the optimum adjustment of a farm organization to different levels of a product price. The price level of the product has implications with respect to the use of both single-period and multiple-period resources. However, whether or not changes in multiple-period resources will be profitable will depend as much upon the length of time the price remains at a particular level as upon the price level per se. Large investments in very durable resources may be profitable with small changes in price relationships of long duration. On the other hand, only changes in the use of single period resources may be profitable if the price change is of short duration, regardless of whether it is of large or small magnitude.

The foregoing model, cast in terms of maximizing farm profits of one production period, does not handle the question of the effect of changes in price level of different duration. To handle this type of question using linear programming, it is necessary to construct a different type of model. The principal change in the model would be that a duplicate set of restraints and activities would be defined for each of a series of production periods. Each set would refer to the use of resources and the production of product in a particular production period. The profit equation would be constructed as before, except that it would take into account the profits resulting in each of the time periods and profits would be maximized for the total series of production periods included in the model. The duration of particular price is taken into account by specifying the product price separately in each production period. This type of programming model is sometimes referred to in the literature as a "dynamic" programming model (7). The supply function derived from this model would be a three dimensional relationship, in which alternative magnitudes and durations of price changes would be considered. The usual concept of a supply function considers only the magnitude of a price change. Basically, its interpretation would still be the same as for the normative supply function derived from the first programming model that was outlined. It would describe the optimum adjustment that a farmer would make at specified points in time in the use of resources under various price conditions differing as to level and duration.

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[6] The same reasoning would apply to declines in the market value of an asset held by a farm apart from the question of price discontinuities.

THE AGGREGATE

Up to this point, the discussion has been in terms of using programming procedures for deriving a normative supply function for a single farm independently of any consideration of its relationship to other farms with which it competes. As agricultural production takes place under conditions approaching perfect competition, the actions of one farm alone will have little impact on any other farm or on the group of farms as a whole. However, all farms in the group face similar problems of farm organization and resource allocation and their collective action will have a considerable impact upon the conditions facing the group as a whole. From the standpoint of maximization of farm profits, it is necessary to obtain some estimate of the effect of the collective actions of the group of farms on product price. If this can be obtained, the optimum organization of a particular farm under a particular set of circumstances can be more closely specified. This requires that the aggregate supply function be related to the demand function for the product.

The aggregate supply of a product at any given price would be the sum of the outputs of the individual farms at that price. Similarly, the aggregate normative supply function would be the sum of the individual farm normative supply functions. An estimate of the aggregate normative supply function for a product from an area or region may be obtained by first deriving the individual farm normative supply functions of a group of representative farm situations from that area or region. Then, by attaching appropriate weights to each of the individual functions, an estimate of the aggregate function can be made. The weights attached to each individual function should reflect the relative importance of the individual farm in the population of farms to which the aggregate supply function is to apply. In this approach to the estimation of an aggregate normative supply function, the designation of the representative farm situations becomes of crucial importance because these situations form the description of the base from which any change in organization is made. To a large extent, this base determines the nature of the normative supply function. The weights and the description of the farms would be obtained from a sample survey. The farms to be included in the analysis should include not only those currently producing the product for which the function is to be derived but other farms operating in the area as well. With a rise in price of the product, it may be profitable for some farmers who are not currently producing the product to enter into its production. Even without a price increase, it may be that production of the product is a profitable alternative for more farmers than are currently producing it.

Since the aggregate supply function is a weighted sum of the representative farm supply functions, the conceptual problems of constructing and applying the programming model are no different. The programming still is done with respect to individual farms or representative farms. However, in deriving the aggregate supply, the concern is primarily with structural changes that have occurred or are expected to occur in the industry. These types of changes tend to be permanent, as do their effect on prices. To the extent that this is true, the static programming model first described rather than the time-dated or dynamic model would be adequate for deriving the individual farm supply functions that are to be aggregated. That is, the assumption that the pattern of prices over future production periods is constant rather than fluctuating or of some other cyclical nature does no great violence to the validity of the analysis.

When programming an individual farm, the supply of inputs as compared with the farm's demand for these inputs is of relatively minor importance because the individual farm demand represents such a small part of the total demand. In dealing with aggregate relationships, this is no longer true. This becomes particularly important with respect to the land input that is fixed locationally. A static programming model which considers all inputs as variable so long as their marginal value product lies above their purchase price or below their sale price could give the result that it is profitable for all farmers to buy some amount of land. If this occurred, the aggregate supply function would overestimate supply because it would be based upon the condition that all farmers have purchased land when in reality there is none available to be bought. To avoid these problems of aggregate inconsistencies with respect to resources where this type of situation does exist, it may be necessary to fix the supply of the resource that is available to the representative farm at its current level. To do this, however, is in contradiction to the basic objective of determining the optimum allocation of resources and farm organization.

Theoretically, the optimum allocation of resources among farms is attained when the marginal value product of all resources are the same for all farms using the same resource. To fix the quantity of the resource at its initial level is not likely to allow this condition to be achieved. However, even under these more restrictive assumptions with regard to the extent to which the quantity of certain resources employed can be changed, estimates of the marginal value product are obtained which do allow for some evaluation of the ability of different farms to compete for the same resource. The farms having the higher marginal value products would be capable of outbidding those having the lower marginal value products. The shortcoming of this approach is that no information is obtained concerning how the marginal value product is affected as additional units of the resource are obtained nor how the organization and product output of the farm will be altered as a greater or lesser quantity of the resource is utilized.

The difficulty at the aggregate level arises from the competition among farms for a given supply of a resource. In the case of the land resource, this competition is for a supply that is essentially fixed in physical quantity in the aggregate but not necessarily fixed for the individual farms using the land. Theoretically, from a purely normative standpoint, the individual farmers would exchange land among themselves, bidding up the market price to the point at which the price of land and the marginal value product of land for each farm are equal. To determine this point, it would be necessary to derive the individual farm's demand curve for the resource, in this instance, land. This could be done with the programming model by varying the price of land and finding the optimum solution to the model at each price. Having derived the individual farm normative demand curves for land, an aggregate demand curve could be obtained by aggregating the individual farm demand curves. The equilibrium price would occur at the point where the supply curve for the resource intersects the demand curve. The demand curves for the resource would be affected by the price level of the product as this is one of the determining factors of the marginal value productivity of the resource. Hence, for each product price there would be a different resource demand curve. It would appear that an iterative process would be involved in actually determining what the equilibrium resource price and pattern of allocation would be.

SUMMARY

In general, the use of linear programming for deriving estimates of individual farm normative supply functions has the advantage of permitting examination of resource use alternatives in considerable detail. At the farm level, specific techniques of production, the use of specific resources, and the production of specific products can be taken into account quite readily, depending upon the amount of detail one wishes to build into the model. The principal benefit derived from this is that it allows movement from the micro- to the macro-level of analysis without loss due to gross aggregation of inputs and outputs. The method has the disadvantage of being somewhat cumbersome to handle when dealing with relationships that are curvilinear rather than linear. Curvilinear relationships can only be approximated by linear segments requiring the specification of a large number of activities and, in some instances, additional equations as well. One example of the difficulties encountered as a result of assuming linear input-output relationships arise with respect to labor. In using linear programming, it is necessary to assume that the resource requirement per unit of output remains constant at all output levels for any particular method of production. In many types of agricultural enterprises, there is a minimum overhead labor requirement which does not increase as output is increased. Thus, the per unit labor requirement declines with increasing output. Economies of scale cannot be handled well in the programming model and can lead to biases in the results.

The assumption of perfectly divisible factors and products present few problems except where investments are considered as with the use of programming outlined here. Investments in such things as buildings and machinery occur in large lumps. When transitions from one type of facility to another are considered in a single model, results are often obtained which would imply the partial use of two different types of production techniques for producing the same product. One possible example would be a solution that shows part of a milking herd being handled under a parlor system and the rest being handled under a stanchion system. Problems such as this can be handled by introducing one system in each of two different models, then comparing the results obtained from the separate models.

In deriving aggregate relationships by aggregating individual farm relationships, a compromise must often be struck between the detail and adequacy with which the microanalysis is carried out as compared with the macro-level analysis. As the detail with which representative farm situations are differentiated is increased, the number of individual farm supply functions that must be programmed is increased also. If major emphasis is upon the aggregate relationships, somewhat less precision may possibly be tolerated with respect to these micro-level problems. However, if major emphasis is upon the micro-level or farm management analysis, they become of much greater concern. These are questions that depend primarily upon the definition of the problem and the objectives of the research.

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Discussion

THERE ARE two distinct problem areas involved in deriving an aggregate supply function based on individual farm supply functions. The first of these concerns appropriate procedures for determining the individual farm supply functions. The second problem area concerns the appropriate means of aggregation.

McKee and Loftsgard specifically limit their discussion to the first of these problem areas. Their discussion is further restricted to the use of linear programming for quantifying the individual farm supply function. The matter of supply function aggregation does come into play, but in a somewhat different context than posed here. The authors do not consider alternative procedures for aggregating individual supply functions, but simply state that "the aggregate supply function is here regarded as being obtained by the horizontal summation of the supply functions of the individual farms." Likewise, they make no attempt to either specify the applications for which such an aggregate function would be used or to evaluate the overall usefulness of the application. Rather, the authors in essence consider the matter of aggregation through a "two-stage question procedure." First they ask the question: "What should be considered in developing an appropriate programming model which is designed to yield a normative supply function for an individual farm?" This is followed by the question: "How should this programming model be altered if you know in advance that the resulting farm supply functions for representative farms are to be aggregated?"

The major part of the McKee-Loftsgard paper is devoted to the first question, and two general programming models are outlined. Both would use the variable price programming technique to specify the optimum profit output levels of a given commodity under varying prices. Although the matter is not completely clear, I gather that both would use the same planning period — a period which is short enough to allow no change in technology or institutional circumstances, but long enough to allow for the purchase of multiple-period inputs. In this connection, I wonder if the authors are serious about their suggestion to allow the purchase of all resources, limited only by capital and borrowing capacity. With only one <u>effective</u> limiting resource (borrowing capacity), the programmed supply function rests upon an extremely narrow base.

The difference in the two models presented involves the length of time over which farm profits are to be maximized. The first model (termed the static model) would maximize profits for a single production period. It assumes that product and factor prices will hold constant long enough to justify the purchase of multiple-period inputs. The second model (termed the time-dated model) would maximize farm profits for each of a sequence of production periods. The latter model avoids the assumption of constant prices over time, for here both price levels and durations would be allowed to vary.

In answer to their second question, the authors rule in favor of the static model when the objective is aggregation of individual farm supply functions. They point out the possibility of a discrepancy between aggregate factor use as prescribed and aggregate factor supply, with land as a specific example.

Neither of the two models outlined are spelled out in detail. Instead, the authors describe in a general way the considerations involved in the formulation of the basic programming matrix. These "considerations" include the choice of process alternatives, the choice of restraints, and the formulation of the profit equation. But the specific nature of the appropriate programming model (as the title leads one to anticipate) is said to be largely a matter of arbitrary judgment. To be sure, the discussion of "considerations" is useful in itself. The authors are to be commended for laying out many of the considerations involved, even though they may be a bit frightening. However, research analysts find little solace in arbitrary judgment as a guideline. If we are not to be told what specific programming model is appropriate, we would at least like to know the criteria for determining propriety. This the authors have accomplished only in part.

At this point I would temper my criticism by indicating my general agreement with the authors' conclusion that "if the major emphasis is upon the aggregate supply relationship, possibly somewhat less precision can be tolerated with respect to the micro-level problems."

My remaining comments relate to the matter of supply function aggregation _per se_. Essentially, I question the usefulness of aggregating normative individual farm supply functions. In so doing, of course, I also question the need for formulating programming procedures designed to yield individual farm supply functions _for aggregative purposes_. My doubts stem from several sources.

First, the prime reason for aggregation is to take into account the effect of collective action upon the optimum resource allocation of the individual farm. But this collective action is not the cumulative result of strict adherence to the profit maximizing norm. To make such an assessment it is more important to know what all farmers _would_ do rather than what they _should_ do under various price levels.

Second, it is illogical to attempt relating aggregate demand and supply for a single commodity back to the individual farm when it is assumed implicitly that prices of all alternative products remain constant.

Finally, I suspect the possibility of a real dilemma as regards the process of aggregation. Let me use a highly oversimplified illustration. Suppose we are interested in deriving an aggregate supply function for milk in Wisconsin and Michigan. Farm A is representative of all farms in Wisconsin while Farm B is representative of all farms in Michigan. We have programmed the optimum profit level of milk production under various milk prices for each farm. These output levels are based on the price for milk at Farm A and at Farm B. Now can we expand each by its appropriate weight and add the supply schedules horizontally? We cannot, for this assumes that Michigan and Wisconsin milk prices are equal. That is, the same milk price must hold in both states at a given point in time to make horizontal summation of the weighted supply schedules valid. Likewise, we cannot attach some historical price differential which is based at least in part upon past area production patterns (as opposed to transportation differentials from some major market), for the production patterns are now variable. Thus there appears an element of circularity; supply depends upon price, but price depends upon supply. This is just one of the aggregation problems to be hurdled.

In view of the above, it may be well to stop and ask how and why normative individual farm supply functions would be aggregated before becoming concerned about programming procedures which are appropriate for the aggregative objective.

Chapter 9

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Budgeting and Engineering Analyses of Normative Supply Functions

THE ASSIGNED TITLE, "Budgeting and engineering analyses of normative supply functions," probably implies the synthesis of supply response estimates from basic input-output data. Use of the phrase "budgeting and engineering" vaguely restricts this paper to studies which build up supply response estimates from micro data instead of estimating them directly from macro-price and output data. This limits the discussion to such works as that of Mighell and Black on interregional competition in milk production or that of Schuh on the influence of cost of production on supply responses for milk in the Detroit milk shed (14, 17). The author is restrained from discussing the type of study represented by his own work on burley tobacco, by Hathaway's study on dry edible beans, or Nerlove's on the use of distributed lags to derive supply estimates for corn, wheat, and cotton (17, 5, 15).

The term "normative," which appears in the title, has unfortunately tended to become an opprobrious epithet reserved in certain circles for inaccurate supply estimates while accurate estimates are labelled "predictive" or "positive."[1] This unfortunate distinction arises from the desire of positivists to avoid purpose or ends as being animistic, teleological and, hence, non-scientific (in their opinion). The use of this distinction implies that the behavior of producers can be accurately predicted without reference to desire for profit, liquidity preference, desires for security as reflected in risk discounts, and the desires for income as reflected in willingness to make long chance investments which condition the behavior of producers. The author feels that appropriate handling of subjective matters involving purpose and ends will produce more accurate (in the positivistic sense) supply response estimates than attempts to eliminate consideration of such matters. Obviously, studies which assume entrepreneurs to maximize what they do not, in fact, try to maximize may produce at least as inaccurate estimates as studies which avoid all maximization. Human behavior (and production decisions are a form of human behavior) is often a compromise between the entrepreneurs concepts about "what

[1] "Normative" is an adjective relating a subject to norms. Restricting normative to mean optimizing profits may destroy a respectable adjective in our vocabulary.

ought to be" (values or norms) and concepts about "what is or can be" (beliefs - facts or predicted facts). It seems obvious that more accurate predictions of facts about supply decisions and responses must, generally speaking, be obtained in studies which take both values and beliefs into account than by non-normative studies. In addition, of course, errors in the process by which "right actions" are determined from value and belief concepts would have to be considered in order to arrive at still more accurate predictions. The point is that the behavior of producers is in part a social phenomenon, "a serious analysis" of which, in Knight's words, requires "a quite complicated pluralism" including but not limited to positivism (12, 13).

A principal problem encountered in synthesizing macro supply estimates from micro data has to do with predicting which inputs or resources are changed and which are not changed. In what follows, it will be taken as self-evident that a supply estimate will have to reflect changes in the inputs which determine output. Arbitrary assumptions about resource fixity do not permit prediction of changes in output resulting from changes in resources arbitrarily assumed fixed. Changes in inputs to be considered include, of course, those necessary in introducing new technologies and in securing the benefits of regional, sector, and farm by farm specialization and diversification. It goes without saying that the problem of predicting when resource flows will and will not occur is a common problem for budgeting, continuous function, simultaneous equation, programming, and Leontief-type studies.

THE GENERAL PROCEDURE

Fundamentally, there are seven more or less related steps in producing a supply estimate by the method under discussion here. While some of these steps may be omitted in a particular study because they have been done previously or are unimportant, they must all be considered. The seven steps are:[2]

1. Securing an appropriate set of input-output coefficients.
2. Devising a method of determining which resource flows can and cannot be varied.
3. Selecting a range over which variation in product price will be considered.
4. Computing optimum outputs (in terms of a selected set of norms) as a function (discrete or continuous) of product price.
5. Repeating steps 1 to 4 for different situations within the industry.
6. Aggregating results from steps 1 to 5 into an estimate of how output for the industry depends on price.

[2] See Schuh (17) for an illustration of how each of these problems can be handled in a budgeting study. Schuh's treatment of step 2 is inadequate for anything but very short lengths of run.

7. Adjusting the results obtained in 6 for their shortcomings as partial equilibrium estimates, i.e. for the influence which expanded use of an input may have on its price and, hence, on marginal costs and on the ability of the industry to expand production.

The three main kinds of data required have to do with (1) input-output relationships, (2) prices, and (3) aggregation. Controlled experiments, surveys, farm accounts, and time and motion studies are common sources of input-output data. Mighell and Black used farm survey and account data to good advantage in their work while Schuh drew heavily on input-output data produced by the controlled experiments reported in USDA Technical Bulletin 815, "Farm surveys and time and motion studies." Time and motion data are of particular value because their "building block" nature permits easy synthesis. In predicting supply responses through time, input-output data of a forward looking nature with respect to new technology must be used if accurate estimates are to be secured. Price data are difficult as input prices may become functions of quantities used. For the most part, various USDA secondary sources and surveys are useful but often inadequate sources of market prices. The law of comparative advantage and the principle of opportunity costs must be utilized in pricing committed resources between the limits imposed by acquisition costs and salvage values which are sometimes market values and sometimes internal (to the industry or firm) costs and values. Programming employs the law of comparative advantage and the principle of opportunity costs to price fixed assets. Data for use in aggregation can often be secured from surveys and the census. Though the author has never used them, the USDA typical family farm studies must have valuable unpublished as well as published data to contribute to supply response studies.

Much time could be wasted discussing the best method of carrying out step 4 — the location of optima — and loyalties to budgeting, programming, and continuous function analysis would probably interfere with the objectivity of such a discussion. So would the unfortunate distinction between normative and positivistic or predictive work. The important points to consider in selecting an appropriate method appear to be (1) the avoidance of arbitrary restrictions on input variability, (2) the maintenance of scope for originality and flexibility in the computations and in conceiving of the patterns of production which will be followed in the future, and (3) the maximization of appropriate or realistic norms.

While substantial problems are involved in executing each of the seven steps,[3] none seems more neglected or more important than the one of avoiding arbitrary restrictions on input variability by securing endogenous determination of when resources flow into and out of the enterprises producing the product under consideration. Thus, the

[3] See Mighell and Black (14) and Schuh (17) for examples of the difficulties and for practical help in overcoming the difficulties encountered in carrying out these steps.

remainder of this paper will concentrate on this problem and its many facets. While the organization to be followed and a small part of the content to be presented is new, most of the content will be a repetition of material presented elsewhere (8, 9, 10, 11).

SECURING ENDOGENOUS DETERMINATION OF RESOURCE FLOWS

Inputs used in producing farm products typically have acquisition costs which exceed their net salvage values. This difference between acquisition costs and salvage value arises, in part, because of (1) the geographical dispersion of producing units from each other and from supply centers, (2) institutional costs involved in transferring ownership, and (3) subjective premium and discounts attached to ownership of certain inputs and to the production of certain products. When the value of a marginal unit of an input useful in the production of a particular product exceeds its acquisition cost, it pays to acquire that unit for use in producing that product. When the value of a marginal unit of an input useful in the production of a particular product is less than its salvage value, it pays to dispose of it or uncommit it. When the value of a marginal unit of an input committed to the production of a particular product is less than its acquisition cost but in excess of its salvage value, it does not pay to change the resource committed insofar as this input is concerned. The problem of concern today is the problem of working this definition of resource commitment into the theory of the firm, costs and supply responses. The main subproblems are now fairly clear; some of them are solved, some are being worked on while others await our efforts. The main subproblems involve:

1. The conversion of stocks to flows — this involves capitalization, maintenance, obsolescence, depreciation, and user costs.
2. Subjective premiums and discounts for acquisition costs, salvage values, and marginal value products or capital values of inputs.
3. Discrete inputs.
4. Optimum rates of flow are different for a fixed discrete input than for the same input when variable.
5. The influence of credit availability on acquisition costs and salvage values for durables.
6. The role played by erroneous expectations with respect to product prices, input prices, technology, institutional arrangements, and the human factor in inducing overcommitment[4] of resources to the production of farm products.
7. The role played by capital gains due to inflation and increased demand, war, and population growth as sources of:

[4] Undercommitment of resources was the point of emphasis in earlier work on capital rationing. Overproduction in terms of producing market rates of return to labor and capital is, however, the outstanding characteristic of American agriculture to be "explained."

a. credit
b. errors in expectations (i.e., such gains are easily confused with marginal value productivities).

8. The role played by capital losses.

Much work has been done on the above subproblems. The references (8, 9, 10, 11) (a) relate the basic theory to classical, neoclassical, and modern literature; (b) show how the theory explains the overcommitment of resources to agricultural production, shifts from one cost structure to another, and irreversible and discontinuous supply responses; (c) show the origin of substantial capital losses; and (d) explain the roles played by advancing nonfarm wage rates, macro adjustments and technological advance in the development of erroneous expectations. Edwards (3), in a rather carefully developed mathematical thesis, has developed theoretical solutions to some aspects of the stock-flow problem and has related credit supply functions for individual firms to acquisition costs and salvage values. Hildebrand and Dvorak (2, 6) have used Edward's theoretical results in programming and have developed an ad hoc but not particularly original method of handling discrete durable inputs; they have not handled the problem of varying optimum flows from committed discrete durables, even on an ad hoc basis.

Marginal cost responses have been developed graphically for the one and two variable input cases and algebraically for the N variable case (3), but have not been aggregated into commodity supply response estimates. However, the success of both micro-synthetic and direct estimates from macro data suggests that the aggregation problem may not have to be entirely solved before effective work can be done.

Some progress has been made on classifying inputs into categories relevant for the type of analysis suggested above. These classifications were reported in the author's "facts and notions" article (11). In the same article, 72 hypotheses about resource flows were tested; despite a regrettable mistake in which undeflated data were reported in terms of 1910-14 dollars, these hypotheses are substantiated and offer much hope for micro-synthetic studies incorporating these hypotheses. Hathaway's data on resource flows and capital gains and losses (4) offer similar encouragement.

The success of Mighell's and Black's work (which handled several of the above problems on an ad hoc basis) offers further reason for hope. Bird, an ARS contract employee at MSU, seems to be making some progress on subjective premiums and discounts, user costs, obsolescence, and expectations.

One of the biggest deterrents to progress is the lack of data (1) on resource flows (farm-nonfarm, among farms in different regions, among farms in a given region, and among enterprises on given farms) and (2) on credit opportunities as influenced by net worth. Compilation of such data for one or two minor commodities should permit completion of one or two supply response studies incorporating the theory suggested above.

Despite the work of Interstate Managerial Study cooperators and Bird's efforts referred to above, the formulation of expectations is poorly understood. Nerlove's distributed lags appear helpful but still inadequate.

More work along the lines carried out in the Interstate Managerial Survey (1, 16) seems to be required before we evolve more adequate theories on the role played by price, technological, institutional, and human factor expectations in the determination of resource flows and supply responses. Unfortunately, much past work has concentrated on risk and uncertainty as a source of capital rationing which restricts output rather than as a source of overcommitment of resources and surplus production (8).

TWO VERY GENERAL CONCLUSIONS

1. Syntheses of macro supply response estimates from micro data have been moderately successful in the past, despite serious difficulties in carrying out the seven steps involved in making such estimates; there is much hope that these studies can be improved by overcoming these difficulties. One of the difficulties involves the problem of obtaining endogenous, as opposed to arbitrary, determination of resource flows.

2. Slow, rather painful progress is being made and will continue to be made on this problem.

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Chapter 10

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Surveys and Studies to Estimate Farmers' "Planned" or "Proposed" Supply Response*

SURVEYS AND STUDIES designed to estimate farmers' "planned" or "proposed" supply responses take us into areas where little is known. A farmer's planned supply response involves estimates of the future. Here, his knowledge is imperfect. Little is known about what these estimates of the future are, how they are constructed, and how they are integrated into a plan of action. We know that as the farmer formulates his plans, he is faced with imperfect knowledge about such variables as prices, technology, yields, institutions, and people. But we do not know how and to what extent these variables are considered in the farmer's planned supply response. Perhaps other variables (in an interfirm sense) such as subjective fixity of factors, age of operator, family composition, equity position, desire for leisure, and level of income, are the ones that determine his planned supply response. Moreover, as a farmer's planned supply response involves estimates of the future, these estimates are subject to error and error gives rise to differences between planned and realized supply response. Little is known as to the extent of this gap. A study of farmers' planned supply responses may uncover portions of supply response that are unplanned; that is, some of the changes in supply response may be due to random variation and little is known as to the extent of this variation.

Thus, in planning a study to estimate farmers' planned or proposed supply responses, we are unable to draw upon a well-developed body of theory. Perhaps, therefore, we had best initiate such studies on a limited scale with the primary purpose of gaining some information and insight as a basis for some meaningful hypotheses.

With this word of caution, the objective of this paper is to suggest how cross-sectional studies using survey techniques may provide estimates of supply response and/or supplemental information about a variable or variables used in other approaches to supply estimation, such as linear programming or time series. Although it is not our objective to evaluate critically or compare alternative techniques or procedures used in supply estimation, it is necessary to describe some of

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the problems associated with the use of linear programming and time series analyses. In our view, no one technique or procedure can provide us with all the knowledge we need about supply.

CONCEPTUAL FRAMEWORK USED AS A BASIS FOR DEVELOPING THIS PAPER

Before we can proceed toward our objective, it is essential to explain briefly the conceptual framework that serves as a basis for developing this paper. It is also necessary to point out general areas in which lack of data (and in some instances of theory) make this framework inadequate in the sense of possibly failing to provide reasonably accurate estimates of supply response.

As shown in Figure 10.1, changes in supply of a commodity can be viewed at both the firm and the aggregate level as consisting of two parts: (1) moving up and down a given supply function a_1, a_2, or a_3, in response to price, and (2) the growth or shift of supply (Δs_1 and ... Δs_2) through time accomplished by investment in plant or fixed resources and the adoption of those new technologies which over the relevant range in prices take on the form of investment in plant. Once adopted, they are often found to be profitable under a wide range in price and their use is not discontinued as prices decline. Thus, the growth or shift of supply over a wide range of prices is considered to be a one-way irreversible street, especially for those commodities whose production involves factors with acquisition costs differing significantly from salvage costs and with fixed costs making up a substantial part of total costs. It is recognized, however, that at some level of price, it is possible that the use of these technologies will be discontinued or the use of plant resources will be drastically reduced or used for other purposes, as shown by the dotted lines in Figure 10.1.

This elementary framework suggests the existence of pure price-quantity relationships. In the real world, we doubt that such pure relationships exist. We suppose that in actuality, supply response is related to a number of variables, among which price is one. This framework further suggests three closely related, though conceptually separable, areas in which critical information is lacking. First, while considerable farm management information is available as to what farmers "should" do to maximize profits in a timeless static sense, there is a paucity of information about the changes farmers plan and do make in time under conditions of uncertainty about prices and other variables. Second, we need more information about technological change, for example, the rate of adoption of existing technologies, the variables that influence adoptions, and the effect of such adoptions on factor combination and output levels. Third, we lack a theory of investment at the firm level. Worse yet, we need to identify the variables that significantly influence investment so they may serve as a

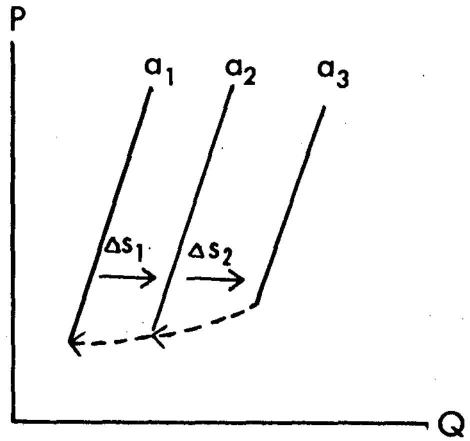

Figure 10.1. Changes in price-supply relations through time.

basis for constructing a theory of investment. Obtaining information on changes in asset structure through time by means of cross-sectional surveys may serve as a basis for formulating some meaningful constructs for building a theory of investment at the firm level.

SUPPLEMENTING LINEAR PROGRAMMING ESTIMATES OF SHORT-RUN SUPPLY

The authors are involved in the Lake States Dairy Adjustment Study.[1] Thus, our remarks are largely in terms of supply estimation for milk, although many of the implications are equally applicable to estimation for other commodities. Variable price linear programming techniques are being used in this study. They have been used in other studies to derive estimates of supply elasticity.[2] It is recognized, however, that the linear programming technique is inherently unsuited to the handling of lumpy variables, and the growth of supply referred to in Figure 10.1 often involves such "lumpy" variables as land, milking parlors, combines, tractors, etc.[3] Thus, we believe that linear programming as now known is essentially applicable only to a unique kind

[1] This study is being made by the Farm Economics Research Division, Agricultural Research Service, U. S. Department of Agriculture in cooperation with the Michigan, Wisconsin, Minnesota, Iowa and Illinois Agricultural Experiment Stations.

[2] See, for example, McPherson, W. W., and Faris, J. E., "Price mapping of optimum changes in enterprises," Jour. Farm Econ., 40:821-34, 1958; and Knudtson, A. C., and Cochrane, W. W., "A supply function for flax at the firm level," Jour. Farm Econ., 40:117-23, 1958.

[3] One way to overcome this difficulty is to program at different levels of lumpy variables, representing the same and/or different technology, and then comparing outcomes. This procedure, however, can result in numerous programming problems and hence may become prohibitively expensive.

of estimation of a short-run, timeless supply schedule. This estimation is of a unique kind because the linear programming model depicts the production behavior that would be rational if profit maximization is the goal or end and if the conditions within which this maximization takes place are realistically described within the model. The maximization of an end or goal subject to constraints is applicable not only to a profit goal. Conceptually, the profit goal can be replaced with another goal or set of goals. But if a multiplicity of goals is admitted, the economic calculation is not operational unless some weights can be attached to these goals. With the present state of our knowledge, there seems little possibility of determining these weights and hence little possibility of actually replacing the profit goal with a wide range of goals in the economic calculation. But another alternative is available. Instead of attempting to maximize some weighted set of goals, other goals could be considered as additional constraints in a profit-maximizing calculation. This could be accomplished if the effect of these other goals are reflected in the availability of resources for farm production, for example, credit capital availability in relation to willingness to borrow and labor availability in relation to willingness to work. Further, if the effects of these other goals on response to a change in price can be related to other nonprice variables, such as age, equity, and family position — possible variables associated with goals other than profit — in addition to the usual production matrix, then it is possible to develop a prediction of human behavior in this limited context. Thus, if goals other than profit maximization are closely associated with age, equity, and family composition, the effect of these other goals can be reflected by stratifying on age, family composition, and equity position.

It may be useful to illustrate briefly how differences may arise between what farmers do and what they could do if they were to follow the dictates of a profit-maximizing model. For example, a farmer who has decided to send his children to college may need to place high demands for income in the present relative to some future date. At low prices for his products, high demands for present income may be evidenced by willingness to work longer hours, which within the limits of his other resources may cause him to favor high labor-using enterprises even though returns per hour of labor are low. At higher prices, however, the resulting higher income may reduce the pressure for higher income and this reduction may be evidenced by unwillingness to continue working long hours. Thus the reservation price on his own output may be related not only to goals other than profit but also the price of the commodity he sells. More explicitly, the farmer's attitude toward the dairy enterprise or his unwillingness to work long hours may limit the dairy herd on a particular farm to 10 cows with milk at $3.50 per cwt., even though it would be profitable, as determined by the linear programming model, to expand to a 15-cow herd. But at a price of $3.00 per cwt. of milk, the increased pressure to maintain income may cause this farmer to work longer hours and expand his herd to 15 cows. Since linear programming solutions are often sensitive to changes in constraints, it is clear that failure to reflect the effect of other goals on availability of resources may change considerably the estimates of elasticity based on the analysis.

Determination of these modified constraints or the effect of other goals might be accomplished for a sample of farms by a questioning process. The objective of the questioning would be to learn from the farmer his estimate of what his response would be to a variety of price situations. As we are attempting to supplement estimates of supply relations obtained from linear programming and as we have argued that linear programming is appropriate only for estimation of short-run supply response, certain limitations must be placed on the kind of response. New investment in plant cannot be allowed. Such items as the number of cows, the number of sows, and the use of labor and operating capital would be allowed to vary only within the limits imposed by existing land, buildings, and major equipment and machines.

In structuring questions to learn what the farmer says he will do in response to different price situations, we need to start the farmer thinking about resources that may limit his ability to change. Thus, initial questioning might take the form of: (1) How many more cows, if any, could you handle with your present housing facility? (2) How many more cows, if any, could you handle with your present hay and pasture? (3) How many more cows, if any, could you handle with your present labor supply? These questions could then be followed by others to ascertain the existing livestock program and organization, the planned changes for the coming year, and reasons for these planned changes. We might then follow with questions as to their price expectations for each major livestock enterprise. In the context of this kind of a supply function, we have a price quantity point (point a in Figure 10.2) identified by an expected price of milk, planned production perhaps measured in terms of number of cows, and the expected or most likely price of other livestock commodities. In the sample with which we are dealing in the Lake States Dairy Adjustment Study, this would be the price of hogs. As we will be comparing the supply functions derived for different farmers, it is essential that each function be identified by the same expected price of hogs. The next question would be to solicit the planned response for milk production if an expected price of (say) $12 for hogs were held with the same degree of certainty as the farmer's expected price of hogs. The answer to this question would then identify the point designated as a′ in Figure 10.2. With this background information, the questioning then might proceed as follows: (1) If a higher (specified) price expectation for dairy were held with the same degree of certainty, what changes in organization would be made? (2) What higher price expectation would be necessary to cause the farmer to change his planned organization (with no change in plant)? (3) What changes would be made at that price? (4) How would these changes be accomplished (effect on other enterprises) and what limits the amount of change?[4] (5) Repeat the above process to determine the price at which no further changes would be made without changes in plant. Above this point, the short-run supply function is presumed to be perfectly inelastic. This same form of questioning would be repeated for prices below the expected price to develop the remainder of the step function below point a′ in Figure 10.2.

As step functions would be derived for individual farmers, these functions could be compared to determine whether age of operator, family composition, equity position, and other characteristics influence supply elasticity. If these characteristics do affect supply elasticity, a measurement of their effect through cross-sectional studies based on surveys can be used (1) to modify elasticities derived from a profit-maximizing linear programming model, or (2) to initially stratify farms on these characteristics and then have the linear programming computations directly reflect supply elasticities unique to groups of farms with different characteristics. In the first instance, we could more readily measure how and to what extent the modified elasticities improved linear programming as a predictive model. In any event, we should have more meaningful short-run farm-management guides, since the restrictions reflect those imposed by the farmers themselves rather than those imposed by research workers.

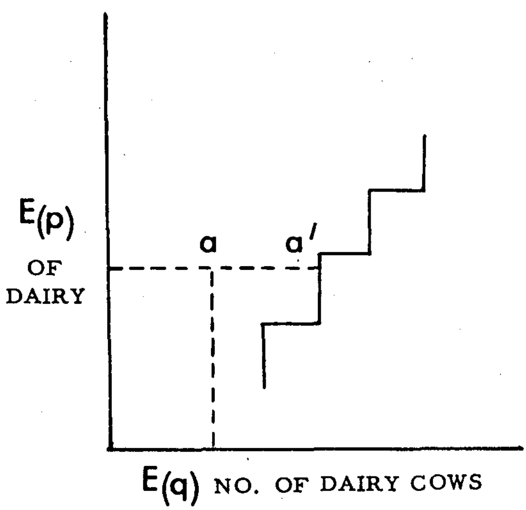

Figure 10.2. An expected short-run price-supply relations

[4] Statements on limits to change and how, if at all, these would be overcome would be checked against the farmer's response to how limiting various resources actually were to an increase in cows.

SUPPLEMENTING TIME SERIES ESTIMATES OF SHORT-RUN SUPPLY

A major problem in estimating supply response through the use of time series analysis is the ever-changing composition of agricultural firms — the movement of people out of agriculture, the recombination of land, capital, and labor, and the associated change in the age composition of the farm labor force. The effect of these changes in combination for a given set of prices undoubtedly changes the combination of enterprises and may change the effectiveness of price in bringing about changes in the composition of agricultural production. There is, of course, no guarantee that a cross-sectional interfirm study based on survey will overcome these problems; however, to the extent that such a survey analysis can identify the combination of firm and entrepreneurial characteristics associated with different magnitudes and kinds of production and the combination of characteristics associated with change and no change in production, it should yield information useful for improving time series estimates of short-run price-supply relations.

An examination of ten-year records of dairy-hog farms suggests that, while dairy cow numbers vary somewhat from year to year, on most farms this year-to-year variation is largely of a random nature. But, on some farms, there appears to be a definite trend in the number of dairy cows, suggesting that they are in process of going into or leaving dairy farming. It is the authors' hypothesis that those going out of dairying have a different combination of observable characteristics than those staying in dairying and hence changes over the years in the number of farms possessing these observable characteristics help to explain changes in dairy cow numbers and production.

Any attempt at identifying these characteristics involves a cross-sectional study over a period of years to (1) identify those farms having trends and (2) remove at least part of the effect of random variations in the number of cows from year to year. A continuing survey of a sample of farms or use of a "producer panel" for several years is an expensive undertaking. Certainly, if only a record of past production is obtained, the period required to allow isolation of the magnitude of random variation might be so long as to make the costs prohibitive. But by combining information as to planned production, actual production, and reasons for deviation between planned and actual production, the length of time perhaps can be greatly reduced.

For those farms having no observable trend in number of cows or no planned change in production, the analysis would proceed by relating the average production (number of cows) to certain firm and entrepreneurial characteristics. Preferably, these characteristics should be readily observable and attainable from other sources, as well as having some stability over time in an intrafirm sense. Certain firm and entrepreneurial characteristics are assumed initially to be stable or near-stable for individual firms over (say) a four-year period. These are age of operator, man-years of labor per 100 acres of land, long-run price expectations, type of farm in terms of a broad classification, and number of acres of land. Of these, age of operator, man-years per 100 acres, and number of acres of land are continuous variables in an interfirm analysis, while type of farm and long-run price expectations are discontinuous.

Any combination of variables that attempts to explain differences in the number of cows may take on a number of forms. The most simple and easily used is a linear form[5]

$$Y = a_0 + a_1 x_1 + a_2 x_2 + a_3 x_3 \mid x_4, x_5$$

in which: Y is total number of cows

x_1 is number of acres of land

x_2 is age of operator

x_3 is man-years of labor available per 100 acres (assumed to reflect family composition)

x_4 is long-run price expectations for dairy relative to hogs

x_5 is a broad classification of type of farm, such as dairy or dairy-hog

This linear form, in addition to simplicity in fitting, has advantages in simplicity of use. Ex post, if the number of farms is known, as well as the average value of each of these variables for the area in question, the linear equation can provide a preliminary estimate of the number of cows for the area. Thus, in a probability sense, that portion of the change in cow numbers associated with a change in structure of farms or in entrepreneurial characteristics can be isolated. Other forms, including products of two or more variables, or logarithmic functions, would require a considerable degree of disaggregation. For example, an equation with one term a product of two variables with a significant and sizable coefficient would require disaggregation to the point that the number of farms with each combination of the variables in the product term is known. The choice between functions must be determined largely by the comparative proportions of the total variation explained by the alternative forms.

To the extent that some farms indicate a trend or have made planned changes in dairy production, analysis beyond the foregoing can add to our information. First, it seems likely that those farms with increasing cow numbers would have a different combination of characteristics than those with decreasing numbers of cows. Hence, we need

[5] The simplified models presented here should in no way be considered as the only type of analysis that can or would be conducted in a cross-sectional study. We hope, however, that they will serve as a starting point from which more complex and more realistic models can be developed.

to know something about the combination of firm characteristics associated with positive changes, no change, and negative changes in cow numbers. Second, because changes in organization usually represent new investment in livestock, buildings and equipment, we need to know something about the investment function for farms making changes in organization.

The study of firm characteristics associated with change in production could be accomplished with an analysis similar to that suggested earlier for absolute production. To minimize the importance of small random changes from year to year, change in product (for example, cow numbers) can be studied as an average over some time period. The relationship can be expressed in this general form:

Average change in total cows = f (number of acres of land, age of operator, man-years per 100 acres, type of farm, long-run price expectations)

This relationship would need to be expressed for each type of farm and the particular long-run price expectations of the entrepreneurs on particular types of farms. Thus, we might have the above relationship for dairy and for dairy-hog farms with (1) long-run price expectations favorable for dairy but unfavorable for hogs (where favorable or unfavorable is defined relative to the date of the first survey), (2) long-run price expectations favorable for hogs but unfavorable for dairy, or (3) long-run price expectations for dairy and hogs similar to existing prices for hogs and dairy.

To illustrate the kind of information that can develop from study and analysis of these relationships, we hypothesize the relationship shown in Figure 10.3 between average change in cow numbers and number of acres of land for dairy farms with long-run price expectations favorable for dairy but unfavorable for hogs.

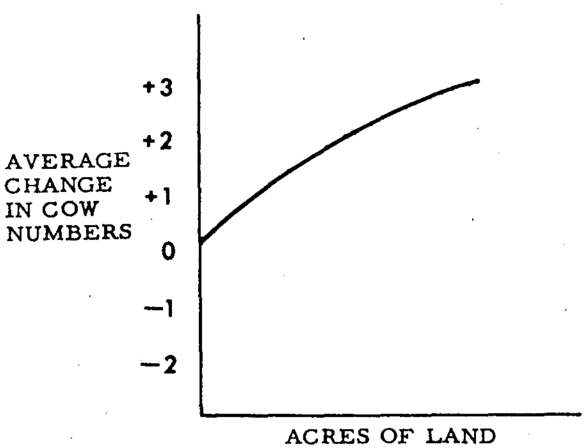

Figure 10.3. Relationship between cow numbers and acres of land.

This increase in number of cows represents new investment, not only in cows but possibly also in feed, buildings, and equipment for the additional cows. To the extent to which level of net cash income reflects the ability to make new investment, we suppose a positive relationship between new investment and income as illustrated in Figure 10.4.

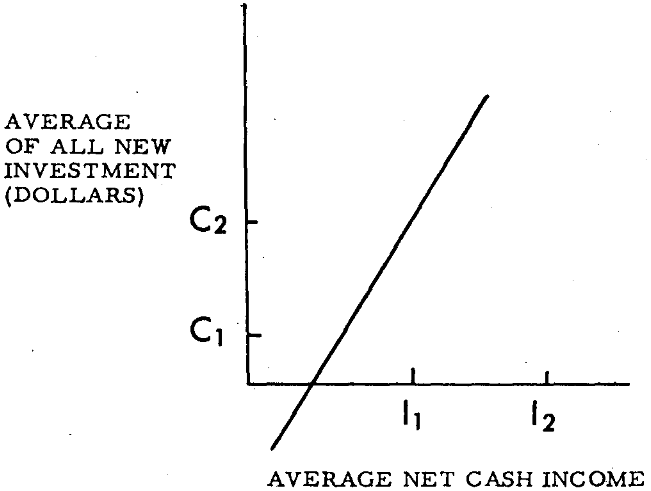

Figure 10.4. Relationship between new investment and net cash income.

With these relationships, price administrators (those for milk, for example) should be in a better position to estimate the effect of a milk price increase on new investments in dairy. Suppose, for example, that a milk price increase augments net cash income from I_1 to I_2. This increase in income is then expected to increase new investment from C_1 to C_2. To determine the effect of this new investment on milk production, we would need to know the relationship between average new investment and the increase in cow numbers as indicated in Figure 10.5.

Thus, with an increase in investment of $C_1 C_2$, we could expect an increase of 200 cows. With knowledge of average production per cow, we can then estimate the total increase in production expected from a given price increase.

To the extent to which an analysis of changes in product in relation to certain entrepreneurial and firm characteristics adds to our information on supply of production response, such information can be used to adjust the solutions from the equation

$$Y = a_0 + a_1x_1 + a_2x_2 + a_3x_3 \mid x_4, x_5$$

(in which Y, or total number of cows, is a function of acres of land, age of operator, man-years of labor per 100 acres, given long-run price expectations and type of farming), when used as a first step in analyzing

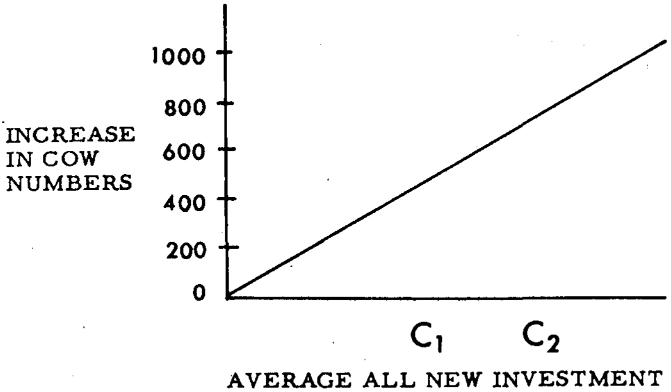

Figure 10.5. Relationship between change in cow numbers and new investment.

those farms evidencing some trend or directional change in cow numbers. Such information could be used also to adjust short-run price quantity estimates from time-series analysis. Empirical information on the composition of new investment may furnish a basis for isolating the part of a supply change that is due to a change in price and the part that is due to a change in investment that causes the supply function to shift. At present, such isolation is a bothersome problem in time series analysis. Moreover, empirical information on the investment process at the firm level can serve as a beginning foundation for constructing a theory of investment for the farm firm that can serve as a useful guide to both firm and policy decisions.

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Discussion

IN BUILDING supply estimates from the micro level, Jensen and Day present cross-sectional studies based on survey techniques. The authors have laid a sound groundwork for their proposal. Farmers make plans in an area of imperfect knowledge. This leads to errors in judgment, resulting in a gap between planned and realized supply response. In order to improve our estimates of farmers' supply response to changing conditions, we need better information.

Jensen and Day have called attention to three areas where critical information is lacking: (1) how farmers respond to changes in price and other variables under conditions of uncertainty, (2) the influence of a change in technologies, and (3) a theory of investment at the firm level.

Previous papers indicate some of the shortcomings of other methods of studying supply response at the micro level. Jensen and Day give emphasis, quite properly, to the difficulties in handling "lumpy" variables (land, equipment, etc.) in linear programming and the further difficulty in recognizing goals other than profit maximization. In time series analysis the difficulty stems from the "ever changing composition of agricultural firms." It is hoped that the proposed cross-sectional survey study would supplement and improve our estimates of supply responses.

Jensen and Day state their objective is to suggest how cross-sectional survey studies may be used to provide estimates of supply response, and to provide information about variables affecting supply response of the farmer, which would be helpful in other approaches to supply estimation. The proposed method would involve the preparation of a questionnaire for farm producer interviews. It might be necessary to make observations on some farms for several years in order to observe the characteristics of those farms where production is increasing and those where production is decreasing.

One question that might be raised regarding a survey to determine how farmers would respond to various changes in price is the validity of the information obtained. Will the farmers' response to the question, "What would you do if the price of milk increased 20 per cent?" be the same as the action he would take if prices really should rise 20 per cent? Is it likely that the farmer would be influenced by current conditions or experiences? Let us assume that the enumerator arrives for the interview (1) just after the farmer received the report that his was the top producing herd in the DHI Association, for the first time, or (2) toward the end of the day when everything pertaining to the dairy enterprise had gone wrong. Would we get the same response from farmers in either case? I am certain the authors recognize this problem. They experienced similar problems in the Interstate Managerial Studies.

The observations over a period of years to study the characteristics of farms that make changes in production could be costly and time consuming. The procedure could be justified on both counts if this technique is necessary to obtain the information. Such studies would have to be made in many areas. For example, in dairying the variables that influence supply response and their relative importance would probably vary among the dairy producing regions in the United States. It might be possible to obtain some of this information from farm record association cooperators where records are available for numerous farms for a number of years. This might reduce the cost and the time required to accumulate useful information.

The task of accumulating the information we need would be enormous if we relied entirely on the proposed survey method. Each major type of farming area might have to be surveyed for each important commodity. Jensen and Day do not suggest this use of the survey method. They want to use it on a "trial" basis to determine if this method could be used to provide better information than is available from other sources.

One important question pertains not only to the survey method, but to all procedures discussed. How can we determine the response of farmers who are not now producing the product in question? Taking dairy production as an example, we need to know how dairymen would respond to various increases in the price of dairy products. It is also important to know at what price other farmers would shift to dairying and how much they would add to total production at various prices.

In spite of the questions raised regarding the cross section survey method, it may prove to be a useful tool in our effort to determine the farmers supply responses and the variables that influence his decision. Any method that would help to shed more light on our problem merits careful consideration.

Chapter 11

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Some Comments on Sampling

THIS PAPER consists of a brief discussion of three separate but related topics: specification of a statistical population; the design of a sample for a specific study;[1] and a simple means of estimating sampling error.

SPECIFICATION OF THE POPULATION AND PARAMETERS

Rigorous use of modern statistical methods in sampling, estimation, and interpretation of results requires detailed specification of parameters (population-values) to be estimated. This means a complete definition of the population and of the data, and complete specification of procedures. Any thorough and careful interpretation of estimates from a sample, including interpretation of estimates of sampling error, must be with reference to a specific set of conditions, because if any of the conditions are changed the results may change. As this paper is limited primarily to sampling, the discussion of specifications will be primarily in reference to those necessary to design and select a sample. Unfortunately, the definition and specification of data, concepts, coverage, and various conditions are often not as fully developed and clarified as they should be.

Definition of the Statistical Population

To define a population one must define the units of observation and the geographical limits.

The Unit of Observation.

A statistical population, for our purposes, is made up of a finite number of units of observation, a unit of observation being, for example,

[1] "Adjustments in dairy farming in the lake states region." This is a cooperative study involving the Farm Economics Research Division of the Agricultural Research Service, USDA, and the states of Iowa, Illinois, Michigan, Minnesota, and Wisconsin. At the time of this writing there are no specific publication plans.

a farm enterprise for which a questionnaire is to be completed. The choice of definition of an observation unit is arbitrary and may involve a compromise between what is desired conceptually for purposes of the study and the practical problems or difficulties in obtaining accurate data for a unit defined in different ways. A definition of an enterprise usually requires a specification of minimum size as well as composition.

Geographical Limits of the Population.

Many farm surveys are limited to the "open country," as it was defined for purposes of the master sample of agriculture. Open country is the area remaining after delineation and deletion of incorporated places and unincorporated settlements having a population of more than about 100 persons and a density of more than about 100 persons per square mile. Whether to limit coverage to the open country is a matter of cost and practical considerations. Selecting an area sample from the open country of a few counties is a quick and inexpensive task. Farms in the nonopen country parts can also be sampled, but the so-called master sample materials are not as well suited for that purpose. That, combined with the difficulty of finding and identifying farms in nonopen country areas, is why the coverage for most farm studies of a research nature is limited to the open country. Perhaps the nonopen country territory should be covered as well — at least places having less than about 2,500 inhabitants.

Decisions must also be made on the broader limits of coverage. Should the area covered be a region, a state, a local area, etc.? That question is obviously related to objectives, costs, and making inferences, which are beyond the scope of this paper. However, when a study is limited to a particular type of operation, such as sugar beet production, census statistics for counties and minor civil divisions can be used to help define the limits of the statistical population to be sampled. For example, for a study of a local sugar beet producing area the statistical population might be defined as a group of minor civil divisions that account for about 90 percent of the production in that area. Should the area be defined to include 95 percent of the population or is 70 percent good enough? Elimination of the peripheral areas where the beet farms are of low density might reduce costs appreciably but how much will the purposes of study be impaired?

With respect to the matter of uncertainty in the making of decisions from survey results, it is clear that definitional or specification errors, as well as sampling errors, response errors, tabulation errors, etc., are a part of the total uncertainty or error picture. A definitional error is the result of defining, for example, the population, a class of the population, or a variable in a way that differs from the corresponding situations about which decisions are made. One would like to have the definitions and data specifications made to serve ideally the ultimate uses, but practical compromises must be made which means the exercise of judgment on what definitional errors to tolerate. This problem of definitional errors is receiving, and should receive, increased attention by statisticians and subject matter specialists since it is an important problem area in the improvement of research technique.

Tabulation Plans

In addition to the definition of a statistical population there should be a clear understanding about analysis or tabulation plans before a sampler makes final recommendations on sample size and design.

SAMPLE DESIGN

Generally speaking, suitable lists are not available for sampling purposes. Hence, if the principles of probability sampling are to be applied, area sampling is indicated. Much literature is available on sampling, so rather than prepare a general paper it seems more appropriate to use a specific survey as a basis for discussion. Some interest has been expressed in a description of the sample for the study "Adjustments in Dairy Farming in Lake States Dairy Region," so that study will be used.

The statistical population for this study was all commercial farms in economic classes I through V, except specialized poultry, fruit, and truck farms. Each state was divided into regions as indicated in Table 11.1, and each region was treated separately for analysis purposes. Table 11.1 gives some general descriptive information about the population and the sample. Available notes reveal very little about how the sampling rates were determined. However, the matter of setting sampling rates will be briefly discussed later.

A geographically stratified random sample of area segments would

Table 11.1. Some Statistics About the Population and the Sample for the Lake States Dairy Adjustments Study

State	Region	No. of counties	Total No. of segments	No. of farms Class I thru V, 1954	Av. No. of farms per segment	No. of segments selected	Sampling rate	Sampling rate times No. of farms
Minnesota	1	17	11,148	38,384	3.4	99	1/113	340
	2	18	10,224	28,964	2.8	96	1/107	271
Wisconsin	1	21	13,182	44,538	3.3	126	1/105	424
	2	25	17,270	52,825	3.1	132	1/131	403
Michigan	1	5	2,441	6,927	2.4	72	1/34	204
	2	5	5,236	13,013	2.5	64	1/82	159
	3	10	7,318	16,909	2.3	76	1/108	157
	4	13	6,844	14,766	2.2	80	1/95	155
	5	9	7,818	16,640	2.1	84	1/103	162
Iowa	1	11	5,763	20,311	3.5	54	1/107	190
	2	6	3,575	11,944	3.3	54	1/66	181
Illinois	1	8	3,186	10,761	3.4	54	1/59	182

have given a sample of segments well distributed over a region, but because of the small size of the sample the average distance between sample segments would have been large. It did not appear advisable to increase the size of the segments, so a means of introducing some clustering of sample segments was sought. A two stage sample design was indicated. A county was not a suitable primary sampling unit because of the small number of counties in a region. Therefore, except for two regions in Michigan, minor civil divisions were used as primary sampling units. In the two Michigan regions, single stage sampling was used because they were small.

Region 2 in Iowa has been chosen to illustrate how the sample was selected. A sampling rate of 1/66 (see Table 11.1) meant that 54 segments were to be selected. As the sampling plan called for three sample segments in each township (minor civil division), 18 sample townships were needed. The six counties in this region listed in a geographical order, the total number of segments in each county, and the random numbers for designating selected townships are shown in Table 11.2.

Table 11.2. Counties in Iowa Region 2 Surveyed in Lake States Dairy Adjustment Study

County	Total number of segments	Random numbers designating sample townships
Winneshiek	718	45, 243, 441, 639
Allamakee	501	119, 317
Clayton	703	14, 212, 410, 608
Dubuque	547	103, 301, 499
Jones	563	150, 348, 546
Jackson	543	181, 379
Total	3,575	

As three segments were to be selected from each sample township, the sample townships were chosen with probabilities proportional to their numbers of segments. There are various ways to do this but considering the form in which the materials were available, it was actually done as follows: The 3,575 segments may be visualized as a continuous array, ordered geographically within townships and with the townships being in a geographical order within counties. A selection of every 198th segment in the array from a random starting point would give 18 segments and hence 18 corresponding townships. The townships so selected would have probabilities of selection proportional to their numbers of segments. That was the method followed. The starting point, a number selected at random between 1 and 198, was 45. Consequently, 45 is the first random number shown in the table above. The other numbers were obtained by adding 198 successively, but when 198 was added to 639 the result, 837, exceeded the number of segments in the first county. Hence, 718 was subtracted from 837 which gives 119, the first number in the second county, etc.

Table 11.3. Partial List of Count Units in Clayton County, Iowa

Township	Count unit	No. of segments	Cumulative no. of segments
1	1	2	2
	2	3	5
	3	3	8
	4	2	10
	5	2	12
	6	3	15
	7	4	19
	8	3	22
	9	1	23
	10	2	25
	11	2	27
	12	4	31
	13	3	34
	14	2	36
2	1	2	38
	2	3	41
	3	2	43
	4	2	45
	---------- etc. ----------		

There is for each county, as part of the so-called master sample materials, a listing of "count units."[2] Part of the listing for Clayton County is reproduced in Table 11.3. Note that the first random number for Clayton County was 14 which falls in the 6th count unit in the first township. This count unit was divided into three segments and one was selected at random for the sample. As the sampling plan called for three sample segments in each selected township, two additional segments were selected at random within the first township. That, in essence, is the way the sample was designed and selected.

The dairy adjustments study presented a sampling problem that is common to many such studies. How does one manage the situation when information on the size of the statistical population is insufficient to provide a satisfactory basis for setting sampling rates? Selecting a sample and finding half or twice the desired number of sample farms may present a number of difficulties. For a local survey, it is possible to design a sample so the field work may be terminated after approximately the desired number of questionnaires have been completed. This may be done in various ways without loss of the basic principles of probability sampling. Perhaps the simplest procedure is to select an unrestricted random sample of segments and number the segments in the order selected. The segments would be enumerated in the order numbered until the desired number of schedules is obtained. Of course,

[2] A count unit is a group of one or more segments. For a description of a count unit and the master sample materials, see Houseman, Earl E., and Reed, T. J., "Application of probability area sampling to farm surveys," Agr. Handbook. 67, 1954.

administratively, if one knew that he had to cover at least 15 segments he would enumerate the first 15 segments in whatever order was the most efficient and then proceed to 16, 17, etc.

Such a sample would lack stratification, but that can be provided for. Suppose a total of 80 segments was to be selected. One could set up, for example, eight strata and impose a restriction, without introducing bias, such that the sample segments numbered 1 through 8 would constitute a stratified random sample of eight segments, one from each of the eight strata. The same would be true for sample segments numbered 9 through 16, etc. The main point being made is that, particularly for a local survey, there are ways and means of keeping a sample statistically efficient and sound but at the same time have a plan that can be successfully administered and a plan that provides for termination of field work when a given number of schedules have been completed. Provision for making call-backs can and should be included in the plans.

Another kind of problem occurs when, for example, three types of farms A, B, and C are to be compared but their proportions in the population are:

A	10 percent
B	30 percent
C	60 percent

It is possible, with complication, to design a sample so that approximately the same number of farms of the three types would be enumerated. Suppose a sample of 50 farms of each type is desired and that the average size of segment is four farms, considering the three types A, B, and C. On that basis the required number of segments for each type, ignoring call-backs, refusals, etc., would be:

A	125 segments
B	42 segments
C	21 segments

Three samples, X, Y, and Z, could be set up:

X	21 segments	enumerate all three types
Y	21 segments	enumerate only types A and B
Z	83 segments	enumerate only type A

Because of the problem and cost of getting 50 farms of type A one might decide to reduce the size of the sample of type A. On the other hand, because of the low frequency of type A farms, one might decide to make the segments in the "Z" sample 3 times the average size. That is, the "Z" sample could be 26 segments averaging 12 farms.

An alternative to the above approach would be:

1. Use a sample of 125 segments.
2. Canvass all segments and contact each farm, ascertain its type, and list on a special form designed to provide a separate listing of each type.
3. Certain lines on the special form would be checked and farms falling on those lines would be included in the sample. All lines for the listing of type A would be checked. One-third of the lines for type B farms and one-sixth of the lines for the type C farms would be checked. Thus, the interviewing and listing could proceed simultaneously.

Actually, for this alternative approach one would probably use larger segments, perhaps 63 segments averaging eight farms instead of 125 segments averaging four farms.

The principles of probability-area sampling can be readily adapted to a wide range of conditions, but to do so successfully, an experienced sampler must work closely with the subject matter specialists. Otherwise, misunderstandings may develop. The sample may not be best for the objectives, the sample may not be properly used in the field, the data may not be properly weighted if weighting is required, etc.

ESTIMATION OF SAMPLING ERROR

Insufficient attention has been given to obtaining estimates of sampling error for interpreting results and planning studies in the future. Therefore, reference is made to a simple means of estimating the sampling error for many selected items, even though the sample design and estimation procedure may be rather involved. In essence, the sample is designed as a composite of several equivalent samples, perhaps eight or ten, set up in such a way that separate estimates may be made from each. The variability among these estimates provides a valid estimate of the sampling error.

As a simple case, consider a sample of 96 segments. Twelve equal sized strata could be formed and eight segments selected at random from each. Such a sample would be equivalent to a composite of eight samples, each being a stratified random sample of 12 segments, one from each stratum. The eight equivalent samples could be separately identified in the sampling operation or they could be established after data collection using appropriate randomization procedures. Separate estimates (probably only for selected items) would be made for each sample. Suppose, for example, that the average number of dairy cows per farm was computed for each of the eight samples, $\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_8$. The variance among these eight averages is $V = \frac{\Sigma(\bar{x}_j - \bar{x})^2}{n-1}$ where n is the number of samples (eight for the case in point) and $\bar{x}$ is the average of the eight means. The estimated variance of the mean of the entire sample is simply $\frac{V}{n}$.

Actually, for the situation just cited, another way of estimating the sampling error might be recommended, especially if an electronic computer is being used. However, the above approach may become a practical necessity when the structure of an estimate and/or the sample design becomes complicated. The point to be noted is that appropriate steps can be taken in the design of a sample so valid estimates of sampling error can be obtained rather expediently. This is important if it means the possibility of getting estimates of sampling error for many or several items. Otherwise, no sampling errors would be available. Moreover, the arithmetic procedure for estimating the sampling error is simple and can be administered by a nonstatistician. A statistician should be consulted, however, regarding the establishment of the "equivalent" samples to make sure that the differences among them will properly reflect various components of sampling error.

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Discussion

I SUSPECT that, as participants in this workshop, most of us would have been disappointed had not the topic of sampling been included as part of our study. A consideration of sampling problems is a key issue faced by the supply analyst. It is appropriate to have a paper on sampling and to focus attention on the statistical means by which observations should be generated to estimate the supply reactions made by producers to changes in product prices.

Houseman has outlined some of the essential components of sampling, organizing his paper around the related topics (1) specification, (2) the design of a sample, and (3) the sampling error. To "specification" may be attributed the translation of the supply response problem into statistical terms, which we do by defining the appropriate population of farms and by defining the parameters of supply reactions to be studied. The sample survey design treated is the Lake States Dairy Adjustment Study, referred to in several other papers presented at this workshop.

Houseman carried out his indicated objectives and gave a fairly good, though brief, discussion of the above three topics. However, his presentation deals largely with sampling of farms in general and is not a well-pointed paper on sampling for supply functions. I find this so-called "area of omission" a basis for criticism.

The logical question at this point is: "What do we expect to find in a pointed paper focusing attention on the statistical means by which observations should be generated to estimate the supply reactions of producers to changes in product prices?" In a pointed discussion, we would expect a "tie up" to be made between the process of sampling and the tools of analysis to be applied to the observations once they have been generated and collected. Data needs, including the degree of accuracy, may be expected to vary with the tools used, so that in the sampling scheme it is more than questions concerning the use of a priori knowledge about the population and the cost of obtaining data. I suggest this is especially true in supply analysis where the tools are: (1) sometimes normative in nature, (2) sometimes positive, but (3) often of a combination normative-positive type. A distinction has been made elsewhere of the categories of tools for supply analysis; hence, we do not need to repeat it here. Regression analysis, especially if cross-sectional or some combination of cross-sectional and time-series data, is probably both a positive and a normative tool of analysis.

A pointed paper would probably relate itself in someway to this categorization of tools. It might, however, be in the form of some reference to:

1. The sampling scheme when supply response is to be derived from inter-firm production functions.
2. The sampling scheme when supply response is to be estimated through variable price programming.
3. The sampling scheme when supply response is to specifically reflect planned and/or realized reactions of producers rather than normative reactions.

Would not the sampling scheme be different for some two or all three of these situations? We would expect to find a partial answer in a pointed paper.

Supply response based on inter-firm production functions needs a sampling scheme chosen with the requirements of production function analyses in mind. Actual random samples are not the most efficient designs for this purpose. A different kind of sample would be more efficient if designed to select firms for adequate coverage of the variation in inputs within the sample and reduction in multicolinearity in observations of inputs of the sample. It is not an easy task to select farms into a sample for minimization of correlations between inputs. For one thing, economic conclusions derived from production functions, including those related to supply response, can be valid only if the firms in the sample are operating such that marginal productivity decreases continuously and is always less than average productivity. For another, the selection and classification of groups of farms of varying managerial capacity to insure that they are on approximately the same production functions present serious problems. It is possible in relevant situations for the investigator to identify a range in managerial capacity, thereby selecting for study groups of firms of either a rather uniform level or situations of randomly distributed divergencies in this inter-firm capacity.

It can be easily seen that the appropriate sampling scheme when supply response is derived from inter-firm production functions involves judgment at all stages of its empirical application. Such judgment could pay handsome dividends in increasing the reliability of estimation. In fact, increases in efficiency which can be derived by such sampling over random or other common types of samples should go up substantially as the extent of this a priori knowledge of the universe of farms increases. This need for prior knowledge explains the real reason why supply response derived from inter-firm production function does not have as great a future as some related techniques. Prior knowledge in advance of selecting farms to obtain the necessary data includes: (1) the expansion path along which production is typically expanded by a population of farms operating more or less on the same production function, (2) the divergencies of individual farms from the norm that belongs in the population, and (3) the information to restrict considerations to firms operating under a declining positive marginal productivity curve as the law of diminishing returns requires.

Few if any researchers have such a priori knowledge available, although this information can be more or less approximated in a carefully designed research study. Even if such were available to the researcher, there is likely to be real difficulty in observing a range of proportions in which resources are combined sufficiently for representing particular portions of the production surface and its derived supply curve. In other words, we know how the sample should be drawn with respect to intercorrelations among the input variables. But conditions of farming and methodological issues are such as to reduce the plausibility of finding the kinds of observations we need in an otherwise homogeneous population in respect to resources available and techniques of production.

Supply response based on activity analysis needs a sampling scheme chosen with the requirements of continuous programming in mind. Probability sampling is appropriate for this purpose. This being the case, the sampling scheme should handle by stratification characteristics that affect the slope and elasticity of the firm supply relationship. We usually consider the following characteristics important, in that they often affect not only the slope and elasticity but also the reversibility of supply curves: Size of farm, tenure arrangement, amount of available capital, risk aversion, age of operator, cropland-pasture ratio, managerial ability of operator, and productivity of relevant resources. The most relevant of these should be the ones held constant in the analysis, at least within strata. The appropriate sampling scheme is expected to identify typical farm units possessing the most relevant of characteristics in varying combinations and also allow estimation of appropriate weights on the basis of the combinations' occurrence in the population.

Much of the same sampling procedure is involved when supply response is to reflect planned and/or realized reactions of producers.

In describing the sampling scheme for the Lakes States Dairy Adjustment Study, Houseman is dealing with stratified sampling. His paper deals mainly with a geographically stratified random sample with some clustering of sample segments introduced. This is certainly a practical approach to probability sampling of farms. Perhaps it accomplishes, to some extent, some of the objectives mentioned above.

PART IV

Regional Competition and Spatial Equilibrium Models

Chapter 12

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Interregional Competition* or Spatial Equilibrium Models in Farm Supply Analysis

PROBLEMS in interregional competition are concerned with supply and demand analysis in an appropriate degree of spatial disaggregation from national levels or a practical degree of aggregation of "contiguous" firms.

The viewpoint of the authors is that important problems in interregional competition exist primarily because of our dynamic economic system. In the absence of change, the need to explain the existing pattern of production is not nearly so great. Hence, our concern here will be with methods as they help us analyze important regional changes in physical production possibilities and factor markets. Changes in farming skills and managerial abilities (so called quality changes in factors), as well as changes in technology, are considered to be reflected in production possibilities.

Analysis of changes evident in the economy or in prospect might have either of these two objectives: (1) to predict regional changes in output, income, and product distribution given certain changes in the economic structure, or (2) to specify optimum regional output and product distribution patterns as goals for change in light of current and prospective changes in the economic system. The latter objective, of course, is normative, but it merges into the former as more and more restraints that simulate the actual adjustment process are imposed in the analytical system. Conversely, we would expect analyses of the former type to produce results that would merge into the second as adjustment "frictions" are removed from the economy.

The ideal information for studies in interregional competition would consist of (1) regional supply functions for individual products, (2) supply and demand functions for individual factors, and even (3) regional consumer demand functions for each commodity. Given this information we could better assess the relative advantages of regions in terms of location and transportation costs as well as natural factors. To support these supply and demand functions, we would have basic information

*The opinions expressed in this paper are those of the authors and do not necessarily represent those of the Farm Economics Research Division, the Agricultural Research Service, the United States Department of Agriculture or Iowa State University. The writers wish to acknowledge the helpful comments received from Walter Butcher during the development of this paper.

which ties the relevant regional supply and demand functions to the shifters representing regional changes in (a) production functions, (b) market structures, (c) institutions, and (d) factor prices.

With this information, we could predict eventual distributions of producing among regions as changes in production functions, prices, and institutions occur. But more important, we could also predict (1) the effects of relevant changes on the demands and prices of resources in the various regions and (2) increases and decreases in regional farm income. This "longer-run, ex ante outlook" would then be the basis for guiding the decisions of individuals, structuring education and vocational guidance, and providing action elements which would facilitate this change and make it less painful.

Quite obviously, we are not soon to have data conforming to such a general equilibrium system unless the many relevant supply and demand functions are synthesized from other data. It is unlikely even that we will soon have this "system of equations model" in national aggregate such that (1) it will provide any great detail as to individual commodities and resources and (2) will take account of the major "shifters" represented by changes in technology, market institutions, and factor prices.

What tools, then, have we for analyzing the many facets of product supply and factor demand? What tools have we for tying together, region by region, the inevitable changes in technology, factor markets, and institutions? These questions must remain unanswered. The authors have not presumed to "provide the perfect tool." While the alternatives may be discussed, it must be realized that they are imperfect due to similar data inadequacies. Although, conceptually, numerous alternatives are available only those that appear most realistic in terms of the computational task and related problems will be discussed. An attempt will be made to indicate some of the problems and limitations involved in using these methods in empirical work. The authors have interpreted their specific assignment as a thorough discussion of programming models as alternatives in analyzing those supply problems that come under the heading of interregional competition.

INTERREGIONAL COMPETITION MODELS

Before turning to the central theme of this paper we shall (1) review briefly regression methods and some special techniques in the area of activity analysis, and (2) indicate summarily the apparent limitations of these methods as tools for analysis of interregional competition problems.

Regression Analysis

It is assumed the technical aspects of regression methods are covered adequately in other papers and that most of the conclusions and observations will apply to interregional competition studies. Therefore, comments are more or less general and are concerned mainly with the problem of incorporating change into regression systems.

At a high level of aggregation, regression analysis can provide some "history of the past," but it is doubtful that it can produce anything that adequately accounts for major changes expected in the future. Unless we are interested in interregional analysis only from the standpoint of academic sophistication, the essential task, from the standpoint of education and public policy, is to predict, even if only roughly, (1) where the types of changes mentioned above will occur and (2) the magnitude of these changes. History and historic coefficients are interesting perhaps to farmers as well as to academicians. Ex post information alone, however, is of little value in guiding their decisions relative to change.

The fundamental problem in the use of regression methods is that of "detail;" detail with respect to regional disaggregation and range of observations, but primarily with respect to technological change and regional development. Supply shifters resulting from technological change cannot be ascertained from past observations. Technology is not a "smooth" function of time. True, judgment shifters could be applied to ex post regression functions but this would involve analysis that may best be carried out by other methods.

There apparently is little opportunity to use time series data and regression procedures to account for important changes of the future; changes such as those which caused much of the cotton production to shift to the Southwest, broiler production to become concentrated in the Southeast, dairy production to retreat in the central Cornbelt, and so forth. Hence, it seems that the major reliance must be placed on other methods of analysis. But there is an opportunity to use regression methods to measure the functional relationships between variables; then, use these relationships in other methods.

The Transportation Programming Model

The transportation programming model has been used quite extensively in recent studies dealing with interregional competition in agriculture; at times as a supplementary tool and at other times as the only tool. Although this type of programming has its uses, it has important limitations in studies of interregional competition because of its lack of generality. Specifically, this method is restricted to problems involving a single commodity or commodities that are perfect substitutes in satisfying requirements or demand. Thus, problems dealing with changes in comparative advantage among regions cannot be handled by this method. Furthermore, nonlinear production coefficients cannot be used as satisfactorily in this model as in others. It is best suited to short-run problems. The fundamental nature and possible uses of this programming method can be explained by referring to the following equations. The usual objective in the transportation model is to:

(1a) $$\text{minimize } f(c) = \Sigma_i \Sigma_j C_{ij} A_{ij}$$

subject to these constraints

(1b) $$\Sigma_i A_{ij} = X_i ,$$

(1c) $$A_{ij} \geq 0 ,$$

(1d) $$\Sigma_i X_i = \Sigma_j Y_j ,$$

in which

X_i = quantity or fixed stock of a homogeneous[1] product available at i-th supply point,

Y_i = quantity of a homogeneous product required at j-th destination,

C_{ij} = unit cost of transporting product from i-th supply point to j-th destination,

A_{ij} = quantity of product transported from i-th supply point to j-th destination.

The transportation method of programming can be best used to solve an allocation problem when: (1) fixed quantities are available at particular locations, (2) fixed quantities are required at particular destinations, (3) one unit of input (product at origin) is transformed into one unit of output (product at destination), (4) the cost of transporting one unit from the origin to the destination is the same for all levels of the activity (A_{ij}), and (5) the sum of the inputs is equal to the sum of the outputs.

A transportation model of the type outlined parallels the short-run market situation in agriculture of a single production period. Specifically, it characterizes the case in which a specific crop has been harvested, storage cannot be expanded, and consumer purchases are invariant for a wide range in prices. Under the assumption of competitive markets, shipments are made as long as transport costs are covered. Market prices are determined by the cost of making the marginal shipments to each market, and the distribution pattern will be such that net revenue will be a maximum for each supply point.[2]

In the comparative statics sense, the transportation model depicted by equations 1a through 1d, could be used to analyze regional changes in technology and factor prices within the marketing system. For example, it would be possible to express the effects of changes in freight rates or processing costs on the regional distribution of particular products.

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[1] The term homogeneous as used here includes perfect substitutes.

[2] For added details see, Henderson, James M., "A short-run model for the coal industry," Review of Economics and Statistics, 38:336-46, 1955.

In actual practice, the transportation model need not be confined to the very narrow orientation suggested by equations 1a through 1d. Certain modifications can be used to give this analytical method a wider range of applicability than has been true thus far in application to agriculture. For example, the inputs, X_i, could be defined as the maximum amount of beef that could be produced in each of a set of regions. The costs, C_{ij}, could be defined as the combined costs of producing beef in the i-th regions and shipping it to the j-th market. Furthermore, if total possible production exceeds the total requirements (that is, $\Sigma_i X_i > \Sigma_j Y_i$), a dummy requirement, Y_{j+1}, can be set up so that restraint 1d will be met. In this case, the cost of shipments to the dummy destination would be set equal to zero, or any other constant, since they represent shipments which in practice would not be made. Also, the model can be given a "longer-run flavor" by incorporating costs due to factors involving some fixity. Thus the transportation programming model could be used to define optimal (minimum cost) production or supply patterns as well as distribution patterns, even to the extent of considering some long-run variables.

The transportation model can be used to solve maximum profit problems as well as problems of minimum costs. In this case, market prices, as well as requirements, are considered to be given. Such an analysis could be useful in determining the differential influence of different price levels on regional farm income, but again from the standpoint of one commodity only.

The types of problems in interregional competition best adapted to analysis by the transportation method of linear programming have been summarized briefly. The chief limitations of this method, compared with other programming methods, are these: (1) Only one commodity or group of close substitutes can be considered. The method cannot account for the effects of opportunity costs and competition among commodities as they relate to possible supply relationships. (2) Insufficient flexibility exists in consideration of fixed and variable resources. In summary, the transportation model can handle only very special cases of more general supply or programming problems. It was not designed to do more and the authors have not intended to imply that it was.

The Input-Output Model

Input-output models imply the opposite extreme with respect to nonfixity of resources and possible supply relationships. This is hardly a tool for meaningful analysis of the production response relationships that characterize problems of interregional competition. First, there is no adequate way to account for change aside from recomputing a matrix of interdependence coefficients for each new point in time at which data become available. Even the so-called dynamic input-output model does not express supply functions that reflect important changes in interregional competition.

The conventional input-output table can provide a useful description of certain interrelationships among geographic, commodity, and other separate agricultural sectors for which measurements are available. Similarly, it can specify flows between agriculture and nonagricultural sectors. For a particular period or point in time, and given the particular mix in which the commodities were exchanged, the interdependence coefficients of an input-output model indicate the association between changes in output in one sector, or in final demand, with changes in other sectors. But because of the mathematical properties of this particular model, it is not useful for projecting differential changes in inputs and outputs among regional sectors — supposing that the focus of the analysis is in spatial stratification of the economy.

These properties can best be illustrated by a quick review of the system. Given the available data, we start with the equality in equation 2a

$$AX = Y \tag{2a}$$

in which X is a vector of outputs for regions and commodity sectors within regions; A is a matrix of input-output coefficients with a_{ij}^{km}, an individual element, defining the amount of the k-th output in the i-th region necessary (associated with a projected increase) for one unit of the m-th output in the j-th region; and Y is a vector of final demand for autonomous or consuming sector.[3]

The main representation of the input-output model is

$$X = A^{-1}Y . \tag{2b}$$

$A^{-1} = C$ is an inverse matrix. Each element, c_{ij}^{km}, indicates the amount by which the k-th output of the i-th region must increase for a one unit increase in the autonomous demand for the m-th product in the j-th region.

A main limitation of input-output models for analyzing output interrelationships characterizing problems in interregional competition is apparent in equation 2b. It is impossible to measure how the changes in supply relations of one region may affect the outputs of other regions. In fact, the method is not one for measuring competitive interactions among sectors. Instead it stresses the complementarity among them. It shows, under a spatial formulation of the model, how outputs of commodities in various regions must be increased together, if the output of or demand for a commodity in a particular region is increased.

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[3] This representation supposes that an initial flow matrix, M has been constructed. The m_{ij}^{km} element indicates the amount of the k-th output of the i-th region flowing into the m-th commodity sector of the j-th region in the period under study. The elements of A, therefore, are computed as $a_{ij}^{km} = m_{ij}^{km} (X_j^m)^{-1}$. For a "net" model, $a_{ij}^{km} = 0$, where i = j and k = m, but all other a_{ij}^{km} are negative or zero. The relationship in (2a) could be expressed also as (I-A) X = Y where I is an identity matrix and flows within a region, which characterize a "gross" model, are assumed.

The ratios between the coefficients in C for the i-th and z-th regions, $(c_{ij}^{km})(c_{zj}^{km})^{-1}$ expresses the "rate of complementarity" of the regions. It shows the "fixed mix" rate at which the k-th outputs of the two regions, i and z, (the ratio of increase of $(\Delta X_i^k)(\Delta X_z^k)^{-1}$ are required to increase with a unit increase in the final demand for the m-th product in the j-th region. Or, if we return to the original requirements matrix, A, the ratio of the two coefficients for the i-th and z-th regions, k-th product, $(a_{ij}^{km})(a_{zj}^{km})^{-1}$ shows the fixed mix rate at which the k-th output of the i-th and z-th regions must increase for a one unit increase in the output of the m-th product of the j-th region. Hence, the system assumes that the commodities produced in the various regions are technical complements, and for any increase in final demand, they must increase in the proportions existing at the time of the observations.

This discussion of the conventional input-output model is in terms of its limitations in studying major problems of interregional competition. The input-output model, of course, has other limitations which are not specific to regional analysis. These limitations include assumptions of constant scale returns, the absence of fixed factors, and others. The problem of "pure scale relationships" for firms is not a major problem when the model is applied to geographic sectors of agriculture. But differential quality of resources such as soil within a region is a problem akin to that of "pure scale" considerations. As output of an agricultural commodity is increased in a region, increments may need to be produced on soils of lower productivity or by farmers of less ability. Furthermore, if all available or suitable land is already devoted to a product, output can be increased only by a more intensive use of resources such as fertilizer on a fixed acreage of land; that is, output is agumented by a nonproportional increase of inputs rather than by "scale" increases. It is not possible to set up an input-output model with "less intensive" and "more intensive" subsectors in one geographic region to allow output to be increased from the latter as demand increases. Outputs from the less intensive and more intensive subsectors would bear the same ratio to each other regardless of the level of demand. This condition, which is forced into quantitative projections by the mathematical properties of the model, is unrealistic.

In the same vein, the "forced condition" — that as one region increases its output, poultry for example, it will be supplied inputs such as grain from other regions in the same proportions as in the past — is also unrealistic. As positive tools for supply analysis, input-output models have the same limitation as regression models in the sense that they must be restricted to observations of inputs and outputs from the past. They are historic or descriptive analyses or relationships as they have existed, and not as they will or might exist under major structural changes. Regression models, however, are not restricted to fixed mix projections of regional outputs and inputs within the historic framework. The supply functions so defined would allow specification of changes in patterns of output, given an increase in demand, among regions. This change in pattern could rest partly on differential qualities of soil and additions of such inputs as fertilizer. Even so, the relationships expressed by the regression model could only be those experienced in the past.

The input-output model is superior to the "transportation model" in this respect: it can consider the impact of changes in demand for many products simultaneously. Through the input-output model, we can "trace back" the impact of demand changes on regional and commodity output levels and the quantity of factor inputs needed region by region. More specifically, we can ascertain how, within the limitations of the model as noted, changes in the demand for one commodity will change the demand for others. The transportation model can be used, if we are concerned with one commodity only, to "discover" what possible impact changes in such things as production techniques and shifts or differential rates of population growth would have on production location, the distribution pattern of products, and regional changes in demand for factors.

OTHER MODELS FOR INTERREGIONAL COMPETITION ANALYSIS

We now turn to the main purpose of this paper, which is to outline other programming models that might be used in supply analysis relating to interregional competition. A model will be outlined that might be feasible for analyzing those interregional supply and competition problems related to change. The nature of this approach will be explained and the difficulties inherent in it (or any method that must extend estimates of change into the future) will be indicated. Before outlining the general model let us pause and review some of the more obvious difficulties involved in such applications.

A programming model could be constructed to consider all competing agricultural products, resources, and fixed factors of different regions. It could incorporate ample detail in terms of the types and number of variables. National agriculture, in fact, could be disaggregated to any extent we wished, even to the degree of treating each farm as a separate entity with its own unique set of resources and products of different qualities. Data collection and computational problems limit such a procedure. It is necessary, therefore, to spatially disaggregate agriculture into a "reasonable" number of regions. The term "reasonable" is yet to be defined, although 104 regions have been used for a particular problem. In programming, the computational limit is related more to the number of equations than to the number of variables.

Size limits aside, programming per se does not provide the means of obtaining the necessary coefficients. It is necessary to know or to estimate the production functions (in various forms) before programming can begin. This knowledge must come from production data and scientists who can supply production relationships and other sources.

The product "supply" and "factor" demand functions derived by programming can be no better than the coefficients obtained from other sources; or no better than the restraints incorporated into the model, such as institutions and time.

Regional Producing Units

The model to be outlined considers regions rather than individual farms to be the producing units. Restraints on outputs are those of the region and various programming activities are defined accordingly. Given the concept of a regional producing unit, we would want to define regions so as to include only those farms that have the same supply or response functions.

For example, an area could be defined in which there are N farms, each with a production function for product Y as indicated by equation 3a

$$(3a) \qquad Y_i = a_i z_i \qquad i = 1, 2, 3, \ldots, N$$

in which $z_i = b_{1i} X_1 + b_{2i} X_2 + \ldots + b_{ji} X_j + \ldots + b_{mi} X_m$ and the X's are the various factors, the b_{ji} are the various weights that define the least cost factor "mix" as it appears to farmers, and a_i is the transformation coefficient of the i-th farm.[4] Theoretically, the farm supply function would be given by equation (3b),

$$(3b) \qquad P_{y_i} = \frac{P_{z_i}}{a_i} \qquad \text{for } y_i \leq m_i$$

in which m_i is the output limit defined by some absolute restraint, such as land, P_{y_i} is the price of the product, and P_{z_i} is the cost of the input mix.[5]

If the ratios $\frac{P_{z_i}}{a_i}$ are equal for all farms, the total supply response for the region can be represented by equation (3c)

$$(3c) \qquad P_y = K \qquad \text{for } Y \leq \Sigma_i m_i \, .$$

Hence, the region can be treated as a single producing unit.

If the ratios $\frac{P_{z_i}}{a_i}$ are not equal, the regional supply response would be characterized as a discontinuous step function such as AB shown in Figure 12.1.

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[4] We have assumed, of course, a Cobb-Douglas type function, which is homogeneous of degree 1. As an approximation of output increases within a limited range, due to expansion by technical units, such an assumption may be quite realistic.

[5] The particular variables that go into the pricing of aggregate z_i will, of course, depend on the length of run.

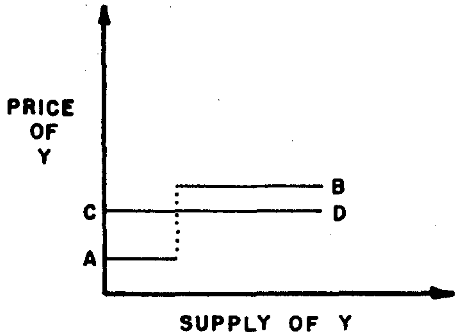

Figure 12.1.

If function AB characterizes the actual one, the use of constant production coefficients to represent an entire region would not be realistic and would assume a regional supply response such as CD in Figure 12.1. If, in fact, the ratios $\frac{P_{z_i}}{a_i}$ in equation 3b are not equal or nearly alike, and if the number of rows in the matrix is not a limiting factor, farms could be stratified, new activities could be defined for each strata with appropriate production coefficients and restraints, and the actual regional production response would be depicted. If the problem is such that the number of rows cannot be increased, the theory of convex sets can be used in some problems to account for intraregional differences in production coefficients. But this matter, which is simply the problem of adjusting the matrix size to the computing facilities, need not be gone into here.

It has not been the intent to imply in the foregoing discussion that only one activity per commodity per region would be considered in the model to be discussed. In this model, any number of activities representing different techniques for the production of the same commodity in each region can be used. These "same product" activities could be of two types: (1) Those representing different activities of the same production function; that is, vectors specifying different factor-output combinations, and (2) activities representing different production functions. If alternative activities are considered for production of a particular commodity in a region, the regional supply response may be similar to that characterized by "step" function AB in Figure 12.1. The lower segments of the step function would represent activities relatively "less efficient" in the use of fixed resources. The converse would be true for the higher segments. Consideration of many production alternatives per product in each region does not mean that farms cannot be represented in aggregate in programming analysis. The essential element for aggregation is that farms are homogeneous in production response. This may mean that they have similar combinations of resources as well as similar production functions.

The authors believe that a model built around a regional producing unit, with certain adjustments for intraregional differences, is sufficiently realistic for interregional competition analysis; especially in consideration of the goal of analysis of prospective changes in regional variables, the data requirements, computational costs, and general "manageability."

SOME FORMAL MODELS

A regional programming model, one in which restraints and activities are defined relative to regions that make up the national aggregate, can be used to define: (1) the production and resource use pattern for a given set of regional factor prices and regional requirements of commodities; (2) the production and resource use pattern for a given set of regional product requirements when we incorporate activities which depict factor supply functions; and (3) production and resource use patterns when we incorporate activities which depict product demand functions and mesh output with regional demands. Regional "supply" relationships can be ascertained by any of these methods by making appropriate adjustments in output or demand.

The following sequence will be used for the remainder of the paper. First, models will be explained that parallel items (1) through (3) above. Second, data will be summarized that has been computed for a model of the first type. Finally, some limitations of and difficulties involved in the use of these models will be outlined.

Model A. For this model, prices of the variable factors, fixed resource levels, and regional requirements of commodities are given. We wish to define (1) the regional production and resource use pattern, (2) commodity flows between regions, (3) regional supply prices for commodities, and (4) the prices (imputed returns) of the fixed resources. Items (1) and (2) are derived by the primal solution and (3) and (4) by the dual solution to the programming problem.

The programming objective is to:

$$\text{(4a)} \quad \min. f(c) = Y_1 C_1 + \ldots + Y_i C_i + \ldots + Y_m C_m + T_1 R_1 + \ldots + T_i R_i + \ldots + T_m R_m$$

in which Y_i is a subvector of product outputs, containing n elements to represent the output levels of n products[6] in the i-th region. C_i is a subvector of n elements representing the per unit variable costs of the n products of the i-th region. T_i is a subvector of n x (m-1) elements which represent the export levels of n products for the i-th region. R_i is a subvector of per unit transfer costs for these exports.

Function 4a is minimized subject to these regional restraints:

[6] As noted before, in some cases two or more of these elements may represent the output levels of the same commodity produced by different methods.

$$\text{(4b)} \qquad y_i^1 f_{ij}^1 + \dots + y_i^k f_{ij}^k + \dots + y_i^n f_{ij}^n \leq f_{ij}$$

in which y_i^k represents the output of the k-th product in the i-th region, f_{ij}^k represents the per unit inputs of the j-th resource used to produce the k-th product in the i-th region, and f_{ij} is the level of the j-th resource available in the i-th region. There are r inequalities or restraints of type 4b per region.

The interregional restraints, the restraints that tie the system together, are defined by restraint 4c:

$$\text{(4c)} \qquad \pm\, y_i^k\, p_i^k + \sum_{i \neq j} {}_i\, t_{ij}^k + \sum_{i \neq j} {}_i\, t_{ij}^k \geq s_i^k$$

in which y_i^k means the same as in equation 4b, p_i^k stands for the output (+) coefficient of the k-th product in the i-th region (or input (-) for this same activity if it represents an intermediate product, for example feed, t_{ij}^k stands for import or export levels of the k-th product with respect to the i-th region (exports have a negative sign and imports a positive sign), and s_i^k represents a constant which defines the requirements of the k-th product in the i-th region. The elements s_i^k representing intermediate goods are zero. The coefficients of all t_{ij}^k terms are unity by definition. There are n restraints or inequalities of type 4c per region.

The length of run nature of model A can be changed simply by shifting factors from the fixed to the variable class, so long as variable factor prices can be taken as given. In this case, a regional restraint or inequality is removed from the system for each resource shifted from the fixed to the variable class. Factor prices also can be set at particular levels to determine how differential changes in factor earnings would affect the production and distribution patterns, and supply prices of commodities by regions.

As factor prices would seldom be expected to be invariant for significant changes in output levels within regions, it is desirable to incorporate factor supply functions, if these functions can be defined, into interregional competition models.

Model B. For this model, prices of variable factors are endogenous (to be determined). All other variables are either exogenous (given) or endogenous as in model A. The objective of this model is to:

$$\text{(5a)} \qquad \begin{aligned} \min. f(c) = {} & F_1 P_1 + \dots + F_i P_i + \dots + F_m P_m \\ & + Y_1 C_1 + \dots + Y_i C_i + \dots + Y_m C_m \\ & + T_1 R_1 + \dots + T_i R_i + \dots + T_m R_m \end{aligned}$$

in which Y_i, C_i, T_i, and R_i have the same meaning as in model A, but some C_i may represent a subvector of zeroes if all resources used by the Y_i activities are either acquired by factor purchasing activities or produced within a region (intermediate products). If this is the case, Y_i represents a "transformation" subvector; F_i is a subvector of factor use levels containing q elements to represent the level of the i-th factor used at each price level in the i-th region;[7] P_i is a subvector of factor prices containing d elements, one element for each factor and for each change in the average supply price of that factor.

Restraints of the type 4b of model A are modified for factor purchasing as in restraint 5b,

$$\text{(5b)} \quad y_i^1 f_{ij}^1 + \ldots + y_i^k f_{ij}^k + \ldots + y_i^m - b_{ij}^1 - \ldots - b_{ij}^z - \ldots - b_{ij}^r \leq 0$$

in which b_{ij}^z represents the quantity of the j-th factor purchased at the z-th price in the i-th region.

Restraints of type 5b are necessary in model B only for purchased factors. Regional resources that are fixed, such as land, would have restraints like 4b for model A.

The optimum solution for model B, that is, when function 5a is minimized, would define a production and resource use pattern that is more realistic than that for model A. This added realism is attained because not all factor prices are taken as invariant. Obviously, changes in the output of predominate crops in a region would result in changes in factor prices. Hence, the incorporation of factor supply functions into the "interregional" competition matrix should produce answers that are more important in regard to regional changes in prospect.

Changes in Demand

Both models, A and B, can be used to define changes in regional production and resource use patterns, given certain changes in regional commodity requirements or "demands" that might be associated with growth or shifts in population. To do this we would simply change the requirement levels in the 4c equations. We could, in fact, "map" the differential changes occurring region by region as one or more of the regional requirements was varied in a continuous manner. Programming routines are available to do this. But when such a procedure is applied to more than a few of the total regions and commodities the computational problem becomes excessive.

Either model A or model B could be used to obtain a more or less general spatial equilibrium solution if regional demand functions were available by commodities. The procedure, though complex, would be as follows: A particular level of output would be selected for each commodity required in each region. A minimum cost solution for supplying these requirements would be obtained. The dual solution would give the

[7] In this formulation, the factor supply function is partitioned into segments and one average price is taken for each segment. In other words, price increases due to increased levels of factors use are approximate. The greater the number of segments used, the more appropriate will be the average factor price.

supply prices associated with the regional requirements assumed. The supplies and associated prices would then be compared with price-quantity relationships denoted by the demand functions to ascertain whether the markets were in equilibrium; that is, "supply price" would be equal to "demand price" for the quantity supplied. If the markets were not in equilibrium, new supply levels would be selected that were in the direction of equilibrium. For example, consider a particular market and product, with the supply price at \$1.50 and the quantity supplied at 100,000 units, as given by the first programming solution. The demand price for this quantity is \$1.25. Hence, to move toward equilibrium, 90,000 units might be selected for the next round of programming. Of course, if the supply price were below the demand price, the target output would be increased in the next programming matrix. Such a procedure would be followed until all markets were in balance (that is, all price-quantities had converged).

However, if intermediate products were not involved, a general spatial equilibrium solution could be more easily worked out if regional market prices were taken as given and the objective criterion were that of maximizing regional profits. If this approach were used, regional outputs could be programmed independently. This would mean a drastic cut in matrix size for programming. The amounts of each product supplied to the different markets would be added together to determine whether markets were in equilibrium; that is, the quantities supplied could be sold at the prices assumed. If markets were not in equilibrium, a new set of prices would be selected which would encourage outputs in the general direction of equilibrium. The programming routine would be repeated again and again with new sets of prices until all markets were brought into balance. In the maximum regional profit approach to general equilibrium we are asking the question: "At <u>these prices</u>, <u>what quantities</u> would be supplied?" In the minimum-cost approach we are asking the question: "At <u>what prices</u> would <u>these quantities</u> be supplied?" The general equilibrium solution would be the same in either case, but the time involved and the matrix size would determine the procedure to be used.

Technology or Other Changes in Supply Structure

The foregoing discussion has emphasized production and output response resulting from changes or shifts in demand. In summary, the general types of programming models outlined could be used as follows: (1) Either a cost minimizing or profit maximizing model could be used to define the optimum pattern of outputs, given the level of national or regional demands for the relevant commodities. (2) Using a cost minimizing model, demand restraints could be varied to trace out the pattern of outputs and their associated supply prices. (3) Using a profit maximizing model, prices could be varied to trace out patterns of supply responses in individual regions and for the national aggregate.

These models are not restricted to analysis of the demand side alone. They can also be used for analysis of changes on the supply side. If analyses were restricted to the demand side, time would have provided sufficient observations, so that very likely this general class of problems could be best solved by regression analysis. But dramatic changes are occurring in production and programming models probably have greatest applicability in analyzing possibilities in regional output response when important changes occur in production functions and institutions. (If changes relating to supply functions originated only through changes in factor prices, it is possible that time series data would provide sufficient observations for sole reliance on regression models.)

If our purpose is to project differential changes in production patterns due to changes in production functions or institutions, we might follow about this procedure: First, we would program to define regional production patterns given the current state of technology, using any of the models or approaches outlined above. Next, we would set up a new programming matrix, in which substitutes would be made for the c_i^k, f_{ij}^k, p_i^k and other elements as affected by changes in production functions, institutions, factor supply or restraints, etc. The general computational procedure as outlined above would then be repeated. The results would allow us to specify how output responses, production patterns, supply prices, and imputed factor returns would be altered if (1) projected coefficients were employed, (2) producers attempted to approximate maximization of certain objectives, and (3) certain conditions in respect to restraints of fixed resources were to prevail.

For such analyses, we would suppose that for the sake of simplicity and manageability of computations in consideration of the size of problems visualized, a comparative statics or "snapshot of time" approach would be used. That is, solutions would be computed for each set of production coefficients, factor prices, and institutional restraints. By this arrangement, the variables in the first set of solutions would not be related to those of a later period in the vein of a true dynamic model. It is not that programming disallows this approach, but it becomes computationally cumbersome for a problem involving more than a few regions and commodities.

Difficulties and limitations abound for regional programming models, just as they do for any other empirical approaches that can now be used. The authors are, because of some experience in trying to manipulate them, perhaps as aware of them as anyone else. Later, some of the difficulties inherent in the application of such models will be summarized. But now, some empirical results generated by a regional programming model are presented. These results entail the types of limitations noted here and elsewhere.[8] The example is not proposed as one illustrating the optimum degree of detail and empirical

[8]Heady, Earl O., and Egbert, Alvin C., "Programming regional adjustments in grain production to eliminate surpluses," Jour. Farm Econ., 41:718-33, 1959.

adequacy. It is presented only as a relatively simple example (although certainly a difficult example in terms of data assembly and computational routine) of the general types of data that can be generated by regional programming models. The emphasis in this model is on regional production patterns. As outlined previously, another step will produce an expression of supply functions and factor demand relationships. The model to be explained employs those techniques existing at the time the study was initiated. Further analysis is being made to define production patterns under potential changes in technology and demand. Also, project plans call for the incorporation of livestock production, but under a greater degree of aggregation, into the model.

AN EMPIRICAL MODEL

The model and results presented below required several man years of data assembly, model construction, and computing. This initial investment would be required for most regional models of some detail. However, once the data have been assembled and converted to appropriate form, professional time spent can be less for other phases of the analysis. Only a few of the possible analytical phases or steps are outlined below.

Nature of the Model

The analysis deals with wheat and feed grains. The objective function for programming was similar to function 4a. Three products were considered for each region: food wheat, feed wheat, and a feed grain composite consisting of corn, oats, barley, and grain sorghums weighted by the average relative acreage of each planted in the particular region. One hundred and four production regions were considered. To make the analysis manageable, the consumption regions were limited to ten. Thus there were three elements in each of the 104 Y_i subvectors and 27 elements in each of the 104 T_i subvectors of functions 4a. Each element, c_j^k, of the subvector C_i, represented the per unit cost of producing the k-th grain in the i-th region and included costs that were due to labor, power, machinery, seed, fertilizer, and related inputs. Land and overhead costs were not included in this cost calculation. Each element, r_{ij}^k, of the subvector R_i, represented the per unit freight cost of transporting the k-th grain from the i-th production region to the j-th consumption region.

Only one resource restraint (F_i in the 4b inequality) was used for each region. This was the maximum acreage planted to the five grains in the last eight years. Because feed wheat was considered as a substitute for other feed grains, only two demand restraints (inequality 4c) were needed for each consumption region. S_j^1 was an estimate of the food wheat disappearance in the j-th consumption region in 1954,

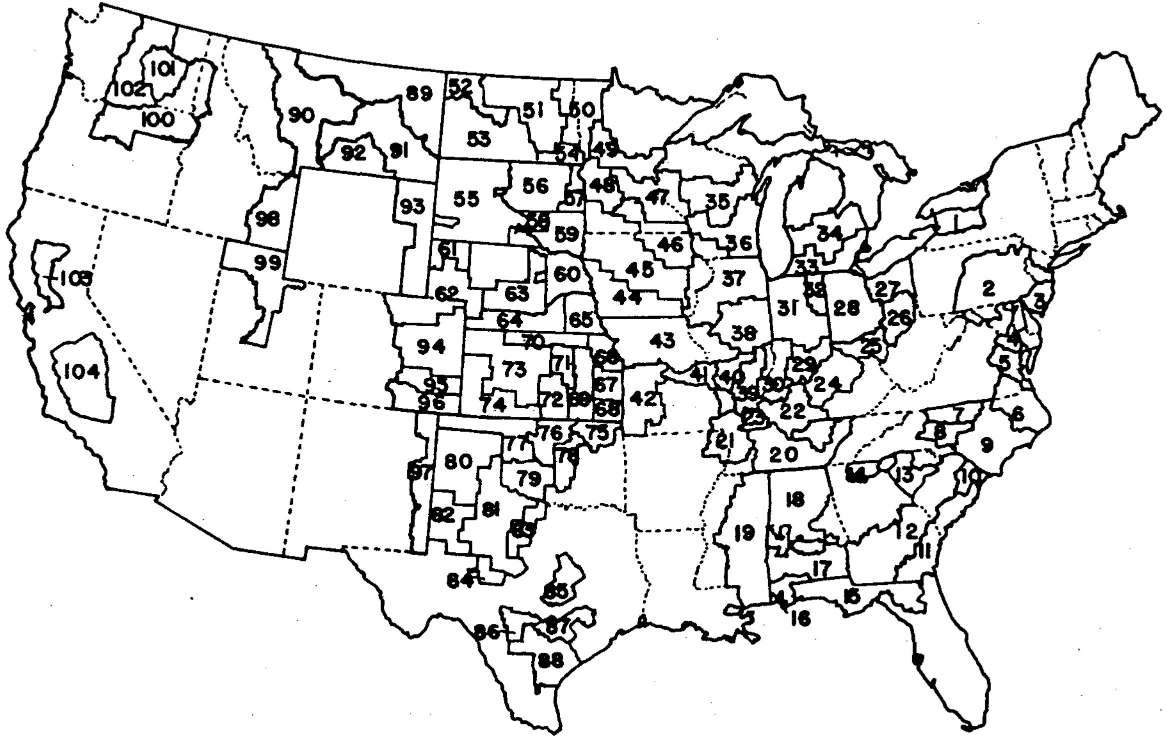

Figure 12.2. Production Regions.

adjusted for normal per capita consumption and including net exports. S_j^2 was an estimate of the feed grain disappearance in the j-th consumption region in 1954, adjusted for normal livestock consumption.

The geographic boundaries of the production and consumption regions are shown in Figures 12.2 and 12.3, respectively. Freight costs for each activity were based on shipping points and destinations near the centers of these production and consumption regions. Freights tariffs existing in May 1954 were used to compute these costs. Production costs and outputs were computed from regional production data and average prices of 1954.[9]

Results

Regional flows of wheat given by the minimum cost solution to the empirical model are shown in Table 12.1. Table 12.2 shows the feed grain shipments between regions. The data presented in these tables represent aggregates of the exports of the production regions outlined in Figure 12.1. For example, the figure 58,989, Table 12.1, represents the total shipments of wheat from four Corn Belt production regions to the Northeast.[10] Except for the Southeast, the historical deficit areas are shown as importers and (except for possibly the Mountain States) their suppliers are the traditional surplus areas. More detailed results to be presented later will show the Southeast supplying its own wheat. This means wheat replaces corn production in this area. Only a few of the normal grain regions of the Mountain States (located in Montana) are shown as wheat producers. We shall comment further on these results later.

Table 12.1. Wheat Shipments

	Destination										
Origin	N. east	App.	S. east	Lake States	Corn Belt	Delta States	N. Plains	S. Plains	Moun.	Pac.	Total exports
						(1,000 bushels)					
Northeast		--	--	--	--	--	--	--	--	--	--
Appalachian	--		--	--	--	--	--	--	--	--	--
Southeast	--	--		--	--	--	--	--	--	--	--
Lake States	69,654	--	--		--	--	--	--	--	--	69,654
Corn Belt	58,989	--	--	--		--	--	--	--	--	58,989
Delta States	--	--	--	--	--		--	--	--	--	--
Northern Plains	82,507	--	--	--	--	59,850		--	--	--	142,357
Southern Plains	--	37,213	--	--	--	--	--		--	--	37,213
Mountain	--	--	--	--	--	--	--	--		--	--
Pacific	--	--	--	--	--	--	--	--	--		--
Total Imports	211,150	37,213	--	--	--	59,850	--	--	--	--	308,213

[9]For added details see Egbert, A. C., and Heady, E. O., "Programming regional adjustments in grain farming to balance production with consumption," USDA Tech. Bul., (in process).

[10]These four regions, as well as the other 100 production regions, were entered into the programming matrix as separate entities. Shipments of the production regions are aggregated in Tables 12.1 and 12.2 to simplify the presentation.

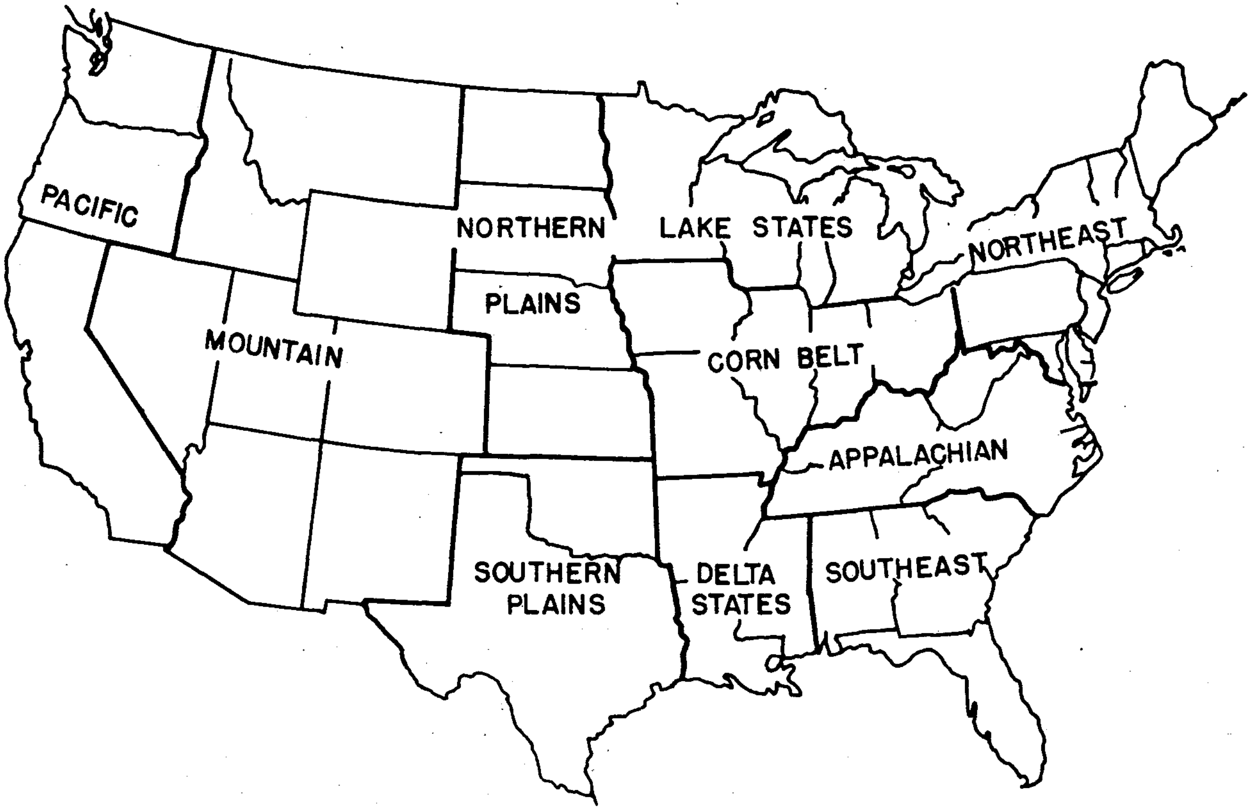

Figure 12.3. Consumption Regions.

Table 12.2. Feed Grain Shipments

Origin	Destination										
	N. east	App.	S. east	Lake States	Corn Belt	Delta States	N. Plains	S. Plains	Moun.	Pac.	Total exports
	(1,000 bushels)[a]										
Northeast		--	--	--	--	--	--	--	--	--	--
Appalachian	--		--	--	--	--	--	--	--	--	--
Southeast	--	--		--	--	--	--	--	--	--	--
Lake States	--	--	--		--	--	--	--	--	--	--
Corn Belt	296,391	118,419	198,923	--		--	--	--	--	--	613,733
Delta States	--	--	--	--	--		--	--	--	--	--
Northern Plains	--	--	--	--	--	129,167		20,806	--	74,266	224,239
Southern Plains	--	--	--	--	--	--	--		--	--	--
Mountain	--	--	--	--	--	--	--	--		--	--
Pacific	--	--	--	--	--	--	--	--	--		--
Total Imports	296,391	118,419	198,923	--	--	129,167	--	20,806	--	74,266	837,972

[a]In corn equivalents

As shown in Table 12.2, only the Corn Belt and the Northern Plains are feed grain exporters. But again, historical deficit areas are shown as importers; namely, the Northeast, Appalachian, Southeast, Delta States, Southern Plains, and Pacific. The shipments from the Northern Plains are primarily from the Corn Belt fringe in Nebraska. This fact makes the feed grain flow pattern more plausible than it might appear to be on the basis of Table 12.2.

Although Tables 12.1 and 12.2 make it appear as though there is little interdependence or competition between these ten broad areas, such is not the case. For one reason, interdependence is built into the model; for another, shifts in regional requirements would change significantly these regional flows.

As mentioned before, one objective of this analysis was to obtain supply prices in each of the ten consumption regions. These prices are shown in Table 12.3. One apparent "inconsistency" in the prices shown in this table is the feed grain supply price, at 1954 price levels, shown for the Mountain States. Here, historically, the price of feed grain has been about the highest in the United States. But perhaps a price of 69 cents is not so unreasonable for this area when considered in light of

Table 12.3. Wheat and Feed Grain Supply Prices, by Regions

Region	Wheat price	Feed grain price
	(dollars)	(dollars)
Northeast	1.48	1.05
Appalachian	1.53	1.16
Southeast	1.39	1.24
Lake States	1.11	0.88
Corn Belt	1.04	0.67
Delta States	1.22	0.98
Northern Plains	0.73	0.65
Southern Plains	1.21	1.08
Mountain	0.77	0.69
Pacific	1.28	1.12

the fact that an "output brake" was imposed by the model because of a wheat and feed grain economy with current overcapacity. Hence, if the historical wheat areas in the Mountain States were "squeezed out" of wheat production by market forces, feed grain production (in this case barley) could easily be expanded. Consequently, the price of feed grains would decline. Aside from this, the regional supply prices shown in Table 12.3 appear to be reasonable, at least in the relative sense.

The regional producing units, the kind and quantity of grain produced by each, and the imputed returns per acre of grain land are shown in Table 12.4. The reader can obtain a more vivid mental image of the specified regional production pattern if the data shown in Table 12.4 are related to Figure 12.2. He should conceive that grain production is absent in much of the Southeast, the Delta States, North Dakota, eastern Kansas, and the Mountain States, and in some other scattered areas. This production pattern represents an "ideal regional distribution of production" arising from "pure" interregional competition in two farm products given invariant consumption at a point in time, in this case 1954.

Furthermore, when interpreting these results the reader should remember that (a) spatial production patterns implied by Table 12.4 were computed under the assumption of techniques (that is, technical coefficients) equal to the average of each region and (b) the coefficients are constant within the delineated regions. Locational variations from the coefficients used would mean that some acreages in the regions not designated for production would "remain" in grain production and some acreages in regions designated for production would be "withdrawn."[11]

The returns per acre of grainland or imputed rents shown in Table 12.4 are actually the maximum net return achievable in each region from the given production opportunities when shipments are made only to the most profitable market (or markets). Production and distribution take place only when the market price (Table 12.3) will at least cover unit supply costs. The regions shown in Table 12.4 for which no production is listed represent areas in which 48.5 million acres are usually planted to grain. To summarize, the data presented in Tables 12.1 through 12.4 characterize a production and distribution pattern that would be expected in a purely competitive economy — one of restricted nature by definition of the model and the data that went into it.

As suggested previously, many modifications can be made in the model to investigate the effects of regional changes in prospect. With adequate funds and computing facilities and time, we could add competing and complementary commodities such as soybeans, cotton, beef, pork, etc., and tie these all together in one huge matrix. We could add factor supply equations if we had them and, looking to the future, production coefficients could be modified to account for technical innovations in prospect. Product demand equations would be used to define a

[11] For details on limitations of the analysis, see Egbert and Heady, op. cit.

Table 12.4. Specific Grain Production Levels and Imputed Returns to Grain Land in Production Regions

Region	Wheat production	Feed grain production	Imputed rent
	(1,000 bushels)	(1,000 bushels)	(dollars)
1	16,197	--	10.49
2	--	97,567	10.88
3	--	19,189	15.82
4	--	13,075	14.35
5	--	7,434	12.00
6	--	20,029	9.53
7	--	--	--
8	--	--	--
9	--	--	--
10	--	5,780[a]	1.92
11	--	--	--
12	--	--	--
13	367	7,665[a]	0.12
14	--	--	--
15	--	--	--
16	2,087	--	8.40
17	14,856	--	4.64
18	24,058	--	3.66
19	--	--	--
20	--	24,651	1.44
21	8,971	8,771	4.87
22	--	36,399	9.41
23	--	8,342	8.85
24	--	11,089	8.98
25	--	27,833	4.68
26	--	17,770	1.15
27	27,691	--	0.78
28	--	233,287	6.46
29	--	--	--
30	--	72,903	8.30
31	--	222,916	6.86
32	26,847	--	10.52
33	5,804	48,348	2.40
34	63,850	--	1.10
35	20,494	--	1.10
36	18,941	61,318	8.83
37	--	356,616	8.72
38	--	231,170	14.30
39	--	39,025	4.01
40	19,630	--	1.36
41	14,756	--	2.14
42	--	--	--
43	57,690	82,317	3.64
44	--	153,258	5.01
45	--	403,933	8.53
46	--	157,062	7.63
47	--	84,314	5.93
48	--	73,085	10.01
49	--	30,795	3.79
50	--	89,054	3.52
51	--	--	--
52	--	--	--

[a]Wheat used for feed

Table 12.4. (Continued)

Region	Wheat production	Feed grain production	Imputed rent
	(1,000 bushels)	(1,000 bushels)	(dollars)
53	--	--	--
54	--	--	--
55	--	31,182	0.27
56	--	68,643	1.29
57	--	--	--
58	--	17,430	0.82
59	--	94,739	3.87
60	--	113,013	6.64
61	5,667	--	2.18
62	10,764	35,771[a]	0.20
63	--	--	--
64	28,104	--	2.46
65	--	148,798	4.03
66	--	--	--
67	--	--	--
68	--	--	--
69	--	--	--
70	17,098	--	0.06
71	18,212	--	0.40
72	37,617	--	2.76
73	72,121	--	1.60
74	3,795	74,266	2.12
75	--	--	--
76	31,628	4,309[a]	6.20
77	--	35,114	3.95
78	--	5,841[a]	1.60
79	20,898	--	4.95
80	--	76,302	15.04
81	14,561	--	3.51
82	--	16,449	5.98
83	3,063	--	2.99
84	--	1,088	3.54
85	--	7,871	5.35
86	--	1,388	1.25
87	--	5,673	2.75
88	--	12,359	10.09
89	--	58,300[a]	1.18
90	--	0.346[a]	0.00
91	--	--	--
92	--	--	--
93	--	--	--
94	--	--	--
95	--	--	--
96	--	--	--
97	--	--	--
98	--	--	--
99	--	--	--
100	79,077	--	10.59
101	5,712	56,911	9.31
102	--	7,083[a]	6.01
103	6,896	--	5.82
104	--	30,643	15.87
Total	677,452	3,548,514	

[a] Wheat used for feed

general spatial equilibrium pattern. Thus a "longer-run ex ante outlook" on American agriculture would be provided to guide the decisions of farmers and those concerned with aiding farmers in the adjustment process.

PROBLEMS AND LIMITATIONS IN THE USE OF INTERREGIONAL PROGRAMMING MODELS

Most of the problems or limitations in the use of interregional programming models fall into three classes: size, data, and research resources. The most important of these, as noted earlier, is research resources. If we had sufficient resources, the other problems would disappear; and we should include human resources as well as funds. But because resources are limited, the other two limitations are highly relevant. Hence, we discuss some of the problems relating to them.

Relatively few variables in an interregional programming model generates a mammouth coefficient matrix. Although only 104 production regions, each with three activities, and ten consumption regions were considered in the wheat and feed grain problem just described, the resulting matrix was of a 124 x 3,120 order. With a matrix of this size, "bookkeeping" alone is a significant problem. In addition, it leads to difficulties in finding and financing computing facilities of adequate capacity. But future advances in computing technology may easily take care of most of the size problems.

Problems relating to data for interregional competition studies may be most significant for some period ahead. Much information on farm production functions or input-output coefficients is needed. A large part of that currently available is very fragmentary, incomplete, or out of date. The problem of defining and obtaining coefficients for approaching technology is especially important. If farm production functions were available by type of farm, managerial level, and soil class, data relating to these variables could be incorporated in the matrix, even to the extent of taking into account changes in scale returns. There is even the possibility of using programming "gimmicks" to consider such things as discontinuities of inputs and interactions between activities or enterprises.

Production functions or coefficients are needed for the marketing sectors of the system as well as for farms in interregional competition models. A dearth of information now prevails. Reliable input-output data in the marketing sectors are difficult to acquire, even when resources are adequate.

If size, data, and research resources did not place limitations on models dealing with interregional competition, could we "exactly" describe the production and distribution pattern, and the general supply situation of 10 or 20 years ahead? Very probably we could not. Technological changes and innovations of the future could be forecast only imperfectly. Even though all measurable economic variables could be "captured," present and future, in one great system, the human psyche or actions of people themselves may distort the "best laid" projections. Hence, the "most exact" programming models might be in error in much the same way that extrapolation into the future with regression models could produce erroneous projections.

Certainly, programming is not the "perfect tool" for interregional competition analysis. However, the authors believe, that given the current state of the arts, it offers one of the better means for analyzing the prospective effects of major changes now occurring in the economy and those now appearing on the horizon. They believe that compared with other formal empirical tools available, it has more inherent flexibility for accounting for change and for considering fixed factors and different lengths of run. Its major limitations arise from data requirements and in defining restraints that are consistent with those of the real world.

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Discussion

IT IS CLEAR that the impact of technology, organizational innovation, economic growth, and demand shifts have not been uniform among regions within this country. This being the case, analyses based on national aggregates do not provide an adequate basis for measuring the effects of structural changes, nor for evaluating the price, income, and efficiency implications of alternative policy and program proposals. The renewed interest of agricultural economists in the structure of interregional competition is commendable and the Egbert and Heady paper is a significant contribution to the growing literature in the area. The Egbert-Heady review of the characteristics and limitations of alternative interregional competition models will prove to be particularly helpful to other researchers and students of the problem.

There are three major conclusions explicitly or implicitly expressed in the Egbert-Heady paper. These are:

1. Programming models presently offer the most effective means of analyzing major structural changes in the economy.

2. The major impediment to the effective application of the various programming models is a critical deficiency of data and computing facilities.

3. Programming models provide a suitable framework for incorporating available coefficients into models approximating general equilibrium systems.

The discussion which follows is primarily devoted to an analysis of the first two of these propositions.

Problems relating to the inclusion of "shifters" in regression models has long been recognized and has been analyzed in other papers presented at this conference. For this reason, as well as the greater flexibility of the programming models, I tend to agree that programming models may offer a superior means of analyzing change. At the same time, the manner of evaluating structural change in static programming models is far from obvious. Clearly other tools have failed to provide the basis for predicting major structural changes, such as the rather recent shifts in the broiler industry. However, it is not clear that programming models could have done better.

Egbert and Heady conclude that transportation models lack generality and must play a minor role in interregional competition studies. Specifically, the transportation model is concerned with spatial allocation of a single commodity when production and consumption for each region are predetermined. The problem is to minimize transportation costs in satisfying the predetermined regional bill of goods. We might accept the single commodity limitation, but the independence of supply, demand, and prices postulated by the transportation model is not a tenable assumption.

The Enke-Samuelson-Berkman spatial equilibrium model may be superior to the pure transportation model in that product price differentials between regions and total consumption within regions are endogenously determined, given predetermined production, along with demand relationships for each region. Thus this model enables the investigator to examine the implications of changes in supply, shifts in demand variables and parameters, and changes in transportation or processing costs for equilibrium shipments and prices. An alternative formulation is to take total consumption in each region as a predetermined variable with supply to be determined endogenously.

Input-output models emphasize the interdependence of the various sectors of the economy. This is obviously a desirable feature and an important advantage over the transportation model and its preoccupation with a single commodity. However, input-output models relate to historical periods and their application to analyses of structural changes is yet to be demonstrated. Thus Egbert and Heady are probably correct that the more general programming models offer a more promising approach to the relevant question at hand.

Perhaps without exception, economists engaged in empirical research decry the lack of "suitable" data, time, and computing facilities. In the case of generalized programming models such as those described by Egbert and Heady, data and computing requirements are indeed imposing. Such models require data essentially in the form of supply, demand, and price parameters by regions. Nevertheless, it would appear that important conceptual problems must be solved before the nature of the data needs for programming are specified and developed.

It should be clear that data, computational, and conceptual problems are interdependent. Empirical models are compromises between incomprehensible and unmanageable but realistic detail and understandable and manageable simplification. In the case of interregional competition problems, simplification must be the rule rather than the exception. I would hypothesize that larger matrices and larger computers are not the answer. Rather, researchers must develop models with relatively high degrees of aggregation which are capable of reflecting the magnitudes of the most relevant variables.

All would agree that supply relationships should be built into interregional programming models. But are we interested in normative or predictive relationships? In a like vein, what length of run is most relevant? Similar questions could be raised with respect to demand. However, work to date seems to suggest no differences as to length of run in demand relationships.

Perhaps the proposed use of the results may dictate the nature of the supply relationship desired. If interest is centered on economic efficiency or normative product-resource flows given pure and perfect competition, as appears to be the case in the Egbert-Heady work, normative supply relationship would appear appropriate. On the other hand, if one is interested in providing guides for program evaluation or administration, the predictive models may be more suitable. Actually both the normative and predictive estimates will be necessary if we are to adequately measure the gains and losses implied by alternative programs. Also the "no man's land" between normative and predictive models may offer a fertile field for investigation.

A conceptual problem of first order importance relates to the quantity to be maximized. We have traditionally limited ourselves to criterion functions consistent with efficiency models assuming pure and perfect competition. We must broaden the scope of our research if we are to play a significant role in policy evaluation and program guidance. We need to measure the effects of alternative institutional restraints, including market or supply controls, on the total and the farm economies and regions within the farm economy. I would hypothesize, for example, that agricultural economists have little concept of the nature of intra-industry and inter-regional income transfers which would result from a program of comprehensive supply controls. In fact, the notion of supply controls is rivaled only by the free or flexible price idea as being the most over-advocated, under-analyzed proposition of our time.

Historically, demand relationships have apparently been much more stable than supply relationships. Thus it is generally conceded that our knowledge of demand is superior to our knowledge of supply and that the critical void is the area of supply. Nevertheless, most available demand coefficients are of the ceteris paribus nature, while in the real world other things simply do not remain the same with a change in one price or quantity. Such an assumption may be tenable in dealing with a single commodity or group of commodities, however the crucial problems in agriculture relate to total supply-demand relationships.

Egbert and Heady are fully aware of the limitations of the empirical results which they present. Yet the rather simple model presented does indicate the tremendous complexity and suggests the substantial understanding that could be derived from a more general model. Also it should be clear from the Egbert-Heady paper that the various approaches to supply analysis can be highly complementary when results from various forms of analyses are incorporated into a generalized programming model so as to allow a general equilibrium analysis. Nevertheless, it is likely that over the near term partial equilibrium analyses such as the Egbert-Heady approach would seem to offer a fruitful field for further research.

Chapter 13

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Interpretation of Regional and Spatial Models

NUMEROUS EXAMPLES exist of empiric studies to estimate equilibria among regions in prices and flows of agricultural products. A few such studies particularly relevant for adjustment problems of Midwestern agriculture are summarized below. Limitations of findings imposed by the models and of the data available for use of the models are then examined in light of these adjustment problems. Some extensions of the models also will be suggested, as well as alternate approaches to research relevant to adjustment problems.

SPATIAL EQUILIBRIUM MODELS

Agricultural economists long have demonstrated a strong interest in problems of interregional trade.[1] Closely associated are studies in the theory of production location.[2] Isard has shown that theories of trade over space and of location are two views of the same phenomenon. He also shows that important features of leading variants in both trade and location theory can be translated into each other and integrated into the general theory of production economics (11).

Interest has been stimulated by development of programming models by Enke, Baumol, and Samuelson. Enke (6) demonstrated in 1951 that with "a relatively simple electric circuit," a model could be used to generate estimates for a single product in each of several regions, (1) net price and (2) quantity, if any, of exports or imports; and, among regions, (3) aggregate trade and (4) volume and direction of interregion trade. Each region is specified to be separated from other regions only by a transport cost per physical unit of product independent of volume or direction of shipment. Price-quantity relations as well as transport rates are taken as givens.

[1]Though a long list might be mentioned, we refer the reader to the last major effort prior to the recent applications of operations research methods, Mighell and Black (14). Farms were budgeted in each of six areas sampled in the Lake States and New England in 1935-36 and 1945-46, the objective being to predict the relative responsiveness of the two areas to likely changes in demand conditions.

[2]Again, a rich literature is available in economics generally and in agricultural economics specifically. See especially Isard (11) and Dunn (5). Some features of Dunn's contribution might be used in probing for boundaries and shifters thereof between regions, a problem neglected or "solved" arbitrarily in the studies outlined below.

Baumol (2) and Samuelson (15) formulated the same problem as one of maximizing an objective function, given demand and supply functions in each of three or more regions separated, as were Enke's regions, by transport rates independent of volume or direction of shipment. In all three models production, consumption, and shipments occur at a single point in each region. Subject to obvious empiric limits imposed by this simplification, the models are adaptable to the study of change in any three of the four sets of data already mentioned, given a postulated change in some part(s) of the fourth set of data. To overcome this limitation, Beckmann (3) has developed a model capable of generating equilibria in the presence of continuous change in distributions of production density and transport costs.[3]

Most agricultural applications of spatial equilibrium models have been of adaptations of the Baumol or Samuelson type.[4] Hence we turn to a brief summary of such a model and its properties. In each of two regions we define for a single product an "excess supply function." It relates to prices of the product differences between quantity supplied and quantity demanded at respective prices. Each such quantity difference represents a surplus exportable at the given price from the region. Figure 13.1 is due to Samuelson. On the vertical axis we represent

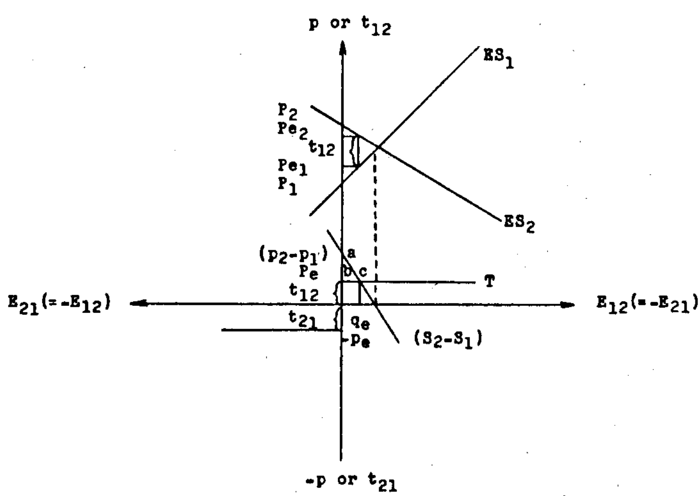

Figure 13.1. Equilibria in prices and quantities: a single product in two markets.

[3]It may be something of an anomaly, however, to regard Beckmann's model as one to use in a study of "interregion relations." The power of his model is in its ability to depict an area of continuous change instead of one divided into regions. However, a careful review of the model reveals that the requirements for making it empirically usable are considerable.

[4]For an alternative type, see Henderson and Schlaifer (10). This "transportation" model is a special case of the more general model of Samuelson, Baumol, et al, though certain computational advantages make its extension for some problems easier than is the case with the more general model.

monetary rates: positive for quadrants I and II; negative for quadrants III and IV. On the horizontal axis we show quantities of product: positive for quadrants I and IV; negative for quadrants II and III.

ES_1 and ES_2 are excess supply functions in, respectively, regions 1 (R_1) and 2 (R_2).[5] In Figure 13.1, prices or transport costs are measured vertically; quantities shipped, horizontally. Equilibrium in R_1 (R_2) yields p_1 (p_2), since at this price supply equals demand and ES_1 (ES_2) is zero. The fact that p_2 exceeds p_1 suggests that if interregion trade develops it will consist of shipment from R_1 to R_2. Whether trade will develop depends on the cost of shipping the product from R_1 to R_2. Equilibrium is established with product flow q_e from R_1 to R_2 and prices p_{e1} and p_{e2} in R_1 and R_2, respectively. The difference in price is exactly equal to the cost, t_{12}, of transporting a unit of product from R_1 to R_2. Should the prices without trade have been reversed, trade would have been reversed. Should they have differed by less than the transport rate, t_{12}, no trade would have developed.

The curve (S_2-S_1) is a locus of points representing the vertical differences (P_2-P_1) at scheduled quantities, E_{12}. Thus it shows in terms of joint equilibria the same information as do ES_1 and ES_2. Ordinates of the curve T are transport rates between regions. At q_e, (S_2-S_1) equals T. Trade at t_{12} would develop so long as the negatively sloped (S_2-S_1) intersects T to the right of the vertical axis. Should the intersection occur between p_e and $-p_e$ no trade would occur between regions. Indeed the gain from trade is given by the area of triangle abc, the difference between total gain oacq_e and transport cost obcq_e. The fact that this gain can be formulated as a function to be maximized makes possible a normative approach to problems of interregion trade.

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[5]For models in which supply for the ith region is taken as fixed, the slope ES_i depends solely on the slope of the demand curve in the ith region. With supply functionally related to price, it can be shown that the slope of the excess supply curve is simply the algebraic sum of the slopes of the supply and demand functions.

We define for a given region an "excess supply" (ES) as the difference at a given price (p) between quantity supplied (s) and quantity demanded (d). The quantity supplied is given by

$$s = a_s + b_s p + u \tag{1}$$

where a_s and b_s are constants in the supply relation and u is a source of random variation in s. The quantity demanded is given similarly by

$$d = a_d - b_d p + v. \tag{2}$$

The excess supply at any given price is thus given by

$$\begin{aligned} ES &= s - d \\ &= a_s + b_s p + u - (a_d - b_d p + v) \\ &= a_s + b_s p + u - a_d + b_d p - v = (b_s + b_d)\,p + a' + w \end{aligned} \tag{3}$$

where a′ represents the collection of constant terms and w the net effect of u and v. Differentiating relation 3 with respect to p we get as the slope of the excess supply function in the ith region:

$$b_{ES} = b_s + b_d \tag{4}$$

Thus, to assume b_s equal to zero is equivalent to assigning to b_{ES} a minimum of likely values.

We digress here to point out that the function T could reflect a transport rate t_{12} that declines with volume of shipment, E_{12}. The general result would be an increased quantity of interregion trade. Should the transport rate from R_2 to R_1 differ from the transport rate from R_1 to R_2, the values of p_e and $-p_e$ (sign neglected) would differ. So long as p_2 exceeds p_1 in pretrade equilibrium, the only consequence of the latter extension is to vary the price differential that can exist without inducing trade between regions.

The gain from interregion trade is a nonlinear function of trade volume. That is, the area of triangle abc increases or decreases with respect to shifts in the curve (S_2-S_1) at a rate that is not constant with respect to changes in E_{12}. Moreover, the extension of the two-region problem to a problem of n regions is not obvious. However, Samuelson (15) shows that at a maximum of gain, the following relation holds between any pair of regions R_i, R_j:

$$-t_{ij} \leq (p_j - p_i) \leq t_{ji} \qquad (i, j = 1 \ldots n)$$

where t_{ij} (t_{ji}) is the rate of transport cost from R_i (R_j) to R_j (R_i) and p_i (p_j) is the price of product in R_i (R_j) at an interregion equilibrium. The problem then may be visualized as one of finding price differences that will maximize total gain from interregion trade, subject to transport costs between regions.[6] Solution to this problem is equivalent to a solution that minimizes the sum of transport costs subject to equilibrium price differences.

The major assumptions imposed on investigations of interregion competition in which spatial equilibrium models are applied are summarized here. There are, first of all, assumptions common to models generally, arising from attempts to simulate real phenomena with either scale replicas or, still more abstractly, variables and relations among variables. Since applications of spatial equilibrium models have employed programming techniques, they employ the special assumptions common to programming models (4): (1) linearity (in space-related variables), (2) divisibility (in quantities of product flow), (3) additivity, i.e., imports in R_j are independent (other than in terms of transport costs and demand) of imports in regions other than R_j and (4) finiteness (of all possible divisions into which a space might be divided, only n divisions are considered, among which k export and (n-k) import).

Because spatial equilibrium models are specialized examples of programming models, still further assumptions are involved. Of greatest relevance for adjustment problems are the following: (1) within regions, spatial homogeneity with respect to opportunity cost in terms of nonoptimized products,[7] and a sum of deviations from the mean of

[6] It is also required that over all regions, total supply equals total demand, unless allowance is made for accumulation or depreciation of stocks.

[7] A related assumption is that resources are assumed immobile as among regions. However, this assumption seems not particularly limiting since resources are easily conceived as products.

quantity-weighted transport cost at each regional basing point, equal to zero; (2) between regions, homogeneity in elasticity of supply of optimized product with respect to price(s) of nonoptimized product(s); and (3) extension to regional entities of normative behavior otherwise ascribed to decision makers at levels of firms and sovereign public bodies.

AGRICULTURAL APPLICATIONS IN SINGLE PRODUCT STUDIES

Judge and Wallace (12) used an adaptation of the Samuelson model to estimate an equilibrium marketing pattern for slaughter beef under production and consumption conditions in 1955. For each of 21 regions in continental United States, beef supply was taken as given and perfectly inelastic with respect to price of slaughter beef. Demand in each region was estimated by the following equation:

$$Y_{Bi} = 78.3543 - 1.0529\, x_{1i} + 0.6509\, x_{2i} + 0.0303\, x_{3i}$$

where for region i,

Y_{Bi} is per capital consumption of slaughter beef in pounds.

X_{1i} is retail price of beef in cents per pound.

X_{2i} is price of pork in cents per pound.

X_{3i} is disposable income per capita in dollars.

A basing point was selected for each region. Truck and rail costs between basing points were estimated. The lower of the two was used to reflect transport costs between regions.

In Figure 13.2 we show the general results that were obtained in shipments that minimize transport costs. For each region the top number indicates the amount of slaughter beef "produced"[8] in millions of pounds. The following "E" numbers indicate either exports from the region (E) or imports into the region (-E), also in millions of pounds. Subscripts refer to regions of either origin (for -E) or destination (for E). Following the E number(s) is the total regional consumption of slaughter beef in equilibrium. The last number is the price of slaughter beef consistent with the interregion system of production, consumption, and shipments. The shipments are so organized that transport costs are minimized subject to the separate demand and supply functions for each region and the constraint that total production for the U.S. equals total consumption for the U.S. within the year.

The Iowa-Nebraska region produces the largest supply of beef and also the largest export of beef. In total supply it is followed closely by the Illinois-Indiana region. However, the Illinois-Indiana region produces only a small export that goes entirely to the deficit region of

[8] The product is beef slaughter cattle in carcass weight.

Michigan and Ohio. The Iowa-Nebraska region exports to regions centered on (in order of importance) Philadelphia, New York, and Roanoke, Virginia. It furnishes the Philadelphia region its sole supply of imports. It furnishes the New York market with more than two-thirds of its imports, the remainder being supplied from the Minnesota-Wisconsin region. Exports to the Virginia-West Virginia-North Carolina region are relatively small. This market is also shared. The Kansas-Missouri region furnishes it with more than twice the volume supplied by the Iowa-Nebraska region. The Kansas-Missouri region also ships a small volume to the Kentucky-Tennessee region, otherwise self-sufficient in beef.

The prices shown in Figure 13.2 are equilibrium prices in the restricted sense permitted by the model. In each region it is the price that equates the given supply with the amount consumed within the region plus or minus the amount imported or exported, where the latter is consistent with a minimum total of interregion transport cost. It is interesting to compare these with prices observed in the regions in 1955. Should they correlate highly one might conclude that (1) transport costs are important as criteria of interregion organization and are well represented despite the simplifying assumptions already referred to; and (2), as a corollary of (1), that the substitution relations not taken into account are not as important as is commonly assumed by agricultural economists. Unfortunately, prices for 1955 are not available for such a comparison. However, they are for the years of 1947 and 1952. The squared correlation coefficients are less than 0.50.

Thus, only a small fraction of geographic price variation is explained by shipments that minimize transport costs. Yet the phenomena of interregion specialization are shown to be important by assuming, with observed supply, the consequences of no shipments. In deficit regions prices become high, while in the surplus regions the product sells at low prices. In the New England region the price goes to more than \$5.00 per pound. In the Iowa-Nebraska region the price drops to seven cents per pound. Such variations as these would generate supply response in each region. However, regional specialization is of evident importance.

These results are of considerable interest in terms of adjustment problems of individual farms and areas. They illustrate that all regions are affected by a change of "givens" in any one. Thus, what happens to consumer income in the Philadelphia region is vitally important to cattle feeders in Iowa-Nebraska. The population of this importing region is 20,213,000. Hence, an increase or decrease of \$1.00 in per capita disposable income increases or decreases the demand for beef by 6.1 million pounds (= .0303 x 20,213,000). More than half of the variation traces back to the Iowa-Nebraska region. In turn, increased or decreased demand for beef from this region reverberates throughout the interregion system.

The assumption of fixed supply restricts the empiric significance of the results to a short time run. The authors ascribe to it a one-year

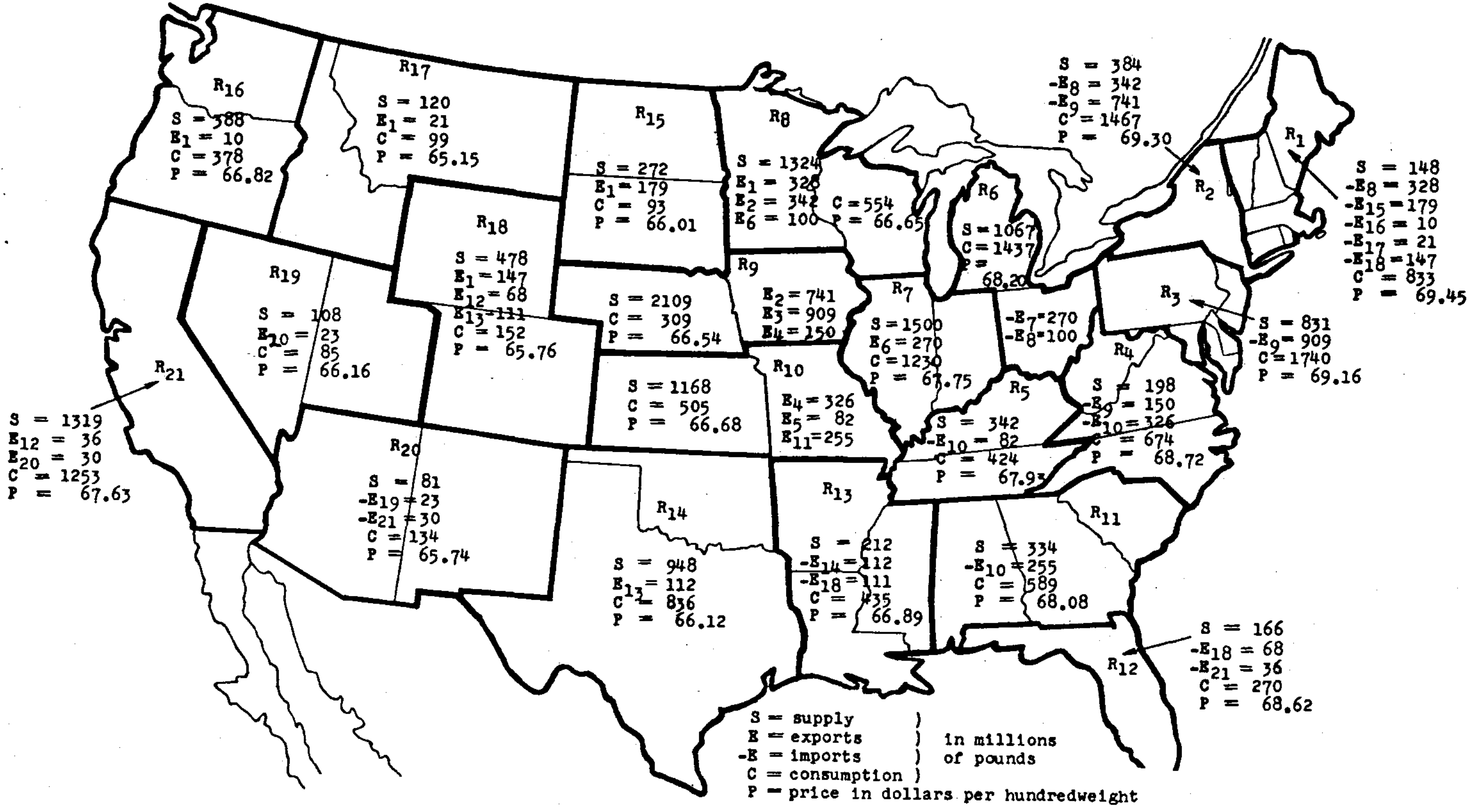

Figure 13.2. Equilibria in prices and interregion flows: beef, 1955, 21 regions of continental U.S. (Source: Judge and Wallace (12).)

significance. This may be a reasonable assumption for slaughter cattle. Few production alternatives in cattle feeding can be changed radically within a year's time. However, even in a one-year period, the feeding rate and weight to which cattle are fed can be varied. Also, since alternatives to cattle feeding vary considerably among regions, one would hardly expect the price elasticity of supply to be the same among regions. Thus, though in the aggregate a fixed supply might be a fairly reasonable assumption, a supply fixed in each region might not be.

Another limitation is imposed by the aggregation represented in a 21-region division of continental U.S. Such aggregation is a necessary characteristic of a regional study.[9] Yet it requires an arbitrary assumption that for product(s) investigated, all production, consumption, export, and import occur at basing points of the regions. Even for a single product the selection of a basing point is more or less arbitrary because of limitations of data. Where several products are involved the problem of selecting an appropriate basing point becomes extremely complex even from a conceptual viewpoint. Data requirements force an arbitrary selection.

A further limitation attaches from the assumption that variation in transport rate is independent of direction from the basing point.[10] Casual observation suggests that the actual structure of transport rates is far different. Not only are the rates not continuous but they also differ by direction. While directional differences might be taken into account, to do so would multiply the computational problems already large for such models as these and larger still on attempts to improve the region-aggregate assumptions.[11]

We return finally to the fixed supply assumed for each region. This assumption presumes not only that in the aggregate the marginal cost of output increases with output at the same rate in each region (infinitely in the above example), but that any scale economies that might exist in producing slaughter cattle are exploited equally in all regions. Otherwise, the difference in horizontal position of marginal cost curves will alone create a difference in price elasticity of supply at given price. With slope constant with respect to shifts, the elasticity coefficient is decreased (increased) with an increase (decrease) in quantity. Since the economic position of the cattle feeding enterprise varies between farms and between regions we suspect large differences in the aggregate supply elasticities.

There may be reason to suspect that in many corn-belt farm organizations, cattle feeding is coming to play an important supplemental role (1). Should this hypothesis be confirmed, the corn belt supply function for fed cattle and the shifters strategic to such a function would differ from functions found in regions where cattle feeding is dominantly "competitive." In supplemental enterprises one would expect less

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[9] See footnote 4 above.

[10] It is commonly assumed too that transport rates are independent of the product. Yet actual rate structures are a complex of product-direction-distance interrelations.

[11] For an attempt with a 104-region system, see Heady and Egbert (9).

response to changes in product price ratios and stronger relations between production levels and inputs in which household values are important determinants.

The programming model yields interesting results from changes introduced in basic data and the postulates. We have already referred to the effects of barring all interregion shipments. The authors also show the effects on regional prices and interregion shipments of an increase and a decrease (of 20 percent) in transport costs; and changes in regional income, population and supplies to represent conditions of 1947, 1952 and projected for 1963, the latter considering, alternatively, (1) increase in population, production and income, at rates similar to those observed between 1947 and 1955, (2) increase in population and income, production fixed at 1955 levels, (3) increase in population and income, production per capita (within regions) fixed at 1955 levels, and (4) increase in population and production, per capita incomes (within region) fixed at 1955 levels. Transport costs were held constant throughout at 1955 rates.

The 20-percent increase (decrease) in transport costs decreased (increased) interregion shipments by only about half of one percent, accentuating (dampening) the interregion price variation as compared with the 1955 equilibria. The combination of lower income and high level beef production in 1947 led to equilibria with lower prices generally and a smaller volume of interregion shipments than found at 1955 equilibrium. Relatively low production in 1952 led to higher prices in equilibrium and a slightly lower volume of interregion movement of beef. Projection (1) converted Kentucky-Tennessee into a surplus region. Interregion shipments increased by 34 percent and prices remained about the same as 1955 equilibrium prices. Projection (2) yielded increased prices in all regions and some changes in interregion flows. California changed from a surplus (in slaughter cattle "production") to a deficit region importing the total surplus of Washington-Oregon and Utah-Nevada and most of the surplus from the Montana-Idaho region. Projection (3) increased prices generally and altered the surpluses and deficits by small amounts but changed no interregion shipment patterns. Projection (4) resulted in lower prices and converted Kentucky-Tennessee into a surplus region.

As a by-product of the various solutions the model generates an estimate of the amount by which the transport cost must be reduced between any pair of regions to induce, _ceteris paribus_, a flow of product from the region of surplus to the region of deficit. The _ceteris paribus_ assumption is, of course, something of an anomaly in a programming model. The very strength of the model lies in its capacity to encompass simultaneous change and (synthetically) to produce a logically valid estimate of results from their joint effects.

Besides introducing explicitly only a limited number of the phenomena that are in fact related to geographic price variation and interregion product flow, the model postulates a behavioral relation in the objective function to be minimized. Hence, departures in behavior from such an optimizing postulate furnishes still another reason for the relatively low value of r^2. The surprising result is that it is in fact as large as it is in the two years observed.

In a comparable study of slaughter hogs, Judge and Wallace (13) report equilibria among the 21 regions outlined in Figure 13.3 in prices and interregion shipments, given 1955 supplies in each region and demand for pork in region i according to $Y_{pi} = 50.9829 - 1.1917\,x_{1i} + 0.3468\,x_{2i} + 0.0359\,x_{3i}$ where for the ith region

Y_p is the per capita consumption of pork in pounds.

X_1 is the price of pork in cents per pound.

X_2 is the price of beef in cents per pound.

X_3 is per capita disposable income in dollars.

Regions producing a surplus in equilibrium are restricted to the Midwest and the Kentucky-Tennessee region. The Iowa-Nebraska region produces the largest total supply and exports a larger percentage of the supply than do any of the other surplus regions. In percentage of supply exported, the Dakota region ranks a close second, followed by the Minnesota-Wisconsin region. However, the total supply from the Dakota region is smallest among surplus regions and the Minnesota-Wisconsin region is surpassed in total by the Illinois-Indiana region. Among surplus regions, the Illinois-Indiana region ranks highest in percent of supply consumed within the region.

It is interesting to note that, as expected, the direction of shipment varies too among the surplus-producing regions. The Iowa-Nebraska region ships east and west. The Dakota's ship exclusively west; Minnesota-Wisconsin and Illinois-Indiana, exclusively east; Kentucky-Tennessee, south and east; and Kansas-Missouri, to Florida, the mid-south and to California. The squared correlation coefficient for equilibrium and actual prices again is less than 0.50. Thus of total variance, among regions, less than half is explained by interregion shipments that minimize transport costs.

The assumption of fixed supply is likely more damaging in the pork study than in the beef study. To counter this the authors provide a quarterly analysis. On this basis the Kentucky-Tennessee region exports less to the West Virginia-Virginia-North Carolina region and more to Florida. The Illinois-Indiana region sends more to New York and less to Michigan-Ohio. Minnesota-Wisconsin ships less to New York and more to Michigan-Ohio. Iowa-Nebraska exports slightly less to California. Kansas-Missouri ships less to Florida and more to West Virginia-Virginia-North Carolina and more to California. Alabama-Georgia-South Carolina become slightly surplus (first quarter) shipping a small quantity to Florida.

By introducing changes analogous to those introduced in the beef

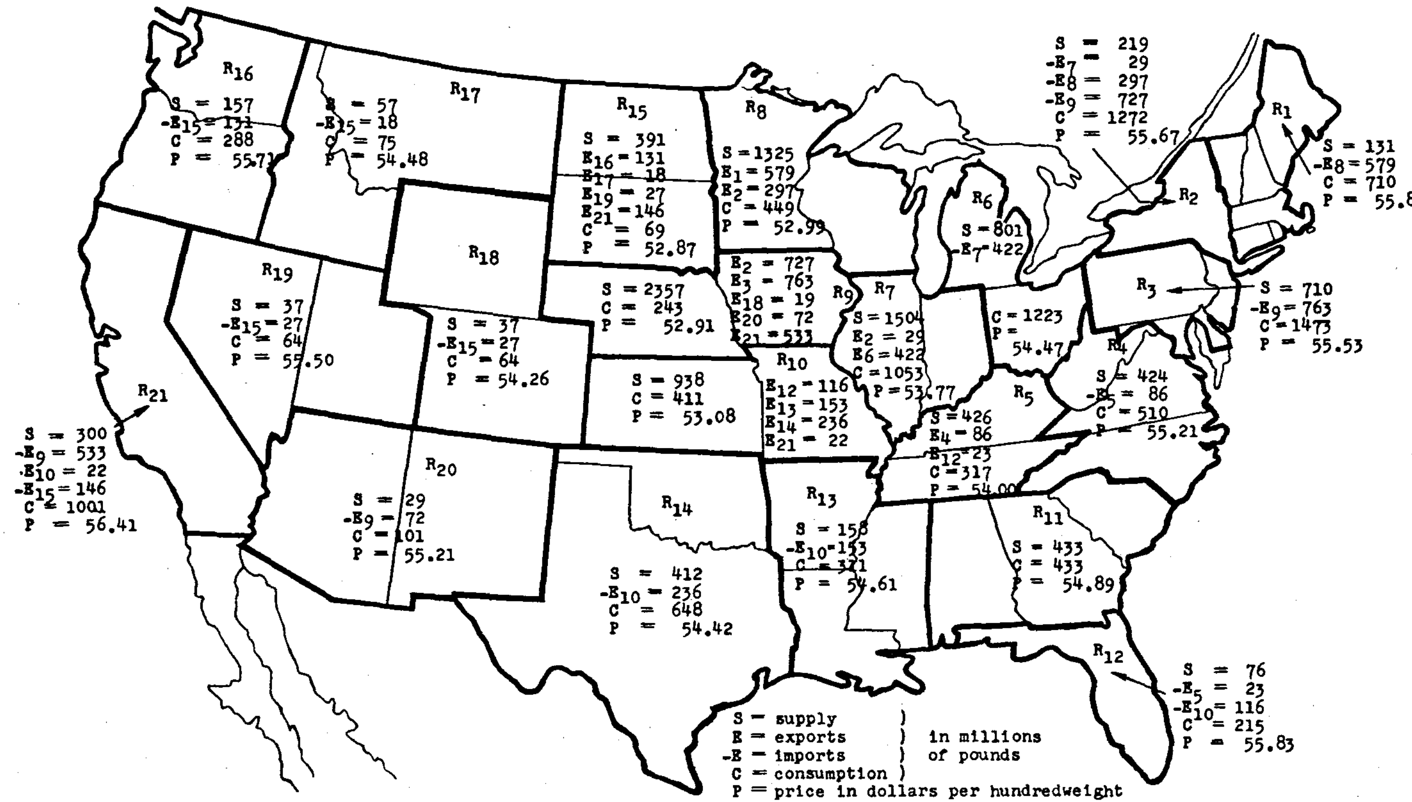

Figure 13.3. Equilibria in prices and interregion flows: pork, 1955, 21 regions of continental U.S. (Source: Judge and Wallace (13).)

investigation, similar sorts of "adjustment" changes are induced.[12] Interpretations are conditioned for hogs by the same properties of the analytical model as were observed for the beef study. The major phenomenal difference was taken into account by the estimation by quarters of equilibria in region prices and interregion product flows.

Other "single-product" studies include the pioneering study by Fox (7) in which he established interregion equilibria in prices and flows of livestock feed. For each of the ten regions in the United States, shown in Figure 13.4, feed consumption was related to price of feed in a price-dependent demand function where feed production and livestock price were taken as given. As a gross equilibrium condition, aggregate feed consumed in all ten regions was required to equal aggregate feed produced. No net change was permitted in feed inventories. Between each of all possible pairs of regions a transport rate for a bushel of corn was estimated from freight charges by mileage blocks observed in a sample of 1950 ICC waybills. For each region all consumption and production were assumed to occur at a single point. Fox then estimated from an aggregate demand function for feed for the United States a demand function for each region, based on the proportion of United States grain-consuming livestock produced by that region.

Then, feed supplies given for each region and completely nonresponsive to price change, an interregion flow was sought such that no individual could make a profit by (further) shipping from one region to another. This is attained by following these rules: (1) "If one region ships to another region, the prices must differ by the amount of the intervening transportation costs;" and (2) "if two surplus regions ship to the same deficit region, the difference between equilibrium prices in the surplus regions will be equal to the difference between their freight rates to the deficit region." Results are shown in Figure 13.4 for demand and supply conditions as of 1949-50. Prices shown are different by the transport rate between the region of origin and the region of destination. All data are in terms of corn equivalent. The squared correlation coefficient relating actual prices and prices generated by the model is 0.49.

Though no specified objective relation is used explicitly by Fox, properties of his model are similar to those of the model used by Judge and Wallace. Thus all the limitations so far noted apply to these results as well. In addition, the aggregation implied by the 10-region division of continental United States renders more tenuous the assumptions based on (1) production, consumption, exports and/or imports from a single point interior to each region and (2) homogeneity within regions. Yet certain advantages attach to the use of fewer regions: the greater relevance of transport costs (compared with nontransport costs) when shipping centers are farther separated, and the lesser computational

[12] In the pork study, a 36-region model also was estimated. In a second study on beef (as yet unpublished), quarterly models were used, but yielded results little different from those obtained with the annual model.

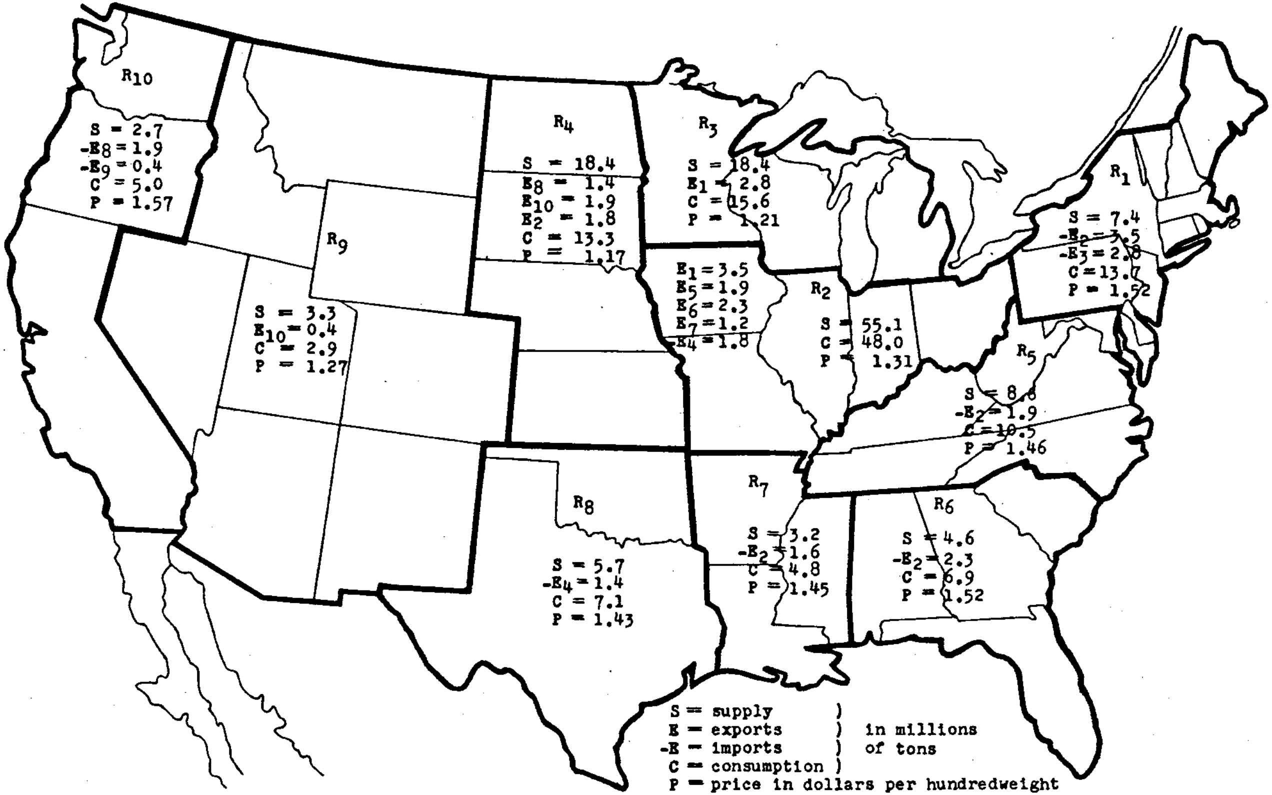

Figure 13.4. Equilibria in prices and interregion flows: livestock feed, 1949-50, 10 regions of continental U.S. (Source: Fox (7).)

requirements when using the fewer regions. Fox reported solutions obtained "...with ease...with no more exotic equipment than a desk calculator, supplemented by the investigator's judgment."

AGRICULTURAL APPLICATIONS IN MULTIPLE PRODUCT STUDIES

As is well known, there exists for resources in agriculture competition between products within regions as well as between regions with respect to single products. It is interesting to include in our survey such attempts as have been made to extend analyses of spatial equilibria to the multiple-product case. We report at the outset that such an extension has been made only under extremely limiting conditions. Indeed the only examples so far reported entail either complementary products or joint products.

An example of the former is found in an extension of the Fox model made by Fox and Taeuber (8). Under conditions approximated in 1949-50 they established (joint) equilibria among regions in both feed and livestock. Initially Fox took as given, in the regional demand for feed, regional livestock production and price of livestock. In the later approach Fox and Taeuber related regional demand for feed to the regional prices of feed and livestock and to regional livestock production. In turn, regional livestock production was related to regional prices, given the human population and disposable income per capita. Livestock supply was related to prices of both livestock and feed. The supply of feed was assumed fixed by region.

By following the same rules used in Fox's simpler model, Fox and Taeuber generated an equilibrium in regional prices, consumption, and (livestock) production. These and the interregion flows are shown in Figure 13.5. Unfortunately, the feed supplies, given for each region, differ slightly from those used in the previous model. Hence a direct comparison is not possible between equilibrium results. It appears that supplies generally are smaller in the later model. However, it seems clear that by introducing the livestock variable in demand and supply, the following results are obtained: (1) the interregion volume of feed shipments is reduced, (2) feed prices generally are lowered, and (3) the interregion price variation is reduced. All would be expected from introducing livestock feeding as an alternative to export in surplus regions and from the reduced relative importance of transport costs in livestock as compared with feed.

Snodgrass and French (16 and 17[13]) using (explicitly) a transportation model solved for a shipping pattern that minimized transport costs for milk among the 48 continental states, given 1953 supplies and

[13] This article contains also references to preceding literature relating to transportation models.

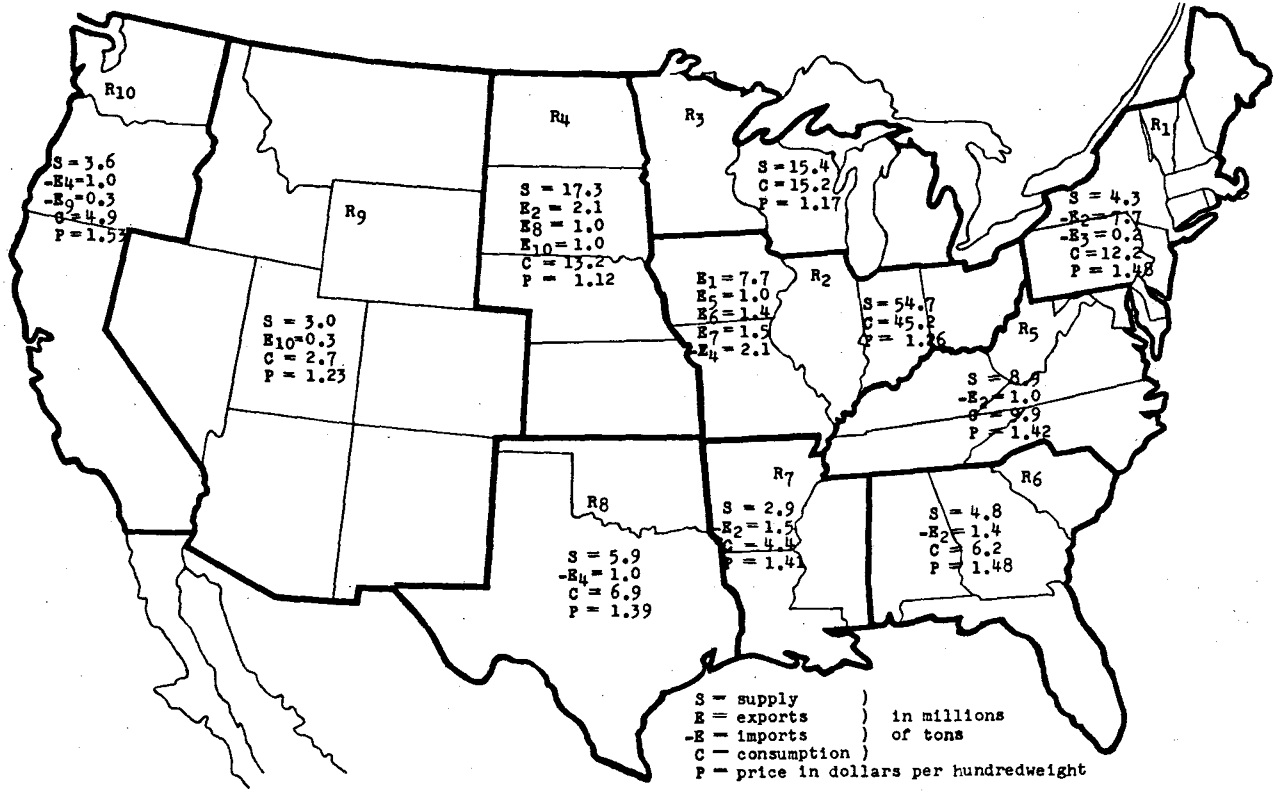

Figure 13.5. Equilibria in prices and interregion flows: livestock feed, 1949-50, 10 regions of continental U.S. (Source: Fox and Taueber (18).)

demands in each state.[14] Then a population increase was projected for each state to represent 1965 conditions. Projected increase in milk production was allocated to each state in proportion to its 1953 contribution to the 1953 total. The new transport cost-minimizing solution yielded results little different from those found for 1953. Michigan switched from a surplus to a deficit state. New Hampshire and Maine switched from deficit to surplus. Relatively more milk was shipped west and total transport costs increased, with increases in total milk shipped and the greater distances involved in shipment. The squared correlation coefficient for actual and equilibrium prices for 1953 is 0.44.

To illustrate the effects of market restrictions, the authors assumed an increase in transport costs imposed by the (importing) states of Rhode Island, New Hampshire, Florida, Alabama, Georgia, Connecticut, Massachusetts, North Carolina, Oregon, New Jersey, Virginia, Pennsylvania, California, and New York. The result was to change Michigan into an exporting state (shipping to New Jersey) and to attract supplies from new sources for Connecticut (from Vermont), for South Carolina (from Wisconsin) and for Virginia (from Wisconsin). The total transport bill increased by 9.5 percent, a considerable result from a relatively small change by a few states. The result would, in the "real world," be modified by production responses in the states imposing the market restrictions and by the chain of reactions set off thereby.

Adjustment implications are shown also by increments to cost from increasing production in each of the regions (states, for the models so far described). Considering milk in the aggregate, transport costs would be increased by more than $3.00 were production to be increased by one "unit" (i.e., 10,000 hundredweight) in Minnesota, Wisconsin, Iowa, North Dakota, South Dakota, Nebraska, or Missouri. On the other hand, transport costs would be increased by less than $1.00 were production to be increased by one unit in California, Nevada, Oregon, Florida, or Arizona. The former are surplus and the latter deficit states. Discontinuities in programming solutions limit these estimates, of course, to a range of adjustment with unknown limits. The only estimate that is certain is for one unit. Yet the effect of transport costs is evident.

As the milk product is disaggregated into its marketable components, differences in the cost increments diminish. To increase production of nonfat dry milk solids would increase transport costs by $2.00 per unit only for two states (Minnesota and Wisconsin) while the increase would be less than $1.00 also only in two states (New York and Vermont). Differences between high and low are reduced likewise for other components of milk, as the bulk of the product is reduced.

Snodgrass and French adapt the disaggregated models to estimate a regional distribution of processing facilities that would minimize the

[14] Note that use of this model requires that within-region demand must be taken as given in terms of quantity. Thus the model does not allow for within-region consumption response to price change induced by inter-region shipments.

sum of transport and processing costs. For each region the processing cost of a milk component is added to transport cost that separates it from each of the other regions. The distribution of processing facilities (and hence product flow) is changed much as would be expected by an increase in transport cost. However, since the milk components are processed in different proportions in the different regions, other changes also occur. One of these is a further reduction in the differences between regions[15] in cost increments consequent to increase of output. In this case the increase was in processing output of each of the various milk components.

In a final investigation, Snodgrass and French take into account the variation between states in feed and labor costs in milk production. A regional distribution of production minimizing the sum of transport, processing and these production costs yields the following results. All manufacturing milk is produced in Minnesota and Wisconsin, wherein no fluid milk is produced. Fluid milk is produced in only eight states:

1. New York, shipping to Massachusetts, Rhode Island, Connecticut, New Jersey, Pennsylvania, Delaware and Maryland
2. Iowa, shipping to Arkansas, Oklahoma, Colorado, Nebraska, Kansas and Texas
3. Ohio, shipping to Virginia and West Virginia
4. Indiana, shipping to Tennessee and Kentucky
5. Alabama, shipping to Florida and Georgia
6. South Carolina, shipping to North Carolina
7. California, shipping to Nevada and Arizona
8. Utah, shipping to New Mexico

As the authors themselves noted, the conversion of Iowa into a surplus producer of fluid milk requires a heroic willingness to forget about opportunity costs imposed by production alternatives other than milk. These are imperfectly reflected in the production costs taken explicitly into account. Also, and closely related to this point, the intra-region heterogeneity would differ among regions so as to impose limits to dairy production that would vary if opportunity costs were taken explicitly into account.

In all models, consumption is taken as given. Hence the interregion reorganizations of production and/or processing assume no consequences in quantities sold within regions. Such a restriction is important. Given a change of transport costs from either a change in form of product or from including processing costs, one would expect a general decline in interregion shipment and an increase in interregion price

[15] To facilitate computations, the regions were reduced to 24 in the adaptation.

variation. These models do not permit these results. The final demands in each region are taken as given so that the only change allowed is in the directions and relative quantities of interregion product flows.

With comparable demand assumptions, Heady and Egbert (9) have developed a model more elaborate from an empiric viewpoint than any so far reviewed. Among 104 United States Bureau of Census subregions they sought, with three models, an equilibrium allocation of land and land-related resources to feed wheat, food wheat, and nonwheat feed grains under demand conditions of 1954. In two of the models the equilibrium was solved for in minimizing the sum of regional costs. In the first, costs included, by region, the unit costs from labor, power, machine, seed, fertilizer and "related inputs." In the second, land rent was added. In each case the minimization was subject to two sets of restraints. The first consisted of a land restriction for each of the 104 subregions. The land supply was set for each subregion at the largest acreage used for these crops in the eight years prior to 1954. Two added restrictions were necessary to equate total annual production (for feed grains and for wheat, respectively) with total annual consumption.

In a third model, activity units for feed and food wheat and for feed grains were priced in each region according to a historically determined price relation, assuming that transportation costs were thus accounted for. Then the equilibrium was sought by maximizing a "revenue" function, comprised of the sum of price-quantity products in each region, summed over all 104 regions, subject to restrictions similar to those of the first model. The essential difference in this from the first model lies in the reflection of transport costs in product prices.

The results are as follows. In the model that minimized costs exclusively of land rents, <u>all</u> grains were withdrawn from such marginal areas as southeastern Colorado, eastern New Mexico, northern Utah, eastern Wyoming, southeastern Montana as well as fringe areas in Texas, Nebraska, Wisconsin, Michigan, Oklahoma, Missouri, Kansas, New York and certain areas in the southeast. When land rents were included, food wheat was restored to Montana and to southwest Missouri (from feed wheat). But all grains were withdrawn from the Oklahoma panhandle and from Pennsylvania.

When transport costs were taken into account, the changes were more dramatic. Food wheat was restricted largely to the Dakotas and to Minnesota and Wisconsin. Wheat areas of Nebraska and the Pacific northwest (including Montana) were shifted from food to feed wheat. Yet even here, it is interesting to note, no substantial change was found necessary for many large and stable areas producing wheat and feed grains.

All these models take into explicit account space ordered phenomena ordinarily abstracted from by models used to study agricultural adjustment. Hence they provide a view of agriculture that differs fundamentally from views provided by other models. The applications so far made are severely restricted by the use of extremely large regional aggregates (e.g., Fox); by sorely naive assumptions on demand and supply of product, in aggregate and in regional distribution (e.g., Snodgrass and French); and by fragmentation in terms of products demanded by computational complexities met in a model that would take account of inter-product as well as interregion relations (see comments by Heady and Egbert). Use of results from the models in suggesting action on adjustment problems must be conditioned further by the fact that they have a normative orientation. Limitations imposed by this property differ according to (1) the value judgments of the adviser and/or (2) how well the normative postulate(s) accord with actual behavior in the adjustment units.

REGIONS AS ADJUSTMENT UNITS

Unless we damage severely ordinary use of the term, we must ascribe to a region the spatial attribute of contiguity. When we speak therefore about the corn-belt region we denote a contiguous space north and east of the middle part of the United States below the western end of the Great Lakes. In contrast, the corn producing industry includes, in addition to parts of (most) farms in the corn belt that produce corn, parts of farms elsewhere that produce corn. The region designation is descriptive merely in terms of a high percentage of farms found within its boundaries. However, criteria for fixing the boundaries for regions are exceedingly vague.

Isard (11, chapter 1) describes the development of a region as a "nucleation," abetted by the gregariousness of human nature, and nurtured by economies provided by such an aggregation that are external to individual action systems. A restricted variety of such economies are external economies made available to firms by financing institutions, labor centers, service agencies, etc., that evolve in the process of "nucleation." As suggested by Isard, the aggregates can even, under certain conditions, develop new "decision foci" and hence new action systems. Further possibilities of economies are created. But as growth continues a retardation occurs in the rate at which the external economies are created. Indeed a cursory view suggests that a stage of external diseconomies has been reached for firms and consumers in many metropolitan areas. Dispersal then occurs subject to costs of transport, diminishing returns and the nonuniform spatial distribution of resources.

In a given stage of development, production differences among regions are partly summed up in the law of comparative advantage. Insofar as the law describes space-related differences in production, it is reflected in the functional relations of Figure 13.1. But Figure 13.1 reflects the combined effect of all determinants of excess supply within regions. Hence the "exportable surplus" of a given region is made to depend on the determinants of demand as well as those of supply for each of the various regions included in the model. Such determinants as are introduced in the models reviewed are restricted to per capita income and to population. While these are important, they by no means exhaust the relevant demand shifters within regions and hence shifters of excess supply of given regions.

Finally, when we view regions as adjustment units we note Isard's "decision foci." Instead of serving merely to reflect any space-related advantages apparent in (historical) data, opportunities for adjustment are created by the development of groups capable of aggregate action in behalf of regions. The results are expressed partly through external economies for firms already in an area by attraction of new firms into an area. Otherwise regional action may be expressed in such ways as regulation of selected aspects of product markets (e.g., market orders) or factor markets (e.g., zoning regulations). Production alternatives are varied, added restraints are imposed (or removed) and resource requirements are affected. Many of these changes are noneconomic or only partly economic in origin. All result from some type of group action and depend on the existence of some sort of region.

SUMMARY

In describing the studies selected above we already have suggested some of the properties of spatial equilibrium models that seem especially important in conditioning the interpretations permitted of their results. This discussion is summarized in the following four points.

1. For all applications it was assumed that the supply of product was fixed in the aggregate and for each of the regions among which equilibria were sought. In the aggregate the assumption seems well founded on an annual basis for the products so far studied. It is sensible to take as given, for example, a regional and total feed supply and to assume that neither will vary within a year in response to change in its price.

It may even be sensible to suppose the aggregate will not vary much next year in response to this year's price. However, it is heroic indeed to assume that the response for a given product will be zero next year — or even the same among regions. Supply elasticities will vary among regions for the many reasons cited in this conference. Especially important, however, are (a) the scale of farms producing the product, (b) the degree of specialization in its production and (c) products competing on the region's farms for resources required in its production.

It may be pointed out that the models estimated are "one-year" models. This is a time period particularly convenient for observation and analysis in agriculture. However, in interpreting results generated by the models, the adjustment-problem solver is led to draw implications for "next-year." He is little interested in "this year" except for its helpfulness in predicting (albeit with severely defined conditions) the course of events next year.

2. With respect to related products, the models applied were considerably less than satisfactory. They assumed, as a matter of fact,

that for commodity x the elasticity of supply with respect to price of non-x is either zero (by ignoring all non-x) or is determined completely by a complement (e.g., the method of Fox and Taeuber). This point is closely related to the first and would not be especially bothersome except for the requirement that the cross elasticities be the same among regions. For most agricultural products, it is elementary to note that this is not likely to be a very good assumption.

Thus it seems necessary to take inter-product competition into account. The brief account by Heady and Egbert precluded a careful appraisal of their method and findings. We note with fascination the tremendous computational problem imposed by matrices "of the order 104 x 316" even though only three products were taken into account in their essentially simple set of models.

3. As in any classification scheme, certain homogeneity properties are assumed for "regions." The point here is subtle in the use of spatial equilibrium models. When implications are drawn from equilibrium product flows or from the "marginal costs" of adjustments reported (e.g.) by Snodgrass and French, an implicit assumption is required for problem-solving purposes that the adjustment can proceed with comparable resource limits in each of the various regions.

A closely related comment might be injected here to indicate a possible direction of extension of the models. Aside from joint equilibria in feed and livestock, all resources were assumed immobile among regions (though perfectly mobile within regions!).

4. Transport costs were assumed either zero from the internal basing point to all points within a region or to yield a zero sum of deviations from a quantity-weighted mean within the region. Again, with a properly selected basing point, such an assumption may be a fairly good one. Yet it seems anomalous in a method that depends on the relative importance of transport costs as the basis for its use.

In conclusion we add that it would be easy at this stage to be negative regarding the application of spatial equilibrium models to studies of agricultural adjustment problems. We have no desire to be so interpreted. Indeed the applications so far made have been ingenious and have shed considerable quantitative light on an area heretofore reserved for (at best) qualitative analysis. To say that we need to refine the models is, therefore, hardly a negative criticism.

We add only one note of pessimism. It may be entirely possible that the agricultural economist is awakening to the quantitative importance of space-ordered comparative advantages at the very time that technological changes are reducing their importance relative to comparative advantages oriented to management differences and relative to advantages induced for regions by group action not captured in the relations so far included in the models.

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Discussion

BAKER'S OBJECTIVE, as I interpret it, was to summarize and evaluate current regional and spatial equilibrium models appropriate for application to adjustment problems of midwestern agriculture. He further restricted his area by emphasizing interregional trade and production location problems, and by categorizing theories of trade over space and of location as essentially the same phenomenon. His general discussion of spatial equilibrium models draws almost exclusively on programming models as applied to problems of interregional trade. In these examples the differentiating variable factor was the transport rate based on an arbitrary "centralized" point, independent of volume or direction of shipment.

Baker presents a description and analysis of representative studies in the area. I believe he has systematically and thoroughly traced the evolutionary development of regional and spatial equilibrium models relating to trade in midwestern agriculture.

In the process of formulating a critique of this paper, I think we would, logically, attempt to evaluate (1) his review of the literature, including his recognition of the limitations of existing work in its application to the problem being discussed, (2) his view of the pertinent adjustment problems which are appropriately conceived at a regional level, (3) his selection of properties of the programming models for discussion, and (4) the extent and validity of his suggestions to improve existing models or to suggest alternate research approaches to adjustment problems.

I believe Baker has covered this assignment with his characteristic thoroughness. His detailed presentation and analysis of pertinent studies and the many bibliographical references provide an excellent background source for those interested in the application of regional and spatial models.

Chapter 14

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Demand Analysis and Data for Regional and Spatial Models of Adjustment*

THE REAL REASON for being concerned here with demand analysis and data for regional and spatial models is that we wish to use these models to predict the interactions of the agricultural production process with the rest of the economic and social web of society. In short, we are concerned with the adjustment process.

ADJUSTMENT DEFINED

What is meant by adjustment? Unfortunately, many vague things. Even more unfortunate in some cases is the fact that analysis said to be adjustment research is designed and executed without explicit recognition of some of the essential elements of any adjustment problem. In the most general sense, to be in adjustment implies that, by some criterion or set of criteria, a satisfactory relationship has been attained between the needs, desires, or goals of the object being adjusted and the nature and organization of that object and its environment. The essentials of this generalized definition are:

1. A criteria for specifying adjustment.
2. The understood object which is "adjusting" or "being adjusted."
3. The nature of the object of adjustment and its environment.
4. The internal organization of the object of adjustment as well as the organization of the environment.

By "nature" is meant those characteristics of the environment and the object of adjustment which do not change during the process of attaining a satisfactory adjustment.[1] By "organization" we mean those

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[1]The facts of nature are of varying order. Elements of the social order can be changed over greater or lesser spans of time: by law, by informal change in social structure, and by technical change. At the other extreme are the more immutable constants of the physical universe.

characteristics which do change or can be changed in attaining a satisfactory adjustment. In economic analysis we usually assume 1 and 2, identify the relevant elements of 3 to be faced as "facts", and operate upon the variables of 4 to determine what constitutes a satisfactory adjustment.

Time is an important dimension of the adjustment process. The chronological period of time involved may vary considerable from one specific adjustment problem to another. The span will depend on the period of time it takes to execute the changes in "organization" (part 4 of the definition above) necessary to attain the optimum equilibrium condition of adjustment.

Usually, in agriculture the criterion of adjustment used is that of optimum income, although social welfare, minimum income, equality of income distribution, market share or dominance, and many other criteria have also been applied.[2] The object of adjustment conceivably could be a firm or group of firms, a region, a conglomerate of subregions, an industry (such as agriculture) or functional sector of an industry (the feed-livestock sector), or an entire national economy. All necessarily have somewhat different organizations and natures. The nature of economic environments will include most of the technical coefficients of production and consumption, particularly in sectors that provide the closest substitutes and necessary complements to the object of the analysis. Important environmental elements often omitted include the asset structure, inventories, and other stocks of the economy. We also tend to overlook the structures in the economic environment which result in action at variance with that conceptualized, such as the deviations from the usually postulated perfectly competitive economic organization of society.

COMMON ERRORS IN CONCEPTUALIZATION

One final important consideration is the question that must be answered in the design of any study: to what end do we do adjustment research, or to what use do we wish to apply our results?

If one wishes his research to provide meaningful adjustment recommendations for national agricultural policy, then he must have as a focus for analysis a socio-economic unit larger than a farm or group of farms. For that matter, even in research which is to provide only farm management recommendations one must consider the aggregative price effects of the sum of individual firm production decisions, if the farm management recommendation is to be reasonably close to an optimum result in any long run adjustment or equilibrium sense.[3]

[2] Some of these other criteria are most often introduced as qualifications to the results of the research.

[3] Also of importance in this problem are the often inconsistent goals of the nation and individual farmer. In a human social structure it is rarely safe to assume that a social aggregate or nation is no more than the sum of its parts. Science must operate with such

In adjustment research, the problem is usually improperly set up to begin with. Often this involves the generally incorrect notion that research formulated at a very low (or micro) level of aggregation can be used directly to illuminate adequately the much higher national or macro level variables of adjustment problems. The reverse also holds. Research done at quite aggregative or macro levels in the economy is rarely ever directly useful in analyses involving micro level variables. Obviously the problems of adjustment are not confined only to one level of aggregation in the agricultural industry. There is a clear need in adjustment problems to design research which functionally relates the micro level analysis to the macro. No doubt it is easier said than done, but it is clearly needed.

Conditionally normative resource allocation models of farms may be aggregated to help specify production and other boundary limits under differing conditions (of say, technology) but they can be used neither to specify the process of adjustment nor to predict production, price, and other equilibrium adjustment reactions with any reasonable degree of reality.[4] To view the rest of the economy through a fixed set of assumed prices as such models typically do is to throw the adjustment "baby" out before you ever fix the "bath water." How else is one to interpret a model that postulates prices which do not respond to any of the imputed changes in production? This is not to say that such resource models do not have uses. They most certainly do. But these are extremely short run in analytical nature. Such models must be greatly adapted for research that focuses on the basic problems of sector or economy equilibrium adjustment. They also are more limited for farm management purposes than we are often willing to admit.

In such models the production unit will appear to have made satisfactory adjustments to current prices and resource problems. But these adjustments will almost invariably involve an increase in the farm's capacity to produce and in production. Not just one, or a few, but many farmers presumably will make these adjustments and the net aggregative result will have been to increase product output, not just for the farm, but for the industry as a whole. The aggregative production response involved in the adjustments of the original time period have an effect on price and necessitate additional rounds of adjustment in subsequent time periods. The usual resource allocation model does not go beyond analysis of the original time period. This means that the aggregative effect of individual firm actions are never considered as part of the adjustment problem. Where new techniques and organization are involved, the aggregative effect is not just a part but an all important part of the adjustment process. To use price in such a manner is not a failure to include time or aggregation in the model but a failure

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postulates but we should not be misled by "a method of analysis" to exclude from the determination of what constitutes an "optimum" for social policy, all individual or societal outlooks on reality that are not implicit in "the method of science."

[4] This is equally true in budgeting, traditional production function analysis as well as linear programming.

to include the demand structure in the calculus of adjustment. At best it makes the entire demand structure exogenous to the model.

This failure to consider the demand structure is implicit in much of the current discussion of adjustment. Reorganization of input mix and technological change, in fact, practically all structural changes, are often discussed as if such changes, as matters of importance, were limited to the farm production organization. This is obviously not true. The non-farm produced services and physical factors with which farm products are combined account for well over half of the final retail value of food products. Surely if, for example, we were focusing on the impact of technical change on the farmer's adjustment problems, we would have to give some consideration to the technical change taking place in the chain of production organizations that connect the farmer with the consumer. Innovations in the "form of the product" presented to the consumer such as were involved in the precooking and prepackaging of foods and in frozen foods have had considerable influence on the rate of growth in demand for some products. This is certainly true of broilers. Indeed, in the case of broilers and some vegetables the packaging and freezing revolution has probably had significant influence on the location of production as well as on the legal and organizational form of many of the farm units producing broilers and certain vegetables. The freezing of fresh orange juice is another innovation in commodity form that has had profound effects at the farm level.

If and when concentrated sterile milk becomes a major market reality we are likely to see rather significant shifts in comparative advantage between areas of the nation in the production of milk. Any reduction by as much as a third in unit costs such as are potentially involved in this innovation will have very direct effects on enterprise organization, resource mix, and locational advantage of farms and entire production areas (9). The transport and communication developments of the last 20 years have had the effect of breaking down old locational advantages and of greatly shrinking the economic space between markets. In the future such diverse things as the St. Lawrence Seaway and the integrated unit-cartonization of truck, rail, and sea transport will have great location advantage impacts. Any major change in storage or transport costs shifts the locational advantages of different production areas and changes the enterprise and input mix that is optimum for a production area and for types of farms within an area.

New consumer durables have considerably altered consumption behavior and the demand for individual farm products. The refrigerator and the home freezer have had important effects in changing the composition of diets and thus have caused shifts in demand.

One could go on. But all this is only to point out that major changes in consumer tastes or in the costs, organization, and form of products, markets, transport, communication, and storage must inevitably influence the economic facts of life faced by a farm unit and result in changes in the nature of the farm organization itself. No consideration of the adjustment process or the problems of an equilibrium adjustment in agriculture is likely to be complete or realistic that does not explicitly include in its analysis the relevant structure of demand and the production processes that connect the farm gate and the dinner table.[5]

We have been dealing here with the characteristics that ideally should be associated with research in an adjustment framework. A few final observations need to be made. Limitations of data and analytical tools usually impose partial equilibrium frameworks on our research. If we are to avoid many common errors, such as those noted above, the research problem should be formulated theoretically in a general equilibrium form before it is cut down to workable empirical size. The cutting down of the conceptualization should not only be carefully done, but as much as possible the cuts should be made on the basis of what is least important analytically to the focus of the specific research. Cutting the analytical framework simply to fit a proven tool of research or to avoid data problems quickly leads to sterile research. We stand to gain far more from bold "half failures" than timid "total successes." Failure to conceptualize adequately the research problem is a crippling affliction and one common to much of our research today on adjustment problems.

The author finds arguments for regionalization or spatialization of the analytical framework convincing. This seems to be one of the first steps necessary to bridge the great void between macro level analysis and the firm-household or micro level. Regions of the United States are, of course, less self-contained economic organizations than the nation so that the analytical framework of regional analysis should extend beyond the confines of the region itself. For instance, a study of adjustment problems of the Lake States dairy industry should take into empirical and analytical consideration the major deficit milk markets to which Lake State surplus milk flows. A complete general equilibrium framework would also include the surplus producing regions that compete with the Lake States.

If the research aims primarily at policy recommendations or general adjustment problems, it is desirable that the results be cast around a series of consecutive time horizons rather than just one. Despite the inherent crudity and hazards involved in projecting a portion of the structural variables of a model, this in some fashion is what must be done. These hazards must be accepted. Adjustment even to past circumstances takes place in unaccomplished, not accomplished, time. Also any consideration of anticipated structural shifts, such as new technical change, must necessarily be given some dated unaccomplished time dimension. Normally this means the construction of more than one model. Approximation of the path of change or process of adjustment will generally provide more valuable information than the

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[5] The same is true, of course, of the market structures on the farm input side. The arguments here have been stated quite cogently by D. E. Hathaway, G. L. Johnson, T. W. Schultz, and others.

analysis of the final static equilibrium. Time is the irreducible and strategic dimension of adjustment.[6]

PROBLEMS FACED AND DATA NEEDED

It is one thing to specify the general requirements of adjustment analysis and quite another to know how to go about meeting these requirements in integrated and concrete empirical terms. The author cannot claim much progress in his own struggles with these problems. What follows is still quite eclectric, and in instances tentative.

Notice first the overall context in which we attempt to develop empirical models of integrated demand and supply structures. Most empirical and analytical tools available for demand analysis are macro in form and in variables specified. Most of the applied supply analysis tools are developed for the micro or firm level. Thus there are really two gaps rather than one in our general need for theoretical concepts and analytical tools. An overall prescription can be written in the following form. A major problem of development of supply functions lies in mastering the problems of aggregation of supply functions. A major problem of demand function development lies in mastering the problems of disaggregation. These are not just problems of empirical weights and index numbers but also problems of meaningful empirical specification of intermediate market organization as well as a theoretical explanation of what is happening in the process of aggregation. The author believes the most profitable line of attack on this problem is through direct specification of the structure of production (market firm) and demand that lies between the farm and the consumer. A tall order and not something that we will see done very quickly. However, let us look at some parts of the problem.

National Demand Aggregates

If it pretends to any significant degree of reality, the least an adjustment study can do is to involve an aggregative statement of demand relationships. Only in this fashion can the price effect of net changes in aggregate production be evaluated, even crudely.

Since we are, perforce, limited to comparative statics as a technique of analysis, we must produce a number of macro demand relationships identified in time with the adjustment periods selected for the related supply models. These must be basically synthetic functions but to the extent that it is possible they should be derived from available empirical demand analysis. For dairy, the industry with which the

[6] At least two forms of time are involved, regular chronological time and the more elusive and functional economic time, such as that of Marshall's traditional three periods of "run." One of the great difficulties of time in an adjustment framework is the necessity to relate in a meaningful, quantitative, and theoretical fashion the different forms of time.

author has been concerned at the time of this writing, Anthony Rojko's work (10) could be adapted to this purpose. For many commodities and industries, nothing as developed as Rojko's analysis of demand is available. Even at the level where the most demand work has been done, the macro level, there are gaps in both analysis and data for many commodities.

Basic demand data and demand functions are usually cast at the retail level, supply functions at the farm level. This is the micro-macro problem, but note the integrative problem it causes in developing analytical structures for adjustment purposes. One function must be translated to the other structural level. The point is that this involves some rather complex problems that are not avoided by the oft used technique of applying, as a single constant shifter, farm-to-retail price margins. Our mythologies to the contrary, margins do change with volume. Nor does assuming one function in an entirely synthetic form solve any empirical problems. The nature of demand for a specific commodity <u>at the farm</u> is as much dependent upon the organization, techniques and behavior of processing, transport, storage, and retailing as it is of basic consumer tastes and demand at retail. Farm adjustments to the market are thus as much adjustments to these former factors as to basic consumer demand.

Regional Aggregates and Data Problems

The very act of regionalization of analysis creates a problem in itself. In some degree regional boundaries are necessarily arbitrary. This is a weakness in any regional model. Until one can specify the boundaries of regions in a systematic fashion <u>from within</u> the model, research results are subject to the qualification that another equally satisfactory (by the criteria of the model), but different equilibrium could be obtained from a different and probably no more arbitrary set of regional boundaries. Aggregation problems make this difficulty more intense. The "most logical" regional structure for one industry is not necessarily the most logical for another. Yet the same regionalization must normally be used for both if they are to be aggregated directly.

Another problem in the eternal tension between computational capacity and the degree of differentiation of the usual spatial model lies in the fact that economic space exists <u>between</u> but <u>not within</u> regions. This drives one toward proliferation of regions in order to obtain greater empirical reality. Heady and Egbert's work (5) with a spatial model of grain production ended with coefficient matrices greater than 100 x 300 in order to obtain 104 regions in their U. S. model. But even at this level of differentiation the problem of an adequate level of empirical reality is obviously far from solved.

Farmers are rarely conscious of adjusting their operations to national conditions. They are conscious of and feel the effect of the national market for their products through particular (usually local) market alternatives to which they may ship their products. Specification of the adjustment problem of farm firms in any particular production area is probably ideally expressed in terms of major local markets and local market structural interrelationships. One cannot begin to list the data needs involved in specifying local market structure. The complex of interdependence involved in some of these local markets is awing. In recent years in the area east of the Mississippi a significant change in milk prices in one major Federal Order milk market has rippled through the rest like a row of dominoes.

Further, the internal cost structures and organization of particular markets should be specified if the national and regional aggregate changes are to be translated accurately to the farm and into farm management data. Only under these conditions can the economic pressures on location of production be introduced as a dimension of the adjustment problem within a given market area. Adjustments as they take place at the intensive versus extensive margin of nonlabor resource use are of very real importance in forming farm management recommendations. As an enterprise moves away from the market it substitutes transport expenditures for rent expenditures; and at the same time because the price of land usually falls, it tends to substitute rent expenditures for other (excluding transport) expenditures. The costs resulting from many market functions are actually mixtures of discounts for space, time, and form preferences and are complex to handle.

Spatial models to date have stated spatial costs entirely in terms of the location of production directly relative to the location of consumption. This is often not an adequate representation even of spatial costs. The location of the intervening market functions where non-farm inputs are combined with the raw farm product should be specified to obtain an accurate minimum cost spatial equilibria.

General Types of Data Used

It is not difficult to classify the demand data most commonly used in handling analytical problems of the sort discussed above.

Historical price data are easily accessible for the national level and even for states. There are, of course, rarely any price data for regions larger than states although states may be aggregated rather easily to such regions. For areas smaller than states local prices can often be had but they are usually quite varied in quality and costly to obtain. Most local markets of any size will have records, but frequently only in sale lots; thus, much laborious work is needed if one is to obtain market prices for any period of time. Central market prices are usually reported in the Wall Street Journal and in newspapers published close to the market. Federal Order markets will usually have good files on such things as prices and volume.

Commodity data on flows between regions and between production areas and points of consumption are not easily had. Typically we use regional production data (actual) in combination with regional demand functions and data (synthetic) to arrive at imputed flows. The smaller the area of the region the more difficult it is to determine empirically the flows of commodities across regional boundaries. The carlot unload data available from a number of major markets is generally inadequate for this purpose. In a few commodities, trade sources and even trade publications have data of at least some use. Overall, however, it is difficult to visualize much improvement in present spatial flow data without major effort by the federal government to collect such data.

Data on transportation costs and alternatives will have to be obtained from varied sources for the different commodities. Transport cost studies are available for a few commodities and areas and types of transport. Undoubtedly in many instances one will be forced to go to major processors to obtain "estimates" of rate structures. One can go to the transportation companies themselves and to their rate books, but actual rate structures are so complicated that this is likely to be a rather costly and tedious process. However, if one needed only a limited number of rates between a few specific points this could be the best approach.

Processing costs and structure data are quite crucial to adjustment models. This type of data is not available in any easily accessible form or centralized location. Usually one is dependent upon the cooperation of industry sources, firms, trade associations, and trade publications. In a few instances studies of processing will provide some guides. In recent years increasing numbers of engineering-economic studies have been done in agricultural processing industries.[7] Such studies are usually the only source of processing firm input-output data which is well articulated for economic research.

Per capita consumption data for demand analysis is available for the United States but in general not for states. The 1955 USDA Household Food Consumption Study does provide data on consumption and income by four large regions of the United States.[8] There are also smaller USDA dietary studies for a number of cities and some rural areas which could be used as a basis for estimates of regional per capita consumption levels. This would be pretty much of a patch work empirically, and one might be better off, depending on how extensive the empirical data were, to develop regional demand functions from the traditional variables of disposable income, prices of the product and prices of close substitutes. Even this leaves out variables known to be

[7] These are almost exclusively the product of Bressler and Sammet of the University of California or of their former students. For one of the most highly developed examples see (2).

[8] The "1955 Household Food Consumption Survey" was planned and executed by the Institute of Home Economics in cooperation with other units of the United States Department of Agriculture and is published in a series of reports, fourteen of which have appeared to date.

important in explaining regional differences in consumption; for example, size and composition of families, age and sex composition of regional populations, race, and that inevitable residual, historically conditioned tastes. Harold Goldsmith, Robert Herrmann, and the author are working at Michigan State University with a portion of the original data cards from the USDA 1955 Household Food Consumption Survey. They are developing family compositional classes for the United States and four regions[9] which, along with other data, they hope to use in testing J. A. C. Brown's general hypothesis (1) concerning the relationship between age and sex composition of households, size of household, income, prices, and per capita consumption of specific foods.[10] This may also provide a means of very roughly estimating per capita consumption for states within the four regions using the empirically derived regional demand function and state data.

Snodgrass and French (11, 12) developed an interesting approach to estimating total state milk consumption figures using available income elasticities and disposable income. This is a rough estimate, to be sure, but it has more basis in empirical fact than most efforts so far. The technique could be used for other commodities.

THE USE OF SPATIAL MODELS

Without some comment on the analytical forms in which data are used in regional adjustment analyses, observations on data mean much less than they might. This is particularly so since it has been necessary to cast this paper in terms of farm products generally rather than specific commodities. No one really needs to be told that the analytical and data problems of particular commodities are extremely varied.

There have been a number of interesting empirical efforts to develop spatial models in agriculture in recent years. The earlier efforts of Fox (3, 4) and Judge (7) were with spatial equilibrium models. More recently Snodgrass and French (12) as well as Henry and Bishop (6) applied the transportation model of programming to milk and to broilers. In 1958 Judge and Wallace (8, 13) built a spatial equilibrium model for beef. And in 1959 Heady and Egbert (5) published a general linear programming allocation model analyzing feed and food grain production location.

There should be no need to review these in any detail since C. B. Baker has done so elsewhere in this volume. However, some of the general characteristics of these models as they relate to analysis of adjustment problems should be made clear.

There are three apparent types of models here: the spatial equilibrium models of Samuelson-Enke genesis, the so-called transportation model from linear programming, and the general linear

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[9] Northeast, North Central, South, and West.

[10] Brown's formulation seemed to test out fairly well on post World War II English data.

programming allocation model. The differences between these models are more apparent than real. The transportation model is a mathematical subset of the Samuelson-Enke spatial equilibrium model. Both the spatial equilibrium model and the entire linear programming approach were developed from the more general mathematics of activity analysis. And, of course, the transportation model differs from the general resource allocation model of linear programming only in computational procedure, not in the basic mathematical formulation. Thus, these models are more alike than different.

Elegant and logical as these models are, empirically they remain severe abstractions in their present state of development. The Heady-Egbert regional grain production model demonstrates the computational and other difficulties intrinsic in moving toward greater differentiation and empirical content. This model contained 104 producing regions as compared to 48 in the Snodgrass-French model and 21 in the Judge-Wallace model. With 104 regions Heady and Egbert were handling coefficient matrices of over 100 x 300 in order. But despite this admirable and massive effort to improve the empirical capacity of this type of model, Heady and Egbert still end up with results that are too aggregate and of clearly limited empirical value. It is a valiant effort, however, and the result is very instructive, for it seems to demonstrate quite clearly the need for specifying in one's analysis the major functional sections of the intervening market structure.

These models are highly synthetic, the Judge-Wallace model perhaps most so. This is due only in part to data limitations. It is in the nature of things that some "predetermined" variables of the analysis are not susceptible of empirical predetermination. A major difference in models is to be noted in the fact that problems of production are not a part of the Judge-Wallace or Snodgrass-French models. Thus no analysis of price-supply response is possible. In the Heady-Egbert model the production-resource problem is the central feature of the analysis. Heady and Egbert on the other hand do not handle the transportation problem, although, as they and others have pointed out, transport costs could be included as a production cost in the standard programming model.

In their present stage of development, all of the types of models described above end with rather unreal empirical conclusions. The reasons for this differ with the models but the models share an important limitation that is at least partly responsible. All implicitly assume a perfect or near perfectly competitive economic world. Institutional restraints and imperfect markets explain much of the actual pattern of production location, prices, and interregional commodity flow. Realization of this limitation is particularly important if analysis of adjustment is one's objective. It is instructive to note that the policy proposals of Cochrane and others imply that we are badly off base in using unqualified competitive models in analyzing agriculture's present major policy problems.

Both the transportation and production problem must be a part of any spatial or regional framework for adjustment analysis. But this cannot be where one stops, for none of these models may be described as adequate for handling the analytical problems of analysis of the adjustment process. Increasingly many of the intellectual and practical problems of science in our generation seem to be those of understanding "process" and the structures associated with "process" (14). Comparative statics and the dating of variables is about as close as one now can get to the analytical dynamics required for any reasonably complete understanding of "process". Even the present tools of comparative statics in many instances are more highly developed than much of the empirical data to which they are applied. This is true both of the production as well as the demand and market structure data used in the spatial models cited above.

Giving up some of the rigor of single system models (such as the spatial models above) for a carefully tailored combination of models which would mesh in one or a few selected common variables or assumptions might be a profitable direction in which to experiment in our empirical research. An integrated sequential system of models should divide the research problem into more manageable pieces and allow one to obtain more sophisticated empirical content and thus probably greater predictive validity. The transportation model can be adapted to handle elements of market structure in addition to transportation costs. Technical change in functions of market structure can probably be handled, at least in a rough manner, within or in conjunction with a transportation model. The demand functions with which we face these models must have a better developed empirical basis than at present. Surely too, we can use the results of the resource allocation model for the product supply dimension of the transportation or spatial equilibrium models. Changes in one model would then be capable of being worked through the other and a price-supply response process of a limited sort would be simulated. It is a common characteristic of applied empirical research that many of the most productive frameworks are less elegant than the theoretical proto-types from which they come. This is not a suggestion that one flee rigor but rather that empirical problems be approached from more of a problem-solving point of view. We are somewhat prone today to be testing tools when we claim to be solving problems.

Any empirical bridging of the structure between the firm-household level and the national economy or macro level must be designed around a particular goal or limited set of research goals. Given the present state of the arts in agricultural economics research, when we say we wish to be able to draw meaningful conclusions adapted for agricultural policy purposes from such a framework, we should recognize that in so designing it we give up some of the potential capacity to draw a very wide range of farm management conclusions from the same model. The reverse is also true. Indeed, in connecting macro and micro levels in the same analysis, some capacity is given up at both levels in order to make the structural connection. No finite construct or set of constructs has infinite capacity.

It is necessary to note in conclusion that the basic theoretical concepts through which the economist must view an adjustment problem are the major limitation to present research. The most urgent need is to develop such concepts, not the improvement of data or adaptation of analytical tools.

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Discussion

IN CONSIDERING Bonnen's paper, I believe it is important to note particularly his characterization of the models discussed. He refers to them as <u>models of the adjustment process</u>. The adjustment process introduces a succession of time periods into the analysis. The succession of time periods extend into the future because he proposes to use the models to predict "the interactions of the agricultural production process with the rest of the economic and social web of society." They are therefore models of integrated demand and supply structures, and would thus empirically bridge the entire structure between the farm and the household levels in the national economy. Both the production and the intervening market functions would be specified to obtain an accurate minimum cost spatial equilibrium.

He justifies broad scope in the models he proposes with the observation that "no consideration of the adjustment process or the problems of adjustment in agriculture is likely to be complete or realistic that does not explicitly include in its analysis the relevant structure of demand and the production process that connects the farm gate and the dinner table."

He recognizes that he identifies the characteristics that ideally should be associated with research in adjustment. He proposes an ideal model as a basis for a theoretical formulation of the research problem in a general equilibrium form before it is cut down to workable empirical size. I am in full agreement with this complete conceptualization of the research problem. I also agree that the cutting down should be carefully done without undue reference to use of a particular analytical tool or reference to shunning data problems.

When Bonnen turns from the general requirements of the models to how to meet these requirements in integrated analysis with empirical data, he joins most of us in finding progress difficult. Regionalization of the analysis is accepted as a first step in bridging the gap between the national aggregative and the farm-household level. A number of macro demand estimates that are identified in time with the adjustments periods selected for the related supply estimates must be made. These must be basically synthetic functions but they should be derived from available empirical demand analysis. Reference is made to several types of basic demand data, including prices, transportation costs, processing costs and structure data, interregional flows of commodities, and per capita consumption.

As to models, neither of the three in common use — transportation, Enke-Samuelson-Berkman spatial equilibrium, or general linear programming model — are considered to be adequate.

With further respect to models, Bonnen concludes that experiment would be profitable in the direction of giving up some of the completely systematic rigor of these present spatial models for "more eclectic combinations of models which mesh on one or two fronts, but obtain greater empirical content and sophistication and thus greater predictive value." Although I am not sure that I understand the implications of some of the words in this quoted suggestion, I believe we might all join in this proposal for further experimenting with model formulations and combinations. I suspect, however, that successful combinations of models cannot depart very far from a considerable degree of systematic rigor. Several suggestions were offered on how combinations might be made. This is the part of the paper that I hope will be expanded in a further development of suggestions on how progress may be made in meeting this general problem.

I am going to close my remarks with further observations on Bonnen's final comment concerning the use of linear programming models for normative allocation of resources on representative farm strata for spatial aggregative purposes. The tendency has been, I believe, to overload the programming model with farm management alternatives that might well be decided outside the model from prior and more simple types of analyses. This applies particularly to a wide range in choice of production practices. We need to make more preliminary studies so that more judgments can be made on choices of alternatives and thereby simplify the programming models that are used for spatial aggregative studies.

PART V

Interpretation of Supply Functions

Chapter 15

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Supply Response and the Feed-Livestock Economy

THIS PAPER will deal with some general relationships in the feed-livestock economy with the intent of showing that it is desirable to look at supply response from a farm management orientation as well as from the perspective of the analyst who deals with aggregative data of a time series nature. After all, we are looking toward a general kind of consistency in the results of analyses employing a variety of methods. If the estimates of relationships within the feed-livestock economy (or any other sector) are to be useful for policy formation, then the various pieces of evidence presented must have a reasonable degree of concordance.

FEED-LIVESTOCK RELATIONSHIPS 1922-41

The role of the feed-livestock sector in the agricultural economy of the nation is an important one in terms of the responsibility sometimes assigned to it to act as an equilibrator. The general nature of this process during the inter-war period (1922-41) can be shown by first tracing the direct or immediate effects of changes in certain key variables through the system (19).

1. During the period 1922-41, a 1 percent change in disposable consumer income was associated with an average change of 0.8 to 0.9 percent in retail prices of meat, dairy, and poultry products. Farm prices of livestock products generally changed about 1.5 percent for a 1 percent change in their retail prices, and the farm price of corn changed about 1 percent in response to a 1 percent change in farm prices of livestock products, if livestock production remained constant. Linking the steps together, during 1922-41, a 1 percent change in disposable income led, on the average, to about a 1.3 percent change in the farm price of corn.

2. In the absence of corn-price supports, a 1 percent change in corn production was directly associated with a 0.6 percent change in the supply of privately held feed concentrates — that is, feed supplies excluding CCC stocks. And a 1 percent change in the supply of feed concentrates (excluding CCC stocks) was associated with an opposite change of 2 percent in the market price of corn.

3. A 1 percent change in the supply of privately held feed concentrates was associated with a 0.9 percent change in total concentrates fed, while a 1 percent change in total concentrates fed was in turn associated with an average change of 0.34 percent in total volume of livestock production.

4. A 1 percent change in the volume of livestock production is associated with an opposite change of about 2 percent in livestock prices and a 1 percent change in the value of livestock products (farm basis). A 1 percent change in the farm value of livestock products is associated with a similar change in the farm price of corn.

The above average relationships are the estimated immediate ones. The longer term cumulative effect of a 1 percent increase in corn production would have been to depress the corn price about 1.2 percent. Over time, the lower corn price would cause an increase of about 0.2 percent in livestock production. The subsequent decrease in livestock prices leads to a corresponding decrease of 0.2 percent in the price of corn. Thus if time lags were disregarded, the total effect of a 1 percent change in corn production would, in the absence of storage or price support operations, have been an opposite change of about 1.4 percent in corn prices.

Thus two forces act in opposite direction on corn price — a change of 1.3 percent in corn price for each change of 1 percent in consumer incomes and a change of 1.4 percent in the opposite direction for each 1 percent change in corn production.

VARIATION IN FEED-GRAIN PRODUCTION

Let us now turn to supply response first for feed grains and then for the livestock. Most analyses of the feed-livestock economy have not explicitly considered feed-grain supply responses to price variables in their models. For example, Foote (9, p. 4) states:

> Acreage used for feed crops normally does not vary greatly from year to year, and changes in yields depend mainly upon weather and the general level of cultural practices. Within the usual framework of price relationships, year-to-year changes in supplies of feed are determined chiefly by nonprice factors.

In his treatment of determining optimal carryover levels of grains, Gustafson (12, p. 17) also treats the year-to-year variation in the production of feed grains as a random variable. However, he indicates that his analysis can accommodate supply functions for feed grains but that the present state of information concerning the economic determinants of acreage planted does not justify such inclusion at this time.

The general recognition that yield variation swamps acreage variation in its influence on feed-grain production,[1] should not discourage

[1] According to the method suggested by Sackrin (20) for measurement, yield variations account for about 90 percent of total corn production variation in the U.S. 1900-1958, with acreage variations accounting for the remaining 10 percent.

economists from attempting to explain acreage changes and at least a part of yield variations in a framework of economic analysis (18), rather than becoming meteorologists. Among other indications of the apparently growing recognition of the influence of weather, we may cite Marion Clawson's suggestion (4, p. 248):

> ...random annual variations in gross farm output, due primarily to weather conditions, have blurred the picture of a comparatively continuous and regular increases.... It seems to me that the first step in any careful analysis of output, whether of total or by farms or commodities, is to estimate first the effect of weather conditions in the year and time period under study.

We cannot help being reminded of Cochrane's review (5) of "The Economic Organization of Agriculture" in which he commented on preoccupation with weather phenomena as an explanation of instability in agriculture. Stallings (21) has recently attacked the problem of adjusting yield data for weather.

VARIATIONS IN LIVESTOCK PRODUCTION

Although livestock production is much less affected directly by such uncontrollable factors as the weather, the explanation of changes in production is not exactly straightforward (see, e.g., 2, 17). Needless to say, an explanation of fluctuations in livestock production is a necessary condition for estimating the demand for feed grains, an important key in determining any storage policy.[2] The beginning inventory of livestock on farms in a given year plays the dominant role in determining livestock production in that year (9, p. 16-18). But an important effect of changes in feed supply in a given year is the indirect effect on later livestock production via the build-up or depletion of breeding livestock inventories. The estimates of Hildreth and Jarrett (14) are consistent with the Foote analysis in terms of the relative importance of changes in feed supply on current marketing and later production.[3]

[2] If the storage problem is viewed in terms of "the inventory problem" [see, e.g., Gustafson (12) and Gislason (11)], then account needs to be taken of not only the fluctuations in feed-grain production (which may be sufficiently close to being random to consider as random), but also fluctuations in demand for feed grains which are certainly not of a random character, being influenced as they are by livestock cycles, general business cycles, wars, defense spending, etc. Historically, the year-to-year changes in domestic demand for feed grains have been smaller than the year-to-year changes in feed grain production. For a discussion of the nature of yield variations see Foote and Bean (10).

[3] In contrast to the work of Foote and Hildreth and Jarrett, Cromarty (6) has (within a model for U.S. agriculture) disaggregated the feed-livestock economy into a category for feed grains and five livestock product categories. It is interesting to note that his supply elasticities appear more plausible than his price elasticities for demand. For example, he reports an estimate of demand price elasticity for hogs of approximately -2.37. Work is also underway by Hassler at the University of Nebraska on a feed-livestock model for determining, among other things, the effects of various allocations of feed grains among the different species of livestock.

CHANGES IN THE FUNCTIONING OF THE FEED-LIVESTOCK ECONOMY

There is now some feeling that the traditional role of the feed-livestock economy to act as an equilibrator to absorb the shocks of fluctuations in feed grain production is changing (3). Immediately following the war the relatively high price and income elasticity for livestock products acted as an important factor in maintaining farm income and preventing an even more rapid accumulation of crop surpluses. The experience of 1955-56 made it clear to many observers that the absorptive capacity of the livestock economy for feed has some limits that need recognition. Cavin (3) suggests that the price structure to producers is endangered whenever the supply of meat for consumption is much in excess of 160 pounds per capita. He reported that the 1955 Household Food Consumption Survey indicated that when incomes rise a shift in purchases occurs to higher priced meats rather than higher quantities. There is other evidence that the demand conditions are changing. Dean and Heady (7) report that both price and income elasticity for hogs have decreased when the 1924-1937 period is compared with the 1938-1956 period.

In this connection Kiehl (15) has also pointed out, "... it would be unfortunate if hogs were given the assignment to 'eat up' our feed-grain surplus."

IMPLICATIONS OF STRUCTURAL CHANGE IN SUPPLY ANALYSES

Changes occurring in the demand for livestock products (and hence the derived demand for feed grains) have been detected by analyses of time series data with a relatively high degree of aggregation <u>and</u> by cross-sectional data on consumer expenditures. Similarly, it seems natural that we check results of time series supply analyses against cross-sectional data from individual farms. The more rapidly the structural changes in production occur, the more important it is to do such cross checking.[4] An earlier paper at this workshop by Cochrane and Learn has dealt with the interpretation of regression analyses when structural changes occur.

An example of some insights that might be obtained by analysis of individual farm data is suggested by a preliminary analysis of data on some hog farms in four Illinois counties: Bureau, Henry, Knox, and Stark. The general problem being investigated is the effect on stability of production of concentration of the production of hogs in the hands of fewer producers. This particular problem is of interest in connection

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[4] It is important to establish some logical relationship between the types of analyses. Kuh (16) cautions: "In general, we cannot estimate dynamic coefficients from cross-sections with any degree of confidence unless there is supporting time series information..."

with the apparent increase in the importance of a cyclical pattern in affecting variation in hog production.[5]

In the four counties mentioned above the census reports indicate that (in addition to a drop in the number of farms) the percent of farms reporting hogs dropped from 83.6 percent to 78.4 percent from 1950 to 1954. With total numbers of hogs on farms increasing, it is reasonable to infer that average size of operation of hog producers is increasing.

A suggestion that production may become more stable, as a result of fewer but larger producers, is given by relating the stability of the production (in terms of annual variation in numbers of litters produced) to the size of operation. Coefficients of variation on the variable, litters produced, were computed for each of 82 hog producers for the period 1946-58. This measure of variation, in turn, was related to the average number of litters per producer during the period. The resulting regression[6] indicates greater stability of production on the part of the larger producers. Thus it appears that a force tending toward stability would be the concentration of production in larger scale operations. This year-to-year variation is, of course, due to many causes; the analysis must obviously proceed to seek to explain the variation.[7]

Analysis of individual farm data is mentioned only to suggest that supply analyses with aggregative time series data may, especially in times of rapid structural change in production, need support from collateral analyses of a cross-sectional type with more detailed individual farm data.

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[5] Dean and Heady (7) report an increase in supply elasticity (to hog-corn price rate) when the period 1924-1937 is compared with 1938-1956 (Table 1, p. 854). This, coupled with a decrease in demand elasticity would tend to accentuate instability. See also Breimyer (1). It should be noted that the greater the influence of endogenous variables in explaining production, the more useful become models which test the time path of certain of the variables to analyze the nature of the system, i.e., whether it is explosive, stable, etc. Among other studies employing this type of analysis related to livestock production, see Foote (8) and Haavelmo (13).

[6] $y = 65.5 - 0.49\bar{x}^{*}$ where y = coefficient of variation and x = average number of litters produced (1946-1958). For details see Swanson and Meline (22) pp. 10-12.

[7] The suggested increase in stability does not necessarily mean that supply elasticity with respect to the hog-corn ratio is not more elastic than it has been previously. See Dean and Heady (7, p. 854). Other factors than the hog-corn price ratio account for year-to-year changes: e.g., cross-elasticity with respect to beef, etc.

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Discussion

SWANSON'S STATED PURPOSE was to show that it is desirable to look at supply response from a farm management as well as a time series perspective. I believe he has made a real contribution in this regard. His classification and discussion of the various phases of analysis of the feed-livestock area suggest several important areas of research at the level of the firm.

My review will be largely from the standpoint of the use of the general relationships he has developed in making aggregate production response estimates and the research needed for such estimates. Frequently at the national level, economists in the Agricultural Marketing Service and the Agricultural Research Service are called upon to make projections of production at some future date under assumed program price and economic conditions. It seemed to me that a few comments from this standpoint might be helpful. These comments center largely on estimates of feed grain production.

Swanson points out that formal aggregative models of feed-livestock economy usually have not explicitly included feed grain supply response. A few time series studies have been made of supply response for corn and other individual crops, but I am not aware of any studies of feed grains as an aggregate. In preparing national estimates, the production from different feed grains are close substitutes over considerable ranges. Consequently, total feed grain production becomes the most important variable. I wonder whether some analysis of aggregate feed grain output, yield, and acreage changes wouldn't be helpful. As one approach, perhaps some variant of the time series approach now used by Griliches[1] in analyzing aggregate output might be used for feed grain output. In this connection, it would seem desirable to take a closer look at the effects of acreage controls of other crops on feed grain acreages.

Swanson warns economists against becoming meteorologists. Although I would agree with this, I was not clear as to how he would suggest that economists handle the effects of weather. I believe there is also the danger that economists will overlook the contributions of meteorology, agronomy, and other physical sciences to supply analysis. In making aggregate estimates of supply, it is important that economists bring to bear enough meteorology and physical science to estimate the effects of both weather and crop practices, such as fertilizer, on yields. In recent publications by economists from the USDA and the University of Illinois,[2] the "normal" yield for corn for 1960 is projected at 49 and 44 bushels, respectively.

A gap like this really overshadows the price-supply relations. Assuming a supply elasticity of, say, 0.2, for example, it would take a price decline of 50 percent to be equivalent to this difference in yield.

In analyzing aggregate production changes in feed grains, it is usually not possible to distinguish between the implications to production of "structural changes" and the movement from one equilibrium point to another as a result of changes in price-cost relations.

This is particularly true in the case of crop yield. Use of fertilizer, for example, is a major factor affecting corn yields. This is related partly to adoption of new technology or use of fertilizer on additional farms. It is related partly to the use by able operators of proper amounts of fertilizer, whose usage will shift because of price relations. In between, of course, is a group of farmers who are "experimenting" with small quantities and who, at least in the aggregate, will find it profitable to use more under the range of prices assumed.

It is not possible to distinguish at the aggregate level among these

[1] Griliches, Zvi, "Estimates of Aggregate U. S. Farm Supply Function," University of Chicago and National Bureau of Economic Research, 1959 (mimeo).

[2] Hieronymus, T. A., "We aren't growing too much feed!" Farm Journal, 83:45, Oct. 1959. Christensen, R. P., Johnson, S. E., and Baumann, R. V., "Production prospects for wheat, feed and livestock, 1960-65," USDA, ARS 43-115, 1959.

situations. Further, I am not convinced that it is of a first order of importance in aggregate production analysis. A decrease in prices, for example, is likely to be in the direction of decreasing the "rate of adoption" by the farmers in the first and third situations and of less usage by farmers now in an equilibrium situation. A rise in prices would have reverse effects.

Such a distinction, however, does have increasing importance as the relative importance of these situations change materially. For this reason, an approximate indication of the stage of the industry is needed. I believe this to be an area in which analysis at the level of the firm can make a real contribution.

An associated area for farm management analysis that would be of assistance in aggregate analysis would seem to be analysis of the relation of income and prices to the rate of adoption of technological developments.

Available evidence indicates that on many farms increased applications of yield-increasing inputs would be profitable even with considerably lower prices. The explanation for this would seem to lie partly with such things as the learning process and risk and uncertainty on investment aspects. How are these factors affected by changes in income and price? The device of the producer panel discussed earlier in the conference would seem to offer some promise for gaining insight into this area.

Finally, the question raised by Swanson as to whether structural changes are increasing or decreasing the elasticity of supply is an important one in constructing aggregate production projections. Analysis of representative farming systems in different stages of technological development from the standpoint of shifts in the optimum combination of production factors in response to price with special emphasis on the importance and flexibility of so-called "fixed" factors might yield valuable insight into this problem.

Some economists reason that because labor has become less important in the process of agricultural production and "cash costs" more important, changes in supply will be more readily affected by price changes. On the other hand, it has been pointed out that in some enterprises and cases, the "cash costs" are composed to a considerable extent of specialized capital goods, such as a corn picker-sheller. If aggregate returns for the enterprise are reduced, the "salvage value" of such machines may be reduced proportionately. Further, as Glenn Johnson emphasizes, the salvage values are often considerably lower than the value in use.

Chapter 16

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Prospective Uses of Estimated Coefficients and Related Stastics

STUDIES of agricultural supply have two broad uses: forecasting and model building. These uses are related, but they are not the same.

FORECASTING

An accurate forecast of future supplies may help a farmer, or a business concern, make a profit. It may help a statesman improve our farm programs and policies.

Such a forecast may cover any period from a few days to several decades. Farmers may try to pick the best day to ship their hogs, or the best month to sell their apples. A dairy concern may need to estimate milk supplies over a decade ahead when designing a new milk plant.

In such cases, forecasts of future supplies are inescapable. The profit or loss of an operation may depend upon the accuracy of the forecast. Therefore, it is natural enough that farmers and businessmen seek the help of the economist and the statistician.

The officer of a farm organization, the administrator, and the congressman must also forecast supplies when considering changes in farm programs. Their aim is not individual profit, but a workable program that will benefit both farmers and the public as a whole.

The effects of farm programs upon the output and supply of farm products is a difficult and controversial subject. But here again, it is impossible to escape forecasts. The only real question is how to make the forecasts more accurate, more timely, and more objective. A poor forecast may wreck a program that is sound in principle.

More and more, the economist is asked to estimate what would happen to agricultural output if price supports were lowered; if the domestic price were maintained at parity and the surplus sold at the world prices; if the market price were unsupported, but the Government paid farmers enough to maintain some income objective; and so on.

The USDA and a committe of land-grant university economists have worked with Senator Ellender's staff on a study of the probable economic results of eliminating production controls, and letting market prices drop to the levels needed to get rid of present surpluses and to balance supplies and demand in the next seven to ten years.

In the future, Congressional committees and Secretaries of Agriculture probably will ask agricultural economists for more forecasts. That is the way to make farm programs less political and more scientific.

MODEL BUILDING

The other main purpose of supply analysis is that of setting up and quantifying an economic model attempting to describe the structure of the economy. Such models may be relatively simple — for example, including only a national aggregate demand function and a national aggregate supply function. On the other hand, there is increasing interest in much more detailed models, breaking down the total economy into dozens or even hundreds of geographical regions or types of farms.

The purpose of these detailed models is not that of forecasting — at least not directly. Such detailed models certainly should help our understanding of micro-economics of agricultural supply. In a general way, this better understanding might well lead to improved forecasts of national aggregates. But the main value of these detailed studies is not forecasting at all.

Their value is similar to the value of the detailed models of demand developed by such men as Walras (13), Pareto (10), and Hicks (7). No practical economist would try to forecast the demand for hogs by first determining the indifference surface of each individual in the economy and trying to compute from these surfaces the national aggregate demand for hogs. Certainly, we all would have much more confidence in a simple analysis relating the national aggregate consumption of hogs to the national average price and the national aggregate income of consumers.

THE DEMAND FOR COEFFICIENTS

Perhaps these remarks are enough to indicate the importance of good quantitative research on agricultural supply. These studies can be of great value to the farmer, the businessman, the congressman, and the theoretical economist. This does not necessarily mean that these people want a number of "coefficients." They may want accurate forecasts of supplies. They may want a more basic understanding of the market mechanism. Statistical coefficients of correlation, regression, standard errors, elasticities, etc., are useful to these people only if these coefficients either help make more accurate forecasts or give a better understanding of the underlying mechanism of the market.

A PLEA FOR GRAPHIC ANALYSIS

Before World War II, Bean (1), Cochrane (3), and others made good use of graphic analysis to study agricultural supplies. Their results helped build a theory of supply, and helped forecast expected changes in supply.

Graphic analysis has been sadly neglected in recent years in favor of more mechanized, routine, conventional methods, based upon computation of coefficients. There seems to be a search for an automatic method, requiring no human thought. The sheer volume of this mass-produced research may at times be awe inspiring. But it probably will not give more accurate predictions of supply than can be obtained much more easily and quickly by simple graphic methods.

Graphic analysis dispenses with most computations of most coefficients. Sometimes, to make the study look impressive and "scientific," the analyst may compute a correlation coefficient. But this is not essential. A good graphic analyst draws a diagram that shows his estimate of the basic relationship. The degree of relationship is shown visually by the closeness of the scatter of dots around the regression line. The interest centers on the shape and slope of the line itself. Here, graphics has a great advantage over algebraic methods. In graphic analysis, it is unnecessary to make the unrealistic assumption that everything is linear — either in absolute numbers or in logarithms.

The electronic computer has made it possible to work with very large, very complicated models. This can be a great boon to research. But much more graphic analysis should be done before punching the data on a tape and pushing the start button on the electronic computer. Otherwise, forecasts made from routine analyses of linear models may often be less accurate than those that could have been made by easy, inexpensive graphics.

STRUCTURAL EQUATIONS VS. SINGLE EQUATIONS

The economic theorist may picture the market mechanism as a set of simultaneous equations. One or more supply equations may be part of this mechanism. Other equations may explain current demand, storage, and other economic variables. In recent years, econometricians have become much interested in such sets of simultaneous equations or "models."

For example, Gerra's (6) bulletin presented a structural model for the egg industry. Rojko (11) has presented a structural model for the dairy industry. Both models included supply equations. They are intended to show the interactions of supply forces and demand forces, and to investigate the nature of economic equilibrium.

Such models can be of great help to the theorist who is concerned with market mechanisms. Some apparently think the structural models will also provide the best forecasts. This question should be tested more adequately — either by Monte Carlo methods, or with actual economic data. Gerra's bulletin includes a brief report of a test with actual data, but his results were not conclusive.

An old-fashioned single least-squares equation is likely to give a better forecast than can be made from one of the structural equations using the same variables. Success in forecasting is the ultimate test of good economic method. In spite of all the modern writing about "least-squares bias," a single least-squares equation gives unbiased estimates of the dependent variable. It is biased only if it is misused as an approximation to one of the structural equations. In like manner, a structural equation is biased if it is used to estimate a dependent variable.

CANONICAL REGRESSIONS

The idea of canonical regression is related to that of structural equations. It was invented by Hotelling (8) in 1936. A good discussion of canonical regression can be found in Tintner (12). It appears to have obvious possibilities in research upon the elasticity of agricultural supplies.

Canonical regression is the regression of one set of variables upon another set of variables. In this case there is no single dependent variable. Rather, one group of variables is dependent upon another group. For example, suppose that the acreages of ten vegetables depend upon last year's prices of the same ten vegetables (and perhaps other factors, such as wage rates for farm labor).

One way to study such a problem would be to make index numbers of vegetable acreage and of vegetable prices. These index numbers could be treated as single variables in the analysis. What weights should be used in constructing such index numbers of acreage and of price? The canonical regression is essentially a method of assigning both sets of weights. Assuming that we want to estimate (forecast) the index of acreage, canonical regression lets us assign weights that minimize the standard error of estimates of that index.

Thus, in a general way, canonical regression should be useful in indicating how a group of production items respond to a set of prices. We would not expect such a regression to give the best forecast of the output of a single commodity. But it, like a set of structural equations, might help the theorist understand the basic mechanism of the market. Also, there is great interest in how total agricultural output responds to the average level of farm prices. This can be studied only in terms of some sort of indexes of output and prices. All indexes are arbitrary. There is something to say for the kind of weights implied by canonical regression.

THE "DECAY RATE"

Works of Friedman (5) and Nerlove (9) have revived interest in what Fisher (4) called "distributed lags." Friedman found that the spending pattern of the typical consumer depends not only upon current income, but upon his income for many periods in the past. Nerlove found that farmers, in planning production, were influenced by prices over a period of several past years. In general, they both found that the most effective periods were the most recent ones — that the effect "decayed" over time.

Data on orange advertising over a period of 50 years suggest that consumers respond not only to current advertising, but also to the advertising of several years in the past. In many cases, the effectiveness seems to decrease by a "decay rate" similar to that of the radio-active material. That is, it loses a constant percentage of its effectiveness in each unit of time. If it loses, say, 40 percent the first year (leaving it 60 percent effective), it will lose 0.40 x 60 percent the second year (leaving it 36 percent effective), 0.40 x 36 = 14.4 percent the third year (leaving it 21.6 percent effective), etc. Of course, this particular pattern of distributed lag (or decay) is not necessary in all cases. But it does seem plausible theoretically, and it does seem to fit several economic series very well.

So the decay rate is one coefficient the researcher is likely to find useful in studying the supply of farm products. Brandow (2) and others have questioned some of the methods used to derive such a coefficient. Probably it is well to try different methods.

TWO-PRICE DEALS

In the past few years, there have been several proposals for farm programs that would result in two different rates of return to the farmer — a higher return for his "domestic quota" of wheat, rice, milk, or turkeys, and a lower return for "extra-quota" amounts.

One of the unsettled questions about such proposals is whether farmers would react to the "blend price" in planning production, or whether they would react to the lower price received for extra-quota production. Theoretically, we might expect farmers to react only to the lower price on extra-quota production, <u>if</u> they were convinced that their future quota was fixed and did not depend on current production, and <u>if</u> the farmer computed his economic interests correctly and reacted strictly as an economic man.

These are big ifs. We need to know much more about how farmers actually do react to two-price deals. Some work has been done in base-rating plans for milk, but more research is needed in this area.

NON-PRICE FACTORS

Finally, economists tend to limit their interest too narrowly to the response of production to prices. There are certain other factors that warrant consideration. A number of years ago onions and tobacco were the two main crops raised in the Connecticut Valley of Massachusetts. Price was certainly a factor in determining the acreage planted to each. But so was inertia, and the unwillingness of American-born farmers to follow the example of their Polish-born neighbors and have their wives and children weed onions on their hands and knees in the hot sun. Price is probably not the main reason that very few onions are grown in the Valley today. It is rather that the Polish immigrants and their children have adopted American culture patterns.

Farmers' production plans are influenced by anything that makes them more or less optimistic about the future. Advertising and promotion generally might induce farmers to expand their operations — whether or not they reason that future prices will be higher. In any case, the economist does not need to limit his interest to the response of farm production to price alone. Rather, he should look for any sort of influences that could be identified and measured.

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Discussion

IN LISTENING to the papers at this workshop I am reminded of the quotation that appears on the title page of H. Theil's book Economic Forecasts and Policy.[1] This quotation, taken from the book The Napoleon of Notting Hill by G. K. Chesterton, goes as follows:

> The human race, to which so many of my readers belong, has been playing at children's games from the beginning — and one of the games to which it is most attached is called, "Keep Tomorrow Dark" and which is also named "Cheat the Prophet." The players listen very carefully and respectfully to all that the clever men have to say about what is to happen in the next generation. The players then wait until all the clever men are dead and bury them nicely. They then go and do something else. That is all. For a race of simple tastes, however, it is great fun.

In spite of our efforts to capture the parameters of supply relations and to predict, our opponent, the real world, has done a good job of keeping tomorrow dark, or at least dim. This is not surprising when one considers that our theories of economic change do not enable us to narrow substantially the class of admissible hypotheses and that by their very nature structural supply relationships are subject to strong random fluctuations. Perhaps if we can, as in the spirit of this workshop, study individual structural equations, we may find bits of order here and there. These can gradually be combined into a systematic picture of the whole, thereby generating a little light along with our heat.

Waugh has discussed the two broad uses of supply functions and has commented on alternative methods of capturing the coefficients of supply relationships. Since many of the other papers and discussions have discussed methods of sampling the coefficient space for desired parameters, I will concern myself mainly with an extension of Waugh's remarks relating to the uses of these studies and how models, methods, and uses interact.

It seems apparent that if we could gain knowledge of domains, such as firm behavior relations, we could make predictions about them, or by understanding the underlying structure or mechanism, control or at least influence them. Therefore, if we could succeed in capturing the relevant variables and their attendant coefficients, this knowledge could be of invaluable use for decision making at the various choice levels. Use of this knowledge as a basis for decision making could run the gamut of providing the necessary information to guide a particular firm in its choice of output level to that of a government that desires to know

[1] Theil, H. Economic Forecasts and Policy. North Holland Publishing Company, Amsterdam, 1958.

in advance the probable consequences of alternative courses of action that may be considered. Therefore, such quantitative knowledge is a prerequisite for intelligent formulation of government policy and for resource allocation by the firm.

Although the estimates have a variety of uses, no particular set of estimated coefficients contains magic numbers that can be used for all purposes. In most cases the definitions and assumptions underlying the model and methods specify the use to which the estimates can be put. For some decisions only changes in the exogenous variables will be relevant. For this situation, knowledge of the reduced forms relation is adequate. However, knowledge of the past structure is necessary if actions under consideration and the expected changes of uncontrolled conditions involve not only changes in exogenous variables but changes in the structure itself, e.g., if we change from a free market situation to that where the price of a commodity is controlled. The important thing about having knowledge of structural relations is that it makes it possible to predict the effect of not only one given structural change but of any well-defined structural change. Of course, for many decision-making purposes knowledge of supply relations is not sufficient. We must also have parameter estimates for other behavior and technical relations.

With tongue in cheek, I will say I have a feeling many of us use estimates for purposes other than for what they are intended. Estimates and analysis from which specific inferences are to be derived should be designed in detail to provide an appropriate base. It is important before "pushing the button" to check directly the appropriateness of the most critical coefficients for the problem at hand.

Thus far we have said that the estimates of supply relations should be useful for decision making on the government and firm level. Given knowledge of these relations it is now in order to consider how to use this information to make the best decisions. Two of the outstanding men who have concerned themselves with this problem are the Dutch economists, J. Tinbergen and H. Theil. Theil considers this problem under the framework of "decision making under uncertainty" and his procedure may be sketched as follows:[2] assume that the variables for which predictions are to be made are connected by a linear model of the type

$$BY + \Gamma Z = U \tag{1}$$

where B and Γ are matrices of structural coefficients, U is a vector of random disturbances, Y a vector of endogenous variables, and Z a vector of predetermined variables. If we then assume the B matrix is nonsingular, we can write the reduced form equation which expresses each Y as a linear function of the Z's. The Z's may then be partitioned in the following categories: (a) instruments or controlled variables,

[2] Ibid., pp. 379-556.

(b) uncontrolled variables, and (c) lagged variables. Since the lagged variables are known at the moment of decision making, they appear in the reduced form as a constant term. It is then necessary to predict the values of the uncontrolled exogenous variables along with that of the disturbances of the reduced form. The reduced form may then be written as follows:

$$Y = \pi_1 \; Z_1 + E \tag{2}$$

where Y is the vector of noncontrolled variables which the policymaker can influence, Z_1 is a vector of the policymakers' instrument or controlled variables, π_1 is a matrix of coefficients and E is a vector of constant terms and is composed of noncontrolled exogenous variables, lagged variables, and reduced form disturbances.

Given the above equation it is possible to make conditional predictions. The decision maker may then proceed to evaluate the alternative values of Y and Z_1 that are available for choice. Assuming that a welfare function exists which describes the ordering of alternative outcomes according to increasing preference, the policymakers' "best" decision is then found by maximizing

$$U = u(Y_1 \; Z_1) \;, \tag{3}$$

subject to

$$Y = \pi_1 \; Z_1 + E \;. \tag{4}$$

In this sense, a very close formal relationship exists between this type of analysis and the classical theory of consumer demand. In Theil's formulation it is easy to see how imperfections in the coefficient matrix π_1, or in E could bring about imperfect predictions and decisions and thus generate welfare losses.

Although the approach by Theil is similar to that of Tinbergen in that it uses econometric models for policy purposes it differs in how decisions are made. Also, Tinbergen[3] neglects disturbances in the equations.

Tinbergen fixes certain desirable target values for the noncontrolled variables on an a priori basis. He then tries to find the instrument values necessary to reach the target or targets. His approach would be to start with a system of linear equations of type 1 and assume that the number of controlled and noncontrolled variables are equal. Therefore, the Γ matrix of coefficients is square. If we also assume that it is nonsingular, we can express each Z in terms of all Y's, i.e., just the reverse of the reduced form equations. If we ignore the disturbances, we can generate for each set of target values the

[3] Tinbergen, J., On the Theory of Economic Policy, North Holland Publishing Company, Amsterdam, 1952, and H. Theil, op. cit., p. 392.

necessary instrument values. In this sense it is similar to structural analyses in input-output models. Of course, in this approach we have the problem of how to handle exogenous variables which are not instruments and how to proceed in decision making when the number of target and instrument variables are not equal.

I have attempted to sketch two approaches to the use of econometric results in decision making. Obviously my brief sketch has not done justice to the penetrating and refreshing approaches of Theil and Tinbergen. Many problems, of course, remain in real world applications. However, I believe it is safe to say our ability for using the estimates greatly exceeds our ability to capture the relevant coefficients.

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Discussion, Chapters 15 and 16

THE PAPERS by Waugh and Swanson demonstrate that many useful ideas can still be stated simply and briefly. It was good to have persons of their stature make this point.

Swanson's paper will be considered first. It could be criticized, perhaps, for deviating from the topic assigned. This is excusable, but it means we were denied his views on alternative models for analyzing the feed-grain economy and their appropriateness. He did, however, have some worthwhile things to say about supply response studies.

The estimates of the effects of changes in certain variables in the feed-livestock economy and their interrelationships are of interest.

The discussion of yield variation due to weather points out an important difficulty in trying to explain production changes. The caution against economists trying to be meteorologists seems well founded. It would probably have been expecting too much to have looked for an alternative way of getting around this problem.

Some of the other ideas presented in the paper that seemed particularly interesting were: (1) the effect of changes in feed supply in a given year on later livestock production via the build up or depletion of breeding livestock inventories, (2) the apparently changing role of the feed-livestock economy as an equilibrator to absorb shock of fluctuation in feed-grain production, and (3) the apparent increase in stability in hog production associated with increased size of operation. Some of these and other relationships mentioned support the viewpoint that data from individual farms can supply us with insights on supply response not obtainable from time series data alone.

We turn now to Waugh's paper. Several times during the conference the question has been raised as to what use we are to put the supply response coefficients. From the title of Waugh's paper, we would expect to find the answers here. The paper does have something to say about this. But largely it is dismissed by saying that farmers and policy makers are not directly concerned with coefficients. We must agree with him that they are of value only if they help us forecast or understand the market. He really does not tell us whether or not he thinks the coefficients forthcoming will do either of these things. He does make a plea for using much simpler methods — particularly graphic analysis. He states, however, the major weakness of this method when he says we can visualize only three dimensions at once. This seems a serious enough limitation in dealing with the complex of variables involved in the supply response function and seems to confine its use largely to preliminary analysis.

Waugh favors "old fashioned" single least squares equations over the structural equations. It may be that, as he states, a structural equation will give a biased estimate of a dependent variable. It was not made clear why this may be so.

One of the principal values of the paper is its emphasis on the use of good common sense as far as it will carry us and his warning that we should not let ourselves get so entangled in the complications of methodology that we lose sight of the problems and the real objectives of our studies.

PART VI

Summary

Chapter 17

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Summary and Conclusions

A SUMMARY of this nature can only attempt to interpret the proceedings and conclusions of this workshop as they are presented in the various chapters. No attempt has been made to prepare a detailed documentary summary.

The workshop topic, "Estimating and Interpreting Farm Supply Functions," was formidable. Established approaches were reviewed rather exhaustively, and new ideas and new approaches were presented. Some concepts and approaches were accepted, others were largely rejected. Probably most important of all are those ideas which will be developed further in the months ahead.

An important factor contributing to the success of this workshop was the consistent conscious orientation to supply analysis as it applies specifically to the problems of change and adjustment which currently face agriculture. The objectives of supply response analysis as established clearly at the outset of the workshop were: (1) to provide guidance for general policy formulation and (2) to indicate how much relevant variables need to be manipulated to effect specific changes.

The requirements for the fulfillment of these two purposes differ considerably. The latter purpose is quite demanding with respect to our knowledge and understanding of the structure of supply response and, in fact, requires a working model which explains (or at least characterizes or empirically represents) human behavior or the decision processes of farm firms. Most of the discussion is worded in terms of prediction rather than manipulation, but for our purposes these are essentially equivalent. Prediction of future supply responses based on changes in both the influencing variables and the supply structure is analytically the same problem as how to manipulate supply outcomes. The operational requirement that a variable used in prediction must be subject to exogenous manipulation is not of concern in our discussion.

Several of the approaches considered can fulfill the first objective of general guidance. This objective involves primarily conclusions of probable aggregate changes in output and direction of necessary changes in resource inputs and firm reorganization. This amounts, essentially, to problem recognition and definition. The major difficulties arise in the application of all approaches to the second more demanding objective of supply response analysis — to understand behavior or predict it with sufficient accuracy to know how to effect specified quantitative changes in input patterns, firm organization, and output.

ANALYSIS OF AGGREGATIVE TIME SERIES DATA

The discussions of alternative methods of analyzing aggregative time series data pointed up the need for developing a better theory of aggregation for supply response analysis. This need arises from two general sources: (1) the problem of aggregating several variables under a single measurable common denominator to derive a single (average) relationship when in fact the relationship between the components aggregated may be quite different; and (2) the problem of aggregating data for firms from estimates for individual firms or firms representative of strata within a defined population. The first problem is that of using aggregative time series data as we ordinarily use the term. The second is the problem of "adding up" estimates derived at what is commonly called the micro level.

Regression

Much of the supply analysis of time series data has been in the form of regression analysis. Exhaustive review of regression applications indicated that regression has rather limited usefulness in supply analysis.

The major single limitation is that it cannot be used for prediction in light of new variables and structures. Regression models based on time series data reflect historic relationships and at best describe present relationships. Although explanation of variations in supply is important, it is not directly predictive, and in supply response analysis, prediction is more important than the record of the past.

Our present concern in supply analysis as it relates to adjustment in agriculture is the incorporation of future structural change into the predictive model. The standard regression model can take into account the effects of changes in the values of strictly shift variables but can never completely take into account the effects of changes in structural variables. Since the regression model cannot adequately take into account structural variables, such as changes in technology, managerial ability, and institutions, it is unsuitable for supply analysis in the present transitional era in American agriculture. Even if we could adequately handle structural changes of the past, we still are faced with the problem of predicting future structural change. The regression model is not likely to serve this purpose, since the current rate of change is so rapid.

Modifications of Standard Regression Approach

While the problems of aggregation were recognized as very serious, major emphasis was placed on the development of better theoretical economic models to accommodate uncertainty and investment in fixed factors.

If we assume that an uncertain variable can be reduced to a certainty equivalent and that group behavior can be treated as the behavior of a single representative and hypothetical decision maker, who acts to maximize profit, we are faced squarely with the initial ground rules of much of the conventional regression of aggregative time series data. Thus, the answer lies in the development of economic theory which will modify the standard regression model and make it more useful in dealing with supply responses in agriculture.

Alternative models were developed to deal with the problems of uncertain expectations and investment in fixed factors and how they affect supply response. These models are presented in Chapter II. While their significance is treated with great modesty, they are worthy of careful study for their detailed specific content.

However, as the author indicates, this consideration of modifications points up the need for more effective economic theory.

Other Techniques

Chapter V explains and discusses the application of recursive programming to the problem of supply response. This technique is a synthesis of time series analysis and linear programming to apply the optimizing principle of production theory without grossly misrepresenting the simple decision processes that govern farmer behavior. It makes its basic modification in the realm of decision making, which in turn determines the process of farm change.

The basic approach is to program supply response but to condition the solutions with dynamic restrictions on the rate of change. This is in contrast to standard normative programming solutions based on pure profit maximization with essentially exogenous constraints. It also permits the determination of "effective" net returns derived from uncertain anticipations. It thus generates predictive supply response estimates.

The concept accommodates many possible flexibility restraints which greatly modify the profit motive. In this system the normal constraints of programming are replaced by a set of equations which characterize the dynamics of adjustments made by farm firms. These equations can incorporate anything from general inertia to capital rationing and uncertainty.

These governing equations are derived or even synthesized from various sources of knowledge and information including a heavy reliance on standard time series analysis. They are dynamic in the sense that they change based on time series experience and incorporate this added knowledge into equations for predicting supply response in the future once the system has been initiated.

This approach is not free of all data and aggregation problems but is based on production theory at the firm level. It is designed to synthesize existing statistical methods and other explicit choice criteria.

Whether this approach can adequately incorporate the process of change to provide accurate estimates of supply response depends largely on whether the governing equations can adequately characterize the decision processes of farmers as a modification of an optimizing model and effectively accommodate uncertainty and alternative goals. The empirical success of initial applications is of great interest but is probably not as important as the variation in approach which it represents. It represents an approach of working from the optimizing structure of supply response, as contrasted with purely empirical prediction, which conceptually at least has operational possibilities.

MICRO SOURCES OF DATA

In the face of the problems of supply analysis based on aggregative data, the next step was to examine the possibility of building upward from the micro level. This approach considerably reduces the problem of aggregation but does not completely remove it as some problems of "adding up" micro response data are encountered. The use of micro sources of data in itself does not essentially change the problems of accommodating uncertainty, alternative goals, and the need for adequately representing the decision processes of farmers.

Production Functions and Cost Functions

Conceptually production functions derived from cross-section physical data can be used for estimating supply response through the application of prices and costs, provided the production structure is not expected to change during the period for which predictions are being made and provided the usual optimizing assumption is acceptable.

Deriving production functions from cross-sectional data eliminates the main drawback of time series regression analysis, the problem of changes in production structure during the time period from which data are drawn. It has its problem, however, in that the data from individual farms in the cross sections may represent points on different production functions.

Another problem arises in attempting to use studies of the type now available for supply response estimation. Inputs and outputs have commonly been aggregated in terms of some measurable common denominator (such as dollar value) in a manner which precludes the refinement and detail of prediction necessary to our current purposes.

This leads to the conclusion that the use of production functions in estimating supply response is limited to the fairly short run. Even in this application, estimates should be restricted to groupings of components with relatively homogeneous production structures.

Cost studies based on the financial records available on farms reflect the relative economies associated with different organization and size but do not provide any empirical basis for predicting changes in supply resulting from changes in size and organization. The findings of such cost studies have relatively short-lived application, as they are based on financial summaries rather than physical production relationships

Programmed Normative Supply Response

Programming offers considerable possibilities for incorporating more micro detail into supply analysis work. In its context here it also permits disaggregation to farm groupings with relatively homogeneous resource bases. Both of these characteristics of linear programming are important although not necessarily nor exclusively associated with programming.

Programmed normative supply estimates based upon the criterion of profit maximization may have low predictive value because we lack knowledge of the constraints that modify the profit motive. Probably the biggest single problem faced in programming supply response, however, is the current inability to obtain and incorporate coefficients which adequately reflect future production possibilities. The adoption of technology in the future can materially change the production structure and resulting supply estimates. Without such coefficients relevant to the future we cannot even very well say what the supply estimates really represent. On the other hand, linear programming models can facilitate deductions from postulated change perhaps better than other approaches.

The general conclusion was reached that with the optimizing criterion, current coefficients, and only arbitrary or experimental constraints, the predictive value of programming is quite limited in respect to supply response. It can, however, be very useful in generating much so-called purely farm management information. It also is useful in identifying the nature and magnitude of the economic adjustment problem in agriculture.

Synthetic Approaches

On several occasions hope and even optimism were expressed for the development of macro supply response from micro sources. This enthusiasm, however, was prudently tempered with several considerations of prime importance.

The incorporation of alternative goals into the prediction models is very important. However, whether this can be done by imposing constraints on the profit maximizing criterion is seriously questionable.

A major area of concern is how the dictates of economic adjustment can be incorporated into the reorganization of farm firms. The "solutions" obtained from most adjustment models involve substantial changes in several resource inputs on individual farms. In order to predict supply response we must have better information on which of these resource categories need to be changed significantly and how these changes can be made. We cannot simply assume that resources will flow into the reorganization process in the quantity or quality and at the time that optimum solutions indicate they should or would. Some people have even argued that the way in which we reshape our institutions that regulate such resource flows could be as important in determining the nature of change as the purely profit incentives that may exist.

How resources can or will flow into farm reorganization is a major area of research investigation very pertinent to supply response which received little attention in this workshop aside from being clearly recognized. Supply response analysis must include this area if it is to measure up to the objectives accepted at the outset. This would inevitably seem to require deeper research into the decision processes of farm firms.

A Farmer Reaction Panel

The idea of predicting supply response from continued observation of the reactions and actions of a "representative" panel of farmers is a unique addition to this area of work in many ways although it is by no means new to economic research in general. It involves many problems of measurement, avoiding predetermination of the results by the manner in which data are obtained, etc., but these are not completely insurmountable problems. The direct use of a reaction panel probably only gives more explicit recognition to many of the problems associated with other data sources.

Such an approach has several important advantages. Working from the micro level and permitting considerable disaggregation allows incorporation of more detail. Such a panel can be classified into smaller groupings that are relatively homogeneous with respect to the starting resource base, a factor which will surely influence rate and nature of change in the relatively near future. This classification would probably also, at the same time, result in a considerable degree of homogeneity in reaction patterns within such groupings. This approach would avoid many problems of "hybridization and averaging" to give more precise empirical predictions.

This is essentially an empirical approach. The observations and measurements and the questions asked of such a panel would be structured by some specific hypotheses regarding decision-making models, but the direct end product would be an empirical prediction. Nonetheless a valuable contribution to decision-making theory could very well result from the feed-back of empirical knowledge of reaction patterns.

REGIONAL COMPETITION AND SPATIAL EQUILIBRIUM MODELS

The only adequate conceptual framework of analysis for problems of adjustment which involve change in various structural variables is a general equilibrium system. Obviously the data and research resources for such a system are not available nor likely to become available soon. Short of general equilibrium, changes in regional production can still be analyzed under a variety of conditions by treating one or more of the sectors of a general equilibrium system as exogenously determined or specified at some level. Most commonly in supply applications (although not exclusively) this has been done on the demand side by taking product prices or aggregate production as given. Factor prices have also been treated in the same manner in some cases. Such models (commonly referred to as regional competition and spatial equilibrium models) can then generate production and resource use solutions from which supply functions can be derived on a regional basis.

The limitations of these models are discussed at length in their presentation. The results they generate are not presented as a blueprint of an adjusted agriculture nor as completely predictive estimates of future change. These limitations include inherent shortcomings in each model, various aspects of the data problem, and the large demands made on human and computational research resources.

Many of the reservations concerning various models are some form of discontent with a partial equilibrium model. Paramount among these is the particular reservation that the demand side is ignored in many cases. More generally, to treat any portion of the total economy as fixed defines away a major part of the purpose. Another reservation in an action form of application is that the models do not predict the process of change.

A common problem in prediction is that coefficients for the future are not known with any certainty. In addition, the impact of even existing technology on firm organization is not known nor can it easily be obtained to incorporate into the production possibilities. Models proposed so far have not adequately accounted for interproduct competition. This could be accommodated in most of the models but would greatly aggravate an already huge computational problem.

The predictive value of these models as they now stand is open to some question. The choice criterion (even with constraints in programming models) does not adequately reflect alternative goals and decision processes. The flow of resources and the impact of change in institutions on resource flows and firm organization are not adequately reflected in some models. This might suggest that agricultural economists are awakening to the quantitative importance of space-ordered comparative advantages at the very time that technological changes are reducing their importance relative to comparative regional advantages resulting from group action not reflected in the relations so far included in the models.

Still a strong sentiment persists that the solutions generated by these models have some real significance as general guides besides their immediate empirical definition of comparative advantage and resource allocation under the specified conditions and assumptions. The meaning and application of this guiding quality was not explicitly established. In considering this approach research resources need to be weighed against not-too-well-defined results. This remains one of the major unresolved questions of the workshop. This question cannot be resolved arbitrarily — but until we can decide exactly what we have in these solutions, we do not have much empirically that we can use. The need for improving this technique, which seems still to be in the process of development and refinement, was repeatedly emphasized.

OBSERVATIONS OF NEEDED DEVELOPMENT

Several problems, limitations, and needed developments were mentioned throughout the formal presentations and the subsequent discussion. The consideration of alternative approaches pointed up questions of how to meet deficiencies, particularly in the following areas:

1. Aggregation
2. Uncertainty
3. Alternative goals
4. Investment at the firm level — particularly related to fixed assets
5. Data which would reflect changes in production structure
6. Decision process — particularly related to change
7. Empirical estimates from readily observable variables

These needs could probably be summed up in three words: "data," "theory," and "prediction." These are time-honored needs that characterize most problems encountered in economic research. We are not, however, back where we started. A great deal of progress has been made in developing and bringing together theoretical models relevant to supply analysis. This, in turn, enriches empirical prediction with many important ideas.

While major gaps in our research approaches still stem from problems of data, theory, and prediction, much that is new has been added. Important contributions have been made in the attack on these areas. Many ideas have been presented on how we might provide the "missing links." A considerable contribution has also been made in spelling out data requirements and in establishing exactly what needs to be done in each of these areas in addition to actually doing some of it in many areas.

Fortunately, research is underway in the North Central region and more is contemplated using some of the alternative approaches discussed. The USDA, in cooperation with five Midwest states, is completing the pilot phase of a dairy adjustment study designed to estimate supply parameters and production response alternatives on dairy farms. Another study, essentially similar in approach, is in the planning stages for hogs and beef cattle in the Corn Belt.

IMPLICATIONS AND IMPACT

We have complete agreement that no one method of approach will provide all the answers and that no one tool is perfect. The needed information on supply response can probably be obtained most effectively through the use of several approaches. This information, in turn, can be applied to the problems of economic adjustment in agriculture.

An important implication is that we must be careful not to be drawn into the sometimes inviting dichotomy between "model building" and "empiricism." In some isolated contexts we might assume that to obtain a solution we have to choose between the development of theory to make our models more complete and the use of more purely empirical prediction and problem-solving approaches. Actually we need both.

"Theoretical" and "empirical" approaches can be complementary in successive phases of development, provided each is duly oriented to the problems of supply response and economic adjustment rather than to their intrinsic satisfactions. The theoretical developments sought and the empirical predictions discussed and urged are very similar. Even the purest form of empirical prediction is not without an implicit model, and if successful, it can greatly contribute to the development of more adequate theoretical models. One example of this possibility has already been noted in regard to the empirical estimates of producer reactions and decision-making theory. Similarly, developments in theoretical model building can lead to the incorporation of new variables, equations, and procedures to improve empirical estimates. We need to pursue both and continue to exchange and incorporate the findings of each into the other.

Much of the impact of the deliberations at this workshop will be diffused, untraceable, and unmeasurable. We might also recognize that much of the apparent impact on research may well be the direct result of the same thinking that led to this workshop. It is still impressive, however, to see a large number of research projects and research committee activities devoted to the problems discussed at this workshop and employing the techniques and approaches considered. Many of these studies explicitly recognize the benefits of combining theoretical and empirical approaches and plan to use both.

EXPANDING RESEARCH EFFORTS

Governmentally sponsored agricultural programs involving supply control aspects have been in operation more than thirty years. No serious continuing effort has been made to determine the economic effects of these programs on farm production, farm prices, or farm income. Sporadic studies have been made, usually long after initiation of the program. Thus, we have practically no reliable historical documentation of the effects of past and current programs to guide future changes.

In contrast, research programs are included by law in many government projects. For example, the interstate highway program authorizes the use of a fixed percentage of the appropriated federal funds for research relevant to highway projects.

Increased allocation of research resources to supply analysis by agricultural experiment stations and the USDA is encouraging. Much of the burden of the agricultural adjustment problem falls within the area of estimating and predicting agricultural supply functions. More and more, the economist is being asked to estimate agricultural output under differing circumstances, such as various levels of price supports, differential pricing, free prices, direct payments, etc. Expanded research efforts are long overdue in this area.

Existing research resources may not be adequate considering the scope of the problems involved. Consideration needs to be given to additional resources for research to enable more precise forecasts. Perhaps a small percentage of all agricultural price-support program funds should be allocated to qualified research agencies to determine the economic effects of the program and to provide data for increased research on agricultural supply functions. Limited amounts of this research can be done on a national or state basis. However, much of it requires regional and interregional coordination to insure optimum results.

Index